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Policy Improvement via Inverse ALM

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Policy Improvement via Inverse ALM

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Abstract

Traditional ALM first sets the policy parameters and then assesses the impact on some sub-set of risk and return measures. We propose a method to 'invert' the traditional ALM approach: first formulate the desired level of risk and return measures and then systematically search through the policy space to find the policy that 'best' meets the objectives and constraints. The method is more 'open minded' than traditional ALM as is shown in a numerical example using an ALM model for a stylized pension fund.

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1. *Introduction*

Asset liability management, typically used by for example pension funds and insurance companies, is challenging. The situation is complicated and the consequences are uncertain but significant.

One has to account for multiple risk and return measures, for example future profits, contributions and probabilities of insolvency, on different time horizons, for example 1 year, 5 years and/or 30 years. There are multiple goals which may differ for each of the stakeholders and there are many policy instruments. Examples of the latter are the investment policy, profit-sharing policy and contribution policy. To make it even more difficult there is uncertainty about for example future economic variables and future regulation.

Often, Asset Liability Management (ALM) models are used when making (and to support) policy decisions. Traditional ALM first sets the policy parameters and then assesses the impact on some sub-set of risk and return measures. We propose a method to 'invert' the traditional ALM approach: first formulate the desired level for risk and return measures and then find the policy parameters. The method is based on the idea that the objectives should drive the policy choice and not the other way around as is often seen in practice, see e.g. Chapman (1999) and Slater (2002) for interesting discussions on this issue.

The remainder of this paper is organized as follows: we first describe the method, which we refer to as 'Inverse' ALM. Then we show a numerical example for a stylized pension fund and propose some extensions of the method. We end with a summary and conclusions.

2. *Inverse ALM using WLS*

In traditional ALM one chooses different policy parameters and the ALM model (a non-linear mapping) shows the effect of the new policies on different risk and return measures. Typically the number of policy- or control variables in the ALM model is limited: the contribution policy, profit-sharing policy and the investment policy for example. There are far more outcomes: risk and return measures such as (the percentiles of) solvency measures and profits at different time horizons. Figure 1 on the left shows the traditional ALM process schematically, depicted as a 'box' with a few buttons and many outputs.

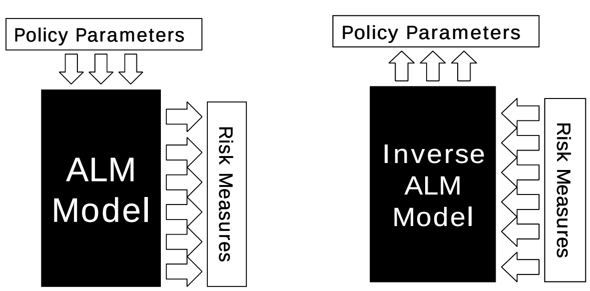

Figure 1 Traditional ALM en inverse ALM

With 'inverse' ALM we propose to 'invert' the traditional ALM approach. First formulate the desired levels of the risk measures and then find the police parameters to get there. Since the number of control variables is limited with respect to the number of risk measures in this context, we will on overall not be able to achieve all desired outcomes of risk and return measures. We can come as close as possible by projecting the desired outcomes on the 'number of control variables' dimensional subspace of achievable outcomes. Figure 2 shows the inverse ALM process schematically, where the desired change in risk measures is ΔR^*, the achievable change in risk measures is $\Delta \hat{R}^*$, and the cosine of the angle between ΔR^* and $\Delta \hat{R}^*$, $\cos\theta$, is a measure of 'realism' of the desired ΔR^* (see Davidson and MacKinnon, 2004).

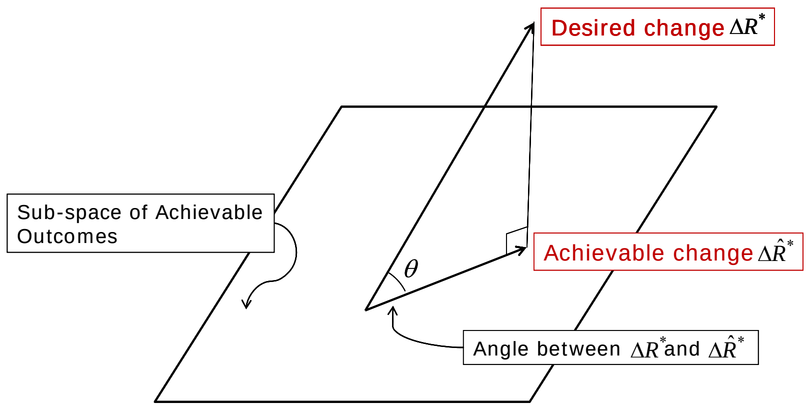

Figure 2 Sub-space of achievable outcomes

First we look at traditional ALM. For small changes in policy parameters *B* (Δ*B*) the traditional ALM is approximately linear[4]. If we look at risk measures *R*, the change in these risk measures caused by a small change in policy instruments *B* equals:

$$\Delta R = X \Delta B \tag{2.1}$$

where *R* is a *N*-vector, *B* is a *M*-vector (we assume the typical case that $N > M$) and where *X* is a non-square ($N \times M$)-matrix of partial derivatives $X_{nm} = \partial R_n / \partial B_m$.

Inverse ALM is also linear:

$$\Delta B^* = X^+ \Delta R^* \tag{2.2}$$

where ΔR^* is the desired change in risk measures, and X^+ is the pseudo-inverse of the matrix *X* of partial derivatives. If $(X'X)$ is well-defined and non-singular (2.2) can be written as:

$$\Delta B^* = \left(X'X\right)^{-1} X' \Delta R^* \tag{2.3}$$

$X^+ = \left(X'X\right)^{-1} X'$ is also called the Moore-Penrose pseudo-inverse (see for example Greene, 2004, pp 45-46). One can recognize (2.2) and (2.3) as an application of least squares regression. When ΔR^* is the *desired change* in the outcomes of risk measures R, equation (2.3) gives the change in the policy parameters to get as close as possible to the desired outcomes. The 'achievable' change in risk measures is given by:

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[4] Actually for small changes in parameters any model is approximately linear.

$$\Delta\hat{R}^* = X\Delta B^* = XX^+\Delta R^* = X\left(X'X\right)^{-1}X'\Delta R^* \qquad (2.4)$$

where $X\left(X'X\right)^{-1}X'$ is the least squares projection matrix.

One could use the singular value decomposition (SVD, see e.g. Bretscher 1997) of the ($N \times M$)-matrix X to obtain X^+. The matrix X can be decomposed as $X = U\Sigma V'$ so that $X^+ = V\Sigma^{-1}U'$. Here U and V are (column) orthogonal $N \times M$ and $M \times M$ matrices respectively, Σ is a $M \times M$ diagonal matrix containing the singular values σ_i $i = 1,..,M$ of X, and Σ^{-1} is a diagonal matrix with elements $1/\sigma_i$, for $\sigma_i > 0$, zero otherwise. An advantage of SVD is that it can also be used to approximate the inverse of the matrix of partial derivatives in case $(X'X)$ is ill-defined or singular.

In the formulas above all risk measures are taken into account as equally important. When some risk measures are considered more important then others, we can use weighted least squares (WLS), which results in formula (2.5)

$$\Delta\hat{R}^* = X\left(X'WX\right)^{-1}X'W\Delta R^* \qquad (2.5)$$

where W is a matrix with elements (the 'weights') W_{nn} $n \in \{1,...,N\}$ on the diagonal, and other elements equal to zero.

Using iteration of the inverse ALM process it should be checked that the change risk measures by a change in policy controls is approximately linear. The appropriate step-size may depend on the risk measures under consideration. For example, order statistics such as value at risk measures will be highly non-linear when the step-size is too large. Stochastic flow analysis works locally but not any more if scenario's cross each other, because then the assumption of local differentiability will not hold any more for order statistics.

Many extensions of this simple 'inverse' ALM are possible. We will propose some extensions in section 4, but we first illustrate the results of inverse ALM using the WLS method proposed above in a numerical example for a stylized pension fund.

3. *Numerical example using WLS*

3.1. The ALM model

We demonstrate the method using an ALM model for a stylized pension fund which is inspired by Grosen and Jørgensen (2000). We choose this relatively simple setting to illustrate the idea without the necessity to focus on technical details of a full-version practical ALM method. We first introduce the following balance sheet of the pension fund at time t:

Assets	Liabilities
A_t	L_t
	E_t

where A_t are the assets, L_t the liabilities and E_t is the equity or buffer of the pension fund at time t. Given this balance sheet we define the funding ratio at time t as:

$$FR_t = \frac{A_t}{L_t} \tag{3.1}$$

With respect to the liabilities we make the following simplifying assumptions. There is a single payment after T years to the average cohort of all participants. The value of the liabilities at time $t+1$ is determined by the liabilities at time t where the liabilities will only grow by indexation and interest until the contract expires. Other factors, such as mortality are assumed to be deterministic and already accounted for in the liabilities at starting time $t=0$, L_0.

The interest and indexation depend on each year's market return. The interest rate, denoted by r, is assumed to be constant and equal to the economy's continuously compounded riskless rate of interest. The yearly indexation is denoted by i_t. The dynamics of the liabilities $L_t, t \in \{1,2,...,T\}$, of the pension fund are then given by $L_{t+1} = L_t(1+r)(1+i_t)$.

For the indexation policy, which results in i_t, we assume there is some minimum guaranteed level of indexation denoted by i_g. Additional indexation depends on the solvency position of the pension fund. We assume the pension fund has some target funding ratio, denoted by FR_{target}. If the funding ratio is above this level, then some percentage α of the buffer above the target level is used for indexation. We will, as in Grosen and Jørgensen (2000), refer to α as the distribution ratio. The liabilities L_t, $t \in \{1,2,...,T\}$, can be written as:

$$L_{t+1} = L_t(1+r)\left[1+\max\left(\alpha\left(FR_t - FR_{target}\right), i_g\right)\right] \tag{3.2}$$

We now turn to the dynamics on the asset side of the balance sheet. The pension fund has a yearly rebalancing portfolio with some percentage, denoted by w_s, invested in risky assets and (1- w_s) is invested in riskless bonds. We assume the process of the risky assets to evolve according to a geometric Brownian motion on a finite time interval *[0,T]* as:

$$dS_t = \mu_S S_t dt + \sigma_S S_t dB_t \tag{3.3}$$

where S_0, μ_S and σ_S are constants. The standard Brownian motion B_t is defined on the probability space (Ω, H, P) with filtration $(H_t)_{0 \le t \le T}$ satisfying the usual conditions, and P denotes the real world probability measure.

The annual (log) returns of the risky assets are conditionally Gaussian distributed:

$$\ln(S_t / S_{t-1}) \mid H_t \sim \varphi(\mu_S - \tfrac{1}{2}\sigma_S^2, \sigma_S^2) \qquad (3.4)$$

Lastly, we need inflation in the economy, denoted i, which is assumed to be constant.

3.2. Policy instruments, risk measures, economic parameters

There are five policy parameters in this simple model. They are related to the indexation policy, the investment policy and contribution or initial value of the assets. One can decide on the guaranteed minimum rate of indexation i_g, the target funding ratio FR_{target}, distribution ratio α, the percentage of risky assets w_s and the funding ratio ($L_0=1$) or assets at start A_0 respectively. In this example the pension fund's initial policy is given by i_g =0, FR_{target} =105%, α=25%, w_s=40% and A_0=125%.

We defined the following risk and return measures: the expected value, the 5th percentile, the 10th percentile and the 25th percentile of 1) the value of the liabilities and 2) the buffer (assets minus liabilities). All risk measures are calculated on four time horizons: 5, 10, 20 and 40 years, and will be considered in today's currency to make them comparable.

The parameter set to generate 1000 economic scenarios are for the risky assets μ_S =8%, σ_S^2=10%, for the interest rate r =4% and for inflation i =1%.

3.3. Results

The ALM results under the initial policy are given in figure 1. It shows the risk measures we defined (expectation and percentiles of the liabilities and the funding ratio) on the four different time horizons: 5, 10, 20 and 40 years.

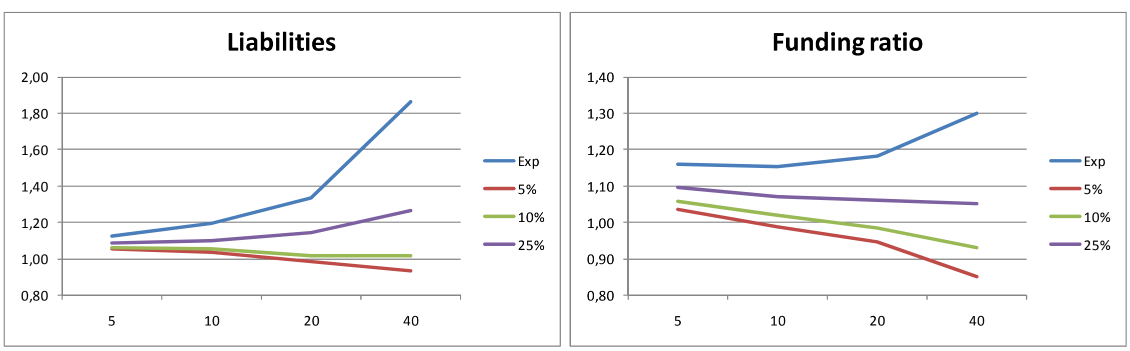

Figure 1 ALM outcomes under initial policy

From figure 1 we conclude that the averages are good: the expected value of the liabilities (in today's currency) is 1,86 after 40 years, the expected funding ratio is 130% on the same time horizon. But the lower percentiles are bad, especially those of the funding ratio: the 10% percentile is 93%, and the 5% percentile 85% at the 40 year horizon. Can we improve these outcomes?

Horizon	Risk measure	€ today	Weight	Target Δ	Achieved Δ	Achieved linear	Achieved new parameters
40	Exp-Liabilities	1.86	1	0.00	-0.02	1.85	1.82
40	5%-Liabilities	0.94	1	0.06	0.01	0.94	0.95
40	10%-Liabilities	1.02	1	0.00	0.02	1.03	1.03
40	25%-Liabilities	1.27	1	0.00	0.01	1.28	1.27
40	Exp-Buffer	0.30	1	0.00	0.01	0.31	0.31
40	5%-Buffer	-0.15	1	0.15	0.08	-0.07	-0.07
40	10%-Buffer	-0.07	1	0.07	0.07	0.00	0.00
40	25%-Buffer	0.05	1	0.00	0.04	0.09	0.10

Table 1 Results and goal differences for 40 year horizon in inverse ALM, other horizons are given in the appendix

To illustrate the inverse ALM method table 1 shows the results for the time horizon of 40 years, the whole table is given in the appendix. The column '€ Today' gives the risk measures under the initial policy. Using the inverse ALM method we first need to define the target values of these risk measures (we choose to weigh all risk measures equally). The 'Target Δ' column says by how much the value of the risk measure should be adjusted according to the pension fund objectives and constraints. For example, the 5% percentile of the liabilities 40 years from now is estimated at 0.94. The target is that the 5% percentile is 1 that is 0.94(€ Today)+0.06(Target Δ), so that the liabilities do not lose any purchasing power in the 5% percentile. The 5% percentile of the buffer is now -0.15, the target is that the 5% percentile is 0 that is -0.15(€ Today)+0.15(Target Δ), so that buffer is nonnegative. Notice that 1) all values are in real euro's, and 2) we can see whether the actual target values can be achieved by adjusting the policy parameters.

Table 1, column 'Achieved Δ' is the weighted least squares result of 'Target Δ' when policy parameters are adjusted in order to get as close to the target as possible. For example, for the 5% percentile of the liabilities the goal was to get this risk measure equal to 1, so we where 0.06 short. By adjusting the policy parameters, we 'Achieved' an adjustment of just 0.01. With the 5% percentile of the buffer, initially equal to -0.15, the target was to adjust the policy parameters such that the risk measure equals zero, 'achieved' is the value of -0.07.

The first conclusion is that improvement can be reached, but not all target values can be achieved, the measure of 'realism' of the desired change in outcomes, $\cos\theta$, equals 74%. As a final check, the last column in table 1 gives the results of the risk measures by re-simulating the ALM model using the new policy parameters.

Table 2 shows the initial values of the policy parameters and those suggested by the inverse ALM method.

Policy parameter	initial policy	new policy
Initial assets, cash in/out	1.25	1.37
Minimum guaranteed indexation	0.0%	-0.2%
Distribution ratio	25%	35%
Target funding ratio	5%	11%
Percentage of risky assets	40%	33%

Table 2 Initial policy parameters and policy parameters suggested by the inverse ALM method

In order to achieve the goals of the pension fund the inverse ALM method suggests to add 12% to the pension fund's assets via a single premium payment; when the buffers are inadequate (< 11%) the liabilities are cut with 0.2% every year; conditional indexation is given when the funding ratio is higher than 111%; the percentage of the buffer above 11% that will be given as indexation is adjusted to 35%; the percentage stocks is reduced to 33%.

Given the example results we can conclude that inverse ALM is an open minded method. Policy variants that at forehand would be unmentionable (negative guaranteed indexation percentage) can show up to be the best way to meet the objectives of the pension fund. Figure 2 shows all results given the initial policy and the new policy. Almost all risk measures have improved on all time horizons. The new policy, although surprising, seems to be a good alternative for the initial policy. But remember the policy suggested includes a single premium equal to 12% of the initial assets.

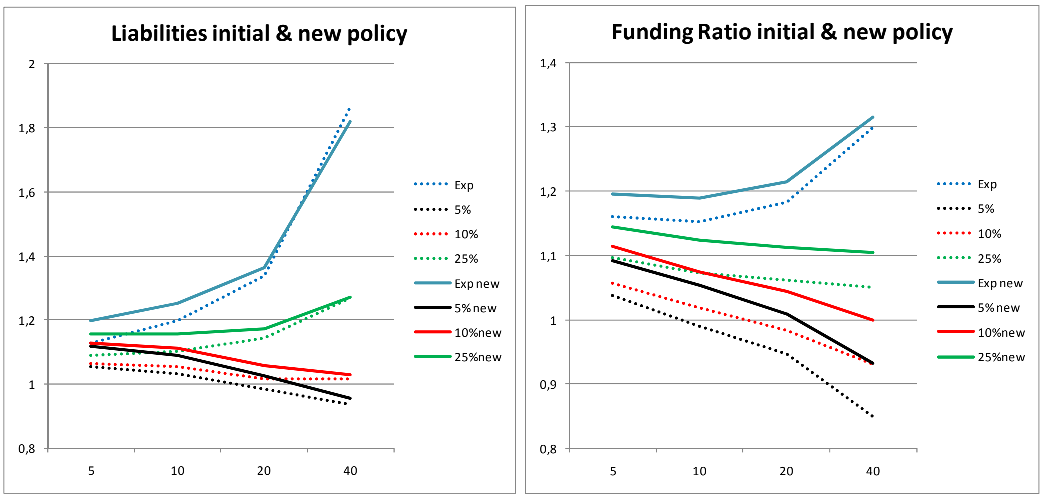

Figure 2 ALM outcomes under new policy (sold lines) and under initial policy (dotted lines)

4. Inverse ALM using LP

In this section we suggest several improvements of the inverse ALM method as proposed in section 2.

First of all, in the numerical example we added 12% by a single premium payment to the assets of the pension fund, which could be done without concern or punishment. The method could easily be adjusted by adding quadratic costs for changes in the policy parameters:

$$\min_{\Delta B}(\Delta R^* - \Delta\hat{R}^*)' W (\Delta R^* - \Delta\hat{R}^*) + \Delta B' C \Delta B \qquad (4.1)$$

where $\Delta\hat{R}^* = X\Delta B$

But in practice, there may also be strict limitations to the change in control variables. Can we really expect a single premium payment as high as x% of the initial assets even if it seems to be the best policy despite using a model that includes quadratic costs as in (4.1)? Therefore one could also add restrictions on ΔB to the model, see for example Judge et al (1966) and Liew (1976) on restrictions in regression analysis.

Another possible improvement over the WLS-method for inverse ALM is to incorporate asymmetric penalties. In the numerical example above the real value of the expected liabilities on the 40-year horizon was 1.86 (see table 1), which is a good result so that we set the target value at 1.86. If we search for new policy parameter using WLS a downside deviation from 1.86 is equally treated as an upside deviation, while an upside deviation in this case is more attractive. One can use linear programming (LP, see e.g. Dantzig and Thapa, 2003) to incorporate asymmetric penalties and add restrictions on the change in policy parameters at the same time. Use for example

$$\min_{\Delta B} \sum_{n=1}^{N} I_{\Delta R_n^* < \Delta R_n} w_n (\Delta R_n^* - \Delta \hat{R}_n^*)^2 \text{ s.t. } \Delta B^d \leq \Delta B \leq \Delta B^u \tag{4.2}$$

Here $I_{\Delta R_n^* < \Delta R_n}$ $n \in \{1,..., N\}$ is an indicator function, equal to one if $\Delta R_n^* < \Delta R_n$ and zero otherwise

4.1. *Numerical examples using LP*

Using again the ALM model of the stylized pension fund as in section 3, we now demonstrate inverse ALM using linear programming to obtain new policy parameters (4.2).

First, we calculated the results as if there are still no restrictions on the change in policy parameters. We just incorporated asymmetric penalties. Table 3 gives the risk and return measures concerning the liabilities and the buffer on the 40 year horizon. Table 4 gives the newly proposed policy parameters.

Horizon	Risk measure	€ today	Weight	Target Δ	Achieved Δ LP unrestricted	Achieved LP unrestricted
40	Exp-Liabilities	1.86	1	0.00	-0.02	1.84
40	5%-Liabilities	0.94	1	0.06	0.02	0.96
40	10%-Liabilities	1.02	1	0.00	0.03	1.04
40	25%-Liabilities	1.27	1	0.00	0.00	1.26
40	Exp-Buffer	0.30	1	0.00	0.48	0.78
40	5%-Buffer	-0.15	1	0.15	0.21	0.06
40	10%-Buffer	-0.07	1	0.07	0.21	0.14
40	25%-Buffer	0.05	1	0.00	0.27	0.32

Table 3 Results and goal differences for 40 year horizon in inverse ALM incorporating asymmetric penalties, other horizons are given in the appendix

Policy parameter	initial policy	new policy
Initial assets, cash in/out	1.25	1.45
Minimum guaranteed indexation	0.0%	0.2%
Distribution ratio	25%	12%
Target funding ratio	5%	18%
Percentage of risky assets	40%	43%

Table 4 Initial and adjusted values of the policy parameters using Linear Programming

As we can see, comparing table 3 with table 1, the expected liabilities and percentiles of the liabilities are more or less the same using the new parameters suggested by inverse ALM using WLS respectively LP. But by using the policy parameters suggested by linear programming (incorporate asymmetric penalties), the expected buffer and percentiles of the buffer are much higher.

But, comparing the policy parameters we see that the policy suggested by LP adds 20% to the initial assets, compared to 12% in the case of WLS. If additional payment is a problem, then nor WLS or the LP without policy restrictions will not give a proper solution.

Therefore, we calculated a third example where we restrict the change in the value of the initial assets $A_0^* \le A_0$. We also increased the weights for the liabilities over the expected buffer, since high buffers as in table 4 are attractive, but we would rather like to achieve higher liabilities. The results using LP, now with restrictions, are given in table 5 and table 6.

Horizon	Risk measure	€ today	Weight	Target Δ	Achieved Δ	Achieved Δ LP restricted	Achieved LP restricted
40	Exp-Liabilities	1.86	5	0.00	-0.02	-0.14	1.72
40	5%-Liabilities	0.94	5	0.06	0.01	0.00	0.93
40	10%-Liabilities	1.02	5	0.00	0.02	-0.01	1.01
40	25%-Liabilities	1.27	5	0.00	0.01	-0.06	1.20
40	Exp-Buffer	0.30	1	0.00	0.01	0.23	0.53
40	5%-Buffer	-0.15	1	0.15	0.08	0.08	-0.07
40	10%-Buffer	-0.07	1	0.07	0.07	0.06	-0.01
40	25%-Buffer	0.05	1	0.00	0.04	0.10	0.15

Table 5 Results and goal differences for 40 year horizon in inverse ALM, other horizons are given in the appendix

Policy parameter	initial policy	new policy
Initial assets, cash in/out	1.25	1.25
Minimum guaranteed indexation	0.0%	0.3%
Distribution ratio	25%	11%
Target funding ratio	5%	3%
Percentage of risky assets	40%	43%

Table 6 Initial and adjusted values of the policy parameters using Linear Programming with restrictions on the adjustment of the initial assets

Adding restrictions, there now is no single premium any more. The investment policy is similar to the policy suggested by LP without restrictions and other weights (table 6 compared with table 4). However, the parameters concerning the indexation policy are slightly different, due to the extra weight on liability results. The guaranteed minimum rate of indexation is slightly higher (0.3% compared to 0.2%). The target funding ratio is only 103%, so that conditional indexation is given earlier (target funding ratio was 118%), the distribution ratio is 11% (was 12% in LP without restrictions).

Starting with lower initial assets does result in lower expected liabilities and buffers (table 5 compared to table 3), given the restrictions inverse ALM shows which goals w.r.t. the risk and return measures are realistic.

By starting to define the goals of the pension fund, inverse ALM systematically searches through the policy space. The numerical examples show that the policies suggested by inverse ALM clearly improve the ALM-results on the different risk and return measures.

5. Summary and conclusions

Traditional ALM first sets the policy parameters and then assesses the impact on some sub-set of risk and return measures. We propose a method to 'invert' the traditional ALM approach: first formulate the desired level of risk and return measures and then find the policy parameters.

We started by using a weighted least squares method to obtain new policy parameters. An example showed that 1) probably not all objectives and constraints set by the pension fund management can be realized, but inverse ALM gives the policy that 'best' meets the targets, and 2) the method is very open minded: policy variants that at forehand would be unmentionable can show up to be the best way to meet the objectives.

We also proposed several extensions: to add quadratic costs, to incorporate asymmetric penalties using linear programming and to add restrictions to the change in policy instruments.

We used a simple model to illustrate the ideas, but the method also seems feasible for realistic ALM models, and is therefore interesting for practitioners. In the examples we defined the targets of the company under consideration as a whole. Another extension could be to use the ALM model to facilitate communication between stakeholders of the company on risk/return measures, by defining targets for the stakeholders separately, see Joseph et.al. (2010).

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Appendix

Horizon	Risk measure	€ today	Target Δ	Achieved Δ	Achieved linear	Achieved new parameters	Achieved LP unrestricted	Achieved LP restricted
5	Exp-Liabilities	1.13	0.00	0.07	1.20	1.20	1.09	1.06
5	5%-Liabilities	1.06	0.00	0.06	1.11	1.12	1.04	1.02
5	10%-Liabilities	1.06	0.00	0.06	1.12	1.13	1.05	1.02
5	25%-Liabilities	1.09	0.00	0.07	1.16	1.16	1.07	1.04
10	Exp-Liabilities	1.20	0.00	0.07	1.26	1.25	1.17	1.11
10	5%-Liabilities	1.03	0.00	0.06	1.09	1.09	1.03	1.00
10	10%-Liabilities	1.05	0.00	0.05	1.11	1.11	1.05	1.02
10	25%-Liabilities	1.10	0.00	0.06	1.16	1.16	1.09	1.05
20	Exp-Liabilities	1.34	0.00	0.04	1.38	1.36	1.32	1.25
20	5%-Liabilities	0.98	0.02	0.04	1.02	1.02	0.99	0.97
20	10%-Liabilities	1.02	0.00	0.05	1.06	1.06	1.03	1.00
20	25%-Liabilities	1.14	0.00	0.03	1.17	1.17	1.14	1.10
40	Exp-Liabilities	1.86	0.00	-0.02	1.85	1.82	1.84	1.72
40	5%-Liabilities	0.94	0.06	0.01	0.94	0.95	0.96	0.93
40	10%-Liabilities	1.02	0.00	0.02	1.03	1.03	1.04	1.01
40	25%-Liabilities	1.27	0.00	0.01	1.28	1.27	1.26	1.20
5	Exp-Buffer	0.16	0.00	0.04	0.20	0.20	0.41	0.24
5	5%-Buffer	0.04	0.00	0.05	0.09	0.09	0.23	0.08
5	10%-Buffer	0.06	0.00	0.06	0.11	0.11	0.26	0.10
5	25%-Buffer	0.10	0.00	0.04	0.14	0.14	0.32	0.15
10	Exp-Buffer	0.15	0.00	0.03	0.18	0.19	0.42	0.25
10	5%-Buffer	-0.01	0.01	0.06	0.05	0.05	0.16	0.03
10	10%-Buffer	0.02	0.00	0.05	0.07	0.07	0.21	0.06
10	25%-Buffer	0.07	0.00	0.04	0.12	0.12	0.29	0.14
20	Exp-Buffer	0.18	0.00	0.03	0.21	0.21	0.49	0.31
20	5%-Buffer	-0.05	0.05	0.07	0.02	0.01	0.12	-0.01
20	10%-Buffer	-0.02	0.02	0.06	0.04	0.04	0.18	0.04
20	25%-Buffer	0.06	0.00	0.05	0.11	0.11	0.28	0.13
40	Exp-Buffer	0.30	0.00	0.01	0.31	0.31	0.78	0.53
40	5%-Buffer	-0.15	0.15	0.08	-0.07	-0.07	0.06	-0.07
40	10%-Buffer	-0.07	0.07	0.07	0.00	0.00	0.14	-0.01
40	25%-Buffer	0.05	0.00	0.04	0.09	0.10	0.32	0.15

Table A Results examples inverse ALM