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Helmuth Cremer and Philippe De Donder

Longevity, Annuities and the Political Support for Public Pensions

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Longevity, annuities and the political support for public pensions[1]

Helmuth Cremer[2]
Philippe De Donder[3]

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[2] Toulouse School of Economics (GREMAQ, IDEI and Institut universitaire de France).
[3] Toulouse School of Economics (GREMAQ-CNRS and IDEI).

Abstract

We develop a model where individuals differ in productivity and in longevity. Benefits from the public pay-as-you-go pension system take the form of a collective annuity, with both a contributive (Bismarckian) component (based on the worker's past earnings) and a flat (Beveridgean) part. Voters choose the size (generosity) of the system and its degree of income redistribution (or type).

While individual longevity does not affect preferences for type and size of the program, these are influenced by both the average longevity and by the correlation between longevity and productivity. When the size of the annuity program is chosen by majority voting for a given type, we obtain that the size of the Beveridgean scheme decreases smoothly with increases in average longevity, while the support for a Bismarckian pension abruptly drops to zero once a threshold is crossed. When both size and type are determined by majority voting, we obtain either large and mostly Bismarckian systems or smaller Beveridgean systems, as is empirically observed. Also, a larger correlation between longevity and productivity makes the collective annuity more redistributive, although sometimes at the expense of its size.

Keywords: generosity, redistributiveness, pay-as-you-go pensions, collective annuity, longevity, Shepsle structure-induced equilibrium

JEL Codes: D78, H55

1 Introduction

There is now a large body of literature dealing with the political determination of the characteristics of a public pay-as-you-go pension system. The seminal paper by Browning (1975) assumed that the only heterogeneity between voters was their age. Subsequent papers (such as Casamatta *et al.* (2000a)) have enriched this approach by assuming that agents belonging to the same cohort differ in income or in productivity. This richer set of individual traits has allowed these papers to study the determination of both the size of the pension system and of its degree of redistribution. For instance, one focus of this literature has been to explain the empirical observation that more redistributive systems have a smaller size (see Casamatta *et al.* (2000b) and Conde-Ruiz and Profeta (2007)).

There is a third dimension of heterogeneity among voters that may play a critical role in the determination of the pension system, namely longevity. It is well known empirically how people of the same age differ in life expectancy. Moreover, lifetime expectancy is positively correlated with income or wealth, as shown by Deaton and Paxon (1999) for the US, Attanasio and Emerson (2001) for the UK and Reil-Held (2000) for Germany. The empirical consequences of life expectancy differences for actual pension systems have been extensively studied in the literature by e.g. Coronado et al. (2000) for the US, Gil and Lopez-Casasnovas (1997) for Spain, Bommier et al. (2005) for France and Reil-Held (2000) for Germany. These papers take the existing characteristics of the pension system as given in order to assess how the heterogeneity of lifetime expectancy (and its correlation with income) affect the redistributive extent of these programs. Not surprisingly, they find that, with richer people tending to live longer lives, and with public pensions not related to individual longevity, there is less redistribution across income levels than meets the eye.

However, little is known on the impact of lifetime heterogeneity and of its correlation with income on the equilibrium characteristics of the public pension system. The two main characteristics of such a program are its size (or generosity), and its apparent degree of redistribution across income levels. As for the latter, the literature (surveyed by Galasso and Profeta (2002)) has contrasted so-called Bismarckian systems, where the pension benefit is proportional to the individual contribution, with Beveridgean systems, where the benefit does not depend on individual contributions. The main objective of our paper is to understand how the distribution of lifetime expectancy affects the equilibrium size and degree of income redistribution of the pension system. To answer this question requires building a political economy model with a bidimensional type space (income or productivity and lifetime expectancy) and a bidimensional policy space (size and degree of redistributiveness of the public pension program). As we now show, and to the best of our knowledge, no paper has yet attempted to build such a model.

Few papers endogenize the public pension program when lifetime expectancy is heterogeneous. Cremer *et al.* (2009) study the design of pension systems and the role played by collective annuities when individuals differ in longevity (as well as in productivity). Their approach is normative and based on a utilitarian social welfare function. Other papers take a positive perspective and study the impact of this longevity heterogeneity when the pension system is determined though some voting procedure. Leroux (2007) studies the choice of the size and characteristics of the pension system in the case where individuals have the same income but differ in their life span. She obtains that a majority is in favor of a pension system awarding the same annuity to everyone if the distribution of longevity is negatively-skewed. Borck (2007) assumes from the outset that richer individuals always live longer lives (so that heterogeneity between agents is truly one dimensional) and shows how individual preferences and equilibrium pension policies are affected by the slope of the relationship between income and life expectancy. Finally, De Donder and Hindriks (2002) assume that individuals differ both in their productivity and survival probabilities. Their focus is on the majority chosen size of the pension system as a function of its (exogenous) redistributiveness. They show that the amount of distortions associated to the pension system need not decrease when the system is made less redistributive, because voters favor a larger pension size.

In our paper, individuals differ in productivity and longevity and choose, by majority voting, both the size of a pay-as-you-go pension system providing a collective annuity and its degree of redistributiveness across income levels. Individuals live at most two periods and differ in their probability to be alive in the second period. In the first period, they choose labor supply and retirement saving which is invested in the private market. [Borrowing constraints are assumed away so that saving can actually be negative.] Agents alive in the second period consume their saving and the pension benefit (if any). The pension system is financed by a linear payroll tax on labor income.

Throughout the paper we assume that pension benefits are paid out as a *collective annuity*, which does not depend on an individual's survival probability. This corresponds to the practice of public pensions, which redistribute ex ante from agents with short life expectancies to those with long life expectancies. This type of redistribution comes on top of the income redistribution that the pension system may achieve through the benefit formula. More precisely, the collective annuity received by any individual has both a contributive, or Bismarckian component (based on the individual's own tax payments when young) and a non-contributive, or Beveridgean component (based on the average contribution in the economy). Voters choose both the generosity (or size) of the pension system (the value of the proportional income tax rate) and the degree of redistributiveness (or type) of the system (measured by the relative importance of the contributive component, dubbed the Bismarckian factor).

We concentrate on the case where there is no private (individual) annuity market. This is in line with the empirical evidence, since most retirees are reluctant to buy an annuity, so much so that this behavioral pattern is often referred to as the "annuity puzzle"; see Brown *et al.* (2005).[1] We assume that private saving has an exogenous rate of return, such as the world interest rate, which is independent of longevity.

It has been well known since Browning (1975) that older voters favor a larger size of the pay-as-you-go system than younger voters because they consider their past contributions as sunk when voting. In order to shed light on factors other than this age bias, we assume that only young people vote. One way to rationalize this assumption is to consider that the public pension characteristics voted upon affect the future retirement

[1] See also Sheshinski (2007). Finkelstein and Poterba (2002, 2004) and Mitchell (1996) show that where they exist, rates of return of individual annuities are much below actuarially fair levels and often significantly less attractive than the implicit return of collective annuities.

benefits of the current workers, and not the benefits of the current retirees.

We are interested in the joint political determination of the size and redistributiveness of the collective annuity program. It is well know that simultaneous voting over a bidimensional policy space has generically no equilibrium (see De Donder *et al.* (2009) for instance). We adopt the voting procedure first proposed by Kramer (1972) and Shepsle (1979), where each policy dimension is a majority voting equilibrium given the other dimension. We then first look at "partial political equilibrium" situations (where voters decide over the size of the program given its type, or over its type given its size) before turning to the "general political equilibrium" allocation (where both type and size are chosen separately by majority voting, and where each dimension is a majority voting equilibrium given the other dimension).

The partial equilibrium results constitute a necessary step to construct the general equilibrium allocation, but are also interesting by themselves. We first show that the individual with the median productivity level is decisive in both partial equilibrium situations. With our formulation, individual longevity does not affect preferences for the size and type of the collective annuity. These are affected mainly by the average longevity, and by the correlation between longevity and productivity. An important driving force behind our results is that the contributive annuity's return impacts the non-contributive annuity's return (thanks to labor supply effects), so that there is a contagion effect from Bismarck to Beveridge, but not the other way round.

When the type of program is set exogenously and people vote over its size, we obtain that (pure) Bismarckian systems are not always larger than (pure) Beveridgean programs. Moreover, the program size is not always monotonic in its type (as measured by the Bismarckian parameter) when the rate of return of the Bismarckian scheme is smaller than the interest rate. The comparative statics analysis of this partial equilibrium provides two important results. First, as long as there is a contributive part to the collective annuity, an increase in the correlation between longevity and productivity decreases the equilibrium size of the program, which reinforces the observation made by the empirical literature that such a correlation decreases the redistributiveness (across income levels) of collective annuity schemes of given size and type. Second, we study which of the two systems, Beveridge or Bismarck, is more resistant to an increase in longevity (which decreases the return of both forms of collective annuity). We obtain that, for the Bismarckian system, there is not impact on the majority chosen size until a threshold return is reached, beyond which the size drops discontinuously to zero. On the opposite, the majority chosen size of the Beveridgean system smoothly decreases with the average longevity. The intuition for the difference between the two systems is that the redistribution embedded into the Beveridgean system creates heterogeneity in voters preferences so that a lower intrinsic return slowly erodes its political support, unlike the Bismarckian scheme whose political support suddenly disappears for all voters when the Bismarckian annuity's return becomes lower than the interest rate.

As for the majority choice of type given the (exogenous) size of the pension system, we obtain that the system becomes more Bismarckian as the contribution rate increases. As this rate increases, the distortion associated with the Beveridgean component increases (labor supply effect), and individuals react by favoring a more Bismarckian system. Strikingly, even agents with no income may favor some contributive component to the collective annuity, in order to increase the return of the non-contributory part (thanks to the contagion effect).

Moving to the joint political determination of the size and redistributiveness of the collective annuity programs, we obtain three types of equilibria. If the Bismarckian return is larger than the interest rate, we have a unique equilibrium with a large, mostly but not always exclusively Bismarckian program. If the Bismarckian return is smaller and the median voter's productivity is low enough, we have a smaller and purely Beveridgean program. Finally, with a low Bismarckian return and a productive median voter, there is no majoritarian political support for a collective annuity program. Observe that these Shepsle equilibria correspond to what is empirically observed: large Bismarckian systems and smaller Beveridgean ones.

We finally study how this general political equilibrium is affected by variations in the longevity distribution. Increasing the positive correlation between income and longevity makes the equilibrium system more redistributive (both because the mostly Bismarckian equilibrium becomes less so, and because of a move from mostly Bismarckian to purely

Beveridgean when a threshold level is crossed), although sometimes at the expense of its size (because the Beveridgean system is always smaller than the mostly Bismarckian one). This is in stark contrast with the partial equilibrium results (where the size of the collective annuity program decreases with the correlation).

In a nutshell, we obtain that one should be cautious when studying the impact of (heterogeneity in) life expectancy on pension systems while not endogenizing the main parameters of these systems, since it may lead to conclusions which are overturned when both parameters are endogenized. This calls for a reevaluation of the empirical literature, which should go beyond a mechanical assessment of the impact of positive correlation between income and longevity on existing pension systems. Moreover, the fact that the counter-factual partial equilibria (larger Beveridgean than Bismarckian systems when the type is taken as exogenous) do not constitute general (Shepsle) equilibria also stresses the importance of adopting a general political equilibrium approach.

The paper is structured as follows. Section 2 presents the model. Section 3 analyzes the private (labor supply and saving) decisions taken by individuals. Section 4 analyzes majority voting over the size of the system, while section 5 studies the political determination of the Bismarckian parameter. Section 6 studies the joint determination by majority voting of both the size and the redistributiveness of the collective annuity program. Section 7 concludes.

2 The model

Consider an economy consisting of agents who live (at most) two periods, working in the first period and retiring in the second one. There is a continuum of individuals differing in productivity w and in life expectancy, measured as the probability p to be alive in period 2.[2] The joint distribution of these two characteristics is denoted by $H(w, p)$, with marginal distributions $F(w)$ for productivity and $G(p)$ for life expectancy. We make no assumption on the correlation between the two characteristics w and p for the time being. However, for the interpretation of our results we will often concentrate

[2] We will use without distinction the terms life expectancy, longevity and survival probability when referring to p.

on the empirically relevant case where life expectancy and productivity are positively correlated so that $\text{cov}(p, w) > 0$. The average productivity is denoted by $\bar{w}$ while the average life expectancy is $\bar{p}$. We assume as usual that the productivity distribution is positively skewed, so that the median w is lower than the average, $\bar{w}$.

Agents take two private decisions, both in their first period of life: how much labor they supply and how much to save on the private market. We assume away any borrowing constraint, so that saving can be negative. There are no private annuity markets. Saving has an exogenous gross return of $1 + r$, with $r > 0$ the exogenous world interest rate.[3]

In their second period of life, individuals take no decision and consume their private saving and public pension benefit (if any). The public pension benefit is financed by a linear payroll tax at rate $\tau \in [0, 1]$. The benefit consists of a collective annuity which depends on the *average* survival probability $\bar{p}$ but not on the individual's probability p. The benefit may be linked in part to the individual's past labor income tax (the contributive, or Bismarckian part) and in part to the average labor income taxes paid in the economy (the non-contributive, or Beveridgean part). The contributory share is denoted by $\alpha \in [0, 1]$ and referred to as the Bismarckian parameter.

We concentrate on decision making by agents in the first period of their life. They first vote over both the generosity (measured by τ) and redistributiveness (measured by α) of the collective annuity program, assuming that both votes set the values of these parameters for (at least) two periods. They then determine their individually optimal labor supply and private saving.[4] As usual, we proceed backwards and we first solve for the individual labor supply and saving decisions, before moving to the analysis of majority voting over the characteristics of the public system.

[3] The assumption that agents can borrow against future income at this exogenous interest rate is of course a strong assumption, made to simplify the algebra. As we explain in the concluding section, imposing borrowing constraints would decrease the most-preferred tax rate of some voters, but would not affect the qualitative results we obtain.

[4] Throughout the paper, we assume that only young people vote. With a pay-as-you-go collective annuity program, the voting behavior of retirees is well known. They favor the proceeds-maximizing contribution rate since their past contributions are sunk while they enjoy the tax proceeds from the current workers. As for the system's type, it is easy to see that they have the same preferences as a younger agent of the same characteristics. Allowing older people to vote then would not bring any new insight.

3 Individual choices

Individual preferences are given by

$$u(c - h(z)) + \beta p u(d), \tag{1}$$

where c is first-period consumption, d is second-period consumption, $h(z)$ measures the disutility of labor supply and β is the discount rate. The function u is increasing and concave while h is increasing and convex. First period consumption net of labor supply disutility is denoted by $x = c - h(z)$.

The first-period budget constraint is given by

$$c = (1 - \tau)zw - s,$$

where z denotes labor supply while s denotes saving. In the second-period, we have

$$d = (1 + r)s + b,$$

where b is the public pension benefit. Pension benefits are given by[5]

$$b = \tau \left[(1 - \alpha)\frac{Ewz}{\bar{p}} + \alpha \frac{wz}{\hat{p}} \right], \tag{2}$$

where Ewz is the average first-period income,[6] $1/\bar{p}$ is the internal rate of return of the non-contributory (Beveridgean) collective annuity while $1/\hat{p}$ with

$$\hat{p} = \frac{Epwz}{Ewz} \tag{3}$$

is the internal rate of return of the contributory (Bismarckian) collective annuity. Observe that the two components of the collective annuity differ not only in their internal rate of return, but also on the basis on which this return is applied: to the individual's labor income for Bismarck and to the economy's average income for Beveridge. In

[5] We assume a pay-as-you go collective annuity, where tax proceeds of the currently young finance pensions paid by current retirees, as is most often the case in reality. In a fully funded system, tax proceeds would be invested on the financial market for one period before the annuity is paid out in the second period of life, so that the right hand side of (2) would have to be multiplied by $1 + r$.

[6] Throughout the paper, Ef denotes $\int f(w,p)dH(w,p)$ for any function f. Similarly, $\text{cov}(f,g)$ denotes $E(fg) - E(f)E(g)$ for any functions f and g.

other words, the non-contributory part of the collective annuity redistributes across income levels while the contributory part does not. Both components redistribute from short-lived to long-lived agents (since both are based on some aggregate longevity).

For future references, note that from (3), we have that $\hat{p} > \bar{p}$ if $\text{cov}(w, p) > 0$. Intuitively, if more productive agents live longer, then the internal rate of return of the Bismarckian public annuity is lower than that of the Beveridgean annuity. This is because the Beveridgean annuity is based on the average life expectancy while the Bismarckian system puts more weight on more productive and longer lived agents (because they contribute more and thus receive a larger benefit than less productive agents).

The first-order condition with respect to private saving s is given by

$$p\beta u'(d)(1+r) = u'(x), \tag{4}$$

or

$$\frac{u'(x)}{u'(d)} = p\beta(1+r).$$

The first-order condition with respect to labor supply z is given by

$$-u'(x)\left[h'(z) - (1-\tau)w\right] + p\beta u'(d)\left(\frac{\tau}{\hat{p}}\alpha w\right) = 0. \tag{5}$$

Using (4) and the fact that $u'(x) > 0$, equation (5) simplifies to

$$(1+\gamma\tau)w = h'(z), \tag{6}$$

where

$$\gamma = \frac{\alpha}{\hat{p}(1+r)} - 1 \tag{7}$$

measures the net *individual* marginal return of the collective annuity scheme. Specifically, $\alpha/[\hat{p}(1+r)]$ is the discounted value of the extra benefits to which individuals are entitled when their contributions increase at the margin. It increases with the share of the contributive component α (since by definition the non-contributive component of the collective annuity is independent of the individual's contribution) and with the internal rate of return of the contributive part, $1/\hat{p}$. The variable γ plays a crucial role in the subsequent analysis.

If the gross individual marginal return of pension $\alpha/\hat{p}$ is equal to the private saving return $1+r$, we have that $\gamma = 0$ and the FOC for labor supply simplifies to $w = h'(z)$, which means that the pension scheme does not affect the labor supply decision. This corresponds to the efficient labor supply decision (no deadweight loss from the public program). If the gross return of pension is lower than the private saving return ($\alpha/\hat{p} < 1+r$), $\gamma < 0$ and the agent works less than in the absence of public pension, while a higher return yields exactly the opposite result.[7]

Interestingly, labor supply does *not* depend on individual life expectancy. Equation (6) shows that labor supply depends only on the net wage, $(1+\gamma\tau)w$, which, as indicated by (7) does not depend on p.[8] On the other hand, labor supply increases with the Bismarckian parameter α, decreases with $\hat{p}$ and decreases with the interest rate r if $\alpha > 0$. All these effects directly follow how the net wage is affected by these variables. Similarly, labor supply may decrease or increase with τ depending on the sign of γ. For instance, if $\gamma < 0$ then the net wage decreases with the payroll tax and so does labor supply.

We summarize these results in the following Proposition.

Proposition 1 *Assume that preferences are represented by (1) and that pension benefits are paid as a collective annuity with a contributive and a flat part as specified by (2). Individual labor supply has the following properties:*
i) it does not depend on individual life expectancy;
ii) it increases with the individual's productivity w, with the Bismarckian parameter α and with the internal rate of return of the contributive collective annuity $1/\hat{p}$;
iii) it decreases with the private saving rate of return r;
iv) it increases with τ when the gross individual marginal pension return is higher than the private saving return ($\gamma > 0$), and decreases with τ when $\gamma < 0$;
iv) with $\alpha > 0$, increasing the covariance between life expectancy and productivity while keeping the marginal distributions of p and w unchanged decreases the retirement age

[7] When z is interpreted as retirement age, a negative γ corresponds to the "implicit tax on continued activity" studied in the pension literature.

[8] With preferences specified by (1), there is no income effect in the labor supply decision.

of everybody.

The indirect utility (incorporating the optimal choices z^* and s^* of all individuals) is given by

$$V(\alpha,\tau,w,p) = u\left[(1-\tau)wz^* - s^* - h(z^*)\right] + \beta p u\left[(1+r)s^* + \tau((1-\alpha)\frac{Ewz^*}{\bar{p}} + \alpha\frac{wz^*}{\hat{p}})\right]. \tag{8}$$

We now move to the choice by majority voting of the size (or generosity) of the public pension scheme.

4 Voting over the generosity of the pension system

In this section, we assume that α is set at an exogenous level. We first compute, and comment on, the first-order condition for the individual most-preferred value of τ, before turning to the majority chosen level. We then perform the comparative static analysis of this level with respect to changes in the distribution of lifetime expectancy and in the type of collective annuity program.

4.1 Individuals most preferred contribution rate

Differentiating a voter's utility (8) yields the following first-order condition

$$\begin{aligned}\frac{\partial V(\alpha,\tau,w,p)}{\partial\tau} &= -u'(x)wz^* + \beta p u'(d)\left(\left[(1-\alpha)\frac{Ewz^*}{\bar{p}} + \alpha\frac{wz^*}{\hat{p}}\right]\right.\\ &\quad \left. + \frac{\tau}{\bar{p}}(1-\alpha)\frac{\partial Ewz^*}{\partial\tau} + \tau\alpha\frac{\partial(1/\hat{p})}{\partial\tau}wz^*\right) = 0. \end{aligned} \tag{9}$$

The first term on the right hand side measures the loss of first period utility when the contribution rate is increased, the term between brackets is the direct impact of a higher tax rate on the pension benefit, the next term is the indirect impact of increasing the tax rate on the Beveridgean tax proceeds (because of labor supply incentives) while the last term is the impact of increasing τ on the return of the Bismarckian part of the pension benefit. Using (4), we can rewrite (9) as

$$wz^*\left[\gamma + \frac{\alpha\tau}{1+r}\frac{\partial(1/\hat{p})}{\partial\tau}\right] + \frac{1-\alpha}{(1+r)\bar{p}}\left(Ewz^* + \tau\frac{\partial Ewz^*}{\partial\tau}\right) = 0. \tag{10}$$

To go beyond this expression, we need to introduce a functional form for the disutility of labor. We assume a quadratic specification so that

$$h(z) = \frac{z^2}{2}.$$

Labor supply is then given by

$$z = (1 + \gamma\tau)w.$$

Average first-period labor income is

$$Ewz = (1 + \gamma\tau)Ew^2,$$

while

$$Epwz = (1 + \gamma\tau)Epw^2.$$

We then obtain that

$$\hat{p} = \frac{Epwz}{Ewz} = \frac{Epw^2}{Ew^2},$$

so that

$$\frac{\partial \hat{p}}{\partial \tau} = \frac{\partial(1/\hat{p})}{\partial \tau} = \frac{\partial \hat{p}}{\partial \alpha} = \frac{\partial(1/\hat{p})}{\partial \alpha} = 0.$$

In words, with a quadratic disutility of labor, the internal rate of return of the Bismarckian public annuity, $1/\hat{p}$, is affected neither by the generosity of the system nor by the Bismarckian parameter. The first-order condition for optimal τ, equation (10), becomes

$$\gamma w^2(1 + \gamma\tau) + \frac{1-\alpha}{(1+r)\bar{p}}(1 + 2\gamma\tau)Ew^2 = 0. \tag{11}$$

We comment this formula before solving it with respect to τ. Observe first that this formula is equivalent to the one obtained when maximizing the individual's lifetime income—i.e., $(1-\tau)wz + b/(1+r)$. In the absence of borrowing constraints, the individuals use τ to maximize their discounted lifetime income (when labor supply z is optimally chosen, trading off higher consumption in exchange of a larger disutility from working) and s to reach their optimal allocation of this income across periods.[9] The first term in (11) measures the marginal impact of increasing τ on the discounted Bismarckian (contributive) part of the pension benefit, net of the first period decrease

[9] We thank Pascal Belan for pointing this to our attention.

in disposable income. It has the same sign as γ. The second term is the marginal variation in the non-contributive (Beveridgean) part of the pension benefit. It corresponds, loosely speaking, to the slope of the Laffer curve (i.e., to the derivative of the first part of (2) with respect to τ). There is a striking asymmetry between these two terms: while $\bar{p}$ (the internal rate of return of the non contributive annuity) affects only the second, Beveridgean, part, $\hat{p}$ (the internal rate of return of the contributory collective annuity) affects both terms. More precisely, $\hat{p}$ affects the second, Beveridgean, term (through γ) because it impacts the (dis)incentive to work of all agents, and hence the return of the non-contributive pension. This phenomenon, which we dub the "contagion effect" from Bismarck to Beveridge, will play a crucial role in the subsequent analysis. Finally, observe that *individual* life expectancy p does not play any role in formula (11). It affects neither the expected marginal utility of second period consumption nor the first period marginal utility, and does not affect the return of any form of saving (on the private market or through the public annuities system).

We now introduce the following notation:

$$\theta = \frac{w^2}{Ew^2}$$

represents the type of an individual (with $\theta^{med} < E\theta = \bar{\theta} = 1$ when the productivity (and thus the income) distribution is positively skewed, as is empirically the case) and $\tau^*(\theta, \alpha)$ denotes individual θ's most preferred tax rate for any given value of α.

4.2 The majority chosen value of τ

We now study equation (11) more closely, starting with the case when $\alpha = 1$. For a purely Bismarckian pension scheme ($\alpha = 1$), equation (11) shows that we obtain a corner solution

$$\begin{aligned} \tau^*(\theta, 1) &= 0 \;\forall\theta \text{ if } (1+r) > 1/\hat{p} = Ew^2/Epw^2, \\ \tau^*(\theta, 1) &= 1 \;\forall\theta \text{ if } (1+r) < 1/\hat{p} = Ew^2/Epw^2. \end{aligned}$$

Individuals perform a simple comparison of rate of returns: they are in favor of a Bismarckian system if its return is larger than private saving return (i.e., if $\gamma > 0$ when

$\alpha = 1$). In the absence of borrowing constraints, they favor the maximum size of the Bismarckian system ($\tau = 1$) and then use the savings market to borrow against their future pension income. Observe that this condition does not depend on individuals' characteristics, so that there is a unanimity either in favor of $\tau = 1$ or of $\tau = 0$.

We now turn to intermediate values of α, where the annuity has both a contributive and a non-contributive component. If the pension system has a larger gross individual marginal return than private savings ($\alpha/\hat{p} = \alpha Ew^2/Epw^2 > 1 + r$, so that $\gamma > 0$), the FOC (11) for τ is always positive. The intuition is as follows. A positive value of γ (for any given α) means that the Bismarckian internal rate of return $1/\hat{p}$ is so much larger than the private savings return that the net individual marginal return of the collective annuity program is positive–i.e., that even though only a part α of the collective annuity is contributory, this part is large enough that the individual's annuity increases more than his tax bill when the tax rate is increased. This explains why the first term of (11) is increasing. Moreover, increasing τ also affects the non-contributory part of the pension. Recall that labor supply is increasing in τ when $\gamma > 0$. This means that the Beveridgean part of the pension is monotonically increasing in τ in that case, so that the second term in (11) is positive. This is the contagion effect from Bismarck to Beveridge at full play. As the two terms of (11) are positive, all individuals favor $\tau = 1$. Observe that the condition that $\gamma > 0$ imposes a lower bound on the value of α: this case may only happen if the Bismarckian factor is large enough.

If the net individual marginal return of the pension system is negative ($\alpha/\hat{p} = \alpha Ew^2/Epw^2 < 1 + r$, so that $\gamma < 0$), we obtain that the first term of equation (11) is negative while the second term is positive (at least when τ is not too high), opening up the possibility of an interior most-preferred value of τ. An individual with no income ($\theta = 0$) does not receive any contributory collective annuity and wishes to maximize the size of the non-contributory collective annuity received by everyone.[10] With labor supply decreasing with τ when $\gamma < 0$, the non-contributory annuity is concave in τ and reaches a maximum for

$$\tau^*(0, \alpha) = \frac{\hat{p}(1+r)}{2(\hat{p}(1+r) - \alpha)},$$

[10] This corresponds to the summit of the Laffer curve mentioned above.

which is increasing in α independently of whether $\hat{p}(1+r)$ is smaller or larger than one. Recall that labor supply increases with the net return of the pension system, γ, which itself increases with α. A larger value of α then decreases the labor supply distortions generated by the public pension scheme, so that the maximum non-contributive collective annuity is attained with a larger value of τ. This is a second instance of the contagion effect from Bismarck to Beveridge. Observe also that, when an individual has no income, her most-preferred contribution rate is not affected by $\bar{p}$, the intrinsic return of the Beveridgean scheme. This is due to the fact that $\bar{p}$ affects the absolute amount of the pension received, but not its sensitivity to the tax rate (up to a multiplicative factor).

Going back to (11), we also observe that the most-preferred tax rate decreases with θ when $\gamma < 0$ and that there exists a threshold value of θ such that people above this threshold most prefer $\tau = 0$. As θ increases, the contributory part of the collective annuity looms larger compared with the non-contributory part, and the individual then favors a lower value of the tax rate (since the non-contributory collective annuity's net marginal individual return is negative). The income threshold above which individuals favor $\tau = 0$ is denoted by[11]

$$\tilde{\theta}(\alpha) = \frac{\hat{p}}{\bar{p}} \frac{1-\alpha}{\hat{p}(1+r)-\alpha},$$

which is increasing in α if $1/\hat{p} > (1+r)$ (i.e., if the Bismarckian collective annuity internal rate of return is larger than the private saving return and so represents a good deal for all agents) and decreasing in α otherwise.

Finally, for the Beveridgean pension scheme ($\alpha = 0$ so that $\gamma = -1$), $\tau^*(\theta, \alpha)$ equals $1/2$ when $\theta = 0$, decreases with θ and reaches zero for the individuals with

$$\tilde{\theta}(0) = \frac{1}{\bar{p}(1+r)}.$$

Intuitively, when the Beveridgean internal rate of return $1/\bar{p}$ is equal to the private saving return $1+r$, all individuals with below-average income (*i.e.*, with $\theta < 1$) favor a positive tax rate with the Beveridgean scheme. If the Beveridgean internal rate of return is larger (resp., smaller) than the private saving return, then individuals with an

[11] The reader can check that the second-order condition for τ is satisfied for all $\theta \leq \tilde{\theta}(\alpha)$.

income slightly above (resp., below) the average gain (lose) through the scheme, so that $\tilde{\theta}(0) > 1$ (resp., $\tilde{\theta}(0) < 1$).[12]

Since the utility function (8) is concave in τ, and since the most-preferred value of τ is (weakly) decreasing in θ for any given α, we can apply the median voter theorem to obtain that the individual with $\theta = \theta^{med}$ is decisive in the vote over τ:

$$\tau^V(\alpha) = \tau^*(\theta^{med}, \alpha).$$

We summarize these results in the following Proposition.

Proposition 2 *Assume that preferences are represented by (1), that pension benefits are paid as a collective annuity with a contributive and a flat part as specified by (2) and that the disutility from working is quadratic. We have the following properties:*
i) the individually optimal value of τ does not depend on individual life expectancy but on productivity θ and on the type of system α through its impact on the net individual marginal return of the collective annuity, γ;
ii) if $\gamma = \alpha/(\hat{p}(1+r)) - 1 \geq 0$, everybody favors $\tau = 1$.
iii) if $\gamma < 0$, preferences depend upon the value of α: a) if $\alpha = 1$ (pure Bismarckian system), then everybody favors $\tau = 0$; b) if $\alpha < 1$, the most-preferred value of τ is strictly positive for an individual with zero productivity, decreases with productivity and is zero above some productivity threshold level. The majority chosen level of τ corresponds to the most-preferred level of the individual with the median productivity. In the special case of a purely Beveridgean system ($\alpha = 0$), a sufficient condition for a positive majority chosen value of τ when the income distribution is positively skewed is that the internal rate of return of the collective annuity is larger than the private saving return ($1/\bar{p} > 1 + r$).

There are two conditions to be satisfied for an individual to have an interior most-preferred size of the collective annuity program: that the system not be purely Bismarckian ($\alpha < 1$) and that the net individual marginal return of the collective annuity

[12] Observe that $\bar{p}$ affects the preferences for τ of individuals with positive income, in contrast to individuals with no income, whose only objective is to maximize the transfer they receive, irrespective of the intrinsic Beveridgean return $\bar{p}$.

(γ) be negative. When both conditions are satisfied, individuals below a threshold productivity level face a trade-off between the redistribution embedded in the non-contributive component of the collective annuity and the low individual return of its contributive component. Individuals with large productivities gain less from the redistributive Beveridgean component, and most-prefer a lower size of the overall collective annuity program.

We now turn to how the majority chosen value of τ is affected by variations in the distribution of lifetime expectancies.[13]

4.3 Comparative static analysis with respect to the distribution of lifetime expectancy

Observe first that individual life expectancy plays no role in the determination of the individual's most-preferred contribution rate. As explained above, all the impact of p is subsumed in the saving decision, so that p affects neither individual labor supply z nor preferences over τ (and α, as we will see later on). As for the lifetime expectancy distribution in the economy, it affects individual preferences through two channels, namely the intrinsic returns of the contributive ($\hat{p}$) and non contributive ($\bar{p}$) collective annuity program. These are not identical as soon as people differ in p with a non zero correlation between income and lifetime expectancy. If these two conditions are satisfied, we can devise variations in the distribution function $H(w,p)$ that either increase $\bar{p}$ while maintaining $\hat{p}$ constant, or that increase $\hat{p}$ with $\bar{p}$ constant.

Variations in the average life expectancy $\bar{p}$ impact the individuals' most-preferred contribution rate if three conditions are met simultaneously: (i) the collective annuity is at least partially Beveridgean ($\alpha < 1$), (ii) the marginal individual return of the Bismarckian annuity is negative ($\gamma < 0$; otherwise, everyone most prefers $\tau = 1$ irrespective of the value of $\bar{p}$) and (iii) the individual has strictly positive income ($w > 0$, otherwise the individual wants to maximize the non-contributive annuity received, and the value

[13] The impact of the distribution of productivities on the size of the collective annuity program should be obvious from the analysis above. First, the median productivity individual is decisive in the vote, with the most-preferred value of τ decreasing with productivity. Second, the most-preferred value of τ increases with the average productivity as long as $\alpha < 1$, as can be seen from the second term in (11). Moreover, if the correlation between productivity and life expectancy is non zero, a change in the distribution function $F(w)$ impacts $\hat{p}$. The impact of $\hat{p}$ on τ^V is studied in the next section.

of τ for which the maximum annuity is attained is independent of $\bar{p}$). If these three assumptions are satisfied, the most-preferred value of τ is (weakly) decreasing with $\bar{p}$ for all voters, so that the majority-chosen value of τ also (weakly) decreases, for any value of α strictly lower than one. The intuition for this result is straightforward: a higher value of $\bar{p}$ decreases the intrinsic return of the Beveridgean collective annuity, so that this annuity becomes less attractive to all voters.

Variations in $\hat{p}$ impact voters' most-preferred value of the contribution rate provided that $\alpha > 0$—i.e., that there is a Bismarckian component in the collective annuity. Individuals' most-preferred contribution rate decreases with $\hat{p}$, because a larger $\hat{p}$ decreases the return of the Bismarckian scheme. There are two differences between the impact of changing $\hat{p}$ and $\bar{p}$ on the majority chosen value of τ. First, while the impact of varying $\bar{p}$ on τ^V is smooth, increasing $\hat{p}$ may have a discontinuous and dramatic impact on τ^V, which falls from one to zero when the increase in $\hat{p}$ decreases γ below zero and the collective annuity is exclusively Bismarckian ($\alpha = 1$). Second, increasing $\hat{p}$ decreases the return of both (contributive and non-contributive) parts of the collective annuity –i.e., it renders both the Beveridgean and the Bismarckian components less attractive, by the afore-mentioned contagion effect.

In the special cases where either all individuals have the same lifetime expectancy, or where they differ but without any correlation between productivity and lifetime expectancy, we have that $\bar{p} = \hat{p}$. In that case, increasing average lifetime expectancy decreases the returns of both contributory and non-contributory collective annuities, resulting in a lower size of the system.

Finally, one can wonder which of the two systems, Beveridge or Bismarck, is more resistant to an increase in longevity. Comparing how these two pure systems (represented by $\alpha = 0$ and $\alpha = 1$, respectively) react to increases in $\bar{p}$ and $\hat{p}$, we obtain that the size of both programs (weakly) decreases when lifetime expectancies increase, but in very different ways. For the Bismarckian system, there is not impact of $\hat{p}$ on τ^V as long as γ remains positive, and then a brusque shift to $\tau^V = 0$ when γ becomes negative. In other words, the Bismarckian system, as in "The oak and the reed" by Lafontaine, breaks but never bends. On the opposite, the majority chosen size of the Beveridgean system smoothly decreases with the average longevity $\bar{p}$. The intuition for the difference between the two systems is that the redistribution embedded into the Beveridgean system creates heterogeneity in voters preferences (unlike for Bismarckian), so that a lower intrinsic return (implied by a larger $\bar{p}$) slowly erodes its political support, rather than suddenly disappearing for all voters when the Bismarckian annuity's return becomes lower than the interest saving rate.

We now move to the study of the majority chosen size of the collective annuity scheme as a function of its type.

4.4 Comparative static analysis with respect to the type of collective annuity

The next proposition summarizes how the majority chosen size of the collective annuity program (τ) is affected by the type of system (α).

Proposition 3 *Assume that preferences are represented by (1), that pension benefits are paid as a collective annuity with a contributive and a flat part as specified by (2) and that the disutility from working is quadratic. We then have the following properties:*
i) If the internal rate of return of the collective annuity is larger than the private saving return ($1/\hat{p} > 1+r$), the majority chosen level of τ is increasing with α, until it reaches one and remains equal to one for larger values of α.
ii) If the internal rate of return of the collective annuity is smaller than the private saving return ($1/\hat{p} < 1+r$), $\tau^V(\alpha)$ may not be monotone in α.

Proof: i) The majority chosen value of τ is interior for low values of α ($\alpha/\hat{p} < 1+r$) and increasing with α (because $\tau^*(0,\alpha)$ increases with α and $\tilde{\theta}(\alpha)$ increases with α, together with the continuity of $\tau^*(\theta,\alpha)$). For large values of α ($\alpha/\hat{p} > 1+r$), everybody most-prefers $\tau = 1$.

ii) The most-preferred value of τ of an individual with zero productivity increases with α, while the productivity level above which individuals most-prefer a zero value of τ is decreasing with α. This means that, although the median productivity individual is always decisive when voting over τ for any given α, her most-preferred value of τ may not be monotone in α.

Individuals with low productivities mainly aim at maximizing the benefit they get from the non-contributory component of the collective annuity. The tax rate that maximizes this benefit increases with α (another instance of the contagion effect from Bismarck to Beveridge) since a larger α induces individuals to work more. Individuals with larger productivities also care about the non-contributory component of the annuity. If the internal rate of return of the collective annuity is larger than the private saving return ($1/\hat{p} > 1+r$), the Bismarckian component of the annuity is intrinsically a good deal, and individuals react to a larger Bismarckian parameter by favoring a larger value of α. In that case, the impact of a larger α on both components of the collective annuity calls for a larger value of τ for all individuals. If the internal rate of return of the collective annuity is smaller than the private saving return ($1/\hat{p} < 1+r$), there is a trade-off between the positive impact of a larger α on the marginal return of the Beveridgean part and the negative impact of increasing the share of the under-performing Bismarckian part. The first effect is larger for low productivity individuals (whose most-favored value of τ increases with α), while the second effect is more important for high-productivity individuals (whose most-favored value of τ decreases with α). For certain individuals, the trade-off is such that their most-preferred value of τ may not be monotone in α. If the decisive individual belongs to this intermediate group, the majority-chosen size of the system may not be monotone in α.

Figures 1 and 2 illustrate part *i)* of Proposition 3. Figures 3 to 5 illustrate part *ii)* of the Proposition. Figure 3 depicts a numerical example where the most-preferred value of τ is not monotone increasing in α for all values of θ. Using the same numerical example and varying the identity of the median (decisive) individual θ^{med}, Figure 4 shows that the majority chosen level of τ may be first increasing and then decreasing in α while Figure 5 illustrates the case where $\tau^V(\alpha)$ is monotone decreasing in α.

We are now in a position to compare the size of pure Beveridgean and Bismarckian systems:

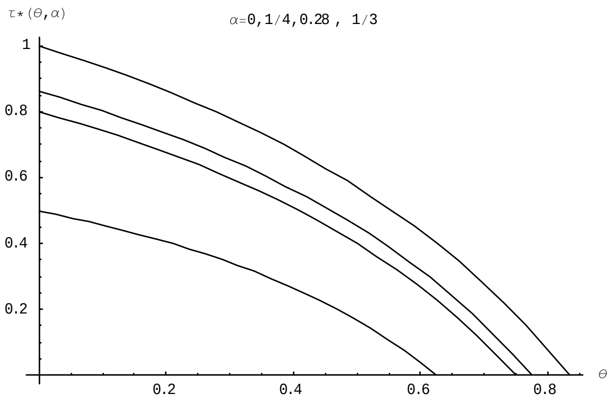

Figure 1: $\tau^*(\theta, \alpha)$ when $1/\hat{p} = 3 > 1 + r = 2$

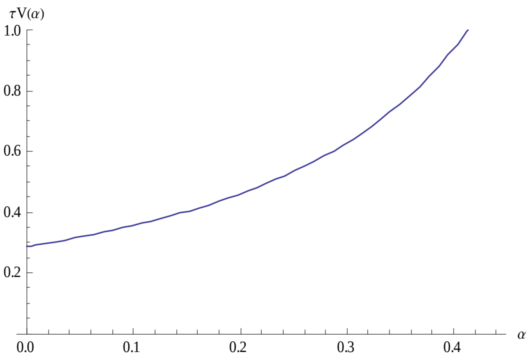

Figure 2: $\tau^V(\alpha)$ when $\theta_{med} = 3/8$ and $1/\hat{p} = 3 > 1 + r = 2$

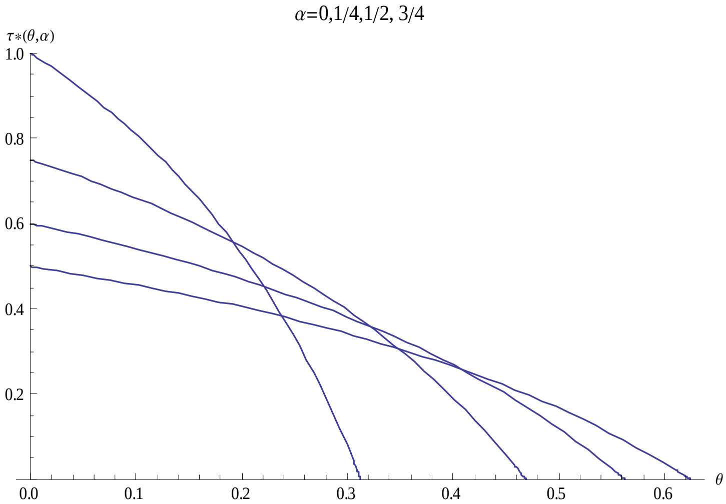

Figure 3: $\tau^*(\theta, \alpha)$ when $1/\hat{p} = 4/3 < 1 + r = 2$

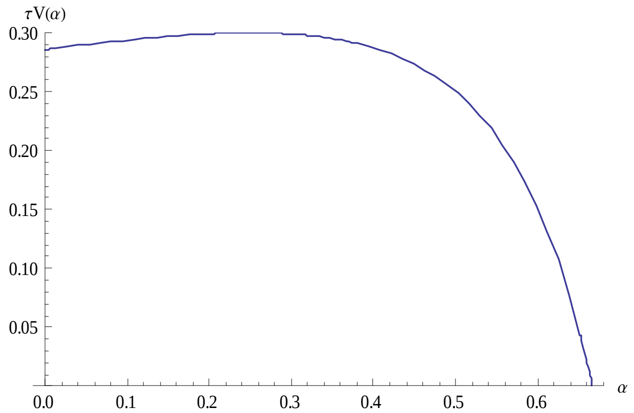

Figure 4: $\tau^V(\alpha)$ when $\theta^{med} = 3/8$ and $1/\hat{p} = 4/3 < 1 + r = 2$

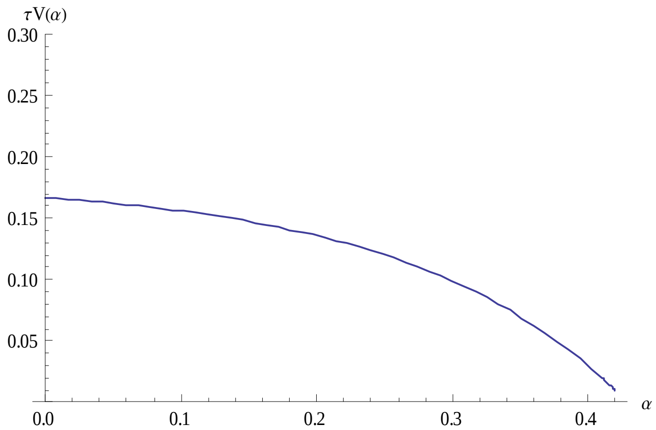

Figure 5: $\tau^V(\alpha)$ when $\theta^{med} = 1/2$ and $1/\hat{p} = 4/3 < 1 + r = 2$

Proposition 4 *Assume that preferences are represented by (1), that pension benefits are paid as a collective annuity with either a contributive or a flat part as specified by (2) with, respectively,* $\alpha = 1$ *and* $\alpha = 0$*, and that the disutility from working is quadratic. Comparing the majority chosen size of the pure Beveridgean and Bismarckian systems (i.e.,* $\tau^V(0)$ *and* $\tau^V(1)$*), we obtain the following results:*

i) The Bismarckian contribution rate is strictly larger than the Beveridgean one if $1+r < 1/\hat{p}$;

ii) The Beveridgean contribution rate is strictly larger than the Bismarckian one if $1/\hat{p} < 1 + r < 1/(\bar{p}\theta^{med})$;

iii) There is no collective annuity ($\tau^V = 0$*) in both the Beveridgean and the Bismarckian systems if* $1 + r > \max(1/\hat{p}, 1/(\bar{p}\theta^{med}))$

Bismarck is always larger than Beveridge when the interest rate $1+r$ is smaller than $1/\hat{p}$ since the Bismarckian annuity has a better return than private saving, and voters unanimously support a contribution rate of one, while the Beveridgean tax rate is always lower than one half (corresponding to the summit of the Laffer curve). A necessary condition for Beveridge to be larger than Bismarck is that $\bar{p}\theta^{med} < \hat{p}$. How likely is this condition to be satisfied? Recall that $\theta^{med} < 1$ with a positively skewed distribution of productivities. A simple benchmark is the case where either all individuals have the same longevity, or where they differ in longevity but with a zero correlation between productivity and longevity. In these two cases, $\bar{p} = \hat{p}$ and the necessary condition is satisfied. The empirically relevant case is a positive correlation between productivity and longevity (since richer people tend to live longer), which tends to increase $\hat{p}$ above $\bar{p}$, enlarging the set of values of interest rate for which Beveridge is larger than Bismarck. For these values of the interest rate, $1 + r$ is large enough that it dominates the return of a Bismarckian annuity (calling for zero contribution rate by all voters) while low enough compared to the intrinsic return of a non-contributive annuity that the median productivity individual favors a positive contribution rate under Beveridge. Finally, if $1+r$ is large enough, both the Bismarckian and the Beveridgean schemes are dominated by private saving, and the majority chosen tax rate is zero for both schemes.

Finally, as for intermediate systems with $0 < \alpha < 1$, recall that Proposition 3 has shown that the majority chosen contribution rate (i) increases with α if $1 + r < 1/\hat{p}$ but may not be monotonically increasing in α when $1 + r > 1/\hat{p}$, which is exemplified in Figure 4.

We now turn to the political determination of the type of collective annuity system, as measured by the parameter α.

5 Voting over the type of the pension system

We assume in this section that τ is not affected by the choice of α but is given exogenously. We proceed as in the previous section, studying first the individually optimal type of collective annuity, then the majority chosen one and how it is affected by variations in the life expectancy distribution.

5.1 Individuals' most-preferred type of collective annuity

Differentiating the utility function (8), we obtain

$$\frac{\partial V(\alpha, \tau, w, p)}{\partial \alpha} = \beta p u'(d) \tau \left[\frac{wz^*}{\hat{p}} - \frac{Ewz^*}{\bar{p}} + \frac{(1-\alpha)}{\bar{p}} \frac{\partial Ewz^*}{\partial \alpha} + \alpha wz^* \frac{\partial (1/\hat{p})}{\partial \alpha} \right]. \qquad (12)$$

By the envelope theorem, the only impact of α on the utility of voters is through variations in the value of the collective annuity served in the second period. The first two terms in (12) measure the direct impact of increasing α on the value of the collective annuity for the individual with productivity w, with the contributive component increasing and the non-contributive component decreasing. The third term measures the indirect impact of α on the Beveridgean component through variations in its tax basis (the average labor income) while the last term shows the impact of α on the internal rate of return of the Bismarckian annuity.

Assuming once again $h(z) = z^2/2$, we have $\partial \hat{p}/\partial \alpha = 0$ and

$$\frac{\partial Ewz^*}{\partial \alpha} = \frac{\tau E w^2}{\hat{p}(1+r)} > 0. \qquad (13)$$

The intuition for this contagion result is that tightening the link between pension benefit and labor supply increases the average first-period income.

In that case equation (12) can be rewritten and divided by Ew^2 to obtain the following FOC[14] for α

$$(1+\gamma\tau)\left(\frac{\bar{p}}{\hat{p}}\theta-1\right)+\frac{1-\alpha}{\hat{p}(1+r)}\tau=0. \tag{14}$$

The second term represents the impact of a higher α on the return of the Beveridgean component and is always positive since a higher α increases labor supply in the economy, and thus the tax base which finances the non-contributive part of the collective annuity. It represents the contagion effect of α on the Beveridgean part of the annuity. The first term measures the marginal variation in the total collective annuity perceived by individual θ as α increases. It is positive for high productivity individuals ($\theta > \hat{p}/\bar{p}$) and negative otherwise.

5.2 The majority chosen type of collective annuity

Let $\alpha^*(\theta,\tau)$ denote individual θ's most preferred value of α for any given τ. Intuitively, all individuals with $\theta > \hat{p}/\bar{p}$ most-prefer Bismarck ($\alpha = 1$) whatever the value of $\tau > 0$. When interior, the most-preferred value of α is given by

$$\alpha^*(\theta,\tau)=\frac{\hat{p}}{2\hat{p}-\bar{p}\theta}+\frac{1-\tau}{\tau}\frac{\hat{p}(1+r)(\theta\bar{p}-\hat{p})}{2\hat{p}-\bar{p}\theta}, \tag{15}$$

where $\theta\bar{p}-\hat{p} < 0 < 2\hat{p}-\bar{p}\theta$ so that the first term is positive and the second term negative. From (15), we first observe that the most-preferred value of α increases with θ. This is because the marginal impact of increasing α on the collective annuity received by an individual (the first term in (14)) increases with the individual's productivity, while its impact on the Beveridgean tax base is independent of θ. Since preferences are concave over α, we can apply the median voter theorem and obtain that the majority voting value of α, denoted by $\alpha^V(\tau)$, is the one most-preferred by the median ability individual:

$$\alpha^V(\tau)=\alpha^*(\theta^{med},\tau).$$

We also obtain from (15) that the most-preferred value of α increases with τ. As τ increases, the distortion associated with the Beveridgean component increases, and

[14] It is straightforward to see that the second-order condition is satisfied.

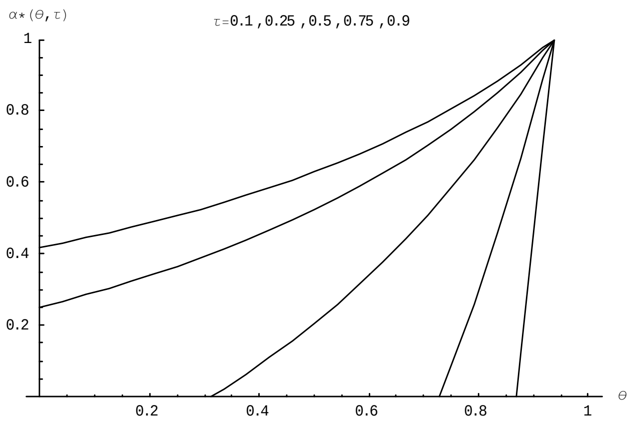

Figure 6: $\alpha^*(\theta, \tau)$ with $\hat{p} = 3/4$ and $r = 1$

individuals react by increasing their most-preferred value of α for two reasons: because the contributive component of the annuity looks comparatively better, and because a larger value of α decreases the distortions associated with a larger τ (another manifestation of the contagion effect). Even for individuals with zero productivity (for which the first reason is not relevant since they receive a zero contributive annuity), the second effect is sufficient for them to favor a larger value of α as τ increases, as can be seen from

$$\alpha^*(0, \tau) = \frac{1}{2} - \frac{1-\tau}{2\tau}\hat{p}(1+r),$$

which corresponds to the maximum non contributory annuity. We obtain that $\alpha^*(0, \tau) = 0$ if $\tau < 1/(1+\hat{p}(1+r))$, but that $\alpha^*(0, \tau)$ is strictly positive for values of τ larger than this threshold, and increases to reach $1/2$ when $\tau = 1$. We know that this individual is only interested in maximizing the size of the Beveridgean component given the value of τ. To offset the larger distortions associated with an increase in τ, this individual increases his most-preferred value of α, even though this decreases the relative share of the non-contributory component in the average collective annuity.

Since $\alpha^*(\theta, \tau)$ increases both with θ and with τ (see Figure 6 for a numerical illustration), we obtain that the majority-chosen level of α increases with τ as well; see Figure 7.

Finally, observe that increasing the rate of return of private savings, r, leads everyone

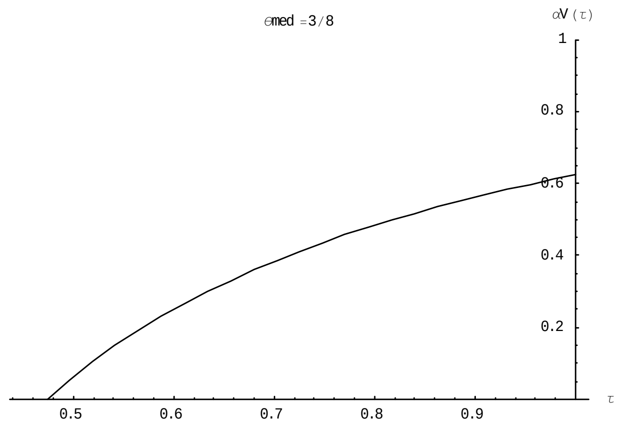

Figure 7: $\alpha^V(\tau)$ when $\theta^{med} = 3/8$, $\hat{p} = 3/4$ and $r = 1$

to favor a more Beveridgean collective annuity (*i.e.*, a lower value of α). This is because increasing r decreases the positive impact of a larger α on the Beveridgean tax base (the average labor supply): see equation (13).

These results are summarized in the following proposition:

Proposition 5 *Assume that preferences are represented by (1), that pension benefits are paid as a collective annuity with a contributive and a flat part as specified by (2) and that the disutility from working is quadratic. For a given level of τ, we obtain the following properties:*
i) The most-preferred value of α increases with θ. Consequently, the majority-chosen value of α is the one most preferred by the median ability individual.
ii) All individuals with $\theta > \hat{p}/\bar{p}$ most-prefer a purely Bismarckian system, whatever the value of τ. Individuals prefer a purely Beveridgean system if their productivity is low enough and the tax rate τ is low enough.
iii) The most-preferred value of α increases with τ (even for individuals with zero income), so that the majority-chosen value of α also increases with τ.

5.3 Comparative static analysis with respect to the distribution of lifetime expectancy

As was already the case for the political determination of the size of the system, the individual lifetime expectancy p plays no role in the determination of its type. Rather, the distribution of longevity affects the majority chosen value of α through two channels: $\bar{p}$ and $\hat{p}$. We first assume that $\text{cov}(w, p) \neq 0$, so that we can study the impact of $\bar{p}$ and $\hat{p}$ on α^V separately.

Raising $\bar{p}$ (while maintaining $\hat{p}$ constant) increases individuals' most preferred value of α, as can be seen from (14). Intuitively, a larger value of $\bar{p}$ decreases the intrinsic return of the non-contributory annuity (raising the first term in (14)), and voters react by moving away from a Beveridgean system. The comparative static analysis is slightly more complex for $\hat{p}$, because of the contagion effect: increasing $\hat{p}$ (with $\bar{p}$ constant) decreases the intrinsic return of the contributive annuity, which decreases both the amount of the contributive annuity, but also of the non-contributive annuity by contagion. Differentiating (15) with respect to $\hat{p}$ shows that the first impact is larger than the second, so that an increase in $\hat{p}$ moves the majority chosen system type towards Beveridge.

Finally, in the case where either all individuals have the same longevity p, or where $\text{cov}(w, p) = 0$, we have that $\bar{p} = \hat{p}$. Interestingly, raising $\bar{p} = \hat{p}$ results in a decrease in individuals' most preferred value of α —that is, a decrease in the return of both contributive and non-contributive annuities drives voter to prefer a more Beveridgean system (for τ given). The intuition for this result is that, as the intrinsic return of both schemes is (i) the same and (ii) simultaneously decreased, the redistributive motive takes a relatively more important role and leads to a more Beveridgean scheme at equilibrium.

We now turn to the joint determination by majority voting of the generosity and of the type of the pension system.

6 Shepsle equilibrium

It is well known that simultaneous majority voting over a multidimensional policy space generically fails to have an equilibrium (see for instance De Donder *et al.* (2009)). We then model the joint determination procedure first suggested independently by Kramer (1972) and Shesple (1979). A policy pair (α,τ) is a Kramer-Shepsle equilibrium, denoted by (α^S, τ^S), if each element in the pair corresponds to a majority voting equilibrium given the value taken by the other element –*i.e.*, if $\alpha^S = \alpha^V(\tau^S)$ and $\tau^S = \tau^V(\alpha^S)$. The two previous sections have made it clear that the individual with the median value of θ is decisive both in the choice of α given τ (Proposition 5) and of τ given α (Proposition 2). Since the median productivity individual is decisive in both votes, we obtain that $\alpha^S = \alpha^*(\theta^{med}, \tau^S)$ and $\tau^S = \tau^*(\theta^{med}, \alpha^S)$. We then get the following proposition which is established in Appendix A.

Proposition 6 *Assume that preferences are represented by (1), that pension benefits are paid as a collective annuity with a contributive and a flat part as specified by (2) and that the disutility from working is quadratic. We obtain the following properties.*
i) There is no Shepsle equilibrium with $0 < \alpha^S, \tau^S < 1$ *— i.e., with interior solutions for both* τ *and* α.
ii) If $\hat{p}(1+r) < 1$, *there is a unique Shepsle equilibrium, with* $\tau^S = 1$ *and* $\alpha^S = min(\hat{p}/(2\hat{p} - \bar{p}\theta^{med}), 1) \in [1/2, 1]$.
iii) If $\hat{p}(1+r) > 1$ *and* $\theta^{med} < \frac{1}{\bar{p}(1+r)}$, *there is a unique Shepsle equilibrium, with* $\alpha^S = 0$ *and* $\tau^S = 1 - \frac{1}{2 - \bar{p}(1+r)\theta^{med}} \in [0, 1/2]$.
iv) If $\hat{p}(1+r) > 1$ *and* $\theta^{med} \geq \frac{1}{\bar{p}(1+r)}$, *there is a unique Shepsle equilibrium, with* $\tau^S = 0$.

Proposition 6 states that there is no equilibrium for which both τ and α take interior values. Consequently, we have either a corner solution or no equilibrium at all. If $\hat{p}(1+r) < 1$, we have a unique equilibrium with a large, mostly but not always exclusively Bismarckian program. If $\hat{p}(1 + r) > 1$ and if θ^{med} is low enough, we have a smaller and purely Beveridgean program. Finally, if $\hat{p}(1 + r) > 1$ and θ^{med} is large enough, there is no majoritarian political support for a collective annuity program.

The intuition for these results runs as follows. If $1/\hat{p}$ is large, meaning that the contributive annuity's intrinsic return is large, the Bismarckian system is very attractive and results in a large contribution rate. The reason why a purely Bismarckian system is not always chosen is that, if her productivity is low enough, the median productivity individual benefits from redistribution and thus favors the introduction of some non-contributive part in the collective annuity. If $1/\hat{p}$ is small, the non-contributive annuity has a low return and voters prefer a purely Beveridgean system provided that the decisive voters' productivity is not too large. The size of this Beveridgean annuity remains quite low because a purely Beveridgean system creates large distortions when its size increases. In other words, a low value of $1/\hat{p}$ discourages voters from introducing a contributive component into the collective annuity, which puts an upperbound on the size of the collective program because in the absence of Bismarckian component there is a lot of distortions (on labor supply) generated by the Beveridgean scheme. Finally, if the Bismarckian intrinsic return is low while the decisive voters' productivity is large, there is no political support for a collective annuity scheme, since a majority of voters would rather rely exclusively on private saving.

Observe that the Shepsle equilibria identified in the proposition correspond to what is empirically observed: large Bismarckian systems and smaller Beveridgean ones. Contrary to the "partial equilibrium" analysis of the majority chosen size of the pure Beveridgean and Bismarckian programs when the type of program is exogenously imposed (see section 4.4), which left open the possibility that the Beveridgean system could be larger than the Bismarckian system, we obtain that, when endogenizing both the size and type of system, Bismarckian programs are always larger than Beveridgean ones. We also obtain conditions on exogenous variables (the rate of return of both types of annuities and the median productivity level) that explain which equilibrium arises.

We now study how the Shepsle equilibrium is affected by variations in the longevity distribution—i.e., by variations in $\bar{p}$ and/or $\hat{p}$. We obtain that $\bar{p}$ (when it is varied while maintaining $\hat{p}$ constant) does not affect whether the Shepsle equilibrium is mostly Bismarckian or purely Beveridgean, but affects how Bismarckian the equilibrium scheme is (in the former case) and the size of the purely Beveridgean annuity in the latter case. In both cases, the comparative statics analysis is intuitive: raising $\bar{p}$ decreases the intrinsic return of the non-contributive annuity, which results in a more Bismarckian system in the former case, and in a smaller pure Beveridgean program in the latter.

The value of $\hat{p}$ affects both whether a (mostly) Bismarckian or a (pure) Beveridgean schemes emerges at equilibrium, and also the exact type of the mostly Bismarckian program. On the other hand, it does not affect the size of the equilibrium program. The impact is also intuitive: raising $\hat{p}$ decreases the intrinsic return of the Bismarckian scheme, which results in the introduction of a larger Beveridgean component when the equilibrium is mostly Bismarckian, and in a discontinuous jump from a large Bismarckian program to a small purely Beveridgean one when a threshold is crossed.

This result sheds light on the impact of the correlation between income and longevity on the redistributiveness of public pension programs. As mentioned in the introduction, the literature has focused on the impact of this positive correlation on the amount of redistributiveness incorporated into existing systems, and has concluded not surprisingly that such a positive correlation decreases the amount of redistribution for a given system's type and size. These results have been reinforced in section 4.4 when the type of system is considered as exogenous while the size is chosen by majority voting. Endogenizing both the type and size of the pension program brings new elements to the fore. Increasing the positive correlation between income and longevity raises $\hat{p}$ (while keeping by assumption $\bar{p}$ constant) and makes the equilibrium system more redistributive, although sometimes at the expense of its size.[15] This result then calls for a reevaluation of the empirical literature, which should go beyond a mechanical assessment of the impact of positive correlation between income and longevity on existing pension systems.

7 Conclusion

This paper has developed a model where individuals differ in productivity and in longevity (modeled as the probability to be alive in the second period of their life). Individuals decide how much to work and to save when young, and consume their saving plus any pension benefit when old. The public pension system takes the form of a collective annuity, with both a contributive component (with the benefit based on the worker's

[15] A similar result happens when $\bar{p} = \hat{p}$ because of zero correlation between productivity and longevity, or because voters all have the same longevity. In that case, increasing $\bar{p}$ has no impact on the type/size of a (mostly) Bismarckian system, but results in a discontinuous jump to a pure Beveridgean annuity when a threshold is crossed. Also, the size of this pure non-contributive annuity decreases with $\bar{p}$.

own contribution) and a flat part (based on the average contribution in the economy). Voters choose both the size or generosity of the system (measured by the payroll tax rate) and its degree of income redistribution (measured by the proportion of the average pension that is contributory).

We first look at majority voting over the size for a given system's type, and we obtain (i) that a Bismarckian system may be larger than a Beveridgean system; (ii) that while the equilibrium size of a Beveridgean system decreases smoothly with average longevity, the support for a Bismarckian system drops discontinuously to zero when the distribution of longevity is such that its return falls below the interest rate; and (iii) that increasing the covariance between income and longevity leads to smaller pension systems (whatever their exogenous type). We then obtain that voters prefer a more Bismarckian (i.e., less redistributive) system when the tax rate is exogenously increased. Finally, we endogenize both the size and type of pension system using majority voting, and we obtain that the unique Shesple equilibrium is either a large (mainly) Bismarckian system, a smaller (purely) Beveridgean pension, or no public pension at all. This equilibrium pattern corresponds to what is observed in reality, with larger Bismarckian than Beveridgean systems. Also, a larger correlation between income and longevity makes the collective annuity more redistributive at equilibrium, although sometimes at the expense of its size. This calls into question the results obtained by the empirical literature, which shows that, *for given size and type of the collective annuity*, a larger correlation reduces income redistribution.

Our analysis makes uses of two simplifying assumptions: we assume away borrowing constraints, so that saving can be negative, and we assume that the disutility from working can be expressed in consumption terms, independently of income. These two assumptions taken together simplify a lot the solving of the model, since preferences for collective annuities are made independent of individual longevity. The first of the two assumptions may strike the reader as especially strong, since it often (but not always) results in some voters favoring a confiscatory payroll tax (even in the presence of labor supply distortions from income taxation). The introduction of explicit borrowing constraints would complicate the model a lot without bringing much new insight. Specifically, rather than favoring confiscatory tax rates, individuals would favor the largest value of the payroll tax consistent with non-negative saving. This would prevent people from favoring extremely large values of the payroll tax, but it would not affect the qualitative results we have obtained in this simpler framework.

As an extension to our approach, we could consider the case where the private market offers annuities which are actuarially fair –*i.e.*, based on each individual's own lifetime expectancy. As we claim in the Introduction, our model without private annuities is a good representation of the current reality, except maybe for countries with mandatory annuitization. The introduction of private annuities would change dramatically our results, because individuals' labor supply and preferences for the collective annuity would depend upon their own longevity. It would be interesting to look at how the political determination of the characteristics of the collective annuity are affected by the presence of private annuity, in the spirit of Cremer *et al.* (2009). We leave this extension for future research.

References

[1] Attanasio O. and C. Emerson, 2001, Differential mortality in the UK, IFS working paper 01/16.

[2] Bommier A., T. Magnac, B. Rapoport and M. Roger, 2005, Droit à la retraite et mortalité différentielle, *Economie et Prévision* 168: 1-16.

[3] Borck R., 2007, On the choice of public pensions when income and life expectancy are correlated, *Journal of Public Economic Theory*, 9-4, 711-725.

[4] Brown J., T. Davidoff and P. Diamond, 2005, Annuities and individual welfare, *American Economic Review*, 95, 1573–90.

[5] Browning, E. K., 1975, Why the social insurance budget is too large in a democracy, *Economic Enquiry* 13: 373-388.

[6] Casamatta G., H. Cremer and P. Pestieau, 2000a, On the political sustainability of redistributive social insurance systems, *Journal of Public Economics* 75: 341-364.

[7] Casamatta G., H. Cremer and P. Pestieau, 2000b, The political economy of social security, *Scandinavian Journal of Economics* 102: 502-522.

[8] Conde-Ruiz I. and P. Profeta, 2007, The Redistributive Design of Social Security Systems, *The Economic Journal* 117 : 686-712.

[9] Coronado J.L., D. Fullerton and T. Glass, 2000, The progressivity of social security, NBER working paper 7520.

[10] Cremer H., J.-M. Lozachmeur and P. Pestieau, 2009, Collective Annuities and Redistribution, *Journal of Public Economic Theory*, forthcoming.

[11] Deaton A. and C. Paxson, 1999, Mortality, education, income and income inequality among American cohorts, NBER working paper 7141.

[12] De Donder Ph. and J. Hindriks, 2002, Voting over Social Security with Uncertain Lifetimes, in Institutional and Financial Incentives for Social Insurance, edited by

C. d'Aspremont, V. Ginsburgh, H. Sneessens and F. Spinnewyn, Kluwer Academic Publishers, Boston, 201-220.

[13] De Donder Ph., Le Breton M. and E. Peluso, 2009, On the (Sequential) Majority Choice of Public Good's Size and Location, CEPR Discussion Paper 7223.

[14] Finkelstein, A. and J. Poterba, 2002, Selection effects in the UK individual annuities market, *Economic Journal*, 112, 28–50.

[15] Finkelstein, A. and J. Poterba, 2004, Adverse selection in insurance markets: policyholder evidence from the UK annuity market, *Journal of Political Economy*, 112, 183–208.

[16] Galasso V. and P. Profeta, 2002, The political economy of social security: a survey, European Journal of Political Economy, 18, 1-29.

[17] Gil J. and G. Lopez-Casasnovas, 1997, Life-time redistribution effects of the spaniels public pension system, University Pompeu Fabra working paper.

[18] Kramer, G.H., 1972, Sophisticated Voting over Multidimensional Choice Spaces, *Journal of Mathematical Sociology*, 2, 165–180.

[19] Leroux M.-L., 2007, The Political Economy of Social Security Under. Differential Mortality and Voluntary Retirement, mimeo available at www.cesifo-group.de/link/vsi07_LA_Leroux.pdf

[20] Mitchell, O., 1996, Administrative Costs in Public and Private Retirement Systems, NBER Working Papers 5734.

[21] Reil-Held A., 2000, Einkommen und Sterblichkeit in Deutschland: Leben Reiche länger?, Discussion paper 00-14, SFB 504, Mannheim.

[22] Shepsle, K.A., 1979, Institutional Arrangements and Equilibrium in Multidimensional Voting Models, *American Journal of Political Science*, 23, 27–59.

[23] Sheshinski E. (2007), *The Economics of Annuities*, Princeton University Press, Princeton, NJ.

A Proof of Proposition 6

i) This can be shown by solving simultaneously (14) and (11), the necessary conditions for an interior solution. This yields

$$\begin{aligned}\alpha^S &= 1,\\ \tau^S &= 1-\frac{1}{1-\hat{p}(1+r)},\end{aligned}$$

which cannot be an equilibrium because it specifies a level $\tau^S \notin [0,1]$ whatever the value of $\hat{p}(1+r) \neq 1$.

ii) Assume that $\hat{p}(1+r) < 1$.

- If $\theta^{med} > \hat{p}/\bar{p}$, $\alpha^*(\theta^{med},\tau) = 1$ (all individuals with $\theta > \hat{p}/\bar{p}$ most prefer Bismarck whatever the value of τ) and $\tau^*(\theta^{med},1) = 1$, so that $\alpha^S = 1$ and $\tau^S = 1$.
- If $\theta^{med} < \hat{p}/\bar{p}$, there are two possible equilibria: $(\alpha = 0, \tau > 0)$ and $(\alpha > 0, \tau = 1)$.
  - First candidate for equilibrium: $(\alpha = 0, \tau > 0)$

    If $\alpha = 0$, solving (11) with $\theta^{med} = w_{med}^2/Ew^2$ gives the majority chosen interior value of τ. Observe that

    $$\theta^{med} < \frac{\hat{p}}{\bar{p}} < \frac{1}{\bar{p}(1+r)} = \tilde{\theta}(0),$$

    so that the majority chosen value of τ is positive. We then replace τ by this value in the first-order condition for α given by equation (14), and we solve it for $\alpha = 0$ to obtain

    $$\frac{\partial V(0,\tau,\theta)}{\partial \alpha} = \frac{-1+\hat{p}(1+r)}{\hat{p}(1+r)(-2+\bar{p}(1+r)\theta^{med})},$$

    which is positive because $\hat{p}(1+r) < 1$ and $\bar{p}(1+r)\theta^{med} < 1$, a contradiction with the assumption that $\alpha = 0$.
  - Second candidate for equilibrium: $(\alpha > 0, \tau = 1)$

    If $\tau = 1$, solving the first-order condition (15) for α gives

    $$\alpha^*(\theta^{med}, 1) = \frac{\hat{p}}{2\hat{p}-\bar{p}\theta^{med}} \in [1/2, 1]$$

since $\theta^{med} < \hat{p}/\bar{p}$. We then replace α by this value in the first-order condition for τ, and we evaluate it at $\tau = 1$ to obtain

$$\frac{\partial V(1, \tau, \theta)}{\partial \tau} = \frac{-1 + \hat{p}(1 + r)}{\bar{p}(1 + r)^2(-2\hat{p} + \bar{p}\theta_{med})}$$

which is positive, confirming that $\tau^S = 1$.

iii) Assume that $\hat{p}(1+r) > 1$ and assume that $\alpha^S = 0$. From the FOC for τ measured at $\alpha = 0$, we infer that

$$\tau^S = 1 - \frac{1}{2 - \bar{p}(1 + r)\theta^{med}},$$

which decreases with $\bar{p}(1 + r)\theta^{med}$ and is non-negative (and at most equal to 1/2) provided that $\bar{p}(1 + r)\theta^{med} < 1$. Evaluating the FOC with respect to α at this value of τ, we obtain

$$\frac{\partial V(\alpha, \tau, w, p)}{\partial \alpha} = \frac{1 - \hat{p}(1 + t)}{\hat{p}(1 + r)(2 - \bar{p}(1 + r)\theta^{med})} < 0.$$