Network for Studies on Pensions, Aging and Retirement

NETSPAR ACADEMIC SERIES

The Liquidity Risk Premium Demanded by Large Investors

Dynamic Portfolio Choice with Stochastic Illiquidity

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DP 12/2015-049

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December 21, 2015

Abstract

Recent empirical work documents large liquidity risk premiums in stock markets. We calculate the liquidity risk premiums demanded by large investors by solving a dynamic portfolio choice problem with stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set. We find that, even with high trading-cost rates and substantial trading motives, the theoretically demanded liquidity risk premium is negligible, less than 3 basis points per year. Assuming forced selling during market downturn enlarges the liquidity risk premium to maximally 20 basis points per year, which is well below existing empirical estimates of the liquidity risk premium.

*We thank Lieven Baele, Itzhak Ben-David, Servaas van Bilsen, Jules van Binsbergen, Fabio Braggion, Nicolae Gârleanu, Vincent Glode, Sebastian Gryglewicz, Jennifer Huang, Frank de Jong, Alberto Manconi, Patrick Tuijp, Patrick Verwijmeren, Bas Werker, Xiaoyan Zhang, and seminar participants at Tilburg University, Netspar's Liquidity Risk Workshop and Netspar Pension Day for helpful comments and discussions. All remaining errors are of course our responsibility. Driessen and Xing are from Finance Department of Tilburg University. Emails: J.J.A.G.Driessen@uvt.nl, r.xing_1@uvt.nl.

The Liquidity Risk Premium Demanded by Large Investors: Dynamic Portfolio Choice with Stochastic Illiquidity

Recent empirical work documents large liquidity risk premiums in stock markets. We calculate the liquidity risk premiums demanded by large investors by solving a dynamic portfolio choice problem with stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set. We find that, even with high trading-cost rates and substantial trading motives, the theoretically demanded liquidity risk premium is negligible, less than 3 basis points per year. Assuming forced selling during market downturn enlarges the liquidity risk premium to maximally 20 basis points per year, which is well below existing empirical estimates of the liquidity risk premium.

JEL classification:

Keywords: Liquidity premium; Liquidity risk; Dynamic portfolio choice; Trading Costs; Price impact.

1 Introduction

Over the last 30 years, a growing literature has empirically analyzed the effect of illiquidity on asset prices. Recently, empirical work has focused in particular on the liquidity risk premium, which is a compensation for exposure to systematic liquidity shocks. Several articles document substantial liquidity risk premiums in realized returns (for example Pastor and Stambaugh (2003)), while other work finds that is difficult to disentangle the liquidity risk premium from the direct effect of transaction costs on prices, sometimes called the liquidity level premium (Acharya and Pedersen (2005)). In addition, the liquidity risk factors are often correlated with other risk factors, such as market risk, volatility risk and the Fama-French (1993) size factor. This makes it nontrivial to empirically pin down the liquidity risk premium. Surprisingly, there is little theoretical work on the size of the liquidity risk premium. In this paper we therefore add to the debate on the liquidity risk premium by analyzing what size for the liquidity risk premium can be justified theoretically. We do this by calculating the liquidity risk premium demanded by large investors, in setting with dynamic portfolio choice, stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set.

Our first key finding is that our setup generates very small liquidity risk premiums, which are well below most empirical estimates. This is even the case under quite extreme assumptions on the degree of liquidity risk and trading frequency. This result provides a benchmark for existing and future empirical work on the liquidity risk premium. In addition, as our setting follows as much as possible the standard portfolio choice framework, our work implies that nonstandard assumptions are necessary in theoretical models in order to have a chance at generating larger liquidity risk premiums.

Our second key finding is that in our setup the liquidity risk premium is always small relative to the liquidity level premium, which is the direct compensation for trading costs of a given asset (Amihud and Mendelson (1986)). Depending on the parameter settings, our model can generate a liquidity level premium of 1% to 2% per year, while the liquidity risk premium is at most 20 basis points. This provides some support for the empirical work that finds evidence for the existence of a substantial liquidity level premium.

Even though there is little theoretical work on the liquidity *risk* premium, several articles have developed theoretical models to understand the size of the liquidity *level* premium, including Constantinides (1986), Liu (2004), Lo, Mamaysky and Wang (2004), Jang, Koo, Liu and Loewenstein (2007). These articles study dynamic portfolio choice problems with transaction costs or other forms of illiquidity, but the degree of illiquidity is always constant and hence they cannot analyze the compensation demanded for liquidity risk. A few articles incorporate liquidity risk in theoretical asset pricing or portfolio choice problems (Acharya and Pedersen (2005), Lynch and Tan (2011), and Beber, Driessen and Tuijp (2012)). We compare to this work in more detail in the literature section.

We now explain our setup in more detail. Our approach is "partial equilibrium". We model an investor solving a multi-period portfolio choice problem with stochastic illiquidity, and obtain liquidity level and risk premiums by calculating how much expected return the investor is willing to give up to remove illiquidity or illiquidity risk. In our setup, we aim to follow as much as possible the most common features of multi-period portfolio choice. We focus on a CRRA agent who solves a multi-period portfolio choice problem, maximizing expected utility of terminal wealth. There are two assets, a risk-free asset and a risky asset with lognormal returns, calibrated to match U.S. equity index data. We allow for predictability of asset returns by having a time-varying expected return that mean reverts over time, which we calibrate using the often-documented predictability of returns by the dividend-price ratio. As noted by Lynch and Tan (2011), incorporating predictability induces the agent to trade more, which in turn makes illiquidity more important.

There are various way to model illiquidity, such as fixed transaction costs, proportional transaction costs, and periods where trading is not possible (see the literature section). We follow Garleanu and Pedersen (2013) and use transaction costs that are a quadratic function of transaction size. This is consistent with the idea that trading has price impact (Kyle (1985)). We choose this type of illiquidity as we want to focus on large investors, who are likely most important for the price formation in asset markets, and thus for the empirically observed liquidity premiums. For these large investors the price impact of trading is a key aspect of illiquidity. To incorporate liquidity risk we allow the price impact of trading to change stochastically over time. This is consistent with empirical findings. For example, Amihud (2002) proposes the ILLIQ measure to estimate price impact and finds substantial time variation in this measure. In addition, this existing work has found that shocks to price impact are negatively correlated to market returns: price impact is higher in bad times. We incorporate such correlation in our setting as it likely amplifies liquidity risk premiums. Acharya and Pedersen (2005) focus on these liquidity covariances as the source of liquidity risk premiums.

We calibrate the parameters of the illiquidity process to match empirical estimates of price impact of large transactions (Bikker, Spierdijk and van der Sluis (2007)). We then solve the dynamic portfolio choice problem numerically by backward induction. We calculate the liquidity level premium as the expected return the investor is willing to give up to remove a constant level of price impact of trading, and the liquidity risk premium as the expected return the investor is willing to give up to remove the time-series variation of the price impact (but not the average level of the price impact). In our benchmark setting the agent has a 10-year horizon and trades annually. More frequent trading leads to lower liquidity premiums as transaction sizes per trading round are smaller and hence total price impact is smaller.

In our benchmark parameter calibration, we find a rather small liquidity level premium of 17 basis points. The main reason for this small liquidity level effect is that investors endogenously choose to trade less in response to the presence of trading costs (as in Constantinides (1986)). Without trading costs investors rebalance their portfolio and trade to profit from the time-varying expected return. With trading costs, investors carefully trade off the benefits and costs of trading. The utility benefits of rebalancing and profiting from time-varying expected returns are rather small according to our calibrations, and hence even small trading costs strongly reduce the amounts traded. To see this quantitatively, we decompose the total premium into a part that directly compensates for average trading costs, which equals 4 basis points, and a part that captures the utility loss of deviating from the optimal weight in the risky asset (13 basis points). Lynch and Tan (2011) also study the effect of predictability on the liquidity level premium and find a somewhat larger effect of 43 basis points, which is still below most empirical estimates of the liquidity level premium.

Our key result is on the liquidity risk premium. In the benchmark setting, the liquidity risk premium is below 1 basis point per year. This effect is due to the negative covariance of the asset return and shocks to the price impact of trading. This effect is small for several reasons. First, since the agent cares only very moderately about the level of trading costs, variation in these trading costs does not affect the expected utility much either. Second, even though the negative covariance between costs and returns implies that trading costs are higher in bad states of the world, the agent can endogenously choose to trade less when trading costs are currently higher than usual. This is a key difference between our approach and the model of Acharya and Pedersen (2005), where agents always trade their entire portfolio irrespective of the state of the world. By looking at a case with zero covariance between price impact and asset returns, we also find that independent variation in the price impact of trading has no meaningful effect on the agent's utility.

We perform various robustness checks to validate this result. We vary risk aversion, the covariance between costs and returns, and the level of price impact costs, and find that all these aspects have only a very small effect of the liquidity risk premium. We then add two nonstandard features to the setup in order to try to generate a larger liquidity risk premium. First, we force the investor to completely build up his risky asset position at time zero and fully sell off this position after some time. Even if we force the investor to perform this building up and selling off every year, the liquidity risk premium is below 2 basis points, while the liquidity level premium is much higher at around 2% due to the much higher trading amounts. The liquidity risk premium remains small in this case because the "forced" buying and selling is fully anticipated in this setting. We therefore consider a second case where we add "liquidity crisis" periods to the model. In each period, if the market return is below (minus) one standard deviation, the agent has to sell part of the risky asset. The size of the amount sold depends negatively on the market return. This generates priced liquidity risk, as the amount traded depends on the market return and thus on the state of the world. However, even in this rather extreme setting, the maximum liquidity risk premium we obtain is 20 basis points per year, while the liquidity level premium is higher at 55 basis points. In sum, our results show that it is difficult to generate a large liquidity risk premium using standard preferences and dynamic portfolio choice.

The paper is organized as follows. Section 2 discusses related literature and contributions. Section 3 describes the dynamic portfolio choice problem with quadratic and time-varying trading costs. Section 4 solves the problem numerically. In Section 5, we calculate the implied liquidity level premiums and liquidity risk premiums under the benchmark setting, setting with fixed frequency of rebuilding the portfolio and setting with forced selling during market downturn separately. Section 6 compare the correlation between market returns and turnovers indicated by our model and that in market data, and followed by conclusions in Section 7.

2 Related Literature and Contributions

Several papers investigate the magnitudes of liquidity and liquidity risk premiums in financial markets, both theoretically and empirically.[1][2] One major thread of the theoretical literature is the analysis of portfolio choice with trading costs. Most papers in this thread assume time constant trading-cost rates, which might be true for explicit costs (e.g. brokerage commissions) but is often not true for implicit trading costs (e.g. bid-ask spreads and price impact costs). As a starting point of this thread, Constantinides (1986) shows that for realistic proportional costs, the per-annum liquidity premium that must be offered to induce a constant relative risk aversion (CRRA) investor to hold the illiquid asset instead of an otherwise identical liquid asset is an order of magnitude smaller than the trading-cost

[1] In the previous research of asset illiquidity there are many different definitions. For example, the existence of non-trading interval in Diamond (1982), Ang, Papanikolaou and Westerfield (2014); the limitation on trading quantities (e.g. Longstaff (2001)); or trading at deterministic times (e.g. Kahl, Liu, and Longstaff (2003); Koren and Szeidl (2003); Schwartz and Tebaldi (2006); Longstaff (2009) etc.). The type of illiquidity we study in this paper is the trading costs, the most common one investigated in both liquidity pricing and portfolio choice literature (e.g. Constantinides (1986); Grossman and Laroque (1990); Vayanos (1998); Pastor and Stambaugh (2003); Lo, Mamaysky and Wang (2004); Acharya and Pedersen (2005) etc.).

[2] Liquidity risk is defined in many different ways in previous literature. For example, Huang (2003) defines it as the randomly arrived liquidity shocks; in Vayanos (2004), it refers to the time variation of needs to liquidate; Ang, Papanikolaou and Westerfield (2014) uses it for the uncertainty of the length of non-trading interval. In this paper, we follow Acharya and Pedersen (2005) and define liquidity risk as the time variation of trading costs which is more consistent with the reality in stock market.

rate itself. In subsequent work, Liu (2004) and Lo, Mamaysky and Wang (2004) use fixed trading costs. Realistic fixed trading costs can still hardly explain the large magnitude of the liquidity level premium. Longstaff (2001) limits the maximum amount of each transaction; and Garleanu (2008) models the illiquidity as the delay of trades.

The influence of trading costs largely relies on the trading frequency and the trading amounts. More trading leads to a larger liquidity level premium. The most popular way to achieve more trading is to add a time-varying investment opportunity set (return predictability or time-varying volatility). With this setting, many papers, such as Jang, Koo, Liu and Loewenstein (2007) and Lynch and Tan (2011), derive more trades and relatively larger liquidity level premiums.

Another choice is to create more trading motives with background risk. For example, Lynch and Tan (2011) include shocks to labor income in their model. Lo, Mamaysky and Wang (2004) and Garleanu (2008) both assume time-varying endowments in each period. Huang (2003) assumes that all investors face liquidity shocks and have to release their positions at some time point.

We add to this literature by letting trading-cost rates vary over time to study the magnitude of the liquidity risk premium, and we include forced selling during market downturns to further explore how this interacts with the time varying trading-cost rates and how it affects the liquidity risk premium.

Few theoretical studies include liquidity risk. The liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005) assumes time varying trading-cost rates. It provides a unified framework for understanding the various channels through which liquidity risk may affect asset prices. The primary limitation of liquidity-adjusted CAPM is that it is a one-period model. The trading frequency and trading amount are determined exogenously. In reality, both of them should be determined endogenously by investors, and these decisions should affect liquidity level and liquidity risk premiums in equilibrium. To make the trading frequency and trading volume endogenous, a multi-period model is required. Beber, Driessen and Tuijp (2012) provide a multi-period extension of Acharya and Pedersen (2005), but continue to assume that investors do not trade at intermediate dates. In contrast, in our model the investor is allowed to rebalance and trade at intermediate dates.

To our best knowledge, Lynch and Tan (2011) and Garleanu and Pedersen (2013) are the only two dynamic portfolio choice papers assuming time-varying trading-cost rates while also having endogenously determined trading amounts and frequencies.

Lynch and Tan (2011) shows that permanent shocks on labor income and return predictability produce an additional trading motive and thus a first-order liquidity level premium. Their numerical results also show that time-varying trading-cost rates further inflate the premium since under their setting the trading-cost rate is high when the agent trades the most. Different from Lynch and Tan (2011), we study the portfolio choice problem of large institutional investors instead of individual investors and allow for price impact of trading. Institutional investors are more likely to be the marginal investors in financial markets. We thus use time-varying quadratic trading costs, instead of the percentage trading costs as Lynch and Tan (2011) do. In addition, institutional investors care more about funding liquidity shocks than labor income, therefore we assume exogenous liquidity shocks rather than labor income shocks. Finally we show that the forced selling of institutional investors during market downturns actually interacts with the time variation of trading costs and enlarges the liquidity risk premium.

Garleanu and Pedersen (2013) define a multi-period mean-variance portfolio choice problem, using additional assumptions on the objective function and return dynamics. Specifically, they assume that price changes (and not returns) are homoskedastic. They focus on the implications for portfolio choice, and do not calculate liquidity level or liquidity risk premiums. Different from their paper, we use a standard multi-period CRRA utility framework, with standard dynamics of returns. In terms of portfolio choice, we do find similar implications as Garleanu and Pedersen (2013). Specifically, we confirm their conclusion that investors "aim in front of the target": when chasing time-varying expected returns, investors balance trading costs, the utility benefits of these time-varying returns, and the extent to which these return opportunities are expected to disappear quickly over time or not. More generally, our paper provides useful implications to the trading cost management of long-term investors, showing how to balance trading costs, rebalancing and investment opportunities.

Our paper provides a benchmark to empirical work on liquidity level and liquidity risk premiums. A number of empirical papers (e.g. Amihud and Mendelson (1986), Amihud (2002), Pastor and Stambaugh (2003) and Acharya and Pedersen (2005)) find substantial differences in expected returns across portfolios sorted on liquidity measures, with a magnitude ranges from 4% to 7% per annum. Some recognize it as the premium for the level of illiquidity (Amihud and Mendelson (1986) and Amihud (2002)), while others understand it as the premium for liquidity risk (Pastor and Stambaugh (2003)) or both (Acharya and Pedersen (2005)). In general, existing theories can hardly explain the large liquidity premiums found empirically.

3 Model

Our model follows the most common features of the dynamic portfolio choice problem in the existing literature. We solve a dynamic portfolio choice problem for a CRRA agent by maximizing his expected utility of terminal wealth. The model has two assets, a risk-free asset and a risky asset with lognormal returns. We incorporate return predictability by allowing the expected return to vary over time. Our model deviates from the common features of dynamic portfolio choice by including quadratic transaction costs (price impact costs) into the setting and allows the price impact of trading to change stochastically over time.

We assume the log risk free rate r_f is constant over time, and the log return of the risky asset is

$$r_{t+1} = \mu_t + \sigma_r u_{t+1} \tag{1}$$

where μ_t is the conditional mean of log return, and σ_r is the volatility parameter. The return shock u_{t+1} is normally distributed with mean zero and standard deviation of one. We assume that μ_t depends on a driving factor F_t which follows an AR(1) process,

$$F_{t+1} = \rho F_t + v_{t+1} \tag{2}$$

$$\mu_t = \mu_0 + aF_t \tag{3}$$

In our model, F_t is the only state variable which drives the time variations of both expected return μ_t and the trading cost parameter λ_t, which will be introduced later. v_{t+1} is the shock on F_{t+1}, and it follows a standard normal distribution. We assume the correlation between the shocks of returns and state variable F_t as $Corr(u_t, v_t) = Corr$, the time persistency parameter of F_t is ρ, and the long-run mean of F_t is zero. Parameter a decides the magnitude of the time variation of μ_t. μ_t is constant over time if $a = 0$. By substituting F_t into the expression of μ_t, we can easily find that μ_t is also an AR(1) processes, and μ_0 is the long-run mean of the expected return.

Trading is costly in our setting. We follow Garleanu and Pedersen (2013) and use quadratic transaction costs. The expression for the dollar costs of trading a dollar amount V_t is

$$TC_t = \frac{1}{2} V_t^2 \sigma_r^2 \lambda_t \tag{4}$$

The trading costs TC_t depend on V_t^2, rather than V_t which is what proportional transaction costs imply. Like Garleanu and Pedersen (2013), we assume the price impact scales with the variance of returns σ_r^2, and multiply this by a stochastic trading cost parameter λ_t. This expression of quadratic transaction costs is consistent with the idea that trading has price impact (Kyle 1985). Under this setting, trading V_t moves the price by

$$PI_t = V_t \sigma_r^2 \lambda_t \tag{5}$$

For a given trading amount V_t, the corresponding proportional trading cost c_t equals half the total price change, which can be written as

$$c_t = \frac{1}{2} PI_t = \frac{1}{2} V_t \sigma_r^2 \lambda_t \tag{6}$$

We assume the natural logarithm of the trading cost parameter λ_t also depends on state variable F_t and follows an AR(1) process,

$$\ln \lambda_t = \ln \lambda_0 + bF_t \tag{7}$$

where b is the sensitivity of $\ln \lambda_t$ to shocks on F_t, and $\ln \lambda_0$ is the long-run mean of $\ln \lambda_t$. λ_t is constant over time if $b = 0$.

The investor has a finite investment horizon T, and his initial wealth W_0 is strictly positive. We assume that the investor maximizes the expected CRRA utility of the terminal wealth, W_T,

$$E_0(\frac{W_T^{1-\gamma} - 1}{1-\gamma}) \tag{8}$$

The weight in risky asset at each time step, $\alpha_t, t = 1, 2, \ldots, T-1$, serves as the control variable. The investor's objective is to maximize the expected CRRA utility of the terminal wealth by choosing the dynamic investment strategy $(\alpha_0, \alpha_1, \ldots, \alpha_{T-1})$,

$$\max_{\alpha_0, \alpha_1, \ldots, \alpha_{T-1}} E_0(\frac{W_T^{1-\gamma} - 1}{1-\gamma}) \tag{9}$$

Then the total trading amount at each time step is

$$V_t = (\alpha_t - \alpha_{t-})W_t \tag{10}$$

where α_{t-} is the weight in risky asset before rebalancing. Substituting equation (10) into equation (4), we get

$$TC_t(W_t, \alpha_t, \alpha_{t-}, \lambda_t) = \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_r^2 \lambda_t \tag{11}$$

We assume that all trading costs are paid from the risky asset[3]. Thus, the level of wealth

[3] Assuming that trading costs are paid out of the risk-free asset instead of risky asset does not make any difference on our result, since it is the weight in risky asset after trading costs are paid that matters in our model.

in next time step is

$$W_{t+1} = (1-\alpha_t)W_t exp(r_f) + (\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_r^2 \lambda_t) exp(r_{t+1}) \tag{12}$$

and the weight of the risky asset before rebalancing in next time step is

$$\alpha_{(t+1)-} = \frac{(\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-})W_t)^2 \sigma_r^2 \lambda_t) exp(r_{t+1})}{W_{t+1}} \tag{13}$$

The value function J at each time step t can be expressed as

$$J(W_t, \alpha_{t-}, F_t, t) = \max_{\alpha_t} E_t(\frac{W_T^{1-\gamma} - 1}{1-\gamma}) \tag{14}$$

and the Bellman equation for this dynamic portfolio choice problem is

$$J(W_t, \alpha_{t-}, F_t, t) = \max_{\alpha_t} E_t[J(W_{t+1}, \alpha_{(t+1)-}, F_{t+1}, t+1)] \tag{15}$$

The problem is solved using backward induction. We search numerically for the weight in risky asset α_t which maximizes the expected utility of the terminal wealth from the last period to the first.

4 Numerical Solution

In this section, we further discuss how we solve this dynamic portfolio choice problem numerically with realistic parameter values comparable with U.S. stock market data. We use the setting with time-varying expected returns as the benchmark setting. The shocks on expected returns provide the trading motives needed to generate liquidity level premium and liquidity risk premiums[4].

[4] If the time-constant expected return is used, the only trading motive is to rebalance the portfolio after price fluctuation. Such trading motive is usually very small (refer to the Appendix). It does not even exist under the market clearing condition in our setting, since both the initial weight and the optimal weight of our risky asset is fixed to 100% and does not change with the price.

4.1 Parameter Values

We assume that the long-term mean of expected annual return is 4% ($\mu_0 = 0.04$) and the standard deviation of return shocks is 10% ($\sigma_r = 0.10$). Risk free rate is 2% ($r_f = 0.02$), and the risk aversion level of our representative investor is 2.5 ($\gamma = 2.5$). Then if there is no trading cost and time variation in expected returns ($\mu_t = \mu_0 = 0.04$), the analytical solution of the optimal weight is

$$\alpha^{LongRun} = \frac{\mu_0 - r_f + \sigma_r^2/2}{\gamma\sigma_r^2} = 100\% \tag{16}$$

for all t. It means it is optimal for the representative investor to invest all his wealth in risky asset. To add the time variation of expected returns to our setting, we set the standard deviation of the shocks to expected returns at 1% ($a = 0.01$)[5].

Since we want to document an upper bound of the liquidity risk premium, we use a high trading-cost rate with substantial time variation for our analysis. For the level of trading costs parameter λ_c, we take the estimates in Bikker, Spierdijk and van der Sluis (2007) as a reference and assume that the price impact of a 1.5 million dollar trade is 40 basis points, $\lambda_c = 26.88$[6], which indicates a trading costs of 20 basis points (half the price impact). This assumption is also consistent with the numbers found in most papers of price impact (for example, Chan and Lakonishok 1997 find a price impact about 54 bps, and Keim and Madhavan 1997 find a price impact about 30 bps to 65 bps). In addition, we allow the trading-cost rate to be 3 times higher in a robustness check. We set the parameter for time variation of the trading-cost rates to 0.3149 ($b = 0.3149$), which is calibrated using the variation in the annual $ln(ILLIQ)$ measure proposed in Amihud (2002)[7]. ILLIQ, as λ in our model, is a measure of price impact calculated as the absolute value of the daily return divided by the daily dollar trading volume. Under this setting, the 95% confidence interval of λ_t is $[0.18 * \lambda_c, 5.64 * \lambda_c]$, which means a 2 standard deviation positive (negative)

[5] Assuming dividend yield as the only predictor, we use the dividend yield data from 1952 to 2010 for the calibration of parameter a. The calibration using monthly data indicates an annual standard deviation of 0.92%, and using annual data, it increases to 1.86%.

[6] It is calibrated using equation (5).

[7] The annual ILLIQ values from 1952 to 2010 are used for the calibration.

shock on λ_t makes it more than five times (less than one fifth) its long-term level.

We set the annual time persistence of the state variable F_t at 0.7 ($\rho = 0.7$), which is calibrated using the monthly data ILLIQ series, and we set the initial wealth at 100 million dollars, ($W_0 = 1$), which is about the median size of U.S. hedge funds. Since hedge funds are more likely to be the marginal traders in the financial market and more likely to experience liquidity shocks than large mutual funds and pension funds do, we choose to use the median size of hedge funds in our benchmark setting. Considering there is only 1 asset in our economy, 100 million dollars holdings of one single asset is large enough to generate a significant price impact of trades. To further make sure we will not underestimate the liquidity risk premium, we also solve the problem with higher level of wealth, which is equivalent to using a higher trading-cost rate λ_t in our setting. We solve the portfolio choice problem for 10 years, with an annual trading frequency. For the calculation of liquidity risk premium, we solve the problem for different values of the correlation between shocks on realized returns and trading-cost rates, $Corr(v_t, u_t) = 0, -0.2, -0.4, -0.6$.

To sum up, in our assumptions we try incorporate a substantial degree of illiquidity, in an attempt to try to generate substantial liquidity level and liquidity risk premiums. We assume high levels for the trading-cost rate, large time variation in it (from one fifth to five time the mean level), large trading motives (time-varying expected returns, and later a fixed frequency of rebuilding the portfolio and exogenous liquidity shocks), a single risky asset (no spreading of trades over stocks to reduce the price impact of trades), large institutional investors (high price impact of trades and high exposure to liquidity shocks), and an annual trading frequency (no spreading of trades within a year to reduce the price impact of trades).

4.2 Numerical Results

The dynamic portfolio choice problem is solved by backward recursion. Gaussian Quadrature is used to deliver the joint distribution of shocks on the state variable F_t and return shocks (v_t, u_t). Four points are used for each shock. Figure 1 shows the weights in risky asset both before and after rebalancing under the reference case with a time constant trading-cost rate. Here we assume zero correlation between the shocks of investment opportunity set and the returns shocks thus there is no hedging demand.

[Insert Figure 1 about here]

Figure 1 plots one simulation of the weights in the risky asset across 10 time steps, for both the weights before rebalancing, α_{t-}, with black circles, and the weights after rebalancing and trading costs, α_{t+}, with red stars. In each time step, the investor trades partially towards the myopic optimal weight, α_t^{Myopic}, the pink crosses. The expression of α_t^{Myopic} is

$$\alpha_t^{Myopic} = \frac{\mu_t - r_f + \sigma_r^2/2}{\gamma \sigma_r^2} \tag{17}$$

which varies over time with the conditional expected return μ_t. If the investor trades the entire way from α_{t-} to α_t^{Myopic}, he needs to pay a large amount of trading costs. On the other hand, if he does not trade at all and keeps the weight at α_{t-}, he loses too much utility by deviating from the α_t^{Myopic}. Therefore, it is optimal to trade partially towards the aim. The optimal amount to trade is decided by the trade-off between the marginal utility gain of getting closer to the aim and the marginal trading costs incurred. We will show later that both the loss of utility caused by deviation from α_t^{Myopic} and the actual trading costs incurred should be compensated in the form of a higher expected return (liquidity level premium). Besides, the investor resists to trade further away from $\alpha^{LongRun}$, the green dash line, since it will generate more trading costs in the future. These results are consistent with the main findings in Garleanu and Pedersen (2013), when trading is costly, the investor should trade partially towards the current aim, and also aim in front of the target (consider the long-run optimal weight, $\alpha^{LongRun}$).

5 Liquidity Level Premium and Liquidity Risk Premium

In the previous section, we have shown that trading costs make the investor deviate from the optimal solution in the frictionless market. In a competitive market, investors should require a premium (higher expected return) to compensate for the loss of utility caused by trading costs. Therefore, in this section, we compute both the liquidity *level* premium, the premium compensates for the level of trading costs, and the liquidity *risk* premium, the premium compensates for the time variation of trading costs.

In this paper, the liquidity level premium is defined to be the decrease in the long-term mean of the expected return, μ_0, on the risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is defined to be the decrease in the long-term mean of the expected return on the risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. This approach is a "partial equilibrium". A positive premium means that the investor should be compensated for the utility loss caused by trading costs or time variation of trading costs, and a negative premium indicates investor benefits from them.

First, we discuss intuitively the sources of liquidity level premium and liquidity risk premium. As we mentioned in previous section, investors should be compensated for both the actual trading costs and the loss of utility caused by the deviation from the optimal weight. Liquidity level premium measures such compensation in term of a higher expected return. It is worth noting that the part of the liquidity level premium that compensates for the actual trading costs depends on the total amount of trading costs, which is the product of cost rate and total trading amount, rather than the cost rate itself. The larger the trading amount, the higher the liquidity level premium.

The liquidity risk premium measures the compensation for the loss of utility caused by the time variation of trading-cost rates, also in terms of the increase in expected return. The time variation of trading-cost rates has three different effects on the utility of the investor, thus it also enters the liquidity risk premium through three different channels: the variance of the cost rates (*Variance Effect*), the covariance between trading costs and realized returns (*Covariance Effect*), and the additional freedom to choose the weight in risky asset introduced by the time variation of cost rates (*Choice Effect*).

1. *Variance Effect:* Since the representative investor is risk averse, the time variation of trading costs should be penalized. A positive premium should be required as a

compensation.

2. *Covariance Effect:* The investor dislikes to pay large amounts of trading costs during market downturns, hence a negative covariance between the trading costs and realized returns $Cov(c_t, r_t)$ should be penalized, and a positive premium should be required as a compensation.

3. *Choice Effect:* In a dynamic setting, investors respond actively to the time variation of cost rates. Investors trade more when cost rates are low and trade less when it is high. In this case, investors can actually benefits from the time variation of cost rates, and if this effect dominates, a negative liquidity risk premium should be found. We show later that the liquidity risk premium is actually negative in some of our settings.

To sum up, the liquidity risk premium is always an aggregate premium for these three different effects instead of just a single one.

5.1 Benchmark Setting

Under the benchmark setting, the only trading motive is the time-varying myopic aim introduced by the time-varying expected returns. To calculate the liquidity level premium and liquidity risk premium, we solve four different cases of the portfolio choice problem:

Case 1: with constant expected return and *no* trading costs
$(a = 0, b = 0, \lambda_t = \lambda_c = 0)$
Case 2: with time-varying expected returns and *no* trading costs
$(a = 0.01, b = 0, \lambda_t = \lambda_c = 0)$
Case 3: with time-varying expected returns and *constant* trading-cost rate
$(a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88)$
Case 4: with time-varying expected returns and *time-varying* trading-cost rates
$(a = 0.01, b = 0.315, \lambda_c = 26.88)$

We calculate the expected utility for each case. The initial position in risky asset is assumed to be at 100%, the long-term optimal weight in risky asset. *Case 1*, the case with constant expected return and no trading costs is used as the reference case for the calculation of both the liquidity level premium and liquidity risk premium. Specifically, for each of *Case 2,3,4*, we find the corresponding level of expected return in the benchmark case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the reference case (it means investor has the same expected utility when holding either one of these two risky assets). Then we compare the corresponding levels of expected returns across different cases. The difference of corresponding expected returns between *Case 2* and *Case 3* is recorded as the liquidity level premium, and the difference of the corresponding expected returns between *Case 3* and *Case 4* is recorded as the liquidity risk premium. The liquidity risk premium for $Cov(\lambda_t, r_t)$, the covariance between trading-cost rates and returns, is calculated as the difference of the corresponding expected returns with nonzero correlation between trading-cost rates and returns, $Cov(\lambda_t, r_t) \neq 0$, and the case with zero correlation, $Cov(\lambda_t, r_t) = 0$.

[Insert Table 1 about here]

Table 1 reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the return shocks and shocks on trading-cost rate (shocks on state variable F_t), 0, -0.2, -0.4, -0.6. Firstly, we find that the magnitudes of the liquidity risk premium are extremely small, ranging from -0.623 to -0.104 bps, significantly smaller than the liquidity level premium, which ranges from 16.43 to 15.68 bps. The effect of time variation of cost rates on investor's utility is substantially smaller than the effect of the trading cost level. The premium for $Cov(\lambda_t, r_t)$ indeed increases as the correlation between returns and cost rates becomes more negative, but the effect is always smaller than 1 bps and accounts for 3.3% of the total liquidity premium at most. The small liquidity risk premiums found in our analysis conflict with the empirical literature on the liquidity risk premium (e.g. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005)), where the covariance between trading-cost rates and returns, $Cov(c_t, r_t)$, generates a large liquidity risk premium.

We find negative liquidity risk premiums in all 4 settings. As we have explained at the beginning of this section, it means the *Choice Effect* dominates the *Variance Effect* and *Covariance Effect*, the investor actually prefers to have time variation in trading-cost rates since he/she can react according to the realized cost rate and thus variation in cost rates increases the expected utility. To better understand how the *Choice Effect* generates a negative liquidity risk premium, we plot the expected utility of the terminal wealth as a function of the trading-cost parameter λ_c under the optimal strategy in Figure 2 (the solid curve), for the reference case with time constant trading-cost rate and zero correlation between return shocks and shocks on F_t. Since trading costs limit investor's rebalancing of the portfolio, the expected utility decreases with the increase of trading-cost parameter λ_c. When λ_c becomes larger, investors choose to trade less. So the increase of λ_c will have a smaller effect on investor's expected utility. Therefore, the expected utility is strictly convex in λ_c, which means the linear combination of any two points on the curve is above the curve. Then if we assume λ_c is stochastic at time 0, instead of deterministic, the expect utility of the investor should always be higher than the expect utility indicated by the curve. The uncertainty in λ_c always increases the expected utility of investor. Because of this, when we introduce time variation into the trading-cost parameter λ, the *Choice Effect* makes the liquidity risk premium negative. Economically, this effect is rather small however.

Secondly, it is also worth noting that the liquidity level premium, ranging from 15.68 to 16.48 bps, is significantly smaller than the 65 bps [8] price impact costs of an average trade under the benchmark setting. This result is consistent with Constantinides (1986), who shows that the liquidity premium is an order of magnitude smaller than the trading-cost rate itself. The reason is that investors trade less if the trading-cost rate is high. Under the benchmark setting, the only trading motive is to trade towards the time-varying myopic aim introduced by the time-varying expected return μ_t. If there is no trading cost, the utility gain of chasing the myopic aim is equivalent to a 22 bps increase in expected return μ_0. It means the upper limit of the liquidity level premium is 22 bps, since if the trading-cost rate is too high, investors will choose to not trade at all and bear all the utility loss of not trading. As Table 1 shows, the total liquidity premium is about 15.27-16.12 bps, slightly smaller than the 22 bps. It indicates that it is optimal for investors to trade slightly

[8] The average trading amount for the benchmark setting is 4.9 million dollars, which corresponds to price impact costs of 65.3 bps (1/2*4.9m*0.01*26.88) according to equation (5).

towards the myopic aim to reduce the utility loss of investing sub-optimally. Table 2, we show that only a small faction of liquidity level premium compensates for actual trading costs, about 4.18 bps (out of 16.34 bps), and a large fraction compensates for the utility loss of deviating from the myopic optimal weight, about 12.65 bps (out of 16.34 bps). This result is in accordance with the conjecture in Garleanu and Pedersen (2013) that investors balance between the trading costs and the loss of utility caused by deviating of the myopic optimal weight to maximize the expected utility.

[Insert Table 2 about here]

To further investigate how the level of trading-cost rate affects the magnitudes of liquidity risk premium as well as the liquidity level premium, we solve the dynamic problem for different values of the trading-cost parameter. Table 2 reports the liquidity level premium, for the parts compensating for trading costs (TC)[9] and compensating for the deviation from optimal weight[10] separately, liquidity risk premium and average trading amounts across different trading-cost rates, from 3.36 (1/8*26.88) to 107.52 (4*26.88). The correlation between return shocks and shocks on F_t, $Corr(u_t, v_t)$ is set to -0.3 in all cases. We see that the liquidity risk premium changes only slightly from -0.494 bps to -0.297 bps when the cost rate becomes 4 times as large as before. Considering a cost rate 4 times large indicates a 1.6% price impact from a 1.5 million $ trade, the liquidity risk premium of -0.297 bps is negligible. The magnitude of the cost rate does not have a significant effect on the liquidity risk premium under our benchmark setting. It is worth noting that since the wealth level and the trading amount are the only two assumptions based on dollar value in our model, and the trading-cost parameter λ_c is calibrated based on the trading amount, an increase in trading-cost parameter λ_c is equivalent to an increase in wealth level in our model. It means the liquidity premiums calculated for the setting with initial level of wealth as 100 million dollars and λ_c=4*26.88 are the same as the setting with initial level of wealth as

[9] We calculate the trading costs generated as a percentage of total wealth for each time step of each simulation, and use the average value across all 10,000 simulations and all 10 steps each as a measure of liquidity level premium compensating for the actual trading costs.

[10] The liquidity level premium compensating for the deviation from optimal weight is calculated by deducting the liquidity level premium compensating for TC from the total liquidity level premium.

400 million dollars and λ_c=26.88. Therefore, the results shown in Table 2 also indicate that the liquidity risk premium is small also for higher levels of wealth.

[Insert Figure 3 about here]

Consistent with the Constantinides (1986) and Garleanu and Pedersen (2013), we find that investors trade less when the trading-cost rate is higher. Table 2 and Figure 3 both show that the average trading amount per year decreases from 4.9 million dollars to 2.0 million dollars when the cost rate becomes 4 time the benchmark level, and it increases to 14.8 million dollars when the cost rate becomes 1/8 of the benchmark level. In addition, Figure 3 shows that the optimal trading amount is decreasing and convex in the cost rate. Trading amount is more sensitive to the cost rate when it is low. The relative importance of liquidity level premium compensating for TC decreases monotonically with the increase of trading-cost rate (from 64% for $\lambda_c = 1/8 * 26.88$ to 14% for $\lambda_c = 4 * 26.88$); and the premium compensating for the deviation from optimal weight increases with the increase of cost rate (from 2.88 bps for $\lambda_c = 1/8 * 26.88$ to 18.05 bps for $\lambda_c = 4 * 26.88$). More interestingly, different from the implications of models in Amihud and Mendelson (1986) and Acharya and Pedersen (2005), Figure 4 shows that rather than being proportional to trading-cost rate, the equilibrium liquidity premium is increasing and concave in the trading-cost rate, and there is an upper limit on the liquidity premium, which is about 22 bps for our benchmark setting. In Amihud and Mendelson (1986) and Acharya and Pedersen (2005), both the trading amount and the trading frequency are exogenous, and hence total trading costs, as the product of cost rate and total trading amount, increases linearly with the cost rate. In our benchmark setting, the trading amount is decided endogenously by the tradeoff between trading costs and utility gain of trading more. Therefore, under our setting, the liquidity premium does not only depend on the trading-cost rate, trading amount and trading frequency, but also on the sensitivity of investor's expected utility on the trading behavior.

[Insert Figure 4 about here]

5.2 Setting with Fixed Frequency of Rebuilding and Releasing

Until now, we assumed that the only trading motive is the time-varying expected returns. In the real world, investors may choose to rebuild their portfolios at a fixed time frequency. One main thread of liquidity literature (e.g. Amihud and Mendelson 1986 and Acharya and Pedersen 2005) is based on this assumption. Following their spirits, we also solve a dynamic portfolio choice problem under the assumption that investors release and rebuild their portfolios at a fixed time frequency in this subsection, in order to see whether this assumption helps to generate a large liquidity risk premium comparable to those found empirically.

[Insert Figure 5 about here]

We solve the problems for different frequencies of rebuilding the portfolio: every year, every 2 years, 5 years and 10 years. And for each frequency, we solve the problem for 3 values of the correlation between return shocks and the shocks on F_t, 0, -0.3 and -0.6. As an example, Figure 5 plots the trajectory of the optimal weights invested in the risky asset for rebuilding and releasing the portfolio every 10 years. The case with zero correlation is plotted. It shows that it is optimal for investors to trade gradually during the rebuilding and releasing of the portfolio to reduce the price impact of trades, as predicted in Garleanu and Pedersen (2013).

Using the same partial equilibrium approach, we calculate the liquidity level premiums and liquidity risk premium for different frequencies of rebuilding the portfolio. Table 3 shows that liquidity risk premiums are still very small, from 0.847 bps to 2.659 bps out of a total liquidity premium from 94.98 bps to 212.15 bps. The assumption of fixed frequency of releasing and rebuilding the portfolio does not help to generate a large liquidity risk premium.

Besides, similar to the increase of the trading-cost rate, as forced releasing and rebuilding of the portfolio becomes more frequent, it is optimal for investor to invest less into the risky asset and thus trade less and pay less trading costs. As Table 3 shows, the liquidity level premium compensating for trading costs and average trading amount per year both decrease as the rebuilding of portfolio becomes more frequent, and the liquidity level premium compensating for the deviation from the optimal weight increases. In addition, the total liquidity premium ranges from 94.98 bps to 212.15 bps.

[Insert Table 3 about here]

5.3 Setting with Exogenous Liquidity Shocks (Forced Selling)

The results in section 5.1 and 5.2 show that the magnitude of the liquidity risk premium is negligible under the setting with time-varying expected returns or rebuilding of the portfolio at a fixed time frequency as a trading motive. Does that mean that the liquidity risk premium is always negligible in financial markets, and all the large liquidity risk premiums documented in the recent liquidity literature are wrong? Not necessarily. During periods of crisis (e.g., the 1987 market crash, the 1997 Asian crisis, the Russian debt crisis of 1998, the hedge-funds meltdown of 2007, and the 2008 financial crisis), market liquidity goes down, trading-cost rates go up substantially, and at the same time, institutional investors are forced to release a large amount of their positions. The large amount of trading costs paid for their forced selling hurts those already wounded investors even more. Because of this, investors are supposed to worry about the high trading costs during market downturn a lot and require large compensation for that.

5.3.1 Assumptions of Exogenous Liquidity Shocks (Forced Selling)

To investigate how large a liquidity risk premium can be generated by the large trading costs during the market downturn, we add exogenous liquidity shocks into our model[11]. To further identify the importance of the correlation between the trading motive and the market condition, we distinguish between two types of liquidity shocks: the liquidity shocks depending on market condition, and the liquidity shocks independent of the market condi-

[11] In the real world, investors are usually forced to release part of their positions when market goes down. For example, the mutual fund literature has a long history of documenting the flow-performance sensitivity (e.g. Warther 1995, Sirri and Tufano 1998, Froot, O'connell and Seasholes 2001, Huang, Wei and Yan 2007 etc.), they all show there are more fund outflows during market downturn; and Brunnermeier and Pedersen (2009) claim that investors' capital and margin requirements are binding when market deteriorates, thus they are forced to reduce their holdings.

tion. Since there is only one risky asset in our model, the changes of the market condition are equivalent to the changes in the price of the risky asset.

For the cases with exogenous liquidity shocks *depending* on realized returns, if the risky asset performs badly (with a realized return r_{t+1} more than one standard deviation, σ_r, lower than the conditional mean μ_t), the investors are forced to release a proportion (or all) of his positions in risky asset. The effect of liquidity shocks on the portfolio weight in the risky asset before rebalancing, $\Delta\alpha_{(t+1)-}$, is as follows:

- *if* $\mu_t - 3\sigma_r \leq r_{t+1} \leq \mu_t - \sigma_r$, *investors are forced to sell a proportion of their positions in the risky asset, the positions released in terms of the change of weight in risky asset is* $\Delta\alpha_{(t+1)-} = \alpha_{(t+1)-} * \frac{r_{t+1}-(\mu_t-\sigma_r)}{2\sigma_r}$, $\Delta\alpha_{(t+1)-} = 0$, *if* $r_{t+1} = \mu_t - \sigma_r$; $\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$, *if* $r_{t+1} = \mu_t - 3\sigma_r$. *If* $r_{t+1} < \mu_t - 3\sigma_r$, *investors release all their positions in the risky asset:* $\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$.

For the cases with exogenous liquidity shocks *independent* of realized returns, we substitute the realized return r_{t+1} by a random variable ε_{t+1} which follows a normal distribution with a mean of 0 and a standard deviation of 1, ε_{t+1} N(0,1):

- *if* $-3 \leq \varepsilon_{t+1} \leq -1$, *investors are forced to sell a proportion of their positions in risky asset, the positions released in terms of the change of weight in the risky asset is* $\Delta\alpha_{(t+1)-} = \alpha_{(t+1)-} * \frac{\epsilon_{t+1}-(-1)}{2}$, $\Delta\alpha_{(t+1)-} = 0$, if $\epsilon_{t+1} = -1$; $\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$, *if* $\epsilon_{t+1} = -3$. *If* $\epsilon_{t+1} < -3$, *investor releases all his positions in risky asset* $\Delta\alpha_{(t+1)-} = -\alpha_{(t+1)-}$.

5.3.2 Calculation of the Liquidity Level Premium and Liquidity Risk Premium

Similar to the benchmark setting, we solve four different cases of the portfolio choice problem to calculate the liquidity level premium and liquidity risk premium.

Case 1: with constant expected return, exogenous liquidity shocks *independent* of realized returns, and *no* trading costs[12]

[12]Since there is no trading cost, investor will instantly trade back to the optimal weight after liquidity shocks without any utility loss or additional costs. Therefore, *Case 1,2* with liquidity shocks are exactly the same as the *Case 1,2* in benchmark setting. In our setting, liquidity shocks affect investors only if there are trading costs.

$(a = 0, b = 0, \lambda_t = \lambda_c = 0, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

Case 2: with *time-varying* expected returns, exogenous liquidity shocks *independent* of realized returns, and *no* trading costs

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 0, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

Case 3: with time-varying expected returns, exogenous liquidity shocks *independent* of realized returns, and *constant* trading-cost rate

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

Case 4: with time-varying expected returns, exogenous liquidity shocks *depending* on realized returns, and *time-varying* trading-cost rates

$(a = 0.01, b = 0.315, \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) > 0)$

As before, the initial position in risky asset is assumed to be 100%. *Case 1*, the case with constant expected return, exogenous liquidity shocks independent of realized returns, and no trading costs is used as the reference case for the calculation of liquidity level premium and liquidity risk premium. For each of *Case 2,3,4*, we find the corresponding level of expected return in the reference case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the reference case. Then the difference of corresponding expected returns between *Case 2* and *Case 3* is documented as the liquidity level premium, and the difference of the corresponding expected returns between *Case 3* and *Case 4* is documented as the liquidity risk premium.

It is worth noting that under the benchmark setting, the only difference between *Case 3* and *Case 4* is that *constant* trading-cost rate becomes *time-varying*. However, under this setting with liquidity shocks, in addition, exogenous liquidity shocks *independent of* realized returns become *dependent on* realized returns. The reason is that besides the covariance between trading-cost rates and realized returns $Cov_t(\lambda_{t+1}, r_{t+1})$, the negative covariance between the square of trading amount (forced selling) and realized returns $Cov_t(V_{t+1}^2, r_{t+1})$ also generates a liquidity risk premium. To calculate the total liquidity risk premium, we need to include both effects, and the interaction of these two effects as well.

5.3.3 Relation to Liquidity-Adjusted CAPM in Acharya and Pedersen (2005)

The liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005) assumes that investors have a fixed investment horizon with end releasing and no rebalancing in between (the same as the assumption in section 5.2), and they use the ILLIQ, which is a measure of price impact of trading (λ in our setup), to measure the effective percentage trading costs (c_t in our setup). Under their assumptions, the liquidity risk premium can be measured by the covariance between the trading-cost rates and the realized returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, since λ_t is proportional to the effective percentage trading costs c_t. In reality, trading motives usually depend on the market condition as our setting with liquidity shocks assumes. If we relax the assumption in Acharya and Pedersen (2005) by allowing the trading amount to be different from the holding amount, (assume the holdings of the investor at time t is H_t, and the trading amount is V_t which is smaller than H_t and varying over time), then we have the actual trading costs $\hat{c}_t$ as a percentage of previous holdings H_{t-1} as

$$\hat{c}_t = \frac{c_t V_t}{H_{t-1}} = \frac{1}{2}\frac{\sigma_r^2 \lambda_t V_t^2}{H_{t-1}} \neq \lambda_t, s.t. 0 \leq V_t \leq H_t \tag{18}$$

Then following the logic of the liquidity-adjusted CAPM in Acharya and Pedersen (2005), liquidity risk should be priced by $Cov_t(\hat{c}_{t+1}, r_{t+1})$, the covariance between the actual trading costs paid as a percentage of holdings and realized returns, instead of $Cov_t(\lambda_{t+1}, r_{t+1})$, the covariance between cost rates and realized returns. Since trading costs $\hat{c}_{t+1}$ depends on both cost rate λ_t and trading amount V_t, if investors have to trade a lot when the realized return r_t is low, the liquidity risk $Cov_t(\hat{c}_{t+1}, r_{t+1})$ is high even if the cost rate λ_t does not change with realized return r_t. Therefore, the correlation between trading amount and realized return $Cov_t(V_{t+1}^2, r_{t+1})$[13] is also an important element of liquidity risk which is not covered by the liquidity-adjusted CAPM in Acharya and Pedersen (2005). In addition, the interaction between $Cov_t(\lambda_{t+1}, r_{t+1})$ and $Cov_t(V_{t+1}^2, r_{t+1})$ could lead to even higher liquidity risk than the simple sum of these two.

[13] According to equation (18), we see trading costs as a percentage of holdings $\hat{c}_t$ actually depends on $Cov_t(V_{t+1}^2, r_{t+1})$, rather than $Cov_t(V_{t+1}, r_{t+1})$ since $\hat{c}_t$ increases with both trading amount V_t and the price impact $PI_t = V_t \sigma_r^2 \lambda_t$, which increases with the trading amount V_t as well.

5.3.4 Decomposition of Liquidity Risk Premium

To separate the liquidity risk premium induced by $Cov_t(\lambda_{t+1}, r_{t+1})$, $Cov_t(V_{t+1}^2, r_{t+1})$ and the interaction effect included in $Cov_t(\lambda_{t+1}V_{t+1}^2, r_{t+1})$, we solve 2 additional cases for the portfolio choice problem with liquidity shocks.

Case 4-1: with time-varying expected returns, exogenous liquidity shocks *dependent on* realized returns, and *constant* trading-cost rate

$(a = 0.01, b = 0, \lambda_t = \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) > 0)$

Case 4-2: with time-varying expected returns, exogenous liquidity shocks *independent of* realized returns, and *time-varying* trading-cost rates

$(a = 0.01, b = 0.315, \lambda_c = 26.88, Cov_t(V_{t+1}^2, r_{t+1}) = 0)$

Again, for *Case 4-1* and *Case 4-2*, we find the corresponding level of expected return in the reference case (*Case 1*) which makes the investor indifferent between holding the risky asset in this case and the one in the reference case. Since the only difference between *Case 4-1* and *Case 3* is that liquidity shocks depend on realized returns, the difference of corresponding expected returns between these 2 cases is recorded as a liquidity risk premium for $Cov_t(V_{t+1}^2, r_{t+1})$. Similarly, since the only difference between *Case 4-2* and *Case 3* is that trading-cost rate becomes time-varying, the difference of the corresponding expected returns between these 2 cases is recorded as the total liquidity risk premium induced by the time variation of trading-cost rates. As in the benchmark setting, the premium for $Cov_t(\lambda_{t+1}, r_{t+1})$ is calculated as the additional liquidity risk premium introduced by the correlation between return shocks and shocks on state variable F_t, the case with zero correlation ($Corr(u_t, v_t) = 0$) is used as the reference case. The difference of the corresponding expected returns between *Case 4* and *Case 3* is the total liquidity risk premium introduced by the liquidity shocks depending on realized returns and the time variation of trading-cost rates together. The premium for $Cov_t(\lambda_{t+1}V_{t+1}^2, r_{t+1})$ is also calculated using the case with zero correlation between return shocks and shocks on state variable F_t as the reference case.

5.3.5 Liquidity Risk Premiums for the Setting with Forced Selling

We solve the dynamic portfolio choice problem with exogenous liquidity shocks for three values of the correlation between return shocks and shocks on trading-cost rates $Corr(u_t, v_t)$, 0, -0.3, -0.6. Table 4 shows liquidity level premiums and liquidity risk premiums in this setting. We find that the total liquidity risk premium under this setting is significantly larger than before, 11.53 bps for the case with $Corr(u_t, v_t) = -0.3$, and 20.83 bps for the case with $Corr(u_t, v_t) = -0.6$. It accounts for a substantial fraction of the total liquidity premium, 18% and 28% correspondingly. Although the liquidity level premium and liquidity risk premium generated here are still substantially smaller than the annual 4%-7% premiums documented empirically, in term of relative importance, this result is comparable to that in Acharya and Pedersen (2005), who find that 1.1% out of a 4.6% liquidity premium compensates for liquidity risk. More interestingly, although the total liquidity risk premium can be as large as 20.83 bps, the liquidity risk premiums for $Cov_t(V_{t+1}^2, r_{t+1})$ and $Cov_t(\lambda_{t+1}, r_{t+1})$ individually are very small, only about 6.03 bps for $Cov_t(V_{t+1}^2, r_{t+1})$, and 2.13 bps for $Cov_t(\lambda_{t+1}, r_{t+1})$. The large total liquidity risk premium mainly comes from the interacted covariance, $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$, 18.61 bps out of 20.83. Therefore, it is the large trading amount and high trading-cost rate during the market downturn together that hurt the investor and make him require a large liquidity risk premium. None of these two effects itself is sufficient to generate a large liquidity risk premium. However, previous researches about liquidity risk premium, such as Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2005) etc., attribute the liquidity risk premiums purely to the covariance between the trading-cost rates and market returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, and neglect the important role of the covariance between the trading amounts and market returns, $Cov_t(V_{t+1}^2, r_{t+1})$ and their interacting effect, which is the main source of the liquidity risk premium according to our analysis.

[Insert Table 4 about here]

In addition, under the setting with exogenous liquidity shocks, the liquidity level premium mainly compensates for the actual trading costs, about 30 out of 50 bps, rather than the deviation from the optimal weight if there is no trading cost. It is because the forced selling caused by liquidity shocks is inelastic to the level of the trading-cost rate, investor can only reduce the amount of the forced sales by investing less in risky assets. That sacrifices too much of the expected returns when compared with the possible trading costs caused by the potential liquidity shocks.

5.3.6 Varying the Cost Rate and the Frequency of Liquidity Shocks

Now, we have shown that the liquidity risk premium can account for as large as 28% of the total liquidity premium when investors are forced to sell during a market downturn. Next, to check the robustness of this finding, we investigate how the relative importance of the liquidity risk premium changes with the level of the trading-cost rate and the frequency of the liquidity shocks.

First, we solve the same problem with exogenous liquidity shocks for different levels of trading-cost rates λ_c, from 3.36(1/8*26.88) to 107.52(4*26.88). The correlation between return shocks and shocks on trading-cost rate is set to -0.3 for all cases. We see from Table 5 that both the liquidity level premium and liquidity risk premium increase with the trading-cost rate. As the cost rate becomes 4 times as large as before, the liquidity risk premium increases from 11.53 bps to 21.54 bps, and the relative importance decreases from 18% of total liquidity premium to 12%. Though it decreases slightly, it still accounts for a significant fraction of the total liquidity premium. Moreover, as we predict, the average trading amount decreases as the trading-cost rate becomes higher, and the % of liquidity premium compensating for trading costs also decreases from 61% to 57%, since the investor chooses to invest less into risky assets to reduce the total trading amount, and thus he pays less trading costs and bears more utility loss caused by underinvestment.

[Insert Table 5 about here]

Secondly, we solve the same problem for lower frequencies of the liquidity threat. Instead of facing a probability of forced selling every year as in the previous setting, the investor faces it now every 2 years or every 5 years. As usual, we solve it for 3 values of correlation between returns and cost rates, 0, -0.3, -0.6. Table 6 shows that both liquidity level premium and liquidity risk premium increases as the liquidity shocks become more frequent. The relative importance of the liquidity risk premium is almost the same for the cases with annual liquidity threats and per 2 years, about 18% of total premium when the correlation is -0.3, and about 28% when the correlation is -0.6. It decreases slightly to 12% and 22% as the liquidity threat becomes more infrequent, every 5 years, and it is negligible for the case with no liquidity threat at all as we have shown in the benchmark setting. Besides, both average trading amounts and the liquidity level premium compensating for trading costs increase as liquidity threat becomes more frequent, and the relative importance of premium for trading costs increases as we expect.

[Insert Table 6 about here]

To sum up, in this section, using the setting with exogenous liquidity shocks, we find that the liquidity risk premium is economically significant if and only if investors are forced to trade during a market downturn, and the trading-cost rate goes up at the same time. The liquidity risk premium, instead of generated by the covariance between trading-cost rates and the return shocks, $Cov_t(\lambda_{t+1}, r_{t+1})$, as claimed in most papers of liquidity risk, is mainly generated by the covariance between the total trading costs (the product of trading amounts and trading-cost rates) and the return shocks, $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$. In addition, we show the importance of liquidity risk premium remains for different levels of trading-cost rates and different frequencies of liquidity threats.

6 The Relation between Market Turnovers and Market Returns

We have shown that our benchmark setup generates very small liquidity risk premiums even under quite extreme assumptions on liquidity levels and liquidity risk. One nonstandard assumption, which has a chance at generating larger liquidity risk premiums, is the forced selling during market downturn. The forced selling introduces a large correlation between the portfolio turnovers and the realized returns. In this section, we compare the correlation of turnovers and returns (liquidity risk) of the U.S. stock market with the correlation predicted by our model. We find that the correlation of turnover and returns in market data is comparable to that in our benchmark setting, but substantially smaller than the extremely negative correlation in our setting with exogenous liquidity shocks (forced selling). This shows that the setting with forced selling in market downturns is quite extreme.

6.1 Market Data

Since both market turnover and market liquidity of U.S. stock market are highly persistent over time, we first do an AR(1) regression for the log values of both turnover and ILLIQ to capture the innovations. We use the data from 1966 to 2010.

$$lnTrn_{t+1} = \alpha^{trn} + \rho^{trn} lnTrn_t + \varepsilon^{trn}_{t+1} \tag{19}$$

$$lnILLIQ_{t+1} = \alpha^{ILLIQ} + \rho^{ILLIQ} lnILLIQ_t + \varepsilon^{ILLIQ}_{t+1} \tag{20}$$

We find that both the market turnover and ILLIQ are quite persistent at annually frequency. The time persistence of market turnover, ρ^{trn} in equation (19), is 0.9987, and the R^2 of equation (19) is 0.9696. The time persistence of ILLIQ, ρ^{ILLIQ} in equation (20), is 0.9735, and the R^2 of equation (20) is 0.9135.

Table 7 reports the correlations between the annual[14] market excess returns, the innovations in market liquidity and the innovations in market turnover from 1966 to 2010, and the covariance between the annually market excess returns and the innovations in market turnover. In accordance with the forced selling during the market downturn, Panel C of Table 7 reports a negative correlation (-0.170) between market excess returns and market turnovers when market excess returns $R_M - r_f$ are negative, but it is not significant because of the small number of observations. The correlation between market excess returns and market turnovers is positive for the entire sample (0.203), which is probably because investors on average trade more during bull markets than bear markets. In addition, Panel B

[14] The correlations of monthly data please refer to the Appendix 8.2.

of Table 7 reports a significant negative correlation (-0.584) between market excess returns and the innovations in lnILLIQ, which is because the market is more liquid during bull markets than bear markets, and a negative correlation (-0.281) between the innovations in lnILLIQ and the innovations in market turnover indicating investors on average trade more when the market is relatively more liquid.

For the level of the market turnover, the market data reports an average annual market turnover about 66%, which is substantially larger than the 5% - 10% in the simulations of our model. Therefore our model is not able to capture the high turnover in stock market. Previous literature suggests it could be caused by noise trades of investors, high-frequency traders and the large variation of investment sentiment, and those are not included in our setting.

6.2 Comparison with Simulated Results

For each setting of our model, we simulate 10,000 trajectories of the stock returns and turnovers. Then for each trajectory of turnovers, we do the AR(1) regression, equation (19), to calculate the innovation in the natural logarithm of turnovers. The correlation and covariance between excess returns and innovations in turnover are calculated across all 10,000 simulations with 10 steps each.

Table 8 reports the correlation between the annual returns and turnover for the simulations of our model, and Table 9 reports the covariance.

Now we compare the simulated results with the market data. In general, the covariances between turnovers and returns of our simulated data in benchmark case are comparable with the market data. For the correlation between the excess returns and innovations in turnover, $Corr(R_m - r_f, \Delta lnTrn)$, Panel A in Table 8 reports that for the benchmark setting with $Corr(u_t, v_t)$ between -0.4 and -0.6, the $Corr(R_m - r_f, \Delta lnTrn)$ for the entire sample ranges from -0.010 to 0.023, which is smaller than the 0.203 in market data. This positive correlation is partially caused by the fact that investor trade more when the market is liquid, but the even higher correlation in market data might be caused by higher investment sentiment during the bull market than bear market. The $Corr(R_m - r_f, \Delta lnTrn)$ for the positive sample ranges from -0.014 to 0.049, slightly higher than the -0.056 in market data, and the $Corr(R_m - r_f, \Delta lnTrn)$ for the negative sample ranges from -0.087 to -0.110, slightly smaller than the -0.170 in market data in terms of magnitude, which is because the forced selling is not included in the benchmark setting. The magnitude of the covariance between the excess returns and innovations in turnover, $Cov(R_m - r_f, \Delta lnTrn)$, in our simulated data, Panel A in Table 9, is comparable to that in the market data, ranging from -51.6 to 16.2 (*10^{-4}).

Panel B in Table 8 reports that for the setting with exogenous liquidity shocks (forced selling) and a $Corr(u_t, v_t)$ as -0.6, the $Corr(R_m - r_f, \Delta lnTrn)$ shoots up to -0.220 for the entire sample, and -0.673 for the negative sample only. It means our assumption of forced selling is extremely strong. The fact that such a strong assumption of liquidity risk can only generate a 20 bps liquidity risk premium strengthens our claim that the actual liquidity risk premium in the market required by investors is very small. Consistently, the magnitude of the covariances reported in Panel B of Table 9 is substantially larger than that in the market data in term of magnitude.

To understand the correlations of turnovers and returns of our simulated data more, there are mainly 4 effects affecting these values of correlation and covariance.

1. Time-varying expected returns: Investor trades more when the conditional expected return is either higher or lower than the average level (high realized return usually comes with low expected return, vice versa). We see this from the column with $Corr(u_t, v_t) = -0.6$ in the Panel A of Table 8 and Table 9 for the benchmark setting where time-varying expected returns play the most crucial role. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are positive for the positive sample and negative for the negative sample as we expected. In addition, in the Panel B of Table 8 and Table 9 for the setting with exogenous liquidity shocks, both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are positive for the positive sample where there is no liquidity shock.

2. Time-varying price impact of trading: Investor trades more when the market is more liquid (high realized return comes with high market liquidity when $Corr(u_t, v_t)$ is negative in our model). See row 'Entire sample' in both Panel A and B of Table 8 and Table 9.

Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ increases as $Corr(u_t, v_t)$ becomes more negative from 0 to -0.6.

3. Forced sales during crisis: The investor is forced to sell when the realized return is too low. See Panel B of Table 8 and Table 9 for the setting with exogenous liquidity shocks. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are negative for the entire sample and the negative sample. It is the most dominant effect in our setting.

4. Wealth effect: A higher wealth level means larger price impact for the same level of turnover (higher realized returns lead to higher wealth level). See column $Corr(u_t, v_t) = 0$ in the Panel A of Table 8 and Table 9 for the benchmark setting. Both $Corr(R_m - r_f, Trn)$ and $Cov(R_m - r_f, Trn)$ are negative when $Corr(u_t, v_t) = 0$ for the entire sample under the benchmark setting.

7 Conclusions

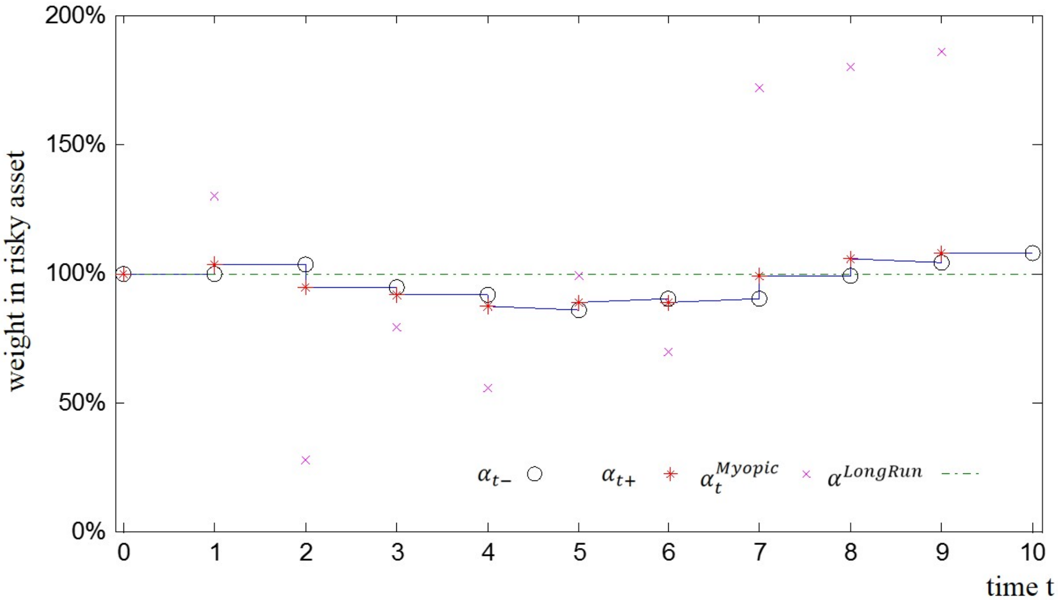
In this paper we solve a dynamic portfolio choice problem with stochastic illiquidity, CRRA utility and a time-varying expected return. Our goal is to generate theoretical predictions for the liquidity risk premium that large investors demand.

We find that the liquidity risk premium generated by the covariance between trading-cost rates and realized returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, which is documented as the main source of liquidity risk (e.g. Pastor and Stambaugh 2003 and Acharya and Pedersen 2005), is negligible, less than 1 bp per year, under our benchmark setting with time-varying expected returns. Larger trading amounts and higher trading frequencies increase the premium for the level of trading costs (liquidity level premium) only but not the liquidity risk premium.

However, once we add exogenous liquidity shocks (forced selling) into the setting, the liquidity risk premium become economically significant and accounts for a large fraction of the total liquidity premium. Forced selling and high trading-cost rate during the market downturn together hurt the investor but the liquidity risk premium generated by any of these two itself is still negligible. It indicates that large liquidity risk premiums documented in previous empirical papers, such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), might largely compensate for the forced selling and high trading-cost rate during market downturn together, rather than simply the high trading-cost rate itself as they claim. Moreover, even with forced selling, the largest liquidity risk premium required by large investors in our setting, 20 bps, is still substantially smaller than those documented in empirical literature (7% in Pastor and Stambaugh 2003 and 1.1% in Acharya and Pedersen 2005).

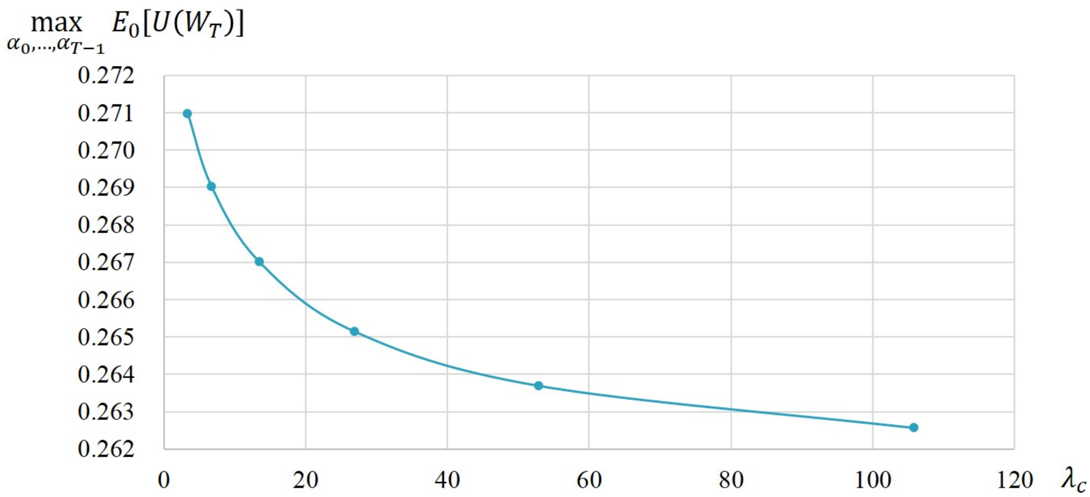
8 Appendix

Figure 1: Weights in risky asset under benchmark setting (1 simulation)



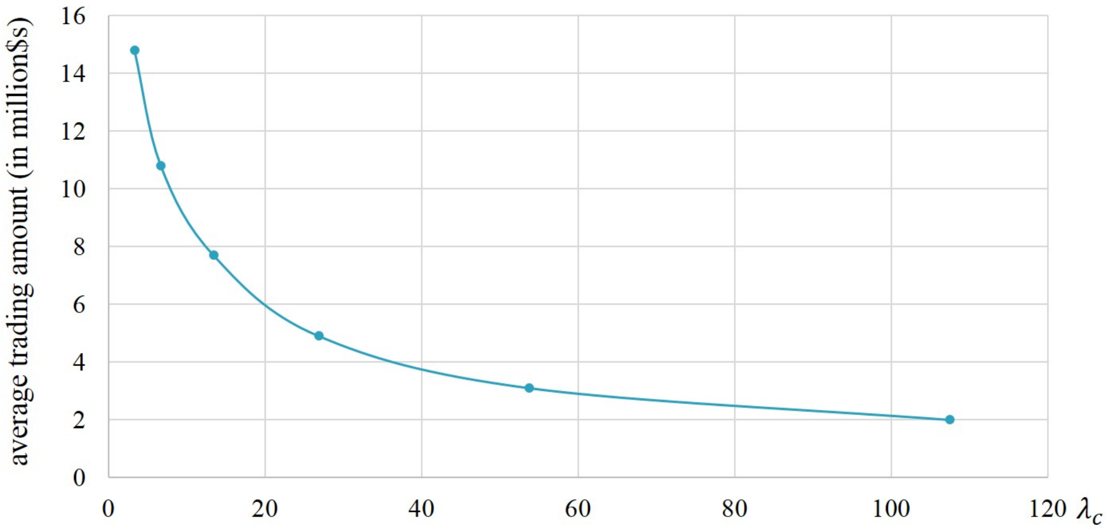
This figure plots one simulation of the weights in risky asset from time 0 to 10 (10 years), for the benchmark setting with time constant trading-cost rate and 0 correlation between returns and costs. The initial weight is set as $\alpha_{0-} = 100\%$. In each time step t, we plot both the weight before rebalancing α_{t-}, the black circle, and the weight after rebalancing and trading costs α_{t+}, the red star. The pink cross denotes the myopic optimal weight in each time step, α_t^{Myopic}, and the green dash line is the long-run optimal weight, $\alpha^{LongRun}$ which equals 100%.

Figure 2: Expected utility for different levels of time constant trading-cost rate λ_c



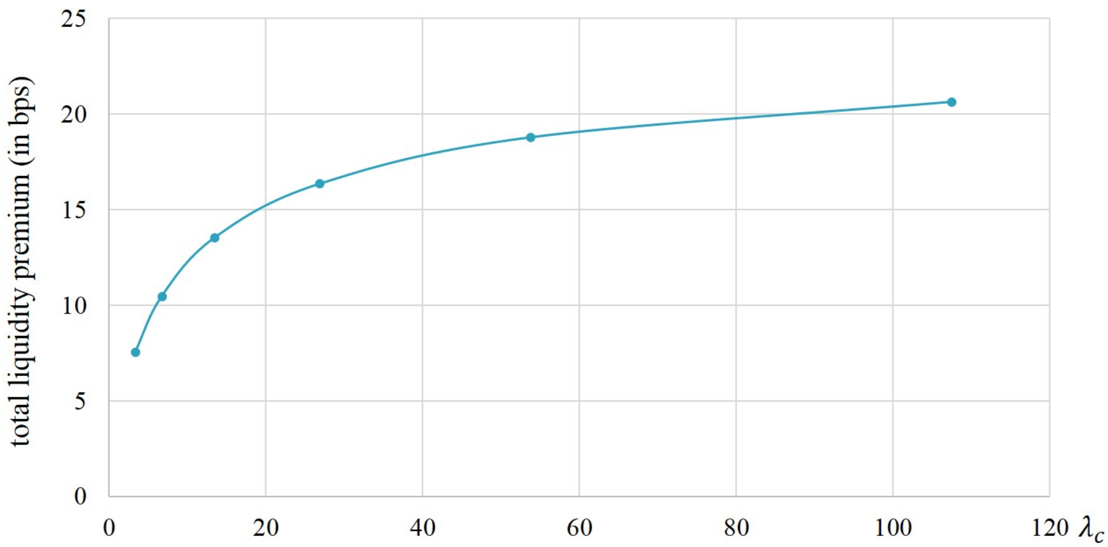
This figure plots the expected utility of the terminal wealth as a function of the trading-cost parameter λ_c under the optimal strategy, $\max E_0 U(W_T)$, for the benchmark setting with time constant trading-cost rate. The fact that expected utility is convex in trading-cost rate indicates that the uncertainty in trading-cost parameter λ_c always increases the expected utility under the benchmark setting when there is no correlation between returns and costs, $Corr(u_t, v_t) = 0$.

Figure 3: Average trading amount as a function of the trading-cost rate



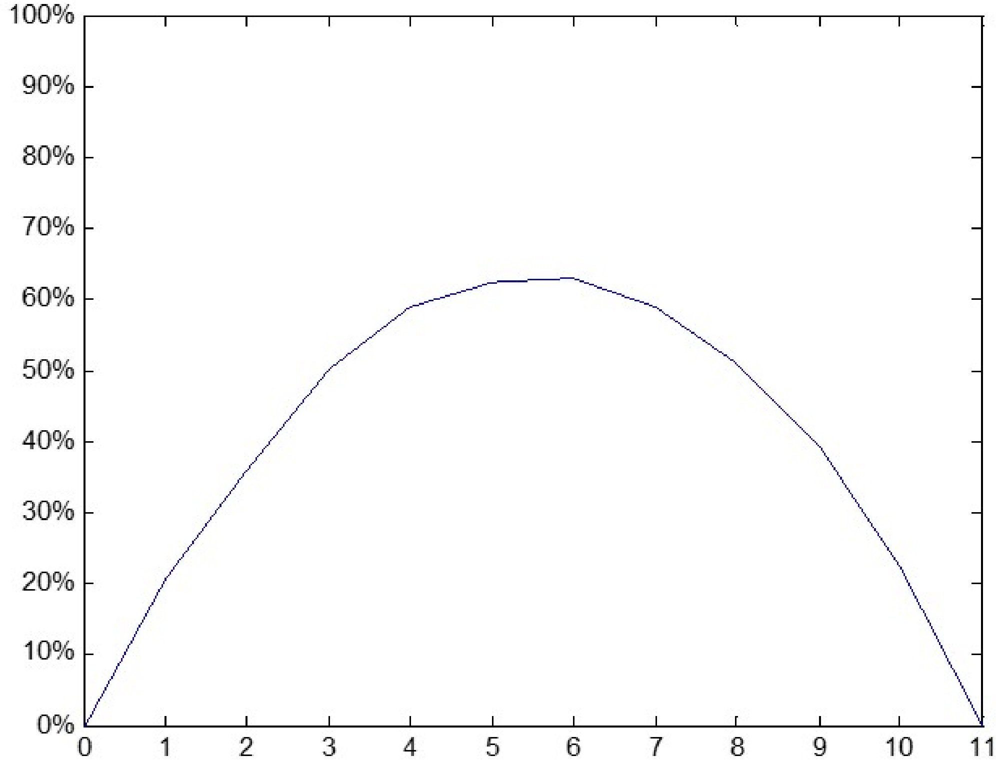
This figure plots the average trading amount as a function of the trading-cost parameter λ_c under the optimal strategy, for the benchmark setting with time constant trading-cost rate. The correlation between returns and costs, $Corr(u_t, v_t) = -0.3$.

Figure 4: Total liquidity premium as a function of trading-cost rate



This figure plots the total liquidity premium as a function of the trading-costs parameter λ_c under the optimal strategy, for the benchmark setting with time constant trading-cost rate. The correlation between returns and costs, $Corr(u_t, v_t) = -0.3$.

Figure 5: Optimal weights for building and releasing the portfolio every 10 years



This figure plots the average trajectory of the optimal weights for building the portfolio from the beginning and releasing all the positions before the end of time period 10. The correlation between returns and costs, $Corr(u_t, v_t) = 0$. The weight is averaged across 10,000 simulations.

Table 1: Liquidity risk premium for the benchmark setting

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the returns and trading costs, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term mean of the expected return, μ_0, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in μ_0 on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between returns and trading costs, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Liquidity Level Premium (Total)	16.43	15.86	16.48	15.68
Liquidity Risk Premium (Total)	-0.623	-0.582	-0.353	-0.104
Total Liquidity Premium	15.80	15.27	16.12	15.58
liquidity risk premium for $Cov(\lambda_t, r_t)$	*0.000*	*0.042*	*0.270*	*0.519*
premium for $Cov(\lambda_t, r_t)$ *as % of total premium*	*0.0%*	*0.3%*	*1.7%*	*3.3%*

Table 2: Liquidity level premium for difference levels of trading-cost rate

This table reports liquidity level premium (for the parts compensating for TC and compensating for the deviation from optimal weight separately), liquidity risk premium and average trading amount across different trading-cost rates λ_c, from 3.36(1/8*26.88) to 107.52(4*26.88). Liquidity level premium for TC is the premium compensating for actual trading costs paid, and liquidity level premium compensating for the deviation from optimal weight is the premium compensating for the utility loss caused by deviating from the none-trading-cost optimal weight. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity level premium for TC as % of total liquidity premium is also reported. All premiums are reported in basis points (bps), and the correlation between the returns and trading costs, $Corr(u_t, v_t) = -0.3$ for all cases.

in bps, $Corr(u_t, v_t) = -0.3$	Average value of trading-cost parameter λ_c					
	3.36	***6.72***	***13.44***	***26.88***	***53.76***	***107.52***
Liquidity Level Premium (compensating for TC)	4.83	5.30	5.35	4.18	3.45	2.88
Liquidity Level Premium (compensating for the deviation from the optimal weight)	2.88	5.45	8.56	12.65	15.69	18.05
Liquidity Risk Premium (Total)	-0.167	-0.316	-0.393	-0.494	-0.370	-0.297
Total Liquidity Premium	7.54	10.44	13.51	16.34	18.77	20.63
liquidity level premium for TC as % of total liquidity premium	*64%*	*51%*	*40%*	*26%*	*18%*	*14%*
avg. trading amount (million$s)	*14.8*	*10.8*	*7.7*	*4.9*	*3.1*	*2.0*

Table 3: Liquidity risk premium with fixed releasing and rebuilding of the portfolio

This table reports the liquidity level premium (compensating for TC and for the deviation from the none-trading-cost optimal weight separately), liquidity risk premium and average trading amount per year for different frequencies of rebuilding the portfolio (per 1, 2, 5 and 10 years), and for 3 values of the correlation between returns and trading costs ($Corr(u_t, v_t) = 0, -0.3, -0.6$) for each frequency. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity level premium for TC as % of total liquidity premium is also reported. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$	Frequency of rebuilding (per X years)			
		1	***2***	***5***	***10***
Liquidity Level Premium (compensating for TC)	0	8.37	11.00	24.97	33.63
	-0.3	7.98	11.26	25.22	36.90
	-0.6	7.85	11.27	25.85	38.14
Liquidity Level Premium (compensating for the deviation from the optimal weight)	0	184.35	171.56	121.10	60.49
	-0.3	195.38	181.77	128.46	64.38
	-0.6	202.44	188.73	134.99	68.28
Liquidity Risk Premium (Total)	0	1.509	0.907	0.847	0.861
	-0.3	1.645	1.163	1.925	1.439
	-0.6	1.859	1.447	2.659	1.846
Total Liquidity Premium	0	194.23	183.47	146.92	94.98
	-0.3	205.01	194.19	155.61	102.72
	-0.6	212.15	201.45	163.50	108.27
liquidity level premium for TC as % of total liquidity premium	*0*	*4%*	*6%*	*17%*	*35%*
	-0.3	*4%*	*6%*	*16%*	*36%*
	-0.6	*4%*	*6%*	*16%*	*36%*
avg. trading amount (million $s)	*0*	*9.49*	*8.75*	*11.39*	*12.43*
	-0.3	*9.26*	*8.83*	*11.46*	*13.06*
	-0.6	*9.18*	*8.87*	*11.68*	*13.39*

Table 4: Liquidity risk premium with exogenous liquidity shocks (forced selling)

This table reports liquidity level premiums and liquidity risk premiums for the setting with exogenous liquidity shocks (forced selling). We report it for 3 values of the correlation between returns and trading costs ($Corr(u_t, v_t) = 0$, -0.3, -0.6) separately. Liquidity level premiums and liquidity risk premiums are reported based on their sources. We report the liquidity level premiums compensating for TC and the deviation from the none-trading-cost optimal weight separately, the total liquidity risk premium, and the liquidity risk premiums for the covariance between trading amounts and realized returns, $Cov_t(V_{t+1}^2, r_{t+1})$, the covariance between trading-cost rates and realized returns, $Cov_t(\lambda_{t+1}, r_{t+1})$, and total covariance including the interaction of these two, $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$, separately. Liquidity risk premium as % of total liquidity premium is also reported. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$		
	0	***-0.3***	***-0.6***
Liquidity Level Premium (compensating for TC)	29.92	31.94	33.19
Liquidity Level Premium (compensating for the deviation from the optimal weight)	19.47	20.72	21.41
Liquidity Risk Premium (Total)	2.22	11.53	20.83
Total Liquidity Premium	51.61	64.19	75.44
liquidity risk premium as % of total liquidity premium	*4%*	*18%*	*28%*
liquidity risk premium for $Cov_t(V_{t+1}^2, r_{t+1})$	*1.17*	*3.80*	*6.03*
liquidity risk premium for $Cov_t(\lambda_{t+1}, r_{t+1})$	*0.00*	*0.75*	*0.93*
liquidity risk premium for $Cov_t(\lambda_{t+1} V_{t+1}^2, r_{t+1})$	*0.00*	*9.31*	*18.61*

Table 5: Liquidity risk premium with exogenous liquidity shocks for different levels of trading-cost rate

This table reports liquidity level premiums, liquidity risk premiums and average trading amount per year under the setting with exogenous liquidity shocks for different levels of trading-cost rate λ_c, from 3.36 (1/8*26.88) to 107.52 (4*26.88). We assume the correlation between returns and trading costs, $Corr(u_t, v_t) = -0.3$. We report the liquidity level premiums (compensating for TC and the deviation from the none-trading-cost optimal weight separately) and the total liquidity risk premiums. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity risk premium as % of total liquidity premium, and liquidity level premium for TC as % of total liquidity premium are also reported. All premiums are reported in basis points (bps).

in bps, $Corr(u_t, v_t) = -0.3$	Average value of trading-cost parameter λ_c					
	3.36	***6.72***	***13.44***	***26.88***	***53.76***	***107.52***
Liquidity Level Premium (compensating for TC)	10.93	15.48	21.79	31.94	49.51	86.53
Liquidity Level Premium (compensating for the deviation from the optimal weight)	2.17	5.91	11.96	20.72	34.58	66.56
Liquidity Risk Premium (Total)	4.88	7.52	9.92	11.53	13.71	21.54
Total Liquidity Premium	17.98	28.91	43.67	64.19	97.81	174.62
liquidity risk premium as % of total premium	*27%*	*26%*	*23%*	*18%*	*14%*	*12%*
liquidity level premium for TC as % of total liquidity premium	*83%*	*72%*	*65%*	*61%*	*59%*	*57%*
avg. trading amount (million $s)	*20.69*	*16.04*	*12.22*	*9.58*	*8.03*	*7.68*

Table 6: Liquidity risk premium for different frequencies of liquidity threats

This table reports liquidity level premiums, liquidity risk premiums and average trading amount per year for different frequencies of liquidity threats (per 1, 2 and 5 years), and for 3 values of the correlation between returns and trading costs ($Corr(u_t, v_t) = 0, -0.3, -0.6$) for each frequency. We report the liquidity level premiums compensating for TC and the deviation from the none-trading-cost optimal weight separately, the total liquidity risk premium. Both liquidity level premium compensating for TC and average trading amount per year are calculated through 10,000 simulations with 10 steps each. Liquidity risk premium as % of total liquidity premium, and liquidity level premium for TC as % of total liquidity premium are also reported. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$	Frequency of liquidity threats (per X years)			
		1	***2***	***5***	***Never***
Liquidity Level Premium (compensating for TC)	0	29.92	20.50	12.28	3.57
	-0.3	31.94	22.13	13.55	4.17
	-0.6	33.19	22.76	13.91	3.93
Liquidity Level Premium (compensating for the deviation from the optimal weight)	0	19.47	13.84	11.47	12.85
	-0.3	20.72	14.14	11.41	12.67
	-0.6	21.41	14.17	10.71	11.75
Liquidity Risk Premium (Total)	0	2.22	1.64	0.47	-0.62
	-0.3	11.53	7.87	3.47	-0.49
	-0.6	20.83	14.49	7.06	-0.10
Total Liquidity Premium	0	51.61	35.97	24.21	15.80
	-0.3	64.19	44.13	28.43	16.34
	-0.6	75.44	51.43	31.68	15.58
liquidity risk premium as % of total premium	*0*	*4%*	*5%*	*2%*	*-4%*
	-0.3	*18%*	*18%*	*12%*	*-3%*
	-0.6	*28%*	*28%*	*22%*	*-1%*
liquidity level premium for TC as % of total liquidity premium	*0*	*58%*	*57%*	*51%*	*23%*
	-0.3	*50%*	*50%*	*48%*	*26%*
	-0.6	*44%*	*44%*	*44%*	*25%*
avg. trading amount (million \$s)	*0*	*9.25*	*7.70*	*6.13*	*4.43*
	-0.3	*9.58*	*7.57*	*6.40*	*4.89*
	-0.6	*9.65*	*7.60*	*6.53*	*4.81*

Table 7: Correlation and Covariance between Market Returns, Innovations in ILLIQ and Turnover (annually)

This table reports summary statistics, the correlations between the annually market excess returns, the innovations in market liquidity ($\Delta lnILLIQ$) and the innovations in market turnover ($\Delta lnTrn$) from 1966 to 2010, and the covariance between the annually market excess returns and the innovations in market turnover. Panel A for the summary statistics, Panel B for the correlations, and Panel C for the correlations and covariance of the annually market excess returns and the innovations in market turnover for the entire sample, sample with positive returns only ($R_m - r_f > 0$), and sample with negative returns only ($R_m - r_f < 0$) separately.

Panel A: Summary Statistics

	Mean	Std.Dev	# obs	Min	Max
Rm-rf	0.056	0.185	45	-0.399	0.321
$\Delta lnTrn$	0.000	0.139	44	-0.318	0.252
$\Delta lnILLIQ$	0.000	0.244	44	-0.431	0.760

Panel B: Correlations for Entire Sample

	Rm-rf	$\Delta lnILLIQ$	$\Delta lnTrn$
Rm-rf	1	-.584***	.203
$\Delta lnILLIQ$		1	-.281*
$\Delta lnTrn$			1

Panel C: Correlation and Covariance of Returns and Turnovers

	$Corr(R_m - r_f, \Delta lnTrn)$	$Cov(R_m - r_f, \Delta lnTrn)$ (*10^{-4})	# obs
Entire sample	.203	51.9	44
$R_m - r_f > 0$	-.056	-6.6	30
$R_m - r_f < 0$	-.170	-25.9	14

Table 8: Correlation of the Returns and Innovations in Turnovers (simulation results)

This table reports the correlation between the annual returns and the innovations in turnovers ($\Delta lnTrn$) for different settings of our model. Panel A for the benchmark setting with time-varying trading-cost rates, Panel B for the setting with exogenous liquidity shocks (forced selling) and time-varying trading-cost rate. For each setting, we report the correlation values for cases with different values of the correlation between returns and trading costs, $Corr(u_t, v_t)$ separately. We also report them for the entire sample, sample with positive returns only ($R_m - r_f > 0$), and sample with negative returns only ($R_m - r_f < 0$) separately. We do 10,000 simulations for each case within each setting.

Panel A: Benchmark setting with time-varying trading-cost rates

$Corr(R_m - r_f, \Delta lnTrn)$	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Entire sample	-0.100***	-0.058***	-0.010***	0.023***
$R_m - r_f > 0$	-0.074***	-0.031***	-0.014***	0.049***
$R_m - r_f < 0$	-0.075***	-0.039***	-0.087***	-0.110***

Panel B: Setting with liquidity shocks and time-varying trading-cost rates

$Corr(R_m - r_f, \Delta lnTrn)$	$Corr(u_t, v_t)$		
	0	***-0.3***	***-0.6***
Entire sample	-0.328***	-0.303***	-0.220***
$R_m - r_f > 0$	-0.087***	-0.047***	0.091***
$R_m - r_f < 0$	-0.683***	-0.662***	-0.673***

Table 9: Covariance of the Returns and Innovations in Turnover (simulation results)

This table reports the covariance between the annual returns and the innovations in turnovers ($\Delta lnTrn$) for different settings of our model. Panel A for the benchmark setting with time-varying trading-cost rates, Panel B for the setting with exogenous liquidity shocks (forced selling) and time-varying trading-cost rate. For each setting, we report the covariance values for cases with different values of the correlation between returns and trading costs, $Corr(u_t, v_t)$ separately. We also report them for the entire sample, sample with positive returns only ($R_m - r_f > 0$), and sample with negative returns only ($R_m - r_f < 0$) separately. We do 10,000 simulations for each case within each setting.

Panel A: Benchmark setting with time-varying trading-cost rates

$\text{Cov}(\text{R}_m - r_f, \Delta lnTrn)$ $(*10^{-4})$	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Entire sample	-93.5	-61.0	-10.9	16.2
$R_m - r_f > 0$	-33.2	-15.9	-7.7	25.7
$R_m - r_f < 0$	-32.1	-18.5	-44.1	-51.6

Panel B: Setting with liquidity shocks and time-varying trading-cost rates

$\text{Cov}(\text{R}_m - r_f, \Delta lnTrn)$ $(*10^{-4})$	$Corr(u_t, v_t)$		
	0	***-0.3***	***-0.6***
Entire sample	-351.6	-327.3	-235.9
$R_m - r_f > 0$	-36.4	-19.9	38.4
$R_m - r_f < 0$	-393.2	-384.3	-391.9

8.1 Robustness Check for the Effect of Rebalancing on Liquidity Risk Premium (Varying the Risk Aversion Level)

Table A1 and A2 show that the liquidity risk premium are still negligible even if the long-run optimal weight is 50% ($\gamma = 5$) and 150% ($\gamma = 5/3$). The need to rebalance does not affect our conclusion.

Table A1: Liquidity risk premium for the benchmark setting with optimal weight as 50%

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting with optimal weight of 50% on risky asset ($\gamma = 5$), for different values of the correlation between returns and trading costs, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term level of the expected return, μ_0, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in μ_0 on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between returns and trading costs, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Liquidity Level Premium (Total)	17.14	17.76	19.26	18.75
Liquidity Risk Premium (Total)	-0.682	-0.635	-0.398	0.069
Total Liquidity Premium	16.46	17.12	18.86	18.82
liquidity risk premium for $Cov(\lambda_t, r_t)$	*0.000*	*0.047*	*0.283*	*0.750*
premium for $Cov(\lambda_t, r_t)$ *as % of total premium*	*0.0%*	*0.3%*	*1.5%*	*4.0%*

Table A2: Liquidity risk premium for the benchmark setting with optimal weight as 150%

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting with optimal weight of 150% on risky asset ($\gamma = 5/3$), for different values of the correlation between returns and trading costs, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the decrease in the long-term level of the expected return, μ_0, on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with trading costs. The liquidity risk premium is the decrease in μ_0 on the trading-cost-free risky asset that the investor requires to be indifferent between having access to the risky asset without rather than with *time variation* in trading costs. The premium for $Cov(\lambda_t, r_t)$ is calculated as the liquidity risk premium additional to the liquidity risk premium for the case with zero correlation between returns and trading costs, $Corr(u_t, v_t) = 0$. All premiums are reported in basis points (bps).

in bps	$Corr(u_t, v_t)$			
	0	***-0.2***	***-0.4***	***-0.6***
Liquidity Level Premium (Total)	15.12	13.70	12.98	11.31
Liquidity Risk Premium (Total)	-0.569	-0.473	-0.354	-0.077
Total Liquidity Premium	14.55	13.22	12.62	11.24
liquidity risk premium for $Cov(\lambda_t, r_t)$	*0.000*	*0.096*	*0.215*	*0.492*
premium for $Cov(\lambda_t, r_t)$ *as % of total premium*	*0.0%*	*0.7%*	*1.7%*	*4.4%*

8.2 Relation between Monthly Market Turnovers and Monthly Market Returns

Since small and active investors usually react to the price changes within a month, the correlation between market returns and innovations in market turnover can be higher in monthly frequency than in annual frequency. In this section, we also document the correlation between monthly market returns and monthly innovations in market turnover for comparison.

Similarly, we do an AR(1) regression for the ln values of both turnover and ILLIQ to capture the monthly innovations in market turnover and liquidity. Both the market turnover and ILLIQ are quite persistent at monthly frequency. The time persistency of monthly market turnover, ρ^{trn} in equation (19), is 0.980, and the R square of equation (19) is 0.959. The time persistency of monthly ILLIQ, ρ^{MILLIQ} in equation (20), is 0.993, and the R square of equation (20) is 0.980.

Figure A1 plots the innovations in monthly market turnover and their corresponding market excess returns for each month from 1966 January to 2010 December, and Figure A2 plots the innovations in monthly market turnover and their corresponding innovations in market ILLIQ. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June). In Figure A1, we could see that on average innovation in market turnover becomes larger when the market excess return becomes either more positive or more negative, and Figure A2 shows there is a weak negative correlation between the innovations in market turnover and market ILLIQ.

Consistent with the figures, Table A3 reports a correlation of 0.106 between the monthly market returns and the innovations in market turnover, a correlation of -0.255 for negative returns, and a correlation of 0.309 for positive returns. Consistent with our expectation, the magnitudes of correlations are larger for monthly frequency, and the correlation for positive returns is more positive since there are more active trades at the monthly frequency.

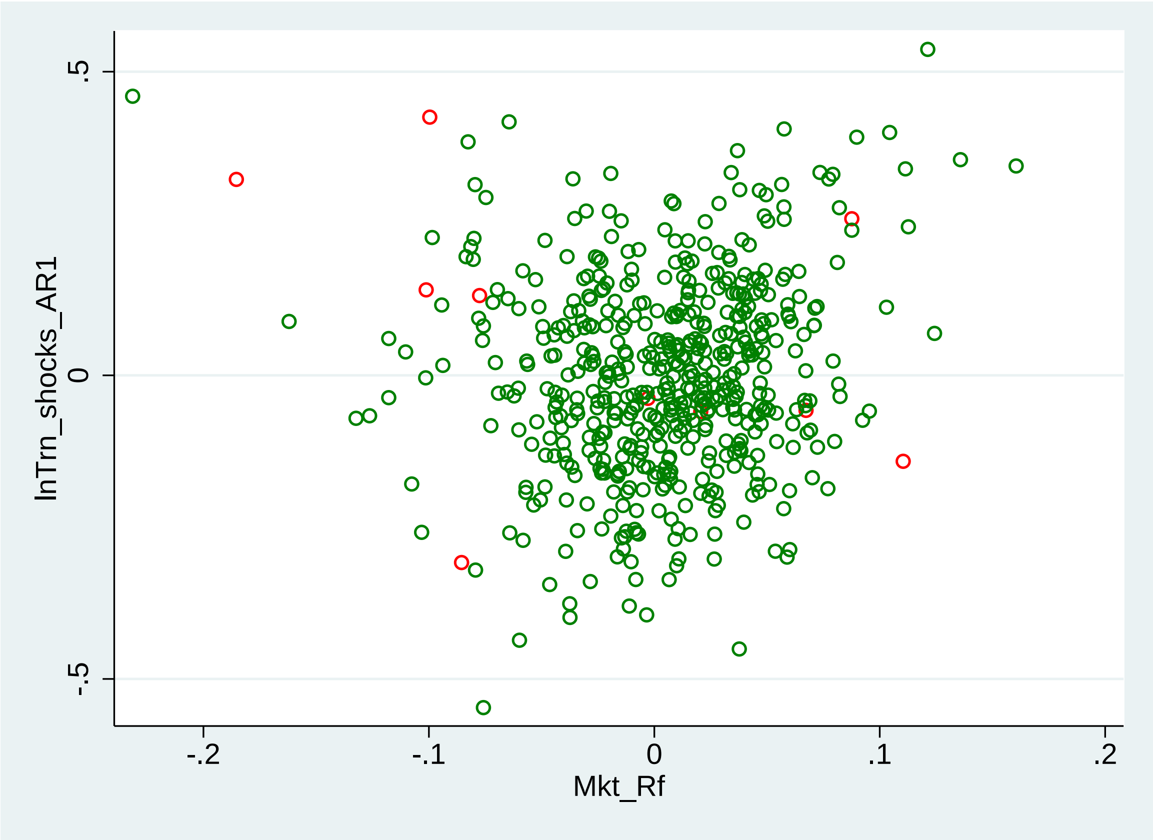


Figure A1: Relation between Innovations of Turnover and Market Returns

This figure plots the innovations in monthly market turnover and their corresponding market excess returns for each month from 1966 January to 2010 December. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June).

Figure A2: Relation between Innovations of Turnover and Market Returns

This figure plots the innovations in monthly market turnover and their corresponding innovations in monthly ILLIQ for each month from 1966 January to 2010 December. The red dots are the observations in 2008 financial crisis (2008 September to 2009 June).

Table A3: Correlation between Market Return, Innovations in ILLIQ and Turnover (monthly)

This table reports the correlation between the monthly market excess returns, innovations in monthly ILLIQ and innovations in market turnover from 1966 January to 2010 December. Panel A for the entire sample, Panel B for the observations with negative market excess returns, and Panel C for the observations with positive market excess returns.

Panel A: Entire sample

	Rm-rf	$\Delta lnILLIQ$	$\Delta lnTrn$
Rm-rf	1	$-.411^{***}$	$.106^{**}$
$\Delta lnILLIQ$		1	$-.156^{***}$
$\Delta lnTrn$			1

Panel B: $Rm - Rf < 0$

	Rm-rf	$\Delta lnILLIQ$	$\Delta lnTrn$
Rm-rf	1	$-.278^{***}$	$-.255^{***}$
$\Delta lnILLIQ$		1	$-.054$
$\Delta lnTrn$			1

Panel C: $Rm - Rf > 0$

	Rm-rf	$\Delta lnILLIQ$	$\Delta lnTrn$
Rm-rf	1	$-.139^{**}$	$.309^{***}$
$\Delta lnILLIQ$		1	$-.172^{***}$
$\Delta lnTrn$			1

References

[1] Adrian, T., and Shin, H., 2010, Liquidity and leverage, *Journal of Financial Intermediation* 19, 418–437.

[2] Acharya, V., and Pedersen, L., 2005. Asset pricing with liquidity risk, *Journal of Financial Economics*, Elsevier, vol. 77(2), 375-410

[3] Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5, 31-56.

[4] Amihud, Y., and Mendelson, H. 1986. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2), 223-249.

[5] Ang, A., S. Gorovyy, and G. B. van Inwegen. 2011. Hedge Fund Leverage. *Journal of Financial Economics.*

[6] Ang, A., Papanikolaou, D., and Westerfield, M. M. 2014. Portfolio choice with illiquid assets. *Management Science*, 60(11), 2737-2761.

[7] Aragon, G. O., and P. Strahan. 2012. Hedge Funds as Liquidity Providers: Evidence from the Lehman Bankruptcy. *Journal of Financial Economics.*

[8] Beber, A., Driessen, J. and Tuijp, P., 2012. Pricing liquidity risk with heterogeneous investment horizons. Working paper, Tilburg University.

[9] Ben_David, I., Franzoni, F. and Moussawi, R. 2012. Hedge Fund Stock Trading in the Financial Crisis of 2007–2009. *Review of Financial Studies.*

[10] Bertsimas, D. and Lo, A.W., 1998. Optimal control of execution costs. *Journal of Financial Markets*, 1(1), pp.1-50.

[11] Brown, D. B., B. I. Carlin, and M. S. Lobo. 2010. Optimal Portfolio Liquidation with Distress Risk. *Management Science* 56:1997–2014.

[12] Brunnermeier, M. K., and S. Nagel. 2004. Hedge Funds and the Technology Bubble. *Journal of Finance* 59:2013–40.

[13] Brunnermeier, M. K., and L. H. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22:2201–38.

[14] Cella, C., Ellul, A., and Giannetti, M. 2013. Investors' horizons and the amplification of market shocks. *Review of Financial Studies*, hht023.

[15] Chan, L. K., and Lakonishok, J. 1997. Institutional equity trading costs: NYSE versus Nasdaq. *The Journal of Finance*, 52(2), 713-735.

[16] Chordia, T., A. Sarkar, and A. Subrahmanyam. 2005. An Empirical Analysis of Stock and Bond Market Liquidity. *Review of Financial Studies* 18:85–129.

[17] Collin-Dufresne, P., Daniel, K., Moallemi, C. C., and Saglam, M. 2012. Dynamic Asset Allocation with Predictable Returns and Transaction Costs. Working Paper, SFI.

[18] Constantinides, G. M. 1986. Capital market equilibrium with transaction costs. *The Journal of Political Economy*, 842-862.

[19] Diamond, P. A. 1982. Aggregate demand management in search equilibrium. *The Journal of Political Economy*, 881-894.

[20] Fama, E.F., and French, K.R., 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427-465.

[21] Gârleanu, N., and Pedersen, L. H. 2013. Dynamic trading with predictable returns and transaction costs. *The Journal of Finance*, 68(6), 2309-2340.

[22] Gromb, D., and Vayanos, D. 2009. Financially constrained arbitrage and cross-market contagion. Department of Finance, London School of Economics and Political Science.

[23] Gromb, D., and D. Vayanos. 2010. Limits of Arbitrage: The State of the Theory. *Annual Review of Financial Economics* 2:251–75.

[24] Grossman, S.J., Laroque, G. 1990 Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica* 58(1):25–51.

[25] He, Z., Khang, I. G., and Krishnamurthy, A. 2010. Balance sheet adjustments during the 2008 crisis. *IMF Economic Review*, 58(1), 118-156.

[26] Huang, M. 2003. Liquidity shocks and equilibrium liquidity premia. *Journal of Economic Theory*, 109(1), 104-129.

[27] Jang, B., Koo, H., Liu, H., and Loewenstein, M. 2007. Liquidity premia and transaction costs. *The Journal of Finance*, 62(5), 2329-2366.

[28] Kahl, M., Liu, J., Longstaff, F.A. 2003 Paper millionaires: How valuable is stock to a stockholder who is restricted from selling it? *Journal of Financial Economics.* 67(3):385–410.

[29] Keim, D. B., and Madhavan, A. 1997. Transactions costs and investment style: An inter-exchange analysis of institutional equity trades. *Journal of Financial Economics*, 46(3), 265-292.

[30] Korajczyk, R.A. and Sadka, R., 2004. Are momentum profits robust to trading costs?. *The Journal of Finance*, 59(3), pp.1039-1082.

[31] Koren, M., Szeidl, A. 2003 Portfolio choice with illiquid assets. CEPR Discussion Paper 3795, Centre for Economic Policy Research, London

[32] Liu, H. 2004. Optimal consumption and investment with transaction costs and multiple risky assets. *The Journal of Finance*, 59(1), 289-338.

[33] Lo, A.W., Mamaysky, H., and Wang, J. 2004 Asset prices and trading volume under fixed transactions costs. *Journal of Political Economy.* 112(5): 1054–1090.

[34] Longstaff, F. A. 2001. Optimal portfolio choice and the valuation of illiquid securities. *Review of Financial Studies*, 14(2), 407-431.

[35] Longstaff, F.A. 2009 Portfolio claustrophobia: Asset pricing in markets with illiquid assets. *American Economic Review.* 99(4):1119–1144

[36] Lynch, A. W., and Tan, S. 2011. Explaining the Magnitude of Liquidity Premia: The Roles of Return Predictability, Wealth Shocks, and StateDependent Transaction Costs. *The Journal of Finance*, 66(4), 1329-1368.

[37] Manconi, A., Massa, M., and Yasuda, A. 2012. The role of institutional investors in propagating the crisis of 2007–2008. *Journal of Financial Economics*, 104(3), 491-518.

[38] Mitchell, M., L. Pedersen, and T. Pulvino. 2007. Slow-moving Capital. *American Economic Review P&P* 97:215–20.

[39] Nagel, S. 2011. Evaporating Liquidity. Working Paper, Stanford University.

[40] Pastor, L., and Stambaugh, R.F., 2003. Liquidity risk and expected stock returns. *Journal of Political Economy*, 111, 642-685.

[41] Scholes, M. 2000. Crisis and Risk Management. *American Economic Review* 90:17–21.

[42] Schwartz ES, Tebaldi C 2006 Illiquid assets and optimal portfolio choice. NBER Working Paper 12633, National Bureau of Economic Research, Cambridge, MA.

[43] Vayanos, D. 1998 Transaction costs and asset prices: A dynamic equilibrium model. *Review of Financial Studies*. 11(1):1–58.

[44] Vayanos, D. 2004. Flight to quality, flight to liquidity, and the pricing of risk. Working paper, London School of Economics.