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How Deep is the Annuity Market Participation Puzzle?

Abstract

Using U.K. microeconomic data, we analyze the empirical determinants of voluntary annuity market demand. We find that annuity market participation increases with financial wealth, life expectancy and education and decreases with other pension income and a possible bequest motive for surviving spouses. We then show that these empirically motivated determinants of annuity market participation have the same, quantitatively important, effects in a life-cycle model of annuity and life insurance demand, saving and portfolio choice. Moreover, reasonable preference parameters predict annuity demand levels comparable to the data. For stockholders, a strong bequest motive can simultaneously generate balanced portfolios and low annuity demand.

JEL Classification: E21, G11, H00.

Key Words: Annuities, portfolio choice, life insurance, bequest motive.

1 Introduction

Why are annuities not voluntarily taken up by a larger number of retirees? The literature on individual consumption/saving and portfolio choice presents a very important participation puzzle: research reveals households' preference not to voluntarily buy annuities at retirement, despite the strong theoretical reasons that point towards high demand for these products. Specifically, in a seminal contribution, Yaari (1965) demonstrates that risk aversion is sufficient to induce a household to buy an actuarially fair annuity as protection against life expectancy risk. More recently, Davidoff et al. (2005) show that complete annuitization is optimal in a more general setting provided that markets are complete, annuities provide net premia over conventional assets and there is no bequest motive. Yet, despite these strong theoretical results, annuity demand remains very low in the data, leading to what is known as the "annuity market participation puzzle."

It is important to understand why this puzzle arises from a theoretical perspective[1] but there is also another, equally strong, empirical reason to explain the puzzle. Specifically, there has been a large shift in pension provision from defined benefit (DB) to defined contribution (DC) plans both in the U.S. and in the U.K. DB plans offer not only a fixed monthly payment but also offer it for life, therefore providing a natural insurance for life expectancy risk. DC plans, on the other hand, make the individual responsible for deciding how quickly to decumulate during retirement.[2] As a result, the issue of annuity provision could become very important for financial planning after retirement.

The pressing need to understand this puzzle has generated a large number of potential explanations. According to Davidoff et al. (2005), the full annuitization result can break down in an incomplete market that fails to provide annuities exactly matching households' optimal consumption path. They find, however, that the optimal level of annuitization remains high even with liquidity shocks (from medical expenses) or when an extreme mismatch exists between desired consumption and available annuity income. In an alternative expla-

[1] Davidoff et al. (2005) imply that an explanation from the psychology and economics literature might be needed.

[2] In the U.K. during the sample period of the data used in this paper (2002-2004) there was mandatory annuitization by age 75 of three quarters of the accumulated assets in a DC plan.

nation, Mitchell et al. (1999) for the U.S. and Finkelstein and Poterba (2002, 2004) for the U.K. make the case that the lack of actuarially fair annuities explains the low participation rate. Mitchell et al. (1999) disagree, arguing that annuity pricing is not sufficient to explain the low take-up because the "money's worth of individual annuities" is actually quite good.

The presence of some annuitization through state social security and private DB plans (Bernheim (1991), Brown et al. (2001) and Dushi and Webb (2004)), and the presence of uncertain medical expenditures (Sinclair and Smetters (2004) and Ameriks et al. (2007)) have recently received substantial attention as determinants of annuity demand. Furthermore, Milevsky and Young (2007) have recently emphasized the lack of flexibility in purchasing an annuity, arguing that buying an annuity limits a household's flexibility to invest in the stock market, while Ameriks et al. (2007) focus on how irreversible annuity purchase decisions can increase the probability of ending up at a nursing home funded by the state. Flexibility in labor supply along with the existing annuity in the form of social security is the explanation offered by Benitez-Silva (2003). Another prominent idea is the presence of a bequest motive, as the preference for leaving bequests may counteract the insurance benefits of annuities (Friedman and Warshawsky (1990)). Nevertheless, the lack of evidence stemming from comparing the choices of households with and without children (for example, Brown (2001)) has cast doubt on the empirical plausibility of this explanation. Other less prominent explanations include inflation risk,[3] non-actuarially fair annuity provision combined with minimum annuity size purchase requirements,[4] and rare events.[5]

Our primary contribution is to take many of the reasons for lower annuity holdings previously proposed in the literature and combine them in a quantitative model, while simultaneously checking that the empirical evidence is consistent with the inputs to, and predictions

[3] In the presence of substantial inflation risk the demand for nominal annuities might be quite low. Nevertheless, this explanation would imply a large demand for real annuities, yet the take-up for real annuities, where they exist, has also been low. Lopes (2006) also finds that the load factors for real annuities are high, thereby negating the value from having real annuities.

[4] See Lopes (2006).

[5] Lopes and Michaelides (2007) argue that the possibility of a "rare event" like the default of the annuity provider cannot by itself explain the puzzle since such a rare event would change behavior for high risk aversion coefficients but a high risk aversion simultaneously makes annuity demand stronger.

from, the model. We first examine the empirical determinants of annuity market decisions. Next, we utilize a comprehensive quantitative model that allows us to assess the ability of different, empirically motivated, factors in explaining the data. Thus, both our reduced form and our structural model can shed light on which explanations are more likely to justify the low annuity market participation in the data.

To achieve the stated goal we first empirically analyze the determinants of voluntary annuity market participation at the household level to determine the characteristics of households that participate (or not) in this market. We confirm that there appears to be a substantial voluntary annuity market participation puzzle because fewer than 6% of households participate in this market. For our multivariate empirical analysis, we separate the sample between stockholders and non-stockholders. We take this route because wealthier and more educated households can better afford and understand annuities, and because we know that stock market participation increases with wealth and education (for instance, Campbell (2006)). Indeed, the annuity market participation rate for stockholders (9.6%) is three times the participation rate of non-stockholders (3.2%).

We also investigate the life insurance decision because the life insurance market is closely related to the annuity market. An annuity continues to pay until death while life insurance pays at death. As a result, life insurance is sometimes called an "inverse annuity." Studying annuity choices in isolation from life insurance choices might thus be a serious omission, especially because life insurance is more widely held among the population (the life insurance participation rate is 40%).

In our multivariate analysis, we find that the factors determining annuity participation are broadly the same for the different groups. Specifically, annuity market participation increases with life expectancy, education and financial wealth. Pension income (or compulsory annuity income) crowds out annuity demand conditional on voluntary annuity market participation, while a possible bequest motive for surviving spouses is a hurdle for voluntary annuitization. We view these empirical findings as interesting in their own right because they increase our understanding of the factors determining annuity market participation.

Surprisingly, there are not many empirical studies investigating the correlates of annuity market participation.[6] One recent study by Brown (2001) uses the U.S. equivalent of the U.K. data we use and has a research objective similar to ours, but it uses a slightly different method. Brown's analysis first calculates the value of having access to an annuity market for each household (based on a life-cycle simulated model) and then relates this value to the intention to annuitize. Brown's approach and ours yield similar insights into some of the empirical correlates of annuity market participation. For instance, as in Brown (2001), we do not find any evidence for a bequest motive when using the presence of children as a variable to proxy for an intentional bequest motive. Like Brown, we find a statistically and economically significant negative impact of being married on the probability to annuitize. Our results differ in other ways, however. For instance, education and subjective survival probabilities turn out to be significant in our analysis while they are insignificant in Brown (2001). Most importantly, we find that wealth has a strong positive impact on the probability to annuitize while it is negative and significant in Brown's analysis. Moreover, Brown (2001) does not consider portfolio choice variables, while we consider the impact of stock and life insurance market participation on the probability of annuitization.

We next construct a quantitative model that replicates these empirical findings and that can therefore be used to quantify the strength of the annuity market participation puzzle. Specifically, we build a model of life-cycle saving, portfolio, life insurance and annuity market choice with Epstein-Zin (1989) preferences over a non-durable good and investigate whether reasonable preference parameters can replicate the observed annuity market participation rate and the level of annuity demand. To do so, we first perform extensive comparative statics and use the wealth distribution and median pension level in the data as exogenous inputs to generate predicted annuity demand after retirement. We find that preference parameters like risk aversion, the strength of the bequest motive, the elasticity of intertemporal substitution and the decision to access the stock market are key determinants of the model's quantitative predictions. Financial wealth, a key endogenous state variable in the model, is directly affected by these parameters and is therefore a key predictor variable in assessing the model's

[6] A recent study by Brown and Poterba (2006) examines variable (or equity-linked) annuities and focuses on the impact of the household's marginal tax rate. Nevertheless, variable annuities only recently developed to a significant part of the total annuity market.

quantitative implications. Contrary to frictionless theoretical models, we find that many households should not purchase an annuity partly because of the state pension income, partly because of the empirical wealth distribution (many households cannot afford an annuity), partly because of the bequest motive and partly because of better opportunities and flexibility in saving through the stock market.

We next use a method of simulated moments to estimate the model separately for stockholders and non-stockholders. We separate the two groups both on account of our multivariate probit findings and due to the large difference in financial wealth profiles across the two groups in the data.[7] For the non-stockholders, we fix the coefficient of relative risk aversion to two and we estimate the discount factor, the bequest parameter and the intertemporal rate of substitution to match the annuity and life insurance participation rates, average wealth and, conditional on annuity market participation, the amount of annuities purchased and the share of wealth annuitized. We also match the share of wealth in life insurance products and in bonds. For the stockholders we add the average share of wealth in stocks during retirement as a moment target and estimate the risk aversion coefficient as well. We find that the life-cycle model is consistent with the empirical findings for reasonable preference specifications. The estimated elasticity of intertemporal substitution is much lower for non-stockholders (0.07) than for stockholders (0.4). We believe these parameter estimates are reasonable estimates for preferences because they are consistent with the empirical evidence in, for instance, Gourinchas and Parker (2002) and, in particular, Vissing-Jorgensen (2002).

We need a bequest motive for both stockholders and non-stockholders, but this motive must be stronger for stockholders. The effect of a strong bequest motive in generating a balanced portfolio comprised of both stocks and bonds has not been stressed in the literature as a sufficient ingredient to explain portfolio allocations. A bequest motive breaks

[7] We do not model the endogenous decision of whether or not to participate in the stock market. Gomes and Michaelides (2005) and Alan (2006) calibrate and estimate, respectively, a life-cycle model and show that households with low financial wealth can be kept out of the stock market with a small fixed cost. Given that in our data the households that do not participate in the stock market are much poorer in terms of financial wealth than stock market participants, we think that a small fixed cost will keep these households out of the stock market as well. We do not model this endogenous choice explicitly here to keep the model relatively simple.

the decumulation of financial wealth and therefore generates a balanced portfolio even at retirement, while in the absence of a bequest motive both financial wealth and the implicit riskless assets (annuities and state pensions) are depleted at similar rates. The need for a bequest motive to explain the data is consistent with recent evidence. De Nardi (2004) emphasizes the effect for matching the observed wealth distribution. Kopczuk and Lupton (2007) use this motive to better understand U.S. wealth data during retirement. The weaker bequest motive for the poorer non-stockholders is consistent with the recent evidence in De Nardi et al. (2010). Moreover, our results are also consistent with Yogo (2008) who needs a bequest motive to generate low welfare gains from annuity market participation in a model with health investments.

Overall, comparing the predictions of the model with the empirical evidence, we find that reasonable calibrations can generate the low annuity demand observed in the data — and that, therefore, the annuity market participation puzzle might not be as deep as previously thought. One caveat to this conclusion comes from the life insurance predictions of the model. When we include life insurance, the model predicts higher annuity market participation (on average 20% as opposed to 10%) than was observed in the data. Moreover, our model shows higher participation rates for non-stockholders' life insurance ownership and their share of wealth allocated to life insurance than was observed in the empirical data, and the model erroneously predicts that these rates will rise during retirement. For reasons of tractability, however, the model of life insurance we propose is necessarily simplistic (insurance expires after one year) while for the same reason we abstract from medical expenditure shocks of either the endogenous (Yogo (2008)) or exogenous (De Nardi et al. (2010)) kind. We think that a more sophisticated treatment of these choices will be an interesting topic for further research.

Our model is closely related to previous life-cycle models with portfolio choice and annuity demand. The model allows for gradual annuitization over time as proposed by Milevsky and Young (2007) and implemented by Horneff et al. (2008, 2010) and Zeng (2008). Horneff et al. (2008) consider a retired household with Epstein-Zin (1989) preferences and a possible bequest motive; they derive the optimal demand for bonds, stocks and nominal annuities. Horneff et al. (2010) study a similar model where nominal annuities are replaced with variable annuities. Zeng (2008) extends the portfolio choice set of the Horneff et al. (2010) model by introducing term life insurance. Our model combines the asset universe of the Horneff et al. (2008) model with life insurance contracts of the type considered by Zeng (2008). None of the previous research estimates the structural parameters of the life-cycle model from household data to match observed annuitization behavior.

The remainder of the paper is organized as follows. In Section 2, we present the multivariate probit (reduced form) results on the actual determinants of annuity market demand (defined as annuity market participation and the level of annuity demand conditional on participation). In Section 3 we perform a number of comparative statics exercises from a calibrated life-cycle model to understand what a quantitative model predicts about the annuity market. In Section 4 we estimate the structural parameters of this model and investigate the strength of the annuity market participation puzzle by comparing the moments in the data to the ones from the model. Section 5 concludes.

2 Empirical Analysis

2.1 Dataset

We derive our empirical data from the English Longitudinal Study of Ageing (ELSA). ELSA is a biannual panel survey among those aged 50 and over (and their younger partners) living in private households in England in 2002. For most of the variables of interest we use data from the first wave of ELSA collected in 2002 and 2003. We restrict our analysis to households with either a retired single person or a couple with at least one retired person. We focus on households with at least one retired person because annuitization is likely to occur during retirement and we are interested in possible substitution effects between public and private pension income and annuities.[8] We focus exclusively on voluntary annuitization, which is recorded in ELSA as a part of the "Income and Assets" module. The survey gives a

[8] With this restriction, we exclude 2,206 non-retired households. We do not view this restriction as important for our analysis because we only observe 14 voluntary annuity contracts for these households in the first wave of ELSA.

definition of annuity income, which should prevent any misinterpretation: "Annuity income is when you make a lump sum payment to a financial institution and in return they give you a regular income for the rest of your life." Further details are provided in Appendix A.

2.2 Descriptive Statistics

2.2.1 Annuities

Table 1 describes the annuity market participation decisions, and also presents a split of this decision between households that participate, or not, in the stock market. We analyze stockholders separately from non-stockholders because we expect that stock market participation might be correlated with the decision to participate in the annuity market - both decisions require a certain level of financial sophistication and financial wealth. According to Table 1, only 5.9% (309 observations) of the households in our sample received income from voluntary annuitization in either the first or second ELSA wave, illustrating the "annuity market participation puzzle."

Table 1 also indicates that there might be an interesting correlation between the decision to participate in the stock market and the decision to purchase an annuity. Stock market participation[9] is around 42.4% of the total sample but the percentage of stock market participants purchasing an annuity (9.6%) is three times the percentage of stock market non-participants (3.2%), with the difference highly statistically significant. Thus, there seems to be some connection between the decisions to participate in the two markets.

Table 2 presents annuity demand statistics conditional on participating in the voluntary annuity market. Specifically, the table reports mean and median annual annuity income, also split across the stock market participation decision. Conditional on having an annuity, the mean annual annuity income is about 3,000 GBP, but this is dominated by a number of very large annuities as the median of about 1,000 GBP shows. Stock market participants tend to demand higher annuities as indicated by a mean (median) annual annuity income of

[9] We define a stock market participant as a household that has stocks in an individual savings account (ISA), or a personal equity plan (PEP), or indirect stock holdings in an investment trust, or direct holdings of stocks. Indirect stock holdings in occupational or private pension schemes are not accounted for.

about 3,650 (1,200) GBP.

2.2.2 Life Insurance

Given the inverse payout structure between a life insurance and an annuity (a life insurance is often called an inverse annuity), participation in one market probably influences one's decision to participate in the other. We therefore also report empirical results for life insurance demand. Table 1 shows that the life insurance participation rate is 40% - this rate is much higher than the participation rate in the voluntary annuity market. We note from Table 1 that there are no significant differences between stockholder and non-stockholder life insurance participation rates (37.5% and 42.2% respectively). Table 2 shows the self-reported, expected life insurance payout. We find that even though the life insurance amount equals around 17,000 GBP for the households that buy life insurance, this amount is progressively higher for stockholders (27,500 GBP) and for participants in both the stock and annuity markets (75,000 GBP).

2.2.3 Wealth and Income

To be informative about annuity take-up decisions, financial wealth should be measured before annuitization takes place. For annuities already observed in the first wave, we capitalize the value of the annuity by multiplying the annual annuity income with the annuity factor and then add this to the household's financial wealth to get total financial wealth.[10] Those households with reported annuity income in the second wave but no reported annuity income in the first wave must have purchased their annuity in the time between the two surveys. We combine the second wave annuity information for these observations with the first wave household variables, thereby achieving the desired match between the annuity and the household characteristics immediately before voluntary annuitization occurs.

[10] We use an annuity factor of 13. The annuity factor was calculated using the Financial Services Authority comparative tables. These tables show the monthly payments offered by the main annuity providers under the open market option. The monthly payments correspond to a purchase price of 100,000 GBP of a single life annuity, with no guarantee, for a 65-year-old male. We use the average monthly payment across providers to calculate the corresponding annuity factor.

Table 2 reports the mean (median) financial wealth[11] of annuitants to be about 135,000 (65,000) GBP, versus 50,000 (14,200) for non-annuitants, already suggesting the importance of financial wealth in purchasing a voluntary annuity. More detailed evidence is displayed in Figure 1. The figure shows average voluntary annuity market participation across the 2.5th, 10th, 25th, 50th, 75th, 90th and 97.5th percentiles of the wealth distribution. While average participation is less than 1% in the bottom 5% of the wealth distribution, it increases steeply to almost 20% in the top 5%. Given that the 10th and 25th percentiles of the wealth distribution are 700 GBP and 3,300 GBP, respectively, it appears that these households have insufficient financial wealth to participate in the voluntary annuity market. Figure 1 decomposes the sample across wealth percentiles into stock market non-participants and participants. Almost all households around the 75th, 90th and 97.5th percentiles of the wealth distribution are stock market participants. The mean (median) wealth of investors who participate in both markets is 174,000 (100,000) GBP (Table 2), considerably larger than the mean (median) wealth of annuity market participants.

The existence of other pension income offers another potential explanation for low annuity market participation. The institutional details of the U.K. pension system have been described elsewhere (for example, Blundell et al. (2002)) and we only summarize its main features. The first tier of the public pension system is the Basic State Pension (BSP). The second tier is earnings-related and can either be provided by the government or the private sector. Both occupational and personal private sector pensions in the U.K. are subject to compulsory annuitization laws (an annuity must be purchased within a certain time from retirement) during the sample period. These compulsory annuities must be distinguished from the voluntary annuities purchased from non-pension wealth on which we focus. Finkelstein and Poterba (2002) indicate that the compulsory annuity market in the U.K. is much larger than the voluntary annuity market: in 1998 the former had a size of 5.4 billion GBP versus 0.8 billion GBP for the latter.

[11] Banks et al. (2010) provide evidence that British households do not reduce housing consumption with increasing age because they stay in their original residence. Correspondingly, we do not use housing wealth in our multivariate analysis because we view the relatively higher liquidity in financial wealth (with respect to housing) as a more relevant criterion for the household decision to annuitize or not.

Public pensions and the compulsory annuities from private pensions may be close substitutes for the voluntary annuity market. Indeed, Attanasio and Rohwedder (2003) find that the earnings-related tier of the U.K. public pension system serves as a perfect substitute for private savings. Table 2 shows mean and median annual pensions for the whole sample and for the sub-samples of annuity and stock market participants. While the level of public pensions changes very little across sub-samples, there is considerable variation in private pensions. Annuity market participants receive higher private pensions (mean 7,236 GBP; median 3,200 GBP) than annuity market non-participants (mean 4,362 GBP; median 1,350 GBP). Figure 2 decomposes the sources of pension income over different percentiles of the wealth distribution. The level of public pensions resembles a flat pension, despite the earnings-related tier of the system. This arises mostly from higher-earning employees opting out from the public second tier (in Figure 2 private (compulsory) pensions increase steeply over the wealth distribution). Compared to the level of public and private pensions, voluntary annuities are small in magnitude and only exist around the 75th, 90th and 97.5th wealth percentiles. Nevertheless, we cannot interpret these results as evidence against the hypothesis that other pension income crowds out voluntary annuities, because we need to control for other variables (like financial wealth).

2.2.4 Health and Life Expectancy

Apart from wealth and existing pensions, an individual's health condition and her life expectancy should also affect the decision to annuitize because annuities hedge longevity risk. These products are in fact priced to reflect the average life expectancy of annuity market participants. If an individual has private information suggesting that she is unlikely to reach the age of an average annuity market participant, she will not buy an annuity simply because the product is overpriced for her. Finkelstein and Poterba (2002, 2004) indeed find evidence for adverse selection in the U.K.'s annuity market: participants in the voluntary annuity market tend to live longer than non-participants. More generally, Rosen and Wu (2004) find evidence from the Health and Retirement Survey indicating that health status affects portfolio choice and stock market participation. Because an annuity is a financial product even more explicitly linked to health status, we expect that health can be a strong predictor of participation in the annuity market.

ELSA allows us to use subjective survival probabilities as a determinant of the annuitization decision. The questionnaire asks individuals of age less than or equal to 65 (69, 74, 79, 84 and 89), "What are the chances that you will live to be 75 (80, 85, 90, 95 and 100, respectively) or more?" and gives a range from 0-100 for possible answers. We compare these subjective survival probabilities with gender- and age-specific objective survival probabilities from the Government Actuary's Department tables (GAD). We see from Table 3 that annuity market participants report a survival probability higher than non-participants by 5%. The difference in objective GAD survival probabilities is 3% and thus slightly smaller. These results are in line with the Finkelstein and Poterba (2002, 2004) self-selection findings for the voluntary annuity market in the U.K. and will justify one of the comparative statics in the structural model where the subjective survival probability is allowed to deviate from the objective one.

2.2.5 Socio-Economic Background

The final two variables possibly affecting annuity market participation decisions are household composition and education. Education might be relevant because annuity products require a basic level of financial literacy.[12] We differentiate between three education levels: low, medium and high. Table 3 shows that annuity market participants are on average much better educated than non-participants. While 61% of the non-participants are in the lowest education group, only one-third of all annuity holders are in the low education category. For the high education level, the order changes: 9% (25%) of non-participants (participants) have a higher education degree. We also investigate household composition to detect a possible bequest motive, which might be a barrier for voluntary irreversible annuitization. The unconditional statistics in Table 3 do not indicate that marital status or the number of children vary between participants and non-participants.

[12] Lusardi and Mitchell (2006) provide evidence that individuals planning for retirement generally exhibit a higher degree of financial literacy than non-planning individuals.

2.3 Econometric Analysis

We next investigate the household's decision to participate in the voluntary annuity market and the amount of purchased annuities conditional on participation in a multivariate regression setup.

2.3.1 Annuity Market Participation

Table 4 displays the results of a Maximum Likelihood estimation of a probit model for the household's decision to participate in the voluntary annuity market. The previous section revealed systematic differences between stock market participants and non-participants - differences that might affect the annuitization decision. Therefore we present estimation results for the two groups next to the estimation results for the whole sample. We use the following as explanatory variables: wealth, income, life insurance market participation, household composition, age, health and life expectancy of the household. The regression for the whole sample also contains a stockholder dummy and its interaction with financial wealth. We focus on marginal effects to assess the quantitative impact in Table 4 because the estimated coefficient in the probit model only shows the qualitative impact of an explanatory variable. We do this for a baseline observation that is defined as a 65-year-old, single, male, with medium education, no children, no life insurance holding, average reported survival probability, average pension income and financial wealth who does not participate in the stock market.

Confirming the earlier descriptive statistics in Table 2, financial wealth is shown to be one of the most important predictors of annuity market participation.[13] A unit increase in log financial wealth, which roughly corresponds to a 100% increase in financial wealth relative to the baseline, significantly increases the annuity market participation probability for the whole sample by 2.3%. The effect is larger in the stockholder sub-sample (5.7%) than in the non-stockholder sub-sample (1.8%). On the other hand, pension income is statistically insignificant for both stockholders and non-stockholders.

[13] For all financial variables, we test for possible nonlinearities by including a squared term. This term is insignificant in all cases.

Changing the baseline household from non-stockholder to stockholder increases the participation in the voluntary annuity market by 2.3%. This marginal effect is the result of a negative and significant effect of stock market participation and a positive and significant effect of an interaction between stock market participation and wealth on annuity market participation.

Turning to health and life expectancy, we find that the health indicators listed in Table 3 are insignificant once we control for the subjective survival probabilities. Correspondingly, we only include the self-reported survival probabilities in the regression because these are a direct measure of the longevity risk targeted by annuities. This variable has differing effects on the annuitization decisions of non-stockholders and stockholders. While statistically insignificant for non-stockholders, a 10% increase in the baseline survival probability of stockholders significantly increases the annuity market participation probability by 0.75%.

Married individuals are significantly less likely to purchase an annuity. The marginal effect for the whole sample suggests that changing the marital status of the baseline household from single to married would decrease the probability that the household would participate in the voluntary annuity market by 3.6% (a significant change). The result is more pronounced in the stockholder sub-sample than in the non-stockholder sub-sample. The number of children (or the presence of children or grandchildren in alternative unreported specifications) does not have a significant effect, which suggests that any bequest motive focuses on the spouse and not on the children.[14] Alternatively, the large impact of marital status could be interpreted as intra-household hedging of longevity risk, instead of relying on the annuity market. However, the explanatory financial wealth and pension income variables are measured on the household level and already comprise the wealth and income of the spouse.

[14]The negative effect of being married could also be explained by "joint-and-last-survivor" or "joint-survivor" types of compulsory annuities, which provide payments until the death of the surviving spouse. Usually, annuity payments are reduced by one half or one third after the death of the first annuitant (Blake, 1999). Brown and Poterba (2000) show that the utility gains from annuitization are smaller for couples than for singles. ELSA does not provide details on the type of annuity a household has purchased. Stark (2003) presents evidence on the importance of joint survivor annuities in the U.K. from a survey of 500 annuitants in the compulsory market. She notes that only 12% of the annuities were of a joint type which suggests a moderate demand.

Therefore, the bequest motive appears to be the more suitable explanation of the importance of the marital status variable. The presence of a life insurance could be a more direct measure of a bequest motive. However, the life insurance dummy variable is insignificant in all regressions. This probably can be explained by the small number of observations (105) that engage in both the annuity market and the life insurance market.[15]

We include dummies for low and high education levels as a measure of financial literacy. Changing the education level of the baseline household from medium to low significantly decreases the participation probability in the whole sample by 2.9%. This is a quantitatively large effect and underscores the importance of financial literacy.

2.3.2 Conditional Annuity Demand

For the whole sample and the two sub-samples of non-stockholders and stockholders, we estimate a linear regression model for annuity demand measured in terms of log annual annuity income. Results are given in Table 4. All non-financial background variables appear insignificant in the conditional annuity demand regressions. These variables affect participation but do not influence demand conditional on participation. The financial variables, however, are significant predictors of annuity demand. The annuity demand elasticity of financial wealth is 0.33 for the whole sample, and 0.32 (0.63) for the sub-samples of non-stockholders (stockholders). While pensions do not significantly affect the annuity demand of non-stockholders, they have a marginally statistically significant and negative impact for stockholders. A 1% increase in compulsory annuities crowds out the demand for voluntary annuities by 0.22%. The stockholder dummy and its interaction with financial wealth in the regression based on the whole sample have the same signs as in the participation probit model, but are only significant at the 10% level.

[15] Using the amount of life insurance coverage instead of a life insurance dummy does not change this result.

2.4 Summary

We provide an in-depth empirical analysis of the voluntary annuity market participation decision and the annuity demand conditional on participation. We reconfirm that there appears to be a substantial voluntary annuity market participation puzzle because less than 6% of households participate in this market. We observe that annuity market participation increases with financial wealth, life expectancy and education. Pension income (or compulsory annuity income) crowds out annuity demand conditional on voluntary annuity market participation, while a possible bequest motive for surviving spouses is a hurdle for voluntary annuitization.

3 Understanding the Implications of a Life-Cycle Model

In the next two sections we investigate the implications of an annual frequency, life-cycle model of annuity, life insurance demand and portfolio choice. We assess the model's consistency with the empirical findings in the previous section. We first outline the most general version of the model; special cases arise by limiting the choices available to households.

3.1 The Model

3.1.1 Bond and Stock Market

The household can save through a riskless asset and the stock market. We use r_f to denote the one period fixed interest rate, $\widetilde{r}_{t+1}$ the random return on the stock market and α_t the share of wealth in stocks, and assume that neither stocks nor bonds can be sold short, therefore α_t has to lie between zero and one. The stock return is lognormally distributed and i.i.d.

3.1.2 Annuity Contracts

We study nominal annuity contracts but for simplicity we assume zero inflation.[16] The current value (AV_t) of a life annuity paying A_t for a household of age t is equal to the present discounted value of all future payouts weighted by the survival probabilities. Let p_s denote the probability that the household is alive at age s, conditional on being alive at age $s-1$. Then

$$AV_t = (1+P)\, A_t \sum_{s=t+1}^{T} \frac{\prod_{j=t+1}^{s} p_j}{(1+r_f)^{s-t}} \tag{1}$$

where P is a load factor that accounts for the fact that an annuity is usually not priced at its actuarially fair value. The load factor is greater than or equal to zero, giving a measure of the "money's worth" of the annuity. If the load factor is zero, then the annuity contract is actuarially fair and the "money's worth" equals one. Empirical evidence by Mitchell et al. (1999) illustrates that the load factor varies between 8% and 20% depending on different assumptions about discounting and mortality tables; a 20% value is suggested as indicative of the transaction cost involved, and this is the baseline value we use in our calibration. Following the empirical evidence on the timing of annuity purchases in our data set,[17] we let households buy incremental annuities between the retirement date (age 65) and age 75.

3.1.3 Life Insurance Contracts

At time t the household can purchase term life insurance. The actuarially fair price of one unit of the life insurance product is then equal to $(1-p_{t+1})$.[18] We use the same load factor as

[16] Recall that our data does not allow us to distinguish between nominal, real and variable annuities. While all of these annuity products are available in the U.K., Stark (2002) shows that more than 70% of all purchased annuities are of the nominal type.

[17] We know the age at which a first annuity was purchased for those 102 observations that had no annuity in the first wave of ELSA but had an annuity in the second wave. The data show that annuities are purchased throughout retirement. 80% of the annuity purchases occur before or at the age of 75, and 98% before the age of 85.

[18] With probability p_{t+1} survival continues into the next period and the insurance gives a payout equal to zero. With probability $(1-p_{t+1})$ the insurance pays out $\exp(r_f)$ in the next period and therefore the current expected value of life insurance equals $(1-p_{t+1})$.

in the annuity market to avoid biasing our results. Therefore, the price of term life insurance per unit equals

$$TI_t = (1 + P)(1 - p_{t+1}) \tag{2}$$

For stockholders we assume that the insurance product is made up of a hybrid security that also pays out if the household survives into the next period. We make this assumption because we observe that stockholders in our data set have a 70% participation rate in the term insurance market and a 30% participation rate in the investment-linked endowment market. To mirror this in our model in a simple way we assume that stockholders who buy life insurance receive a payout that is a weighted average of the investment and term insurance components, and the weights are determined by the participation rate in each market. The investment return is always received and is given by $0.3exp(\max\left\{\frac{\widetilde{r}_{t+1}-r_f}{\exp(r_f)}, 0\right\})$ (see Bacinello (2001)). In case of death the return is enhanced by the term insurance component so that the household then receives (per unit of insurance)

$$0.3exp(\max\left\{\frac{\widetilde{r}_{t+1} - r_f}{\exp(r_f)}, 0\right\}) + 0.7\exp(r_f)$$

The price paid for this hybrid insurance (H_t) is then a weighted average between the term insurance price given by (2) and a special case of the formula given by Bacinello (2001), further explained below, for the one period contract:

$$EP_t = (1 + P)\left[\exp(-r_f) + \frac{c}{1 + r_f}\right] \tag{3}$$

where $c = F(d_1) - F(d_2)$ with $d_1 = \frac{r_f + 0.5\sigma^2 - \ln(1+r_f)}{\sigma}$ and $d_2 = d_1 - \sigma$, with σ denoting the standard deviation of the stock market return. Therefore, we have that

$$H_t = 0.3EP_t + 0.7TI_t.$$

3.1.4 Budget Constraint

During retirement the household has liquid financial wealth (cash on hand) X_t that can be used to purchase the annuity, purchase the life insurance product or save through the bond or stock markets. The household is also endowed with pension income in each period, L. The annuity decision is irreversible, even though the household can add positive amounts every period. Similarly, the household can purchase only positive amounts of the life insurance product. At time t (in the most general version of the model), there are three state variables (age, cash on hand and the amount of annuities) and four control variables (consumption/saving, the share of wealth in stocks (α_t), the share of wealth invested in new annuities (α_{At}) and the share of wealth allocated to the life insurance product (α_{lt})).

Cash on hand evolves according to

$$\begin{aligned} X_{t+1} &= (1-\alpha_{At}-\alpha_{lt})(X_t-C_t)[\alpha_t \exp(\widetilde{r}_{t+1})+(1-\alpha_t)\exp(r_f)] \\ +1(\alpha_t &> 0)0.3\alpha_{lt}(X_t-C_t)\exp(\max\left\{\frac{\widetilde{r}_{t+1}-r_f}{\exp(r_f)},0\right\})/H_t+L+A_{t+1} \end{aligned} \tag{4}$$

The first line denotes the returns from stock and bond investments, while the second reflects the next period pension (L) and annuity income (A_{t+1}) and the endowment policy that is only operative for the stockholders (captured by the indicator function $1(\alpha_t > 0)$). If the individual dies in period $t+1$, then next period cash on hand is augmented by the life insurance payout (equal to $\alpha_{lt}(X_t-C_t)\exp(r_f)/TI_t$ for the non-stockholders) and the household does not receive a pension or an annuity payout in that instance.[19] The stockholders' payout in that instance equals

$$0.7\alpha_{lt}(X_t-C_t)\exp(r_f)/H_t+0.3\alpha_{lt}(X_t-C_t)\exp(\max\left\{\frac{\widetilde{r}_{t+1}-r_f}{\exp(r_f)},0\right\})/H_t.$$

The annuity payout is the second continuous state variable and it evolves as

$$A_{t+1}=A_t+\alpha_{At}(X_t-C_t)\frac{A_t}{AV_t} \tag{5}$$

3.1.5 Preferences

We model household saving, portfolio and annuity choices from retirement onwards at an annual frequency. The household lives for a maximum of T (35) periods after retirement. Household preferences are described by the Epstein-Zin (1989) utility function:

[19] Consistent with Zeng (2008) we find that the timing of the inheritance or, in the context of the model, whether L and A_{t+1} are received at the time of death, can affect substantially the behavior of the household with an operable bequest motive. We think that this could be an interesting area for further research.

$$V_t = \left\{ (1-\beta) C_t^{1-1/\psi} + \beta \left(E_t (p_{t+1} V_{t+1}^{1-\gamma} + b(1-p_{t+1}) X_{t+1}^{1-\gamma}) \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{6}$$

where β is the time discount factor, b is the strength of the bequest motive, ψ is the elasticity of intertemporal substitution (EIS) and γ is the coefficient of relative risk aversion. The specification of the bequest motive is potentially a controversial issue in (6). Cocco et al. (2005) and Yogo (2008) make a similar assumption, while De Nardi (2004) and Lockwood (2009) assume a more complicated version.[20] Kopczuk and Lupton (2007) assume that utility from leaving a bequest is linear in wealth. The state variables in each period are current cash on hand, the annuity payment and age. In each period t, $t = 1, ..., T$, the household chooses optimal consumption C_t and the share of savings allocated to new annuities (α_{At}), the stock market (α_t) and the life insurance product (α_{lt}) with all shares being between zero and one.

3.1.6 Wealth Distribution and Pension Income

To eventually compare the predictions of the model with the observed annuity demand and participation rates, we need (among other exogenous inputs) an initial wealth distribution and a reasonable pension level, and we take both of these from the data. At the same time, based on our empirical results, we also condition these exogenous inputs on stock market participation status. We solve two different models, one in which stock market participation is allowed and another where access to the stock market does not exist, therefore requiring different inputs for wealth and pension income depending on the stock market participation status. We make this choice following the literature showing that wealth and stock market participation are positively correlated and that, to a first approximation, non-stockholders are poorer than stockholders so that a small fixed cost of participation can keep non-stockholders out of the stock market (see, for example, Gomes and Michaelides (2005), Alan (2006) or the evidence summarized in Guiso et al. (2002) and Campbell (2006)). This assumption is consistent with our data showing that mean financial wealth at retirement for

[20] The specified function in De Nardi (2004) is $\phi_1(1 + \frac{X}{\phi_2})^{1-\gamma}$.

stockholders is approximately five times the mean wealth of non-stockholders.[21] For non-stockholders the pension level is set at a constant 7,711 GBP per year and for stockholders at 11,523 GBP per year, corresponding to the empirical averages for the two groups. Using these exogenous inputs we start a simulation from age 65 onwards. For each age we compute the average annuity participation rate, the life insurance participation rate, the average portfolio demand and the aggregate demand for annuities.

3.1.7 Solution Technique and Other Parameters

This problem cannot be solved analytically. Given the finite nature of the problem, a solution exists and can be obtained by backward induction. The numerical appendix offers some details on the solution method. The maximum age that can be reached is 100, but agents will face a probability of death each period. We assume a constant interest rate equal to 2%. The mean equity premium is set at 4% with a standard deviation of 18%.

3.2 Results

3.2.1 Annuity and Life Insurance Policy Functions

We solve four different models depending on whether access to the stock market and the life insurance market is allowed. In the interest of space we present policy functions only for the models with access to life insurance, but the qualitative aspects of annuity demand policy functions are similar across all four models. Figure 3 plots the annuity demand choice at ages 65 (retirement) and 75 (the last year for which annuity purchases are allowed) for non-stockholders in the baseline case (at the estimated structural parameters from the next section: $\beta = 0.94, \gamma = 2, \psi = 0.07, b = 0.01$). For both cases the demand for annuities is zero for low wealth levels, reflecting mainly the annuity in the form of pension income received during retirement. Higher wealth levels generate an increasing demand for annuities for a given level of purchased annuities. However, for a given level of cash on hand, new annuities are a decreasing function of existing annuities. From the shape of the policy functions it

[21] Median wealth differences are even more extreme, with median wealth for non-stockholders being 5,000 GBP while median wealth for stockholders is 48,000 GBP.

should be noted that the wealth distribution is a necessary input before pronouncing the presence of an annuity market participation puzzle. In an economy where all households are very poor, the model predicts that no annuity demand will be generated and therefore the lack of annuity market participation becomes a prediction of the model.

In the absence of a bequest motive, there is no demand for life insurance products. In Figure 3 we can see that life insurance is mostly held by poorer households while the demand for life insurance decreases as liquid wealth rises. Thus, unlike the annuity demand schedule, life insurance demand decreases with higher wealth levels. This implication points towards a prediction of the model that generally implies that annuity and life insurance demands should be negatively correlated. Specifically, for a given annuity level, as wealth rises, annuity demand increases and life insurance demand decreases.

Figure 4 repeats the same graphs for the baseline case for stockholders (again at the estimated parameters: $\beta = 0.99, \gamma = 5, \psi = 0.4, b = 6.0$). The figure illustrates a number of intuitions raised in the literature. First, incremental annuitization can be important, as there are zero annuities purchased at retirement whereas there is a larger participation rate at age 75. The difference between the two ages implies that annuity market demand gradually increases with older age. Second, the share of wealth invested in incremental annuities is increasing in wealth for a given annuity level, while a certain wealth level needs to be reached before the household can purchase an annuity.

In Figure 4 we can see that (i) life insurance demand and participation rise with age, (ii) poorer households all participate in the life insurance market and (iii) the demand for life insurance decreases as liquid wealth rises. Thus, unlike the annuity demand schedule, allocations of wealth to life insurance decrease with higher overall wealth levels, which is the same prediction as from the non-stockholder version of the model.

3.2.2 Portfolio Choice Policy Functions

The share of wealth invested in the stock market as a function of cash on hand and age is familiar from the literature on life-cycle portfolio choice.[22] Specifically, pension income

[22] For instance, see Cocco et al. (2005), Gomes and Michaelides (2005) and Polkovnichenko (2007).

is treated like an implicit bond because it is certain. The share of wealth in stocks is a decreasing function of cash on hand because the investor allocates all financial savings to the stock market for diversification. For higher ratios of financial wealth to pension income, the portfolio becomes more diversified with more riskless assets added to the portfolio. However, given that there is no background risk (like uncertainty about medical expenditures) in the model, the portfolio remains heavily invested in the stock market.[23] An additional (unreported) prediction here is that, for a given level of cash on hand, a higher level of pre-purchased annuities increases the share of wealth invested in stocks as the annuity can be seen as an additional riskless asset.

Nevertheless, our results stress the importance of the bequest motive for asset allocation decisions. One might think that the presence of a bequest motive acts towards making the horizon of the investor longer, therefore generating a higher allocation of the financial portfolio in stocks. This intuition is wrong. As in Cocco et al. (2005), the fixed state pension (or the purchased annuity) is viewed as an implicit riskless asset. In the absence of a bequest motive, financial wealth is decumulated but the presence of the fixed pension income still makes the portfolio heavily biased towards stocks for diversification reasons. In the presence of a strong bequest motive, however, the household optimally does not decumulate financial wealth. The present value of state pension income does get depleted, however, because this is not determined by the preference for bequests. Therefore, the portfolio becomes much more balanced between bonds and stocks. The tendency to reduce stock market risk over time exists but the portfolio might remain balanced throughout retirement in the presence of a strong bequest motive. This analysis confirms the findings in Cocco et al. (2005), who show the importance of the bequest motive in generating balanced portfolios, but we are going to show next that this behavior can coincide with low demand for annuities.

3.2.3 Simulated Profiles

Given that we have computed policy functions for annuity and life insurance demands as a function of financial wealth and given the initial observed wealth distribution in the data, we

[23] In fact, for low levels of risk aversion we have the well-known complete portfolio specialization in stocks result, see, for example, Heaton and Lucas (1997).

can simulate the evolution of individual consumption, portfolio choice, annuity, life insurance demand and financial wealth for the remainder of a household's lifetime. We also calculate and report the annuity equivalent wealth (AEW) that will make an individual without access to the annuity market indifferent between staying outside the market or purchasing the optimal annuity for the given preference configuration and economic environment.[24] The maximum welfare when annuities are set to zero is calculated by solving the consumer's problem. We set annuities equal to zero, giving a value function equal to V (at retirement). The optimal decision with a potentially positive annuity is given by the value function V^* for the first time an annuity is purchased. We then solve for the percentage change in liquid wealth that will equate the two value functions for a given level of wealth as

$$V^*(X) = V(X + \Delta X) = V\left(\frac{X}{AEW}\right).$$

The AEW is therefore given by $X/(X + \Delta X)$; a number like 99% means that the household is willing to give up 1% of its wealth to be able to purchase an annuity, that is, annuities are welfare-improving to individuals.

We also report the share of annuitized wealth conditional on annuity market participation. This is defined as $AW_t = \frac{AV_t/(1+P)}{AV_t/(1+P)+X_t}$. The conditional life insurance payout is defined as the average payout conditional on having purchased a life insurance policy. The share of wealth in life insurance is the average over the life insurance policy functions. α_{lt} and the other shares of wealth (in bonds and stocks) are defined in a similar way.

The left-hand side of Table 5 reports these statistics for non-stockholders for different preference parameters (risk aversion, the elasticity of intertemporal substitution and the bequest motive) that are in the range of estimated parameters reported later on in the paper. The top part of the table reports the results when a life insurance market exists, while the bottom part reports results without a life insurance market. A similar reporting strategy is followed for the stockholders on the right hand side of Table 5. It is important to note that results across the four groups are not directly comparable because the discount factor changes each time to reflect the estimated discount factor from the section on structural

[24] This calculation follows Brown (2001).

estimation.

We start with the non-stockholders who have access to the life insurance market. As expected, higher risk-aversion increases annuity market participation and the total level of annuity demand,[25] while a stronger bequest motive decreases annuity demand. The conditional share of annuitized wealth can rise to between 37% and 53%. Surprisingly, the results illustrate that even in the absence of a bequest motive there exist configurations of parameters where the model still predicts low participation. For the non-stockholder case, when $\gamma = 2$ and $\psi = 0.1$, for instance, only 4.21% of households choose to participate in the annuity market. This result seems very surprising given the existing theoretical literature on the annuity market participation puzzle. What explains this finding? This preference parameter configuration implies a weak motive to save, the wealth distribution is skewed to the left for this group with many poor households and the pension system already provides a substitute for the provision of longevity insurance. The combination of all these factors means that very few households choose to participate in the annuity market.

This explanation is consistent with the other finding from Table 5: that as risk aversion increases, the insurance value of annuities rises substantially and annuity market participation can rise up to 17.3% (for $\gamma = 3$ and $\psi = 0.5$). The table also illustrates that lower annuity demand can be generated if one is willing to admit some preference for leaving bequests. Specifically, for ($\gamma = 2$, $\psi = 0.1$, $b = 0.2$) annuity market participation is around 3.62%. In the absence of a bequest motive, life insurance participation is zero. With the bequest motive included, poorer households participate very strongly in the life insurance market, with the participation rate rising from 0% to above 85% for a very weak bequest motive ($b = 0.2$).

Comparing the results above to the bottom part of Table 5 where access to life insurance is not allowed, we find that our conclusions are relatively unchanged. The low value of annuity access is shown by the AEW, which is very close to 100% for both sets of models.

The right hand side of Table 5 reports results for stockholders, including two factors that

[25] The reported average level of voluntary annuity demand falls but the total annuity demand rises since there are more participants in this scenario. We report this statistic because this will be more directly comparable to the empirical section which reports per capita annuity income conditional on participation.

were not necessary in the non-stockholder model. First, we report the share of financial wealth in stocks. Second, we expand the range of preference parameters for which we report results to reflect the range of estimated parameters from the next section. Thus, the bequest parameter rises from $b = 0.2$ to $b = 6$. The basic qualitative results are similar to the non-stockholders' case. Annuity demand and participation both increase when risk aversion increases and decrease when the strength of the bequest motive increases. Two comparative statics results stand out. First, the share of wealth in stocks can be almost 100% when annuity market participation is close to zero ($\gamma = 2$, $b = 6$). This occurs because the equity premium makes saving through the equity market more valuable than the annuity insurance. This portfolio manifestation of the equity premium puzzle must be taken into account as one does not want to explain low annuity market participation by generating a counterfactual implication about asset allocation in equities. Second, life insurance participation is substantially lower for this richer group of households, ranging between 15% and 35%. Stockholder households are generally wealthier than non-stockholder households, so this finding is consistent with the policy function results that illustrate a declining share in life insurance as households become richer.

The effect of EIS is ambiguous/non-monotonic because it depends on how the EIS affects total saving. This in turn is determined by the difference between the expected rate of return and the discount rate. The expected rate of return is affected by risk aversion through the share of wealth invested in stocks (as explained in Campbell and Viceira (1999)). If the discount rate is lower than the expected rate of return, a higher EIS generally increases saving and therefore results in higher annuity market participation. It is important to note that the effect of EIS is influenced by the (endogenous) portfolio choice that determines the rate of return on the portfolio and thus the difference between the rate of return and the discount rate. For low levels of risk aversion the household is more aggressive and earns a higher mean return on the portfolio, whereas this is reversed for higher levels of risk aversion.

3.3 Summary

We use a life-cycle model to understand both qualitatively and quantitatively the importance of preference parameters affecting the demand for annuities and life insurance. Risk aversion, the strength of the bequest motive, the elasticity of intertemporal substitution and the decision to access the stock market are key determinants of the model's quantitative predictions. Financial wealth, a key endogenous state variable in the model, is directly affected by these parameters and is therefore a key predictor variable in assessing the model's quantitative implications. Contrary to frictionless theoretical models, there exist reasonable preference parameter configurations that generate very low annuity market participation, once an empirically reasonable initial wealth distribution is used to simulate the model.

4 How Deep is the Puzzle?

In this section we evaluate the extent to which the model's predictions are at odds with the data. We employ a Method of Simulated Moments (MSM) estimator to pick the structural parameters that minimize the distance between some selected moments in the data and in the model. Consistent with the empirical evidence from the previous sections, we separate our analysis between stockholders and non-stockholders, and within these two groups we differentiate between households with and without access to a life insurance market.

4.1 Matching Data to Model

We are interested in comparing the average demand for annuities and life insurance products simulated from the model with the average demand in the data. To maintain computational tractability our model is necessarily simpler than reality. One key difference is that the maturity of the life insurance contract is longer in the data than in the model. We can compute the current value of a term insurance in the data (V_t^{TI}) for a household of age t as follows

$$V_t^{TI} = (1+P)\,LP\left(\frac{1-p_{t+1}}{1+r_f} + 1(M>1)\sum_{s=t+2}^{t+M} \frac{\prod_{j=t+1}^{s-1} p_j\,(1-p_s)}{(1+r_f)^{s-t}}\right). \tag{7}$$

where M denotes remaining maturity, and LP is the payout if the household dies within the maturity of the contract. While we observe in the data whether households have a term insurance or an endowment plan, we do not observe the remaining maturity of these contracts. The oldest sample member with a life insurance contract is of age 88 (recall that the sample is truncated at age 89), so we assume $M = 12$.[26] Then, with our assumptions about survival probabilities, the risk free rate and P, and given the observed LP in the data, we can compute V_t^{TI} in the data. For the non-stockholders we can therefore match the term insurance in the data V_t^{TI} relative to financial wealth with its model counterpart (α_{lt}).

In the data we also have investment-linked life insurance products. Let V_t^{EP} denote the current value of an investment-linked endowment plan with remaining maturity M. In contrast with a term insurance, the endowment contract also pays out at maturity if the household survives. At the end of each policy year, the insurance company grants a bonus that depends on the performance of the underlying investment portfolio. The bonus is determined in such a way that the total interest rate credited to the insured equals the return on the investment portfolio, $\widetilde{r}$, but never falls below the minimum guaranteed interest rate, which we set to r_f. Thus, if LP is the initial sum insured, the endowment plan payout in period s is $LP_s = LP\left(\prod_{j=1}^{s}(1+\delta_j)\right)$ with $\delta_j = \max\left\{\frac{\widetilde{r}_j - r_f}{1+r_f}, 0\right\}$. Using the Black-Scholes option pricing framework, Bacinello (2001) shows that

$$V_t^{EP} = (1+P)\,LP\left((1-p_{t+1})\,v + 1(M>1)\sum_{s=t+2}^{t+M}\prod\nolimits_{j=t+1}^{s-1} p_j\,(1-p_s)\,v^{s-t} + \prod\nolimits_{j=t+1}^{t+M} p_j v^M\right) \tag{8}$$

with discount factor $v = \exp(-r_f) + (1+r_f)^{-1} c$, where $c = F(d_1) - (1+r_f)\exp(-r_f)\,F(d_2)$, $d_1 = \sigma^{-1}\left(r_f + 0.5\sigma^2 - \log(1+r_f)\right)$, $d_2 = d_1 - \sigma$, and F denotes the cumulative distribution function of a standard normal random variable.

Of the stockholders in our sample who have a life insurance contract, around 30% have an endowment plan and about 70% hold a term insurance. Almost all non-stockholders with a life insurance hold a term insurance. To reflect this composition in the data, and assuming the endowment fund is fully invested in stocks, we calculate the current value of life

[26] Assuming $M = 20$ does not change the results.

insurance products in the data as $0.7V_t^{TI} + 0.3V_t^{EP}$ for stockholders. This can be matched to the model-equivalent of this value given by $\alpha_{lt}(X_t - C_t)$

We now can describe the wealth and portfolio choice moments we are matching in our Method of Simulated Moments exercise. For the most general model with access to the stock market and the life insurance market we match the averages over ages 65-89 of:[27]

1. Financial wealth: X_t
2. Annuity market participation: $1(A_t > 0)$
3. Conditional annuity demand: $A_t | A_t > 0$
4. Conditional share of wealth annuitized: $AW_t | A_t > 0$
5. Life insurance market participation: $1(LP > 0)$
6. Share of wealth in life insurance: α_{lt}
7. Share of wealth in bonds: α_{Bt}
8. Share of wealth in stocks: α_{St}.

For the restricted model without access to the stock market we drop moment (8) and set $S_t = 0$. For the restricted model without access to the life insurance market, we drop moments (5), (6), and (7) and for the simplest model without access to either the stock or life insurance markets we drop moments (5-8).

In the non-stockholder versions we estimate three parameters: the bequest parameter, the discount factor and the elasticity of intertemporal substitution. We set risk aversion equal to two, which is consistent with the available empirical evidence for large parts of the population (Gourinchas and Parker (2002), for instance). For the stockholders we also estimate the risk aversion coefficient because we need a higher coefficient of risk aversion to generate a balanced portfolio between bonds and stocks.

[27] To compute averages from the simulated data we derive the demographic weights that would be implied by the household's survival probabilities. We then weight each cohort by the respective demographic weight. The conditional survival probabilities are taken from the U.K. GAD for 2002-2004.

4.2 Non-Stockholders

Table 6 (left-hand side) reports the estimated structural parameters for the two different models (with and without access to the life insurance market).[28] The elasticity of intertemporal substitution is estimated at 0.07 or 0.25 depending on the presence/absence of a life insurance market. There is evidence for a bequest motive ($b = 0.01$ or $b = 0.11$). Even though the bequest coefficient is near zero, bequests are essential in this model to match the life insurance participation and the annuity rates observed in the data (life insurance participation is zero when $b = 0$). The weak bequest motive is also consistent with the results in De Nardi et al. (2010). Moreover, the elasticity of intertemporal substitution is consistent with studies based on intertemporal Euler equations (Vissing-Jorgensen (2002)).

The predicted annuity market participation rate for this group of non-stockholder households is 3.89% (3.25% in the absence of life insurance) versus 3.0% in the data. Conditional on participation, the annual annuity purchased is around 3,830 GBP (3,170 GBP without life insurance) in the model and 1,920 GBP in the data. Life insurance participation is 68.5% (40.7% in the data). We think that the intuition for these results is clear. Non-stockholders

[28] The parameter vector (θ) is chosen to minimize the quadratic form $Argmin_\theta D'\Omega^{-1}D$. Under regularity conditions given in Duffie and Singleton (1993), $\sqrt{T}(\hat{\theta} - \theta) \to N(0, W_H)$. The different components of the quadratic are defined as follows,

$$D = \left(\frac{1}{T}\sum_{t=1}^{T} m(Y_t) - \frac{1}{TH}\sum_{t=1}^{TH} m(\tilde{Y}_t)\right)$$

$$\Omega = Var(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} m(Y_t))$$

$$W_H = (1 + \frac{1}{H})\left(E\left[\frac{\partial m(\tilde{Y}_{[TH]})'}{\partial\theta}\right]\Omega^{-1}E\left[\frac{\partial m(\tilde{Y}_{[TH]})}{\partial\theta}\right]\right)^{-1}$$

$m(Y_t)$ denotes the different moments chosen, variables $Y, (\tilde{Y})$ denote actual (simulated) data, T is the sample size and TH is the total size of simulated data. Following the rules of thumb in Michaelides and Ng (2000) we use $H = 10$. The derivatives are computed numerically and E is the population average (sample analog used in the estimation). Following De Nardi et al. (2010) we use a diagonal matrix for weighting the moment conditions because, even though the optimal weighting matrix is asymptotically efficient, it can be severely biased in small samples. The diagonal weighting scheme uses the inverse of the matrix that is the same as Ω along the diagonal and has zeros off the diagonal of the matrix.

are poor households that optimally choose not to annuitize, or annuitize a small fraction of their wealth, because pension income already provides substantial insurance against longevity risk.

Figure 5 (left-hand side) compares the implications of both models to the data during retirement. Both models generate a close fit for the wealth evolution during retirement, the annuity market participation and the conditional share of wealth being annuitized. Even though the conclusions with regards to annuity demand levels are robust to the presence or absence of life insurance, the figures illustrate that the model predicts a stronger demand for life insurance than is evident in the data. Both participation and the share of wealth in insurance are predicted to rise in retirement, contrary to the more stable values in the data. We think that this is an artifact of the one-period life insurance assumption (as poor households decumulate their wealth and approach their pension level they buy the one-period life insurance contract to satisfy their bequest motive). We think that an interesting topic for further research will be to better understand life insurance choices.

It could be argued that our results arise from certain exogenous assumptions in the model. For instance, we use a load factor of 20% which might be considered very high. We therefore investigate the robustness of our conclusions to such maintained assumptions. Table 6 reports the results from changing these assumptions while maintaining everything else as in the estimated model. A lower pension (set at the 25th percentile; column "lowp" in Table 6) increases annuity market participation from 3.89% to 5.01% (3.25% to 4.65% without life insurance).

One interesting comparative static involves reducing the number of years for which an annuity can be purchased. In the column titled "once", we set that time to one year so that annuities can only be purchased at the time of retirement. We find that annuitizing only once does not significantly change annuity demand, reflecting the fact that we do not include any time-variant inflation or interest rate considerations in the model. Similarly, extending the maximum age at which an annuity can be purchased from 75 to 85 (column "incr"), does not affect annuity demand, indicating that households optimally stop accumulating annuities before age 75. We next investigate the implications of a lower subjective survival probability (the household expects the survival probability to be 10% lower than the objective one; column "lows"). This expectation drives annuity demand down to zero for both models. For the model that includes life insurance, however, the demand for life insurance dramatically increases from 68.5% to 85.6%. Thus, the expected decreased annuity demand is accompanied by the large increase in life insurance demand in this comparative static, emphasizing the close link between life insurance and annuity products. We also investigate what happens when an actuarially fair annuity policy exists (column "fair"). This change increases annuity participation from 3.89% to 8.75% and voluntary annuity demand from 3,830 GBP to 5,550 GBP. Overall, these results indicate that there is a range of possible outcomes that the model can generate depending on exogenous assumptions. However, we view as robust the basic message that there exist preference parameters that can replicate the observed low annuity demand in the data as part of the posited structural model.

4.3 Stockholders

We report the results for stockholders on the right-hand side of Table 6. Across the two models (with and without life insurance) the estimated preference parameters are of similar magnitude. The elasticity of intertemporal substitution is around 0.4 and the bequest parameter equals 6.0 in both models, with risk aversion slightly higher (6.5) when life insurance is not present (5.0 when it is). The only parameter in which we observe a clear deviation between the models is the discount factor, which is equal to 0.99 in the model with life insurance and 0.89 in the model without. Given the substantial differences between the moments being matched, we view these preference parameters as empirically plausible in both models.

The level of annuity market participation for stockholder households is around 22% (16% without life insurance) against 10.1% in the data, while life insurance participation equals 36.1% (30.6% in the data). The annual annuity income equals 4,730 GBP with life insurance and 4,350 GBP without (3,660 GBP in the data). We view these predicted outcomes as quite close to their observed counterparts.

Figure 5 (right-hand side) illustrates how closely our predicted behavior for stockholders matches its empirical counterpart. In the model with life insurance, wealth is not decumulated (due to the bequest motive) and the level of financial wealth is slightly higher than its empirical counterpart. In the same model, annuity market participation is predicted to be higher than in data in the later part of life. The conditional share of wealth being annuitized is slightly higher in the data, while the share of wealth in stocks is higher in the model. On the other hand, life insurance participation is now better matched, while the remaining part of the household portfolio (bonds) is higher in the data than in the model. In the absence of life insurance, there is some decumulation of financial wealth, the annuity market participation rate is now much closer to its empirical counterpart and the conditional share of annuitized wealth matches well with the data. The share of financial wealth in stocks is higher in the data than in the model (with the maximum difference at around 25%, shrinking to zero by age 89).

In Table 6 we offer some further comparative statics to investigate the robustness of the conclusions to changes in the economic environment. A lower (25th percentile) pension level (column "lowp" in Table 6) affects the annuity market participation rate in the expected way by increasing participation from 22.1% to 39.7% (and from 15.9% to 37.5% with no life insurance), while it substantially crowds out the participation in the life insurance market (from 36.1% to 22.4%). When markets become actuarially fair, the annuity market participation rate rises from 22.1% to 40.6% (column "fair"), while a lower expected survival probability reduces annuity demand from 22.1% to 2.78% (column "lows"). Overall, we interpret these findings as being supportive of the robustness of the results.

4.4 Do these Findings Square up with the Literature?

We use three types of ex ante heterogeneity in preferences to reconcile the low take-up of annuities. First we look at heterogeneity in the elasticity of intertemporal substitution (EIS) between households. A low elasticity of intertemporal substitution for non-stockholders and a higher elasticity of intertemporal substitution for wealthier stockholders help reconcile our model with the data, and it is consistent with the empirical evidence offered in Vissing-Jorgensen (2002). Gomes and Michaelides (2005) calibrate a portfolio choice model and argue that this type of heterogeneity can explain saving behavior and stock market participation over the working part of the life cycle, while Gomes et al. (2009) estimate similar preference parameters to explain wealth accumulation through tax-deferred accounts in U.S. data. Guvenen (2006) uses this type of heterogeneity to explain the estimates of low elasticity in studies using aggregate data. Overall, we view our EIS estimates as consistent with the empirical evidence.

We can introduce the second type of heterogeneity in two different ways. We could increase heterogeneity in risk aversion while maintaining the original discount rates; to generate balanced portfolios we need a higher risk aversion for stockholders than for non-stockholders. Alternatively, we could decrease the discount factor even further for non-stockholders while maintaining the original levels of risk aversion. Making the non-stockholders more impatient would counteract the higher risk aversion and would cause them to decumulate wealth during retirement and demand lower annuities. These two changes would produce the same observable effect.

Third, we increase heterogeneity in the strength of the bequest motive. Most studies testing for bequest motives compare households with and without children or use the elicited responses of households expecting to leave inheritances. These studies find little explanatory power for the annuitization decision (for example, Brown (2001)). Our results indicate that perhaps combining the predictions for another market (like life insurance), or focusing on other implications of bequests through a more structural model, might be other ways to offer evidence for a bequest motive. In particular, we think that the effect of a strong bequest motive in generating a balanced portfolio in stocks has not been sufficiently stressed in the literature. Here the bequest motive can generate a much slower decumulation of wealth during retirement, while for the same reason it can generate balanced portfolios. As financial wealth is depleted at a slower rate than the implicit riskless assets in the form of pensions, diversification dictates that the household holds a balanced portfolio. Cocco et al. (2005) find this effect for CRRA preferences but they do not compare the resulting profiles to asset allocation profiles from the data. Furthermore, the need for a bequest motive is consistent with recent evidence like that put forth by De Nardi (2004), who emphasizes the need for a bequest motive to match the observed wealth distribution. Kopczuk and Lupton (2007) also use this motive to better understand U.S. wealth data during retirement. Interestingly, our results are consistent with De Nardi et al. (2010) who find that poorer households have a very weak bequest motive relative to wealthier households. Our results are also consistent with Yogo (2008) who needs a bequest motive to generate low welfare gains from annuity market participation in a model with housing and health investments for the U.S. Health and Retirement Survey.

The estimated discount factor seems low at 0.88 for one of the four groups, and it is indeed on the low end of the estimates in the literature. For example, Gourinchas and Parker (2002) estimate it between 0.93 and 0.96. A recent paper by Love (2010) that includes life insurance choices estimates it between 0.9 and 0.92. We think that a higher discount rate during retirement can be a plausible mathematical representation of marginal utility shifts caused by adverse health shocks in that period of the life-cycle. Given the recent emphasis on medical/health shocks (for instance, Pang and Warshawsky (2010), Ameriks et al. (2008) and Yogo (2008)) we think a version of the model that more explicitly deals with how health affects wealth and in turn portfolio choice decisions may generate similar results with a slightly higher discount factor.

5 Conclusion

We provide an in-depth empirical analysis of the characteristics of households that participate (or not) in the U.K. voluntary annuity market. We document that annuity demand increases in financial wealth, education and life expectancy, while it decreases in pension income and a possible bequest motive for surviving spouses. We then estimate a life-cycle model of household portfolio choice, life insurance purchases, and annuity demand after retirement. The model emphasizes the role of access to stock market opportunities, bequests, risk aversion and the elasticity of intertemporal substitution (and through these financial wealth) as the main determinants of annuity demand. Comparing the predictions of the model with their empirical counterparts, we find that reasonable preference parameters can generate the low annuity demand observed in the data. We emphasize that by assuming that all purchased annuities are of the nominal (fixed payout) type, we are assuming essentially an incomplete market. According to Davidoff et al. (2005) we should not expect full annuitization (and participation) in an incomplete market. We show that we can match the observed percentages once ex ante heterogeneity is permitted in risk aversion, the elasticity of intertemporal substitution and the bequest motive.

In this paper, we match average annuity and life insurance market participation rates and the average demand for annuities conditional on participation with our model. It was more difficult to match all features of the data when using the models with an explicit life insurance choice. An ambitious future research project could try to match participation and demand statistics along different percentiles of the wealth and pension distribution with a more comprehensive version of our model that would include multi-period life insurance contracts.

Appendix A The Data

The "Income and Assets" module of ELSA[29] is distributed to all financial units within a household. A financial unit is either a single person, or a couple if the latter declares to share their income and assets. If a couple treats their income and assets separately, it will consist of two financial units. Since we want to use the annuity information on the least aggregated level, we prepare the data on a financial unit level and employ individual specific information (like age, gender, education, and health) of the person who filled in the "Income and Assets" module. Financial information (like wealth and income) is collected at the household level. The first wave of ELSA comprises 12,100 individuals. Our sample consists of 5,233 households because we exclude households without a member in retirement (2,206 observations), partners from couples who report joint income and assets (3,536 observations) and observations with missing values for our variables of interest to be discussed below (1,125 observations).

[29] The data were made available through the U.K. Data Archive. ELSA was developed by a team of researchers based at the National Centre for Social Research, University College London and the Institute for Fiscal Studies. The data were collected by the National Centre for Social Research. The funding is provided by the National Institute of Aging in the United States, and a consortium of U.K. government departments coordinated by the Office for National Statistics. The developers and funders of ELSA and the Archive do not bear any responsibility for the analyses or interpretations presented here.

Appendix B Numerical Solution

There are three state variables (age, cash on hand and purchased annuities) and four control variables (consumption, share of wealth in stocks, share of wealth in life insurance and share of wealth in incremental annuities) in the most general version of the model. The household problem is therefore given by

$$V_t(X_t, A_t) = \underset{c_t,\alpha_t,\alpha_{At},\alpha_{lt}}{MAX} \left\{ (1-\beta)C_t^{1-1/\psi} + \beta \left(E_t(p_{t+1}V_{t+1}^{1-\gamma}(X_{t+1}, A_{t+1}) + b(1-p_{t+1})X_{t+1}^{1-\gamma}) \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

where the evolution of the state variables is given in (4) and (5).

We solve the model recursively backwards[30] starting from the last period. In the last period ($t = T$) the policy functions are trivial and the value function corresponds to the bequest function. We solve for four control variables in every year. For every age t prior to T, and for each point in the state space, we optimize using grid search. From the Bellman equation the optimal decisions are given as current utility plus the discounted expected continuation value ($E_t V_{t+1}(.)$), which we can compute because we have obtained V_{t+1}. We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the risky asset returns. We discretize the state-space along the two continuous state variables and use tensor product splines to perform the interpolation of the value function for points that do not lie on the state space grid, with more points used at lower levels of wealth where the value function has high curvature. Equivalently, we use a denser set of grid points for low values of wealth for the two accounts because the consumption function exhibits a kink at the points where liquidity constraints are no longer binding. Once we have computed the value of each alternative we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation, we obtain this period's value function ($V_t(.)$), which is then used to solve the previous period's maximization problem. This process is iterated until $t = 1$.

[30] We use a value function approach to solve the problem (unlike Zeng (2008) who uses an Euler equation approach).

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Tables

Table 1: Annuity, stock and life insurance market participation

	S = 1	S = 0	Total
A = 1	213	96	309
Col-%	9.6	3.2	5.9
A = 0	2007	2917	4924
Col-%	90.4	96.8	94.1
L = 1	832	1272	2104
Col-%	37.5	42.2	40.2
L = 0	1388	1741	3129
Col-%	62.5	57.8	59.8
Total	2220	3013	5233
Row-%	42.4	57.6	100.0

Notes to Table 1: The table presents the number of sample members in sub-samples defined by participation in the voluntary annuity market (A), the stock market (S) and the life insurance market. "A = 1" ("A = 0") refers to annuity market (non-) participants in 2002 or 2004, "S = 1" ("S = 0") refers to stock market (non-) participants in 2002, while "L = 1" ("L = 0") refers to life insurance market (non-) participants in 2002. The sample consists of retired households in the first (2002) wave of the English Longitudinal Study of Ageing (ELSA).

Table 2: Financial wealth and annual income by annuity and stock market participation

	All		A = 1		A = 0	
	Mean	Median	Mean	Median	Mean	Median
Financial wealth	55031	15800	135017	65000	50011	14200
Annual pension	9328	7305	12182	9036	9149	7228
Annual public pension	4796	4732	4945	4940	4787	4723
Annual private pension	4532	1440	7236	3200	4362	1350
Annual annuity income	179	0	3032	984	0	0
Life insurance payout	17154	3000	55936	10000	15177	2532
Stock share percentage	16	0	24	14	16	0
	S = 1		S = 1 and A = 1		S = 1 and A = 0	
	Mean	Median	Mean	Median	Mean	Median
Financial wealth	101937	47586	173619	99300	94330	44474
Annual pension	11523	9132	14142	11660	11245	8978
Annual public pension	4521	4628	4943	4948	4476	4628
Annual private pension	7002	4145	9199	6600	6769	4000
Annual annuity income	351	0	3656	1200	0	0
Life insurance payout	27523	5900	74984	10000	23027	5000
Stock share percentage	38	32	35	28	38	33
	S = 0		S = 0 and A = 1		S = 0 and A = 0	
	Mean	Median	Mean	Median	Mean	Median
Financial wealth	20470	5000	49368	18420	19519	5000
Annual pension	7711	6315	7832	6551	7707	6304
Annual public pension	4999	4784	4952	4940	5001	4784
Annual private pension	2712	500	2880	693	2706	500
Annual annuity income	53	0	1648	484	0	0
Life insurance payout	10372	2000	14376	2000	10266	2000

Notes to Table 2: The table presents mean and median wealth and income statistics (in GBP) and stock allocation percentages for the whole sample (“All”) and sub-samples defined by participation in the voluntary annuity market (A) and the stock market (S). “A = 1” (“A = 0”) refers to annuity market (non-) participants in 2002 or 2004 while “S = 1” (“S = 0”) refers to stock market (non-) participants in 2002. The life insurance payout statistics are conditional on participation in the life insurance market. The sample consists of 5,233 retired households from the first (2002) wave of the English Longitudinal Study of Ageing (ELSA).

Table 3: Socio-economic background, health and life-expectancy

	All	A = 1	A = 0
Age	69.3	68.2	69.4
Female (%)	53	42	54
Married (%)	56	57	56
Number of children	2.04	1.98	2.04
Low education (%)	59	34	61
Medium education (%)	30	41	30
High education (%)	11	25	9
Survival probability (%)	52	57	52
Objective GAD probability (%)	53	56	53
Bad health condition (%)	19	14	19
Medium health condition (%)	62	60	63
Good health condition (%)	19	27	18

Notes to Table 3: The table presents averages for all sample members ("All"), voluntary annuity market participants ("A = 1") in either 2002 or 2004, and annuity market non-participants ("A = 0"). The sample consists of 5,233 retired households from the first (2002) wave of the English Longitudinal Study of Ageing (ELSA).

Table 4: Estimation results for the annuity market participation decision and the conditional annuity demand

	All				Non-Stockholders				Stockholders			
	Annuity market Participation		Conditional log annuity demand		Annuity market Participation		Conditional log annuity demand		Annuity market Participation		Conditional log annuity demand	
Variable	estimate	t-value	estimate	t-value	estimate	t-value	estimate	t-value	estimate	t-value	estimate	t-value
Intercept	-	-	3.9301	0.84	-	-	5.0530	0.63	-	-	-0.5309	-0.09
Age / 10	**0.0162**	1.88	-0.0433	-0.03	-0.0035	-0.33	-0.7438	-0.31	**0.0448**	2.48	0.7194	0.41
Age^2 / 100	-	-	0.0012	0.01	-	-	0.0561	0.33	-	-	-0.0567	-0.44
Low education	**-0.0289**	-2.80	0.0391	0.19	**-0.0212**	-1.71	0.2381	0.70	**-0.0437**	-2.12	-0.0804	-0.31
High education	**0.0304**	1.90	-0.1086	-0.55	0.0287	1.02	0.2517	0.41	0.0396	1.49	-0.1236	-0.60
Female	**-0.0229**	-2.36	-0.0663	-0.37	-0.0169	-1.49	-0.0112	-0.03	**-0.0360**	-1.82	-0.1302	-0.62
Married	**-0.0359**	-3.41	0.1363	0.68	**-0.0275**	-2.13	0.2310	0.70	**-0.0579**	-2.76	0.0940	0.37
Children	0.0069	0.57	0.1786	0.87	0.0152	0.90	0.4835	1.17	-0.0013	-0.06	0.0848	0.35
Life insurance holder	-0.0070	-0.75	-0.1446	-0.78	-0.0079	-0.71	-0.2622	-0.78	-0.0043	-0.22	-0.1231	-0.54
Survival probability	0.0272	1.46	0.3672	1.07	0.0031	0.14	0.3907	0.59	**0.0749**	1.95	0.3325	0.83
Log pension	-0.0031	-0.61	-0.1197	-1.61	-0.0090	-1.40	-0.0409	-0.40	0.0068	0.58	**-0.2162**	-1.90
Log financial wealth	**0.0232**	4.62	**0.3290**	2.33	**0.0180**	3.58	**0.3213**	2.11	**0.0566**	5.58	**0.6277**	6.81
Stockholder	**0.0234**	1.86	**-2.8853**	-1.71	-	-	-	-	-	-	-	-
Stockholder x f. wealth	-	-	**0.2839**	1.74	-	-	-	-	-	-	-	-
Nr. of observations	5233		309		3013		96		2220		213	
Fit of the model	Corr. pred.: 94%		R^2: 24.48%		Corr. pred.: 97%		R^2: 14.57%		Corr. pred.: 90%		R^2: 22.48%	

Notes to Table 4: The table reports estimation results from a probit model for the annuity market participation decision and from a linear regression model for the (log) annuity demand conditional on participation. Parameters in bold are statistically significant at the 10% level. For the probit model, the marginal effects given are calculated for a 65 year-old, single male, with no children, medium education, average subjective survival probability, average pension and average wealth who does not participate in the stock and life insurance markets. For the linear model, heteroskedasticity-consistent standard errors are computed. Based on data for retirees in ELSA 2002.

Table 5: Average simulated portfolio choice variables for varying preference parameters

	Non-Stockholders								Stockholders							
Bequest parameter, b	0	0	0	0	0.2	0.2	0.2	0.2	0	0	0	0	6	6	6	6
Relative risk aversion, γ	2	2	3	3	2	2	3	3	2	2	6	6	2	2	6	6
Elast. intertemp. subst., ψ	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.2	0.5	0.2	0.5	0.2	0.5	0.2	0.5
With life insurance	β = 0.94								β = 0.99							
Annuity m. participation	4.21	8.44	8.71	17.3	3.62	4.27	6.28	3.62	3.60	15.4	44.5	69.3	0.65	0.65	28.9	26.1
Condit. annuity demand	4.63	5.40	5.75	3.93	3.46	5.23	6.00	3.46	2.14	4.79	6.24	5.65	2.51	2.55	5.17	5.09
Cond. sh. of w.lth in ann.	47.6	52.9	51.7	49.5	36.8	38.9	46.9	36.8	29.1	38.3	55.2	52.4	4.96	2.45	30.3	15.4
Life insu. m. participation	0.00	0.00	0.00	0.00	78.3	85.7	76.9	78.4	0.00	0.00	0.00	0.00	15.2	6.72	36.4	35.2
Condit. life insu. payout	0.00	0.00	0.00	0.00	7.37	15.7	9.78	7.37	0.00	0.00	0.00	0.00	16.2	12.5	25.3	51.3
Sh. of wealth in life insu.	0.00	0.00	0.00	0.00	66.0	66.6	61.0	66.0	0.00	0.00	0.00	0.00	6.13	3.33	19.7	25.7
Share of wealth in bonds	99.9	99.8	99.7	99.4	34.0	33.3	38.9	34.0	26.7	3.69	10.8	6.11	1.91	8.54	15.8	28.0
Share of wealth in stocks	-	-	-	-	-	-	-	-	73.2	95.8	86.4	87.8	92.0	88.1	63.7	45.7
Annuity equival. wealth	99.9	99.7	99.6	99.1	99.9	99.9	99.8	99.9	99.9	99.7	94.9	86.2	99.9	99.9	98.6	99.0
Without life insurance	β = 0.88								β = 0.89							
Annuity m. participation	3.84	4.24	6.29	4.93	3.04	2.77	4.91	4.25	0.65	4.05	26.1	14.1	0.00	0.00	18.0	8.19
Condit. annuity demand	4.01	4.39	6.87	6.31	2.53	2.55	5.87	4.75	2.37	2.07	5.59	4.28	0.00	0.00	4.73	3.50
Cond. sh. of w.lth in ann.	45.6	51.7	54.0	59.1	32.6	29.8	44.4	39.5	35.3	30.0	50.3	49.1	0.00	0.00	33.5	26.7
Share of wealth in stocks	-	-	-	-	-	-	-	-	60.9	56.4	72.0	53.4	99.6	99.9	65.6	69.0
Annuity equival. wealth	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	98.7	99.7	100	100	99.4	99.9

Notes to Table 5: The table shows means of selected portfolio choice variables simulated from the model using the preference parameter constellations given in rows two, three and four. The model is solved with and without access to the stock market (stockholders and non-stockholders, respectively) and with and without access to the life insurance market. The preference parameters (b, γ, ψ) are set in a range that captures the estimated parameters from the empirical section of the paper. The parameter β is set to the estimated parameter obtained from estimating the model with and without access to the stock and life insurance markets.

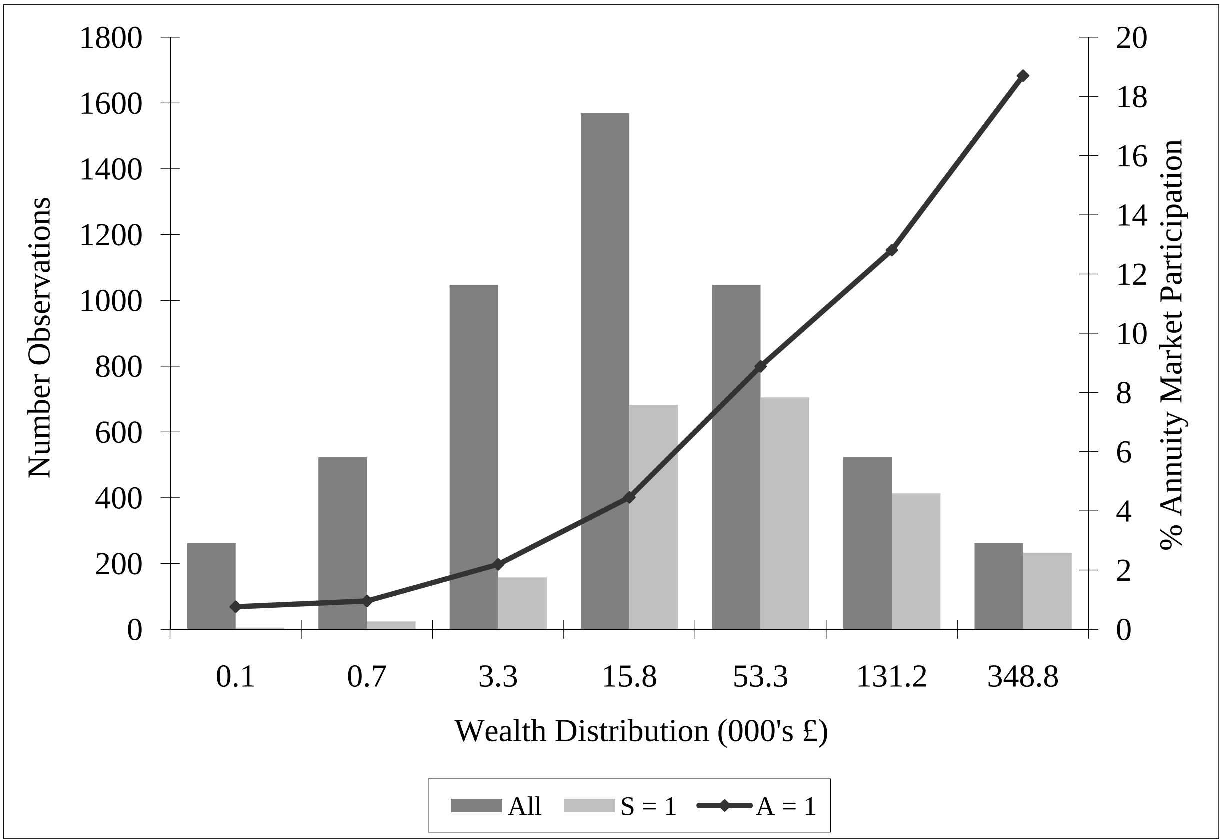
Table 6: Method of Simulated Moments (MSM) estimation results and robustness analysis

	Non-Stockholders								Stockholders							
	data	stdev	MSM	lowp	once	incr	lows	fair	data	stdev	MSM	lowp	once	incr	lows	fair
With life insurance	β	se(β)	b	se(b)	γ	se(γ)	ψ	se(ψ)	β	se(β)	b	se(b)	γ	se(γ)	ψ	se(ψ)
	0.94	0.07	0.01	0.06	2	-	0.07	0.01	0.99	0.05	6	0.01	5	0.01	0.4	0.07
Annuity m. participation	3.01	17.1	3.89	5.01	3.89	3.89	0.00	8.75	10.1	30.2	22.1	39.7	16.3	22.6	2.78	40.6
Condit. annuity demand	1.92	5.30	3.83	4.92	3.16	3.83	0.00	5.55	3.66	8.41	4.73	4.06	5.00	5.02	2.51	5.42
Cond. sh. of w.lth in ann.	38.0	31.9	43.7	49.0	40.4	43.7	0.00	54.5	28.2	26.8	17.5	18.7	17.3	18.3	8.10	21.5
Life insu. m. participation	40.7	49.1	68.5	53.9	68.5	68.5	85.6	69.3	30.6	46.1	36.1	22.4	36.1	36.1	52.5	36.1
Condit. life insu. payout	5.80	21.4	1.38	1.08	1.37	1.38	1.82	1.53	21.0	129	41.6	11.8	41.6	41.6	48.1	41.6
Sh. of wealth in life insu.	23.6	38.5	50.4	30.0	50.3	50.4	75.2	51.2	15.3	33.1	22.4	4.25	22.4	22.4	41.0	22.4
Share of wealth in bonds	75.4	38.8	49.5	69.9	49.7	49.5	24.8	48.6	51.3	33.3	20.4	25.8	21.0	20.2	23.3	18.3
Share of wealth in stocks	-	-	-	-	-	-	-	-	31.6	29.1	56.7	69.0	56.3	56.9	35.7	58.2
Annuity equival. wealth	-	-	99.9	99.9	99.9	99.9	100	99.7	-	-	99.3	98.6	99.5	99.3	99.9	97.6
Without life insurance	β	se(β)	b	se(b)	γ	se(γ)	ψ	se(ψ)	β	se(β)	b	se(b)	γ	se(γ)	ψ	se(ψ)
	0.88	0.03	0.11	0.02	2	-	0.25	0.05	0.89	0.02	6	0.03	6.5	0.02	0.4	0.01
Annuity m. participation	3.01	17.1	3.25	4.65	3.25	3.25	0.00	8.75	10.1	30.2	15.9	37.5	14.2	15.6	8.77	24.2
Condit. annuity demand	1.92	5.30	3.17	3.89	1.84	3.17	0.00	5.55	3.66	8.41	4.35	3.58	4.74	4.25	2.98	6.24
Cond. sh. of w.lth in ann.	38.0	31.9	37.0	41.5	28.9	37.0	0.00	54.5	28.2	26.8	31.3	31.5	32.0	30.7	25.9	39.2
Share of wealth in stocks	-	-	-	-	-	-	-	-	38.5	29.2	58.4	71.3	58.3	58.6	44.9	60.1
Annuity equival. wealth	-	-	99.9	99.9	99.9	99.9	100	99.8	-	-	99.6	98.8	99.7	99.6	99.9	98.6

Notes to Table 6: The columns "data" and "stdev" contain means and standard deviations of the variables in the data for the age group 65-89. The column "MSM" contains the simulated means from the model using the estimated preference parameters. These are given in the shaded areas of the table for the four cases with and without access to the stock and life insurance markets. The columns "lowp", "once", "incr", "lows", and "fair" show robustness results for MSM using a lower pension, single annuitization time at retirement, incremental annuitization until age 85, lower survival probability and actuarial fair valuation of annuities as described in the text.

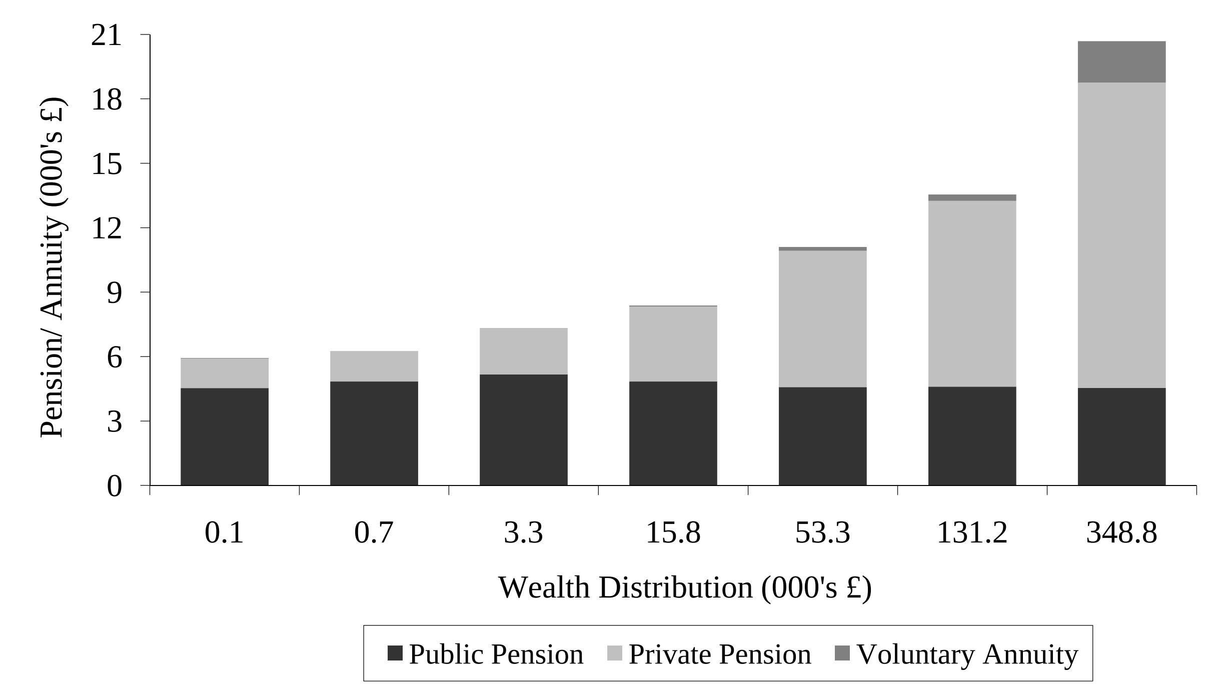
Figures

Figure 1: Wealth distribution, stock market participation and annuity market participation



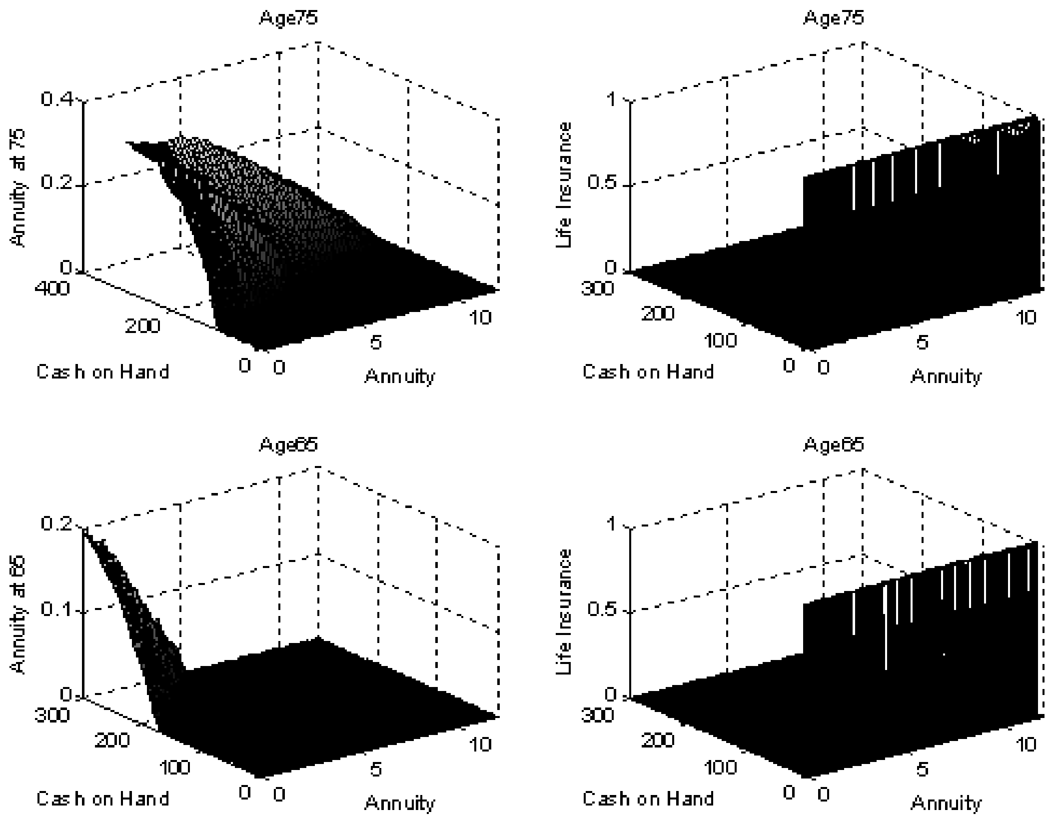
Notes to Figure 1: The columns show the number of households (measured on the ordinate on the left hand side) around the 2.5%, 10%, 25%, 50%, 75%, 90% and 97.5% percentiles of the wealth distribution in the whole sample ("All") and the sub-sample consisting of stock market participants ("S = 1"). The figure shows on the ordinate on the right hand side the average percentage of households participating in the voluntary annuity market ("A = 1") among the households located around a certain percentile of the wealth distribution. The sample consists of 5,233 retired households from the first (2002) wave of the English Longitudinal Study of Ageing (ELSA).

Figure 2: Decomposition of annual pension income into public and private sector pension income and annual annuity income over the wealth distribution



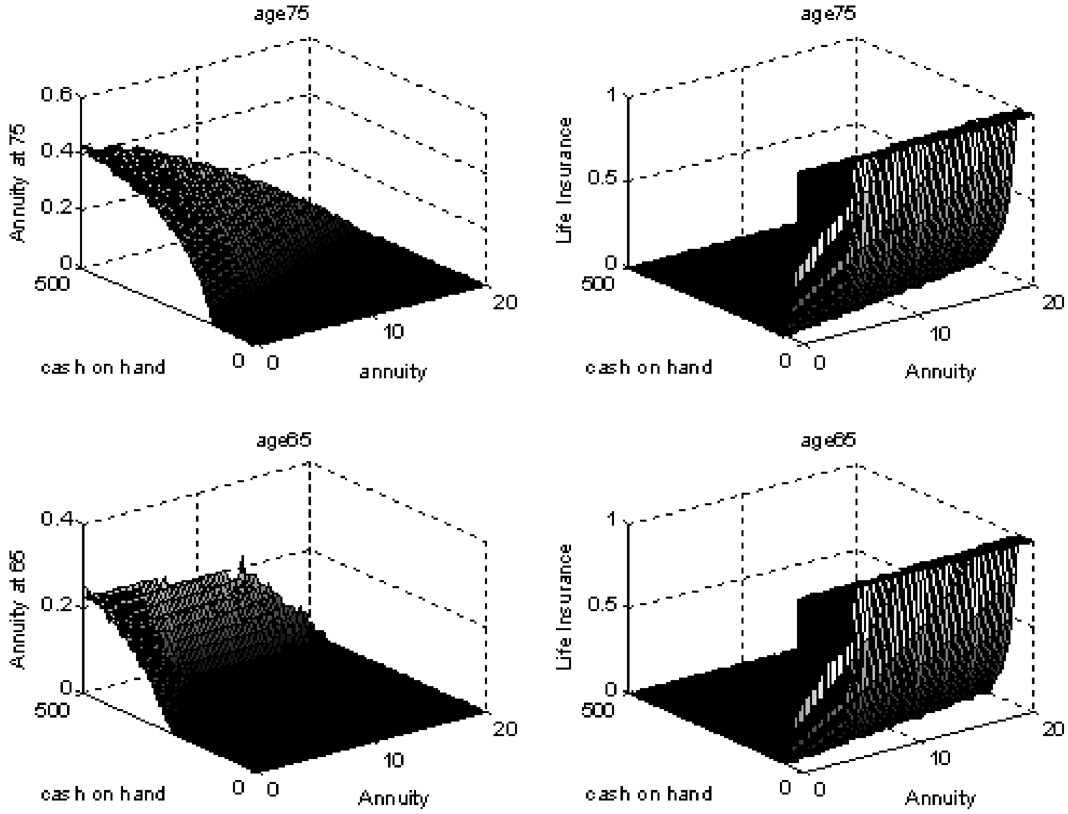
<u>Notes to Figure 2:</u> The figure decomposes the average total annual pension income of households around a certain percentile of the wealth distribution into income from public pensions, private (individual or occupational) pensions (excluding voluntary annuities) and voluntary annuitization. The wealth distribution is generated to represent from the left to the right 5%, 10%, 20%, 30%, 20%, 10% and 5% of the observations. Correspondingly, the abscissa shows the 2.5%, 10%, 25%, 50%, 75%, 90% and 97.5% percentiles of the wealth distribution. The sample consists of 5,233 retired households from the first (2002) wave of the English Longitudinal Study of Ageing (ELSA).

Figure 3: Annuity and life insurance demand as a function of liquid wealth and annuity levels at age 75 and at retirement (age 65) for non-stockholders



Notes to Figure 3: This figure shows the policy functions for the share of wealth invested in annuities at age 75 (the last year for which annuities can be purchased) and at retirement (age 65) for non-stockholders for the baseline case outlined in Section 3.2.1.

Figure 4: Annuity and life insurance demand as a function of liquid wealth and annuity levels at age 75 and at retirement (age 65) for stockholders



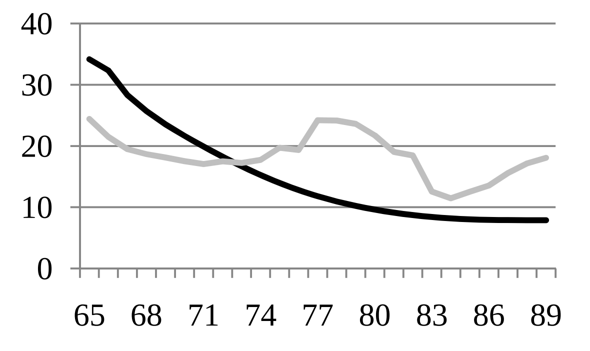
Notes to Figure 4: This figure shows the policy functions for the share of wealth invested in annuities at age 75 (the last year for which annuities can be purchased) and at retirement (age 65) for stockholders for the baseline case outlined in Section 3.2.1.

Figure 5: Average age profiles: model (black line) versus data (grey line)

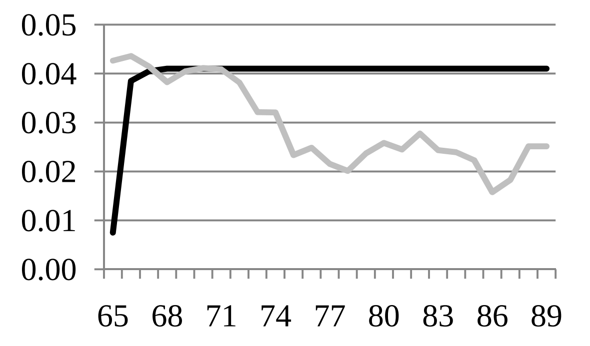
A. With access to the life insurance market

Non-Stockholders

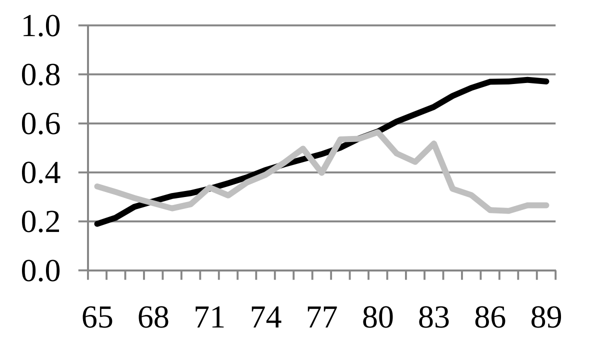
Financial wealth (in 1,000 £)



Annuity market participation (in %)

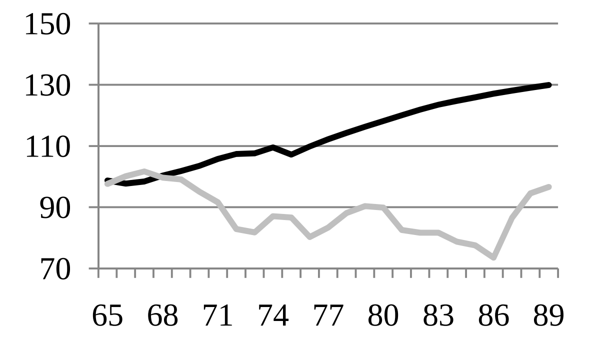


Conditional share of wealth annuitized (in %)

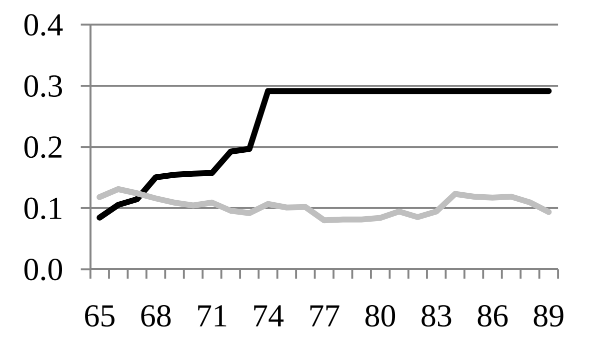


Stockholders

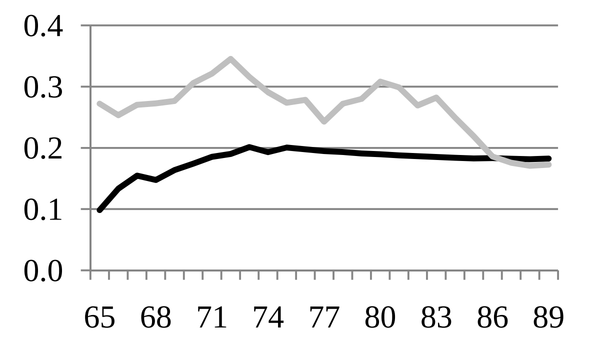
Financial wealth (in 1,000 £)



Annuity market participation (in %)



Conditional share of wealth annuitized (in %)



Share of wealth in stocks (in %)

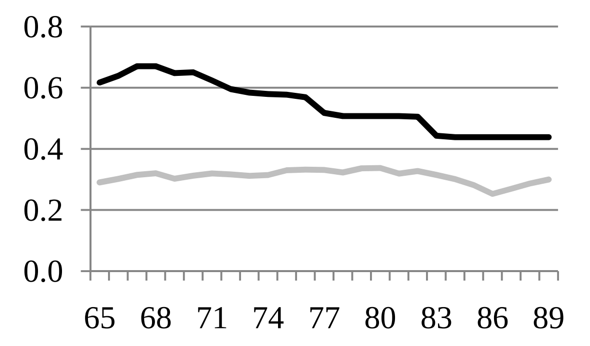
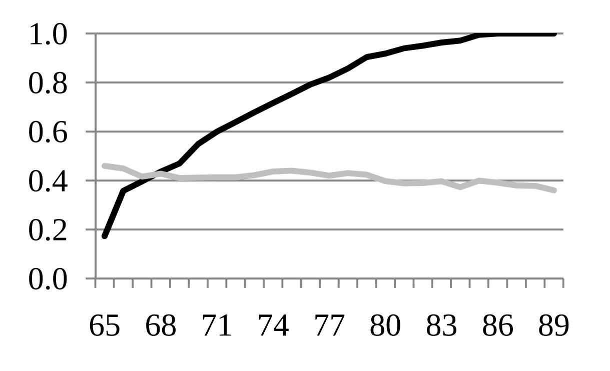


Figure 5: Average age profiles: model (black line) versus data (grey line) – continued

A. With access to the life insurance market – continued

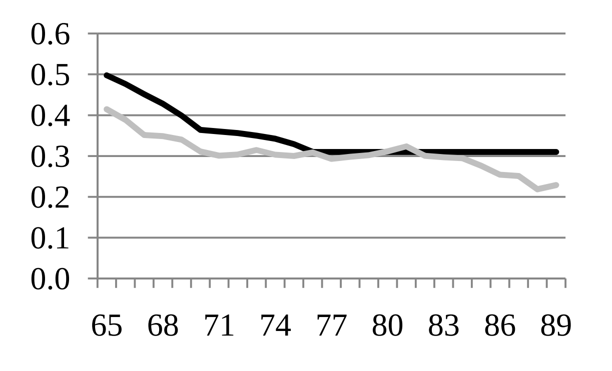
Non-Stockholders

Life insurance market participation (in %)



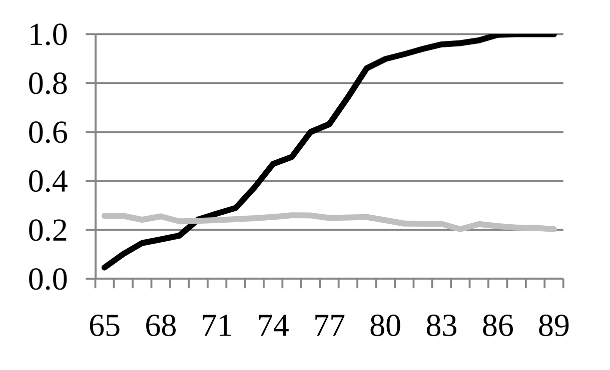
Stockholders

Life insurance market participation (in %)



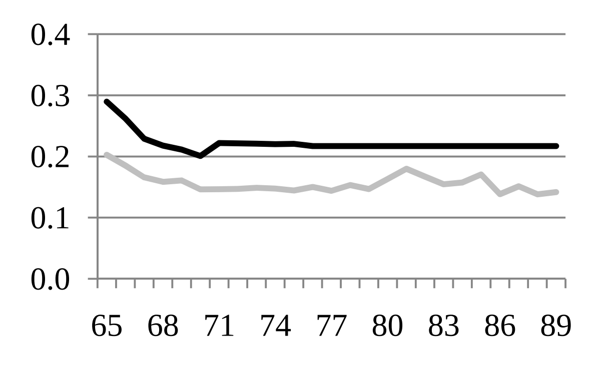
Non-Stockholders

Share of wealth in life insurance (in %)



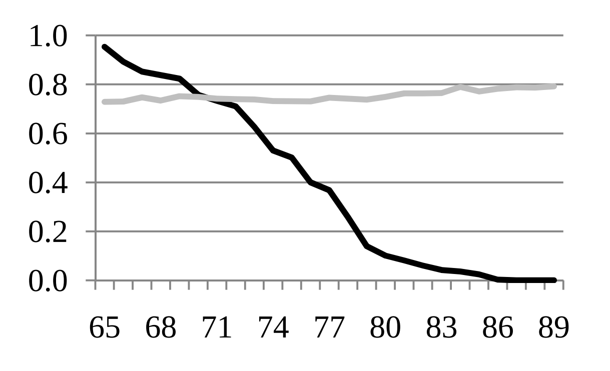
Stockholders

Share of wealth in life insurance (in %)



Non-Stockholders

Share of wealth in bonds (in %)



Stockholders

Share of wealth in bonds (in %)

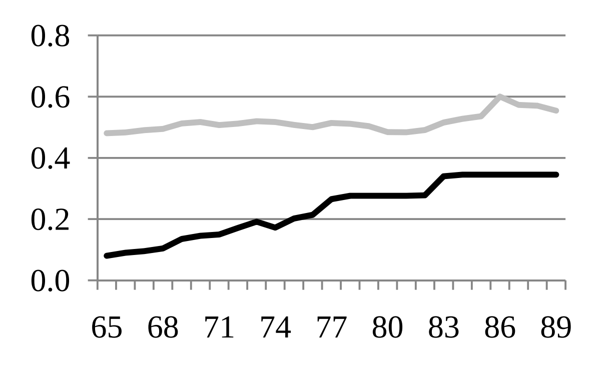
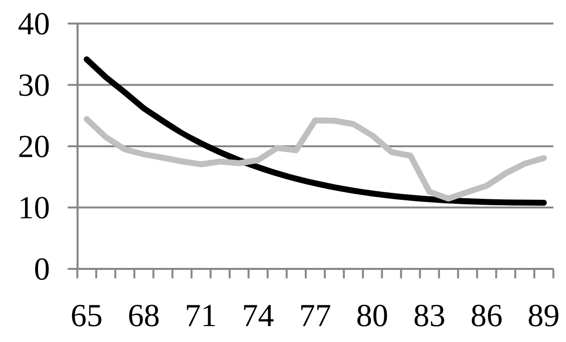


Figure 5: Average age profiles: model (black line) versus data (grey line) – continued

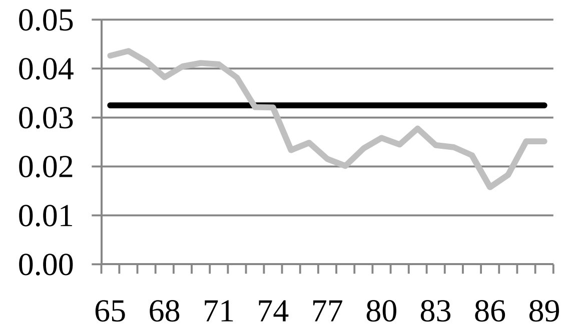
B. Without access to the life insurance market

Non-Stockholders

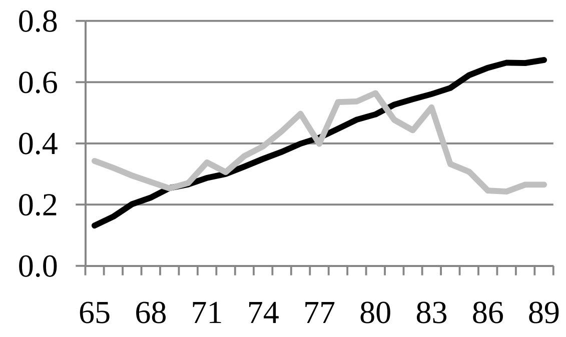
Financial wealth (in 1,000 £)



Annuity market participation (in %)

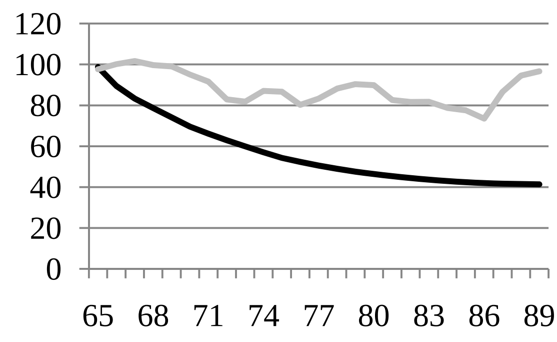


Conditional share of wealth annuitized (in %)

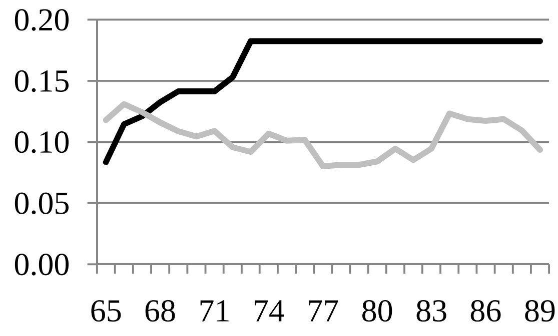


Stockholders

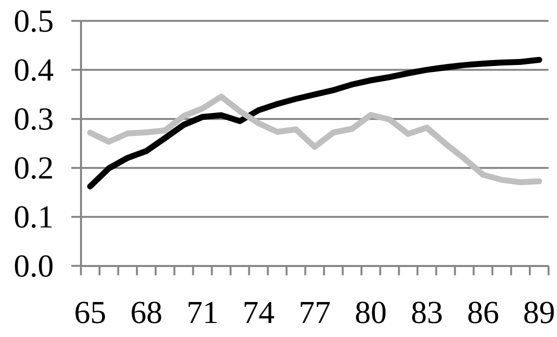
Financial wealth (in 1,000 £)



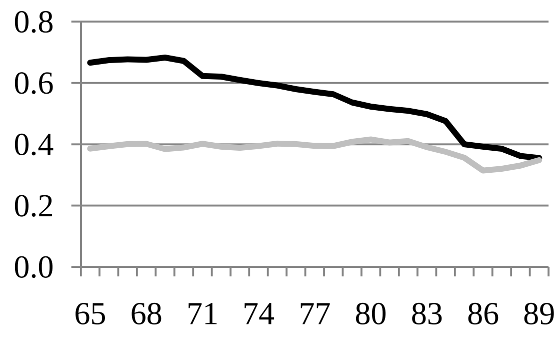
Annuity market participation (in %)



Conditional share of wealth annuitized (in %)



Share of wealth in stocks (in %)



<u>Notes to Figure 5:</u> This figure plots for each variable the average age profile in the data (grey line) against the model prediction (black line) using the baseline parameter calibrations given in Table 6. The data stops at age 89 (all ages over 89 are coded 90 in ELSA to avoid household identification issues) but the model has been solved on the assumption that households might live for a maximum of 100 years.