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Quest for the VaR

Historical Bootstrap Method

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Quest for the VaR: Historical Bootstrap Method

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1 Introduction

There have always existed forces that the human mind cannot grasp. Where in ancient times societies named the cause of events that could not be fully explained "God(s)", nowadays those who are practicing econometrics like to refer to these forces as the DGP; the Data Generating Process. Modeling this DGP can cause many problems. Therefore, data behaviour has been labeled by econometricians. Some examples of labels include *dependence* or *spurious regression* and are argued for through the use of statistical evidence. This particular thesis will address a more practical problem which econometric theory is not very fond of: a small sample size.

The reason for conducting this research is that many companies are interested in the behaviour of portfolios. They are keen on getting insight into how their stocks and bonds will perform in the future. This is due to stricter control by the government which deemed this measure necessary because of the financial crisis that started in November 2008. The solvability requirement discussed in Solvency II for member countries of the European Union calls for tight capital and risk management within companies (More information about these guidelines can be found on the websites of the 'European Central Bank' and the 'De Nederlandse Bank').

Our goal is to obtain a distribution and Value-at-Risk (Jorion, 2006) of the behaviour of our future portfolio: in the medium- or long-run. This is not a straightforward task because of a few problems with our data. The first problem has to do with the fact that it is not possible to use very old data; economic shocks from 1934 cannot be used as the economy at that time is very different from the economy nowadays. One could argue that the underlying DGP has changed or that the data used in the DGP has evolved, and because of that the economic shocks are not representative. From this follows that the distribution of possible shocks at that time is not necessarily the same as how the shocks are absorbed in our current position. Therefore it is necessary to find a method that can provide us with enough plausible yearly or quarterly scenarios to find a distribution of the performance of our portfolio. The second problem is that our current sample does not consist of enough scenarios to be representative.

Therefore, we introduced a new tool to estimate both the distribution and Value-at-Risk of longer time periods in the future based on the bootstrap method (Efron, 1979) and Historical Simulation (Pritsker, 2006). We refer to this method as the Historical Bootstrap Simulation. The general idea of the new method is to use data from at least a period of one month to obtain medium/long-term shock scenarios. We have to weigh the consequences of choosing monthly data over weekly or daily data. Choosing smaller time intervals gives us a larger data set, however, modeling data of smaller time intervals is more complicated. There will be more temporary shocks and more dependence, whilst monthly data is expected to be on the verge of having no serial correlation, but will still provide us with a representable sample size.

To construct these quarterly or yearly shocks we will turn to the bootstrap method, originally designed by Efron (1979), who generalized the method from the so-called Jackknife method (Miller, 1974a, 1974b). The bootstrap method can be used to research a distribution or a statistic. Very briefly explained: by resampling historical data B times the bootstrap method tries to derive the underlying distribution of the data. Normally one takes a test statistic every time the data gets resampled and then researches its properties. Our method is interested in an unconventional statistic and a derived value (the VaR), therefore it pushes us away from the theoretical research that statisticians and econometricians are interested in.

The difference of the research done in this thesis and research done in most of the bootstrapping literature is the addition of an 'in between' step. We want to construct the scenarios from our data - constructing yearly or quarterly data from monthly data of risk drivers - before mapping the risk drivers onto the portfolio performance in $\mathbb{R}$. Our statistic will therefore not be derived from resampling actual historical scenarios, but from generated scenarios.

We will build our research as follows. First, we will introduce the setting and problems shortly in section

2, setting the necessary assumptions for our further work. In section 3 we will discuss the construction of yearly data assuming our sample of scenarios to be independent and identically distributed (IID). This section will also concern itself with the actual bootstrap statistic and its consistency; we will go deeper into the asymptotic properties when researching its validity. In section 4 we will lay out the methods of modelling the data; we will test for dependence structures and how to look at these in panel data sets. The methods will also be applied to our own data set. Section 5 we will research the method empirically and use a Monte Carlo process to generate our data. We will test the method and see how it works when we already have preknowledge about circumstances. In section 6 we apply the method to see the effects of varying lengths of the sample observations and see how the VaR reacts to these. Lastly follows a short conclusion.

2 Setting

This section will address the problems of our research and the assumpations that we have to make. The topic of this thesis is a new method called the Historical Bootstrap Simulation. Some of the problems that arise have been discussed in the introduction. The main problem is having a shortage of data and the reason for this can be summarized in the following assumption:

Assumption 1 Because of changes in the economy and hence in our DGP the most representative data for our current state is data retrieved from up to a number of years ago that we will call P. This gives us a set of $P \times 12$ monthly shock scenarios.

For further simplicity we will state a more general and stronger assumption. This will also be an important statement for our statistical research.

Assumption 2 The data from the last P years has been generated by the same DGP.

In our recent setting this gives us a data set that contains extreme data since the data of the past P years can include very negative shocks due to the recession that started at the end of 2008: when we are considering yearly data we will probably use a sample that includes these shocks. A nice side-effect of having a data set that includes large negative shocks will be that decisions made on this research will be resistant to the appearance of future large negative shocks.

The method we will research in this thesis is a variation of the bootstrap; we will use the bootstrap to construct a certain number of quarterly and yearly scenarios from monthly scenarios. A previously researched method of proceeding would be to use the Value-at-Risk (VaR) to calculate the distribution of monthly outcomes and bootstrap these values to obtain a distribution for longer periods. Research in this area has been done by for example van den Goorbergh et al. (1999). We will bootstrap the underlying risk drivers of the portfolio instead of the VaR. Risk drivers include the ECB-curve, swap-curve, inflation and volatility. Generally speaking, we can say that risk drivers are macroeconomic and financial indicators. The idea behind our method is that if we can simulate a part of the economy, we can relate this to the change in a portfolio. Hence, when bootstraping the economy we obtain one scenario for several portfolios. As we work with the driving forces behind the porfolio behaviour we can easily compare the results of different portfolios, whilst the use of the historical simulation already uses the products of the historical scenarios of the portfolios. What this research does is simply shifting the uncertainty from the shocks of a single portfolio to the shocks in risk drivers.

Another assumption has to be made about the relation between the economy and the performance of the

portfolio:

Assumption 3: The relation between the risk drivers and performance can be described as a function $Z(X)$ where X is the vector containing the shocks of the following period of time such that for $X^m \in \mathbb{R}^m$:

$$Z : X^m \to \mathbb{R}$$

Note that we do not state anything about the nature of the mapping $Z(X)$ other than its continuity. We also do not state that $Z(X)$ is the same function for shocks over periods of different lengths.

Assumptions 1, 2 and 3 will be used throughout the paper and we will not propose a setting in which one of these assumptions is violated.

3 Assuming IID monthly data

This section focuses on the construction, validation and comparison of our Historical Bootstrap Method. As our method is closely related to the bootstrap method, we will first explain the classical bootstrap and how it is used in conventional research. Secondly, we will lay out the fundamentals of the Historical Bootstrap Method. Thirdly, we will validate our method. Lastly, we will compare the method of Historical Simulation to our variation of the bootstrap.

3.1 The IID Bootstrap

The bootstrap method itself is a non-parametric tool to conduct inference. The idea is closely related to that of a Monte Carlo simulation, but where the Monte Carlo simulation generates its own data from a self-implemented function, the bootstrap does not generate its data and only uses a set of observations that are taken from real life.

The most basic and understandable variation in the bootstrap family is the IID bootstrap. This bootstrap variation owes its name to the requirement that the sample used for the bootstrap is independent and identically distributed. Hence, there is no sign of dependence of any kind, for example the ARIMA or GARCH structures. For the sake of simplicity we will first assume that we are dealing with a single timeseries. Therefore we do not have to worry about problems regarding Cross-Sectional Dependence (CSD). Note that the size of the sample is N. The method of this bootstrap is pseudo-coded in the following algorithm.

Algorithm 3.1: The IID Bootstrap

1. *Draw (with replacement) a new sample of size N from the original sample:* $X^* = \{x_i^* : i = 1, ..., n\}$.
2. *Calculate the statistic of interest* T^* *and store the value.*
3. *Repeat the previous steps B times.*

Horowitz (2001) describes the bootstrap procedure and its consistency very thoroughly. The rest of this subsection will both focus on the notation and the properties of the IID bootstrap procedure as these will be used throughout the rest of the thesis. Please note that this is the IID bootstrap method, we only explain the conventional research method in order for us to be able to compare the Historical Bootstrap Simulation to this one.

The function in which we are interested is F_0, which is the 'true' cumulative distribution function (CDF). From this CDF we observe our sample of size N. When we use the bootstrap we have to make sure that

we approximate F_0. Our original data will be the set called $X = \{x_i : i = 1, ..., n\}$. In every iteration of the bootstrap we first draw our bootstrap sample with replacement from the original sample, which will be denoted as X^*. We also note that F_0 is a member of a (in)finite-dimensional family of distribution functions called $\mathfrak{J}$. A general member of this family of distributions is called F. Most research focuses on a statistic called $T_n = T_n(x_1, ..., x_n)$, which is a function of the data. The next step is to construct the finite-sample CDF of T_n: $G_n(x, F_0) \equiv P(T_n \leq x)$. If this fuction $G(x, F)$ does not depend on F we speak of a pivotal statistic. Unfortunately, many econometric statistics and in particular the one discussed in this thesis are not of a pivotal nature. In this case the CDF $G(x, F)$ does depend on the distribution F. The bootstrap, however, provides us with a tool to approximate the finite sample distribution $G_n(x)$ by constructing G_∞ with $\hat{F}_n$. Estimation of $\hat{F}_n$ can be done in several ways: parametric and non-parametric. The latter one is only possible when pre-existing knowledge about parameters and distribution is present. The nonparametric bootstrap uses the empirical distribution function (EDF) which is defined as follows.

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(x_i \leq x) \tag{1}$$

With $I(x_i \leq x)$ being the indicator function equal to 1 when the statement is true and 0 otherwise. When we have estimated $\hat{F}_n$ we can calculate the bootstrap distribution $G_n^*(x, \hat{F}_n)$.

To prove the consistency of the bootstrap we have to proof F_n to be a consistent estimator of F_0;

$$F_n(x) \to F_0(x) \text{ almost surely or in probability} \tag{2}$$

Horowitz (2001) provides us with the following definition (relying on convergence in probability) for consistency of a bootstrap estimator:

Definition 3.1
Let P_n denote the joint probability distribution of the sample $\{X_i : i = 1, ..., n\}$. The bootstrap estimator $G(\cdot, F_n)$ is consistent if for each $\varepsilon > 0$ and $F_0 \in \mathfrak{J}$

$$\lim_{n \to \infty} = P_n[\sup_{\tau} |G_n(\tau, F_n) - G_\infty(\tau, F_0)| > \varepsilon] = 0 \tag{3}$$

To validate the bootstrap and the used statistic we can use the theorem of Mammen (1992). Which states sufficient and necessary conditions. His work on using the Central Limit Theorem (Casella et al., 2002) provides us with a theorem that can easily be checked. We do have to note that the necessary and sufficient conditions are only valid for estimating the distribution of a linear function of F_0 with F_n being the EDF.

Theorem 3.1 - Mammen (1992)
Let $\{X_i : 1, ..., n\}$ be a random sample from a population. For a sequence of functions g_n and sequences of numbers t_n and σ_n, define $\overline{g_n} = \frac{1}{n} \sum_{i=1}^{n} g_n(X_i)$ and $T_n = (\bar{g}_n - t_n)/\sigma_n$. For the bootstrap sample $\{X_i^* : i = 1, ..., n\}$, define $\bar{g}_n^* = \frac{1}{n} \sum_{i=1}^{n} g_n(X_i^*)$ and $T_n^* = (\bar{g}_n^* - \bar{g}_n)/\sigma_n$. Let $G_n(\tau) = P(T_n < \tau)$ and $G_n^*(\tau) = P^*(T_n^* < \tau)$, where $P*$ is the probability distribution induced by bootstrap sampling. Then $G_n^*(\cdot)$ consistently estimates G_n if and only if $T_n \xrightarrow{D} N(0, 1)$.

More general is the theorem of Beran and Ducharme (1991) which gives general conditions under which a bootstrap estimator is consistent.

Theorem 3.2 - Beran and Ducharme (1991)
Let ρ be a metric on the space $\mathfrak{J}$. Then $G(\cdot, F_n)$ is consistent if for any $\epsilon > 0$ and $F_0 \in \mathfrak{J}$:
(i) $\lim_{n\to\infty} P_n[\rho(F_n, F_0) > \epsilon] = 0$
(ii) $G_\infty(\tau, F)$ is a continuous function of τ for each $F \in \mathfrak{J}$
(iii) for any τ and any sequence $\{H_n\} \in \mathfrak{J}$ such that $\lim_{n\to\infty} \rho(H_n, F_0) = 0$, $G_n(\tau, H_n) \to G_\infty(\tau, F_0)$

If we turn to bootstrapping panel data and assume all observations are IID, it is trivial that the recently described algorithm will be a valid method to use. We do have to make a note of the difference of bootstrapping a time series and panel data literature. Bootstrapping panel data requires us to look at the data from another point of view. The N observations are now vectors. On the one hand we have cross-sectional resampling which does not take into account time but only resamples along the line of different individuals (Kapetanios, 2008). If one wishes to use the IID bootstrap, it is still possible under the condition that data does not have cross-sectional dependence. Temporal independence is not required as any dependence structure will be captured automatically in the vector. On the other hand, we can resample over the time dimension as the bootstrap draws different scenarios from the indexes representing time. This thesis will only discuss this last mentioned bootstrap variation. If we still want to use the IID bootstrap we have to make sure that there can only be cross-sectional dependence at time t. Cross-sectional dependence between a_{it} and a_{jt-1} with $i \neq j$ will make it necessary to capture the dependence structure in the bootstrap, hence making the IID bootstrap inadequate. Temporal dependence is not allowed as this cannot be captured in the vector.

A difficulty that arises when one works with resampling multivariate vectors is that dependence structures per individual might be different. Hence, correcting for a dependence structure might not be in the interest of the other individuals as for them the correction is not necessary.

3.2 The Historical Bootstrap Simulation

This section will be focused on explaining our new method: the Historical Bootstrap Simulation. When $X = \{x_i : i = 1, ..., n\}$ is a time series of which the observations are IID we can use the IID-bootstrap to construct quarterly and yearly data if we use the statistic $T = \sum_{i=1}^{k} x_i$ for values of $k = 3$ and $k = 12$ respectively in Algorithm 3.1. However, we are dealing with a panel set. This implies that in order for us to use the IID bootstrap algorithm, we cannot have temporal dependence in our panel and cross-sectional dependence is restricted to a relationship between x_{ri} and x_{ti} with r and t being different risk drivers at the same time i.

Our objective is to construct - through a bootstrap - a set $\{Y_j : j = 1, ..., B\}$ where Y_j is the vector containing yearly shocks at the j^{th} time we resampled. B is the number of times we resampled and is sufficiently large such that we obtain a CDF of the performance of our portfolio. We want to draw this set from smaller shocks from the set $X = \{x_i : i = 1, ..., n\}$ where $x_i \overset{IID}{\sim} G$ for all $i = 1, ..., n$. Please note that x_i is the vector of size m - the number of risk drivers - that describes all the shocks at time i. From this point on we will take the past 10 years as sample, which means that we will have a sample of 120 monthly observations.

The pseudo-code of our variation of the bootstrap can be found in the following algorithm:

Algorithm 3.2: New Bootstrap
1. Draw (with replacement) a new sample of vectors of size k: $X^* = \{x_i^* : i = 1, ..., k\}$.
2. Calculate the yearly scenario $Y^* = \sum_{i=1}^{k} x_i^*$ and store the value.

3. Repeat the previous steps B times.

4. Calculate the statistic that you are interested in from the set $Y^* = \{Y_j : j = 1, ..., B\}$.

Of course one could resample depending on the period k that one is interested in. If we were to be interested in quarterly data we could resample X^* of length $k = 3$. We could also use a set of longer periods (2-month or quarterly data) to construct scenarios of length k. From a programming point of view, resampling X^* larger than needed would be ineffective with respect to running times. Another important remark to make is that we should be aware not to choose k large. This is because of Assumption 2. We can only use data from up to P years ago - in our case 10 years - because the DGP before that time is assumed to have a different DGP. We can therefore argue that wanting to predict 30 years ahead would imply that the DGP over a period of 40 years would not change which violates our assumptions.

In comparison to the classic bootstrap algorithm (Algorithm 3.1) we can note a few things. First, the statistic T^* from Algorithm 3.1 is represented here as $Y_j^* = \sum_{i=1}^{k} x_i^*$. Secondly, we can state that just as in the classical method we are interested in the distribution of our statistic Y_j^*. Thirdly, we must add that the statistic we are interested in eventually, the VaR in our case, is not directly calculated from X but from the generated sample Y^*.

We are particularly interested in the Value-at-Risk, which is a percentile in the tail of our distribution. It is a number and it represents the probability $x\%$ that the outcome of the portfolio over a certain period is smaller than a certain number V (Jorion, 2006). The Value-at-Risk (VaR) is an often used statistic in the world of finance as this is a good representative of the state of the performance of a portfolio. One of the difficulties of estimating a VaR is that it is a quantile deep in the tails of the distribution and as observations are scarce (see Assumption 2), this is exactly why the Historical Bootstrap Simulation would be useful. We can generate as many yearly scenarios as we need to approximate the true distribution of Y. With large B our estimated VaR will be more accurate than the one taken only from the historical observations.

An important note to make about our sample is that we only have a panel set of size $n = 120$ along the time dimension. This is quite a small sample. Hence, we first can state to note that our asymptotic accuracy will not be of a very high standard. Our x_i is drawn from a monthly multivariate distribution, therefore we have 120^{12} combinations when we work with monthly data and want to construct yearly data. If we find evidence that we have a dependence problem in our data we could use 2-monthly or 3-month scenarios provided that they are independent, or insignificantly dependent. This will leave us with an even smaller sample size of either 60^6 or 40^4 scenarios. The rule here is that if we use observations of length l and we want to generate scenarios of length k we have $(\frac{120}{l})^{\frac{k}{l}}$ combinations.

3.3 Validation of the Historical Bootstrap Simulation

There are three steps to consider in our method which we will prove separately.

Validation steps

1. Y_j^* can be estimated from X_i^* and is drawn from the same distribution as Y_j
2. $G_n^*(Y_j^*)$ estimates $G_n(Y_j)$
3. $Z : Y_i \to \mathbb{R}$ is estimated correctly

As we have $n = 120$ months we have 12 yearly observations. These shocks can be constructed by $Y_j = \sum_{i=1}^{i+12} x_i$ with $i = 1, 13, ..., 109$. Generally speaking, a shock of period k can be constructed by $Y_j = \sum_{i=h}^{h+k} x_i$ for all $0 < h < h+k < n$. When h is chosen such that the constructed Y_j's are not overlapping we have $Y_j \overset{IID}{\sim} F$. Of course this could also be done for another set of IID scenarios over a longer period and

another target period.

In the IID case we have to proof the following, general theorem.

Theorem 3.2: Let $X^* = \{x_i^* : i = 1, ..., n\}$ be the bootstrapped sample. Let $Y^* = \sum_{i=h}^{h+k} x_i^*$ with $0 < h < h + k < n$. For every h, fixed k and corresponding Y from the original sample; $Y, Y^* \sim F$ hence are drawn from the same distribution.

Proof: As x_i is IID for all $i = 1, ..., n$, our sample is a random sample. Denote the MGF of x as $M_x(t)$, we know $M_{x_i}(t)^k = M_{Y_j}(t)$ (Casella et al., 2002).Therefore $Y_j = \sum_{i=k}^{h+k} x_i$ is a random variable for any $k > 0$. From the bootstrapped subsample X^*, Y_j^* can be constructed $Y^* = \sum_{i=h}^{h+k} x_i^*$ with $x_i^* \in X^*$. Now Y^* has the property: $M_{x_i^*}(t)^k = M_{Y^*}(t)$. As the Moment Generating Functions for Y and Y^* are equal we can state that these statistics have the same cumulative distribution function (Casella et al., 2002). ■

From a more practical point of view we can describe the situation as following. The theorem implies that at any time t our probability distribution of scenarios for time $t + k$ is the same; it is a random variable. Consider the following: $Y_1 = \sum_{i=1}^{12} x_i$. Hence, the shockscenario of the first year is simply the sum of shocks over the first twelve observations. Now also note that we can construct $Y_2 = \sum_{i=13}^{24} x_i$ and so on. We know that Y_1 and Y_2 are drawn from the same distribution because they have the same moment generating functions. Hence if we want to construct a path from our last moment in time $t = 121$ to $t = 132$ by using historical data from the set X, we can construct a set of possibilities $\{Y_i^* : i = 1, ..., B\}$ as we can draw (with replacement) X^* of size 12 and $X^* \subset X$. This will generate Y^*'s that are drawn from the same probability distribution as the Y's we have observed.

The second step would be to proof that the CDF of $\{Y_j^* : j = 1, ..., B\}$; $F_n(Y) \to F_0(Y)$ as $n \to \infty$. This can be confirmed by the Glivenko-Cantelli theorem (1933):

Theorem 3.3 Glivenko-Cantelli
Let $Y_1, ..., Y_B$ be iid random variables with marginal distribution $F : \mathbb{R} \to [0, 1]$. The definition of the empirical cumulative distribution function $\hat{F}_B : \mathbb{R} \to [0, 1]$ is given in equation (1). The empirical distribution converges uniformly to $F(x)$:

$$\sup_{x \in \mathbb{R}} |\hat{F}_B(Y) - F(Y)| \to 0 \text{ as } n \to \infty \tag{4}$$

Lastly, we have the third step which is the most easy step to validate. When we have a sequence and X_n converges in distribution to X then $H(X_n)$ converges to $H(X)$. In our case we just substitute $H(X)$ for $Z : X \to \mathbb{R}$. This only holds when the mapping $H(X)$ is continuous.

3.4 Comparison

A method exists in finance which is called 'Historical Simulation' (HS). Variations are the BRW (Boudoukh et al., 1998) and the 'Filtered Historical Simulation' (FHS). Both HS and FHS methods have been extensively researched by Pritsker (2006). These simulation methods are used to research the VaR. This is a popular tool for banks to measure risk of a portfolio (Pérignon et al, 2010). The measurement Value-at-Risk (Jorion, 2006) is the maximum expected loss of a portfolio dependending on two parameters: the length of the period and a confidence interval. With the method of Historical Simulation one can choose these parameters and

calculate the Value-at-Risk. The summary of all risk into one single number makes it readable for both the data analyst researching the problem and decision makers who are not involved in the research.

The HS takes the portfolio returns in percentages over a number of recent observations. The first parameter we have to decide on is the length of the period that our observations cover. HS focuses on short-term movements and movements of three years ago might not be as relevant as the ones from last week. Then we have to order the observations with respect to the outcomes. Now it is also possible to see where the lowest $x\%$ quantile lies. For example, if we have a hundred observations we know the 5% - 5th percentile - lies between the 5th and 6th lowest observations. If the quantile lies between two observations that lie in other intervals we can use interpolation to obtain the correct value.

Historical Simulation in Value-at-Risk analysis is used because it has the advantage that it is non-parametric. A disadvantage is that, because of the limited sample size and non-parametric nature the relation between the percentiles and the length of the period over which want to calculate. The VaR over are difficult to estimate (Hendricks, 1996). Two variations of the historical simulation approach are designed because there were problems with the standard Historical Simulation approach. Firstly, the HS method assumes IID-ness of observations. Secondly, a small sample makes it very difficult to estimate extreme percentiles of the distribution. The variations address the problems and adjust for the difficulties.

The first variation of the Historical Simulation method we have mentioned is the BRW method and argued for in a paper by Boudoukh et al. (1998). It explains that in addition to the classical bootstrap method, exponential smoothing is used to assign weights to the historical data. The weights are added to the standard HS method because the standard HS does not deal with changing volatility; every observation is assigned a probability of $\frac{1}{N}$. Exponential smoothing assigns higher probabilities to more recent observations than to older events. The so called 'hybrid approach' is described in the following algorithm:

Algorithm 3.3: Hybrid Algorithm

1. Let $R(t)$ denote the realized return from $t-1$ to t. The K most recent returns form the sample: $R = \{R(t-i) : i = 0, ..., K-1\}$ with assigned weights: $W = \{w_i = [\frac{1-\lambda}{1-\lambda^k}]\lambda^i : i = 0, ..., K-1\}\lambda^k$. Note that the weights add up to 1.

2. Order the weights in ascending order w_{ij} in which $j = 1, ..., K$ is added after ordering.

3. We want to obtain $x\%$ VaR hence we have to accumulate returns starting at the lowest return and keep adding until we have found the weight at point s such that $\sum_{j=0}^{s-1} w_{ij} < x$ and $\sum_{j=0}^{s} w_{ij} \leq x$. Then, if needed, by linear interpolation between $w_{i,s-1}$ and w_{is} we can obtain $x\%$ exactly from the distribution.

The other variation is the Filtered Historical Simulation. This variation uses the technique of the parametric bootstrap method (Davidson et al., 1997) which makes the FHS a semi-parametric method. By filtering the dependence from the data, theoretically we obtain ε_t's that are IID. Then by bootstrapping the residuals with the earlier discussed IID bootstrap method, we can rebuild paths from our current state with the estimated parameters and the resampled residuals. These paths then give us B (the times we use the bootstrap) generated observations of which we can take the $x\%$ VaR value. Barone-Adesi et al. (1999) deliberate on the FHS method when the GARCH dependence structure is used. A method of estimating the (p,q) in the GARCH(p,q) model is the quasi-maximum likelihood estimation. More about this method can be found in a paper by Bollerslev and Wooldridge (1992).

The first main difference between the Historical Simulation methods and our newly discussed method is that HS uses portfolio returns, while our method uses general underlying risk drivers and maps the change of the economy to the portfolio performance. A second difference is that most research in the HS literature is concerned with short-term data: daily and weekly data. Therefore variations such as the BRW and FHS

are constructed to adjust for dependence. Our method focuses on monthly data. This is mainly due to our interest in medium/long-term portfolio performance and the promise of a sample with data that leans towards IID-ness.

It is because of these reasons that we are not interested in the Historical Simulation on its own and have constructed a new method that uses both the Historical Simulation from the world of finance and bootstrap method from the world of statistics.

4 Modelling

This section will focus on the most common problem encountered when choosing a model and testing for a dataset consisting of risk drivers: IID-ness. Most tests are focused on testing for a kind of dependence structure. Research has been conducted to construct tests that will not imply dependence structures. Testing for independence can be done in different ways and research has been done by for example Diks (2001), but these tests use non-parametric econometric techniques that are quite complicated and have to be used carefully.

We want to focus on tests that can tell us whether we can use the Historical Bootstrap Simulation or not. As we are working with high-dimensional data, standard time-series models do not give us all the information we need and vector autoregression models are not practical. There are two dependence structures that are commonly used in modeling time-series of financial and macro-economic data: ARIMA(p,d,q) and GARCH(p,q). We will first discuss the types of dependence and how to test for them.

However, modeling is not our main focus. We want to find indicators that can tell us if our assumption of IID-ness in a panel data set is arguable. We are interested in tests that can help us find an answer to the question whether we can use the IID-bootstrap in a panel data set, even though there might be some evidence for dependence in a few of the individual risk drivers. The data set we are interested in consists of 102 observations per risk driver. Our risk drivers are the ECB interest rates, the Swap-rates, the volatility and the inflation-rates, giving us a panel set of size $N \times M$: 102x172. We will refer to this set as 'our set'.

4.1 The Results of Dependence

Implying an ARMA(p,q) structure has been a very popular modeling choice for econometricians. This modeling structure is extensively discussed in many school textbooks and a particularly clear explanation about the technical properties can be found in Davidson (2000). Quickly summarized: the structure takes into account an autoregressive part (AR(p)) stating a relation between Y_t and its predecessors. The moving average part (MA(q)) states a relationship between the previous error terms. An ARMA(p,q) model looks like:

$$Y_t = \alpha + \beta_0 Y_{t-1} + ... + \beta_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} \tag{5}$$

The theory behind an ARMA(p,q) structure is that after estimating p and q we obtain a model in which our ε_i's are independent and identically distributed. Of course this will rarely occur in practice, which is why we will only find the most optimal model. This can be done by using the Box-Jenkins method (1970). This method checks the extended ARIMA(p,d,q) model, where the I(d) stands for integration of order d.

First the Box-Jenkins (1970) method checks the order of integration d, which can be done by the (augmented) Dickey-Fuller (1979) test using the Pantula principle (1989). The method uses the autocorrelation and partial autocorrelation functions (ACF and PACF) to determine the range of the number of lags (p and

q) we have to include. We use these to determine a starting point for the range we should search for our optimal model.

Next we could use information criteria (IC) to determine the optimal model in our range. The most common criteria are the Akaike (1974), Schwarz (1978) and Hannan-Quinn (1979) criterion. Where n is the number of parameters in the model and $\hat{u}'\hat{u}$ is the minimized sum of squares.

$$\begin{aligned} AIC &= \ln(\frac{\hat{u}'\hat{u}}{n}) + \frac{2p}{n} \\ SBC &= \ln(\frac{\hat{u}'\hat{u}}{n}) + \frac{p \ln n}{n} \\ HQC &= \ln(\frac{\hat{u}'\hat{u}}{n}) + \frac{p \ln \ln n}{n} \end{aligned} \tag{6}$$

It is important to note that the latter two have been constructed with the assumption that the errors are Gaussian distributed, which is almost never the case, especially in medium- and long-term data. Therefore the first one, the Akaike criterion, will be a good alternative. The rule states to choose the model with the smallest value.

Generally speaking, the ARIMA(p,q) strives to model the relation as a function of the past. From (5) we can derive that $CORR(Y_t, Y_{t-1}) > 0$ if the data truly follows an ARMA(p,q) model. Put in other words: the correlation would be significantly different from 0: $\rho(Y_t, Y_{t-i}) \neq 0$.

Another commonly used structure we could test for is the ARCH(p,q) model- autoregressive conditional heteroskedasticity - or GARCH(p,q) - Generalized ARCH. This dependence structure is often used to deal with problems of volatility modeling in financial data. There are tests, amongst them the White-test (White, 1980), to determine the presence of heteroskedasticiy. This model accounts for a structure in the heteroskedasticity. The lag length of the AR part, q, can be found by testing as has been explained earlier. The GARCH(p,q) model is build as follows:

$$\begin{aligned} Y_t &= \beta_0 + \sum_{i=1}^{q} \beta_i Y_{t-i} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \gamma_1 \sigma_{t-1}^2 + \gamma_p \sigma_{t-p}^2 \end{aligned} \tag{7}$$

Needing $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \leq 0$. We observe that if the data would fit this model, correlation would be found in the residuals. For estimating q we can therefore use the Ljung-Box (1978) test which relies on the correlation:

$$Q = n(n+n) \sum_{k=1}^{h} \frac{\hat{\rho}^2}{n-k} \tag{8}$$

Where n is the number of observations, $\hat{\rho}_k^2$ is the estimated correlation at lag k. We also have h which is the total number of lags that is tested. This test is performed on the residuals of the fitted model. The degrees of freedom is h, but would be $h = m - p - q$ if we performed the test on the residuals of an ARIMA(p,d,q) model. Where m would be the original h from equation (8). Note that Q is rejected when it is larger than $\chi^2_{1-\alpha,h}$ with α as the significance level.

We have been discussing the methods we could use to check if there is dependence in our data and which kind of structure the dependence takes. Another, normally forgotten dependence structure we will test is Cross-Sectional Dependence (CSD), which is the dependence of two series $Y_u = \{Y_{ui} : i = 1, ..., n\}$ and

$Y_v = \{Y_{vi} : i = 1, ..., n\}$ to have a relation.

If we observe two events; Y_i and Y_j then these two events are independent if the following definition holds.

Definition 2.1 If Y_i and Y_j are two statistically independent events then:

$$P(x_i \cap x_j) = P(x_i)P(x_j) \tag{9}$$

Hence independence of two events states that the occurence of one event does not influence the occurence of another, and vice versa. A necessary - but not sufficient - condition for two random variables to be independent is given below:

Necessary Condition If X and Y are two independent random variables then the following holds:

$$CORR(X, Y) = \rho(X, Y) = 0 \tag{10}$$

Hence we can check for any X and Y whether they are significantly correlated or not. Our method allows correlation between two different risk drivers of the following form: $\rho(X_{rt}, X_{st}) \neq 0$, but not of the kind $\rho(X_{rt}, X_{s,t-i}) \neq 0$ for any $i = 1, .., n$.

A method to take into account cross-sectional dependence is the factor model approach. Peseran (2006) and Bai (2009) are amongst those whom have recently been working in this area. We will not go further into the literature but we just want to note that when it comes to bootstrap variations there are two approaches to adjust for CSD. The first one does not take any so-called 'common factor' into account but uses a blockbootstrap to capture the dependence (Palm, 2011). The second is more like the parametric bootstrap and estimates the factors of the factor model (Trapani, 2013).

4.2 Panel Data

Our main question is whether or not we can use the IID-bootstrap, even if there is some dependence in the data. A simple example: if 10 of our risk drivers show temporal correlation but 162 do not, then using another variation which accounts for dependence - such as the blockbootstrap - also has an effect on resampling the risk drivers that are IID. This would disrupt our attempt to capture the multivariate process in the non-parametric bootstrap. We will argue that using larger periods - of 2 months or quarterly data - could be a solution if they are shown to have no dependence structure. But the downside is that our already small sample will decrease in size. Hence, we would like to find the most convenient length of periods to divide our 10 years worth of data into in order to be able to both use the IID bootstrap and have a reasonable sample size.

As we have already shown some common methods to test for dependence(structures) we now want to show simplistic methods of testing for dependence(structures) in paneldata. We have to keep a few things in mind: (i) we are not interested in finding an optimal model. We merely want to find statistical evidence that there is need for a dependence structure. (ii) We are interested in the possibility of a dependence structure within one year. (iii) The choice of significance lies with the practitioner, he or she must decide what to allow in the data, because there exists a trade-off between less/no dependence against the number of available observations. (iv) Related to the previous point of making a trade-off is the function $Z(X)$. If there is dependence in a subset of risk drivers but the effect of the risk drivers does not have a large effect, one might choose to ignore the evidence for dependence because bootstrapping the other risk drivers correct might be more important.

The first thing we want to do is search for temporal dependence, hence creating an autocorrelation series. The results statistics of all the combined series are summarized in Table 1.

Table 1: Results ACF 1-Month

	$\alpha = 5\%$	$\alpha = 1\%$	Ratio $\alpha = 5\%$	Ratio $\alpha = 1\%$
1	21	12	0.123	0.07
2	104	89	0.608	0.521
3	18	4	0.105	0.023
4	0	0	0	0
5	45	26	0.263	0.152
6	64	18	0.374	0.105
7	3	0	0.018	0
8	1	0	0.006	0
9	1	0	0.006	0
10	0	0	0	0
11	2	0	0.012	0
12	6	0	0.035	0
Total	265	149		
Ratio	0.129%	0.072%		

What is remarkable is that the most persistent dependence can be found in data concerning inflation. We have $\rho(X_t, X_{t-1}) = 0$ but $\rho(X_t, X_{t-2}) \neq 0$ which explains a third of the significant observations when we test with $\alpha = 5\%$ and half of the significant observations when we test $\alpha = 1\%$. We also have to note that the ECB interest rates for longer periods show significant dependence between X_t and X_{t-6}. Because of the remarkable dependence one could wonder if the cause might be found in the way that the data is built: inflation for example is not an observable and therefore approximated by a CPI (Consumer Price Index). The cause of the significant correlation between X_t and X_{t-2} might be because of the method used to measure this CPI. The same might be said for the strange correlation found in ECB interest rates (periods for ≤ 7 years for $\alpha = 5\%$ and ≤ 10 for $\alpha = 1\%$). A remarkable fact about the found correlation is that it is not high: both problems addressed before have correlations in the interval $[-0.33, 0.33]$. It is up to the practitioner to choose to ignore or adjust the evidence for significant correlation or find other risk drivers. For example the practitionar can find another approximation for inflation that does not have this structural dependence problem.

An interesting comparison can be made if we take periods of two months. The statistics of the results are displayed in Table 2.

Table 2: Results ACF 2-Month

	$\alpha = 5\%$	$\alpha = 1\%$	Ratio $\alpha = 5\%$	Ratio $\alpha = 1\%$
1	10	2	0.058	0.012
2	0	0	0	0
3	60	21	0.351	0.123
4	0	0	0	0
5	2	0	0.012	0
6	1	0	0.006	0
Total	73	23		
Ratio	0.071%	0.022%		

If we want to check for dependence in a year we only have check the first 6 lags. We see that even with

$\alpha = 5\%$ we get reasonable results. From our correlation tests we see that the dependence can be found in the first lag of the inflation, which makes sense now we have clustered our data. But overall the table shows that the data of longer length has less evidence of dependence. Remarkable is that when we take 2-month shocks, the problem with significant second lags in the inflation risk driver vanishes almost completely. The significant dependence of the 3^{rd} (previously 6^{th}) lag in the ECB interest rate, however, persists.

Now we have already checked the autocorrelation function, which the Box-Jenkins method associates with 'moving average' components. The next step would be to look at the partial autocorrelation function. In the Box-Jenkins method this function is associated with the number of autoregressive components in the data.

Table 3: Results PACF 1-Month

	$\alpha = 5\%$	$\alpha = 1\%$	Ratio $\alpha = 5\%$	Ratio $\alpha = 1\%$
1	70	63	0.409	0.368
2	100	89	0.585	0.52
3	64	0	0.374	0
4	4	1	0.023	0.006
5	49	0	0.287	0
6	76	64	0.444	0.374
7	65	0	0.38	0
8	0	0	0	0
9	0	0	0	0
10	15	0	0.087	0
11	1	0	0.006	0
12	0	0	0	0
Total	444	217		
Ratio	0.216%	0.106%		

Table 4: Results PACF 2-Month

	$\alpha = 5\%$	$\alpha = 1\%$	Ratio $\alpha = 5\%$	Ratio $\alpha = 1\%$
1	7	5	0.041	0.029
2	0	0	0	0
3	26	0	0.152	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
Total	33	5		
Ratio	0.032%	0.005%		

If the so-called 'best choice' based on the results of the tests are still arguing that adjustments for dependence have to be made, we advise to look into the parametric bootstrap (Davidson et al., 1997). This method first estimates the parameters in the chosen, optimal model and then uses the IID-bootstrap to resample the residuals. We will not deliberate on this method any further.

5 A General Approach: Generated Data

As we are working with small samples, testing the algorithm is a very interesting part of our research. Before using our own data set in section 6, we will use the Monte Carlo Simulation (1949) to generate a random sample of monthly scenarios that is IID. From the generated sample we will construct the matching 2-monthly

data. We choose to generate our monthly data from a standard Gaussian distribution. Because the monthly data will be standard normally distributed, the derivation of the underlying distributions of other values is very straighforward. Appendix A shows the algorithm that we implemented.

We perform this test because in theory we should get the same results: the VaR value generated from the monthly and 2-monthly data should be the same as $B \to \infty$. But in practice the derived VaR might be different. We would like to get an indication to which extent this is the case.

To test our method we want to do the following:

1. We take our most recent state: $X(0)$, which is not the shock scenario but the actual values of our risk drivers. This is the 'starting point'.

2. From this recent state we construct a vector-Brownian-Motion with length T . So $T = 12$ for monthly data. We then know the end values of our risk drivers $X(T)$.

3. From the previous step we can state that we assumed that our shocks are Gaussian distributed. This is too strong an assumption for our case, but as was said earlier, is necessary in order to interpret our results better.

4. We construct the 2-monthly data from our monthly data by adding the corresponding shocks. We get $X_2(1) = X(1) + X(2)$, $X_2(2) = X(3) + X(4)$ etc.

5. The bootstrap is used to construct yearly scenarios Y and Y_2, where Y is constructed from monthly data and Y_2 from 2-monthly data.

6. We perform the historical simulation with a predefined uniform random vector α in order to make $Z(x)$ a linear function: $Z(x) = \alpha' \times (X(T) - X(0))$.

7. Out of every bootstrap we now get a single value as result, this result captures the performance of the portfolio. By ordering these from low to high, we obtain the Profit & Loss function.

8. We can now use the P&L function to calculate the VaR by taking the value such that $x\%$ is on the left side of the value. If the VaR-value lies between two values we use linear interpolation to obtain the actual VaR-value.

The theory states that using monthly and 2-monthly data should not result in different outcomes. In the construction of the 2-monthly data we add the corresponding shocks of the monthly data we obtain $X_2(1) = X(1) + X(2)$ etc. We therefore know that if our monthly shocks are standard Gaussian generated, our 2-monthly shocks are drawn from a $N(0, 2)$ distribution. The eventual yearly shocks will then be drawn from a $N(0, 12)$ distribution, whether you take twelve monthly or six 2-monthly shocks. When we use the linear function $Z(X)$ we do not touch the fact that the P&L value is normally distributed: the profits and losses are disributed as $N(0, 12 * \sum_{i=1}^{m} \alpha_i)$ for monthly and $N(0, 6 * \sum_{i=1}^{m} \alpha_i)$ for 2-monthly data, where m is the number of risk drivers that are involved.

We first take a small number of risk drivers. If we choose $m = 3$ and $\alpha = [1\ 1\ 1]$ we know that the values in the P&L function are distributed with $N(0, 36)$. Hence by standardizing we can retrace where our VaR-values are lying on the Gaussian curve and the percentage of the scenarios that lie on the left. We have made a table with the results of running the Monte Carlo 10 times and the bootstrap 2000. Also note that we are taking the VaR at 5%.

On average the results are quite promising. Because the VaR lies in the tail of the distribution, 2000 scenarios might not provide us with enough information to estimate the VaR accurately. We also run the algorithm with original $m = 172$ and a random α to compare the VaR-values of the monthly and 2-monthly datasets and behaviour when $B \to \infty$. This is the result of generating our data once and running the bootstrap 10.000 times:

Because the VaR is only a single value we cannot say anything more about testing whether the two values

Table 5: Monte-Carlo and HBS (B=2000, MC=10)

VaR-value		Standardized		Actual x%	
1-Month	2-Month	1-Month	2-Month	1-Month	2-Month
-12.0330	-13.6293	-2.01	-2.27	2.28	1.16
-9.6383	-9.3409	-1.61	-1.56	5.37	5.94
-10.0608	-8.9786	-1.68	-1.50	4.65	6.68
-12.8846	-13.6260	-2.15	-2.27	1.58	1.16
-10.1640	-9.0286	-1.69	-1.50	4.55	6.68
-6.6956	-6.3669	-1.12	-1.06	11.12	14.46
-10.4364	-10.3463	-1.74	-1.72	4.09	4.27
-12.2649	-12.7127	-2.04	-2.12	2.07	1.7
-9.1808	-11.2158	-1.53	-1.87	2.70	3.07
-9.4216	-9.3881	-1.57	-1.56	5.82	5.94

Table 6: Monte-Carlo and HBS (B=10.000, MC=1, RD=172)

Monthy VaR	- 67.750565
2-Monthly VaR	- 71.656422

are statistically the same. We can however see that the values are very close to each other which could be an indication that both VaRs are converging to the same value. This is promising hence we will run the algorithm again obtaining more values. We let our Monte Carlo Series generate a different sample every time and can then obtain the VaR-values for these. As the function $Z(X)$ is predetermined, the final P&L function would be existing of values that are drawn from the same distribution for every generated sample. We obtain a sample of VaR-values and could test for example if the sample mean for the monthly and 2-monthly data is the same.

To check for accuracy we run the algorithm with $m = 3$ and $\alpha = [1\ 1\ 1]$. Keep in mind that in real life we only have a single sample. We will not be able to test if the two VaR-values are the same for different samples with the same DGP. Hence, from the results we want to know if having a different sample will lead to different results and if this increase in bootstrap iterations has improved the workings a lot. The results are displayed in Table 7.

The results show us in the first two columns that the values lie close to each other. We have to keep in mind that differences further in the tails have less effect on the last two columns because of the shape of the normal distribution. When we look at averages the odd result is that from Table 5 the monthly and 2-monthly averages are 4.42% and 5.11% and from Table 7 we have 6.52% and 6.48% respectively. Hence, accuracy does not seem to be improving, but the averages of Table 7 do lie closer to each other.

In theory we can say that the averages converge to a value, this can be explained by the Law of Large Numbers (Casella et al., 2002). However, we saw that due to the small size of our sample, the accuracy of convergence is not very high, which is what we expected. We also noticed that the differences between working with monthly data and 2-monthly data are not very large. Hence, the averages of both monthly and 2-monthly data converge to the same value. Another remark: because we are working in the tail we recommend bootstrapping a large number of times. Normal combinations of Monte Carlo Simulations and bootstrapping would be 499x499 or 499x999, but in our case we would recommend nothing less than $B = 1000$. However, we saw that accuracy between $B = 2000$ and $B = 10.000$ does not improve very much.

Table 7: Monte-Carlo and HBS (B=10.000, MC=20)

VaR-value		Standardized		Actual x%	
1-Month	2-Month	1-Month	2-Month	1-Month	2-Month
-8.2150	-7.0897	-1.37	-1.18	8.53	11.90
-8.2731	-8.9359	-1.38	-1.49	8.38	6.81
-11.8920	-11.1268	-1.98	-1.85	2.39	3.22
-10.4468	-10.2313	-1.74	-1.71	4.09	4.37
-9.1638	-8.4839	-1.53	-1.41	6.06	7.93
-12.1330	-11.2671	-2.02	-1.88	2.17	3.01
-6.3813	-6.0261	-1.06	-1.00	14.46	15.87
-12.5788	-13.1798	-2.10	-2.20	1.79	1.39
-8.5094	-9.0989	-1.42	-1.52	7.78	6.43
-9.3989	-11.5949	-1.57	-1.93	5.82	2.68
-10.4822	-10.1205	-1.75	-1.69	4.01	4.55
-8.9478	-10.1437	-1.49	-1.69	6.81	4.55
-7.6206	-7.2762	-1.27	-1.21	10.03	11.31
-9.7787	-10.6905	-1.63	-1.78	5.16	3.75
-5.8460	-6.7931	-0.97	-1.13	16.60	12.92
-10.2629	-10.0300	-1.71	-1.67	4.36	4.75
-7.6044	-7.4654	-1.27	-1.24	10.03	10.75
-8.8245	-9.0524	-1.47	-1.51	7.08	6.55
-11.7947	-10.3094	-1.97	-1.72	2.44	4.27
-10.9461	-11.6061	-1.82	-1.93	3.44	2.68

6 Empirical Work: Historical Data

Next we will perform our Historical Bootstrap Simulation on historical data to research the trade-off between the presence of some depedency in a dataset and the reduction of the sample size. The latter one caused by increasing the lengths of the observations. The program can be found in Appendix B.

We test for data covering one and two months. Our goal is to construct yearly data. The algorithm works as follows:

1. The monthly and 2-monthly data is read in.

2. After the alpha-vector is constructed. We do not want this vector to vary in the rest of the alogrithm.

3. There are four loops in the algoritm; the two inner loops produce the yearly shock scenarios for each risk driver. The first does so for the monthly data, the second one for the 2-monthly data.

4. The middle-loop is constructed around the two inner loops. We construct R and $R1$ - the random vectors containing the indices of observations that we add to obtain a yearly scenario - x times in this middle-loop such that we get a vector containing the numbers of the scenarios that form a random year.

5. After we have run through the middle-loop we construct the VaR-values separately for monthly and 2-monthly data. This happens while still in the outer-loop.

6. We have programmed it in such a way that it would be possible to run the whole construction of the VaR-values more times. The program first asks how many times one wants to run the whole program and bootstrap.

7. The outer-loop controls how many times the whole program is ran by. All the VaR-values are stored and can be used later on.

Next we run the algorithm. We are interested in a few aspects. Note that we will not be able to say something about the distribution as it is not previously known. The DGP that is the underlying distribu-

tion might be a very difficult distribution that can be estimated by (non-)parametric econometric methods. However, we are not interested in the distribution as our method is constructed such that it does not rely on it directly.

We run the algorithm twice, first with $B = 2000$ and secondly with $B = 5000$. We run the entire bootstrap 20 times for each B with α as a vector of ones such that we compare the results. The results are displayed in Table 8. First we notice the VaR-values of the 2-monthly data to be higher regardless of our choice for B. As we could see in section 4, 2-monthly data was found to have less evidence for containing a dependence structure, we would therefore say that the higher VaR-value would be more reliable. The difference between $B = 2000$ and $B = 5000$ is the variance of the results. As we have already pointed out, we cannot say anything about the correctness of our output, we only see that the variance of our output decreases: more iterations in the bootstrap create a P&L curve with higher density, this gives us more information in the tails and more accurate VaR-values.

Table 8: HBS

	B=2000		B=5000	
	1-Month	2-Month	1-Month	2-Month
	-130.92	-77.50	-129.39	-78.47
	-136.57	-78.01	-131.96	-81.15
	-125.86	-76.49	-126.73	-78.09
	-128.99	-77.53	-128.97	-78.64
	-131.51	-78.21	-127.58	-76.74
	-121.11	-77.07	-126.91	-76.88
	-123.59	-76.38	-127.98	-77.40
	-128.66	-79.99	-126.68	-82.37
	-123.73	-83.24	-126.07	-76.74
	-129.59	-79.42	-129.23	-76.98
	-129.12	-79.16	-123.93	-78.85
	-133.17	-79.15	-125.30	-77.39
	-121.78	-75.47	-130.07	-78.12
	-130.71	-75.04	-131.54	-77.16
	-136.13	-79.77	-125.79	-80.53
	-131.13	-80.14	-132.08	-79.80
	-128.60	-78.25	-130.02	-78.40
	-130.05	-74.06	-130.17	-78.85
	-128.34	-77.10	-130.44	-80.66
	-131.38	-80.89	-126.27	-79.72
mean	-129.05	-78.14	-128.47	-78.59
variance	17.48	4.72	5.63	2.69

We run the algorithm with $B = 10.000$ (Table 9). We see that even though we have 10.000 values on the P&L-curve we cannot say that the VaR-values come close. If we compare our results to Table 8 we conclude that there is no noteworthy difference between the mean of our output and the results in Table 9. The results of Table 9 are not even a standard deviation away from the mean in Table 8.

Table 9: HBS (B=10.000)

VaR	-131.17926
VaR1	-77.674224

We can conclude from this that there is indeed a difference between deriving the VaR-value from monthly and 2-monthly data. Note that the difference in output depends on the values in α: high values might be assigned to risk drivers with little evidence for dependence and low values might be assigned to those with a much evidence for dependence, this will lead to a smaller gap between the VaR values. Therefore, it is important for the practitioner to know the function $Z(X)$ before proceeding with the implementation of the algorithm.

7 Conclusion

This section concludes the thesis. Firstly, we will summarize the results of this thesis. Secondly, we will propose topics for further research.

We have started with stating the assumptions upon which we build our reasearch in section 2. Following this we have explained and explored our method in the section 3. In this section we have also related our method - the Historical Bootstrap Simulation - to existing research. We have explained the IID-bootstrap variation and addressed its properties. However we also stated that this variation and other bootstrap variations are mostly used to conduct inference. The Historical Bootstrap Simulation differs from this because we are not interested in inference but rather in constructing shock scenarios of medium- and long-term length. This makes our statistic a very unconventional one. We also looked at the historical simulation method and a addressed a few variations. Here we found that this method is used for short-term data, therefore the research in that area and the variations that have been constructed are not of particular interest in this thesis.

From section 4 onwards we have looked at the empirical part of our method. First, we made a short overview of the most commonly used dependence structures and how we could find evidence that our data should be modelled and cannot be assumed IID. Because in practice we do not work with a time-series but with a panel-data set we have looked at evidence for dependence in a structured way. In section 5 we have programmed the Historical Bootstrap Simulation with a Monte Carlo series to generate the data such that we have preknowledge about the DGP. Therefore we could see how the algorithm performes in small samples. By generating a panel set with only 3 risk drivers we could conclude that we must need a large B in order to obtain enough information in the tails that we are interested in. The last section is dedicated to running the Historical Bootstrap Simulation on real data. We found that a higher B will provide us with more accurate outcomes, but we cannot make statements about the correctness of the VaR-values. Also, we can see that if we have a very high B the results are not converging, while from section 5 we could see that in the case that both mothly and 2-monthly data are independent, convergence of the results should be the case.

We can say that there are enough topics in this field that are interesting for research. For example, one could look at a way to implement the parametric bootstrap, which could be a useful tool if the data set shows a lot of evidence for dependence. Another topic would be linking other non-parametric and already existing bootstrap methods to this research in the case that the data set shows evidence for dependence. Also, in our empirical part we have not compared the output of the Historical Bootstrap Simulation to other existing methods. Finally one could conduct empirical research on the Historical Bootstrap Method where the function $Z(X)$ is non-linear.

8 Appendix A

The Historical Bootstrap Simulation with Monte Carlo
Code: Scilab

```
   clear()
x = input("How many times do you want to simulate in the Monte Carlo Simulation?")
y = input("How many times do you want to simulate in the Bootstrap?")

rd = 172
alpha = rand(rd, 1)
total_PL = []
total_PL1 = []
VaR = rand(x,1)
VaR1 = rand(x,1)

for v = 1:x
F = rand(120, rd, 'normal')
P = []
N = []

F1 = []
P1 = []
N1 = []

for w = 1:60
    for z = 1:rd
    t = 2*w-1
    s = 2*w
    F1(w,z) = F(t,z)+F(s,z)
    end
end

for i=1:y
    M = []
    T = []
    M1 = []
    T1 = []
    R = round(rand(12,1)*119+1)
    R1 = round(rand(6,1)*59+1)

        for j = 1:12
            A = F(R(j),1:rd)
            M = [M; A]
            T = sum(M,1)
        end
```

```
        for z = 1:6
            A1 = F1(R1(z),1:rd)
            M1 = [M1; A1]
            T1 = sum(M1,1)
        end

        N = [N; T]
        N1 = [N1; T1]
end

PL_values_now = N*alpha
PL_sort = gsort(PL_values_now)

PL_values_now1 = N1*alpha
PL_sort1 = gsort(PL_values_now1)

percentage = 95/100*y
p1 = floor(percentage)
p2 = ceil(percentage)
p_middle = percentage-p1

if p1==percentage then
    VaR = PL_sort(percentage)
    VaR1 = PL_sort1(percentage)
end

if p1~=percentage then
    VaR = PL_sort(p2)+(p_middle*(PL_sort(p1)-PL_sort(p2)))
    VaR1 = PL_sort1(p2)+(p_middle*(PL_sort1(p1)-PL_sort1(p2)))
end

total_PL =[total_PL; PL_values_now]
total_PL1 =[total_PL1; PL_values_now1]
end
```

---

9 Appendix B

The Historical Bootstrap Simulation
Code: Scilab

```
   clear()
ECB=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RD1.csv',";",",")
SWAP=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RD2.csv',";",",")
INF=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RD3.csv',";",",")
VOL=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RD4.csv',";",",")
SPR=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RD5.csv',";",",")

ECB1=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RDA1.csv',";",",")
SWAP1=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RDA2.csv',";",",")
INF1=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RDA3.csv',";",",")
VOL1=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RDA4.csv',";",",")
SPR1=read_csv('C:\Users\Kaya\Documents\ProjectBootstrap\RDA5.csv',";",",")

x = input("How many times do you want to simulate the bootstrap: B?")
y = input("How many times do you want to run the entire bootstrap?")

F = strtod(ECB)
G = strtod(SWAP)
H = strtod(INF)
I = strtod(VOL)
J = strtod(SPR)
F1 = strtod(ECB1)
G1 = strtod(SWAP1)
H1 = strtod(INF1)
I1 = strtod(VOL1)
J1 = strtod(SPR1)

F = F($-101:$,1:30)
G = G($-101:$,1:30)
H = H($-101:$,1:100)
I = I($-101:$,1:11)
J = J($-101:$,1:1)
F1 = F1($-50:$,1:30)
G1 = G1($-50:$,1:30)
H1 = H1($-50:$,1:100)
I1 = I1($-50:$,1:11)
J1 = J1($-50:$,1:1)

TotalVaR = []
TotalVaR1 = []
```

```
alpha = rand(172, 1, 'uniform')

for q = 1:y
N = []
N1 = []
total_PL = []
total_PL1 = []

for i = 1:x
    M = []
    T = []
    M1 = []
    T1 = []
    R = round (rand(12,1)*101+1)
    R1 = round (rand(12,1)*50+1)

        for j = 1:12
            A = F(R(j),1:30)
            B = G(R(j),1:30)
            C = H(R(j),1:100)
            D = I(R(j),1:11)
            E = J(R(j),1)
            M = [M; A B C D E]
            T = sum(M,1)
        end
        for z = 1:6
            A1 = F1(R1(z),1:30)
            B1 = G1(R1(z),1:30)
            C1 = H1(R1(z),1:100)
            D1 = I1(R1(z),1:11)
            E1 = J1(R1(z),1)
            M1 = [M1; A1 B1 C1 D1 E1]
            T1 = sum(M1,1)
        end

        N = [N; T]
        N1 = [N1; T1]
end

PL_values_now = N*alpha
PL_sort = gsort(PL_values_now)

PL_values_now1 = N1*alpha
PL_sort1 = gsort(PL_values_now1)
```

```
percentage = 95/100*x
p1 = floor(percentage)
p2 = ceil(percentage)
p_middle = percentage-p1

if p1==percentage then
    VaR = PL_sort(percentage)
    VaR1 = PL_sort1(percentage)
end

if p1~=percentage then
    VaR = PL_sort(p2)+(p_middle*(PL_sort(p1)-PL_sort(p2)))
    VaR1 = PL_sort1(p2)+(p_middle*(PL_sort1(p1)-PL_sort1(p2)))
end

TotalVaR = [TotalVaR; VaR]
TotalVaR1 = [TotalVaR1; VaR1]
end
```

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