Network for Studies on Pensions, Aging and Retirement

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Discussion Paper 03/2010-044

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First draft: April-17-2009, this version: March-30-2010

Abstract At the end of the deferment period a deferred annuity's policyholder can choose between receiving annuity payouts or the capital accumulated. Considering stochastic mortality improvements, the lump-sum option could be of potential value for the policyholder. Whenever mortality improves less than expected at contract inception, the policyholder will choose the lump-sum and buy an annuity at current market prices. Otherwise, he will retain the more favorable deferred annuity. We use a realistically calibrated life-cycle model and calculate the welfare gains of deferred annuities considering stochastic mortality. Our results are relevant for individual retirement planning, pension system design, and insurance pricing.

Keywords Stochastic Mortality, Deferred Annuitization, Retirement Decisions, Dynamic Intertemporal Utility Maximization

JEL-Classification D14, D81, D91, G11, G22, J11, J26

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Acknowledgements: Financial support from Deutsche Forschungsgemeinschaft via the SFB 649 „Economic Risk" is gratefully acknowledged. For their comments and suggestions I would like to thank Jiajia Cui, Nadine Gatzert, Katja Hanewald, Moshe A. Milevsky, Antoon Pelsser, Peter Schotman, Anthony Webb, and the conference participants at Annual Conference of the German Insurance Science Association and Netspar Pension Workshop 2009 Stockholm, and seminar participants at Humboldt-Universität zu Berlin.

1. Introduction

A life-annuity guarantees the policyholder (annuitant) an income stream as long he is alive in exchange for paying a certain amount of money (premium) to the insurer. Buying an annuity avoids the risk of outliving one’s money. Conversely, if the individual chooses to self-annuitize (i.e., manage his own pay-out plan), the individual may end up with zero wealth and no income while still alive (see Horneff et al., 2008a and the references cited therein). Consequently, in the literature, annuitization is regarded as quite valuable for risk-averse individuals (see Section 2).

A deferred annuity, i.e., an annuity that does not pay out immediately but at some predefined future date, typically comes with an option-like right for the insured. At the end of the deferral period, he may either choose to receive annuity payouts or receive the accumulated capital as a lump-sum.

The mortality table used to calculate these payouts is typically agreed upon at contract inception. Considering stochastic mortality improvements, such an option can be of potential value. Whenever mortality improves less than originally expected, the annuitant will choose the lump-sum and buy an annuity on the market at a better price. However, if mortality improves more than expected, he will choose to retain the deferred annuity. A deferred annuity thus protects the individual from the risk of high annuity prices in the future while providing the opportunity to make a better deal in case of low prices.[1]

For the perspective of an insurance company financial pricing approaches for deferred annuities and inherent options were developed in Milevsky and Promislow (2001), Boyle and Hardy (2003), Pelsser (2003), Biffis and Millossovich (2006), Ballotta and Haberman (2006), and Toplek (2007).

In this contribution, we take the perspective of a risk-averse individual facing incomplete markets who wants to maximize expected utility. We use a realistically calibrated life-cycle consumption/saving/asset allocation model and calculate the welfare gains of deferred annuities under stochastic Lee-Carter mortality taking borrowing and short-selling constraints into consideration. Our results are of considerable interest for individual retirement planning and for policymakers, especially if legislation makes annuitization, at

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[1] In this respect the deferred annuity's lump-sum option differs from the typical lump-sum option inherent in some U.S. employer-sponsored defined benefit (DB) plans. The lump-sum that can be cashed out of these DB plans is calculated based on the actuarial value of future benefit payments valid at the time of exercising the option (see U.S. Department of Labor, 1998). In consequence, a typical DB plan does not protect the individual from adverse mortality developments.

least in part, mandatory as has occurred in the United Kingdom (see Cannon and Tonks, 2008). Our results also reveal the maximal price above the expected value a risk-averse individual would pay for deferred annuities and the willingness to pay for the mortality-related option.

Our results confirm the findings of the optimal annuitization literature: annuitization is found to be welfare enhancing considering deferred annuities with a lump-sum option in a stochastic mortality environment. This option related to fluctuations in mortality, however, appears to be of little value to individuals, with higher values found for middle-aged, patient, and risk-averse individuals.

The remainder of this article is structured as follows. Section 2 contains a literature survey. The stochastic process for mortality is introduced in Section 3. The formal model is developed and calibrated in Section 4. Results are presented in Section 5 and a summary and discussion are found in Section 6.

2. Related Literature

Initiated by the seminal work of Yaari (1965), a broad literature has developed that investigates optimal strategies involving *immediate annuities* under *deterministic mortality*, for example, with respect to the amount of wealth to be annuitized, optimal timing of annuity purchases, or the type of annuity to be purchased.[2] According to this literature, being able to insure longevity risk via annuitization generally increases the utility of a risk-averse individual. In reality however, annuitization rates are much lower than one would expect from the results of these models (see, e.g., Moore and Mitchell, 1997), a contradiction that is called the "annuity puzzle." The literature suggests several explanations for this puzzle, including: annuities may be too expensive due to adverse selection; annuities may induce a suboptimal consumption profile or asset allocation, bequest motives, the crowding-out effect of government pensions, intra-family risk sharing, the insolvency risk of the

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[2] See Richard (1975), Kotlikoff and Spivak (1981), Merton (1983), Friedman and Warshawsky (1988), Brugiavini (1993), Mitchell et al. (1999), Brown and Poterba (2000), Brown (2001), Milevsky and Young (2003, 2007), Dushi and Webb (2004), Vidal and Lejárraga (2004, 2006), Davidoff et al. (2005), Babbel and Merrill (2006), Hainaut and Devolder (2006), Post et al. (2006), Gupta and Li (2007), Lopes and Michaelides (2007), Inkmann et al. (2007), Horneff et al. (2008b, 2008c, 2008d, 2009), Huang et al. (2008), Purcal and Piggott (2008), Yogo (2008), Chai et al. (2009), Davidoff (2009), Koijen et al. (2009), Peijnenburg et al. (2009).

insurer, and various forms of background risk. Furthermore, behavioral biases, like framing or ambiguity aversion, are found to induce a low demand (Agnew et al., 2008; Albis and Thibault, 2008; Brown et al., 2008).

In the context of *immediate annuitization, stochastic mortality* is analyzed by Menoncin (2008), Van de Ven and Weale (2008), Cocco and Gomes, 2009, Schulze and Post (2009), and Stevens (2009). Menoncin (2008) studies consumption and asset allocation decisions under stochastic mortality of an agent having access to longevity bonds. The model does not impose borrowing or short-selling constraints and allows for continuous trading in the longevity bond. The individual can adjust his mortality risk hedging portfolio continuously. Menoncin (2008) does not account for the irreversibility of annuitization decisions or for the fact that hedging opportunities for most individuals are far less than perfect. It is shown that a longevity bond is always welfare enhancing and that the share invested should decrease over the lifetime because the uncertainty surrounding future mortality developments decreases with the decreasing length of the planning horizon. Cocco and Gomes (2009) extend the model used in Menoncin (2008) by allowing for stochastic labor income and a retirement phase. In this setting, the allocation of savings to longevity bonds exhibits a hump-shaped profile over the life-cycle, due to the dominating impact of labor income risk in early life. Schulze and Post (2009) analyze the annuity demand of an individual who is able to buy annuities only at a certain age with no opportunity to cancel or sell the contract later, thus taking into consideration the irreversibility of annuitization decisions. The authors show, given shocks to mortality rates are mean preserving, that annuity demand in a two-period model framework is not influenced by introducing stochastic mortality if the argument of the utility function (consumption) is stochastically independent of mortality risks. However, in their analysis of situations involving mortality-driven insolvency risk of the annuity provider or background risk induced by a mortality dependent government pension income stream, they find that annuity demand may, dependent on the severity of the insolvency risk, increase or decrease compared to a situation without stochastic mortality or without such dependencies. In a multi-period framework Stevens (2009) shows that the presence of stochastic mortality generally reduces the attractiveness of immediate annuities available at some date in the future, because their payouts become subject to risk. Van de Ven and Weale (2008) find that annuity demand could as well increase with higher uncertainty around the expected life span when general-equilibrium effects are taken into account.

The optimal demand for *deferred annuities* under *deterministic mortality* is studied in Horneff and Maurer (2008) and in Gong and Webb (2010). Horneff and Maurer (2008) find that optimal strategies involving deferred annuities are very similar to strategies involving immediate annuities. Because they do not consider stochastic mortality in their model, they do not take into consideration the option features included in deferred annuities. Gong and Webb (2010) show that under reasonable assumptions about actuarial unfairness, deferred annuities might be preferred over immediate annuities due to a better mortality credit versus loading tradeoff.[3]

The demand for *deferred annuities* under *stochastic mortality* is studied in Stevens (2009). In this analysis, however, the deferred annuity does not include a lump-sum option that means the possibility of asymmetrically benefiting from mortality developments. Overall, the results in this case show only small welfare gains compared to immediate annuities, since both products have rather similar exposure to stochastic mortality risk.

Milevsky and Kyrychenko (2008) study the welfare and asset allocation implications of an option, similar to the one considered here, that is included in some variable annuity contracts. The so-called guaranteed minimum income benefit (GMIB) option allows the annuitant to convert a fixed amount of money at a specified date via guaranteed annuity rates or to take the money

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[3] Stevens et al. (2010) study optionality features in defined benefit plans with respect to single life annuities versus joint and survivor annuities under stochastic mortality. Their focus is, however, the design of pension plans regarding the management of mortality risk.

and buy annuities on the market. However, Milevsky and Kyrychenko (2008) do not consider stochastic mortality. In their analysis, the option's value is solely driven by the stochastic investment return of the money invested in the annuity, which determines whether or not the annuitant should exercise the GMIB option.

In summary, in the literature, immediate and deferred annuitization has been analyzed under deterministic and stochastic mortality. We extend this literature by considering a deferred annuity having a lump-sum option in a stochastic mortality environment. In this setting we are able to analyze a product feature that allows the policyholder to asymmetrically benefit from uncertain mortality developments.

3. The Stochastic Process for Mortality

Several models for stochastic mortality are discussed in the literature (see, e.g., Cairns et al., 2008; Plat, 2009a). The model we use is one of the earliest proposed and now one of the most widely used—the Lee-Carter model (Lee and Carter, 1992). According to this model, the log of the central death rate $m_{x,t}$ for a given age x at time t is given by

$$\ln(m_{x,t}) = a_x + b_x k_t \, , \tag{1}$$

where a_x and b_x are age-specific constants and k_t, the mortality index, is a random variable, whose realization defines a complete mortality table for given values of a_x and b_x.[4]

As done by Cocco and Gomes (2009), we follow Lee and Carter (1992) and Lee (2000), with assuming that k_t follows a random walk with drift.[5] Thus k_t is given by

$$k_t = k_{t\text{-}1} + \theta + \varepsilon_t \, , \tag{2}$$

where ε_t is normally distributed with $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{Std}[\varepsilon_t] = \sigma_\varepsilon$. The final variable of interest for the expected-utility maximization and annuity pricing framework, the one-period survival probability for age x at time t, $p_{x,t}$, is then given by[6]

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[4] As in Bauer and Weber (2008), we ignore age-specific mortality shocks.

[5] Checks with our data confirmed that central assumptions of this ARIMA (0,1,0) model are not violated (difference stationary for k_t, normality and independence of Δk_t).

[6] The conversion of central death rates into survival rates is based on the approximation given in Cairns et al. (2008).

$$p_{x,t} = 1 - m_{x,t} / (1 + 0.5m_{x,t}), \tag{3}$$

which means that the individual and the insurer hold symmetric beliefs as to the distribution of future mortality[7] and that there is no difference between individual mortality and aggregate mortality.

4. Preferences, Decisions Alternatives, and Optimization Problem

4.1 Preferences

The individual derives utility from consumption C (all monetary variables are in nominal terms) over his stochastic lifespan. The intertemporally separable utility function $U(C)$, following the standard discounted utility model, is defined as:

$$U(C) = \sum_{t=0}^{T-x_0} \delta^t \left(\prod_{i=0}^{t} p_{i+x_0,0} \right) U_t(C_t), \tag{4}$$

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[7] For annuity demand under asymmetric mortality beliefs (i.e., information uncertainty) and heterogeneity of mortality rates in the population of annuitants, see Brugavani (1993) and Sheshinski (2007).

where T denotes the maximum lifespan, x_0 the individual's current age, δ the subjective discount factor, and $p_{x,0}$ the individual's probability of surviving from age x to $x+1$ given the mortality table information at $t=0$. The individual has no bequest motives; thus, the one-period CRRA-utility function $U_t(C_t)$, with γ as the coefficient of relative risk aversion, is given by:

$$U_t\left(C_t\right)=\begin{cases}\log\left(\dfrac{C_t}{\left(1+\pi\right)^t}\right), \text{for } \gamma=1\\ \dfrac{\left(\dfrac{C_t}{\left(1+\pi\right)^t}\right)^{1-\gamma}-1}{1-\gamma}, \text{otherwise}\end{cases} \tag{5}$$

as long as the individual lives; 0 otherwise. Nominal consumption at time t, C_t, is adjusted for inflation at rate π.

4.2 Decision Alternatives

4.2.1 General Decisions in Each Period

At each point in time, t, the individual must decide on the amount of wealth to be consumed, C_t, which implicitly determines savings, S_t. Wealth at time t is denoted by W_t. Savings $S_t = W_t - C_t$ are invested at the risk-free return R_f. The individual cannot borrow money. Initial wealth is given by W_0.

4.2.2 Only Immediate Annuities are Available

We compare two annuitization decision alternatives. Under the first, the individual can buy only immediate annuities with nominally fixed and constant payouts at age 65 ($t = 65 - x_0$). He annuitizes amount A_I, with $0 \leq A_I \leq W_{65-x_0}$. For every \$1 annuitized, the annuity pays out a_I. The insurance company prices the annuity according to the principle of equivalence, given information about the mortality index k_{65-x_0}, but may include a loading factor L, with $L \geq 0$. Given that the individual and the insurer hold symmetric beliefs regarding the distribution of k_t, the annuity payout per \$1 annuitized is derived at age 65 according to:

$$1 = (1+L) \cdot a_I \cdot \mathrm{E}_{65-x_0}\left[\sum_{j=1}^{T-65} \frac{\left(\prod_{i=0}^{j-1} p_{i+65,65-x_0+i+1} \right)}{(R_f)^j} \right], \qquad (6)$$

where E_t denotes the expected value operator with respect to the information available at time t.[8] Immediate annuities, as well as deferred annuities (in the

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[8] Pricing the annuity as in Equation (6) does not explicitly account for the possibility that mortality risks may be systematic, i.e., be stochastically dependent of capital market returns. This independence assumption follows the model used in Gründl et al. (2006) and can also be justified by the almost negligibly small risk premiums for systematic

pay-out phase), are irreversible decisions, i.e., the policies cannot be sold or canceled.

Figure 1 illustrates how the individual's consumption and wealth evolve over time (conditional on survival) given that only immediate annuities are available.

-- *Figure 1 about here* --

The utility the individual receives in this scenario (the first annuitization decision alternative) serves as the benchmark utility, i.e., it is the utility in a world without deferred annuities.

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mortality risk estimated in Friedberg and Webb (2007). Note, however, that the loading factor L, can be interpreted as an implicit risk premium charged for systematic mortality risk. See also Dahl and Møller (2008); Ludkovski and Young (2008); Delong (2009); Bayraktar et al. (2009) for financial pricing approaches under stochastic mortality, and Van de Ven and Weale (2007, 2008) for a general equilibrium analysis of annuity pricing under stochastic mortality. For the case of stochastic investment returns, see, e.g., Huang et al. (2009) or Nielsen and Zenios (1996).

4.2.3 Immediate and Deferred Annuities are Available

Under the second alternative, both deferred annuities and immediate annuities are available. In contrast to (future) immediate annuities, a deferred annuity allows the policyholder at present to lock-in a mortality table used for the calculation of an annuity starting to pay out at some future date. Hereby, it protects the policyholder from adverse mortality developments which would increase the prices for immediate annuities sold at this future date. However at the same time, due to the lump-sum option inherent in a deferred annuity, the policyholder can still participate in favorable mortality developments.

The deferred annuity considered here is a variant of a variable annuity that allows the policyholder a maximum amount of flexibility during the accumulation phase with respect both to consumption purposes and the amount to be annuitized at retirement age. The only parameter that is fixed at $t = 0$ is the future conversion factor a_D, i.e., the payout per \$1 of wealth annuitized at age 65 should the individual not opt for taking the lump-sum. This flexibility is achieved by a variable annuity having the following contract characteristics:[9]

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[9] For an overview of contract characteristics and options of variable annuities, see, e.g., Bauer et al. (2008).

- Single premium payment, paid at $t = 0$;
- Money that is invested at $t = 0$ is accumulated in a fund earning the risk-free return R_f (as savings outside an annuity would earn),
- Guaranteed minimum death benefit (during the deferral period) (GMDB) equal to the fund value
- Guaranteed minimum withdrawal benefit (GMWB) smaller or equal to the fund value (during the deferral period);
- Guaranteed minimum income benefit (GMIB) granting the annuity payout per $ of the fund value at retirement age or the right to take the fund value at the end of the deferral period as a lump-sum.

In summary, then, because the amount invested at $t = 0$ earns the same return as private savings, withdrawals are possible, and in the case of death during the deferral period all remaining money would be paid out to heirs; the resulting contract structure is identical to the situation before age 65 where no annuities (or only immediate annuities) are available, i.e., private savings are perfectly replicated in the product. Formally, the fund value of the deferred annuity at the beginning of each period can identically be denoted by W_t before taking out money, and by S_t. afterward. Consequently, during the deferral period, we will abstract from the existence of the contract. The only difference between this situation and the one where only immediate annuities are available is that, at retirement age, the individual can choose between the conversion factors of the deferred annuity and, by taking out the fund value as a lump-sum, the conversion factors given by the market in annuitizing his money.

Thus, at age 65, the individual can flexibly annuitize his wealth through this annuity by converting amount A_D or refuse to annuitize and buy an immediate annuity with a price based on mortality information available at $t = 65 - x_0$ for paying the premium A_I, when exercising the lump-sum option.

Figure 2 illustrates the evolution of the individual's consumption and wealth over time (conditional on survival) given that both immediate and deferred annuities are available.

-- *Figure 2 about here* --

The deferred annuity's payout per \$1 of wealth annuitized is given by:

$$1 = (1+L)\cdot a_D \cdot \mathrm{E}_0\left\{\sum_{j=1}^{T-65}\frac{\left(\prod_{i=0}^{j-1} p_{i+65,65-x_0+i+1}\right)}{(R_f)^j}\right\}. \qquad (7)$$

This pricing mechanism is very similarly to that of Equation (6); the only difference being in the expected value operator, which is now conditional on the information available at $t = 0$. Note that in order to derive the *maximal* increase in utility an annuitant could derive from this product, we do not include any price adjustment that accounts for the option inherent in this annuity. In other words, we are concerned with how much the individual would be willing to pay to have this option.

In general, the budget restriction at age 65 under the second alternative is $0 \leq A_I + A_D \leq W_{65-x_0}$, with neither A_I and A_D allowed to be negative. However, at age 65, depending on realization of the mortality index k_{65-x_0}, the individual will utilize only one of the two annuity products. Whenever $k_{65-x_0} < \mathrm{E}_0(k_{65-x_0})$, i.e., mortality rates are smaller than expected at $t = 0$, the individual will utilize the deferred annuity since doing so will result in obtaining a higher payout for the annuity than that available on the current market. In case $k_{65-x_0} > \mathrm{E}_0(k_{65-x_0})$, he will buy annuities priced at current market rates. If $k_{65-x_0} = \mathrm{E}_0(k_{65-x_0})$, both types of annuity have the same payout per \$ of premium and the individual is indifferent between them, as choosing an immediate annuity will yield the same utility as retaining the deferred one.

4.3 Calibration of Model Parameters

The Lee-Carter stochastic process for mortality is calibrated using U.S. data from 1950-2005 obtained from the Human Mortality Database. The model is estimated separately for males and females for the age group 30-100. Following Brouhns et al. (2002), we estimate the parameters with an iterative maximum likelihood algorithm, thereby assuming a Poisson distribution for the number of deaths. As done by Plat (2009a), we utilize the software package "LifeMetrics" for estimation.[10] The estimated parameters are given in Appendix A.

For the risk-free return, we use the sample mean of U.S. T-Bill returns as a proxy. Using the same sample period as for the Lee-Carter estimation (1950 to 2005), R_f is set to 1.0493 (see Morningstar, 2007). For inflation, we use the same sample period, resulting in a value of 0.0390 (see Morningstar, 2007).

The coefficient of relative risk aversion γ is set to 1, 2, 3, 4 or 5 and the subjective discount factor δ is set to 0.93 or 0.99, both of which are typical values in the literature (see, e.g., Laibson et al., 1998).

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[10] See www.lifemetrics.com.

The loading factor L is either set to 0 (no loading) or to 0.1, which is in the range of pricing markups for the U.S. annuity market reported in Mitchell et al. (1999).

4.4 Objective Function and Solving Technique

The individual's objective is to maximize the expected utility of consumption:

$$\max_{C_t, P_I, P_D} \mathrm{E}_0\left(U\left(C\right)\right), \tag{8}$$

subject to consumption constraints:

$$\begin{aligned}
&C_0 = W_0 - S_0 \\
&C_t = \underbrace{S_{t-1} R_f}_{W_t} - S_t \quad \forall \quad t \in \left\{1, 2, ..., 64 - x_0\right\} \\
&C_{65-x_0} = \underbrace{S_{64-x_0} R_f}_{W_{65-x_0}} - S_{65-x_0} - A_I - A_D \\
&C_t = \underbrace{S_{t-1} R_f + a_I A_I + a_D A_D}_{W_t} - S_t \quad \forall \quad t \in \left\{66 - x_0, 67 - x_0, ..., T - x_0\right\},
\end{aligned} \tag{9}$$

subject to borrowing constraints:

$$
\begin{aligned}
&C_0 = W_0 - S_0 \\
&0 \le S_t \le W_t \quad \forall \quad t \in \{1, 2, \ldots, 64 - x_0, 66 - x_0, 67 - x_0, \ldots, T - x_0\} \\
&0 \le S_{65-x_0} + A_I + A_D \le W_{65-x_0}
\end{aligned}
\tag{10}
$$

and subject to no-short-sale constraints:

$$0 \le A_I,\ 0 \le A_D. \tag{11}$$

The optimization problem (Equations (8)–(11)) is solved backward via stochastic dynamic programming. The Bellman equation for this problem depends on three state variables: time t, wealth W_t, and the mortality index k_t. The Bellman equation (with V denoting the value function) is given for $t = 0, 1, \ldots, T - x_0 - 1$ by

$$V_t\left(W_t, k_t\right) = \max_{C_t, A_I, A_D} \left\{ U_t\left(C_t\right) + \delta\, \mathrm{E}_t\left[p_{x,t}\left(V_{t+1}\left(W_{t+1}, k_{t+1}\right)\right)\right]\right\}, \tag{12}$$

subject to the constraints of Equations (9)–(11).[11] In the last period, remaining wealth is consumed, and the value function is given by $U_{T-x_0}\left(W_{T-x_0}\right)$. The Bellman equation (Equation (12)) cannot be solved analytically; hence a numerical technique is used. First, at each point in time t, the W_t-state and the

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[11] Note that the decision on the optimal values for A_I and A_D is made only at $t = 65 - x_0$.

k_t-state spaces are discretized into a grid of $N \times M$ points, W_t^n, with $n = 1, 2, \ldots, N$, and k_t^m, with $m = 1, 2, \ldots, M$. To calculate the distribution of the one-period survival probabilities $p_{x,t}$, the distribution of the mortality index k_t is discretized using Gaussian quadrature methods. Since in the last period (i.e., at $t = T - x_0$), the value function $V_{T-x_0}\left(W_{T-x_0}\right)$ is given by $U_{T-x_0}\left(W_{T-x_0}\right)$, the numerical solution algorithm starts at the penultimate period (i.e., at $t = T - x_0 - 1$). For each (W_t^n, k_t^m) combination, Equation (12) is solved with the MATHEMATICA® 7.0 implemented nonlinear optimizer `NMaximize`, yielding the optimal decisions $C_t^{nm}\,(W_t^n, k_t^m)$, $P_I^{nm}\left(W_{65-x_0}^n, k_{65-x_0}^m\right), P_D^{nm}\left(W_{65-x_0}^n, k_{65-x_0}^m\right)$, and the function value of $V_t(W_t^n, k_t^m)$. Next, a continuous function is fitted to the points $V_t(W_t^n, k_t^m)$, which delivers a continuous approximation of the value function $V_t(W_t, k_t)$. Finally, the problem is rolled back to the preceding period.

5. Results

5.1 The Welfare Gain Measure

To calculate the welfare gain of deferred annuitization, we use an equivalent wealth variation measure (see, e.g., Munk, 2000; Brown, 2001). The general idea is to compare the expected utility of an individual having access only to immediate annuities with an individual who has access to both immediate and deferred annuities and express it in monetary terms.

The reference point for our analyses is the welfare gain at some point in time t the individual achieves through the availability of only immediate annuities $WG_{t,\,I}$. This welfare gain is calculated by comparing the expected utility of an individual having no access to any type of annuity versus an individual having access to immediate annuities. For the point in time where the decision about whether to invest into a deferred annuity or to wait needs to be made, i.e., for $t = 0$, $WG_{0,\,I}$, is derived in Equation (14), i.e., by solving Equation (13) for $\Delta W_{0,\,I}$ and dividing it by W_0 to obtain a relative measure:

$$V_0\left(W_0, k_0 \mid A_I \geq 0, A_D = 0\right) = V_0\left(W_0 + \Delta W_{0,I}, k_0 \mid A_I = 0, A_D = 0\right), \tag{13}$$

$$WG_{0,\,I} = \Delta W_{0,\,I} \,/\, W_0. \tag{14}$$

$WG_{0,\,I}$ measures how much expected utility increases at present, i.e., at $t = 0$, translated into monetary terms when the individual can access the immediate annuity market (vs. having no access). Note that due to the CRRA-feature of the one-period utility function (Equation (5)), $WG_{0,\,I}$, for each combination of model parameters, is a constant, i.e., independent of W_0.

The welfare gain in the case that both immediate and deferred annuities are available (vs. no annuities at all), $WG_{0,\,ID}$, is derived according to Equations (15) and (16):

$$V_0\left(W_0, k_0 \mid A_I \geq 0, A_D \geq 0\right) = V_0\left(W_0 + \Delta W_{0,ID}, k_0 \mid A_I = 0, A_D = 0\right), \tag{15}$$

$$WG_{0,\,ID} = \Delta W_{0,\,ID} / W_0. \tag{16}$$

To measure the sole impact of introducing deferred annuities into the market, the incremental welfare gain $WG_{0,\,D}$ is given by:

$$WG_{0,\,D} = WG_{0,\,ID} - WG_{0,\,I}. \tag{17}$$

5.2 Numerical Results

To illustrate the impact of randomness in future mortality rates on future annuity payouts, we first show, in Figure 3, the distribution of payouts from an immediate annuity a_I and the fixed payout of the deferred annuity a_D at age 65 for an individual aged 30 at $t = 0$.

-- *Figure 3 about here* --

Figure 3 illustrates the option inherent in the deferred annuity. If the payout falls to the left of the dashed horizontal lines depicting the fixed payout from the deferred annuity a_D, the individual would stay with the deferred annuity.

If, however, the payout falls to the right of the dashed lines, the individual would exercise the lump-sum option and buy immediate annuities.

The randomness of payouts influences both the welfare gains finally realized from immediate annuitization $WG_{65-x_0,I}$ or deferred annuitization $WG_{65-x_0,ID}$ at age 65, as well as the optimal amounts of money, A_I or A_D, to be annuitized. An example of both impacts, again for an individual initially aged 30, is shown in Figures 4 and 5. Here, the welfare gains and optimal amounts of money to be annuitized are shown as a function in the realized value of the mortality index at age 65 k_{65-x_0}.

-- *Figure 4 about here* –

-- *Figure 5 about here* --

Figure 4 shows that the welfare gain of annuitization is increased by the availability of deferred annuities when the mortality index realizes at relatively low values. This is the case when mortality has decreased more than expected and the conversion factor from the deferred annuity grants better rates than the market. Furthermore, Figure 4 illustrates that stochastic mortality has an impact not only on the price of annuities, but on the utility evaluation as well, because the survival probabilities work as weights for future utility (compare

Equations (4) and (12)). Due to this, the welfare gain for an individual who stays with the deferred annuity, even though the conversion factor is a constant, is not independent from the realized mortality index. If the individual annuitizes via the deferred annuity, the realized survival probabilities, i.e., the weights for future utility, are comparably high, and thus the welfare gain of deferred annuitization increases the smaller the realized mortality index becomes. This effect also explains why even in case of annuitizing via fixed-conversion-factor deferred annuities, optimal annuitization as shown in Figure 5 is a function in the realized mortality index.[12] For immediate annuities, this is of course also the case, because the mortality credit and thus the conversion factor a_I increases for higher values for the realized mortality index, leading to an upward-sloping curve for $WG_{65-x_0,I}$.

We next analyze the welfare gain at the point in time when the decision about investing savings in the deferred annuity fund must be made ($t = 0$). In particular, we look at the impact of model parameters on the welfare gain. As

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[12] Note, however, that the non-annuitized part of wealth in the parameter combination used in Figure 5 is always being completely consumed, i.e., $W_{65-x_0} - A_I - A_D - C_{65-x_0} = 0$. This gives a 100% annuitization rate after considering consumption. For other parameter combinations, especially while setting the coefficient of relative risk aversion γ to 1, small amounts of savings can be observed, i.e., annuitization is below the standard 100% result.

a measure of welfare gain we concentrate on the incremental welfare gain $WG_{0,D}$ the individual experiences through the availability of deferred annuities (compare Equation (17)). Figure 6 plots the incremental welfare gain as a function in the initial age of the individual x_0, the relative risk aversion parameter γ, and the subjective discount factor δ.

-- *Figure 6 about here* --

Figure 6 reveals the striking result that the incremental welfare gain at $t = 0$ is small, ranging between 0.09% and 0.39% of the individual's initial resources. Deferred annuitization can improve welfare at age 65 considerably, compared to immediate annuitization (compare Figure 2), but, from the perspective of the present, i.e., the age when the decision on investing savings into a deferred annuity has to be made, the incremental welfare gain is small. Two factors are responsible for this effect. First, the probability of realizing very large welfare gains from deferred annuities is rather small, as can be seen from the 99% confidence band for the realization of the mortality index at age 65, shown in Figure 4 for an individual aged 30. Second, the incremental welfare gain possibly realized at age 65 is evaluated at present time, i.e., after being discounted for many periods with the subjective discount factor δ and the survival probabilities (compare Equation (4)). The discounting effect is confirmed by comparing the curve for the subjective discount factor $\delta = 0.93$ with the higher welfare gains curve showing $\delta = 0.99$.

Both effects result in a hump-shaped age profile of the incremental welfare gain. For younger individuals, future welfare gains are heavily discounted, yielding an increasing function in age first. The older the individual is at $t = 0$, the fewer periods there are for mortality to fluctuate (the 99% confidence band for k_{65-x_0} becomes smaller). Due to this, the option value of deferred annuities decreases in initial age, which explains the decreasing part of the function,[13] where the effect of less heavily discounting is overcompensated by the shrinking option value.

Increasing risk aversion leads to larger incremental welfare gains because optimal annuitization increases, and the welfare gain of annuitization increases.

As a further variation in the model input parameters, we look at the impact of gender and the loading factor L on incremental welfare gains $WG_{0,D}$. The

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[13] This confirms the results of Menoncin (2008), who shows that the demand for mortality hedging instruments is decreasing over the life-cycle.

results can be found in Table 1, together with the welfare gains $WG_{0,\,ID}$ for initial ages 30 and 50.

-- *Table 1 about here* --

Gender has an impact on the welfare gains of deferred annuitization $WG_{0,\,ID}$, resulting in higher gains for males. Males have lower survival probabilities and thus the mortality credit of the annuity is larger for them. For males the incremental gains $WG_{0,\,D}$ are also higher at age 50, confirming that annuitization is more utility enhancing for them. The incremental gains are nearly identical for both genders at age 30 because the originally higher gains for males at annuitization age 65 are (due to their lower survival probabilities) more heavily discounted to $t = 0$, which is more pronounced for younger individuals.

The impact of the loading factor on the welfare gains of deferred annuitization $WG_{0,\,ID}$, is straightforward. Making annuities more expensive decreases their attractiveness. The incremental welfare gain $WG_{0,\,D}$, however, is only barely affected by introducing a loading. Deferred annuitization becomes less attractive but, at the same time, the benchmark for measuring the incremental gain, the welfare gain in a world with only immediate annuities $WG_{0,\,I}$, also decreases with a positive loading factor.

With respect to the pricing of deferred annuities, Table 1 indicates that the price markups above the expected value of payouts an insurer could charge would be fairly small. The maximal price markup can be calculated by setting the incremental welfare gain $WG_{0,D}$ (i.e., the amount of money the individual is willing to give up in order to have access to deferred annuities) in relation to the amount of money invested in the deferred annuity fund at $t = 0$, i.e., savings S_0. With fairly priced (expected value of payouts = price) annuities, the range for maximal price markups is 0.05% to 0.42% of the money paid into the deferred annuity fund at $t = 0$. If both immediate and deferred annuities already have a 10% loading factor, the additional price markup ranges between 0.04% and 0.42%.

In a final variation in model parameters we consider the impact of adverse selection effects. Hereby we concentrate on the subpopulation that is generally more unfavorably affected by adverse selection. That means we continue to analyze individuals having the average life expectancy as given by the calibrated mortality process used before. So far we considered annuities being priced fairly from their perspective. In reality, however, annuity prices will typically reflect the tendency that annuitants tend to live longer than the population average. Prices typically would be higher than the values calculated so far.

A deferred annuity in this setting could actually help to mitigate the effects of a certain amount of adverse selection pricing. Finkelstein and Poterba (2004) find that for UK annuities adverse selection effects are generally more pronounced for annuities being bought at higher ages. Thus, when committing to an annuity at a younger age, for example, through the purchase of a deferred annuity, the annuitant can expect to receive comparatively better rates. In this case, the conversion rates from the deferred annuity will match his life expectancy more closely than immediate annuities' rates. For the following calculations (again as in Table 1 for the individuals aged 30 or 50) we thus price deferred annuities as before (no adverse selection, loading factor $L = 0$), while immediate annuities offered in later life come with a loading for adverse selection ($L > 0$). Based on Mitchell et al. (1999), we use a fixed adverse selection loading of 9% for males and 5% for females.[14] The welfare gains in this setting are given in Table 2.

-- *Table 2 about here* --

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[14] These values are based on the adverse selection differential inherent in U.S. annuities' money's worth ratios reported in tables 3 and 5 in Mitchell et al. (1999). For our calculation we use the differential of individuals aged 65 and average between treasury and corporate discount rates results.

As expected, compared to the results shown in Table 1 for a loading factor $L = 0$, the welfare gains of deferred annuitization $WG_{0,ID}$, decrease. While deferred annuities are priced as before, the individual, in case the lump-sum option is exercised, is confronted with immediate annuities that come with a loading for adverse selection. In consequence, the lump-sum option is exercised less often. The incremental gains $WG_{0,D}$, however, reflecting the relative attractiveness of deferred annuities compared to the now more expensive immediate annuities, increase, ranging now between 0.23% and 5.31%. This shows that the potential of deferred annuities to protect against adverse selection pricing could indeed be very valuable for an individual having population average life expectancy.[15] Note, however, that these results should be interpreted cautiously since in tendency they indicate rather an upper bound of the incremental welfare gains. First, this is because it was assumed that deferred annuities do not include any loading for adverse selection. Especially at higher ages (e.g., the individuals in Table 2 aged 50) this will not be the case anymore given increase in adverse selection loadings

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[15] Furthermore, also individuals with typical annuitant mortality would benefit from the deferred annuity's feature to secure average mortality rates at younger ages. However especially in this case, for the utility evaluation, one would also need to model their mortality differential, i.e., their higher expected survival rates as well the specific stochastic process driving changes in their rates. For modeling approaches see, e.g., Brouhns et al. (2002) and Plat (2009b).

found by Finkelstein and Poterba (2004). Second, one needs to consider that the lump-sum option can be exercised at an age that is more prone to adverse selection effects, giving insurers an incentive to charge a loading for adverse selection also for deferred annuities.

6. Summary and Discussion

Deferred annuities including a lump-sum option improve the welfare of a risk-averse individual in the presence of stochastic mortality. Our analysis confirms the results in the optimal annuitization literature for the case of both deterministic and stochastic mortality for immediate and deferred annuities without a lump-sum option.

The incremental gains, that is the option value connected to stochastic mortality, of deferred annuities appear to be small. In pricing these products, an insurer can expect that CRRA-individuals are willing to pay only around 0.04% to 0.42% of the money invested in the deferred annuity fund at the beginning of the deferral period in exchange for an option right related to stochastic mortality improvements (given that the benchmark investment, the immediate annuity, comes with the same initial loading factor L). In general, the incremental gains and possible price markups are higher for individuals who are 45 to 60 years of age, are more patient and have greater risk aversion. Considering adverse selection increases in tendency the welfare gains of deferred annuities.

In contrast to actual price markups for options related to deferred annuitization in variable annuities (GMIB's), the price markups calculated here seem to leave no room for a market because the actual markups are in the range of 0.5% to 0.75% *per annum* of the fund value during the deferment period (see, e.g., Bauer et al., 2008). It should be noted, however, that the products usually also allow investment in risk assets, such as mutual funds. Thus, the price charged needs to cover more than the stochastic mortality driven part of the option, including, for example, minimum interest rate guarantees, which are much more valuable from the individual's perspective (see Milevsky and Kyrychenko, 2008).

A possible policy implication of our results is that for the sole purpose of managing individuals' risk of stochastic mortality improvements, i.e., not risks resulting from stochastic investment returns, mandatory annuitization schemes (thus markets with limited potential for adverse selection) should not necessarily require the purchase of deferred annuities because the option value from the individual's perspective is very small and may be easily overcompensated by price markups by insurance companies. Furthermore it is not guaranteed that all individuals would exercise the option inherent in deferred annuities rationally and thus realize the welfare gain at all (see Bütler and Teppa, 2009; Chalmers and Reuter, 2009). In addition, general equilibrium effects as analyzed for immediate annuities in Van de Ven and Weale (2008) would also needed to be considered.

Our work could be extended by additionally considering shocks to individual mortality, e.g., due to health risks as in Yogo (2008), Davidoff (2009), Horneff et al. (2009), and Pang and Warshawsky (2010). In this case, the option value inherent in deferred annuities will increase because the variation of mortality from the individual’s perspective will increase. Another idea for future research is to consider deferred annuities where the amount of money is already fixed at $t = 0$. In this case, the welfare gains of deferred annuitization could either increase or decrease. Increases could occur due to the higher mortality credits of such products (see Gong and Webb, 2010) because, usually, if death occurs during the deferment period no money is returned (while payouts in case of survival are higher). Decreases could occur due to the higher utility costs of inflexibility with respect to consumption needs during the deferment period and the amount of money to be annuitized at retirement age. Finally, one could account for some kind of “guarantee risk”, that means to consider that the conversion of the capital saved in the deferred annuity could be done at worse rates than originally agreed at contract inception. This could be the case when the regulatory environment explicitly requires some kind of longevity risk sharing between the annuity provider and the annuitant. Furthermore, there could be some kind of implicit guarantee risk: In case of severe mortality developments, threatening the stability of annuity insurers, insurers’ could become insolvent (for immediate annuities see Babbel and Merril, 2006; Schulze and Post 2009), or government could ex post allow insurers to (partially) walk away from their guarantee obligations, thereby protecting the stability of the overall financial sector. Considering such risks, would reduce welfare gains of deferred annuities.

Appendix A. Estimated Parameters for the Lee-Carter Model

	Males		Females			Males		Females	
Drift of $k_t : \theta$	-0.6469		-0.8001						
Standard deviation: σ_ε	0.9276		1.1891						
Age x	a_x	b_x	a_x	b_x	Age x	a_x	b_x	a_x	b_x
30	-6.2885	0.0062	-7.0779	0.0172	66	-3.4840	0.0200	-4.0889	0.0131
31	-6.2445	0.0057	-6.9874	0.0165	67	-3.4022	0.0201	-3.9982	0.0136
32	-6.1981	0.0071	-6.9210	0.0178	68	-3.3211	0.0188	-3.9095	0.0134
33	-6.1522	0.0076	-6.8452	0.0170	69	-3.2512	0.0180	-3.8317	0.0135
34	-6.1251	0.0083	-6.7823	0.0173	70	-3.1561	0.0182	-3.7182	0.0143
35	-6.0572	0.0092	-6.7025	0.0177	71	-3.0919	0.0167	-3.6475	0.0134
36	-5.9948	0.0097	-6.6222	0.0170	72	-2.9913	0.0175	-3.5231	0.0148
37	-5.9289	0.0099	-6.5439	0.0170	73	-2.9200	0.0164	-3.4379	0.0147
38	-5.8350	0.0108	-6.4380	0.0167	74	-2.8404	0.0168	-3.3472	0.0159
39	-5.7888	0.0130	-6.3785	0.0183	75	-2.7558	0.0167	-3.2408	0.0167
40	-5.7047	0.0137	-6.2880	0.0181	76	-2.6797	0.0158	-3.1523	0.0163
41	-5.6198	0.0133	-6.2053	0.0174	77	-2.6030	0.0151	-3.0638	0.0161
42	-5.5253	0.0153	-6.1022	0.0181	78	-2.5248	0.0148	-2.9606	0.0170
43	-5.4484	0.0155	-6.0179	0.0176	79	-2.4400	0.0144	-2.8583	0.0169
44	-5.3730	0.0167	-5.9495	0.0176	80	-2.3402	0.0147	-2.7522	0.0163
45	-5.2787	0.0178	-5.8614	0.0176	81	-2.2672	0.0130	-2.6701	0.0149
46	-5.1905	0.0176	-5.7724	0.0172	82	-2.1734	0.0133	-2.5531	0.0153
47	-5.0996	0.0186	-5.6842	0.0176	83	-2.0833	0.0131	-2.4439	0.0155
48	-5.0038	0.0186	-5.5907	0.0166	84	-1.9955	0.0130	-2.3374	0.0155
49	-4.9336	0.0207	-5.5211	0.0178	85	-1.9135	0.0124	-2.2358	0.0152
50	-4.8309	0.0218	-5.4255	0.0180	86	-1.8272	0.0120	-2.1341	0.0145
51	-4.7455	0.0213	-5.3471	0.0167	87	-1.7460	0.0116	-2.0349	0.0139
52	-4.6500	0.0221	-5.2529	0.0168	88	-1.6742	0.0106	-1.9470	0.0130
53	-4.5715	0.0223	-5.1765	0.0164	89	-1.5873	0.0101	-1.8468	0.0121
54	-4.4895	0.0232	-5.1029	0.0165	90	-1.5049	0.0093	-1.7357	0.0123
55	-4.3990	0.0225	-5.0233	0.0153	91	-1.4428	0.0073	-1.6695	0.0095
56	-4.3177	0.0224	-4.9364	0.0146	92	-1.3533	0.0066	-1.5642	0.0091
57	-4.2361	0.0225	-4.8590	0.0141	93	-1.2725	0.0057	-1.4724	0.0081
58	-4.1346	0.0214	-4.7565	0.0130	94	-1.2064	0.0041	-1.3845	0.0074
59	-4.0628	0.0224	-4.6865	0.0136	95	-1.1410	0.0031	-1.3080	0.0060
60	-3.9603	0.0222	-4.5843	0.0138	96	-1.0689	0.0020	-1.2336	0.0047
61	-3.8863	0.0212	-4.5062	0.0126	97	-1.0133	0.0010	-1.1625	0.0037
62	-3.7809	0.0212	-4.3944	0.0132	98	-0.9750	-0.0008	-1.1062	0.0022
63	-3.7134	0.0212	-4.3289	0.0131	99	-0.9377	-0.0031	-1.0744	-0.0007
64	-3.6387	0.0211	-4.2548	0.0134	100	-0.8901	-0.0041	-1.0005	-0.0012
65	-3.5461	0.0214	-4.1517	0.0141					

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Table 1: Welfare Gain of Deferred Annuitization $WG_{0,\,ID}$, Incremental Welfare Gain of Deferred Annuitization $WG_{0,\,D}$, and Optimal Savings S_0 / W_0 at Time t = 0 and Impact of Gender, Loading L, Relative Risk Aversion γ, and Subjective Discount Factor δ

				γ									
				1		2		3		4		5	
				δ		δ		δ		δ		δ	
Loading L	Age x_0	Gender		0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99
0	30	Male	$WG_{0,\,ID}$	1.01%	9.48%	4.51%	11.82%	7.12%	12.82%	8.76%	13.30%	9.82%	13.58%
			$WG_{0,\,D}$	0.05%	0.31%	0.14%	0.34%	0.20%	0.35%	0.23%	0.36%	0.26%	0.37%
			S_0 / W_0	92.62%	97.48%	95.36%	97.59%	96.20%	97.64%	96.61%	97.67%	96.85%	97.69%
		Female	$WG_{0,\,ID}$	0.85%	8.24%	3.71%	9.38%	5.60%	9.77%	6.69%	9.97%	7.37%	10.09%
			$WG_{0,\,D}$	0.04%	0.30%	0.13%	0.32%	0.19%	0.33%	0.22%	0.34%	0.24%	0.34%
			S_0 / W_0	92.75%	97.66%	95.48%	97.74%	96.32%	97.77%	96.73%	97.79%	96.97%	97.80%
	50	Male	$WG_{0,\,ID}$	5.41%	19.13%	13.83%	23.44%	18.55%	25.19%	21.08%	25.97%	22.54%	26.39%
			$WG_{0,\,D}$	0.13%	0.34%	0.24%	0.37%	0.28%	0.39%	0.32%	0.40%	0.34%	0.41%
			S_0 / W_0	91.71%	96.21%	94.32%	96.43%	95.14%	96.53%	95.55%	96.60%	95.80%	96.64%
		Female	$WG_{0,\,ID}$	4.49%	16.28%	11.27%	18.39%	14.48%	19.04%	16.00%	19.31%	16.83%	19.48%
			$WG_{0,\,D}$	0.12%	0.32%	0.22%	0.34%	0.27%	0.35%	0.29%	0.36%	0.30%	0.36%
			S_0 / W_0	92.05%	96.60%	94.63%	96.76%	95.44%	96.84%	95.84%	96.89%	96.09%	96.92%
0.1	30	Male	$WG_{0,\,ID}$	0.62%	6.74%	3.24%	8.79%	5.34%	9.65%	6.65%	10.05%	7.47%	10.27%
			$WG_{0,\,D}$	0.04%	0.30%	0.14%	0.34%	0.20%	0.36%	0.24%	0.37%	0.27%	0.38%
			S_0 / W_0	92.62%	97.48%	95.39%	97.62%	96.25%	97.68%	96.66%	97.72%	96.91%	97.74%
		Female	$WG_{0,\,ID}$	0.43%	5.14%	2.29%	6.04%	3.63%	6.33%	4.37%	6.46%	4.82%	6.55%
			$WG_{0,\,D}$	0.03%	0.29%	0.14%	0.32%	0.19%	0.33%	0.23%	0.34%	0.25%	0.35%
			S_0 / W_0	92.75%	97.66%	95.51%	97.77%	96.37%	97.82%	96.78%	97.84%	97.02%	97.86%
	50	Male	$WG_{0,\,ID}$	3.48%	13.99%	10.19%	17.83%	14.19%	19.35%	16.29%	20.00%	17.46%	20.34%
			$WG_{0,\,D}$	0.11%	0.33%	0.23%	0.36%	0.28%	0.38%	0.32%	0.39%	0.34%	0.40%
			S_0 / W_0	91.71%	96.21%	94.41%	96.51%	95.26%	96.64%	95.68%	96.72%	95.94%	96.77%
		Female	$WG_{0,\,ID}$	2.48%	10.76%	7.39%	12.53%	9.90%	13.03%	11.02%	13.22%	11.59%	13.34%
			$WG_{0,\,D}$	0.10%	0.31%	0.22%	0.33%	0.26%	0.35%	0.29%	0.35%	0.31%	0.36%
			S_0 / W_0	92.05%	96.60%	94.72%	96.84%	95.56%	96.95%	95.98%	97.01%	96.23%	97.04%

Table 2: Welfare Gain of Deferred Annuitization $WG_{0,ID}$, Incremental Welfare Gain of Deferred Annuitization $WG_{0,D}$, and Optimal Savings S_0 / W_0 at Time $t = 0$ and Impact of Adverse Selection Pricing, Loading L for Deferred Annuities = 0 and for Immediate Annuities = 0.09 (Males) or 0.05 (Females)

			γ									
			1		2		3		4		5	
			δ		δ		δ		δ		δ	
Age x_0	Gender		0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99	0.93	0.99
30	Male	$WG_{0,ID}$	0.95%	9.10%	4.34%	11.40%	6.87%	12.38%	8.47%	12.85%	9.49%	13.12%
		$WG_{0,D}$	0.34%	2.40%	1.11%	2.65%	1.56%	2.78%	1.86%	2.86%	2.06%	2.91%
		S_0 / W_0	92.62%	97.48%	95.36%	97.59%	96.21%	97.64%	96.62%	97.67%	96.86%	97.69%
	Female	$WG_{0,ID}$	0.80%	7.90%	3.56%	9.01%	5.38%	9.39%	6.44%	9.59%	7.09%	9.71%
		$WG_{0,D}$	0.21%	1.55%	0.70%	1.66%	0.97%	1.71%	1.15%	1.74%	1.27%	1.77%
		S_0 / W_0	92.75%	97.66%	95.48%	97.74%	96.33%	97.78%	96.73%	97.80%	96.97%	97.81%
50	Male	$WG_{0,ID}$	5.22%	18.63%	13.48%	22.90%	18.13%	24.63%	20.63%	25.41%	22.06%	25.82%
		$WG_{0,D}$	1.68%	4.49%	3.18%	4.91%	3.81%	5.11%	4.20%	5.23%	4.45%	5.31%
		S_0 / W_0	91.71%	96.21%	94.33%	96.44%	95.15%	96.54%	95.56%	96.61%	95.82%	96.65%
	Female	$WG_{0,ID}$	4.31%	15.81%	10.94%	17.89%	14.10%	18.53%	15.59%	18.80%	16.39%	18.96%
		$WG_{0,D}$	1.00%	2.70%	1.88%	2.86%	2.23%	2.94%	2.43%	2.98%	2.56%	3.00%
		S_0 / W_0	92.05%	96.60%	94.63%	96.77%	95.45%	96.85%	95.85%	96.90%	96.10%	96.93%

Figure 1: Evolution of the Individual's Consumption and Wealth Over Time When Only Immediate Annuities are Available

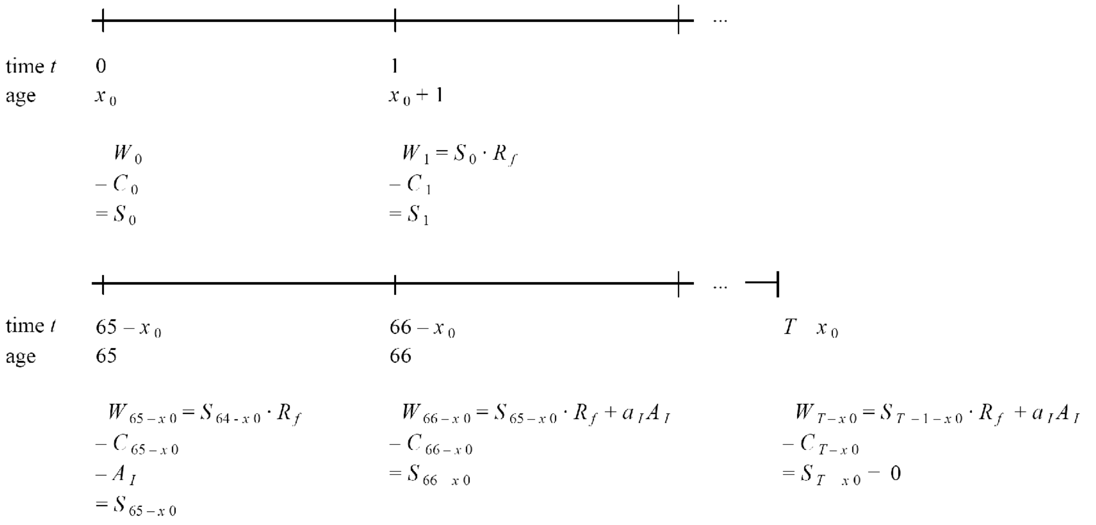

Figure 2: Evolution of the Individual's Consumption and Wealth Over Time When Both Immediate and Deferred Annuities are Available

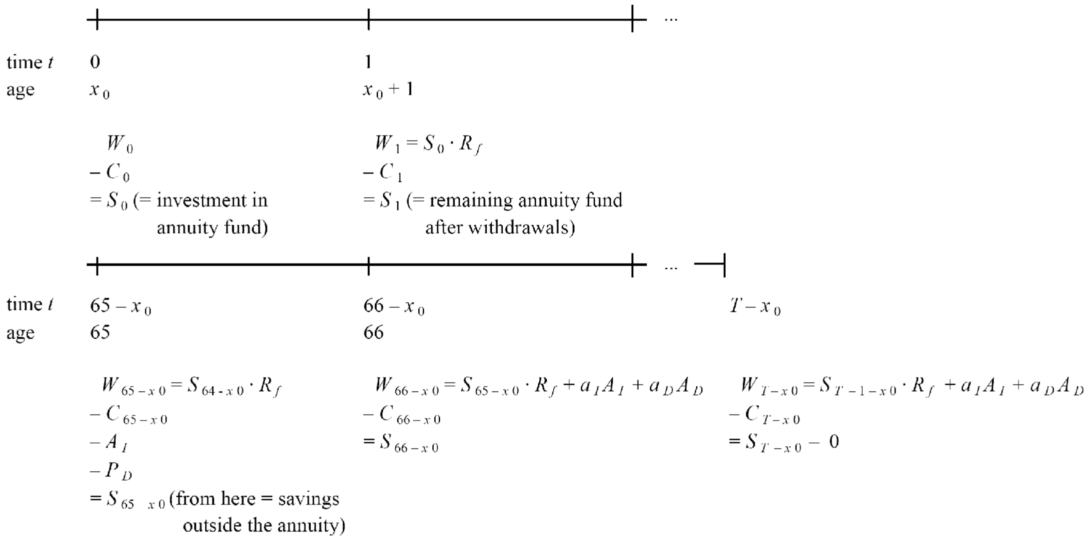

Figure 3: Distribution of Payouts per \$1 of Wealth Annuitized for Immediate Annuity a_I and Deferred Annuity a_D at Age 65 for a at $t = 0$ 30-Year-Old Individual; Loading $L = 0$

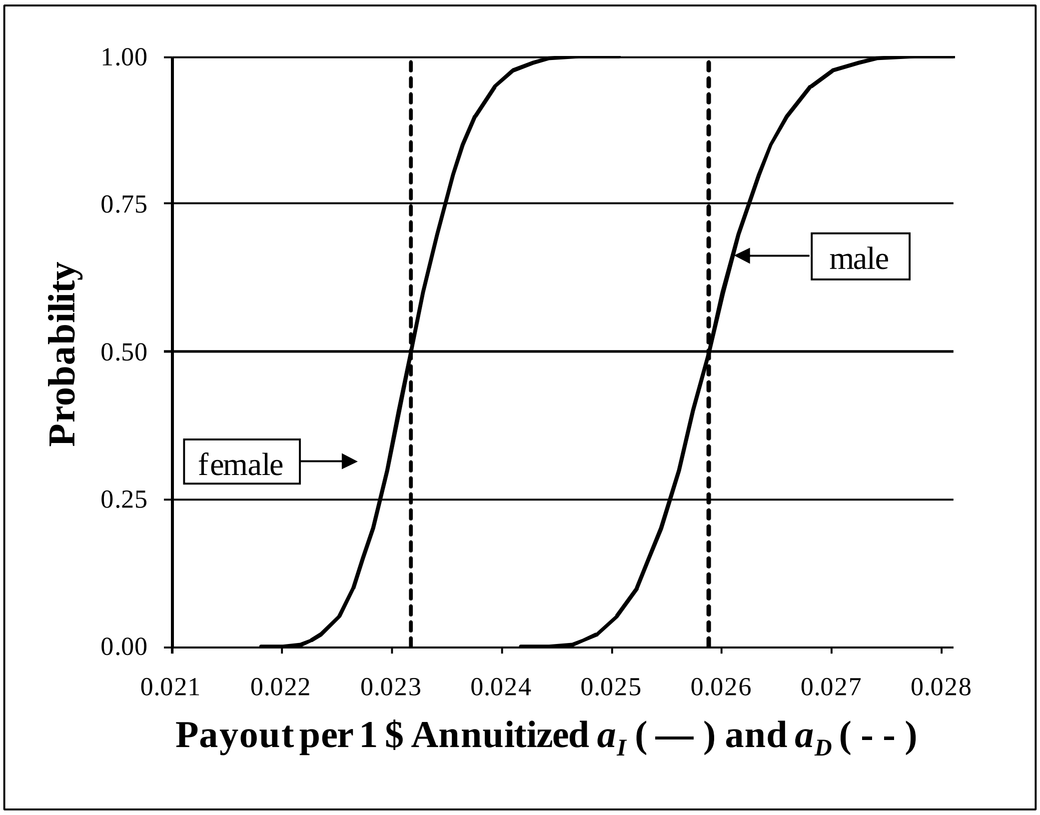

Figure 4: Welfare Gain of Immediate Annuitization $WG_{65-x_0,I}$ vs. Deferred Annuitization $WG_{65-x_0,ID}$ at Age 65; Initial Age x_0 = 30, Gender = Male, γ = 2, δ = 0.99, Loading L = 0

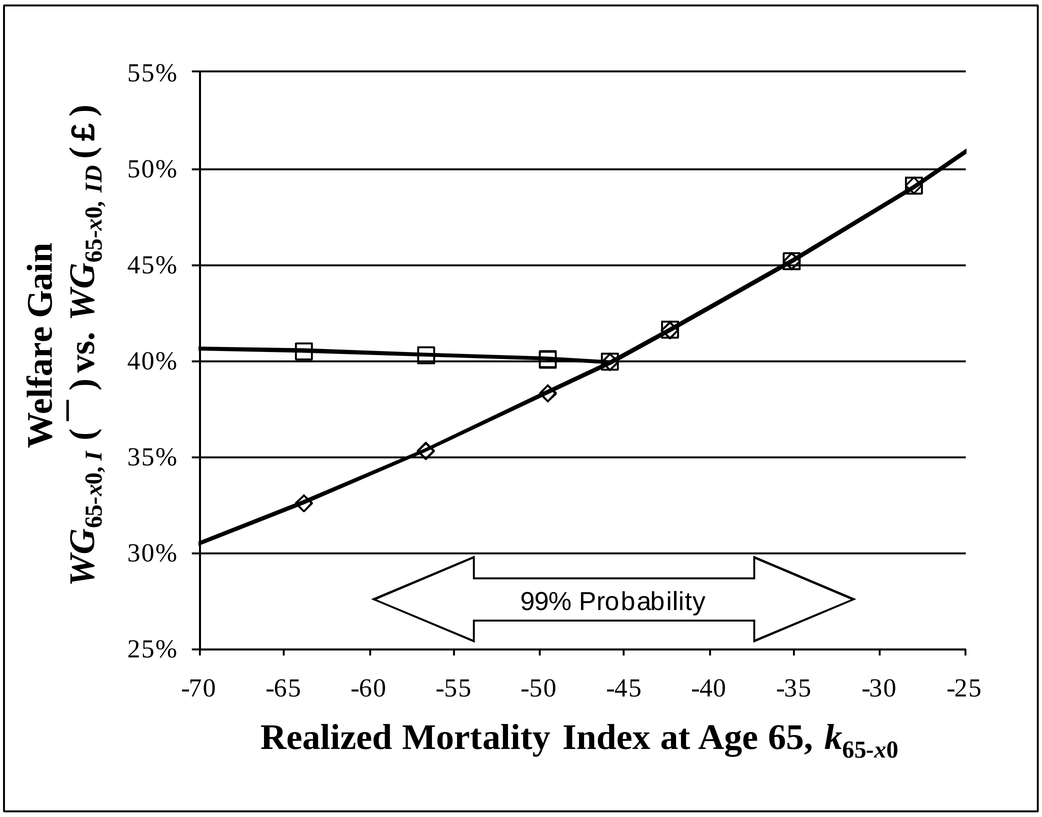

Figure 5: Optimal Amount of Money Annuitized at Age 65 as a Fraction of Wealth at Age 65 A_I / W_{65-x_0} vs. A_D / W_{65-x_0}; Initial Age x_0 = 30, Gender = Male, γ = 2, δ = 0.99, Loading L = 0

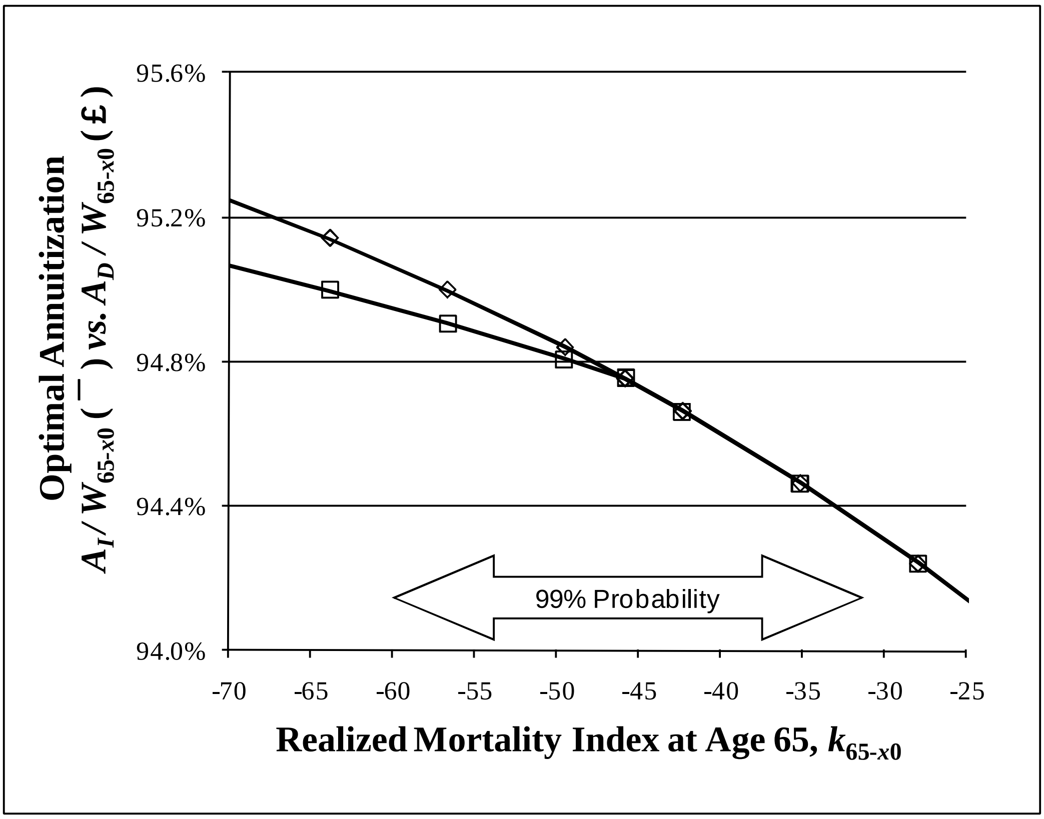

Figure 6: Incremental Welfare Gain of Deferred Annuitization $WG_{0,D}$ at Time $t = 0$ as a Function in Age at $t = 0$ x_0; Gender = Male, Loading $L = 0$