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Life Satisfaction and Age

Dealing with Underidentification in Age-Period-Cohort Models

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Life satisfaction and age: dealing with underidentification in age – period – cohort models

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Abstract

Recent literature typically finds a U shaped relationship between life satisfaction and age. Age profiles, however, are not identified without forcing arbitrary restrictions on the cohort and/or time profiles. In this paper we report what can be identified about the relationship between life satisfaction and age without applying such restrictions. Also, we identify the restrictions needed to conclude that life satisfaction is U shaped in age. We find that a U shaped relationship between life satisfaction and age is indeed supported by the data, but only under the untestable condition that the linear time trend is negative and that the linear trend across birth cohorts is practically flat.

Keywords: Aging, life satisfaction, well-being, happiness

JEL codes: D91, I31

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1 Introduction

An increasing body of literature documents a U shaped relationship between life satisfaction and age over most of the life cycle [see Clark (2007), Blanchflower and Oswald (2008), Frijters and Beatton (2008), Blanchflower and Oswald (2009), Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009), Gwozdz and Sousa-Poza (2010) and Stone, Schwartz, Broderick, and Deaton (2010) for example]. Life satisfaction is found to decrease to midlife and to increase subsequently towards retirement.[1] This result is suggestive, because the elderly are generally less wealthy and less healthy than the middle aged.

Studying the relationship between of life satisfaction on age is interesting in its own right, but it is of additional interest to economists. A baseline life cycle model for example predicts that consumption does not decrease after retirement. People correctly anticipate a drop in income upon retirement and would save enough to maintain a constant level of consumption. It is often observed however that consumption drops right after retirement. This empirical regularity is inconsistent with baseline versions of the life cycle model and suggests that people do not save enough [see e.g., Banks, Blundell, and Tanner (1998)]. The alleged U shaped relationship between life satisfaction and age suggests however that people do not experience a welfare drop at retirement, as the typically observed drop in consumption would suggest. Instead, the evidence suggests that people become more satisfied when they retire. Maybe, the retirement – consumption puzzle is not much of a puzzle after all and people should perhaps have saved even less?[2]

A problem with the many recent studies that find a U shaped relationship between life satisfaction and age is the impossibility of uniquely identifying age, period and cohort effects from panel data, without imposing arbitrary restrictions on the age, time and/or cohort profiles [Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009) review part of the literature and explicitly mention the identification problem. See also Glenn (2009) in

[1] Some find an additional decrease in life satisfaction towards very old age (e.g., Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009) and Gwozdz and Sousa-Poza (2010)). We also find this.

[2] More generally, it is important to develop knowledge about the complex process of preference formation, to be better able to understand or anticipate behavior. Directly observing welfare levels in different circumstances may provide insights in this issue, in addition to data on observed (consumer) behavior.

his commentary on Blanchflower and Oswald (2008)]. An identification problem arises due to the equality *year of birth + age = calender year* and has been a topic of debate in social science from the 1970s onwards [see for example Hall (1971), Mason, Mason, Winsborough, and Poole (1973), Heckman and Robb (1985), Deaton and Paxson (1994), Berndt, Griliches, and Rappaport (1995), Attanasio (1998), Jappelli (1999) and McKenzie (2006)]. To date however, no generally accepted solution to this problem has been proposed. Intuitively, it is impossible to observe the life satisfaction of *a single person* with the same age at two different different points in time to identify time effects. Similarly, it is not possible to observe a single person with different ages at a single point in time to isolate the age effect. Angrist and Pischke (2009) discuss a similar problem on page 6 of their recent book and refer to it as a fundamentally unidentified question (i.e., this question is, as they call it, FUQed).

The equality year of birth + age = calendar year also frustrates inference with respect to the age dependence of life satisfaction. In a cross section of households a U shape in age may be due to cohort effects that are not accounted for. In other words, the old in a cross section may not be more satisfied than the middle aged because they are old, but merely because they belong to cohorts that have lived through rougher times (WWII for example) and have adjusted their point of reference (i.e., older cohorts may be more satisfied with less because they have experienced how bad it can get. This is a form of habit formation). The availability of panel data allows to account for differences across cohorts by including cohort effects (or fixed effects). Using FE or modeling cohort effects however, does not really solve the problem. This empirical strategy immediately confounds age and time effects (which was not the case in a cross section). Within transforming the data gets rid of the cohort effect, and makes that the within transformed time trend $(t - \bar{t}_i)$ is equal to the within transformed age trend $(a - \bar{a}_i)$. Therefore, they cannot be both included in a regression and are thus not separately identified. Consequently, from observing one cohort (or an individual) when they age one can never be fully sure whether the change in life satisfaction is due to the aging process or due to passage of time.

It is therefore problematic to discuss age effects without considering cohort and time effects at the same time. Moreover, all three of them seem important so it is not warranted to dismiss either one of the three a priori. As we noted before, cohort effects may arise by shifting reference points. Where young generations take air travel and iphone 4G's for granted, older generations still experience feelings of satisfaction from these things by comparing the lives they currently lead to their living standards before. Time effects arise when a population as a whole becomes more or less satisfied. Economic growth or improvements in health care are likely to benefit all.

Because the complete identification of age, period and cohort effects is not possible, we might want to impose (perhaps reasonable) restrictions on either one of the three profiles to reach identification. By doing so a second issue becomes important: how do imposing different assumptions matter for final inferences about the age profile? In this paper we show that imposing slightly different identifying assumptions can have far reaching consequences on the substantive results. We hereby follow Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009) who also show the fragility of the inference regarding the age effects on life satisfaction. Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009) however, subsequently place assumptions on the model themselves by modeling the cohort effects with substantive variables, such as life expectancy of the birth cohort.[3]

In this paper we take a different route. Instead of solving the identification problem, we report *what can be identified*, and hence, what can be concluded from the data without making assumptions (other than assumptions on functional form). It has been shown that time, age and cohort profiles can be fully identified in deviation from a linear trend in time, age and year of birth respectively, whereas only two (out of three) of the respective linear trends (age, period, cohort) can be identified [see Heckman and Robb (1985) and McKenzie (2006) for example]. Consequently, without imposing arbitrary assumptions on the age, period and/or cohort profiles it is possible to test whether life satisfaction data is consistent with a U shape in age.

We find that the data is indeed consistent with a U shape over most of the life cycle. This contrasts findings of e.g., Mroczek and Spiro (2005) who report an inverted U shaped relationship between life satisfaction and age. They do however use a different data set then

[3]Glenn (2009) argues that for identification of the age profile, "at least in the case of the United States data on happiness, the dropped variable should be period".

we do in this paper. Whereas we find that the age profile is mainly U shaped around a linear trend in age, the linear trend itself cannot be uniquely inferred from the data. This has important implications. For example, we show that the data is also consistent with a generally negative relationship between life satisfaction in age.

The paper is organized as follows. Section (2) discusses the fact that age profiles are not identified without imposing assumptions on the cohort and time profiles. Furthermore, it shows how sensitive the age profiles are to making different kinds of assumptions. In section (3) we discuss which elements of the age, cohort and time profiles can be identified without making assumptions. Section (4) discusses the data we use in this paper and shows and discusses our empirical results. Finally, section (5) concludes.

2 Cohort, year and age effects: identification, fragile inference and normalization

Cohort, calender year and age effects are not identified from panel data. An identification problem arises due to the equality: year of birth + age = calender year. In this paper we follow the recent literature and only consider the case where cohort, time and age effects are additive. We rule out interactions between age, time and cohort effects a priori. Studying interactions between age, period and cohort effects is perhaps an interesting avenue for further research.

The simplest additive model is the linear model:

$$y_{it} = \phi + \gamma t + \beta a_{it} + \delta c_i + \varepsilon_{it} \tag{1}$$

where t is time/year, a_{it} is the age of individual i at time t, and c_i is year of birth of individual i. Due to the exact collinearity between these three variables, the slope parameters γ, β and δ are not separately identified. By excluding either t, a_{it}, OR c_i from the regression equation we reach identification. The remaining parameters can be seen as composite parameters (we exclude time t here for illustrative purposes, but one could also exclude any of the other two terms without loss of generality):

$$\begin{aligned} y_{it} &= \phi + \gamma(c_i + a_{it}) + \beta a_{it} + \delta c_i + \varepsilon_{it} \\ &= \phi + (\beta + \gamma) a_{it} + (\delta + \gamma) c_i + \varepsilon_{it} \end{aligned} \tag{2}$$

This normalized model is identified because a_{it} and c_i are not perfectly collinear if panel data is available. The regression results however should be interpreted with care, because of the composite nature of the parameters.

Another way of interpreting this model is that we have imposed the arbitrary assumption $\gamma = 0$ in (1). This interpretation is perhaps less desirable as there is no obvious justification for this assumption (which is effectively what makes the assumption arbitrary). Moreover, interpreting the parameters as composite parameters reminds the reader about the identification issue at hand.

The linear structure of (1) does not seem fully satisfactory, as there may be nonlinearities in the age, cohort or in the time profiles. One can use splines, use polynomials or use dummy variables to account for the nonlinearities. An additive model that fully accounts for the possible nonlinearities is the additive dummy variable model.

$$y_{it} = \phi + \sum_{\tau=2}^{T} \gamma_\tau D_\tau^T (t = \tau) + \sum_{\alpha=2}^{A} \beta_\alpha D_\alpha^A (a_{it} = \alpha) + \sum_{\kappa=2}^{C} \delta_\kappa D_\kappa^C (c_i = \kappa) + \varepsilon_{it} \tag{3}$$

where $D_\tau^T (t = \tau)$, $D_\alpha^A (a_{it} = \alpha)$ and $D_\kappa^C (c_i = \kappa)$ are time, age and cohort dummies respectively. The dummies are one if the argument between parentheses is true. The variables t, a_{it} and c_i are normalized so that they start at 1. That means for example that $a_{it} = 1$ is the youngest age group and $a_{it} = A$ is the oldest age group we consider.

Just as in model (1) the identification problem is also important in model (3). This is true because the linear model (1) is nested in (3). Consequently, estimating all the parameters of (3) is not possible. To reach identification one has to "break" the exact collinearity present in model (3). This can be done by restricting the parameters of model (3) such that the (unidentified) linear model (1) is no longer nested in the dummy variable model (3). This is the only strategy to reach complete identification of the profiles and it has been done many times in the literature. For example, Clark (2007) uses the British BHPS data where he uses FE to control for cohort effects, 5 year age blocks, and a full set of wave/time dummies (this in addition to a set of controls). Blanchflower and Oswald (2008) on the other hand model age in five year blocks (or in second degree polynomials), birth cohorts in ten year blocks and a full set of time dummies. By grouping individuals of different ages together in five year blocks Clark (2007) breaks the collinearity problem and reaches identification.[4] However, it is important to note that the model with five year blocks in age, cohorts and time is not identified, just like the fully flexible dummy variable model. Clark (2007) subsequently argues that the U shape in age is robust to controlling for cohort effects with FE.

Breaking the collinearity in model (3) by imposing arbitrary restrictions on the parameters however, may lead to very different substantive results [see also Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009)]. To see this, we adopt, more or less, the identification strategy of e.g., Clark (2007) (using the 5 year age blocks). Subsequently, we redo the analysis by using two year age blocks instead of the five year blocks. We show the results of the analysis in figure (1). The impact of such a seemingly minor change change in the identifying assumptions is striking. We confirm Clark (2007)'s result by using 5 year age blocks –a U shape in age–, yet, we obtain a completely different result if we use 2 year age blocks. Using the 2 year age blocks we find that life satisfaction is a decreasing function of age, apart for a slight recovery around retirement. Clearly, by using the same data, but by applying two slightly different identifying strategies, we could have been persuaded to draw two radically different conclusions.

From figure (1) we can conclude that different *arbitrary* assumptions lead to very different final outcomes. Obviously, if many different kinds of arbitrary assumptions produce very different conclusions one should start worrying about the relevance of such conclusions ["A fragile inference is not worth taking seriously (Leamer 1985)"].

The fragility of inference about age effects are is a direct consequence of the identification problem discussed earlier. The next section elaborates on how this works. Furthermore, we

[4]For exact identification one needs to restrict only one of the parameters in 3. If one restricts more than one parameter, like Clark (2007) and Blanchflower and Oswald (2008) do, the model is overidentified.

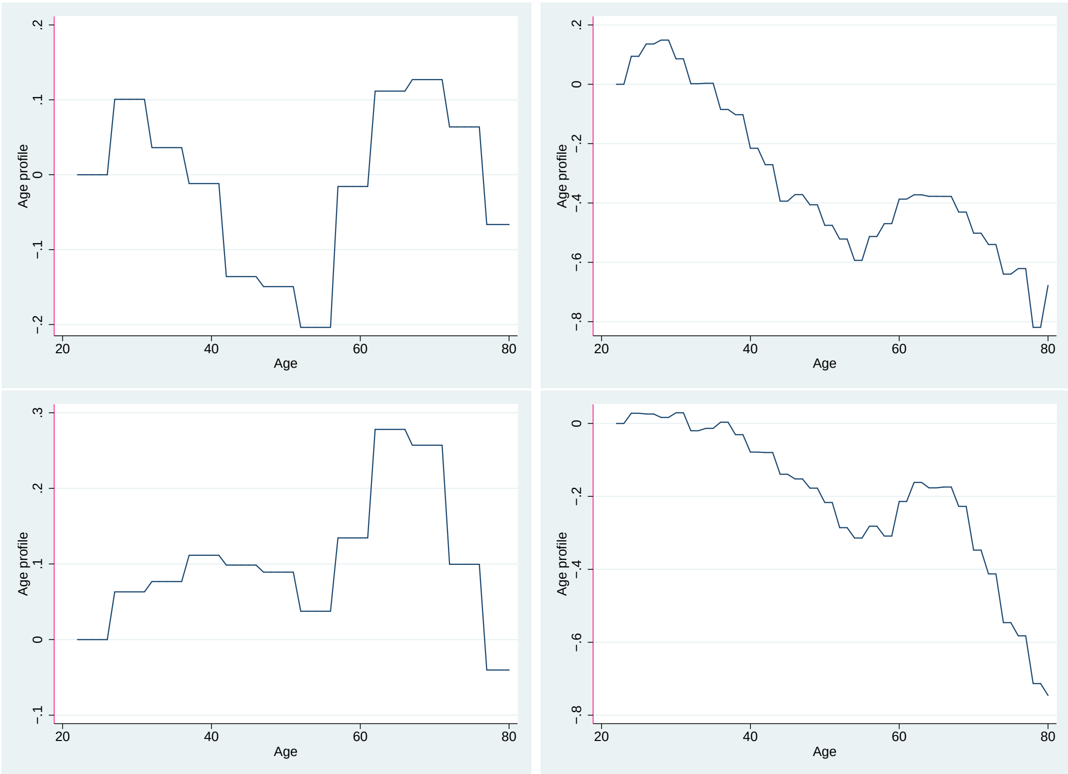

Figure 1: We use GSOEP data to construct the graphs. Further data description follows in section (4). TOP PANELS: The left panel replicates the results of Clark (2007) and uses five year age brackets, cohort dummies and a full set of time dummies. The right panel contrasts the results of Clark (2007) and includes two year age brackets, cohort dummies and a full set of time dummies.
BOTTOM PANELS: The left panel uses five year age brackets, FIXED EFFECTS and a full set of time dummies. The right panel includes two year age brackets, FIXED EFFECTS and a full set of time dummies.

draw on a result of Heckman and Robb (1985) who show that in the additive model, the nonlinear terms are identified, and only two out of three linear terms are identified (see also McKenzie (2006)). In other words, the age, time and cohort profiles are identified *in deviation from a linear trend*, that is in itself not identified. Due to the equality year of birth + age = calender year, only two of the three linear trends can be identified.

In this paper we report what *can* be identified from the data without relying on largely arbitrary restrictions. Our results put the often found U shape in age in perspective: the data is not really informative about whether people become more or less satisfied when they age.

3 Normalization and identification

Both the linear model, as well as the fully flexible dummy variable model are not identified from panel data. In both models, restrictions on (at least one of) the parameters must be imposed for identification. Restricting just one parameter associated with one of the dummy variables may have far reaching consequences for final outcomes (see also for example Wunder, Wiencierz, Schwarze, Küchenhoff, Kleyer, and Bleninger (2009) and figure (1) of the previous section). Instead, McKenzie (2006) shows that the "second differences" in (age, period, and cohort) effects can be identified "without any normalization restriction, providing information on the shape of the age, cohort and time effect profiles" [see also e.g., Heckman and Robb (1985) and Attanasio (1998)].

Complete knowledge of the second differences (or for the continuous case, the second derivatives) means complete knowledge of the *shape* of the age, cohort and time profiles *in deviation from* or perhaps better *orthogonal to* a linear trend. To see this we decompose, without loss of generality, the age profile $f(a)$ as a sum of a linear term βa and a nonlinear term $f_{\perp}(a)$ that is orthogonal to the trend βa and integrates to zero over the available ages in the sample[5]:

$$f(a) = \beta a + f_{\perp}(a) \tag{6}$$

Equation (6) implies:

$$f''(a) = f''_{\perp}(a) \tag{7}$$

This proves that the second derivative $f''_{\perp}(a)$ is identified if $f''(a)$ is identified. The fact however, that we have imposed that $f_{\perp}(a)$ is orthogonal to a linear trend in age and integrates to zero over all ages in the sample, implies that the identification of $f''_{\perp}(a)$ implies the identification of $f_{\perp}(a)$.[6]

[5] So:

$$\int_a f_{\perp}(a)\ a\, da = 0 \tag{4}$$

$$\int_a f_{\perp}(a)\ da = 0 \tag{5}$$

[6] PROOF: By taking the second derivative of $f_{\perp}(a)$ with respect to a one loses a constant and a linear term in a. Hence, to go from $f''_{\perp}(a)$ to $f_{\perp}(a)$ one needs information on this constant and the linear trend. The two

In other words, we can identify a whole lot about the shape of the age, period and cohort profiles without making any assumptions. The data is fully informative about the nonlinearities around the linear trends that are themselves not identified. This result has a few important implications. First, by analyzing the nonlinearities around the unknown linear trends in age, we can test whether life satisfaction is consistent with a U shape in age. A rejection of this test is clear evidence that the U shapes that are typically found are spurious, and consequences from imposing unwarranted identifying assumptions. Second, because the linear terms (even in a nonlinear model) are not identified, we can never be fully certain whether life satisfaction is upward or downward sloping or flat (except if we can somehow use additional identifying information). By concluding that life satisfaction is U shaped in age, therefore, one makes implicit assumptions about the linear trend in age. Any conclusion about the age profiles that has been reported so far, suffers from this problem.

For life satisfaction there seems no obvious (theoretical) justification for imposing such assumptions and one should be careful in making them. We advocate in this paper that a better way of proceeding is to write the additive dummy variable model in terms of composite parameters in some way, just like we did in equation (2).

In what follows we basically apply the same decomposition of the age (and time and cohort) profiles as in (6) to the dummy variable model (3). We first restrict the age, time and cohort profiles of model (3) in such a way that the coefficients with the dummy variables add up to zero over the available range and are orthogonal to a linear trend [see Deaton and Paxson (1994) and Attanasio (1998) who impose the same normalization, but only on the

restrictions on $f_{\perp}(a)$, $\int_a f_{\perp}(a)\, a da = 0$ and $\int_a f_{\perp}(a)\, da = 0$ provide the identifying information to derive the constant and the linear trend. For example, when $f_{\perp}''(a) = 6a$, $f_{\perp}(a) = a^3 + \overline{\beta}a + \overline{c}$. Because on the available domain $a = a_1, ..., a_A$, $f_{\perp}(a)$ should average out to zero AND is uncorrelated to a linear trend we generate two (identifying) restrictions on $f_{\perp}(a) = a^3 + \overline{\beta}a + \overline{c}$. Consequently, the parameters $\overline{\beta}$ and $\overline{c}$ can be derived. The argument is perfectly general and implies that if $f_{\perp}''(a)$ is known, $f_{\perp}(a)$ is as well.

time profile]. The restricted parameters are marked with a tilde ($\sim$).

$$\frac{1}{T}\sum_{t=1}^{T}\tilde{\gamma}_t = 0, \quad \frac{1}{T}\sum_{t=1}^{T}\tilde{\gamma}_t \times t = 0$$
$$\frac{1}{A}\sum_{t=1}^{A}\tilde{\beta}_a = 0, \quad \frac{1}{A}\sum_{a=1}^{A}\tilde{\beta}_a \times a = 0$$
$$\frac{1}{C}\sum_{t=1}^{C}\tilde{\delta}_c = 0, \quad \frac{1}{C}\sum_{c=1}^{C}\tilde{\delta}_c \times c = 0$$

The restrictions on the parameters can be imposed in estimation by normalizing the dummies as follows and include them, instead of the original dummies, in the regression model:

$$\begin{aligned}
\tilde{D}_{\tau}^{T}(t) &= D_{\tau}^{T}(t=\tau) + (\tau-2)\,D_1^T(t=1) - (\tau-1)\,D_2^T(t=2) \quad \text{for } \tau = 3,...,T \\
\tilde{D}_{\alpha}^{A}(a_{it}) &= D_{\alpha}^{A}(a_{it}=\alpha) + (\alpha-2)\,D_1^A(a_{it}=1) - (\alpha-1)\,D_2^A(a_{it}=2) \quad \text{for } \alpha = 3,...,A \\
\tilde{D}_{\kappa}^{C}(c_i) &= D_{\kappa}^{C}(c_i=\kappa) + (\kappa-2)\,D_1^C(c_i=1) - (\kappa-1)\,D_2^C(c_i=2) \quad \text{for } \kappa = 3,...,C
\end{aligned}$$

Subsequently we include the linear terms t, a_{it} and c_i as additional regressors.

$$y_{it} = \phi + \gamma t + \beta a_{it} + \delta c_i + \sum_{\tau=3}^{T}\tilde{\gamma}_{\tau}\tilde{D}_{\tau}^{T}(t) + \sum_{\alpha=3}^{A}\tilde{\beta}_{\alpha}\tilde{D}_{\alpha}^{A}(a_{it}) + \sum_{\kappa=3}^{C}\tilde{\delta}_{\kappa}\tilde{D}_{\kappa}^{C}(c_i) + \varepsilon_{it} \quad (8)$$

It is important to note that model (8) is not a restricted version of (3), but only a different parameterization of the dummy variable model (3). Otherwise, the two models are equivalent.

The profiles in deviation from the respective linear trends in age, time and cohort are fully identified, i.e., the $\tilde{\gamma}$, $\tilde{\beta}$ and $\tilde{\delta}$ parameters. The three linear trends however, are not separately identified. We normalize the underidentified regression model (8) by imposing the equality calender year (t) = year of birth (c_i) + age (a_{it}).

$$\begin{aligned}
y_{it} &= \phi + \gamma(c_i + a_{it}) + \beta a_{it} + \delta c_i + \sum_{\tau=3}^{T}\tilde{\gamma}_{\tau}\tilde{D}_{\tau}^{T}(t) + \sum_{\alpha=3}^{A}\tilde{\beta}_{\alpha}\tilde{D}_{\alpha}^{A}(a_{it}) + \sum_{\kappa=3}^{C}\tilde{\delta}_{\kappa}\tilde{D}_{\kappa}^{C}(c_i) + \varepsilon_{it} \\
&= \phi + (\beta+\gamma)\,a + (\delta+\gamma)\,c + \sum_{\tau=3}^{T}\tilde{\gamma}_{\tau}\tilde{D}_{\tau}^{T}(t) + \sum_{\alpha=3}^{A}\tilde{\beta}_{\alpha}\tilde{D}_{\alpha}^{A}(a_{it}) + \sum_{\kappa=3}^{C}\tilde{\delta}_{\kappa}\tilde{D}_{\kappa}^{C}(c_i) + \varepsilon_{it}
\end{aligned} \quad (9)$$

This specification illustrates the identification problem and offers guidance for interpreting the outcomes. More specifically, the trends we find should be interpreted as composite trends. In the remainder we estimate the parameters of this model and interpret the results. Subsequently, we estimate the same model but after including a set of controls.

4 Data and results

For estimating parameters we use data from the German Socio Economic Panel (henceforth, GSOEP). GSOEP collects data on mostly labor related issues. Moreover, it collects satisfaction data on a variety of topics. A unique feature of the data is the lang time span over which the data was collected, i.e., starting in 1984 in the former West German states, to today.

For the analysis a number of data selections has been applied. First, we only consider household heads and only the ones living in the former Western states of the country. Households in the former East are considerably poorer. The selection is made to maintain a degree of homogeneity in the sample after the German reunification in 1990. Second, we excluded households of which the head was younger than 22 (2764 households) years of age, or older than 80 (7211 households). The excluded groups were relatively small, producing noisy age effects associated for these groups. Third, households with very high income were excluded (236 households). Finally, the 1984 wave (cross sectional weights are zero) and 1985 wave (no information on marital status) were not used in the analysis. After applying these selections we have about 145,000 observations left.

Further data selections resulted from the fact that for some observations, life satisfaction (about 15,000 households) or any one of regressors (another 16,000 households) were missing. We also excluded these observations from the analysis. Furthermore, about 3,000 additional households were assigned a cross sectional weight of zero. These households were also effectively excluded from the analysis. The analysis is done on 110,719 observations.

We apply cross sectional weights reported by GSOEP in the analysis (the cross sectional weights are called *w11101*). This seems important because GSOEP in different periods of time oversampled certain groups (Haisken-DeNew and Frick 2005). For example, after 2002 high income groups were oversampled. Applying cross sectional weights accounts for this. For our purposes this is particularly important because the average age in the sample markedly increases over time if we would not apply the weights. This produces a correlation between age and time that normally should not be there, at least not to this extent.

The primary variable of interest is life satisfaction. Individuals were asked to rate their life satisfaction on a 10 point scale in response to the following question:

How happy are you at present with your life as a whole?[7]

The average household head rates his or her live with a 7. The standard deviation in the sample is 1.8 indicating quite some variability.

4.1 Empirical results

This section reports and interprets all the regression results. First we estimate the parameters of model 9. The estimated parameters with the linear trend in age and year of birth are reported in column 1 of table 1. The detrended age, cohort and time profiles are graphically presented in figure 2.

[Table 1 about here]

The detrended age profile is U shaped from age 30 to age 70 (with an additional dip at old age). Estimating the profiles relative to a linear trend does not suffer from the identification problem mentioned before. In other words, the second order derivatives of the profiles are identified and are positive over most of the life cycle. This finding contradicts the literature that finds an *inverted* U shape in age [e.g., Mroczek and Spiro (2005). Mroczek and Spiro (2005) however, use a different data set for their empirical analysis].

The detrended time profile has two peaks. The profile first peaks around 1990 right after the German reunification, and peaks a second time around 2000 at the end of the ICT bubble.

[7] Or in German: *Wie zufrieden sind Sie gegenwärtig, alles in allem, mit Ihrem Leben?* So, satisfied, "zufrieden" in German, is translated here by the term happy.

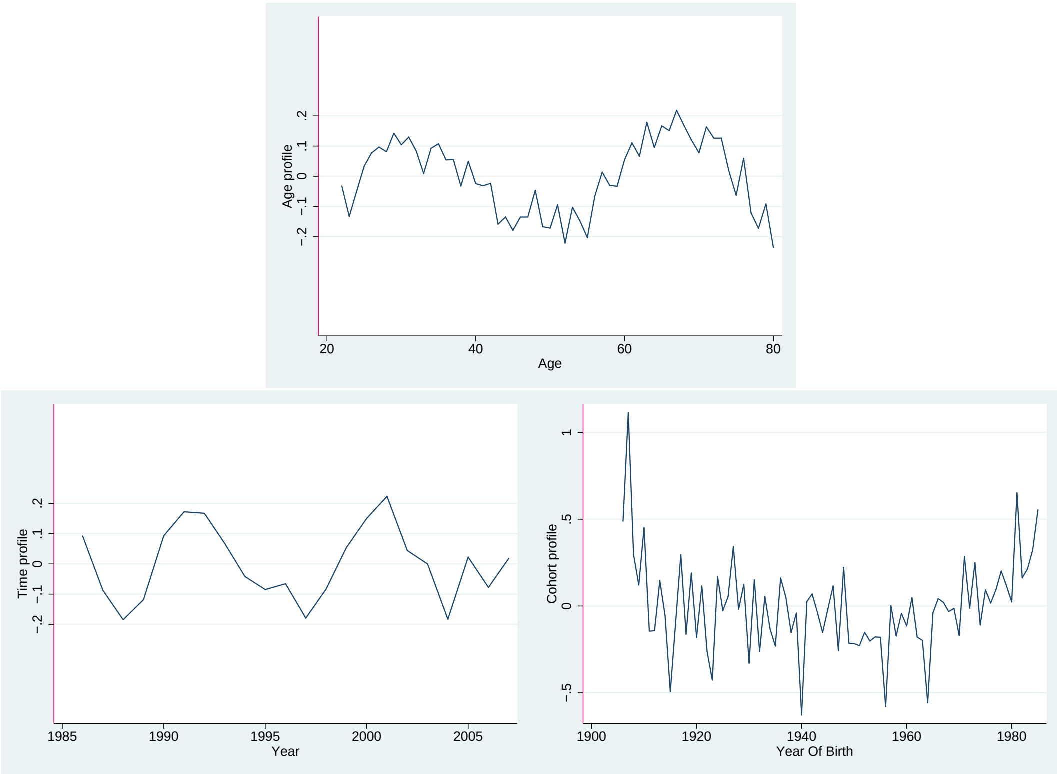

Figure 2: From left to right: age, time and cohort effects *relative* to a linear trend. Two of the three linear trends are included in the regression [see table (1)]

The detrended cohort profile is very spiky and does not show much of a clear pattern.

So far we find that the data is largely consistent with a U shape in age (at least from age 30 to age 70). This, however, does not imply that life satisfaction is U shaped in age. In column 1 we have included a linear trend in age and in year of birth. From equation (9) we know that the parameters with these trends should be interpreted as composite effects. The parameter with age should be interpreted as the sum of the linear effect in age and the linear effect in time. Its negative value, therefore, does not imply that people become less happy as they age, or that people have become less happy over time. Instead, it means that either people become less happy as they age, or that people have become less happy over time: at least one of these must be true! In addition, they may also both be true. Furthermore, the parameter with the variable year of birth is the sum of the cohort effect and the time effect:

$$\text{composite age - time effect: } \widehat{\beta + \gamma} = -0.021 \tag{10}$$

$$\text{composite cohort - time effect: } \widehat{\delta + \gamma} = -0.020 \tag{11}$$

Whereas the three trends cannot be separately identified, the above is already very informative. Apart from the nonlinearities involved, there is a negative trend in age OR in time, AND there exists a negative trend in year of birth OR in time.[8] Moreover, the respective estimates -0.021 and -0.020 are quantitatively meaningful.

How can we interpret the results so far? One way would be to suppose that age is truly U shaped in age over most of the life cycle. From the left panel of figure (2) we know already that the data is largely consistent with a U shape. By supposing that life satisfaction is truly U shaped in age we basically need to assume that β, the linear effect of age, is equal, or at least close to zero. Equation (10) and (11) show that imposing $\beta = 0$ has direct repercussions for the linear time trend and the linear trend in year of birth. Equation (10) implies that under $\beta = 0$ the -0.021 is a pure time effect. Over the sample period (1986 - 2007) we then estimate a drop of $21 \times 0.021 = 0.441$. Such a time effect is striking and, basically, reiterates part of the paradox Easterlin proposed (Easterlin 1974). It seems a priori strange that life satisfaction persistently decreases over time, despite significant improvements in income and health care for example. Nevertheless, this finding is a direct consequence of the supposition that there is no trend in age.

Furthermore, if $\beta = 0$ we find that $\gamma = -0.021$ and, consequently, that $\delta = 0.001$. This effect is small and basically means that if there is no trend in age, there is also no meaningful trend over cohorts. A δ of 0.001 means for example that the 1980 cohort is $60 \times 0.001 = 0.06$ points *more satisfied* with life than the 1920 cohort, a negligible effect. If this is true it means that whereas younger cohorts are significantly richer, healthier and better educated, they are not more satisfied with life. This is a priori hard to believe, but here also, it directly follows from the absence of a trend in age. All in all, we find that a U shape in age is supported by

[8] Equation (10) and (11) can be combined to get an estimate of the composite age $-$ cohort trend.

the data, but only under the untestable assumption that life satisfaction is decreasing over time and that there is no meaningful trend over cohorts.

The data however also supports many other hypotheses. Could it true for example, that life satisfaction is actually decreasing in age? In other words, does the data reasonably support the hypothesis that people lose good health and lose their youth more generally, and thus become less satisfied with their life as a whole when they age? To see what this implies for the time and cohort effects respectively suppose that β, the parameter measuring the linear age effect, is sufficiently negative. $\beta = -0.016$ would secure a decrease in age of a full point from age 20 to 80. Consequently, if $\beta = -0.016$ we must have that $\gamma = -0.005$. This constitutes to a unimportant negative trend over time (over the sample period life satisfaction has dropped by $21 \times -0.005 = 0.105$). Moreover, $\beta = -0.016$ implies a positive effect over generations. $\delta = 0.015$ means that the 1980 cohort is $60 \times 0.015 = 0.900$ points more satisfied with life than the 1920 cohort. So, younger cohorts are quite a lot more satisfied than the old ones.

Both possibilities/hypotheses described above seem not unlikely. To us however, the latter hypothesis is even more intuitive. More generally, we find that the data cannot discriminate between either one of the hypothesis, and we advocate that one should be careful in making an uninformed choice between either one of the two (or any other hypothesis that is supported by the data).

4.2 Robustness: beyond age, period and cohort effects

In the subsequent section we extend the baseline specification of column 1 by allowing for fixed individual effects in column 2 and column 4. Furthermore we test whether the detrended age, cohort and time profiles can be explained by including a set of possibly important regressors in column 3 and column 4.

One potential issue with interpreting satisfaction data is that every individual might have his or her own individual specific way of responding to such questions. This possibility makes it hard to directly compare satisfaction statements across individual households. We have performed a fixed effects regression on equation (9) and reported the results in column 2 of table (1). Because year of birth is constant over time we do not obtain estimates for the composite parameter associated with year of birth nor for the detrended cohort profile. The detrended profiles are presented in figure 3.

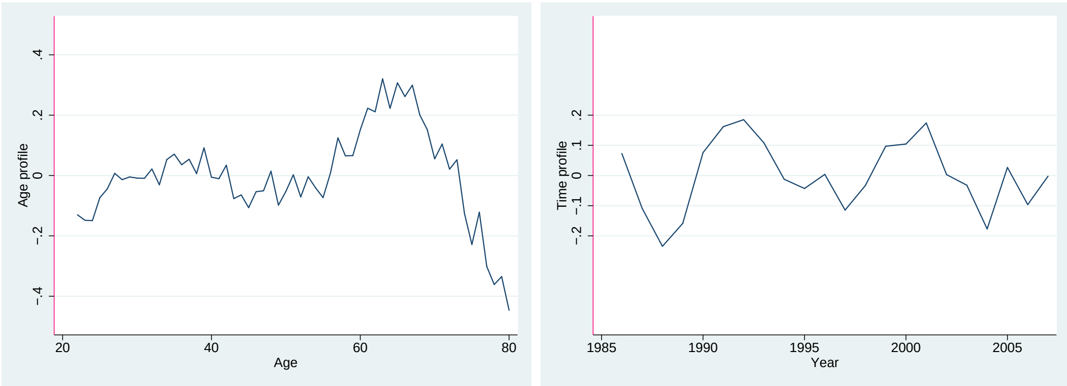

Figure 3: From left to right: age, time and cohort effects *relative* to a linear trend. These are the fixed effects results of column 2 in table (1). The cohort effects are not reported as they are picked up by the fixed effects.

The clearest difference between the column (1) - figure (2) results and the column (2) - figure (3) results is a more pronounced dip at old age. The left panel also seems to suggest that the U shape in the detrended age profiles disappears. This is not really true however. Because the profiles are constructed to be orthogonal to a trend the profile is a little flatter at the beginning, to account for the extra dip at old age. Still, the profile is consistent with a U shape over most of the life cycle. One can see this by adding a negative trend to the profile, which will "tilt" the figure clockwise. By doing so a U shape emerges from age 30 to 70. However, adding a negative trend will make the decrease in life satisfaction towards old age even more pronounced. The result is also reflected by the *more negative* composite age - time trend we find in column 2.

4.2.1 Can the profiles be explained by substantive variables?

So far, the analysis showed that life satisfaction data is consistent with a U shape in age, but that it is also consistent with an across-the-board decrease in age for example. This section studies what happens to the profiles, and the trends, if we include a set of regressors. The regressors we include measure income, family size (to proxy family life), marital (civil) status, and employment characteristics. The results are shown in column (3) - figure (4) and column

(4) - figure (5), where the column (4) - figure (5) results allow for fixed effects.

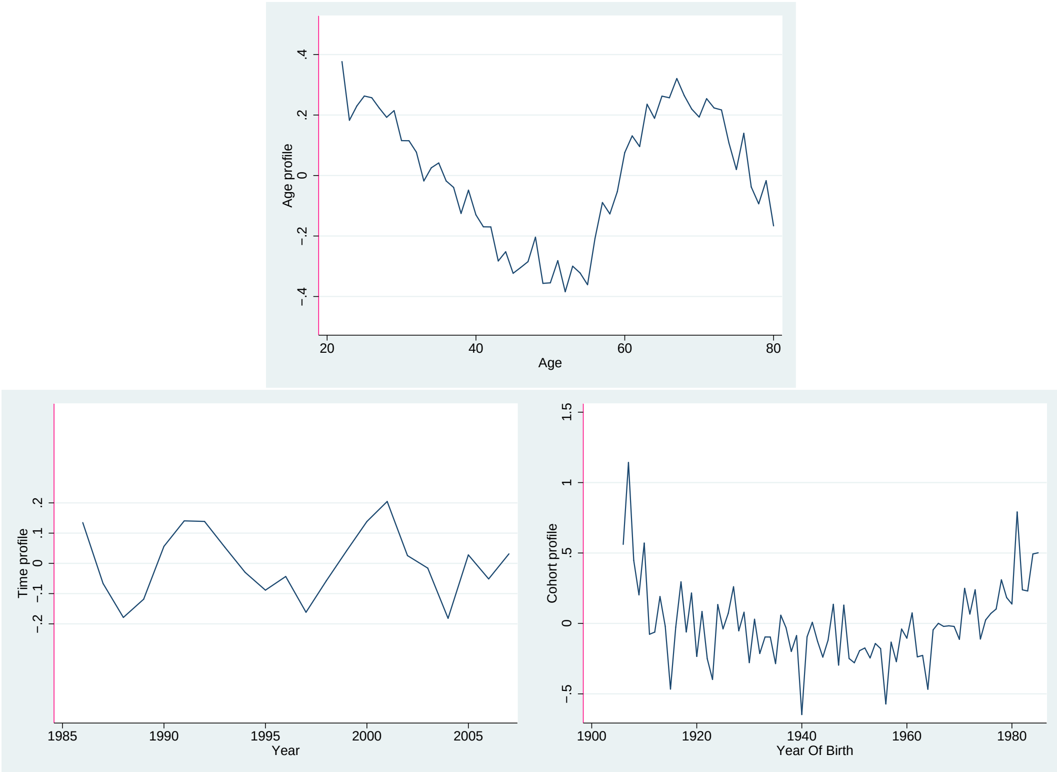

Figure 4: From left to right: age, time and cohort effects *relative* to a linear trend. These are the results of column (3) in table (1)

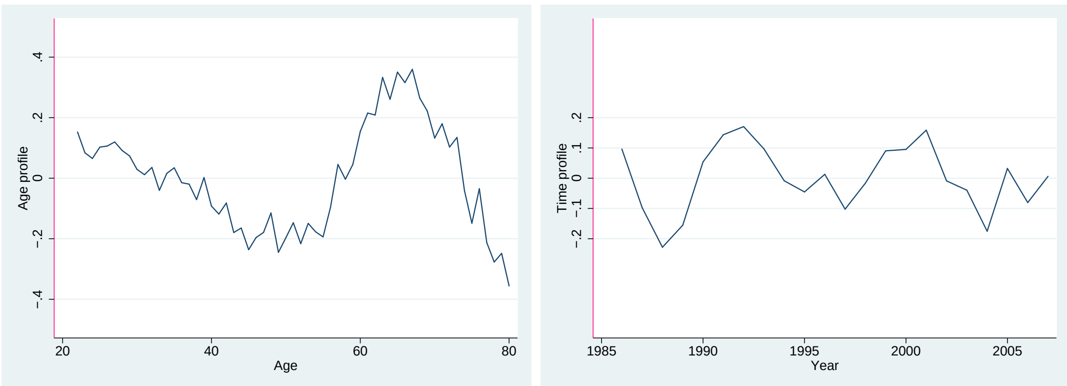

Figure 5: From left to right: age, time and cohort effects *relative* to a linear trend. These are the fixed effects results of column (4) in table (1). The cohort effects are not reported as they are picked up by the fixed effects.

The figures show clearly that the regressors cannot explain the U shape in the detrended age profiles. Effectively, the dip around middle age becomes even more pronounced. The estimates of the included regressors largely have the expected sign and (not surprisingly perhaps) explain more of the variation in life satisfaction than the age, time and cohort effects together. Still the R^2's of these regressions are low.[9] This however should be partly attributed to the noise levels in satisfaction data (or stated preferences data more generally) and is not fundamentally problematic. Finally, the estimated trend variables are largely invariant to including covariates, so the interpretation of the previous section still applies.

The estimated coefficients on substantive variables are very similar between the results in column 2 and the FE results in column 4. The fixed effects coefficients however are on the whole a little smaller. This similarity in results suggests that reporting behavior (as captured by the fixed effect) is not too correlated with personal characteristics, and suggests that at least on average, life satisfaction data can be meaningfully compared between individuals.

All in all, we find that there is a nonlinear pattern in life satisfaction over the life cycle that cannot be explained by the regressors we have included. This conclusion is a weaker than concluding that there is a U shape. There seems to be more to it than just income, family matters, or work. Or perhaps we should say that there is more to it than income, family matters, or work that is accounted for in this way. It is not unlikely income matters *more* in some phases of the life cycle than in others. Could it be that income is more important during midlife than after retirement? So, even if households are richer during midlife than after retirement, it might be that they derive less satisfaction out of it. Or otherwise, becoming widowed is bad when you are old, but it is arguably worse when you are young. The additive way in which we include the regressors our regressions is perhaps too restrictive as the feelings of satisfaction you extract from certain elements of life are likely to be context dependent. We could think of allowing the slope parameters to depend on age, in an attempt to get an idea in which phase in life certain elements of personal life are important. Clearly, however, the identification problem that is the topic of this research is also a problem there. In other words, the complete truth will not be revealed unless plausible assumptions can be imposed or when additional information is used.

[9] We have reported the overall R^2's for the pooled regressions in column 1 and 3. These two R^2's can be readily compared. In column 2 and 4, the fixed effects results, we have reported the R^2's of the within transformed model. These R^2's cannot be readily compared with the reported R^2's in column 1 and 3.

5 Conclusions

Age, period and cohort effects are not separately identified because of the equality year of birth + age = calender year. This paper criticizes a literature that claims to find a U shape of life satisfaction over age that cannot be identified without imposing arbitrary assumptions on the existence and the strength of linear trends in age, time and year of birth. In this paper we instead report what can be identified without making such arbitrary assumptions. Using data from the GSOEP we show that the data is indeed consistent with a U shape in age over most of the life cycle. However, we show that the data is also consistent with a general decrease in life satisfaction over the life cycle for example. The data does not bear any useful information to support one conclusion but not the other.

Following insights from e.g., Heckman and Robb (1985), Deaton and Paxson (1994), Attanasio (1998) and McKenzie (2006) we identify the age, period and cohort profiles that are *orthogonal to* a linear trend (in age, period or year of birth respectively). These profiles are identified with panel data and are already very informative. Although we cannot be sure whether life satisfaction is truly U shaped in age, we conclude that life satisfaction is certainly not flat or trending linearly over the course of life. This empirical fact cannot be explained by (additively) including regressors on income, family life, marital–, or employment status. This is interesting and proposes an interesting avenue for future research. It might be that the effects of certain covariates are age dependent. For example, becoming widowed is bad when you are old, but it is arguably worse when you are young. Studying the age dependence of the slope parameters however is not easy as suffers from the same identification problem that is the topic of this paper.

Another interesting avenue for further research is to search for clever ways to disentangle age, period and cohort effects completely.[10] For the moment however, we must be satisfied with just knowing that the development of life satisfaction over the life cycle is not linear.

[10]See e.g., Hall (1971) who studies the market for second hand trucks. Unlike with people, the same vintages (i.e., cohorts) of trucks are produced at different points in time. The age effect for example, can be identified by comparing price levels of the same vintage, at the same point in time, but with different ages. This feature allows Hall (1971) to completely identify age, period and vintage effects of the price of trucks.

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Table 1: TIME AGE AND COHORT EFFECTS ON LIFE SATISFACTION

	(1) life satisfaction	(2) life satisfaction	(3) life satisfaction	(4) life satisfaction
age	-0.021***	-0.033***	-0.017***	-0.033***
	(-9.769)	(-13.12)	(-7.033)	(-13.41)
year of birth	-0.020***	–	-0.022***	–
	(-9.510)	–	(-10.54)	–
ln real h.h. income			0.719***	0.438***
			(19.66)	(13.12)
2 person h.h.			-0.245***	-0.087*
			(-4.880)	(-1.748)
3 person h.h.			-0.321***	-0.120**
			(-5.257)	(-2.011)
4 person h.h.			-0.397***	-0.163**
			(-5.522)	(-2.232)
>4 person h.h.			-0.515***	-0.176*
			(-6.116)	(-1.831)
registered unemployed			-0.596***	-0.451***
			(-8.668)	(-7.470)
reg. part time empl.			0.364***	0.199***
			(7.206)	(4.494)
voc. training			0.263***	0.095*
			(4.089)	(1.837)
irreg. part time empl.			0.215	0.097
			(1.456)	(0.514)
not employed			0.213***	0.077
			(3.315)	(1.501)
married, separated			-0.899***	-0.642***
			(-10.91)	(-7.979)
single			-0.304***	-0.244***
			(-4.908)	(-4.081)
divorced			-0.487***	-0.229***
			(-7.109)	(-3.244)
widowed			-0.344***	-0.402***
			(-4.389)	(-3.584)
female			0.229***	–
			(4.878)	–
constant	47.332***	8.576***	44.640***	5.475***
	(11.21)	(69.11)	(11.05)	(21.06)
fixed effects	no	yes	no	yes
R^2	0.025	–	0.105	–
R^2 of within model	–	0.027	–	0.051
F tests (p-values)				
detrend. age dumm.	0.00	0.00	0.00	0.00
detrend. time dumm.	0.00	0.00	0.00	0.00
detrend. cohort dumm.	0.00	–	0.00	–
observations	110719	110719	110719	110719
number of individuals	14362	14362	14362	14362

NOTES. Models are estimated with OLS. Dependent variable in all regressions is life satisfaction. Robust-clustered (on person identifier) t-statistics in parentheses *** p<0.01, ** p<0.05, * p<0.10. baseline groups: full-time employed, married-living together, male. Estimates of the detrended time, age and cohort effects are not shown in the table [For these effects see figure (2) for column 1, figure (3) for column 2, figure (4) for column 3 and figure (5) for column 4].