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Patricio Dalton and Sayantan Ghosal

Behavioral Decisions and Welfare

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Patricio S. Dalton
Tilburg University, CentER, Netspar

Sayantan Ghosal
University of Warwick

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Abstract

Allowing for the possibility that individuals may not always act in their best interest, what is the connection between choice and welfare? This paper addresses this concern by studying the normative implications of a general model of individual decision-making that allows for suboptimal behavior. In a behavioral decision, the individual may not always internalize all the consequences of own choices on himself. We show that behavioral decisions impose clear restrictions on choice data (characterized by Chernoff's axiom and a minimal consistency axiom). We show that, for fixed preferences, behavioral and rational decision-making are, typically, distinguishable. Moreover, we show that under specific circumstances, it is possible to identify the divergence of choice and welfare on the basis of choice data alone.

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1 Introduction

Standard normative economics employs the revealed preference approach to extract welfare measures from choice data alone. The *preferences revealed* from the individual's choices are assumed to be identical to the *normative preferences* representing the individual's best interest. There is, however, considerable empirical evidence that in an array of different situations, individuals systematically act against their own best interest, establishing a potential wedge between normative and revealed preferences.[1]

Allowing for the possibility that a decision-maker (DM) may not always maximize a fixed preference relation over different feasible action sets, what is the connection between choice and welfare? One approach, advocated in an influential contribution by Bernheim and Rangel (2009) (hereafter, BR) (and also Rubinstein and Salant, 2008) (hereafter, RS)), is to construct a welfare criterion that never overrules choice based on pairwise coherence: x is (strictly) unambiguously chosen over y if y is never chosen when x is available. A different approach rejects choice altogether as a foundation for normative analysis and proposes alternative measures of individual welfare based on individual's happiness or experienced utility (Kahneman et al., 1997), opportunities (Sugden, 2004) or capabilities (Sen, 1985). However, a consensus regarding the appropriate criteria for behavioral welfare analysis has yet to be reached.

This paper contributes to this ongoing discussion by studying the connection between choice and welfare in presence of DMs who do not always act in their best interest. To what extent can choices alone be used as valid guidance for welfare?

To address this concern, we propose a general framework of individual decision making that allows for suboptimal behavior (i.e. behavior that does not maximize welfare), we characterize and compare choice data consistent with behavioral and rational decision-

[1] Loewenstein and Ubel (2008) point out that in the "heat of the moment," people often take actions that they would not have intended to take (Loewenstein, 1996). Koszegi and Rabin (2008) and Beshears et al (2008) review empirical evidence of systematic mistakes people make. Bernheim and Rangel (2007) record situations in which it is clear that people act against themselves: an anorexic refusal to eat; people save less than what they would like; fail to take advantage of low interest loans available through life insurance policies; unsuccessfully attempt to quit smoking; maintain substantial balances on high-interest credit cards; etc.

making, and show a possible way to identify welfare from choice data alone in specific settings.

The idea of the framework is simple. We study a DM who chooses among mutually exclusive actions. Each action has an effect on utility (equated to welfare) both directly and through its effect on a psychological state, p. The DM's utility depends both on actions and psychological states which are determined by actions through a feedback effect. We interpret p as a psychological state such as mood, self-image, temptation or inebriation, but it could really be anything that affects utility.

We consider two types of decision procedures: a Standard Decision Procedure (SDP) and Behavioral Decision Procedure (BDP). In a SDP, the (rational) DM fully internalizes the feedback from actions to psychological states, and chooses an action and, as a consequence, a psychological state, that maximizes his welfare. In a BDP, in contrast, a (behavioral) DM neglects the effect of his choice of action on his psychological state, and chooses an action taking as given his psychological state at the moment he decides, although psychological states and actions are required to be mutually consistent.

As an illustration, consider a DM who chooses a bundle consisting of both material status and health status, who is fully aware of the risk to his health from a single minded pursuit of material status and who has revealed his preferences for health by, for example, paying for costly treatments. A rational DM will internalize the possible trade-off between his material status and health status when choosing his material status. A behavioral DM, in contrast, will take his health status as given and strive to achieve the highest possible material status without internalizing how his choice to pursue material status affects his health.

Despite the simplicity of this framework, as argued in Section 3, it is general enough to unify seemingly disconnected models in the literature, from more recent positive behavioral economics models to older ones. In addition, it encompasses the standard rational model as a special case (SDP). We first provide a new equilibrium existence result in pure actions without complete and/or transitive preferences. A result like that is important on its own, since incomplete and non-transitive preferences are a common token in behavioral economics models.

We then begin our normative analysis by characterizing the structure of choice data under both decision procedures, BDPs and SDPs. We show that observed choices are compatible with a BDP only if the choice data satisfy one simple testable condition (Chernoff's (1954) axiom or Sen's (1971) axiom α): the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. In turn, if choice data satisfy both Chernoff's axiom and a minimal consistency axiom (heuristically, the choice correspondence has the property that an excluded alternative is also excluded in at least one pairwise comparison with an included alternative), then it can be rationalized as the outcome of a BDP. Chernoff's axiom, is weaker than the condition (Arrow's (1959) axiom) that completely characterizes a SDP, i.e. the choice correspondence is exactly the same as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. Choice data generated by a behavioral DM may not satisfy menu independence (and hence violate Samuelson's Weak Axiom of Revealed Preferences) but rule out pairwise cycles.

In contrast to BR, our starting point is that psychological states (ancillary conditions or frames in their framework)[2] are normatively relevant. The relevant normative benchmark for us are the outcomes of a SDP. For fixed preferences and feedback map, we provide the conditions under which the choice correspondence generated by a SDP and a BDP are distinct. Moreover, we show that in smooth settings, the two decision procedures are, generically, distinguishable, with the implication that the behavioral DM typically chooses suboptimally.

The axiomatic characterization on its own has important normative implications. First, it implies that not any choice data that violate Arrow's axiom of choice is consistent with the outcome of a BDP: a BDP imposes clear restrictions on choices. Second, taken together with the generic distinguishability result mentioned above, choice data that violate Arrow's axiom of choice but satisfy Chernoff's axiom and the minimal consistency axiom, at least suggest the strong possibility that the DM may not be choosing in his best interest.

Arguably, although the axiomatic characterization, taken together with the distinguisha-

[2] An ancillary condition is "an exogenous feature of the choice environment that may affect behavior, but is not taken as relevant to a social planner's evaluation" (BR, pp. 4)

bility result, is normatively informative, it doesn't provide a complete answer to the identification of the DM's welfare. For example, a behavioral DM may systematically choose against his best interest and the choices generated from his behavior may still satisfy Arrow's axiom (as the example of addiction in next section shows). In our quest for a more complete answer, we go beyond our previous analysis and ask under what conditions (if any) we could tease out whether the behavioral DM chooses suboptimally or not. We address this question in two different ways:

(a) If we assume a fixed preference relation, we show that when a SDP and a BDP are indistinguishable from each other, the ranking over actions using the binary relation proposed by BR coincides with the normative ranking over actions proposed in our framework. Indistinguishability is, thus, sufficient to reconcile choice with welfare. However, since the two decision procedures are generically distinguishable, these rankings will not typically coincide.

(b) If we assume that we can only observe choice data, provided that we can observe such data across two different scenarios, we provide conditions under which it is possible to infer if the DM choice is suboptimal. As this result requires observation of choice data in two different choice scenarios, it is limited to specific set of circumstances.

The remainder of the paper is organized as follows. Section 2 introduces our framework with the aid of a simple example. Section 3 develops the general framework together with a dynamic interpretation and states the existence result. Section 4 provides an axiomatic characterization of our theory together with an analysis of indistinguishability. Section 5 discusses the link between choice and welfare in light of relevant existing literature. The last section concludes. The details of the existence proof and the dynamic interpretation of our framework are contained in the appendix.

2 Example (Addiction)

Consider a DM who is considering whether to drink alcohol. The psychological state will either be sober (if he does not drink) or inebriated (if he does). The payoff table below provides a quick summary of the decision problem:

	inebriated	sober
alcohol	$1-2$	$1+0$
no alcohol	$0-2$	$0+0$

In this example, the payoffs are an additive function of the action-based payoff and the psychological state-based payoff. Alcohol generates utility of 1; no alcohol generates utility of 0. Sobriety generates utility of 0; inebriation generates utility of -2.

An DM who uses a SDP to solve this problem recognizes that he has to choose between the on-diagonal elements. Alcohol goes together with the psychological state of inebriation. No alcohol goes together with the psychological state of sobriety. Hence, the off-diagonal paths are not options.

However, the behavioral DM mistakenly believes that (or at least acts as if) he can change his alcohol consumption without changing his psychological state. Consequently, the behavioral DM decides to consume alcohol (since alcohol is always better, conditional on a fixed psychological state) and ends up inebriated (with net payoff -1). This is a mistake in the sense that the DM would be better off if he chose to drink no alcohol and ended up sober (with net payoff 0). In this sense, by using a BDP the DM imposes an externality on himself. Thus, the outcomes of a BDP can (although not necessarily) be welfare dominated. The rest of the paper works out the normative implications of this latter point.

3 The General Framework

3.1 The Model

A decision scenario $D = (A, P, \pi)$ consists of a set A of actions, a set P of psychological states and a map $\pi : A \to P$ modelling the feedback effect from actions to psychological states. It is assumed that $\pi(a)$ is non-empty for each $a \in A$. A decision state is a pair of an action and psychological state (a, p) where $a \in A$ and $p \in P$.

Although a natural starting point is to assume that preferences over A are indexed by p, following Harsanyi (1954), we assume intra-personal comparability of utility. We assume, not only that the DM is able to rank different elements in A for a given p but also that he is able to assess the subjective satisfaction he derives from an action when the psychological state was p with the subjective satisfaction he derives from another action when the psychological state is p'. In other words, we assume that the individual is able to rank elements in $A \times P$. This formulation is critical in order to make meaningful welfare comparisons.

The preferences of the DM are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. The expression $\{(a,p), (a',p')\} \in \succeq$ is written as $(a,p) \succeq (a',p')$ and is to be read as "(a,p) is weakly preferred to (equivalently, weakly welfare dominates) (a',p') by the DM".

A *consistent state* is a decision state (a,p) such that $p = \pi(a)$.

The two decision procedures studied here are:

1. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *standard decision procedure* (SDP) is one where the DM chooses a pair consistent decision state (a,p), $a \in A'$ and $p = \pi(a)$. The outcomes of a SDP, denoted by S, are

$$S = \left\{(a,p) : (a,p) \succeq \left(a',p'\right) \text{ for all } \left(a', \pi(a')\right), a' \in A', p = \pi(a)\right\}.$$

2. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *behavioral decision procedure* (BDP) is one where the DM takes as given the psychological state p when choosing $a \in A'$. Define a preference relation $\succeq_p$ over A as follows:

$$a \succeq_p a' \Leftrightarrow (a,p) \succeq \left(a',p\right) \text{ for } p \in P.$$

The outcomes of a BDP, denoted by B, are

$$B = \left\{(a,p) : a \succeq_p a' \text{ for all } a' \in A', \, p = \pi(a)\right\}.$$

In both, SDPs and BDPs, a decision outcome must be a consistent decision state where the action is chosen from some feasible set of actions. In a SDP the DM internalizes that his psychological state is determined by his action via the feedback effect. In a BDP the DM takes the psychological state as given although the chosen action and the psychological state have to be mutually consistent.

So far we have implicitly assumed that both SDP and BDP are well-defined i.e. lead to well defined outcomes. In what follows, we check for the existence of solutions to a SDP and a BDP in situations where the underlying preferences are not necessarily complete or transitive and underlying action sets are not necessarily convex. Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" (encompassing endowment effects, loss aversion and willingness to pay-willingness to accept diversity) to be outcome rational. Tversky and Kahneman (1979, 1991) argue that reference dependent preferences may not be convex. So we allow preferences to be incomplete, non-convex and acyclic (and not necessarily transitive) and we show existence of a solution to a BDP extending Ghosal's (2011) result for normal form games.

Proposition 1. *Suppose the map $\pi : A \to P$ is increasing in a. Under assumptions of single-crossing, quasi-supermodularity and monotone closure,*[3] *a solution to a BDP exists.*

Proof. See Appendix. ■

3.2 A Dynamic Interpretation

We interpret the outcomes of a SDP and a BDP as corresponding to distinct steady-states associated with an adaptive preference mechanism where the DM's preferences over actions at any t, denoted by $\succeq_{p_{t-1}}$, depends on his past psychological state where p_t is the psychological state for period t. The statement $a \succeq_{p_{t-1}} a'$ means that the DM finds a at least as good as a', given the psychological state p_{t-1}. The DM takes as given the psychological state from the preceding period.

Note that an outcome of a BDP corresponds to the steady state of an adjustment dynamics where the DM is myopic (i.e. does not anticipate that the psychological state at $t+1$ is affected by the action chosen at t).

Let $h(p) = \{a \in A : a \succeq_p a', a' \in A\}$. For ease of exposition, assume that $h(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response.[4]

[3] These terms are all defined in the appendix below.

[4] Under the assumptions required to prove Proposition 1, as shown in the appendix, $h(\cdot)$ is increasing map of p so that the sequence of short-run outcomes is an (component-wise) increasing sequence (as by assumption contained in a compact set) and therefore, converges to its supremum which is necessarily a BDP. So the existence result covers not only cases where a solution to a BDP (equivalently, a steady-state solution to the myopic preference adjustment mechanism) exists but also ensures that short-run outcomes converge to a BDP.

Long-run outcomes are denoted by a pair a, p with $p = \pi(a)$ where a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = h(\pi(a))$. In other words, long-run behavior corresponds to the outcome of a BDP (see also Von Weizsacker (1971), Hammond (1976), Pollak (1978) who make a similar point for the case of adaptive preferences defined over consumption).

In contrast, in a SDP, the DM is farsighted (i.e. anticipates that the psychological state at $t + 1$ is affected by the action chosen at t). The outcome of a SDP is one where a is defined to be the steady state solution to $a \in \{a \in A : a \succeq_{\pi(a)} a', a' \in A\}$ and $p = \pi(a)$: in this case, the DM anticipates that p adjusts to a according to $\pi(\cdot)$ and taking this into account, chooses a. Note that in this simple framework, in a SDP the DM instantaneously adjusts to the steady-state outcome so that p_0, the initial psychological state, has no impact on the steady state solution with farsightedness.[5]

In Appendix 1, we show that our framework also extends to situations that allow DMs (i) to anticipate short-run psychological states that arise from their actions but not the long-run psychological states, and (ii) to make partial prediction of changes in psychological states as a function of their chosen actions (i.e projection bias introduced by Loewenstein et al., 2003).

3.3 Reduced Form Representation

Various interpretations can be given to p. It can be a psychological state, reference point, expectations or, more generally, any dimension of the object of choice that the individual, for some reason, can (mistakenly) take as given at the point of making a choice. Are all of these interpretations consistent with our general theoretical framework?

Our analysis assumes that DMs' preferences depend on both current action and psychological state. In some cases, the action causes the psychological state. This is the case of a reference point or an emotional state like fear, anxiety or stress that quickly adjusts to current actions. But in other situations, the psychological state precedes the action, and in this sense, our definition of "consistent decision state" is an equilibrium concept. This is

[5] Non-trivial dynamics would be associated with farsighted behavior if underlying preferences or action sets were time variant.

the case where the psychological state concerns expectations, endowments or beliefs.[6]

For example, in Tversky and Kahneman (1991)'s theory of reference-dependent preferences over consumption, a could be a consumption bundle and p is a reference point (another commodity bundle). If the DM chooses a when the pre-decision reference point is p, the post-decision reference point shifts to a. In this sense, the model of decision-making studied here corresponds to a situation where "the reference state usually corresponds to the decision-maker's current state." (Tversky and Kahneman, 1991, pp. 1046). Shalev (2000), Köszegi and Rabin (2006, 2007) and Köszegi (2010) also consider models of endogenous reference-dependent preferences.[7] Caplin and Leahy (2001) analysis of anticipatory feelings is also related to our paper as these can be interpreted as a specific example of a psychological state.[8]

By using similar reasoning, it follows that our general framework, unifies seemingly disconnected models in the literature, from situations where the psychological state corresponds to beliefs (Geanakoplos et al., 1989; Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007) and aspirations (Dalton et al., 2011; Heifetz and Minelli, 2006).

Given our interpretation of the outcomes of a SDP and a BDP as corresponding to distinct steady-states associated with an adaptive preference mechanism, as already argued our model can be seen as a reduced form representation of adaptive preferences over consumption (Von Weizsacker, 1971; Hammond, 1976 and Pollak, 1978, already referred to above), the theory of melioration where consumers fail to take into account the effect of current choices on future tastes (Herrnstein and Prelec, 1991) and projection bias (Loewenstein et al., 2003) where a DM tends to exagerate the degree to which their future tastes

[6] A similar notion of equilibrium is proposed by Koszegi (2010) and Geanakopolos et al (1989).

[7] Our paper complements this literature by studying the situations in which the DM doesn't internalize the endogeneity of the reference points and by providing testable restrictions in which actual choice data can in principle be compared.

[8] Caplin and Leahy (2001) provide a set of axioms so that the representation of underlying preferences with anticipatory feeling is possible in an expected utility setting. Given our emphasis on testable restrictions our axiomatic characterization complements their work.

will resemble their current state.[9]

Below we present two further examples that illustrate how our framework encompasses models of status-quo bias and dynamic inconsistency.

Example 1: Status-quo Bias

Consider a DM who is considering whether to switch to a different service provider (e.g. gas and electricity) from his current one. The psychological state (in this case the reference point) will either be current supplier (if he sticks with the current supplier) or the alternative supplier (if he makes the change). There are two payoff relevant dimensions of choice with outcome denoted x_1 and x_2 and preferences $u(x) = x_1 + v(x_1 - r_1) + x_2 + v(x_2 - r_2)$ where $v(\cdot)$ is a Kahneman-Tversky value function with $v(z) = z$ if $z \geq 0$, $v(z) = \alpha z$, $\alpha > 2.5$ if $z < 0$ and $v(0) = 0$. The cost of switching is equal to 0.5. The status-quo option is defined by $q = (0, 1)$ and the alternative option is $a = (2, 0)$. The payoff table below provides a quick summary of the decision problem:

	status quo	alternative
current supplier	1	$2 - 2\alpha$
alternative supplier	$3.5 - \alpha$	1.5

In this example, again, the payoffs are an additive function of the action-based payoff and the psychological state-based payoff.

A DM who uses a SDP recognizes that he has to choose between the on-diagonal elements. Sticking with the current supplier goes with the reference point status quo. Choosing the alternative supplier goes together with the reference point of the alternative. Hence, the off-diagonal paths are not options and the outcome of a SDP will be to switch to the alternative supplier.

However, the behavioral DM mistakenly believes that (or at least acts as if) he can choose between the two suppliers without changing his psychological state. Consequently, there are two payoff ranked outcomes: one where the behavioral DM sticks with the current

[9] Projection bias provides a possible explanation of why DMs may use a BDP instead of a SDP in some particular situations. For example, projection bias can explain why behavioral DMs get trapped in addiction or overconsumption of durable goods. However, projection bias cannot account for all the models encompassed in BDPs. This is the case, for instance, for models of cognitive dissonance.

supplier and the reference point is status-quo and the other where he switches suppliers and the reference point is the alternative. The former choice is a mistake in the sense that the DM would be better off if he chose to switch and ended up with the alternative as the reference point.

Example 2: Dynamic Inconsistency

Consider a three period problem $t = 0, 1, 2$ where a DM has preferences defined over a single consumption good $c_t, t = 0, 1, 2$. The DM is endowed with a single unit of the consumption good at $t = 0$ but has no endowment of the consumption good in either of the subsequent two periods. The DM obtains no utility from consumption at $t = 0$ but obtains utility from consumption at $t = 1, 2$ with an instantaneous linear utility function c. Assume that the DM quasi-hyperbolically discounts the future with $0 < \beta < 1$ and $\delta = 1$.

There are two assets: (i) an illiquid asset I where one unit invested yields nothing at $t = 1$ and $R > 1$ units of the consumption good at $t = 2$, (ii) a liquid asset where one unit invested at $t = 0$ yields 1 unit of the consumption good if liquidated at $t = 1$ and nothing at $t = 2$ or if not liquidated at $t = 1$ yields $R' > R$ units of the consumption good at $t = 2$. We assume that $\beta < \frac{1}{R'}$. The DM at $t = 0$ will choose which asset to invest in in order to maximize $\beta (c_1 + c_2)$. At $t = 1$ the current self of the DM will maximize $c_1 + \beta c_2$.

To represent this decision problem in our framework we proceed as follows. The psychological states of the DM at $t = 0$ are $p_1 =$"tempted to liquidate at $t = 1$" and $p_2 =$"not tempted to liquidate at $t = 1$" (corresponding to not liquidate). Note that at $t = 1$, if L was chosen at $t = 0$, the current self of the DM would be tempted and liquidate if $\beta R' < 1$ i.e. $\beta < \frac{1}{R'}$. Clearly, the current self of the DM cannot be tempted to liquidate if at $t = 0$ the DM has invested in the illiquid asset.

Therefore, the action "invest in the illiquid asset" goes with the psychological state $p_2 =$"not tempted to liquidate at $t = 1$" while the action "invest in the liquid asset" goes with the psychological state $p_1 =$"tempted to liquidate at $t = 1$". The DM at $t = 0$ has to decide whether to invest in the liquid or the illiquid asset. A quick summary of his decision problem at $t = 0$ is:

	tempted	not tempted
liquid	1	R'
illiquid	R	R

If the DM follows a SDP, he will correctly anticipate that the asset chosen today will affect his psychological state at $t = 1$ and will choose to invest in the illiquid asset and obtain a payoff of $R > 1$. In a SDP the DM exhibits self-control by using the illiquid asset as a pre-commitment device. If the DM follows a BDP, he will believe that (or act as if) the asset chosen today will not affect his psychological state at $t = 1$. Interestingly, there is no pure action solution to a BDP.[10] If the psychological state is "tempted", he will choose to invest in the illiquid but if the psychological state is "not tempted" he will invest in the liquid asset. There is, however, a random solution where the behavioral DM chooses to invest in the liquid asset with probability $p = \frac{R'-R}{R'-1}$: if a behavioral DM believes that the distribution over psychological states is $\left\{\frac{R'-R}{R'-1}, \frac{R-1}{R'-1}\right\}$, he is indifferent between investing in either the liquid or the illiquid asset and is willing to randomize between the two actions. By computation, it is easily checked that the expected payoff from such a random action is less than R, the payoff of a standard DM.

3.4 Stackelberg vs. Nash in an Intra-self Game

In a formal sense, we could also interpret the distinction between a SDP and a BDP as corresponding to the *Stackelberg* and, respectively, the *Nash* equilibrium of dual self intra-personal game where one self chooses actions a and the other self chooses the psychological state p and $\pi(a)$ describes the best-response of the latter for each $a \in A$.

In a Stackelberg equilibrium, the self choosing actions anticipates that the other self chooses a psychological state according to the function $\pi(\cdot)$. In a Nash equilibrium, both selves take the choices of the other self as given when making its own choices.

[10] We achnowledge that Proposition 1 (existence) doesn't cover situations with payoffs as in this example, where there are no pure action solutions to a BDP. However, given that the outcome of a BDP can be interpreted as a Nash equilibrium of a two person game (see Section 3.4. below), as long as A and P are finite, a behavioral decision outcome involving randomization always exists. A different possibility, referring back to the dynamic interpretation of model, is that in such situations, the sequence of short-run outcomes will cycle.

Consistent with the dynamic interpretation of the general framework, in the definition of a SDP, internalization (i.e. rationally anticipating the actual effects of one's actions) is equivalent to the DM anticipating equilibrium (e.g. one's own actions is what one expected it to be, or what others expected it to be) and behaving accordingly.

Given this interpretation, it follows that in the welfare analysis reported below, only the preferences of the self that chooses actions is taken into account.

4 Axiomatic Characterization and Distinguishability

In general, suppose that we observed the choices of a DM. How could we test whether his behavior is consistent with a BDP or an SDP?[11] In what follows, we show that both decision procedures are characterized by two familiar observable properties of choice.

Fix $\succeq$, $\pi : A \to P$ and a family $\mathcal{A}$ of non-empty subsets of A. Define two correspondences, $\mathfrak{S}$ and $\mathfrak{B}$, from $\mathcal{A}$ to A as

$$\mathfrak{S}(A') = \left\{a : (a,p) \succeq \left(a', p'\right) \text{ for all } a' \in A',\, p' = \pi(a') \text{ and } p = \pi(a)\right\}$$

and

$$\mathfrak{B}(A') = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\},$$

so, the choices corresponding to a standard and behavioral decision procedure, respectively.

Suppose that we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$. We say that SDP (respectively, BDP) rationalizes C if there exist P, π and $\succeq$ such that $C(A') = \mathfrak{S}(A')$ (respectively, $C(A') = \mathfrak{B}(A')$).

4.1 Choice Data compatible with a SDP

Consider the following condition introduced by Arrow (1959) (henceforth Arrow's axiom):

Arrow's axiom. If $A' \subseteq A$ and $C(A) \cap A'$ is non-empty, then $C(A') = C(A) \cap A'$.

When the set of feasible alternatives shrinks, the choice from the smaller set *consists precisely* of those alternatives chosen in the larger set and remain feasible, if there is any.

The following result provides a complete characterization of choice data compatible with a SDP.

[11] We are grateful to Andres Carvajal for his helpful suggestions on this section of the paper.

Proposition 2. *Choice data are rationalizable as the outcome of a SDP if and only if Arrow's axiom is satisfied.*

Proof. (i) We show that if choice data are rationalizable as the outcome of a SDP, then, *Arrow's axiom* holds.

Fix $\succeq$, $\pi : A \to P$. If

$$a \in \mathfrak{S}(A') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}$$

and $A'' \subseteq A'$, it follows that

$$a \in \mathfrak{S}(A'') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A'',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}.$$

Therefore, $\mathfrak{S}(A') = C(A') \cap A'' \subseteq C(A'') = \mathfrak{S}(A'')$.

It remains to check that $C(A'') = \mathfrak{S}(A'') \subseteq C(A') \cap A'' = \mathfrak{S}(A') \cap A''$.

Suppose there exists $a' \in C(A'') = \mathfrak{S}(A'')$ but $a' \notin \mathfrak{S}(A') \cap A''$. Then, $a' \in A'$ but $a' \notin \mathfrak{S}(A')$. However, by construction, both $(a',p') \succeq (a,p)$ and $(a',p') \preceq (a,p)$ for $p' = \pi(a')$ and $p = \pi(a)$. Therefore, $a' \in \mathfrak{S}(A')$, a contradiction.

It follows that $C(A'') \subseteq C(A') \cap A''$ and therefore, $C(A'') = C(A') \cap A''$ as required.

(ii) We show that if choice data satisfy *Arrow's axiom*, they are rationalizable as the outcome of a SDP.

To this end, we specify $\pi : A \to P$ so that it is one-to-one and onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $(a,p) \succeq (a',p')$ for all $a' \in A'$, $p = \pi(a)$ and $p' = \pi(a')$.

Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A') \Rightarrow \mathfrak{S}(A')$ and therefore, $C(A') \subseteq \mathfrak{S}(A')$.

We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{S}(A') \subseteq C(A')$.

Suppose to the contrary, there exists $a' \in \mathfrak{S}(A')$ but $a' \notin C(A')$. It follows that $(a',\pi(a')) \succeq (b,\pi(b))$ for all $b \in A'$. Since $a' \notin C(A')$, by construction this is only possible if for each $b \in A'$, $a' \in C(A''_b)$ for some A''_b with $\{a,b\} \subseteq A''_b$ and therefore, $A' \subseteq \cup_{b \in A'} A''_b$. But, then, by *Arrow's axiom* $a' \in C(A')$ a contradiction. Therefore, $\mathfrak{S}(A') \subseteq C(A')$.

As $C(A') \subseteq \mathfrak{S}(A')$, it follows that $C(A') = \mathfrak{S}(A')$ as required. ■

Standard choice theory is falsifiable if *Arrow's axiom* holds. Proposition 2 shows that choice data are compatible with SDP if and only if they are also compatible with the standard choice theory.[12]

4.2 Choice Data compatible with a BDP

Consider the following condition introduced by Chernoff (1954) and Sen (1971) (Sen's Axiom α) (henceforth, Chernoff's axiom):

Chernoff's axiom. For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $C(A') \cap A''$ is non-empty, then $C(A') \cap A'' \subseteq C(A'')$.

The choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set.

Next, consider the following condition which we refer to as a minimal consistency:

Minimal consistency axiom. For all $A' \subseteq A$ such that $A'/C(A')$ is non-empty, there exists $b \in A'/C(A')$ such that $b \notin C(\{b, a'\})$ for each $a' \in C(A')$.

The choice correspondence has the property that an excluded alternative is also excluded in at least one pair-wise comparison with an included alternative.

The following example shows that (i) choice data may satisfy *Chernoff's axiom* but not the minimal consistency axiom, and (ii) choice data may satisfy *Chernoff's axiom* and the minimal consistency axiom but not *Arrow's axiom.*

Example 3. (i) Suppose $A = \{a_1, a_2, a_3\}$. If $C(A) = \{a_1, a_2\}$ but $C(\{a_1, a_2\}) = \{a_1, a_2\}$, $C(\{a_1, a_3\}) = \{a_1, a_3\}$ and $C(\{a_2, a_3\}) = \{a_2, a_3\}$, then C satisfies *Chernoff's axiom* but not the minimal consistency axiom. For choice data to satisfy the minimal consistency axiom in this example, it must be the case that $C(\{a_1, a_3\}) = \{a_1\}$ and $C(\{a_2\}) = \{a_2, a_3\}$.

(ii) Now suppose $C(A) = \{a_1\}$ but $C(\{a_1, a_2\}) = \{a_1, a_2\}$, $C(\{a_1, a_3\}) = \{a_1\}$ and $C(\{a_2, a_3\}) = \{a_2, a_3\}$, then C satisfies *Chernoff's axiom* and the minimal consistency axiom but not *Arrow's axiom.*■

The following result provides a complete characterization of choice data compatible with

[12] Masatlioglu and Ok (2005)'s axiomatic characterization of rational choice with status quo bias (exogenous to the actions chosen by the decision-maker) satisfies *Arrow's axiom* among other axioms.

a BDP.

Proposition 3. *(i) If choice data are rationalizable as the outcome of a BDP, Chernoff's axiom is satisfied. (ii) If both Chernoff's axiom and the minimal consistency axiom are satisfied, choice data are rationalizable as the outcome of a BDP.*

Proof. (i) We show that if choice data are rationalizable as the outcome of a BDP, then *Chernoff's axiom* holds.

Fix $\succeq$, $\pi : A \rightarrow P$. If

$$a \in \mathfrak{B}(A') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\right\}$$

and $a \in A'' \subseteq A'$, it follows that

$$a \in \mathfrak{B}(A'') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A''\right\}.$$

Therefore, $C(A') \cap A'' \subseteq C(A'')$ as required.

(ii) We show that if choice data satisfy both the *Chernoff's axiom* and the *minimal consistency axiom,* they are rationalizable as the outcome of a BDP.

To this end, we specify $\pi : A \rightarrow P$ so that it is one-to-one and onto.

Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$.

Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A')$, then $a \in \mathfrak{B}(A')$ and therefore, $C(A') \subseteq \mathfrak{B}(A')$.

We need to check that for the above specification of $\succeq$, $\pi : A \rightarrow P$, $\mathfrak{B}(A') \subseteq C(A')$.

Suppose to the contrary, there exists $a' \in \mathfrak{B}(A')$ but $a' \notin C(A')$. It follows that $a' \succeq_{p'} b$ for all $b \in A'$ and $p' = \pi(a')$. Since $a' \notin C(A')$, by construction this is only possible if $a' \in C(A''_b)$ for some A''_b with $\{a', b\} \subseteq A''_b$ with strict inclusion for at least one b (by the *minimal consistency axiom*).

Let $A'' = \cup_{b \in A'} A''_b$. It follows that $A' \subset A''$ and $a' \in C(A'')$. But, then, by *Chernoff's axiom* $a' \in C(A')$ a contradiction. Therefore, $\mathfrak{B}(A') \subseteq C(A')$.

As $C(A') \subseteq \mathfrak{B}(A')$, it follows that $C(A') = \mathfrak{B}(A')$ as required. ■

In the following example, we show that the outcome of a BDP generates choice data compatible with *Chernoff's axiom* and the minimal consistency but not *Arrow's axiom*:

Example 4. Suppose $A = \{a_1, a_2, a_3\}$. Set $P = \{p_1, p_2, p_3\}$, $\pi(a_1) = p_1$, $\pi(a_2) = p_2$, $\pi(a_3) = p_3$, and $\succeq$ such that:

	p_1	p_2	p_3
a_1	3	1	2
a_2	2	2	1
a_3	1	3	1

Then, $C(A) = \mathfrak{B}(A) = \{a_1\}$, and $C(\{a_1, a_2\}) = \mathfrak{B}(\{a_1, a_2\}) = \{a_1, a_2\}$, $C(\{a_1, a_3\}) = \mathfrak{B}(\{a_1, a_3\}) = \{a_1\}$, $C(\{a_2, a_3\}) = \mathfrak{B}(\{a_2, a_3\}) = \{a_2, a_3\}$ so that the resulting choice data are compatible with *Chernoff's axiom* and the minimal consistency axiom but not *Arrow's axiom.*■

Moreover, by example, we show that if $C(\cdot)$ satisfies *Chernoff's axiom* but not the *minimal consistency axiom* it cannot be rationalized as the outcome of a BDP.

Example 5. Suppose $A = \{a_1, a_2, a_3\}$. If $C(A) = \{a_1, a_2\}$ but $C(\{a_1, a_2\}) = \{a_1, a_2\}$, $C(\{a_1, a_3\}) = \{a_1, a_3\}$ and $C(\{a_2, a_3\}) = \{a_2, a_3\}$, then C satisfies *Chernoff's axiom* but cannot be rationalized as the outcome of a BDP. Observe that $C(\{a_1, a_3\}) = \{a_1, a_3\}$ implies $a_3 \succeq_{\pi(a_3)} a_1$ and $C(\{a_2, a_3\}) = \{a_2, a_3\}$ implies that $a_3 \succeq_{\pi(a_3)} a_2$ and therefore, $C(A) \subset \mathfrak{B}(A) = \{a_1, a_2, a_3\}$. ■

Observe, in addition, that if $A' \subseteq A$ and $C(A) \cap A'$ is non-empty, then $\{C(A) \cap A'\} \cap C(A')$ is the empty set, and such data cannot be rationalized either as the outcome of a BDP or SDP. When the set of feasible alternatives shrinks, the choice from the smaller set *does not include* any alternative selected from the larger set and remains feasible, if there is any.

There is an emerging literature that provides axiomatic characterizations of decision-making models with some specific behavioral flavor. Relevant contributions to this literature are Manzini and Mariotti (2007, 2011), Cherepanov et al. (2008), Masatlioglu et al. (2009). We argue that a BDP is observationally distinguishable from each of these models on the basis of choice data alone.

To start with, choice data consistent with the different procedures of choice proposed by each of these papers can account for pairwise cycles, while choice data consistent with BDP cannot: pairwise cycles of choice are simply inconsistent with Cherfnoff's axiom. For example, suppose $A = \{a, b, c\}$ and $C(A) = \{a\}$, $C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$

but $C(\{c,a\}) = \{c\}$. This choice function can be rationalized, for example, by Manzini and Mariotti's (2007) Categorize then Choose (CTC) choice procedure, but is not consistent with a BDP. The choice data would be consistent with BDP if, for example, $C(\{c,a\}) = \{c,a\}$. Moreover, the Rationalized Shortlist Method (RSM) proposed by Manzini and Mariotti (2007) cannot accommodate menu dependence, whereas a BDP can.

Like us, Masatlioglu et al. (2009) model of Limited Attention allows for violations of menu independence, but in a form very different from (and incompatible with) Chernoff. They define a consideration set (a subset of the set of feasible alternatives) and assume that the DM only pays attention to elements in the consideration set. In their paper revealed preferences are defined as follows: an alternative x is revealed preferred to y if x is chosen whenever y is present and x is not chosen when y is deleted. That is, the choice of an alternative from a set should be unaffected if an element which is not in the consideration set is deleted. If choice changes when an alternative is deleted, then the latter alternative was in the consideration set and clearly the chosen alternative was revealed preferred to it. This is a violation of menu independence, but in a form that is incompatible with Chernoff. Such data cannot be rationalized as an outcome of a BDP, precisely because in a BDP (and also in a SDP), if x is chosen whenever y is present, x must be chosen when y is deleted.

4.3 Menu Independence and WARP

In this subsection, we show that choice data are rationalizable as the outcome of a SDP if and only if it satisfies menu independence and hence, Samuelson's Weak Axiom of Revealed Preferences (WARP).[13]

Fix A the set of alternatives and a family $\mathcal{A}$ of non-empty subsets of A. Suppose, as before, we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$.

A menu is simply an element of $\mathcal{A}$ i.e. a non-empty subset A' of A. A menu-specific revealed preference (Sen, 1997) for any $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow a \in C(A')$. Clearly $R_{A'}$ is incomplete. Menu independent choice requires the existence of a binary relation R_o over A such that for all non-empty $A' \subseteq A$ and for all $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow aR_oa'$. WARP requires

[13] Recall that WARP states that if an alternative a is chosen from some menu of alternatives where some other alternative a' is present (i.e., a is directly revealed preferred to a'), then it can never be the case that alternative a' is selected from any other menu including both a and a'.

that for all non-empty $A', A'' \subseteq A$ and for all $a', a'' \in A' \cap A''$, if $a' \in C(A')$ and $a'' \in C(A'')$, then $a' \in C(A'')$.

Consider the following condition introduced by Sen (1971) (Sen's Axiom β):

Sen's axiom β. For all $A', A'' \subseteq A$, if $A' \subseteq A''$ and $a, a' \in C(A')$, then $a \in C(A'')$ if and only if $a' \in C(A'')$.

We are now in a position to provide a link between SDP, menu independent choice and WARP:

Proposition 4. *Choice data satisfy menu independence (and hence, WARP) if and only if they are rationalizable as the outcome of a SDP.*

Proof. Choice data are menu independent if and only if satisfy both Sen's Axiom α and Sen's Axiom β (Sen, 1997, Theorem 5). As the outcomes of a SDP satisfy Arrow's Axiom, it also satisfies Sen's Axiom α. It remains to show that the outcomes of a SDP satisfy Sen's axiom β.

Suppose $S(A') = C(A')$ for all non-empty $A' \subseteq A$. Suppose $a, a' \in A' \cap A''$, $A', A'' \subseteq A$. If $a, a' \in C(A')$, and $a \in C(A'')$, then $a' \in C(A'')$. Fix $\succeq$, $\pi : A \to P$. Suppose $a, a' \in \mathfrak{S}(A') = C(A')$ and $a, a' \in A''$. We show that if $a \in \mathfrak{S}(A') \cap \mathfrak{S}(A'')$, then $a' \in \mathfrak{S}(A') \cap \mathfrak{S}(A'')$.

Suppose, to the contrary, that $a' \notin \mathfrak{S}(A') \cap \mathfrak{S}(A'')$. By construction, both $(a', p') \succeq (a, p)$ and $(a', p') \preceq (a, p)$ for $p' = \pi(a')$ and $p = \pi(a)$. By assumption

$$a \in \mathfrak{S}(A'') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A'',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}.$$

Therefore, $a' \in \mathfrak{S}(A'')$, a contradiction.

It follows that if $a \in C(A') \cap C(A'') = \mathfrak{S}(A') \cap \mathfrak{S}(A'')$, then $a' \in \mathfrak{S}(A') \cap \mathfrak{S}(A'') = C(A') \cap C(A'')$ so that the outcomes of SDP also satisfy Sen's axiom β and further, WARP is also satisfied. ■

Finally, contrary to what happens in a SDP, choice data consistent with a BDP may violate menu-independence, hence WARP. As we have already shown, choice data can be rationalized as the outcome of a BDP if and only if satisfies Sen's Axiom α. The following example shows that the outcome of a BDP, however, may not satisfy Sen's Axiom β.

Example 6. Suppose $A = \{a_1, a_2, a_3\}$. Consider $A' = \{a_1, a_2\}$ and $A'' = A$. Suppose

$P = \{p_1, p_2, p_3\}$, $\pi(a_1) = p_1$, $\pi(a_2) = p_2$, $\pi(a_3) = p_3$, and $\succeq$ such that:

	p_1	p_2	p_3
a_1	3	1	2
a_2	2	2	1
a_3	4	0	1

In this case, $C(A') = \mathfrak{B}(A') = \{a_1, a_2\}$ and $C(A'') = \mathfrak{B}(A'') = \{a_2\}$.■

The violation of menu independence in a BDP comes from the fact that alternatives may not be irrelevant even though they may never be chosen. In Example 4, for instance, alternative a_3 is never chosen but it makes the DM deviate from choosing a_2 given p_2. Since the DM fails to internalize that the choice of a_3 will trigger p_3 but a_3 is not preferred to the other alternatives given p_3, he will end up choosing neither a_2 (as he did when a_3 wasn't available) nor a_3. So the presence of a_3, although never chosen, is not irrelevant if the DM doesn't fully internalize the endogeneity of psychological states.

4.4 Distinguishability

How relevant is the distinction between a BDP and a SDP? In this subsection, we derive the necessary and sufficient conditions under which outcomes of a BDP and a SDP are indistinguishable from each other and show, in smooth settings, that the two decision procedures are, generically, distinguishable.

A BDP is indistinguishable from a SDP if and only if $B = S$. Note that indistinguishability is, from a normative viewpoint, a compelling property. What matters for welfare purposes is the ranking of consistent decision states, which is the preference relation that a standard DM will use to make a decision. When $B = S$, the outcomes (consistent decision states) of a SDP coincide with that of a BDP, and therefore whether or not the DM internalizes the feedback effect has no normative implications at all.

If $\pi(a) = \pi(a')$ for all $a, a' \in A$, a BDP is, by construction, indistinguishable from a SDP.[14] So suppose $\pi(a) \neq \pi(a')$ for some pair of distinct actions a, a'.

Consider the following conditions:

$\hat{C}1$: For $(a, p = \pi(a))$, $(a', p' = \pi(a'))$ if $(a, p) \succeq (a', p)$, then $(a, p) \succeq (a', p')$;

[14] In this case, p is exogeneous to individual choice and therefore, both, standard and behavioral decision makers rank actions in the same way.

$\hat{C}2$: For $(a, p = \pi(a))$, $(a', p' = \pi(a'))$ such that $(a, p) \succeq (a', p')$, $(a, p) \succeq (a', p)$.

Fix the consistent states (a, p), (a', p'). Condition $(\hat{C}1)$ states that if the action a weakly dominates the action a' at the psychological state p, then the pair (a, p) also weakly dominates the pair (a', p'). Condition $(\hat{C}2)$ states that if the pair (a, p) weakly dominates the pair (a', p'), then the action a weakly dominates the action a' at the psychological state p.

Clearly under $(\hat{C}1)$, $B \subseteq S$ and if $B \subseteq S$ that $(\hat{C}1)$ has to hold is also immediate (from negating $(\hat{C}1)$ and the definition of B and S). Similarly, under $(\hat{C}2)$, $S \subseteq B$ and if $S \subseteq B$ $(\hat{C}1)$ has to hold is also immediate (from negating $(\hat{C}2)$ and the definition of B and S). It follows that $(\hat{C}1)$ and $(\hat{C}2)$ are necessary and sufficient conditions for indistinguishability:

Lemma 1. *Suppose that both B and S are non-empty. Then, (i) $B \subseteq S$ if and only if $(\hat{C}1)$ holds. (ii) $S \subseteq B$ if and only if $(\hat{C}2)$ holds.*

Note that preferences in Example 1 (status-quo bias) violate $(\hat{C}1)$ but satisfy $(\hat{C}2)$ while the preferences in the introductory example about addiction violate both $(\hat{C}1)$ and $(\hat{C}2)$. Shalev (2000) shows (in Theorem 1 of his paper) that in the static case his loss averse preferences satisfy both $(\hat{C}1)$ and $(\hat{C}2)$. Geanakoplos et al. (1989) construct examples where, with one active player, both $(\hat{C}1)$ and $(\hat{C}2)$ are violated.

To further understand the conditions under which indistinguishability occurs, it is convenient to look at smooth decision problems where decision outcomes are characterized by first-order conditions. We show that for the case of smooth decision problems, behavioral decisions are generically *distinguishable* from standard decisions.

A decision problem is smooth if (a) both A and P are convex, open sets in $\Re^k$ and $\Re^n$ respectively, (b) preferences over $A \times P$ are represented by a smooth, concave utility function $u : A \times P \to \Re$ and (c) the feedback map $\pi : A \to P$ is also smooth and concave.

A set of decision problems that satisfies the smoothness assumptions is *diverse* if and only if for each $(a, p) \in A \times P$ it contains the decision problem with utility function and feedback effect defined, in a neighborhood of (a, p), by

$$u + \lambda p$$

and

$$\pi - \mu(a' - a)$$

for each a' in a neighborhood of a and for parameters (λ, μ) in a neighborhood of 0.

A property holds generically if and only if it holds for a set of decision problems of full Lebesgue measure within the set of diverse smooth decision problems.

Proposition 5: *For a diverse set of smooth decision problems, a standard decision procedure is generically distinguishable from a behavioral decision procedure.*

Proof: Let $v(a) = u(a, \pi(a))$.

The outcome $(\hat{a}, \hat{p})$ of a SDP satisfies the first-order condition

$$\partial_a v(\hat{a}) = \partial_a u(\hat{a}, \pi(\hat{a})) + \partial_p u(\hat{a}, \pi(\hat{a}))\partial_a \pi(\hat{a}) = 0 \qquad (1)$$

while the outcome (a^*, p^*) of a BDP satisfies the first-order condition

$$\partial_a u(a^*, p^*) = 0, p^* = \pi(a^*). \qquad (2)$$

For $(a^*, p^*) = (\hat{a}, \hat{p})$, it must be the case that

$$\partial_p u(a^*, p^*)\partial_a \pi(a^*) = 0. \qquad (3)$$

It is easily checked that requiring both $(\hat{C}1)$ and $(\hat{C}2)$ to hold is equivalent to requiring that the preceding equation also holds.

Consider a decision problem with $(a^*, p^*) = (\hat{a}, \hat{p})$. Perturbations of the utility function and the feedback effect do not affect (2) and hence (a^*, p^*) but they do affect (3) and via (1) affect $(\hat{a}, \hat{p})$. Therefore, $(a^*, p^*) \neq (\hat{a}, \hat{p})$ generically. ■

Eq. (3) shows in a simple quick way that BDP and SDP are indistinguishable only in isolated cases (e.g. when $\pi(a^*)$ or $u(a^*, p^*)$ are just constants).[15]

[15] Note that if payoffs over actions have a value function component a la Kahneman and Tversky (where the psychological state is a reference point), the decision problem isn't necessarily smooth or even concave. We note that the first-order approach adopted in Proposition 5 can be extended to non-smooth decision problems as long preferences are concave overall (even though an individual component such as a value function may be non-concave). This would cover cases where $u(a, p) = f(a) + g(a - p)$ where $g(.)$ is a Kahneman-Tversky value function with loss aversion and $u(a, p)$ is concave in a for any fixed p and $v(a) = f(a) + g(a - \pi(a))$ is concave in a. This would be the case when $f(a)$ is concave and $g(.)$ is piece-wise linear with a kink at zero. Essentially, we will need to work with the subgradient of $v(.)$ and $u(.)$ and note that at an action a is an interior optimum of $v(.)$ if and only if it is contained in the subgradient of $v(a)$ and for each fixed p, an action a, p is an interior optimum of $u(a, p)$ if and only if it is contained in the subgradient (with respect to a) of $u(a, p)$ (Hiriart-Urruty and Lemarechal (2001)).

5 Discussion on Choice and Welfare

The recent work on welfare analysis of non-rational choice relies on ordinal (i.e. choice data) information alone to derive a partial preference ordering based on pairwise coherence (Bernheim and Rangel, 2009 (BR); Rubinstein and Salant, 2008 (RS); and earlier by Sen, 1971). BR (and also RS) generalize the standard revealed preference approach to allow for inconsistencies on choice correspondences such as preferences reversals. BR adopt the normative position that what matters for welfare is a binary relation constructed solely on actions using choice data. In that sense, psychological states (ancillary conditions or frames in BR and RS) are assumed to be normatively irrelevant.

Since BR's welfare criterion never overrules choice, it cannot account for well documented cases in which the DM consistently chooses sub-optimally. Choices can be consistent in BR's sense, but still may not represent the true preferences of the individual. For example, observed choices of a DM who suffers from projection bias and consistently mispredicts the consequences of his actions, will be considered appropriate for welfare analysis in BR's sense, but they will not represent DM's true preferences.

Our framework directly addresses this problem by taking the position that psychological states are normatively relevant. Therefore, the preferences of a standard DM provide the relevant normative benchmark. These normative preferences $\succeq$ *over the set of consistent decision states* directly induce a unique ranking of actions, $(a, \pi(a)) \succeq (a', \pi(a'))$. In the example studied in Section 2, the action "no alcohol" and the psychological state "sober" welfare dominates all other consistent decision states; in the three period decision-problem with dynamic inconsistency (Example 2), the relevant benchmark are the preferences of the DM at $t = 0$.

Taken together, the axiomatic characterization and the generic distinguishability result (Lemma 1 and Proposition 5) are, at least partially, normatively informative: whenever a choice correspondence satisfies Chernoff's axiom and the minimal consistency axiom but violates Arrow's axiom, it is typically generated by a DM who chooses against his best interest. However, it doesn't provide a complete answer to the identification of the DM's welfare. For example, a behavioral DM may systematically choose against his best interest and the choices generated from his behavior may still satisfy Arrow's axiom (as the example of addiction in Section 2 shows). In our quest for a more complete answer, we ask under what conditions (if any) we can tease out whether the behavioral DM chooses suboptimally or not. We address this issue in two different ways: under the assumption of a fixed preference relation and under no assumption on preferences.

1. Divergence between choice and welfare for a fixed preference relation

Under what conditions does the ranking over actions using the binary relation in BR agree with some fixed underlying preference relation $\succsim$ *over the set of consistent decision states*? Clearly, one necessary condition (condition $\widehat{C}$) for this to happen is that there are no a and a' such that (i) $(a, \pi(a)) \succ (a', \pi(a'))$ and (ii) for all p, $(a', p) \succ (a, p)$ (as in this case for BR a' is preferred a).[16] It is possible to find examples in which this condition holds, and both rankings (the one derived from a SDP and BR's ranking) agree. For example, consider a decision-problem with reference-dependent preferences, as discussed in Example 1. The preference relation $\succsim$ *over* consistent decision states (a, a) is a frame–independent ranking of actions. In this context, Munro and Sugden's (2003) definition of loss aversion implies condition $\widehat{C}$ (i.e. if a' is preferred to a in the reference neutral sense, then a' is preferred to a when the reference point is a'). Therefore, in this case, the ranking derived form a SDP agrees with BR's ranking. However, as the example in Section 2 shows, condition $\widehat{C}$ may fail. The BR's binary relation derived by pairwise coherence would rank "alcohol"(a') over "no alcohol" (a), whereas the preferences derived from a DM using a SDP would rank "no alcohol" (a) over "alcohol" (a').

This divergence of choice and welfare is not trivial, since as we have shown in Proposition 5, the outcomes of a BDP and a SDP typically diverge. Thus, there is at least an argument for further non-choice data (such as psychological data) to potentially qualify BR's approach. For example, Green and Hojman (2008) study divergence between choice and welfare which relies on use of cardinal information.

[16] Note that $\widehat{C}$ implies $\hat{C}2$ and further that the conjunction of (i) and (ii) is ruled out whenever the decision problems are indistinguishable. A formal proof is provided in Proposition 2 of Dalton and Ghosal (2011), where a distinction is made between a pre-decision and a post-decision frame and, using this distinction, the relation between the normative implications of decision problems with endogeneous frames to choice with frames and ancillary conditions studied by Bernheim and Rangel and Rubinstein and Salant is examined.

This conclusion, however, relies on the assumption of a fixed preference relation. But can we infer the divergence between choice and welfare without making any assumption on preferences and/or feedback effects? In what follows, we construct an scenario in which such a divergence can be inferred.

2. Divergence between choice and welfare comparing choice data across two different choice scenarios

Suppose A is the set of alternatives and let $\widetilde{\mathcal{A}}$ denote the set of subsets of A consisting of singletons so that for each $a \in A$, $\{a\} \in \widetilde{\mathcal{A}}$. Consider the following two choice scenarios:

Choice Scenario 1. The DM is asked to choose between a situation where only action a is available and another one where only action a' is available. In other words, the DM has to choose between singleton choice sets, i.e. all pairs $\{a\}$ and $\{a'\}$ in $\widetilde{\mathcal{A}}$. For example, if a is smoking and a' is not-smoking, $\{a'\}$ is a situation in which the option of smoking is not available, and the only available option is "not-smoking" (i.e. go for dinner to a non-smoking restaurant) whereas $\{a\}$ is a situation in which the option of "not-smoking" is not available and the only available option is to smoke (i.e. go for dinner to a restaurant that only admits smokers).[17]

Choice Scenario 2. The DM is asked to choose between the two actions used in the preceding pairwise comparison when both actions are available, i.e. actions in $\{a, a'\}$ for each such pair of actions. For example, choose between smoking and not smoking over dinner in a restaurant where both choices are available.[18]

For each pair of actions $a, a' \in A$, suppose we observe two non-empty correspondences $\tilde{C}(\{a\}, \{a'\}) \subseteq (a, a')$ and $C(a, a') \subseteq (a, a')$. Consider the following condition:

$\widetilde{C}$. $\tilde{C}(\{a\}, \{a'\}) \cap C(a, a')$ is empty for some $a, a' \in A$.

Clearly, if the DM is using a SDP in both choice scenarios (respectively, BDP), $\tilde{C}(\{a\}, \{a'\}) = C(a, a')$. Suppose $\widetilde{C}$ is satisfied for some pair of actions $a, a' \in A$. Then the DM cannot be using a SDP (respectively, BDP) in both choice scenarios. Without loss of generality

[17] This type of scenarios could exist, for example, if there is a law that gives the option to owners of restaurants to decide whether to have "smoke free restaurants", i.e. $\{a'\}$ or "smoke friendly restaurants", i.e. $\{a\}$.

[18] This type of scenario can exist before the implementation of the law we referred to in the previous footnote.

assume that $\tilde{C}(\{a\},\{a'\}) = a$ but $C(a,a') = a'$. Suppose that the DM is using a SDP in Choice Scenario 1. Then, $\tilde{C}(\{a\},\{a'\}) = \mathfrak{S}(a,a') = a$ and there exists a P and $\pi : A \to P$ such that $(a, \pi(a)) \succ (a', \pi(a'))$ but both $(a', \pi(a)) \succ (a, \pi(a))$ and $(a', \pi(a')) \succeq (a, \pi(a'))$ so that $C(a,a') = \mathfrak{B}(a,a') = a'$. Therefore, a welfare dominates a' even though in Choice Scenario 2 the DM chooses a'. Conversely, suppose that the DM is solving a BDP in Choice Scenario 1. Then, $C(a,a') = \mathfrak{S}(a,a') = a'$ and there exists P and $\pi : A \to P$ such that $(a', \pi(a')) \succ (a, \pi(a))$ but both $(a, \pi(a')) \succ (a', \pi(a'))$ and $(a, \pi(a)) \succeq (a', \pi(a))$ so that $\tilde{C}(\{a\},\{a'\}) = \mathfrak{B}(a,a') = a'$. Therefore, a' welfare dominates a even though in Choice Scenario 1 the DM chooses a.

Therefore, if there is a pair of actions a, a' such that $\widetilde{C}$ holds, then the DM's observed choice in one of the two choice scenarios is welfare dominated.[19]

This result can be generalized in the following way. Fix A the set of alternatives and a family $\mathcal{A}$ of non-empty subsets of A. Suppose, as before, that we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$. Consider the following two scenarios:

Choice Scenario 1, the DM ranks each pair of non-empty subsets $A', A'' \subseteq A$;

Choice Scenario 2, which reveals a ranking over sets of actions in $\mathcal{A}$ as follows: for any $A' \subseteq A$ and non-empty $C(A')$ such that $C(A') \subset A'$, the set $C(A')$ is said to be weakly preferred to the set $A'/C(A')$.

Let $\mathcal{R}_1$ denote the binary ranking of pairs of non-empty subsets $A', A'' \subseteq A$ revealed in *Choice Scenario 1* and let $\mathcal{R}_2$ denote the binary ranking of pairs of non-empty subsets $A', A'' \subseteq A$ revealed in *Choice Scenario 2*. Clearly $\mathcal{R}_2$ is incomplete as might (though, obviously, not necessarily) $\mathcal{R}_1$.

The following proposition examines the conditions under which $\mathcal{R}_1$ and $\mathcal{R}_2$ coincide where both are defined and states the welfare implications when the two rankings do not coincide.

Proposition 6. *Suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ do not necessarily coincide where both are defined. Then, the DM cannot be choosing in his best interests in one of the two choice scenarios.*

Proof. For each $A' \subseteq A$ and $A'' \subseteq A'$, we say that $A'' \in K(A')$ iff $A'' \mathcal{R}_1 A'$ with $A'' \subseteq A'$.

[19] It is of interest to note that if DM is behavioral, $\widetilde{C}$ could be satisfied even in those cases where choice data satisfies Arrow's axiom.

Clearly, if the DM is using a SDP in both choice scenarios (respectively, BDP), $C(A') \in K(A')$ so that $\mathcal{R}_1$ and $\mathcal{R}_2$ must coincide where both are defined. So suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ do not necessarily coincide where both are defined. It follows that there exists $A' \subseteq A$ such that $C(A') \notin K(A')$ i.e. $(A'/C(A'))\,\mathcal{R}_1 A'$ but $A'\mathcal{R}_2\,(A'/C(A'))$ and $\sim (A'/C(A'))\,\mathcal{R}_2 A'$. As the DM cannot be using a SDP (respectively, BDP) in both choice scenarios assume that the DM is using a SDP in Choice Scenario 1. Then, $C(A') = \mathfrak{B}(A')$ for all A' and $C(A') \notin K(A')$ for some A'. Therefore, for some $A' \subseteq A$: (i) there exists a pair of actions $a, a' \in A'$ with $a' \in C(A)$ but $a \notin C(A')$, and (ii) $\pi : A \to P$ such that $(a, \pi(a)) \succ (a', \pi(a'))$ but both $(a', \pi(a)) \succ (a, \pi(a))$ and $(a', \pi(a')) \succeq (a, \pi(a'))$ so that a welfare dominates a' even though in Choice Scenario 2 the DM chooses a'. Conversely, suppose that the DM is solving a BDP in Choice Scenario 1. Then, $C(A') = \mathfrak{S}(A')$ for all A' and $C(A') \notin K(A')$ for some A'. Therefore, for some $A' \subseteq A$: (i) there exists a pair of actions $a, a' \in A'$ with $a' \in C(A')$ but $a \notin C(A')$, and (ii) $\pi : A \to P$ such that $(a', \pi(a')) \succ (a, \pi(a))$ but both $(a, \pi(a')) \succ (a', \pi(a'))$ and $(a, \pi(a)) \succeq (a', \pi(a))$ but so that a' welfare dominates a even though in Choice Scenario 1 the DM chooses a. ■

6 Concluding Remarks

All of the welfare economics we know is based on the assumption that people choose what is best for them, and that we can accordingly use these choices as a guide to welfare policy. Once we build realistic behavioral features into our models, this foundation is lost. Can we still extract some normatively relevant information from choices in a context in which DMs may not be utility maximizers?

Arguably, this is an ongoing puzzle of utmost importance and we don't claim to give a complete answer to this question. However, we believe that this paper contributes with some ammunition towards a better understanding of the normative implications of behavioral economics.

This paper makes use of a simple, yet unifying platform that encompasses different existing work in the literature on behavioral economics. We find the distinction between behavioral and standard decision procedures as a convenient tool to study general properties of models of sub-optimal behavior in light of standard models of rational decision making.

Together with the result that the outcomes of behavioral and standard decision procedures are observationally distinguishable in most decision scenarios, our axiomatic characterization has clear normative implications. If choice data satisfy Chernoff's axiom and the minimal consistency axiom but violate Arrow's axiom, then typically these data are generated by a DM who is not necessarily choosing in his best interest. Moreover, the DM can be systematically choosing against his best interest even if choice data satisfy Arrow's axiom (as the example of addiction in Section 2 shows). We show that, in principle, it is possible to infer the divergence of choice and welfare based on choice data alone, although this can be done in very limited settings.

This paper has formally shown that the use of welfare metrics based solely on choice data is moot.

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A Appendix 1: the dynamic interpretation

Predicting short-run, but not long-run, psychological states

So far we have assumed that DMs fail to anticipate that their future psychological state depend on their current choices including the immediate future. We will now extend our framework to situations where DMs may anticipate short-run psychological states that arise from their actions but not the long-run psychological states.

Let $h^2(p) = h(\pi(h(p)))$ and define $h^t(p) = h\left(\pi(h^{t-1}(p))\right)$ iteratively $t = 1, 2, ...$ Fix a $p_0 \in P$. A sequence of *short-run* outcomes compatible with T-period (for some fixed, finite $T \geq 1$) forecasting is determined by the relations $a_t \in h^T(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a best-response that anticipates the short-run psychological states within a T-period horizon.

Long-run outcomes compatible with T-period forecasting are denoted by a pair a', p' with $p' = \pi(a')$ and a' is defined to be the steady-state solution to the short-run outcome function i.e. $a' = h^T(\pi(a'))$.

It follows that long-run behavior corresponds to the outcome of a BDP where the feedback effect is defined to be $\pi'(a) = \pi(h^{T-1}(a))$.

Partial prediction

Next, we extend our framework to situations where DMs may make partial prediction of changes in psychological states as a function of their chosen actions. There are many different ways of modelling partial prediction. We adopt a simple approach: we will assume that each decision maker predicts that the psychological state will respond to their chosen actions with probability q, $0 \leq q \leq 1$. It will be convenient at this point to assume that the binary relation $\succeq$ has a (expected) utility representation $u : A \times P \rightarrow \Re$. Let $v(a) = u(a, \pi(a))$.

Let $h(p; q) = \{a \in A : a \in \arg\max_{a \in A} qv(a) + (1 - q)u(a, p)\}$. In what follows, we will assume that that $h(p; q)$ is unique.

Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1}; q)$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response.

Long-run outcomes are denoted by a pair a, p with $p = \pi(a)$ and a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = h(\pi(a); q)$.

It follows that long-run behavior corresponds to the outcome of a BDP where the preferences are represented by a utility function $w(a, p) = qv(a) + (1 - q)u(a, p)$. This formulation is formally equivalent to the modelling of projection bias in Loewenstein et al. (2003).

Note that the above representation is consistent with incomplete learning: as long as the DM doesn't fully learn to internalize the feedback effect from actions to psychological states, there is a way of relabelling variables so that the steady-state preferences corresponding to an adaptive preference mechanism are the outcomes of a BDP.

B Appendix 2: Proof of Proposition 1[20]

Recall that the preferences of the DM is denoted by $\succeq$ a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. As the focus is on incomplete preferences, in this section, instead of working with $\succeq$, we find convenient to specify two other preference relations, $\succ$ and $\sim$. The expression $\{(a, p), (a', p')\} \in \succ$ is written as $(a, p) \succ (a', p')$ and is to be read as "(a, p) is strictly preferred to (a', p') by the DM". The expression $\{(a, p), (a', p')\} \in \sim$ is written as $(a, p) \sim (a', p')$ and is to be read as "(a, p) is indifferent to (a', p') by the DM". Define

$$(a, p) \succeq (a', p') \Leftrightarrow \textit{either } (a, p) \succ (a', p') \textit{ or } (a, p) \sim (a', p').$$

Once $\succeq$ is defined in this way, the results obtained in the preceding sections continue to apply. In what follows, we do not require either $\succeq$ or $\succ$ or $\sim$ to be transitive.

[20] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both for showing the existence of an optimal choice and using Kakutani's fix-point theorem.

Suppose $\succ$ is

(i) acyclic i.e. there is no finite set $\{(a^1, p^1), ..., (a^T, p^T)\}$ such that $(a^{t-1}, p^{t-1}) \succ (a^t, p^t)$, $t = 2, ..., T$, and $(a^T, p^T) \succ (a^1, p^1)$, and

(ii) $\succ^{-1}(a, p) = \{(a', p') \in A \times P : (a, p) \succ (a', p')\}$ is open relative to $A \times P$ i.e. $\succ$ has an open lower section[21].

Suppose both A and P are compact. Then, by Bergstrom (1975), it follows that S is non-empty.

Define

$$a \succ_p a' \Leftrightarrow (a, p) \succ (a', p).$$

The preference relation $\succ_p$is a map, $\succ: P \to A \times A$. If $\succ$ is acyclic, then for $p \in P$, $\succ_p$is also acyclic. If $\succ$ has an open lower section, then $\succ_p^{-1}(a) = \{a' \in A : a \succ a'\}$ is also open relative to A i.e. $\succ_p$ has an open lower section. In what follows, we write $a' \notin \succ_p (a)$ as $a \not\succ_p a'$ and $a' \in \succ_p (a)$ as $a' \succ_p a$.

Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \varnothing\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

We make the following additional assumptions:

(A1) A is a compact lattice;

(A2) For each p, and a, a', (i) if $\inf(a, a') \not\succ_p a$, then $a' \not\succ_p \sup(a, a')$ and (ii) if $\sup(a, a') \not\succ_p a$ then $a' \not\succ_p \inf(a, a')$ (quasi-supermodularity);

(A3) For each $a \geq a'$ and $p \geq p'$, (i) if $a' \not\succ_{p'} a$ then $a' \not\succ_p a$ and (ii) if $a \not\succ_p a'$ then $a \not\succ_{p'} a'$ (single-crossing property)[22]

(A4) For each p and $a \geq a'$, (i) if $\succ_p (a') = \varnothing$ and $a' \not\succ_p a$, then $\succ_p (a) = \varnothing$ and (ii) if $\succ_p (a) = \varnothing$ and $a \not\succ_p a'$, $\succ_p (a') = \varnothing$ (monotone closure).

Assumptions (A2)-(A3) are quasi-supermodularity and single-crossing property defined by Milgrom and Shannon (1994).

[21]The continuity assumption, that $\succ$ has an open lower section, is weaker than assuming that that preferences have both open upper and lower sections (Debreu (1959)), which in turn is weaker than the assumption that preferences have open graphs. Note that assuming $\succ$ has an open lower section is consistent with $\succ$ being a lexicographic preference ordering over $A \times P$.

[22]For any two vectors $x, y \in \Re^K$, the ussual component-wise vector ordering is defined as follows: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, .., K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, .., K$.

Assumption (A4) is new. Consider a pair of actions such that the first action is greater (in the usual vector ordering) than the second action. For a fixed p, suppose the two actions are unranked by $\succ_p$. Then, assumption (A4) requires that either both actions are maximal elements for $\succ_p$or neither is.

The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified in Ghosal (2011) who also shows that assumptions (A1)-(A4), taken together, are sufficient to ensure that $\Psi(p)$ is non-empty and compact and monotone in p i.e. for $p \geq p'$ if $a \in \Psi_1(p)$ and $a' \in \Psi_1(p')$, then $\sup(a, a') \in \Psi_1(p)$ and $\inf(a, a') \in \Psi_1(p')$.

To complete the proof of Proposition 1, define a map $\Psi : A \times P \to A \times P$, $\Psi(a, p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each (a, p), $\Psi_1(p) = \{a' \in A : \succ_p (a') = \phi\}$ and $\Psi_2(a) = \pi(a)$. It follows that $\Psi_1(p)$ is a compact (and consequently, complete) sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. By assumption 1, it also follows that for each a, $\pi(a)$ has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{\pi}(a)$ and $\underline{\pi}(a)$ respectively. Therefore, the map $(\bar{a}(p), \bar{\pi}(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{p}) = (\bar{a}(\bar{p}), \bar{\pi}(\bar{a}))$ is a fix-point of Ψ and by a symmetric argument, $(\underline{a}(p), \underline{\pi}(a))$ is an increasing function from $A \times P$ to itself and $\left(\underline{a}, \underline{p}\right) = \left(\underline{a}(\underline{p}), \underline{\pi}(\underline{a})\right)$ is also a fix-point of Ψ; moreover, $(\bar{a}, \bar{p})$ and $\left(\underline{a}, \underline{p}\right)$ are respectively the largest and smallest fix-points of Ψ.