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The Public Market Equivalent and Private Equity Performance

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Abstract

We provide a formal and rigorous justification for the Kaplan and Schoar (2005) public market equivalent (“PME”) measure of historical performance of private equity (“PE”) funds. The PME provides a valid economic performance measure when the investor (“LP”) has log-utility preferences and the return on the LP’s total wealth equals the market return. Notably, the PME is valid regardless of the risk of PE investments, and it is robust to variations in the timing and systematic risks of the underlying cash flows along with potential GP manipulations. Formally, we show that the PME is a generalized method-of-moments estimator using the stochastic discount factor implied by Rubinstein (1976)’s version of the capital asset pricing model.

Keywords: Private equity, public market equivalent, PME, log utility CAPM, ex-post performance evaluation, generalized method of moments.

JEL Classification: G12, G23, G24, G31.

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This analysis considers the practical problem of evaluating the performance of private equity (“PE”) funds, such as buyout (“BO”) or venture capital (“VC”) funds. These funds hold privately-held companies, without quoted market valuations, and hence they have no regularly quoted month-to-month or quarter-to-quarter market returns. This raises the problem of evaluating performance based on their cash flows alone. A natural starting point is to calculate the internal rate of return (“IRR”). The IRR, however, has several well-known problems: It is an absolute performance measure that does not adjust for the market return or the risk of the investments. It implicitly assumes investors can reinvest cash flows at the same rate. The IRR may not exist, and it may not be unique. Moreover, investors in PE funds (“LPs”) have been concerned that PE funds manipulate their IRRs by deliberately choosing the timing and size of their investments. Consequently, in their evaluation of the Kauffman Foundation’s VC investments, Mulcahy, Weeks, and Bradley (2012) recommend that LPs “Reject performance marketing narratives that anchor on internal rate of return (IRR), top quartile, vintage year, or gross returns,” and instead “Adopt PME as a consistent standard for VC performance reporting.”

The public market equivalent (“PME”),¹ introduced by Kaplan and Schoar (2005), is an alternative performance measure that evaluates performance based on cash flows alone. The PME calculation works as follows. Let $X(t)$ denote the cash flow from the fund to the LP at time t . This cash-flow stream is divided into its positive and negative parts,

¹An earlier but slightly different measure that was also originally called the public market equivalent, but later became known as the ACG Index Comparison Method, was introduced by Long and Nickels (1996). Long (2008) discusses various versions of these measures and their relationships. Rouvinez (2003) develops the PME⁺ version to account for portfolio constraints arising from the LPs’ trading and reinvesting.

called distributions, $dist(t)$, and capital calls, $call(t)$. A distribution is a cash flow that is returned to the LP from the PE fund (net of fees) after the fund successfully sells a company. Capital calls are the investments by the LP into the fund, including the payment of ongoing management fees. Distributions and capital calls are then discounted using the realized market returns over the same time periods, and the PME is the ratio of the two resulting valuations:

$$PME = \frac{\sum_t \frac{dist(t)}{1+r_M(t)}}{\sum_t \frac{call(t)}{1+r_M(t)}}. \quad (1)$$

The sum runs over the life of the fund, and $r_M(t)$ is the realized market return from the inception of the fund (at $t = 0$) to the time of the distribution or capital call. A PME greater than one suggests that the value of the distributions exceeds the cost of the capital calls, meaning that the LP has benefitted from the investment in the fund.

From a standard valuation perspective the PME looks odd. In the standard capital asset pricing model (“standard CAPM”) the present value (“PV”) is calculated by discounting expected cash flows using the discount rate:

$$r_{CAPM} = r_F + \beta (E[r_M] - r_F). \quad (2)$$

Here, r_F is the risk-free rate, beta measures the cash flow’s systematic risk ($\beta = cov(r_M, r_{PE})/var(r_M)$), and $E[r_M] - r_F$ is the expected market risk premium. The PV is then:

$$PV = \sum_t \frac{E[X(t)]}{(1 + r_{CAPM})^t}. \quad (3)$$

This standard CAPM is the textbook approach to valuation. Investments that earns higher returns than their cost of capital, r_{CAPM} , have positive PVs.

The PME calculation in equation (1) seems quite different from the standard CAPM. The standard CAPM has no role for the realized market return, which is used as the discount rate in the PME calculation. Instead, the standard CAPM uses the expected market risk premium. Moreover, the PME calculation seems to be missing the beta that adjusts for the systematic risk of the cash flows.

Informally, the PME is sometimes justified as the LP's valuation if capital calls had instead been invested in the market portfolio and distributions are reinvested into the market. Alternatively, when the beta is one, the discount rate in the standard CAPM is just the expected market return, which looks similar to the discount rate in the PME calculation, ignoring the difference between expected and realized market returns. This intuition led Kaplan and Schoar (2005) to argue that the PME is valid when the beta equals one. Obviously, the beta may not be one, particularly for buyout funds, which invest in highly levered transactions.

In contrast, we provide a formal analysis of the PME measure. We show that it is a surprisingly robust and applicable performance measure, under three conditions, neither of which is particularly controversial nor restrictive: (1) markets are frictionless and the "law-of-one-price" holds; (2) the LP has log utility (or more generally, Kreps-Porteus (1978) preferences with relative risk aversion of one); and (3) the return on the LP's total wealth portfolio equals the return on the public market.

Under these conditions, the PME is valid economic measure, with several benefits. It is simple to calculate and apply. This simplicity arises because the PME is robust to differences

in risk and leverage, and changes in these, so the PME can be calculated without any knowledge of any risk measures (such as betas). Further, the PME is difficult to manipulate by GPs.

1 Stochastic Discount Factors

To analyze the PME, it is useful to review the concept of a stochastic discount factor (“SDFs”). This material is standard, and the details can be found in any advanced asset-pricing textbook. Cochrane (2001) and Skiadas (2009) provide excellent discussions of SDFs.

Starting at time $t = 0$, an investor can value a risky future cash-flow stream as follows. At each point in time, t , a state of the world is realized, denoted s_t , with probability $p(s_t)$, as assessed by the investor. The market return varies across states, and the return on the market earned from time 0 to time t , in state s_t , is denoted $r_M(s_t)$. The stochastic discount factor (“SDF”), denoted $m(s_t)$, then gives the investor’s valuation (at time 0) of a future cash flow of one dollar at time t , in state s_t , scaled by its probability. From this basic building block, the investor can value any risky cash-flow stream. For example, the value of the cash flow $X(s_t)$ is:

$$PV = \sum_t \left[\sum_{s_t} p(s_t) m(s_t) X(s_t) \right] = \sum_t E [m(s_t) X(s_t)] . \quad (4)$$

Here, $m(s_t)$ is the stochastic discount factor that discounts the cash flow at time t in state s_t . Since the state s_t occurs with probability $p(s_t)$, the investor’s PV in this state is $p(s_t)m(s_t)X(s_t)$.

The SDF is a fundamental building block of valuation models. Any set of prices or

valuations that are internally consistent (formally, when the “law-of-one-price” holds) can be generated by some SDF. For risk-averse investors, cash flows that are higher when the market is down are more valuable, because they insure the investor against downturns. Since the SDF values cash flows across states, the SDF should be larger in those states where the market return is poor.

To illustrate, the SDF representation of the standard CAPM is:

$$m(s_t) = a - b[1 + r_M(s_t)] , \tag{5}$$

where a and b are constants that reflect the risk-free rate, the market risk, and the market price of risk (see Dybvig and Ingersoll (1982) and Hansen and Richard (1987)). This linear SDF implies that the risk of any investment is summarized by its correlation with the market, as measured by β , and it is unnecessary to track cash flows across individual states. This is a main advantage of the standard CAPM, and this model provides the foundation of most valuations that are done in practice.

1.1 Rubinstein CAPM

Rubinstein (1976) derived an alternative capital asset pricing model (“Rubinstein CAPM”). Suppose that markets are frictionless, and that an investor is infinitely lived and chooses lifetime consumption and investment plans, subject to budget constraints, to maximize lifetime expected utility as represented by the time-separable logarithmic (log) utility function:

$$U(c(s_t)) = \sum_t \beta^t E[\ln(c(s_t))] . \tag{6}$$

With log-utility preferences, the investor's SDF is $m(s_t) = [c(s_t)/c(0)]^{-1}$, where $c(s_t)/c(0)$ is the investor's consumption growth from time 0 to state s_t at time t . With log-utility, the investor consumes a constant fraction of total wealth, so $c(s_t)/c(0) = 1 + r_W(s_t)$, where $r_W(s_t)$ is the return on the investor's total wealth. Assuming $r_W = r_M$, it follows that:

$$m(s_t) = \frac{1}{1 + r_M(s_t)}. \quad (7)$$

This non-linear SDF is different from the linear SDF from the standard CAPM, so it implies a different valuation model. Under the Rubinstein CAPM, equation (4) implies that cash flows are valued by discounting with realized market returns, and the PV of $X(s_t)$ is:

$$PV = \sum_t E \left[\frac{X(s_t)}{1 + r_M(s_t)} \right]. \quad (8)$$

Both the Rubinstein and standard CAPMs imply that idiosyncratic risks are diversified away, and only systematic risk is priced, although the two models differ in their pricing of this risk. Under the standard CAPM, the systematic risk is summarized by beta. A higher beta gives a larger discount rate, and this constant rate is used for cash flows across all states. In contrast, in the Rubinstein CAPM, the discount rate varies across states, even when the systematic risk remains constant. Cash flows that pay off when the market is low are discounted at a smaller rate, giving a higher PV, as they should have.

Formally, the PV of a one-period cash flow under the Rubinstein model is $E[X/(1+r_M)]$, which equals $E[X] \times E[1/(1+r_M)] + COV[X, 1/(1+r_M)]$. The last term is the covariance term between the cash flow and the (inverse) market return. Hence, discounting with realized market returns implicitly captures systematic risk.

1.2 Comments

The investors's standard (time-separable) log-utility preferences can be generalized. When the investors's preferences belong to the scale-invariant Kreps-Porteus (1978) class with relative risk aversion of one, the SDF remains $m(s_t) = 1/(1 + r_W(s_t))$, and the analysis holds (see Skiadas 2009 and Kim 2013). Log utility is one example of such preferences, along with the more general preferences from Epstein and Zin (1989) and Albuquerque, Rui, Eichenbaum, and Rebelo (2013).

We assume $r_W = r_M$. This assumption holds, in equilibrium, when markets are complete and all investors have the same beliefs and preferences, with constant relative risk aversion of one. More generally, without complete markets and with heterogeneous agents, the equality $r_M = r_W$ may fail. In this case, an investor with log-utility should discount cash flows using r_W instead of r_M as the stochastic discount rate.

Several other SDFs have been proposed for evaluating PE and VC performance. Driessen, Lin, and Phalippou (2012) propose a GMM estimator with the SDF on the form $m = 1/\prod(1 + r_F + \alpha + \beta r_M)$ where r_F is the risk free rate and r_M is the return on the market, which can be viewed as a measure of r_W for some constants α and β . Ang, Chen, Goetzmann, and Phalippou (2013) build on this approach and estimate the unobserved time series of quarterly PE returns from observations of investor cash flows. Korteweg and Nagel (2013) value VC investments using the SDF, $m = \exp[a - b \ln(1 + r_M)]$. These different SDFs imply different risk adjustments, and determining which of these SDFs that provides the most appropriate risk adjustment is an important empirical question that goes beyond the scope

of this paper.

2 Practical Example

The following example illustrates the application of the two valuation methods. Over a three-year period, the market return is given by the binomial tree in Panel A of Table 1. In each year, the market either appreciates by 20% or depreciates by 10%, with a 50% probability as illustrated in Panel C. This is obviously a simplified market structure, but it simplifies the calculations and nothing hinges on this simplification.

In the Rubinstein CAPM, the market determines the risk-free rate and market risk premium. The risk-free rate is $r_F = 2.86\%$ and the market risk premium is $E[r_M] - r_F = 2.14\%$.²

Consider a PE investments with the cash flows given in Panel B of Table 1. Initially, at time 0, an LP invests \$1,000 into the PE fund. During the first year, the fund matures, and after the second year, depending on the market return, it returns either \$405, \$585, or \$845. After the third year, the fund returns either \$364.50, \$526.50, \$760.50, or \$1,098.50.³ We do not explicitly model fees, but these cash flows can be interpreted as the net-of-fees cash flows. The expected cash flows, calculated using the probabilities in Panel C of Table 1, are reported in Table 2.

Valuing the cash flow using the Rubinstein CAPM is straightforward. After observing the cash flows from a number of funds, under various market conditions, these cash flows are

²The risk-free rate is calculated using $(1 + r_F)^{-1} = E[m]$, which holds generally for SDFs. Given the risk-free rate, the market risk premium can be calculated directly as $E[r_M] - r_F$.

³These cash flows are constructed by assuming that the PE fund's underlying valuation increases by 30% when the market appreciates and decreases by 10% when the market depreciates; and half the PE fund's value is return after the second period and the remainder is returned at the end. But this underlying structure is unimportant for valuing the resulting cash flow, and our analysis holds for arbitrary cash flows.

discounted using the corresponding market returns and averaged. The valuations for each year are shown in Table 2. The total PV is \$107.68.

The standard CAPM is more complicated. First, calculate the average cash flows in each year, $E[X]$, as reported in Table 2. Next, the beta must be determined. Below, we show that beta is 1.29, but in practice it must be estimated from publicly-traded comparable companies. The standard CAPM discount rate is then, $r_{CAPM} = r_F + \beta \times (E[r_M] - r_F) = 2.86\% + 1.29 \times 2.14\% = 5.64\%$, and the expected cash flows are discounted at this rate. The resulting PV is \$106.99. Despite the simpler calculation with the Rubinstein model, the risk adjustments are similar and the two models produce almost identical valuations.

2.1 Valuation using SDFs

It is also illustrative to value the PE cash flows using the SDFs, shown in Table 3. The standard CAPM has the SDF, $m = a - b(1 + r_M)$, where a and b are determined by the risk-free rate and the market risk premium. For each year we can solve for a and b .⁴ In year one, we have $a_1 = 1.944$ and $b_1 = 0.926$; in the second year, $a_2 = 1.871$ and $b_2 = 0.840$; in the third year, $a_3 = 1.801$ and $b_3 = 0.762$. Using these values, we calculate the SDF as shown in Panel A of Table 3. In contrast, the Rubinstein SDF is simply $m = 1/(1 + r_M)$, as shown in in Panel B of Table 3.

To value the PE cash flows using each SDFs we multiply the cash flows by the SDF and take the average, following equation (4). The resulting valuations are numerically identical

⁴The a and b are calculated as follows. In each year, the following equations hold for all SDFs: $E[m] = (1 + r_F)^{-1}$ and $E[m(1 + r_M)] = 1$. Substituting the linear expression for m into these two equations results in two equations with two unknowns, a and b , which are solved by inverting a two-by-two matrix.

to those reported in Table 2, with PVs of \$107.68 and \$106.99. In fact, for the standard CAPM, we determined the beta of 1.29 from the SDF, by solving for the implied beta, so the two valuations are identical by construction.

2.2 Effects of Leverage

Since the Rubinstein CAPM remains valid regardless of the investment risk, it is robust to changes in leverage. PE funds cannot inflate their PME's above 1 by leveraging up.⁵

To see this, consider an investor that invests \$1,000 in the market, but only puts up half of this amount, and borrows the remaining \$500. This is a three-year investment, and the debt is three-year bullet debt, for simplicity, so the principal and compound interest are repaid at year three. With a three-year risk-free rate of 8.82%,⁶ the investor's final debt payment is \$544.09, and the resulting cash flows are shown in Panel A of Table 4. It is simple to value this leveraged market investment using the Rubinstein CAPM. Each cash flow is discounted by the corresponding market return and averaged, using the probabilities in Panel C of Table 1. Not surprisingly, the total PV of this leveraged investment equals \$0, as shown in Panel C of Table 4.

Alternatively, consider a leveraged investment in the PE cash flow. Again, let the investment be \$1,000 where half is financed with risk-free bullet debt, as above. The resulting

⁵A formal derivation of this result is as follows: Let PV_X be the PV of the PE cash flows, X . The debt is fairly priced, so $PV_D = 0$. The PV of the levered deal is $PV_L = PV_X + PV_D = PV_X$, as illustrated in this example. To see the effect of leverage on the PME, divide the debt's cash flow into its positive and negative parts, denoted DP and DN . Fair debt pricing implies $PV_{DP} - PV_{DN} = 0$ so $PV_{DP} = PV_{DN}$. With the unlevered $PME = PV_{DIST}/PV_{CALL}$, the leveraged $PME_L = (PV_{DIST} + PV_{DP})/(PV_{CALL} + PV_{DN})$, so increasing leverage brings PME's closer to one.

⁶The three-year risk-free interest rate is calculated by valuing the cash flow that pays \$1 in each branch at year three. Using the Rubinstein SDF, the value is \$0.9190, and the risk-free rate is found by solving $(1 + r_F)^{-1} = 0.9190$, implying $r_F = 8.82\%$.

cash flows are given in Panel B of Table 4. In year zero, the investor’s initial investment is \$500. In years 1 and 2 the cash flows are identical to the cash flows from the original PE investment, from Panel B of Table 1. In year 3, these original cash flows are reduced by \$544.09 to service the debt.⁷ Panel C of Table 4 shows the PV of the resulting levered PE cash flow of \$107.68, which is identical to the PV of the unlevered cash flow.

Overall, these examples show that the Rubinstein CAPM is robust to changes in risk and leverage. PE funds cannot artificially inflate their performance by leveraging deals with (correctly priced) debt.

3 Public Market Equivalent

The PME is closely related to the Rubinstein CAPM. With a sufficient number of observed deals for each PE fund, the expectation in equation (8) can be estimated using a generalized method of moments (“GMM”) estimator that replaces the expected values with the realized cash flows. Specifically, index the fund’s deals by $i = 1, 2, \dots, N$, and separate each deal’s cash flows into capital calls, denoted $call_i(t_i)$, and distributions, $dist_i(t_i)$.⁸ Let $DIST$ and $CALL$ denote the numerator and denominator in the PME definition, so $DIST$ is:

$$DIST = \sum_t \frac{dist(t)}{1 + r_M(t)} = \sum_i \frac{dist_i(t_i)}{1 + r_M(t_i)}, \quad (9)$$

⁷The cash flow at year three may now be negative, but we assume that the investor can commit to honor the resulting liability, so the debt remains risk free. The valuation remains unchanged, however, with risky debt as long as this risk is fully anticipated and priced initially.

⁸To simplify the notation, we assume that each deal has only a single capital call and distribution, but the analysis holds with an arbitrary number of cash flows, although the notation becomes more cumbersome. Specifically, let $i(c)$, for $c = 1, \dots, C(i)$ index the individual cash flows associated with deal i . Replacing $\sum_i \frac{dist_i(t_i)}{1+r_M(t_i)}$ with $\sum_i \left(\sum_c \frac{dist_{i(c)}(t_{i(c)})}{1+r_M(t_{i(c)})} \right)$ and noting that $E \left[\left(\sum_c \frac{dist_{i(c)}(t_{i(c)})}{1+r_M(t_{i(c)})} \right) \right] = PV_{dist}$ generalizes the derivation to an arbitrary number of cash flows for each deal.

where $dist(t)$ equals the sum of the distributions at time t across deals. For a sufficiently large N (and under suitable regularity conditions ensuring the law of large numbers), the average of the observed distributions converges to their expected value, which equals the PV of the distributions under the Rubinstein CAPM:

$$\frac{1}{N}DIST \sim E \left[\frac{dist(s_t)}{1 + r_M(s_t)} \right] = PV_{DIST}. \quad (10)$$

A similar argument holds for the capital calls, and after canceling the $1/N$ terms, the PME equals:

$$PME = \frac{DIST}{CALL} \sim \frac{PV_{DIST}}{PV_{CALL}}. \quad (11)$$

The asymptotic sampling distribution of the PME can be computed with suitable further assumptions.

Hence, under the conditions outlined above, the Kaplan and Schoar (2005) PME measure is a consistent estimate of the PV of the distributions divided by the PV of the capital calls.

A PME greater than one implies that the investment has been valuable for the LP.

4 Discussion and Conclusion

The paper provides an axiomatic foundation of the public market equivalent (PME) measure of private equity performance. We show that under three conditions, the PME provides a valid economic risk-adjusted performance measure. A main advantage of the PME is that it is simple to calculate and apply. This simplicity arises because the PME is calculated without explicitly stating the risk of the investment (such as its beta), and the PME is valid regardless of this risk. It does not require different treatments of cash flows associated

with capital calls and distributions, despite their different properties. The PME does not rely on the LP following any particular trading strategy, such as reinvesting distributions dollar for dollar into the market portfolio. As a valid economic measure, it is robust to manipulation. The Rubinstein CAPM may also provide a more general and robust approach to evaluating ex-post performance than the standard CAPM in other situations where these benefits are important, such as when evaluating hedge fund or mutual fund performance. It seems particularly suited for valuing the irregular cash flows of PE investments.

The PME has important limitations. It is more applicable as an ex-post measure of past performance than for evaluating future investment opportunities, ex-ante, because correlations between cash flows and future market returns are more difficult to project. The PME provides the value at the margin, which is relevant for making a small additional investment in PE. This value at the margin, however, does not provide sufficient information for evaluating substantial asset-allocation decisions to PE, which also require a consideration of the illiquidity and investment capacity. Further, with larger PE allocations, the wealth portfolio of the investor evaluating the PE investments also changes, which affects the benchmark return that is used in the calculation of the PME measure itself. One way to address this issue is to use the return on a portfolio of publicly-traded securities that mimics the return on the investor's wealth portfolio, and add a suitable premium as compensation for the illiquidity of the PE investment, as in Driessen, Lin and Phalippou (2012). The PME may be less flexible for adjusting for other specific risk exposures than, for example, the multi-factor CAPM. When PE returns reflect a general premium earned by small-value stocks (as suggested by

Phalippou 2013) it is useful to also evaluate PME's using the return on a small-cap index to understand PE performance relative to this benchmark.

As a final comment, several different valuation methods have been proposed for PE performance. At this point, it is too early to recommend any single one. Our general recommendation would be to evaluate performance using several models and measures, such as calculating PME using different benchmark indices, to gain a better understanding of the performance relative to different benchmarks.

References

- Albuquerque, Rui, Martin Eichenbaum, and Sergio Rebelo (2013) “Valuation Risk and Asset Pricing,” *working paper*.
- Ang, Andrew, Bingxu Chen, William Goetzmann, and Ludovic Phalippou (2013) “Estimating Private Equity Returns from Limited Partner Cash Flows,” *working paper*.
- Cochrane, John (2001) *Asset Pricing*, Princeton University Press: NJ.
- Driessen, Joost, Tse-Chun Lin, and Ludovic Phalippou (2012) “A New Method to Estimate Risk and Return of Nontraded Assets from Cash Flows: The Case of Private Equity Funds,” *Journal of Financial and Quantitative Analysis*, 47, 3, 511–535
- Dybvig, Philip H., and Jonathan E. Ingersoll Jr. (1982) “Mean-variance theory in complete markets,” *Journal of Business*, 233–251.
- Epstein, Larry G. and Stanley E. Zin (1989) “Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework,” *Econometrica*, 57, 937–969.
- Hansen, Lars Peter, and Scott F. Richard (1987) “The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models,” *Econometrica*, 587-613.
- Ingersoll, Jonathan, Matthew Spiegel, William Goetzmann, and Iwo Welch (2007) “Portfolio Performance and Manipulation and Manipulation-proof Performances Measures,” *Review of Financial Studies*, 20, 1503–1546.
- Jagannathan, Ravi, and Robert A. Korajczyk (1986) “Assessing the market training performance of managed portfolios,” *Journal of Business*, 217-235.
- Kaplan, Steven and Antoinette Schoar (2005) “Private Equity Performance: Returns, Persistence, and Capital Flows,” *Journal of Finance*, 60, 1791–1823.
- Kim, Soohun (2013) “IMRS of scale invariance recursive utility functions,” *working paper*.
- Korteweg, Arthur and Stefan Nagel (2013) “Risk Adjusting the Returns to Venture Capital,” *working paper*.

- Kreps, David M., and Evan L. Porteus (1978) “Temporal Resolution of Uncertainty and Dynamic Choice Theory,” *Econometrica*, 46, No. 1, 185-200.
- Ljungqvist, Alexander and Matthew Richardson (2003) “The cash flow, return and risk characteristics of private equity,” *working paper*.
- Long, Austin and Craig Nickels (1996) “A Private Investment Benchmark,” *University of Texas Investment Management Company*.
- Long, Austin (2008) “The Common Mathematical Foundation of ACG’s ICM and AICM and the K&S PME,” *working paper: Alignment Capital Group*.
- Mulcahy, Diane, Bill Weeks, and Harold Bradley (2012) “We Have Met the Enemy ... And He is Us,” *working paper: Kauffman Foundation*
- PEI (2002) “Inside IRR,” *Private Equity International*, January.
- Phalippou, Ludovic (2013) “Performance of Buyout Funds Revisited?” *Review of Finance*.
- Robinson, David and Berk Sensoy (2012) “Cyclicality, Performance Measurement, and Cash Flow Liquidity in Private Equity,” *working paper*.
- Rouvinez, Christophe, (2003) “Private equity benchmarking with PME+,” *Private Equity International*, August, 34–38.
- Rubinstein, Mark (1976) “The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets,” *Journal of Finance*, 31, 551–571.
- Sorensen, Morten, Neng Wang, and Jinqiang Yang (2013) “Valuing Private Equity,” *working paper*.
- Skiadas, Costis (2009) *Asset Pricing Theory*, Princeton University Press: NJ.

TABLE 1: Market returns and PE cash flow. Panels A and B show binomial trees with the market return and the PE cash flows. Each tree start at time zero and span three years, with each branch having a 50% probabilities. The resulting probability of each branch, used when calculating averages, are in Panel C.

Panel A: Market Return: $1+r_M(s_t)$

Year	0	1	2	3
				172.8%
			144%	
		120%		129.6%
	100%		108%	
		90%		97.2%
			81%	
				72.9%

Panel B: PE Cash Flow: $X(s_t)$

Year	0	1	2	3
				\$1,098.50
			\$845	
		\$0		\$760.50
	-\$1,000		\$585	
		\$0		\$526.50
			\$405	
				\$364.50

Panel C: Probability: $p(s_t)$

Year	0	1	2	3
				1/8
			1/4	
		1/2		3/8
	1		1/2	
		1/2		3/8
			1/4	
				1/8

TABLE 2: Present Value. The table shows average cash flows of the PE investment, and its present values, calculated using both the standard and Rubinstein CAPMs.

Year	0	1	2	3	Total
$E[X(s_t)]$	-\$1,000.00	\$0.00	\$605.00	\$665.50	\$270.50
Standard CAPM	-\$1,000.00	\$0.00	\$542.37	\$564.62	\$106.99
Rubinstein CAPM	-\$1,000.00	\$0.00	\$542.53	\$565.14	\$107.68

TABLE 3: Stochastic Discount Factors (SDFs). The two panels show binomial trees, corresponding to the binomial trees for the market return and cash flows in Table 1. Panel A shows the SDF under the standard CAPM, calculated as $m = a - b R_M$, where a and b are constant given in the text. Panel B shows the SDF under the Rubinstein CAPM, calculated as $m = 1/(1+R_M)$.

Panel A: SDF for Standard CAPM: $m = a - b (1+r_M(s_t))$

Year	0	1	2	3
				\$0.484
			\$0.662	
		\$0.833		\$0.814
\$1.000			\$0.964	
		\$1.111		\$1.060
			\$1.191	
				\$1.246

Panel B: SDF for Rubinstein CAPM: $m = 1 / r_M(s_t)$

Year	0	1	2	3
				\$0.579
			\$0.694	
		\$0.833		\$0.772
\$1.000			\$0.926	
		\$1.111		\$1.029
			\$1.235	
				\$1.372

TABLE 4: Levered Cash Flows. Panels A and B show cash flows from levered market and PE investments. Each levered cash flow is given by a three-year investment of \$1,000 that is financed with \$500 debt. For simplicity, the debt is bullet debt, which is repaid in full, with compound interest, in year 3, when the investment is also liquidated. The three-year risk-free interest rate is 8.82%, and the total debt payment in year 3 is \$544.09. Panel C shows valuations of the two cash flows, using the Rubinstein CAPM.

Panel A: Cash Flow for Levered Market Investment

Year	0	1	2	3
				\$1,184
			\$0	
		\$0		\$752
	-\$500		\$0	
		\$0		\$428
			\$0	
				\$185

Panel B: Cash Flow for Levered PE Investment

Year	0	1	2	3
				\$554
			\$845	
		\$0		\$216
	-\$500		\$585	
		\$0		-\$18
			\$405	
				-\$180

Panel C: PV using Rubinstein CAPM

Year	0	1	2	3	Total
Levered Market	-\$500.00	\$0.00	\$0.00	\$500.00	\$0.00
Levered PE	-\$500.00	\$0.00	\$542.54	\$65.14	\$107.68