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Minimum Rate of Return Guarantees

A Valuation for Customers with Habit Formation in Preferences

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Minimum rate of return guarantees: A valuation for customers with habit formation in preferences

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Abstract

This paper constructs two mathematical models in order to determine whether investing in a minimum rate of return guarantee is welfare improving for customers with habit formation in preferences. The value of the minimum rate of return guarantee is determined by comparing the simulation results of two models. The first model determines the welfare for a customer when a minimum rate of return guarantee is unavailable, while the second model finds the welfare when a minimum rate of return guarantee is introduced. Using a measure called the certainty equivalent surplus consumption, we find that these guarantees are an interesting investment strategy for customers with and without habit formation in preferences. However, the welfare improvement is dependent on the specific contracts and the different settings that are considered.

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Table of contents

1. Introduction

Pension providers and insurance companies offer a broad range of embedded options in their contracts. To be more specific, these firms offer minimum rate of return guarantees, which are guarantees to credit the account balance with a pre-specified minimum return every period. In addition, minimum rate of return guarantees may offer some surplus distribution if returns are high during the investment period. These guarantees have been introduced in retirement asset allocations for the following reason. The characteristics of these guarantees offer insurance against equity exposure that could lead to a massive shortfall in retirement wealth, while still offering some equity exposure in order to capture the equity premium. The provider of these options now bears the risk of a fall in investment returns, for which the providers will collect payments. Hansen and Miltersen (2002) discuss how the issuer of the contract can collect payments; this will be explained at a later stage. The research question of this paper is the following: Does investing in a minimum rate of return guarantee increase the welfare of customers with habit formation in preferences? This question will be answered throughout this paper for both customers with habit formation and customers without habit formation. The utility function for a customer with habit formation does not only depend on the consumption level, but also on a minimum subsistence level, called the habit level. A customer, whose preferences reflect habit formation, would be interested in a minimum rate of return guarantee, because it provides the investor with a lower bound, which helps to consume at least the habit level. Different forms of habit formation are found in the empirical literature and these forms will be discussed in the next section of this paper.

In order to get more insight in the question stated above, this paper will formulate a life-cycle model from which a consumption path will be derived. This consumption path will be valued by the utility function of a customer with habit formation in preferences. In this life-cycle model, a financial market is considered where investors can invest in a risk-free asset and a risky asset. Additionally, the model assumes that investment opportunities are constant over time. Furthermore, the so-called "difference-habit model'' is utilized, which is described by Munk (2008) and De Jong and Zhou (2014). In this "difference-habit model", the utility of a customer depends on the difference between the consumption level and the habit level. The model by Munk (2008) provides us with closed-from solutions for the optimal consumption and investment choice. These optimal strategies will be determined for different types of customers, which allows us to calculate a utility level for these different customers. Since utility levels can only be compared in terms of better and worse, I will use a measure called the certainty equivalent surplus consumption (CESC). This measure allows us to compare the different consumption paths and attach comparable values. More about this measure will be explained in a later section of this paper.

In order to test whether a customer values a minimum rate of return guarantee, the model must be adapted such that it includes this guarantee. Hansen and Miltersen (2002) provide a modeled description of the minimum rate of return guarantee and explain how surpluses will be distributed. The minimum guaranteed rate is determined using the condition that the issuer of the guarantee, on average, earns zero profits in a risk neutral world. I consider a guarantee in which the customer has an option on the final bonus reserve, which is paid out to the customer if it is positive at maturity. The customer pays for the guarantee by paying an annual fee to the issuer. Furthermore, the customer provides an initial deposit and is allowed to consume out of the customer's account. This model is combined with the model provided by Munk (2008) and enables us to derive the optimal strategies for customers with habit formation when investment opportunities are constant and a minimum rate of return guarantee is available. By simulating the model described above, we can attach CESC-values to the different optimal consumption paths. The results from simulating the model without the guarantee will be compared to the simulation results of the model including the guarantee. In this way, it is possible to determine if the introduction of the minimum rate of return guarantee increases the welfare of different customers.

By simulating the model without the minimum rate of return guarantee for different customers, the effects of habit formation on the optimal strategies can be determined. It turns out that habit customers allocate a smaller share to risky assets due to the fact that wealth must be partly reserved for future consumption of the habit level. Additionally, their consumption path is steeper. These results will be compared to the results from simulating the model that includes the guarantee. The results are compared for three types of customers: A customer without habit formation, a customer with weak habit formation, and a customer with strong habit formation. For different settings and contracts, we find that the introduction of the minimum rate of return guarantee increases the customer's welfare. The welfare is increased for most contracts and there is an interesting contract for every setting that is considered. Thus, including a minimum rate of return guarantee in the investment strategy is welfare improving given the specifications that are considered in this paper.

The rest of the paper will be structured in the following way. Section 2 provides a literature review that will discuss minimum rate of return guarantees in more detail and provides some empirical research on the topic. Furthermore, it will contain a theoretical explanation of habit formation and provide empirical evidence on the existence of this phenomenon. Section 3 presents the models that will be utilized in this paper. Additionally, this section will provide closed-form solutions for the optimal consumption and investment choice. The results of the simulations for the model without the minimum rate of return guarantee will be presented and discussed in section 4. Section 5 provides the simulation results for the model that includes the guarantee and compares these results to the results presented in section 4. At last, in section 6, the paper's main findings will be summarized and I will propose some interesting ideas that could be the subject of future research.

2. Literature review

This section provides a theoretical explanation and discusses the literature associated with the subject of this paper. Note that this section will only discuss a selection of papers and does not claim to be a representation of the entire literature on the subject. First, section 2.1 provides an explanation on minimum rate of return guarantees and discusses the associated literature. Thereafter, Section 2.2 presents some theory on the subject of habit formation and provides papers that discuss this phenomenon.

2.1 Minimum rate of return guarantees

Embedded options, such as minimum rate of return guarantees and bonus options, pervade the wide range of contracts offered by pension funds and insurance companies. This section will discuss minimum rate of return guarantees and bonus options in more detail and offers insight in the literature on these options.

A minimum rate of return guarantee promises to credit the account balance with a pre-specified minimum rate of return every period. According to Jensen, Jørgensen, and Grosen (2001), a typical contract specifies a claim on the surplus that is generated by the investments, which is called a bonus option. On top of this, the contract may contain a surrender option, which gives the right to terminate the contract before maturity. Miltersen and Persson (2003) mention that such a contract specifies a benchmark return together with a periodic minimum rate of return. Miltersen and Persson evaluate these minimum rate of return guarantees in combination with a mechanism to distribute surpluses. The mechanism describes how investment returns above the minimum rate are distributed between the insurer and the customer. There are different mechanism through which surpluses can be distributed. On the one hand, Grosen and Jørgensen (2000) describe a manner where profits are distributed according to a so-called smoothing mechanism. In this way, the surpluses are distributed to the customer gradually. However, at maturity, the customer does not receive any undistributed surplus. On the other hand, Miltersen and Persson (2003) distribute a fraction of the excess return to the customer. On top of that, the contracts pay out the amount of the bonus reserve at maturity if this bonus reserve is positive. Hansen and Miltersen (2002) also utilize the model that is described by Grosen and Jørgensen (2000) since this mechanism is frequently used in practice. In the case that bonus payments are connected to the investments of the company, there is an incentive for the company to lower the fluctuations of their portfolio. This is due to the fact that the company has to cover the deficit if the final value of the bonus reserve is negative.

Determining the guaranteed minimum rate of return is very important for pension funds and insurance companies since this rate can represent a value and form a potential hazard to the solvency of the company. These guarantees could represent a value if the return on the company's investments is high relative to the guaranteed minimum return. In this case the minimum guaranteed rate is far lower than the market rate of return, therefore the risk associated with these guarantees can be neglected and they do not constitute a threat to solvency. However, if pension funds and insurance companies experience significantly lower returns on their investments, unfortunate events could be triggered. Bacinello (2001) provides an example for the Italian case in which Treasury Bonds and fixed-income securities earned up to 20% per annum to match the high level of inflation. The commonly guaranteed rate was around 3% per annum, which is a completely inadequate rate, causing the marketability of such products to be seriously jeopardized. According to Hansen and Miltersen (2002), this practice of setting a conservative minimum rate of return and compensating the customer through bonus payments was adapted by most Danish pension funds. In principle, companies could set extremely conservative initial terms in order to move the guarantee far out of the money. However, this practice of setting conservative terms was discouraged by the competition among these companies. It is clear that the initial terms cannot be too conservative and that setting a high minimum rate of return can seriously threaten solvency. Therefore, a closer examination of the minimum guaranteed rate and a proper valuation of these guarantees is very important.

2.2 Habit formation

Pension plans often guarantee that benefits will never decrease and that benefits can increase if it is allowed by the financial position of the fund. Third-pillar pension products have similar features, such as the minimum rate of return guarantee explained above. According to standard models of life-cycle consumption, there is no place for such guarantees. This is due to the fact that a negative shock to wealth should cause a lower consumption permanently. However, these guarantees do exist in the form of the minimum benefit guarantee or the promise of never decreasing benefits. The existence of these guarantees in practice could be explained by habit formation. A customer with habit formation derives utility from consumption above the habit level, which is determined by the customer's past consumption level.

De Jong and Zhou (2014) offer insight in the theory of habit formation and the effect of habit formation on the utility preferences of customers. The literature describes two kinds of habit formation. On the one hand, we speak of internal habit formation if the determinants of the habit are internal, which means that the customer's consumption habit is affected by the customer's own past pattern of consumption. On the other hand, if the habit depends on the aggregate level of consumption or the consumption of a reference group, it is called external habit formation.

Different kinds of habit formation models are used in empirical research. These models can be distinguished along two dimensions. First, models can include an external habit specification in which the habit depends on the aggregate consumption level or an internal habit specification in which a customer's habit level depends on the customer's own past pattern of consumption. Second, some models use the "ratio-habit model" where surplus consumption is described by the ratio between the consumption level and the habit level, while others use the "difference-habit model" in which the utility function includes the difference between the consumption level and the habit level.

It is now clear how habit formation is defined, but in what way does habit formation affect the behavior of customers? De Jong and Zhou (2014) use an internal habit formation model and find that habit formation provides a stronger incentive to save, leads to a lower consumption early on, and causes a higher growth of the consumption level. Habit formation consumption policies then result in less consumption smoothing. Additionally, a lower level of investments will flow into equity since it is not the appropriate asset class to hedge against variations in the habit level and to ensure that future consumption will exceed the habit level as stressed by De Jong and Zhou (2014) and Munk (2008). Gomes and Michaelides (2003) introduce a life-cycle model of consumption and portfolio choice for customers with internal habit formation in preferences. They find that habit formation in preferences tends to increase wealth accumulation since the intertemporal incentive for consumption smoothing is stronger. This causes households to participate in the stock market earlier on and invest almost fully in stocks. Polkovnichenko (2007) finds that if there is only a small chance of a severe income shock, much more conservative portfolios are formed. The model also indicates that low to moderately wealthy households allocate a larger share to stocks as wealth increases.

Models that include habit formation in utility preferences have become increasingly successful and important in explaining asset pricing facts and other puzzles. Campbell and Cochrane (1995), Abel (1990, 1999), and Constantinides (1990) show that habit formation models can be utilized to provide a partial solution to the equity premium puzzle. This is the case, because habit formation leads to a higher disutility if there is a large decline in consumption. Campbell and Cochrane (1995) also explain other asset pricing facts, such as the pro-cyclical variation of stock prices (see also Campbell and Shiller (1988)), the predictability of excess stock returns over the long horizon, and the fact that the variation of stock market volatility is countercyclical. Sundaresan (1989) and Constantinides (1990) can explain a high equity premium combined with low levels of risk aversion using habit formation models. Habit formation models are also used to rationalize the variability of expected returns on currencies (Backus, Gregory, & Telmer, 1993). Wachter (2006) proposes a consumption-based model that produces realistic means and volatilities of bond yields and explains the expectations puzzle. Additionally, it helps to explain the high equity premium and excess stock market volatility. Furthermore, the empirical evidence shows that habit formation models can explain macro-economic facts. Boldrin, Christiano, and Fisher (2001) explain business cycle facts. Shore and White (2003) explain the equity home bias. The literature also explains how consumption reacts to monetary shocks (Fuhrer, 2000) and savings and growth (Boldrin et al., 2001). Finally, Verdelhan (2010) uses a framework similar to that of Campbell and Cochrane (1995) in order to resolve the interest rate parity puzzle.

We now know that habit formation models change the behavior of investors and that these models are able to explain asset pricing phenomena and other macro-economic facts. It is then very important to find evidence on the existence of habit formation in preferences. First, some evidence on the existence of internal habit formation is presented. Dunn and Singleton (1986), Eichenbaum, Hansen, and Singleton (1986), and Eichenbaum and Hansen (1990) present evidence for internal habit persistence in monthly data using the Euler equation. Muellbauer (1988) finds similar results using quarterly consumption data. Winder and Palm (1989) present evidence of habit persistence in the Netherlands, while Braun, Constantinides, and Ferson (1993) find evidence of habit formation in aggregate Japanese consumption data. Moreover, Ferson and Constantinides (1991) find evidence of internal habit formation in monthly, quarterly, and annual data. Furthermore, Heaton (1993) uses an explicit consumption process and finds some evidence of habit persistence. At last, Heien and Durham (1991) use panel data to test the habit hypothesis. The results show highly significant evidence of internal habit formation.

Second, there is some empirical research that investigates both internal and external habit formation at the same time. On the one hand, Grishchenko (2010) presents a generalized model and finds that the results are more consistent with internal habit formation than with “catching up with the Joneses” (external habit formation). On the other hand, Korniotis (2010) finds evidence supporting external habit formation, while the evidence on internal habit formation is only weak.

Finally, there is some research that finds no evidence of habit formation at all. Meghir and Weber (1996) find that there is no support for intertemporal non-separability of preferences. Accordingly, Dynan (2000) uses annual household panel data and finds no evidence of habit formation at the annual frequency.

3. The model and optimal strategies

The central question of this paper is: Will an investor with habit formation in utility preferences be interested in a minimum rate of return guarantee? In order to answer this question, the model by Munk (2008) is utilized. Munk (2008) presents a model that assumes that investment opportunities are constant. This model will be discussed in this section and the key equations will be explained in some more detail.[1] Furthermore, a minimum rate of return guarantee including a bonus distribution scheme, as discussed in the paper by Hansen and Miltersen (2002), will be implemented in the model. The model considers a financial market consisting of a risk-free asset and a risky asset. Section 3.1 describes a model to determine the optimal consumption path and risky asset weight assuming that investment opportunities are constant. Section 3.2 will add a minimum rate of return guarantee to the model discussed in section 3.1.

3.1 Constant investment opportunities and the optimization problem

In this section, a model is described that uses several simplifying assumption in order to retrieve a general solution for the consumption level and the risky asset weight. First, I assume that the risk-free rate is constant over time. Second, I assume that the volatility of risky asset returns remains constant over the life-cycle. Third, I assume that the Sharpe-ratio remains constant over time. The price of the risky asset is represented by P_t. The evolution of the risky asset price is as follows:

$$\Delta P_{t+\Delta t} = P_{t+\Delta t} - P_t = P_t[(r + \sigma\lambda)\,\Delta t + \sigma\,\varepsilon_{t+\Delta t}\,\sqrt{\Delta t}], \tag{1}$$

where $\Delta P_{t+\Delta t}$ is the change in the price over the period, r is the risk free rate of return, Δt represents the length of a time step, σ is the constant volatility of the risky asset, λ the constant expected excess return over volatility, n is the number of time periods such that $t\epsilon\{0, \Delta t, 2\Delta t, \dots, n\Delta t\}$, $T = n\Delta t$ is the maturity date, and $\varepsilon_{t+\Delta t}\sqrt{\Delta t}$ is the discrete time approximation of a Brownian motion for which $\varepsilon_{t+\Delta t}$ is distributed as follows

$$\varepsilon_{t+\Delta t} = \begin{cases} +1 & with\,probability \quad \frac{1}{2} \\ -1 & with\,probability \quad \frac{1}{2} \end{cases}.$$

As noted above, r, σ, and λ are assumed to be constant. λ is also called the market price of risk or Sharpe-ratio since it is calculated as the expected excess return of an asset to the volatility of an asset.

In standard life-cycle models it is conventional to assume a time-separable utility function, which means that the consumption in one period does not affect the utility level in another period. However, when we consider an investor with habit formation, this conventional assumption does not hold. Here, we use an internal habit formation model to describe a price-taking investor. The utility level over the life-cycle of this investor can be calculated as

$$U(c) = E[\Sigma_{t=0}^{T}\, e^{-\delta t}\, u(c_t, h_t)\,\Delta t], \tag{2}$$

[1] The continuous time equations presented by Munk (2008) are mostly replaced by their discrete counterparts. Note that sometimes the continuous time equations are used as an approximation.

where $\delta \geq 0$ can be described as the subjective rate of time preference, $u(c_t, h_t)$ is the utility level at time t, c_t is the consumption stream of the investor, and h_t is the habit level, which can be defined by

$$h_t = h_0 \, e^{-\beta t} + \alpha \, \Sigma_{s=0}^{t} \, e^{-\beta(t-s)} \, c_s \, \Delta s, \tag{3}$$

where h_0 is the initial habit level, c_s is the investor's own level of consumption at time s, α is a positive constant that determines how strong past consumption influences current consumption and can be interpreted as a scaling parameter, and β is a positive constant determining how fast the effect of past consumption on the habit level decreases and is called the persistence parameter. $\beta > \alpha$ is required to ensure that the habit level will decrease when the investor consumes at the habit level. Note that Δs, the length of the time step, is equal to Δt.

A utility function usually takes the form of Constant Relative Risk Aversion (CRRA). In the case of habit formation, the utility function does not only depend on the consumption level, but also on the level of past consumption, which is expressed in the habit level. Here, we use a "difference-habit model", where the surplus consumption is calculated as the difference between the consumption level and the habit level in a certain period. In this case, we use a power-linear instantaneous utility function, which is defined as

$$u(c, h) = \frac{1}{1-\gamma} (c - h)^{1-\gamma}, \tag{4}$$

where γ is a positive constant called the coefficient of relative risk aversion, which is not dependent on the habit level. The habit level can be viewed as a minimum consumption rate established by past rates of consumption. It is important to note that as the consumption level equals the habit level, the instantaneous utility level will be minus infinity. This leads to the constraint that the consumption level must be above the habit level at all times.

Over the specified time horizon, the investor maximizes the total level of utility with respect to a consumption path and a portfolio process. The investor has to allocate his wealth between the risky asset and the bank account. π_t is the share invested in the risky asset. The remaining wealth is allocated to the risk-free bank account. The evolution of wealth can then be defined by

$$\Delta W_{t+\Delta t} = [W_t(r + \pi_t \, \sigma \, \lambda) - c_t] \, \Delta t + W_t \, \pi_t \, \sigma \, \varepsilon_{t+\Delta t} \, \sqrt{\Delta t}, \tag{5}$$

where c_t and π_t are determined by the information at that time. It is important to note that the wealth process must be positive at all times. The level of wealth can be calculated as

$$W_{t+\Delta t} = W_t + \Delta W_{t+\Delta t}. \tag{6}$$

Munk (2008) provides two processes from which the optimal consumption path and portfolio process can be derived. These processes are different depending on the assumption of constant investment opportunities. In the case that investment opportunities are constant, r and λ do not change over time. Then, the two processes, F and G, are deterministic functions of time and are defined as

$$F_t = \Sigma_{s=t}^{T} e^{-(r+\beta-\alpha)(T-t)} \Delta s \approx \frac{1}{r+\beta-\alpha} (1 - e^{-(r+\beta-\alpha)(T-t)}) \tag{7}$$

and

$$G_t = \Sigma_{s=t}^{T} \exp\{-(\frac{\delta}{\gamma} + \left[1-\frac{1}{\gamma}\right] r + \frac{1}{2\gamma}\left[1-\frac{1}{\gamma}\right] ||\lambda||^2)(s-t)\} \, (1+\alpha F_s)^{1-\frac{1}{\gamma}} \Delta s. \tag{8}$$

The optimal consumption path and portfolio process are then given by (as approximation of the discrete solution, the continuous time formulas are used)

$$c_t^* = h_t^* + \left((1+\alpha F_t)^{-\frac{1}{\gamma}} \frac{W_t^* - h_t^* F_t}{G_t}\right) \tag{9}$$

and

$$\pi_t^* = \frac{1}{\gamma} (\sigma^T)^{-1} \lambda \frac{W_t^* - h_t^* F_t}{W_t^*}. \tag{10}$$

Due to the approximation by using discrete time, some problems exist with the optimal paths. When the end of the horizon is reached ($t = T$), the two processes, F and G, will move to the values 0 and 1 respectively. This causes the investor to optimally consume the habit level on top of the total remaining wealth, causing the wealth to be negative at the end of the horizon. To counteract this, I assume that the investor consumes the total remaining wealth in the last period. This assumption is problematic in some cases since the consumption level could fall below the habit level and this is not optimal for the investor.[2] A similar problem exists for the optimal weight in risky assets. In the last period, the risky asset weight converges to a pre-specified level since we assume that the coefficient of risk aversion, the volatility of risky assets, and the Sharpe-ratio are constant. Therefore, I assume that the weight in risky assets will be zero in the last period since the total remaining wealth is consumed in that period and there is no remaining wealth to invest.

The optimal risky asset weight depends on several factors. First, the level of risk aversion is important in determining the optimal risky asset weight. A higher level of risk aversion leads to aversion of bad outcomes and thus a lower optimal weight in risky assets. Second, the volatility of risky asset returns, which represents the risk of investing in these assets. The more volatile the returns, the higher the risk of investing in risky assets, causing the optimal investment in risky assets to go down. Third, the Sharpe-ratio indicates the performance of investing by adjusting for risk. A higher risk-adjusted performance leads to a higher optimal weight. Fourth and last, the weight invested in risky assets depends on the free wealth, $W_t^* - h_t^* F_t$. Habit formation in utility preferences lowers the optimal weight since the investor has to take into account that at least the habit level must be consumed in the future. The next section adds a minimum rate of return guarantee to the model described above.

3.2 A modeled description of the minimum rate of return guarantee

This section describes the model for the minimum rate of return guarantee, which is based on the model by Hansen and Miltersen (2002). The model described in this paper will slightly differ from the one provided by Hansen and Miltersen (2002) in that customers are allowed to consume out of the accumulated wealth in the option account, which will be explained in more detail later on. Moreover, I

[2] It is assumed that $c_t - h_t > 0.3$ must always hold. This assumption is introduced since the lower consumption in the last period could lead to a very small difference between the consumption level and the habit level, causing the chance of extreme utility levels ruining the results.

assume that the pension fund is of closed form and consists of only one customer, which is why the optimal portfolio allocation provided by Munk (2008) and described by equation (10) will be used. In the one customer case, which is discussed here, the company's balance sheet can be simplified. This is done by only including the balance sheet accounts that affect the customer's contract, while the hedging activities by the company are excluded from the balance sheet. A more detailed description of the different balance sheet accounts is provided below.

Account X: This is the account that keeps track of the value of the investment by the company on behalf of the customer and is the only account on the asset side of the balance sheet. The initial amount that is deposited into this account is assumed to evolve in the following way

$$\Delta X_{t+\Delta t} = [X_t(r + \pi_t\, \sigma\, \lambda) - c_t]\, \Delta t + X_t\, \pi_t\, \sigma\, \varepsilon_{t+\Delta t}\, \sqrt{\Delta t}, \tag{11}$$

where X_t denotes the value of the investment at date t and is calculated by

$$X_{t+\Delta t} = X_t + \Delta X_{t+\Delta t}. \tag{12}$$

In order to make the two different models compatible, the formulas (11) and (12) look similar to equations (5) and (6). Note that the variables have already been explained in the previous section. Therefore, they will not be explained again.

Account A: This is the most important account for the customer. The initial wealth of the customer is credited to this account. This account earns the minimum rate of return and possibly some distributed surplus. This bonus is distributed only when the buffer ratio[3] is above a pre-specified minimum level.[4] The payment from the bonus reserve to this account is known as the distributed surplus, while the remaining value of the bonus reserve is called the undistributed surplus. The bonus reserve account, account B, will be discussed in more detail shortly.

Account C: This account is used by the company to collect payments from the customer for issuing the contract and guaranteeing a minimum rate of return. The company can collect these payments through the direct payment method or the indirect payment method. According to Hansen and Miltersen (2002), it is common to collect the payments via the direct way in Denmark. In the case that the bonus reserve is negative at maturity of the contract, the deficit will be covered by a payment from this account to the bonus reserve.

Account B: This account is called the bonus reserve. First, the investment return from account X is distributed to this account, which can either be positive or negative. Thereafter, the required amounts are distributed to the customer's account (account A) and the company's account (account C). The amount left in this account is thus determined residually. The determination of the bonus reserve will become clearer when it is discussed in mathematical terms.

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[3] The buffer ratio is calculated as the bonus reserve, Account B, divided by the sum of the customer's account and the company's account, i.e., Account A plus Account C.

[4] Hansen and Miltersen (2002) argue that this minimum level is typically equal to 10%, i.e., $\theta = 0.10$.

Now, the distribution to the different accounts will be discussed. First, the development of the sum of the company's account and the customer's account is modeled. Thereafter, the individual accounts will be modeled separately. At last, the evolution of the bonus reserve will be discussed in more detail.
From now on, the sum of the company's account and the customer's account will be referred to as the combined account. It is important to understand what payment is received by the combined account. In the case of the indirect payment, the combined account receives the minimum guaranteed rate of return as specified by the contract. On top of this, there is a possibility that the combined account receives a bonus, which is mainly dependent on the level of the buffer ratio. The combined account thus receives either the minimum guaranteed rate or a fraction, $\varphi + \rho$, of the excess bonus reserve, depending on which amount is larger. The value of the combined account is compounded continuously at the following rate

$$\max\left\{g, \ln\left(1 + (\varphi + \rho)\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t} + C_{t-\Delta t}} - \theta\right)\right)\right\}, \tag{13}$$

where g is the pre-specified guaranteed minimum rate of return, φ is the share of the excess bonus reserve that is received by the customer's account, ρ is the share of the excess bonus reserve received by the company's account, B_t is the value of the bonus reserve at time t, A_t is the value of the customer's account at time t, C_t is the value of the company's account at time t, and θ is the minimum level that the buffer ratio must reach before any surplus will be distributed. Note that φ and ρ must be non-negative and together cannot be larger than 1, i.e., $\varphi + \rho \in [0,1]$.

In the case that the minimum rate of return is smaller than the fraction of the excess bonus reserve, i.e., $g < ln(1 + (\varphi + \rho)(B_{t-\Delta t}/(A_{t-\Delta t} + C_{t-\Delta t}) - \theta))$, the combined account evolves as

$$\begin{aligned}(A_t + C_t) &= (A_{t-\Delta t} + C_{t-\Delta t})e^{\ln\left(1+(\varphi+\rho)\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t}+C_{t-\Delta t}}-\theta\right)\right)} \\ &= (A_{t-\Delta t} + C_{t-\Delta t}) + (\varphi + \rho)(B_{t-\Delta t} - \theta(A_{t-\Delta t} + C_{t-\Delta t})).\end{aligned} \tag{14}$$

This shows that indeed a fraction $\varphi + \rho$ of the excess bonus reserve is distributed to the combined account. Here, $B_{t-\Delta t} - \theta(A_{t-\Delta t} + C_{t-\Delta t})$ represents the excess bonus reserve. Note that consumption is not yet included in this formula. It will be included when the final development of the combined account is presented.

Now that the payments to the combined account are determined, it is important to determine how the individual accounts evolve. These individual accounts evolve in a similar fashion. The customer's account, account A, receives the guaranteed minimum rate of return or a part of the excess bonus reserve. However, note that the fraction received by the customer's account is smaller than that received by the combined account. Only a fraction φ of the excess bonus reserve goes to account A, while the combined account receives a fraction $\varphi + \rho$. Moreover, in the case of a direct payment, a fee, ξ, decreases the rate of return. In the case of a direct payment fee, the rate of return is given by

$$\max\left\{g, \ln\left(1 + \varphi\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t} + C_{t-\Delta t}} - \theta\right)\right)\right\} - \xi. \tag{15}$$

Thus, there are two ways through which the company can collect a payment for issuing the contract and guaranteeing a minimum rate of return. The payments are specified by the variables ρ and ξ. In the case that the company receives the payment via the direct way, the contract is specified with a positive direct payment fee, i.e., $\xi > 0$. Note that in this case there is no indirect payment, i.e., $\rho = 0$. In the case that the company receives the payment via the indirect way, the contract is specified with a positive ρ, while $\xi = 0$. It is also possible to combine the payment methods. However, this paper does not consider this case. Note that the contract never contains an upfront premium and that the payments are made over time. It is now possible to model the evolution of the combined account, the customer's account, and the company's account. The evolution of the combined account is given by

$$(A_t + C_t) = \left((A_{t-\Delta t} + C_{t-\Delta t}) - c_{t-\Delta t}\right)e^{\max\left\{g, \ln\left(1+(\varphi+\rho)\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t}+C_{t-\Delta t}}-\theta\right)\right)\right\}}, \tag{16}$$

where $c_{t-\Delta t}$ is the consumption in the previous period. It is very important to note that the customer now consumes out of his/her account and not out of the company's account. The development of account A can be described in a similar way and is modeled as

$$A_t = (A_{t-\Delta t} - c_{t-\Delta t})e^{\max\left\{g, \ln\left(1+\varphi\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t}+C_{t-\Delta t}}-\theta\right)\right)\right\}-\xi}, \quad \xi \epsilon [0,1]. \tag{17}$$

The company's account, account C, can now simply be modeled by subtracting the value of the customer's account from the combined account and can thus be written as

$$C_t = (A_t + C_t) - A_t. \tag{18}$$

Now that it is clear how these accounts evolve, it is time to describe how the amount in the bonus reserve is determined. It is important to note that this account starts off at zero value, since the customer did not build up any reserves at the start of the contract. Then, after each year, the return that is earned on the investments is credited to this account, while the payments to account A and C are withdrawn according to equation (16). To summarize, the earnings on the investments made on behalf of the customer are first put into the bonus reserve and then distributed to the customer's account and the company's account. Thus, account B is determined as

$$B_t = B_{t-\Delta t} + (X_t - X_{t-\Delta t}) - ((A_t + C_t) - (A_{t-\Delta t} + C_{t-\Delta t})). \tag{19}$$

The model for the minimum rate of return guarantee is described by equations (16) – (19). However, the model described in section 3.1 must be adapted in order to be compatible with the model explained in this section. The optimal equations from section 3.1, which change in the presence of the guarantee, are now written as

$$c_t^* = h_t^* + \left((1 + \alpha F_t)^{-\frac{1}{\gamma}} \frac{A_t - h_t^* F_t}{G_t}\right) \tag{20}$$

and

$$\pi_t^* = \frac{1}{\gamma}(\sigma^T)^{-1}\lambda \frac{A_t^* - h_t^* F_t}{A_t^*}. \tag{21}$$

The optimal strategies are now depending on the value of the customer's account instead of the level of wealth. Therefore, equations (5) and (6) are superfluous, while the other equations still remain important. If the bonus reserve is positive at maturity, it is assumed that this amount will be consumed in the last period. The consumption level at maturity is then given by

$$c_T^* = \left((1+\alpha F_T)^{-\frac{1}{\gamma}} \frac{A_T - h_T^* F_T}{G_T}\right) + B_T^+, \tag{22}$$

where B_T^+ is the amount in the bonus reserve at maturity in the case that this amount is positive. Since this amount is not paid out until maturity, the customer consumes the full amount in the last period.

In order to determine the value of the contract to the customer, it is important to price the contract. The price is determined such that the contract does not yield any profits in a risk-neutral setting. This can only be the case if we assume that the market for these contracts is characterized as a competitive one. The profits of the company are accumulated in account C; however, in the case that the bonus reserve is negative at maturity, the deficit must be paid out of the company's account. The measure of abnormal profits equal to zero is equivalent to the present value of total profits being equal to zero. The profit of the issuer is given by

$$V_0(C_T - B_T^-) = e^{-rT} E^Q[C_T - B_T^-] = 0,\text{[5]} \tag{23}$$

where B_T^- is the value of the bonus reserve deficit at maturity and therefore the value of the company's account at maturity is equal to $C_T - B_T^-$. In the case that the profit of the issuer is positive, i.e., $V_0(C_T - B_T^-) > 0$, it is possible for another issuer to offer a contract with better terms and still have a positive profit. Due to the competitive market, the profit will eventually become zero. Equation (23) will then determine the relationship between the minimum guaranteed rate (g), the direct payment fee (ξ), and the share of the bonus reserve received by the customer (φ) in the direct payment case. In the case of an indirect payment, equation (23) determines the relationship between the minimum guaranteed rate (g), the share of the bonus reserve received by the company (ρ), and the share of the bonus reserve received by the customer (φ).

[5] A more detailed explanation about this calculation can be found in appendix A.

4. Results without a minimum rate of return guarantee

In order to study the effects of introducing the minimum rate of return guarantee, numerical examples are utilized in this section. These numerical examples help us to determine if a customer would be interested in buying such a guarantee. Utility levels can help us to compare the results; however, it is only possible to determine which result is favorable. The certainty equivalent surplus consumption (CESC) allows us to compare the results and determine how much better/worse a certain outcome is. The CESC is the fixed level of consumption above the habit level that leads to the same utility level. It is important that we measure this consumption level on top of the habit level, since a habit customer does not care about the consumption level on its own, but only cares about the consumption on top of the habit level as can be seen from equation (4). The CESC is found by rearranging the terms in equation (2), which is done as follows:

$$U(c) = E\left[\Sigma_{t=0}^{T} e^{-\delta t} u(c_t, h_t) \Delta t\right]$$
$$= E[\Sigma_{t=0}^{T} e^{-\delta t} \frac{1}{1-\gamma}(c_t - h_t)^{1-\gamma} \Delta t] = \Sigma_{t=0}^{T} e^{-\delta t} \frac{1}{1-\gamma}(CESC)^{1-\gamma} \Delta t,$$

this implies that

$$CESC = \left(\frac{E[\Sigma_{t=0}^{T} e^{-\delta t}(c_t - h_t)^{1-\gamma}]}{\Sigma_{t=0}^{T} e^{-\delta t}}\right)^{\frac{1}{1-\gamma}}. \tag{24}$$

The CESC will be the measure on which the results are based and it is used to ultimately determine the value of the minimum rate of return guarantee. Note that all results are averaged over the total number of simulations used. The results are described in the sections that follow. Section 4.1 presents the optimal strategies when investment opportunities are constant and there is no minimum rate of return guarantee available. Section 4.2 discusses the optimal strategies when alternative parameters are considered, investment opportunities are constant, and there is no minimum rate of return guarantee available.

4.1 Optimal strategies without a minimum rate of return guarantee

This section determines the value of the optimal consumption path for customers with various levels of the habit parameters. It is assumed that investment opportunities are constant and a minimum guarantee is not available. We find that a stronger form of habit formation, as measured by the average habit level over the period, leads a steeper consumption path and a more conservative investment strategy. Below, these results are explained in more detail.

I use simulations to study the effects of habit formation on the optimal strategies discussed earlier. The basic customer is assumed to have a risk aversion parameter $\gamma = 5$, a time preference rate equal to $\delta = 0.02$, an initial wealth level equal to $W_0 = 100$, and an initial habit level of $h_0 = 0$. The customer is studied during his/her working life, starting at the age of 24 and ending at 67, which is a 43-year horizon. The length of the time step equals $\Delta t = 1$. Furthermore, the risky asset has a Sharpe-ratio of $\lambda = 0.3$ corresponding to a long-term expected excess return of 6% and a volatility of 20%. These values are comparable to those used by Munk (2008), who argues that these values are empirically reasonable. Furthermore, for the different customers in this section, the results are averaged over the number of simulations used, which is 1000.

Table 1 presents values that are important for determining the optimal consumption path. These values are calculated for the basic customer described above for various levels of the scaling and persistence parameter. A higher level of the scaling parameter α will lead to stronger habit formation, while a higher level of the persistence parameter β leads to weaker habit formation. The column labeled "Average habit level" shows the mean of the habit level over the whole period. Obviously, stronger habit formation in preferences corresponds to a higher average habit level over the period. The column labeled MPC contains the marginal propensity to consume out of wealth in the first period and is calculated as $(1+\alpha F_t)^{1/\gamma}/G_t$. The column MPC signals the shape of the consumption path of a customer: A low marginal propensity to consume signals a steep consumption path, while a high marginal propensity to consume signals a smoother consumption path. We find that stronger habit formation leads to a lower marginal propensity to consume and thus a steeper consumption path. The last column shows the certainty equivalent surplus consumption. The CESC is lower for habit customers since these customers have a positive habit level, while customers without habit persistence have a habit level of zero.

Table 1. Optimal choices with constant investment opportunities

α	β	MPC	Average habit level	CESC
0	0	4,43	0.00	4,8235
0.1	0,2	2,61	2,39	2.9668
0.1	0,3	3,14	1,59	3.5949
0.2	0,3	1,85	3,33	2.1675
0.1	0,4	3,43	1,16	3.9109
0.2	0,4	2,43	2,41	2.9371
0,3	0,4	1,43	3,82	1.8680
0,1	0,5	3,62	0,89	4.1094
0,2	0,5	2.80	1,84	3.3446
0,3	0,5	1,98	2,89	2.5072
0,4	0,5	1,17	4.10	1.5831

The table presents values that are important for determining the optimal strategies for various levels of the scaling and persistence parameter, α and β. The parameter values are as follows: $t=0$, $T=42$, $\delta=0.02$, $\gamma=5$, $h_0=0$, $W_0=100$, $\sigma=0.2$, $\lambda=0.3$ corresponding to an expected excess return of 6%, and $r=0.03$. MPC is the marginal propensity to consume and is calculated as $(1+\alpha F_t)^{1/\gamma}/G_t$ and the "Average habit level" is calculated as the mean of the habit levels over the entire period. Note that the results presented in this table are averages over the total number of simulations (1000).

Figure 1 shows the optimal weight in the risky asset portfolio for various combinations of the habit parameters. It is clear that a customer with stronger habit formation is investing more conservatively; this is caused by the fact that a customer with stronger habit formation must be able to consume a higher minimum level in the future. In order to consume this minimum level, the habit level, the customer must be more conservative. Hence, the optimal risky asset weight is lower. On the other hand, in the no habit formation case, the optimal risky asset weight remains constant over time.

Figure 2 presents the optimal consumption paths for customers with various combinations of the scaling and persistence parameters. In the case of no habit formation, the customer has a relatively smooth consumption over time. However, as the strength of the habit formation grows, the consumption path becomes steeper. Habit customers will consume a relatively low amount at the start of the working life since the habit level forms a restriction on consumption and causes a customer to consume out of a lower level of free wealth, i.e., a lower level $W_t - h_t F_t$. In the first part of the working life, the habit customers consume a low amount relative to the customer without habit formation, causing habit customers to save money in the early stage of the working life relative to customers without habit formation. Then, at the

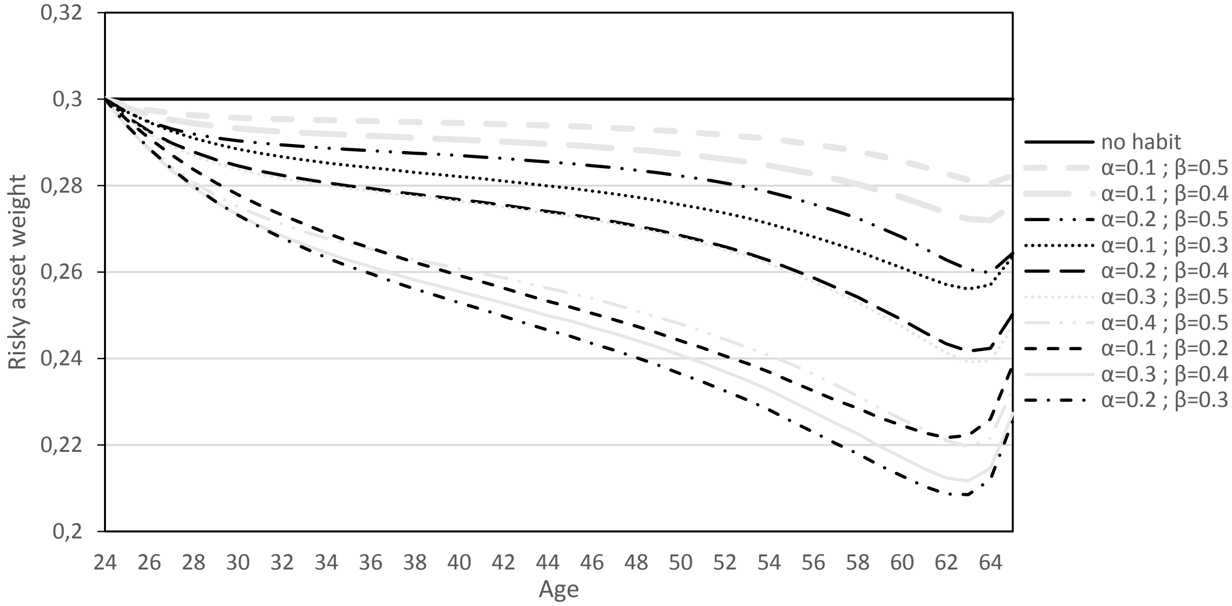

Figure 1. Optimal investment choices for various levels of the scaling and persistence parameter. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. First, note that $t = 0$ corresponds to an age of 24. Second, note that the risky asset weights are presented until the age of 65 since the risky asset weight is set to zero in the last period as explained in section 3.1. Third, note that the risky asset weights are all equal in the first period, which is due to the fact that the initial habit level equals $h_0 = 0$. Note that for every customer, the optimal weights are averages over the total number of simulations (1000).

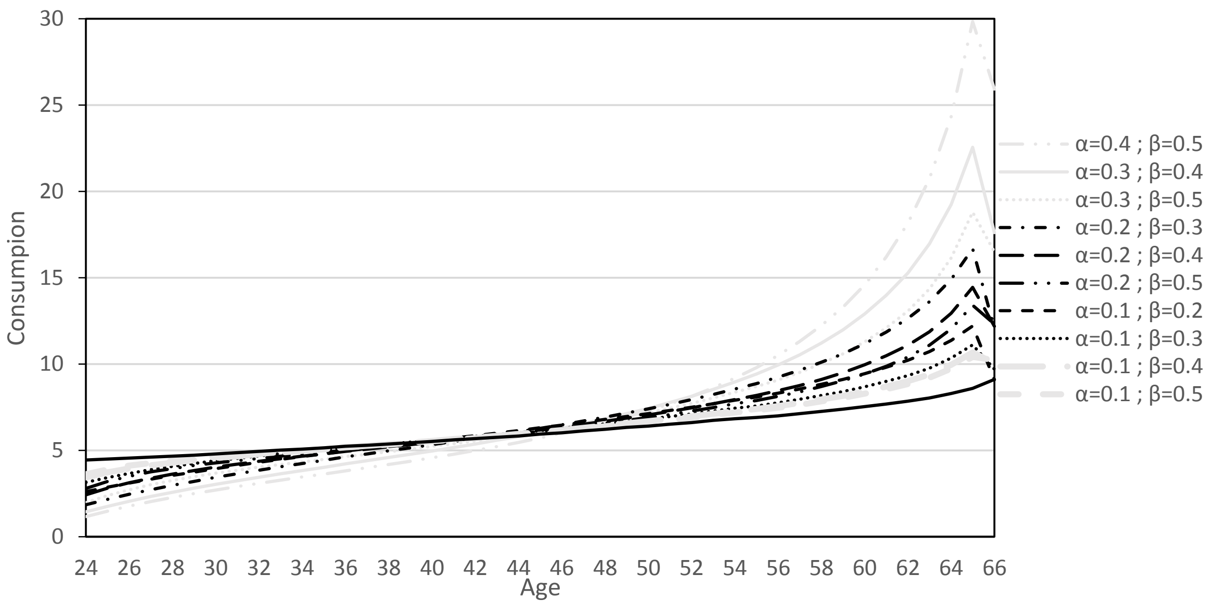

Figure 2. Optimal consumption paths for various levels of the scaling and persistence parameter. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. Note that there is a drop in consumption in the last period for habit customers, which is due to the conversion to discrete time. This conversion led to adaption of the optimal consumption in the last period as described in section 3.1. Note that for every customer, the optimal consumption path is averaged over the total number of simulations (1000).

end of the period, the habit customers have higher consumption, which is caused by the lower consumption in the early stages of the working life. In essence, this effect is stronger for customers with a higher average habit level; however, this is not true in all cases. Note that consumption drops in the last year, which is caused by the conversion to discrete time as explained in section 3.1.

4.2 Optimal strategies without a minimum rate of return guarantee for alternative parameters

This section checks whether the optimal strategies change if alternative parameter values are considered. The changes in four different parameters are considered: The risk aversion coefficient γ, the constant risk-free interest rate r, the Sharpe-ratio λ, and the initial habit level h_0. For these changes, information comparable to table 1 is presented and the optimal investment and consumption choice are derived. Note that the numbers presented in the tables are averages over the total number of simulations (1000). Furthermore, the figures show the effect of a change in one of the four parameters on the optimal strategies. The optimal strategies are averaged over all 11 different customers considered in the tables and the total number of simulations for each customer (1000). Thus, the figures present the general effect of a change in one of the four parameters on the optimal strategies, while not showing the effects for the different customers individually. The remainder of this section is structured as follows. First, section 4.2.1 presents these results for alternative values of the risk aversion coefficient. Second, section 4.2.2 considers changes in the risk-free interest rate. Third, section 4.2.3 provides the results for alternative Sharpe-ratios. Fourth, section 4.2.4 shows the outcomes for different initial habit levels.

4.2.1 Alternative risk aversion levels

Here, information comparable to table 1 and the optimal strategies are presented for alternative risk aversion levels. The results show that a lower risk aversion coefficient lowers the marginal propensity to consume in most cases. However, the changes are only marginal. Furthermore, a lower risk aversion level leads to a higher optimal weight in risky assets, a higher and steeper consumption path, and a higher certainty equivalent surplus consumption. A higher risk aversion coefficient leads to the opposite effect. It should be noted that these results are averages and that a lower risk aversion coefficient is therefore not by definition better. Next, these results are discussed in more detail.

Here, we consider the case in which the risk aversion coefficient is changed, while all other parameters remain the same. Table 2 presents the values important for determining the optimal strategies for different strengths of habit formation. On the left hand side, a more risk-seeking customer is considered, i.e., the risk aversion coefficient is lower. A lower level of risk aversion causes the customer to allocate a larger part to risky assets, causing the expected return on investments to be higher. As we can see, the marginal propensity to consume is very close to the case in which $\gamma = 5$. However, in most cases MPC is slightly higher. On the other hand, the average habit level is clearly larger, signaling that the optimal consumption path shifts upward as the customer becomes less risk-averse. Also the certainty equivalent surplus consumption is considerably higher in the case of lower risk aversion[6]. This effect is stronger for customers with a higher average habit level and weakest for the customer without habit formation, as can be seen when the first CESC-column is divided by the same column in table 1. On the right hand side, a more risk-averse customer is considered. In this case, the marginal propensity to consume is lower; however, the differences are only small. Again, there is a considerable difference between the average

[6] Again, note that these numbers are averages and that a lower risk aversion coefficient does not necessarily lead to a higher CESC. A lower risk aversion coefficient leads to a higher volatility of returns, which causes the worst cases to be worse than the case in which the risk aversion is higher.

habit levels when comparing the high risk aversion case ($\gamma = 10$) to the average risk aversion case ($\gamma = 5$). This result signals that the optimal consumption paths shift downward, which is consistent with the results found in the low risk aversion case ($\gamma = 2$). The CESC is now relatively low and the effect is still stronger for customers with a high average habit level, as can be seen when the second CESC-column is divided by the same column in table 1. However, the differences are smaller in this case.

Table 2. Optimal choices with constant investment opportunities for alternative risk aversion levels

		γ=2			γ=10		
α	β	MPC	Average habit level	CESC	MPC	Average habit level	CESC
0	0	4,51	0,00	6,0681	4,29	0,00	4,4752
0,1	0,2	2,62	4,22	3,8911	2,54	2,02	2,5881
0,1	0,3	3,17	2,89	4,5492	3,04	1,33	3,3119
0,2	0,3	1,84	6,14	3,0273	1,80	2,78	1,7077
0,1	0,4	3,48	2,14	4,9459	3,32	0,96	3,6051
0,2	0,4	2,45	4,50	3,7799	2,35	1,99	2,6643
0,3	0,4	1,42	7,28	2,5257	1,40	3,15	1,6508
0,1	0,5	3,67	1,66	5,1953	3,50	0,74	3,7893
0,2	0,5	2,83	3,47	4,2868	2,71	1,52	3,0435
0,3	0,5	1,99	5,55	3,3092	1,92	2,37	2,2351
0,4	0,5	1,16	8,04	2,2103	1,14	3,34	1,3752

The table presents values that are important for determining the optimal strategies for various levels of the habit parameters and different levels of the risk aversion coefficient. The parameter values are as follows: $t = 0$, $T = 42$, $\delta = 0.02$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an excess return of 6%, and $r = 0.03$. MPC is the marginal propensity to consume and is calculated as $(1 + \alpha F_t)^{1/\gamma}/G_t$ and the "Average habit level" is calculated as the mean of the habit levels over the entire period. Note that the results presented in this table are averages over the total number of simulations (1000).

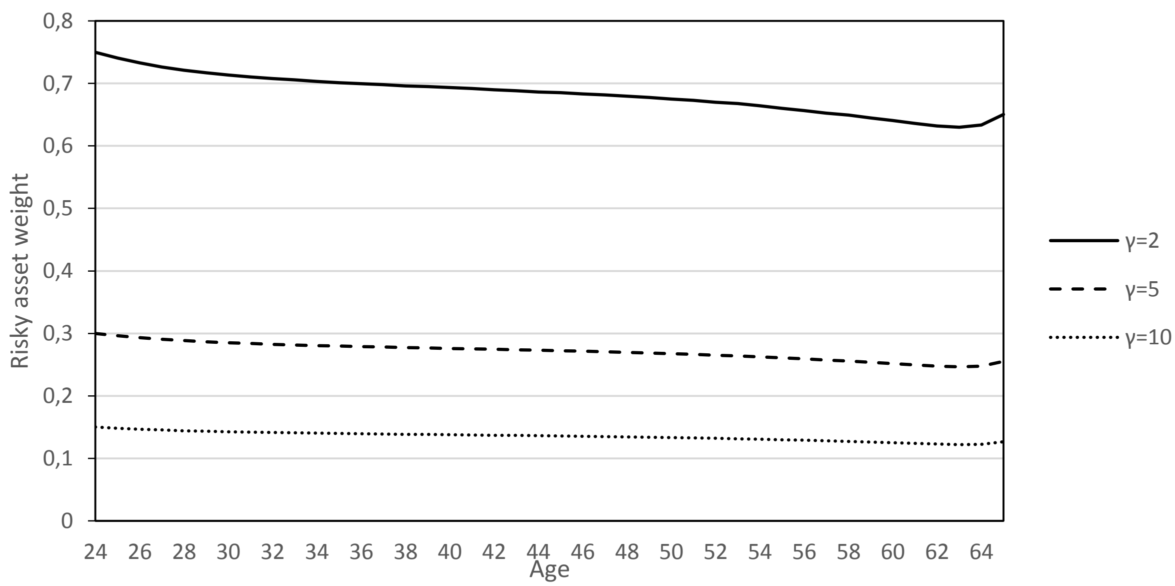

Figure 3. Optimal investment choices for different values of the risk aversion coefficient. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. First, note that $t = 0$ corresponds to an age of 24. Second, note that the risky asset weights are presented until the age of 65 since the risky asset weight is set to zero in the last period as explained in section 3.1. Third, note that the risky asset weights are all equal in the first period, which is due to the fact that the initial habit level equals $h_0 = 0$. Note that the optimal weights are averaged over the different customers and the total number of simulations for the different levels of the risk aversion coefficient.

Figure 3 shows the optimal investment strategy for three different levels of the risk aversion coefficient. As mentioned earlier, the lines are averaged over the various customers and the total number of simulations, allowing us to determine the general effect of a change in risk aversion. It is clear that a higher risk aversion coefficient causes the customer to decrease the allocation to risky assets, while the opposite is true for a lower risk aversion coefficient. Furthermore, the shape of the optimal investment path remains the same; however, the changes over time are more extreme.

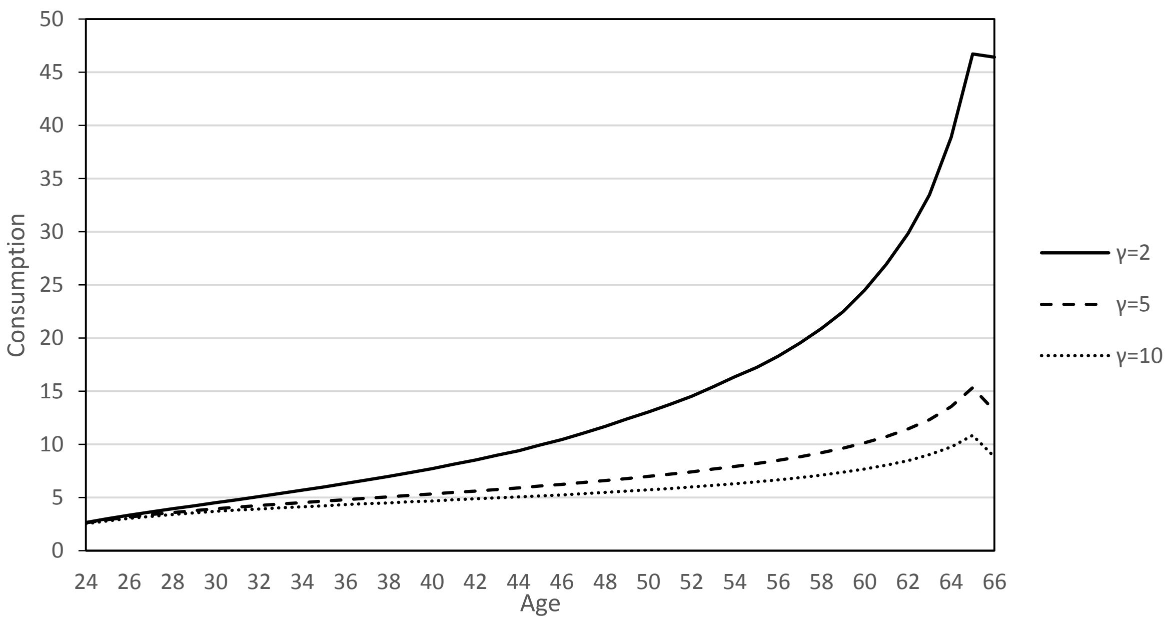

Figure 4. Optimal consumption paths for different values of the risk aversion parameter. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. Note that there is a drop in consumption in the last period for habit customers, which is due to the conversion to discrete time. This conversion led to adaption of the optimal consumption in the last period as described in section 3.1. Note that the optimal consumption paths are averaged over the different customers and the total number of simulations for the different levels of the risk aversion coefficient.

Figure 4 presents the optimal consumption paths for the three different levels of the risk aversion coefficient. Again, the lines are averaged over the different combinations of the habit parameters. As mentioned above, a lower risk aversion coefficient leads to an upward shift in the consumption path, which is due to the higher allocation to risky assets causing the expected return to rise. The opposite holds for a higher risk aversion coefficient, i.e., more risk-averse customers. On average, a more risk-seeking customer will allocate a larger part of wealth to risky investments and have a steeper consumption path. On top of this, the certainty equivalent surplus consumption will, on average, be higher.

4.2.2 Alternative risk-free interest rates

This section considers alternative risk-free interest rates and the effect on the optimal strategies. We find that a lower risk-free rate causes a decrease in the marginal propensity. Moreover, the consumption path shifts downward and the certainty equivalent surplus consumption is lowered, while the effect on the optimal investment path is only minor. The opposite is true for a higher risk-free rate. More details on the results can be found below.

Table 3 presents important values in determining the optimal strategies for various combinations of the scaling and persistence parameter and different risk-free interest rates. On the left hand side, a lower risk-free interest rate is considered. A lower risk-free interest rate will cause the expected return on the customer's portfolio to decrease, while a higher risk-free interest rate will increase the expected return. On the one hand, we see that a lower interest rate leads to a lower marginal propensity to consume. On the other hand, a higher interest rate will cause the MPC to increase. Furthermore, the average habit level is lower if the risk-free rate is lower, the opposite holds for a higher risk-free interest rate. Finally, we find that the CESC is clearly lower if the risk-free interest rate is lowered and considerably higher when the interest rate is higher. Again, this effect is stronger for customers with a higher average habit level, as can been seen when the respective CESC-column is divided by the same column in table 1. Moreover, this effect is stronger for the increase in the risk-free interest rate than for the decrease.

Table 3. Optimal choices with constant investment opportunities for alternative risk-free interest rates

		r=0,01			r=0,05		
α	β	MPC	Average habit level	CESC	MPC	Average habit level	CESC
0	0	3,38	0,00	3,3994	5,61	0,00	6,5213
0,1	0,2	1,90	1,58	1,8912	3,44	3,49	4,2250
0,1	0,3	2,35	1,07	2,4963	4,04	2,29	4,9299
0,2	0,3	1,32	2,14	1,4740	2,49	5,05	3,2286
0,1	0,4	2,59	0,78	2,7388	4,38	1,66	5,3244
0,2	0,4	1,80	1,57	2,0320	3,16	3,57	4,0495
0,3	0,4	1,02	2,37	1,2499	1,94	5,99	2,6560
0,1	0,5	2,74	0,60	2,8879	4,61	1,27	5,5763
0,2	0,5	2,10	1,12	2,3385	3,60	2,70	4,5668
0,3	0,5	1,46	1,84	1,7333	2,59	4,41	3,4668
0,4	0,5	0,82	2,46	1,0608	1,60	6,63	2,2585

The table presents values that are important for determining the optimal strategies for various levels of the habit parameters and different levels of the risk-free interest rate. The parameter values are as follows: $t = 0, T = 42$, $\delta = 0.02, \gamma = 5, h_0 = 0, W_0 = 100, \sigma = 0.2,$ and $\lambda = 0.3$ corresponding to an excess return of 6%. MPC is the marginal propensity to consume and is calculated as $(1 + \alpha F_t)^{1/\gamma}/G_t$ and the "Average habit level" is calculated as the mean of the habit levels over the entire period. Note that the results presented in this table are averages over the total number of simulations (1000).

Figure 5 shows the optimal investment choice for the three different levels of the risk-free rate that are considered. Obviously, the interest rate does not have a big effect on the optimal investment choice since the lines are very close together. In the first half of the working life, it seems that a lower interest rate causes the optimal risky asset weight to be lower than the optimal risky asset weight for the higher interest rate case. However, the opposite is true in the later stage.

Figure 6 presents the optimal consumption paths for the different risk-free interest rates that are considered. As already mentioned above, a lower interest rate causes the optimal consumption path to be lower, which is due to the lower expected return on the investment portfolio since both the risk-free rate and the total return on the risky asset is lower. A higher interest rates causes the opposite effect and shifts the consumption path upwards. It is noteworthy that the change in the consumption path is caused by a change in the risk-free return, while in the case with an alternative risk aversion level, the change is mainly caused by a change in the investment portfolio. Both will lead to a change in the expected return, but the cause is different.

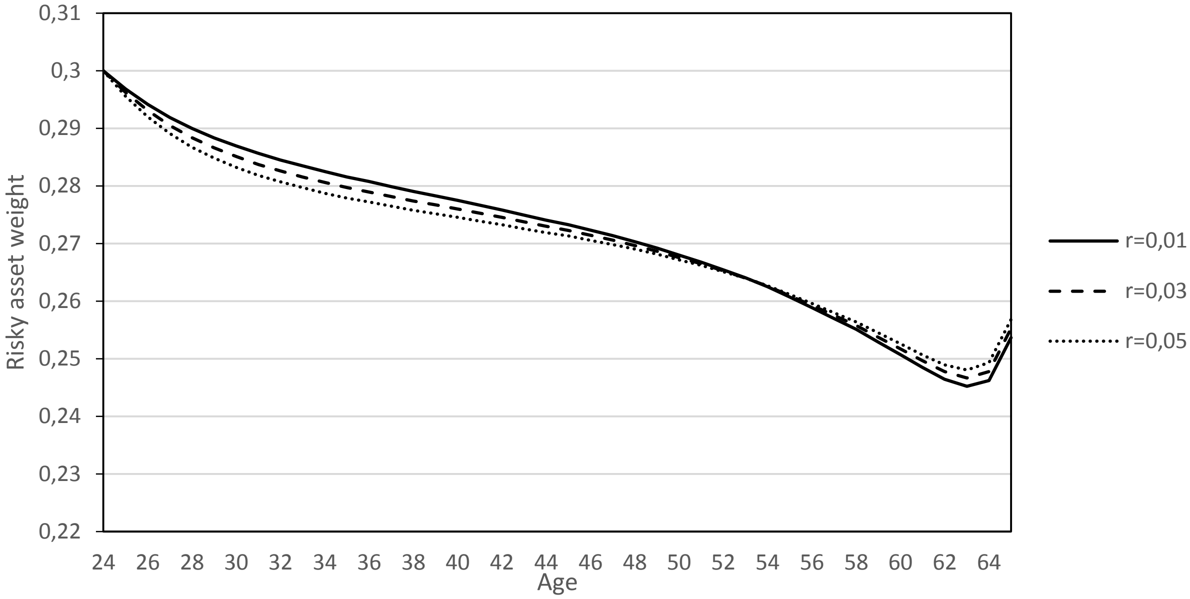

Figure 5. Optimal investment choices for different values of the risk-free interest rate. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, and $\lambda = 0.3$ corresponding to an expected excess return of 6%. First, note that $t = 0$ corresponds to an age of 24. Second, note that the risky asset weights are presented until the age of 65 since the risky asset weight is set to zero in the last period as explained in section 3.1. Third, note that the risky asset weights are all equal in the first period, which is due to the fact that the initial habit level equals $h_0 = 0$. Note that the optimal weights are averaged over the different customers and the total number of simulations for the different levels of the risk-free interest rate.

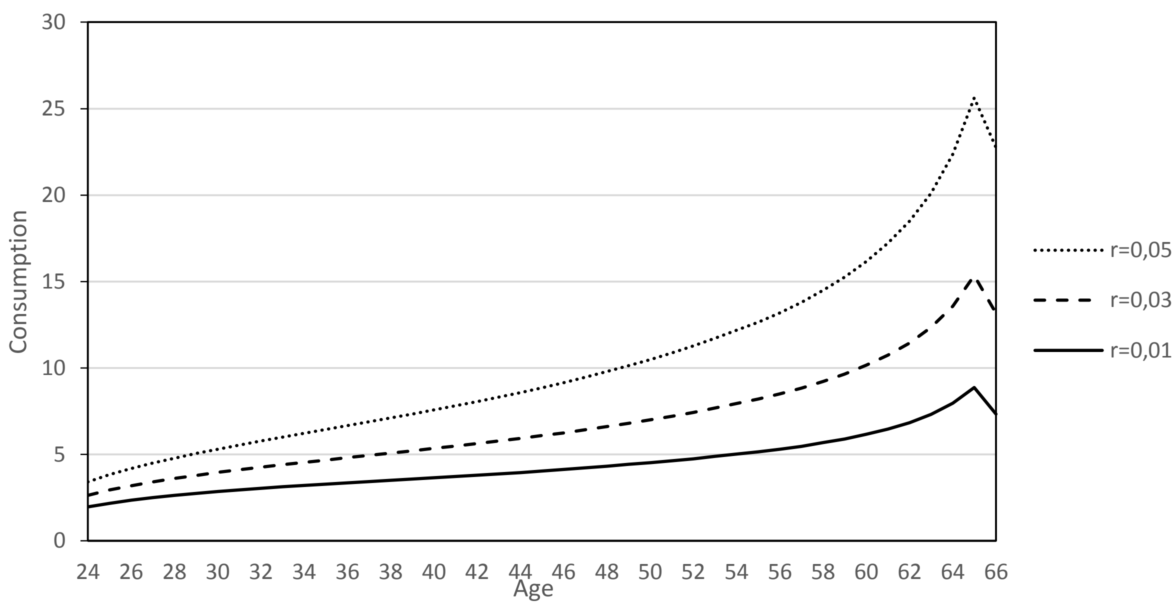

Figure 6. Optimal consumption paths for different values of the risk-free interest rate. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, and $\lambda = 0.3$ corresponding to an expected excess return of 6%. Note that there is a drop in consumption in the last period for habit customers, which is due to the conversion to discrete time. This conversion led to adaption of the optimal consumption in the last period as described in section 3.1. Note that the optimal consumption paths are averaged over the different customers and the total number of simulations for the different levels of the risk-free interest rate.

4.2.3 Alternative Sharpe-ratios

This section describes the effects of a change in the Sharpe-ratio on the optimal investment and consumption choice. It is found that a lower Sharpe-ratio decreases the marginal propensity to consume, the consumption path, the certainty equivalent surplus consumption, and the optimal risky asset weight. On the contrary, an increase in the Sharpe-ratio leads to higher values for these variables. Below, the results are discussed more extensively.

Table 4 considers changes in the Sharpe-ratio and will present values that are important in determining the optimal strategies. On the one hand, a decrease in the Sharpe-ratio to $\lambda = 0.2$ is considered, corresponding to an expected excess return of 4%. On the other hand, an increase in the Sharpe-ratio to $\lambda = 0.4$ is considered, i.e., an expected excess return of 8%. A lower Sharpe-ratio leads to lower values for MPC, Average habit level, and CESC relative to the average Sharpe-ratio case. The marginal propensity to consume is only slightly lower. The lower average habit level signals that consumption is lower over the period. As mentioned, the certainty equivalent surplus consumption is lower, caused by both a decrease in the expected excess return and a lower optimal risky asset weight. Both causes affect the total expected return on the investment portfolio, ultimately leading to a lower CESC. An increase in the Sharpe-ratio causes the opposite changes with a slightly larger magnitude. In this case, the relative increases/decreases in CESC are almost identical for all customers, as can be seen by dividing the respective CESC-columns by the same column in the table 1.

Table 4. Optimal choices with constant investment opportunities for alternative levels of the Sharpe-ratio

		λ=0,2			λ=0,4		
α	β	MPC	Average habit level	CESC	MPC	Average habit level	CESC
0	0	4,16	0,00	4,4460	4,83	0,00	5,3966
0,1	0,2	2,45	1,98	2,7742	2,83	3,11	3,3107
0,1	0,3	2,94	1,31	3,3196	3,41	2,10	4,0109
0,2	0,3	1,74	2,73	2,0898	2,01	4,43	2,4491
0,1	0,4	3,22	0,95	3,6088	3,73	1,53	4,3674
0,2	0,4	2,28	1,96	2,7157	2,64	3,22	3,2697
0,3	0,4	1,35	3,09	1,7355	1,55	5,19	2,0598
0,1	0,5	3,39	0,73	3,7908	3,94	1,18	4,5910
0,2	0,5	2,62	1,49	3,0890	3,04	2,47	3,7290
0,3	0,5	1,86	2,33	2,3204	2,15	3,94	2,7866
0,4	0,5	1,10	3,26	1,4715	1,27	5,68	1,7489

The table presents values that are important for determining the optimal strategies for various levels of the habit parameters and different levels of the risk-free interest rate. The parameter values are as follows: $t = 0$, $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, and $r = 0.03$. MPC is the marginal propensity to consume and is calculated as $(1 + \alpha F_t)^{1/\gamma}/G_t$ and the "Average habit level" is calculated as the mean of the habit levels over the entire period. Note that the results presented in this table are averages over the total number of simulations (1000).

Figure 7 shows the optimal investment strategy for three different levels of the Sharpe-ratio. A lower Sharpe-ratio leads to a lower expected excess return, making risky investments less attractive. This in turn leads to a lower share allocated to risky assets. On the contrary, an increase in the Sharpe-ratio leads to a higher allocation to risky investments. Note that the difference between the risky asset weight in the early stage and the later stage is larger if the initial weight is higher.

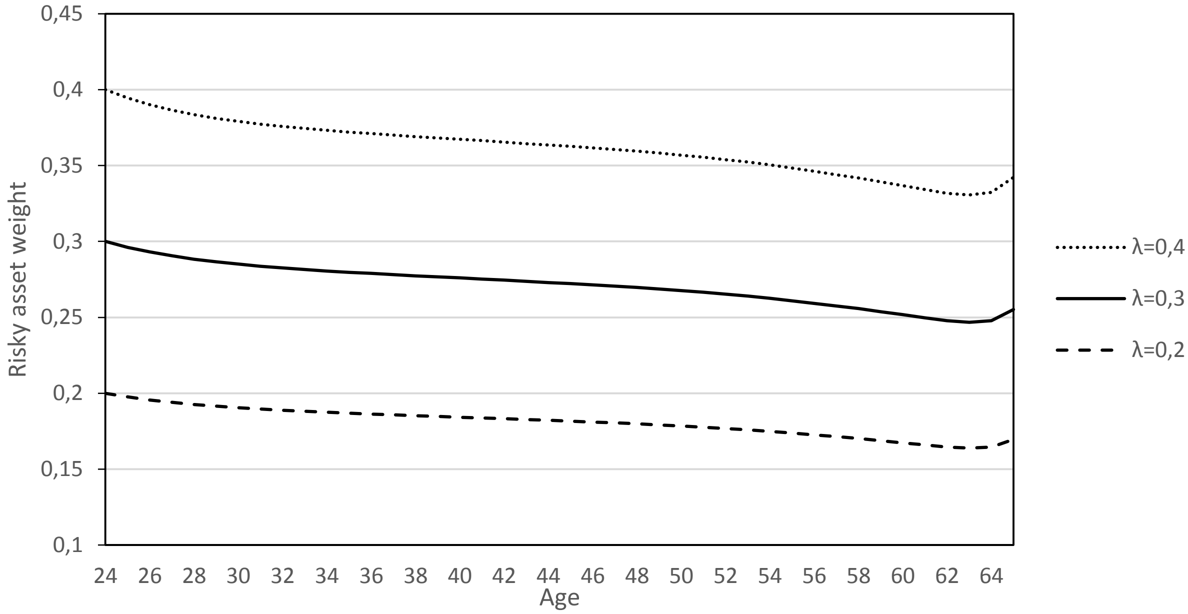

Figure 7. Optimal investment choices for different values of the Sharpe-ratio. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, and $r = 0.03$. First, note that $t = 0$ corresponds to an age of 24. Second, note that the risky asset weights are presented until the age of 65 since the risky asset weight is set to zero in the last period as explained in section 3.1. Third, note that the risky asset weights are all equal in the first period, which is due to the fact that the initial habit level equals $h_0 = 0$. Note that the optimal weights are averaged over the different customers and the total number of simulations for the different Sharpe-ratios.

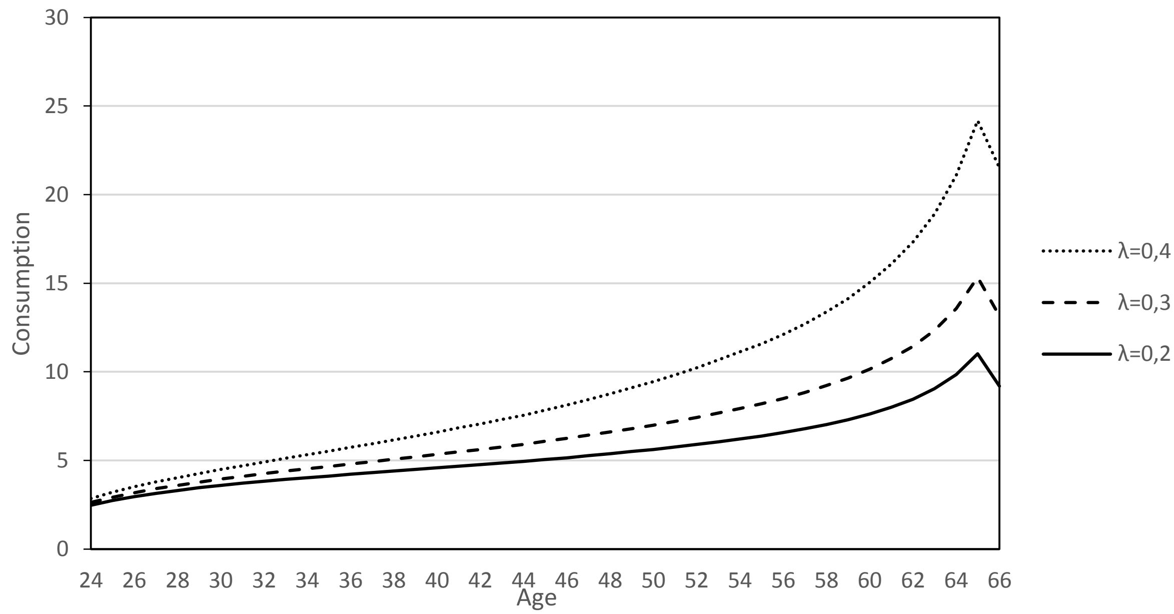

Figure 8. Optimal consumption paths for different values of the Sharpe-ratio. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $h_0 = 0$, $W_0 = 100$, $\sigma = 0.2$, and $r = 0.03$. Note that there is a drop in consumption in the last period for habit customers, which is due to the conversion to discrete time. This conversion led to adaption of the optimal consumption in the last period as described in section 3.1. Note that the optimal consumption paths are averaged over the different customers and the total number of simulations for the different Sharpe-ratios

Figure 8 shows the optimal consumption paths for the three different Sharpe-ratios mentioned before. It is clear that a higher Sharpe-ratio leads to a higher wealth level through a higher expected return on the investment portfolio, causing an upward shift in the consumption path. Of course, the opposite is true for a decrease in the Sharpe-ratio. Furthermore, the upward shift is larger in magnitude than the downward shift. Note that the two causes of a change in the consumption are correlated since a change in the Sharpe-ratio affects the wealth level both through the Sharpe-ratio itself and the share allocated to risky assets.

4.2.4 Alternative initial habit levels

Lastly, this section considers alterations in the initial habit level and studies the effects on the optimal strategies. We find that a higher initial habit level does not affect the marginal propensity to consume. Furthermore, an increase in the initial habit level increases the average habit level, while the certainty equivalent surplus consumption, the optimal asset weight in the early stages of the working life, and the consumption in the later part of the working life are lowered. These effects are larger in magnitude for a larger increase in the initial habit level. Note that a change in the initial habit level does not affect the customer without habit formation.

Now, we will take a closer look at the effects of a change in the initial habit level. Customers without habit formation will not be affected by this change since their habit level always equals $h_t = 0$. The initial habit level will not affect the marginal propensity to consume, but it will affect the optimal consumption level since habit customers have to take the habit level into account when determining their optimal consumption. I will not consider a decrease in the initial habit level, because a negative habit level does not make any sense. On the left hand side, the initial habit level is increased to $h_0 = 1{,}5$. As mentioned before, an increase in the habit level does not affect the MPC. The increase in the initial habit level lowers the CESC since habit customers are more restricted by this increase in terms of free wealth. The effect is smaller for habit customers with a higher persistence parameter since a higher level of β causes the effect of the initial habit level to decrease faster. The effects of an increase in the initial habit level to $h_0 = 3$ are similar; however, they are larger in magnitude.

Table 5. Optimal choices with constant investment opportunities for alternative initial habit levels

		h_0=1.5			h_0=3		
α	β	MPC	Average habit level	CESC	MPC	Average habit level	CESC
0,0	0,0	4,43	0,00	4,8235	4,43	0,00	4,8235
0,1	0,2	2,61	2,45	2,6662	2,61	2,51	2,3641
0,1	0,3	3,14	1,65	3,3911	3,14	1,71	3,1872
0,2	0,3	1,85	3,30	2,0103	1,85	3,26	1,8225
0,1	0,4	3,43	1,21	3,7537	3,43	1,27	3,5965
0,2	0,4	2,43	2,42	2,7824	2,43	2,43	2,6275
0,3	0,4	1,43	3,75	1,7155	1,43	3,68	1,5608
0,1	0,5	3,62	0,94	3,9791	3,62	0,99	3,8488
0,2	0,5	2,80	1,87	3,2168	2,80	1,89	3,0889
0,3	0,5	1,98	2,88	2,3840	1,98	2,87	2,2604
0,4	0,5	1,17	4,02	1,4633	1,17	3,93	1,3404

The table presents values that are important for determining the optimal strategies for various levels of the habit parameters and different initial habit levels. The parameter values are as follows: $t = 0$, $T = 42$, $\delta = 0.02$, $\gamma = 5$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. MPC is the marginal propensity to consume and is calculated as $(1 + \alpha F_t)^{1/\gamma}/G_t$ and the "Average habit level" is calculated as the mean of the habit levels over the entire period. Note that the results presented in this table are averages over the total number of simulations (1000).

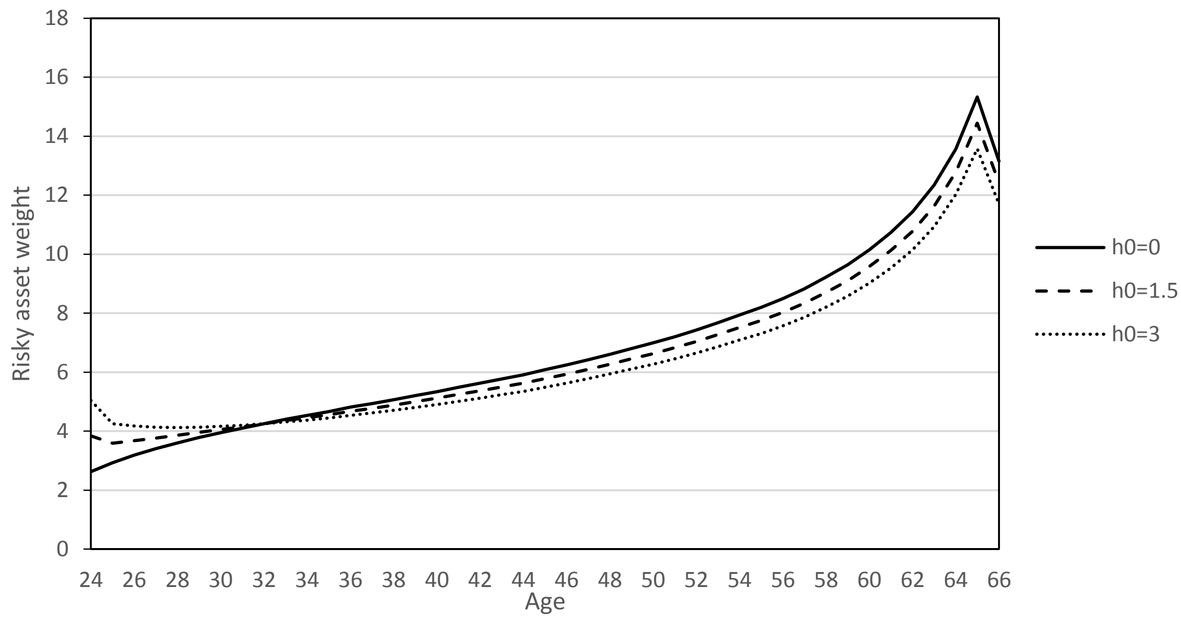

Figure 9. Optimal investment choices for different values of the initial habit level. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. First, note that $t = 0$ corresponds to an age of 24. Second, note that the risky asset weights are presented until the age of 65 since the risky asset weight is set to zero in the last period as explained in section 3.1. Note that the optimal weights are averaged over the different customers and the total number of simulations for the different initial habit levels.

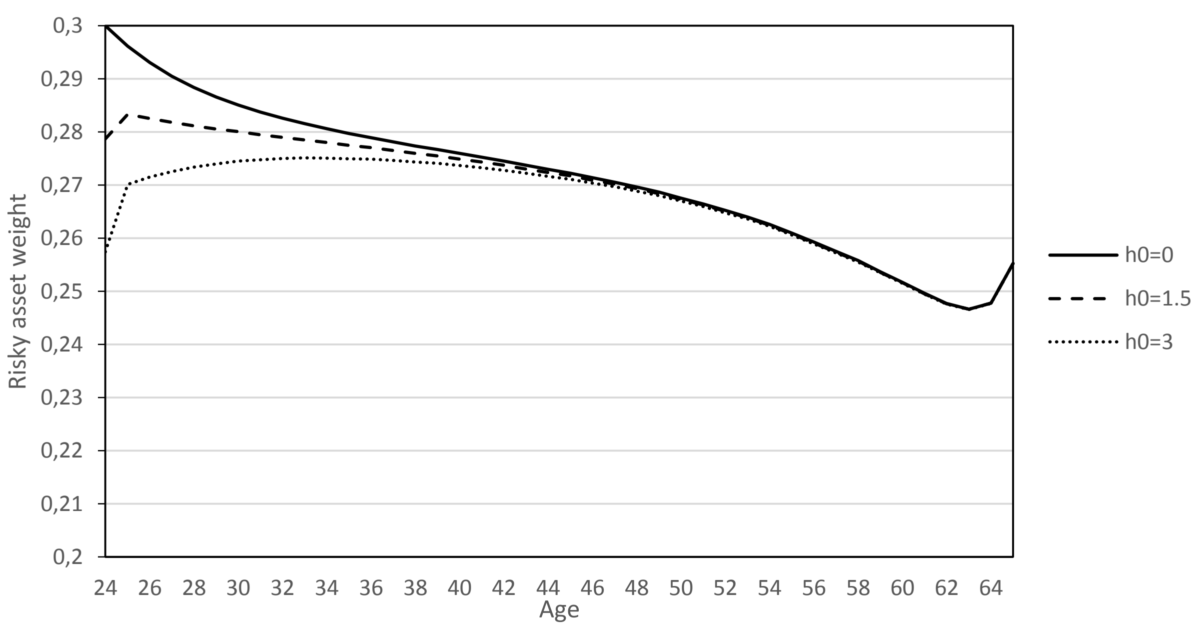

Figure 10. Optimal consumption paths for different values of the initial habit level. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $W_0 = 100$, $\sigma = 0.2$, $\lambda = 0.3$ corresponding to an expected excess return of 6%, and $r = 0.03$. Note that there is a drop in consumption in the last period for habit customers, which is due to the conversion to discrete time. This conversion led to adaption of the optimal consumption in the last period as described in section 3.1. Note that the optimal consumption paths are averaged over the different customers and the total number of simulations for the different initial habit levels.

Figure 9 presents the optimal portfolio choice for three different initial habit levels. As the initial habit level increases, free wealth decreases, lowering the optimal risky asset weight. As time passes, the habit level depends less on the initial habit level, which causes the lines to converge. At the end of the working life, the optimal risky asset weights are equal for the three different initial habit levels.

Figure 10 shows the optimal consumption paths for the three different initial habit levels. Due to an increase in the initial habit level, the consumption levels in the early working life are higher. The higher consumption in the early working life causes the wealth to decrease faster. On top of this, the expected return on the investment portfolio is lower since the higher initial habit level causes the optimal risky asset weight to go down. This means that the optimal consumption levels are lower for the rest of the working life.

5. Results with a minimum rate of return guarantee

This section discusses the results retrieved from the simulation of the model described in section 3.2. The simulations are used to determine the terms of the contract such that the abnormal profits equal zero[7]. As suggested by Hansen and Miltersen (2002), the distribution of the bonus will take place if the buffer ratio exceeds 10%, i.e., $\theta = 0.1$. Using the simulations, the combinations of the parameters g, ρ, φ, and ξ are determined such that the condition in equation (23) is fulfilled. In order to find these combinations, the parameters ρ, φ, and ξ are pre-specified. Then, the minimum rate of return g is determined such that (23) holds. This is done through the use of the trial-and-error method. The trial-and-error process is continued until we find the minimum rate with 5 decimals that leads to the abnormal profit closest to zero. Using these minimum rates of return, the optimal strategies can be determined and the certainty equivalent surplus consumption can be calculated.

The remainder of this section is structured as follows. Section 5.1 presents the optimal strategies for three different customers and compares these results to section 4.1 in order to determine the value of the minimum rate of return guarantee. Thereafter, section 5.2 presents the optimal strategies for the three different customers and compares these results to the outcomes in section 4.2. Note that the results presented in the rest of this section are averaged over the number of simulations.

5.1 Optimal strategies with a minimum rate of return guarantee

The results in this section focus on customers with three different combinations of the habit parameters: A customer without habit formation ($\alpha = 0, \beta = 0$), a customer with a weaker form of habit formation ($\alpha = 0.1, \beta = 0.5$), and a customer with a stronger form of habit formation ($\alpha = 0.2, \beta = 0.3$). In order to compare the results later, the values attached to the parameters are the same as discussed in section 4.1. In addition, the initial values for the different accounts are: $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, and $X_0 = 100$. The bonus reserve account and the company's account are initially zero, because there are no built op reserves at the start and there exists no upfront payment. Moreover, the initial amount credited to the customer's account must be equal to the initial wealth, i.e., $A_0 = W_0 = 100$. Note that the number of simulations used is 1000.

This section is structured in the following way. Section 5.1.1 presents the minimum guaranteed rate, the certainty equivalent surplus consumption, and the optimal consumption path for a customer without habit formation. Section 5.1.2 presents these results in a similar way for a customer with a weaker form of habit formation. Section 5.1.3 provides the results for a customer with a stronger form of habit formation. All results are presented for both the indirect and the direct payment method and are compared to the results in section 4.1.

5.1.1 Optimal strategies for a customer without habit formation

For a customer without habit formation, this section provides the minimum rate of return and the corresponding certainty equivalent surplus consumption for various indirect payment and direct payment contracts. On top of this, the optimal consumption path is presented for the worst case, the best case, and the case without a minimum rate guarantee and for both the indirect and direct payment method. We find that the worst contract terms lead to a decrease in the certainty equivalent surplus consumption and a lower consumption path with a larger consumption in the last period when compared to the case without the guarantee. On the other hand, the best contract increases the CESC by 4.95% in the indirect payment

[7] The abnormal profits are defined as the net present value of profits in a risk-neutral setting. Note that the issuer of the contract earns a risk-premium and therefore earns a profit unequal to zero.

case and by 5.37% in the direct payment case making the guarantee a valuable investment to the customer without habit formation. Furthermore, the consumption path is slightly flatter than in the case without the guarantee, but slightly steeper than in the worst-case scenario. Below, the results are explained in more detail.

Table 6 presents the values for the minimum rate of return for a customer without habit formation and different combinations of the parameters ρ and φ. We find that for low values of φ, an increase in φ leads to an increase in the minimum rate of return as ρ stays constant. This is due to the fact that for a higher minimum rate of return, the probability of the customer receiving the minimum rate is very large. Hence, the company receives almost all of the distributed surplus. We find that for higher values of φ, an increase in φ (i.e. a larger share of the distributed surplus going to the customer), leads to a decrease in the minimum rate of return that can be offered. As φ gets larger, the probability that the customer gets a return higher than the minimum rate increases. This leads to a larger probability that the bonus reserve will be negative at maturity, which is why the company can afford a lower minimum rate of return. As the company gets a larger share of the distributed surplus and thus a higher payment, it can afford to set a higher minimum rate of return.

Table 6. Values for g for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	0.03990	0.04206	0.04185	0.04067	0.03912	0.03770	0.03643	0.03537	0.03426	0.03327	–
0.2000	0.04216	0.04292	0.04294	0.04220	0.04116	0.04012	0.03913	0.03827	0.03745	–	–
0.3000	0.04292	0.04319	0.04330	0.04293	0.04221	0.04140	0.04058	0.03986	–	–	–
0.4000	0.04319	0.04334	0.04344	0.04328	0.04274	0.04208	0.04143	–	–	–	–
0.5000	0.04334	0.04344	0.04350	0.04351	0.04306	0.04249	–	–	–	–	–
0.6000	0.04344	0.04350	0.04354	0.04357	0.04331	–	–	–	–	–	–
0.7000	0.04350	0.04354	0.04357	0.04359	–	–	–	–	–	–	–
0.8000	0.04354	0.04357	0.04359	–	–	–	–	–	–	–	–
0.9000	0.04357	0.04359	–	–	–	–	–	–	–	–	–
1.0000	0.04359	–	–	–	–	–	–	–	–	–	–

The table presents the minimum rates of return for different variants of the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. Note that a customer without habit formation is considered. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 7 presents the values of the certainty equivalent surplus consumption for the different variants of the indirect payment method and the corresponding minimum rates presented in table 6. This table allows us to determine which variant of the indirect payment method the customer favors. Here, the contract that is best for the customer is the one with parameters $\varphi = 0.4$ and $\rho = 0.4$. The worst contract terms are $\varphi = 0.0$ and $\rho = 0.1$. Note that the contract values around the best contract are almost identical, while the difference between the worst and the best contract is quite substantial. As we can see, a higher minimum rate of return leads to a higher value to the customer in almost all cases. Sometimes a slightly lower minimum rate of return leads to a higher value to the customer, which is caused by a higher value of φ. A higher φ leads to a higher probability of a surplus distribution to the customer's account, which may offset the effect of a lower minimum rate of return. However, we must keep in mind that this is only a minor effect.

Table 7. Certainty equivalent surplus consumption values for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	4.8030	4.9693	5.0250	5.0209	4.9963	4.9737	4.9532	4.9372	4.9210	4.9071	–
0.2000	4.9625	5.0142	5.0423	5.0511	5.0428	5.0292	5.0148	5.0016	4.9893	–	–
0.3000	5.0141	5.0320	5.0441	5.0584	5.0588	5.0513	5.0412	5.0326	–	–	–
0.4000	5.0320	5.0417	5.0482	5.0567	5.0625	5.0597	5.0539	–	–	–	–
0.5000	5.0417	5.0481	5.0519	5.0549	5.0614	5.0618	–	–	–	–	–
0.6000	5.0481	5.0519	5.0543	5.0562	5.0599	–	–	–	–	–	–
0.7000	5.0519	5.0543	5.0561	5.0574	–	–	–	–	–	–	–
0.8000	5.0543	5.0561	5.0573	–	–	–	–	–	–	–	–
0.9000	5.0561	5.0573	–	–	–	–	–	–	–	–	–
1.0000	5.0573	–	–	–	–	–	–	–	–	–	–

The table presents the certainty equivalent surplus consumption for different variants of the indirect payment method and the minimum rates of return presented in table 6. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. Note that a customer without habit formation is considered. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

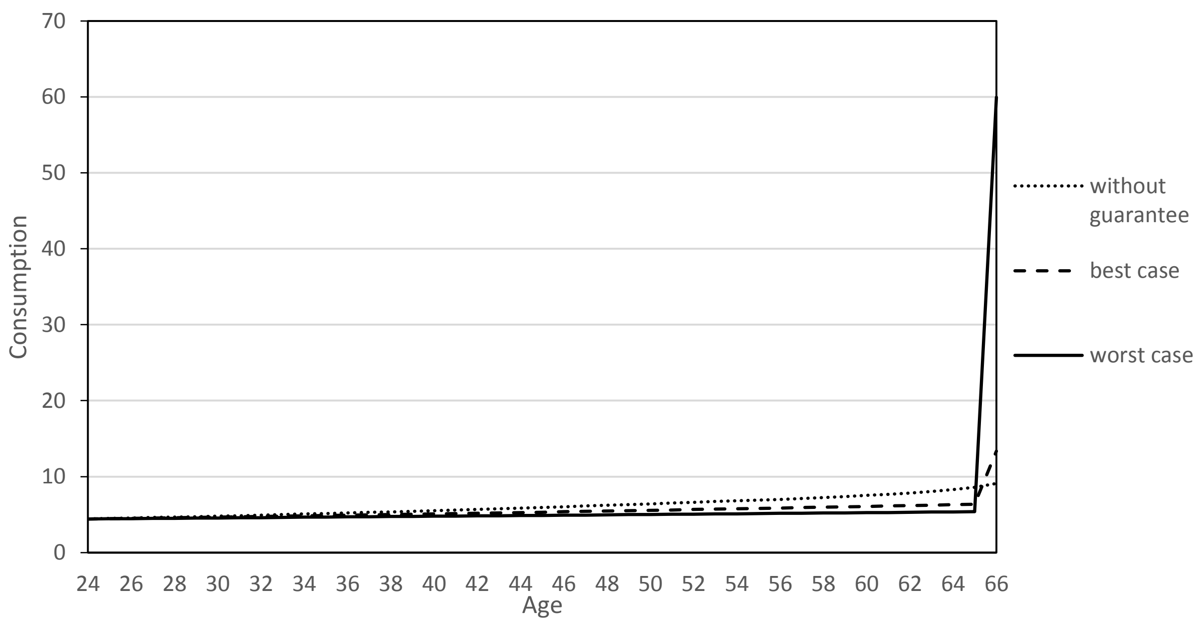

Figure 11. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer without habit formation and the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\rho = 0.1$. The best case is the contract with parameters $\varphi = 0.4$ and $\rho = 0.4$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

The optimal consumption paths change when a minimum rate of return guarantee is introduced, but does this increase the certainty equivalent surplus consumption of the customer, i.e., do customers without habit formation value this option? Without the option, the customer without habit formation has a certainty equivalent surplus consumption of 4.8235. As found in table 7, the worst-case scenario yields a CESC of 4.8030, which is slightly lower, causing this particular contract to be worthless to the customer.

However, all other contracts presented in table 7 are valuable to the customer. In the best-case scenario, the CESC increases by about 4.95%, making the contract interesting to the customer. Thus, in the indirect payment case, a minimum rate of return guarantee could be a very interesting investment for a customer without habit formation.

Next, let us consider the direct payment case for a customer without habit formation (i.e. $\alpha = 0$ and $\beta = 0$). Table 8 presents the values for the minimum rate of return for a customer without habit formation and various direct payment contracts. Logically, the minimum rate offered is increasing in the level of the direct payment. A higher direct payment increases the company's account, which leads to a higher minimum rate of return. Striking is that the minimum guaranteed rate is not considerably influenced by φ. Hansen and Miltersen (2002) argue that a larger φ leads to a larger amount in the customer's account, account A. On top of this, a higher φ leads to a higher probability of the bonus reserve being negative at maturity. The direct payment made to the company is then based on the larger amount in account A, which brings a larger payment to the company's account. This larger payment offsets the higher probability of a negative bonus reserve at maturity, causing the minimum rate to be more or less independent of φ. As can be seen in table 8, this offsetting effect is more apparent for higher direct payments.

Table 8. Values for g for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	0.04047	0.04192	0.04198	0.04139	0.04062	0.03998	0.03941	0.03889	0.03845	0.03807	0.03771
0.0050	0.04666	0.04702	0.04712	0.04666	0.04612	0.04559	0.04511	0.04467	0.04428	0.04394	0.04365
0.0075	0.05046	0.05053	0.05056	0.05031	0.04992	0.04955	0.04918	0.04885	0.04859	0.04837	0.04817
0.0100	0.05345	0.05345	0.05345	0.05329	0.05304	0.05275	0.05250	0.05226	0.05206	0.05188	0.05170
0.0125	0.05613	0.05613	0.05612	0.05602	0.05589	0.05569	0.05549	0.05531	0.05514	0.05498	0.05483
0.0150	0.05869	0.05869	0.05869	0.05866	0.05858	0.05847	0.05833	0.05820	0.05808	0.05796	0.05784
0.0175	0.06120	0.06120	0.06120	0.06120	0.06116	0.06111	0.06102	0.06093	0.06083	0.06074	0.06066
0.0200	0.06370	0.06370	0.06370	0.06370	0.06370	0.06367	0.06364	0.06358	0.06350	0.06343	0.06336
0.0225	0.06620	0.06620	0.06620	0.06620	0.06620	0.06620	0.06618	0.06616	0.06613	0.06608	0.06602
0.0250	0.06870	0.06870	0.06870	0.06870	0.06870	0.06870	0.06870	0.06869	0.06868	0.06866	0.06863

The table presents the minimum rates of return for different variants of the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer without habit formation is considered. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 9 presents the CESC values for various contracts of the direct payment method. The CESC values correspond to the minimum rates presented in table 8. In the direct payment case, the optimal contract is $\varphi = 0.4$ and $\xi = 0.0125$. The worst contract for the customer is $\varphi = 0.0$ and $\xi = 0.0025$. Here, a higher minimum rate does not per se lead to a higher value to the customer. A higher minimum rate goes hand in hand with a higher direct payment. If the increases in both values are close, the value to the customer tends to decrease. This can be explained by the fact that both a higher minimum rate and a higher direct payment lead to a lower probability of a positive bonus reserve, lowering the value of the contract. As in the indirect payment case, there is only a marginal difference between the values of the contracts around the optimal contract and the value of the optimal contract. Furthermore, the worst contract in the direct payment case leads to a lower CESC than the worst contract in the indirect payment case, while most of the contracts in the direct payment case lead to a higher value than the best contract in the indirect case.

Table 9. Certainty equivalent surplus consumption values for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	4.6602	4.8724	4.9459	4.9612	4.9580	4.9541	4.9501	4.9463	4.9435	4.9419	4.9395
0.0050	4.9263	4.9928	5.0409	5.0469	5.0425	5.0359	5.0288	5.0212	5.0144	5.0087	5.0043
0.0075	5.0151	5.0392	5.0683	5.0761	5.0731	5.0679	5.0626	5.0577	5.0544	5.0514	5.0482
0.0100	5.0476	5.0569	5.0749	5.0815	5.0811	5.0775	5.0737	5.0697	5.0673	5.0651	5.0625
0.0125	5.0591	5.0633	5.0745	5.0800	5.0827	5.0809	5.0781	5.0750	5.0720	5.0696	5.0674
0.0150	5.0627	5.0644	5.0718	5.0782	5.0814	5.0823	5.0807	5.0793	5.0778	5.0758	5.0740
0.0175	5.0630	5.0637	5.0677	5.0736	5.0771	5.0798	5.0798	5.0794	5.0781	5.0768	5.0758
0.0200	5.0628	5.0632	5.0649	5.0687	5.0730	5.0755	5.0777	5.0782	5.0772	5.0763	5.0752
0.0225	5.0627	5.0629	5.0637	5.0655	5.0684	5.0716	5.0736	5.0753	5.0763	5.0763	5.0753
0.0250	5.0626	5.0627	5.0632	5.0640	5.0654	5.0675	5.0698	5.0716	5.0730	5.0736	5.0738

The table presents the certainty equivalent surplus consumption for different variants of the direct payment method and the minimum rates of return presented in table 8. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer without habit formation is considered. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 9 presents the CESC values for various contracts of the direct payment method. The CESC values correspond to the minimum rates presented in table 8. In the direct payment case, the optimal contract is $\varphi = 0.4$ and $\xi = 0.0125$. The worst contract for the customer is $\varphi = 0.0$ and $\xi = 0.0025$. Here, a higher minimum rate does not per se lead to a higher value to the customer. A higher minimum rate goes hand in hand with a higher direct payment. If the increases in both values are close, the value to the customer tends to decrease. This can be explained by the fact that both a higher minimum rate and a higher direct payment lead to a lower probability of a positive bonus reserve, lowering the value of the contract. As in the indirect payment case, there is only a marginal difference between the values of the contracts around the optimal contract and the value of the optimal contract. Furthermore, the worst contract in the direct payment case leads to a lower CESC than the worst contract in the indirect payment case, while most of the contracts in the direct payment case lead to a higher value than the best contract in the indirect case.

Figure 12 shows the optimal consumption choice for a customer without habit formation in the direct payment case. Again, the figure showing the optimal investment choice is excluded, because the guarantee does not affect this choice. As in the indirect case, three cases are considered: The worst-case scenario (i.e. $\varphi = 0.0$ and $\xi = 0.0025$), the best-case scenario (i.e. $\varphi = 0.4$ and $\xi = 0.0125$), and the case without the minimum rate guarantee. The results are very similar to the indirect case; the consumption is smoothed in both the worst and best case in comparison to the case without the guarantee. Also, there is a spike in consumption in the last period for both the worst and the best case. However, there are some small differences. In the worst-case scenario, the difference between the minimum rate and the direct payment is lower than the minimum rate in the indirect case, causing the optimal consumption to be lower over time. Consequently, there is an even larger spike in absolute terms in the last period. In the best-case scenario, the consumption path is slightly higher since the difference between the minimum rate and the direct payment is larger than the minimum rate in the indirect case, while the spike in the last period is somewhat smaller both in absolute and relative terms.

Introducing the direct payment leads to slightly different consumption paths when compared to the indirect case. The worst-case scenario yields a CESC of 4.6602, which is considerably lower than the CESC-value without the guarantee. Hence, in the worst-case scenario, the contract is not interesting for the customer. In all other scenarios, the contract is actually valuable. The best contract increases the

CESC by 5.37% when compared to the case without the guarantee. A customer without habit formation would thus be very interested in a minimum rate of return guarantee as long as it does not receive the worst contract terms.

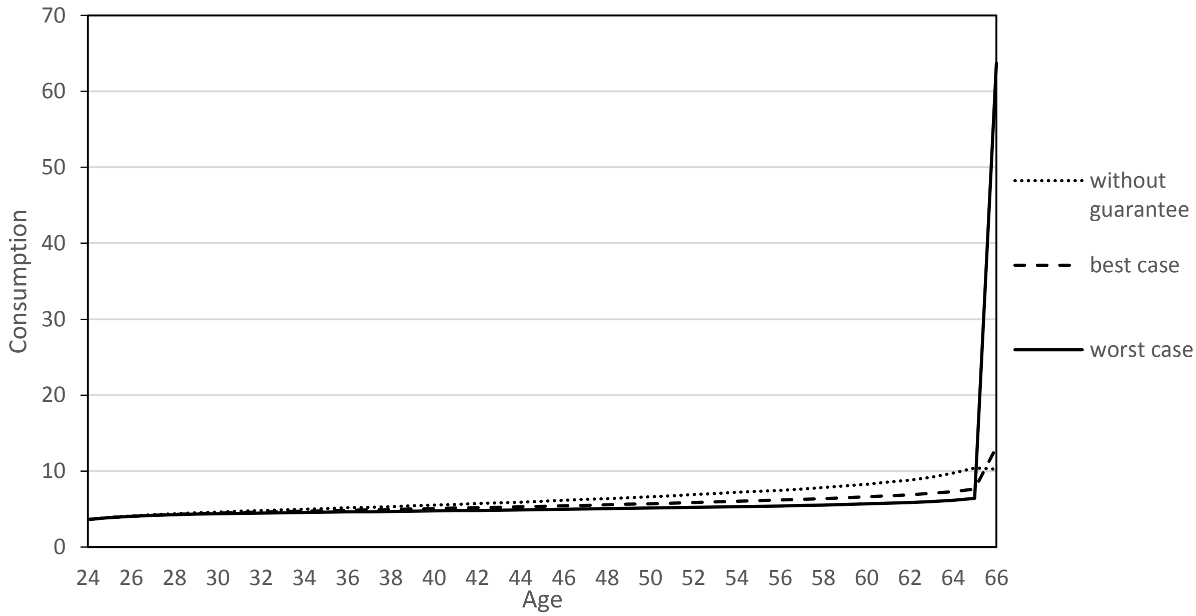

Figure 12. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer without habit formation and the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\xi = 0.0025$. The best case is the contract with parameters $\varphi = 0.4$ and $\xi = 0.0125$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

5.1.2 Optimal strategies for a customer with a weak form of habit formation

In this section, we consider a customer with a weak form of habit formation, i.e., $\alpha = 0.1, \beta = 0.5$. We provide the minimum rate of return and the corresponding CESC-values for different variants of the indirect and direct payment method. Also, the optimal consumption paths are presented for the worst case, the best case, and the case without the guarantee. The results show us that the worst contract terms lead to a lower CESC for both the indirect and direct contract, while the best contract terms increase the CESC by 3.84% for the indirect payment method and by 4.22% for the direct payment method. As in the previous section, the worst-case scenario leads to the lowest consumption path, while the best-case scenario leads to a slightly higher consumption path. Again, the consumption spike in the last period is larger for the worst contract than for the best contract. The remaining part of this section explains the results in more detail.

Table 10 presents the values for the minimum rate for various indirect payment contracts. The minimum rates offered are slightly lower for a customer with weak habit formation than for a customer without habit formation. The company can only afford a lower minimum rate since the optimal weight in risky assets is lower for a customer with habit formation. Due to this lower optimal weight, the total expected return on investments is smaller, which lowers the average amount in the bonus reserve at maturity and the probability of an indirect payment. As mentioned before, if a larger share of the distributed surplus goes to the customer, the minimum rate decreases. Also, a higher ρ leads to a higher minimum rate of return.

Table 10. Values for g for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	0.03900	0.04113	0.04095	0.03983	0.03835	0.03701	0.03579	0.03478	0.03372	0.03279	–
0.2000	0.04123	0.04201	0.04204	0.04133	0.04032	0.03932	0.03838	0.03757	0.03681	–	–
0.3000	0.04201	0.04230	0.04242	0.04205	0.04135	0.04057	0.03979	0.03910	–	–	–
0.4000	0.04230	0.04246	0.04257	0.04241	0.04187	0.04123	0.04061	–	–	–	–
0.5000	0.04246	0.04257	0.04263	0.04264	0.04219	0.04165	–	–	–	–	–
0.6000	0.04257	0.04263	0.04267	0.04270	0.04244	–	–	–	–	–	–
0.7000	0.04263	0.04267	0.04270	0.04272	–	–	–	–	–	–	–
0.8000	0.04267	0.04270	0.04272	–	–	–	–	–	–	–	–
0.9000	0.04270	0.04272	–	–	–	–	–	–	–	–	–
1.0000	0.04272	–	–	–	–	–	–	–	–	–	–

The table presents the minimum rates of return for different variants of the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. Note that a customer with a weak form of habit formation is considered, i.e., $\alpha = 0.1, \beta = 0.5$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 11 presents the CESC-values for different indirect payment contracts. The values in this table correspond to the minimum rates of return presented in table 10. The parameter values $\varphi = 0.0$ and $\rho = 0.1$ lead to the worst contract for the customer, while the contract $\varphi = 0.5$ and $\rho = 0.5$ is most favored by customer. Note that the values of the different contracts are almost identical, with the exception of the worst contract. Similar to the case in which the customer has no habit formation, in most cases the value to the customer is higher when the minimum guaranteed rate is larger. As mentioned earlier, in some cases a slightly lower minimum rate leads to a higher CESC, which is due to the larger φ.

Table 11. Certainty equivalent surplus consumption values for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	4.0647	4.1900	4.2314	4.2300	4.2136	4.1989	4.1850	4.1747	4.1641	4.1555	–
0.2000	4.1861	4.2270	4.2481	4.2551	4.2498	4.2410	4.2321	4.2239	4.2166	–	–
0.3000	4.2270	4.2419	4.2515	4.2617	4.2627	4.2583	4.2525	4.2475	–	–	–
0.4000	4.2419	4.2499	4.2555	4.2618	4.2660	4.2646	4.2616	–	–	–	–
0.5000	4.2499	4.2554	4.2584	4.2607	4.2656	4.2672	–	–	–	–	–
0.6000	4.2554	4.2584	4.2603	4.2618	4.2647	–	–	–	–	–	–
0.7000	4.2584	4.2603	4.2617	4.2627	–	–	–	–	–	–	–
0.8000	4.2603	4.2617	4.2627	–	–	–	–	–	–	–	–
0.9000	4.2617	4.2627	–	–	–	–	–	–	–	–	–
1.0000	4.2627	–	–	–	–	–	–	–	–	–	–

The table presents the certainty equivalent surplus consumption for different variants of the indirect payment method and the minimum rates of return presented in table 10. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a weak form of habit formation is considered, i.e., $\alpha = 0.1, \beta = 0.5$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

The optimal investment choice is again not affected by the introduction of a minimum rate of return guarantee, while the change in the optimal consumption choice is presented in figure 13. As before, three different cases are considered: The worst case (i.e. $\varphi = 0.0$ and $\rho = 0.1$), the best case (i.e. $\varphi = 0.5$ and

$\rho = 0.5$), and the case without the guarantee. As we have seen in section 4.1.1, the consumption path is steeper for a customer with habit formation. This is also true for the consumption paths for the worst and best-case scenario. As with the customer without habit formation, the consumption paths are smoother and there is a spike in consumption in the last period. Note that in this case, the spike is slightly larger in absolute terms in the worst-case scenario, while it is smaller in the best-case scenario. In relative terms, the spikes are smaller in both scenarios when compared to the customer without habit formation.

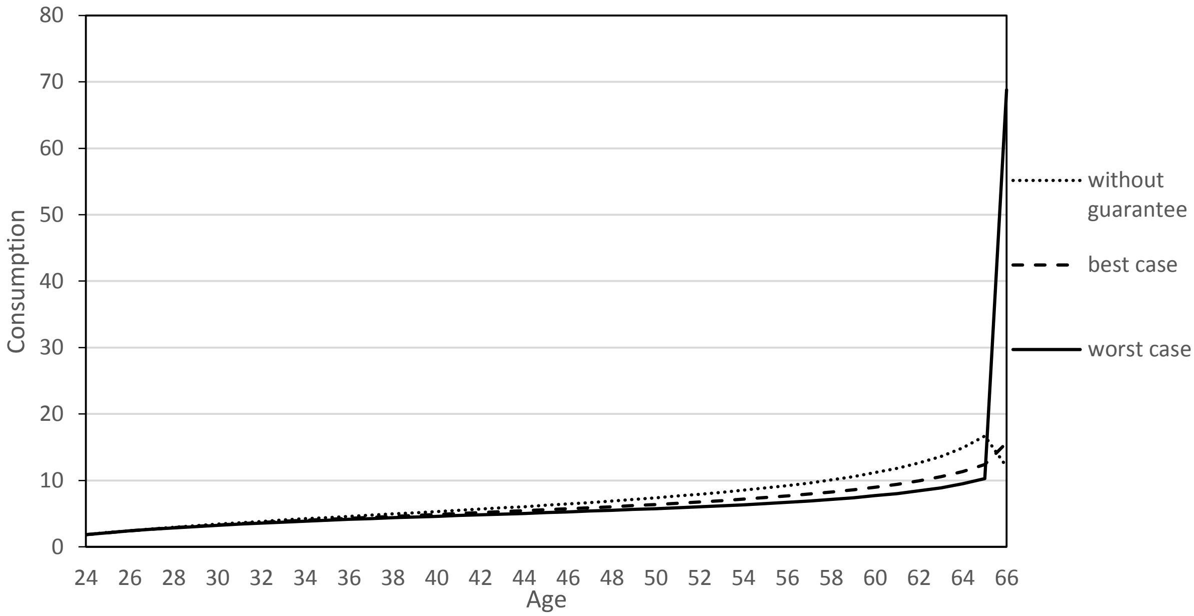

Figure 13. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer with a weaker form habit formation and the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0.1$, $\beta = 0.5$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\rho = 0.1$. The best case is the contract with parameters $\varphi = 0.5$ and $\rho = 0.5$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

In the worst-case scenario, the CESC-value is 4.0647, while it is 4.1094 without the minimum rate guarantee. The worst-case scenario is thus still unfavorable for the customer. On the other hand, the best-case scenario leads to a CESC of 4.2672, increasing the CESC by 3.84% compared to the case where the minimum rate guarantee is not available. The minimum rate of return guarantee is thus relatively less valuable to a customer with a weak form of habit formation.

Now, we consider the case in which the company receives a direct payment. The minimum rates offered by the company for different contracts with a direct payment are presented in table 12. As we can see, the minimum rate increases with the direct payment to the company. Furthermore, it seems that the minimum rate is relatively independent of φ. As argued before, a higher φ leads to a higher amount in the customer's account, which in turn leads to a higher direct payment to the company's account since the direct payment is based on this larger amount. Again, as the direct payment gets larger, the minimum rate is more independent of φ.

Table 13 presents the CESC-values that correspond to the minimum rates of return presented in table 12. The worst-case scenario, with a direct premium, $\varphi = 0.0$ and $\xi = 0.0025$, is worse than the worst contract when an indirect payment is considered. This is true for both a customer without habit formation and weak habit formation. The best contract for a customer with weak habit formation, $\varphi = 0.5$ and $\xi = 0.0150$, is again better than the best contract in the indirect payment case. So, for the two different customers that we have discussed until now, the direct payment method is favored over the indirect payment method. The value to the customer does not necessarily increase with a higher minimum guaranteed rate. A higher minimum rate is accompanied by a higher direct payment and as explained before, if these values are close, the value to the customer decreases.

Table 12. Values for g for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	0.03983	0.04110	0.04118	0.04066	0.04000	0.03941	0.03889	0.03842	0.03803	0.03766	0.03732
0.0050	0.04580	0.04615	0.04627	0.04584	0.04533	0.04486	0.04440	0.04399	0.04363	0.04332	0.04305
0.0075	0.04959	0.04966	0.04970	0.04946	0.04910	0.04876	0.04841	0.04811	0.04787	0.04766	0.04748
0.0100	0.05260	0.05260	0.05260	0.05246	0.05222	0.05195	0.05171	0.05149	0.05129	0.05112	0.05096
0.0125	0.05528	0.05528	0.05527	0.05518	0.05505	0.05487	0.05468	0.05451	0.05435	0.05420	0.05406
0.0150	0.05784	0.05784	0.05784	0.05781	0.05773	0.05763	0.05751	0.05738	0.05727	0.05716	0.05704
0.0175	0.06035	0.06035	0.06035	0.06034	0.06031	0.06026	0.06019	0.06010	0.06001	0.05993	0.05985
0.0200	0.06285	0.06285	0.06285	0.06285	0.06284	0.06282	0.06279	0.06274	0.06267	0.06260	0.06254
0.0225	0.06535	0.06535	0.06535	0.06535	0.06535	0.06534	0.06533	0.06531	0.06529	0.06524	0.06519
0.0250	0.06785	0.06785	0.06785	0.06785	0.06785	0.06785	0.06784	0.06784	0.06783	0.06782	0.06779

The table presents the minimum rates of return for different variants of the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a weak form of habit formation is considered, i.e., $\alpha = 0.1, \beta = 0.5$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 13. Certainty equivalent surplus consumption values for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	3.9693	4.1114	4.1693	4.1851	4.1878	4.1876	4.1873	4.1869	4.1870	4.1870	4.1865
0.0050	4.1622	4.2079	4.2453	4.2518	4.2503	4.2485	4.2446	4.2407	4.2373	4.2348	4.2328
0.0075	4.2301	4.2462	4.2682	4.2748	4.2744	4.2722	4.2697	4.2679	4.2667	4.2653	4.2640
0.0100	4.2562	4.2621	4.2752	4.2812	4.2816	4.2801	4.2780	4.2762	4.2748	4.2739	4.2732
0.0125	4.2652	4.2678	4.2757	4.2804	4.2825	4.2823	4.2810	4.2793	4.2777	4.2766	4.2757
0.0150	4.2680	4.2690	4.2742	4.2790	4.2815	4.2827	4.2827	4.2818	4.2814	4.2805	4.2793
0.0175	4.2683	4.2687	4.2714	4.2753	4.2784	4.2806	4.2816	4.2815	4.2811	4.2808	4.2802
0.0200	4.2681	4.2684	4.2695	4.2722	4.2750	4.2775	4.2792	4.2801	4.2799	4.2794	4.2791
0.0225	4.2680	4.2682	4.2687	4.2700	4.2722	4.2741	4.2762	4.2774	4.2787	4.2788	4.2786
0.0250	4.2680	4.2681	4.2684	4.2689	4.2700	4.2715	4.2728	4.2747	4.2758	4.2768	4.2769

The table presents the certainty equivalent surplus consumption for different variants of the direct payment method and the minimum rates of return presented in table 12. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a weak form of habit formation is considered, i.e., $\alpha = 0.1, \beta = 0.5$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

The optimal consumption path is affected by the minimum rate guarantee that uses the direct payment method, while the optimal risky asset weight is not affected by the introduction of the guarantee. Figure 14 shows the altered consumption paths for the worst-case scenario (i.e. $\varphi = 0.0$ and $\xi = 0.0025$), the best-case scenario (i.e. $\varphi = 0.5$ and $\xi = 0.0150$), and the case without the guarantee. As we have learned from the previous cases, the consumption paths become smoother once a minimum guarantee is introduced. Furthermore, the worst-case scenario leads to a lower consumption path than the best-case scenario. Finally, the introduction of the guarantee causes a spike in consumption in the last period. In the worst case, this spike is larger than the spike for both the indirect payment case and the direct payment case for a customer without habit formation, while the spike is smaller in the best-case scenario.

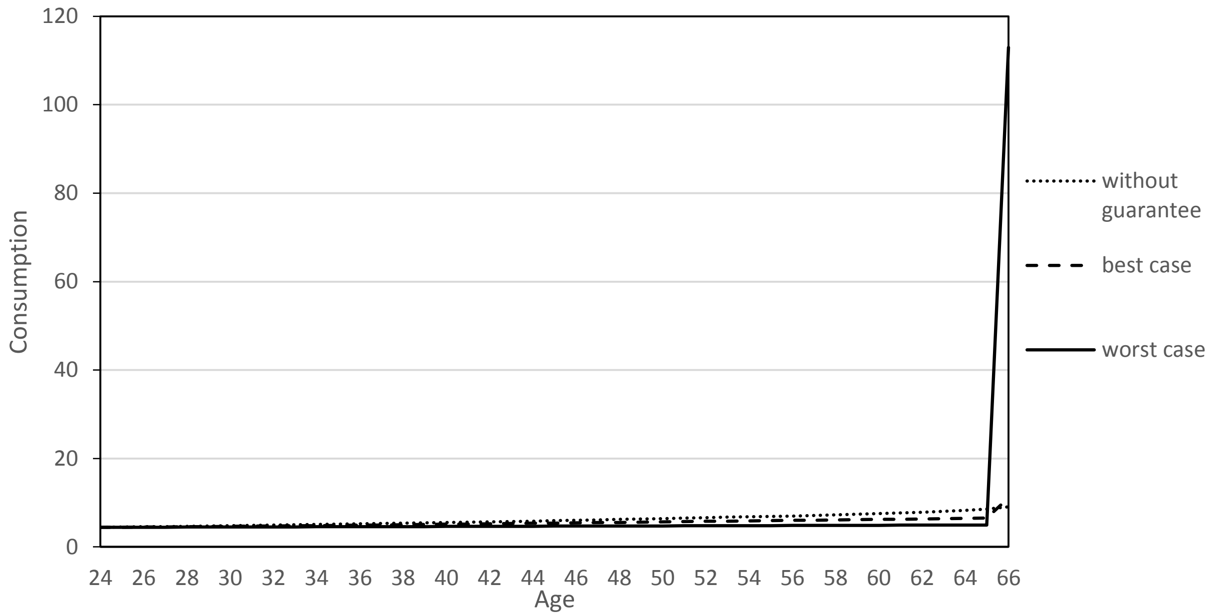

Figure 14. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer with a weaker form of habit formation and the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0.1$, $\beta = 0.5$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\xi = 0.0025$. The best case is the contract with parameters $\varphi = 0.5$ and $\xi = 0.0150$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

Without the guarantee, the customer with weak habit formation has a CESC of 4.1094. The worst contract for the customer leads to a CESC of 3.9693, which is a decrease of 3.41%. However, the worst contract is the only contract that is unfavorable for the customer, all other contract lead to an increase in the CESC-value. In the best-case scenario, the CESC increases with 4.22%. Again, the increase is lower for the customer with weak habit formation than for the customer without habit formation. So, a customer without habit formation gains relatively more from the introduction of the guarantee.

5.1.3 Optimal strategies for a customer with a strong form of habit formation

Here, we consider a customer with a stronger form of habit formation, i.e., $\alpha = 0.2$ and $\beta = 0.3$. The minimum guaranteed rate and the corresponding certainty equivalent surplus consumption for various indirect and direct contracts are presented in table 14 and 15. The optimal consumption paths are provided for the same cases that were considered in the previous sections. Again, it is found that the worst contract decreases the CESC. In the best-case scenario, the CESC increases by 7.51% for the indirect payment method, while the CESC increases by 7.57% in the direct payment case. The changes in the optimal consumption paths are similar to the changes in the previous sections. A more detailed description of the results is found below.

Table 14 presents the minimum rates of return that are offered by the company for different indirect contracts. Now that the habit formation is stronger, the allocation to risky assets will be even lower. As we can see from figure 1, the customer that is considered here, has the lowest optimal share in risky assets. As mentioned before, a lower weight in risky assets leads to a lower return on investments, which is why the company offers a lower minimum rate of return. As before, for low values of φ, an increase in φ leads to a higher minimum rate. For higher values of φ, an increase in φ lowers the minimum rate offered. As the share of distributed surplus to the company increases, i.e., ρ increases, the minimum guaranteed rate is larger.

Table 14. Values for g for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	0.03651	0.03847	0.03857	0.03778	0.03669	0.03566	0.03472	0.03396	0.03313	0.03243	–
0.2000	0.03853	0.03936	0.03956	0.03907	0.03828	0.03752	0.03679	0.03617	0.03558	–	–
0.3000	0.03936	0.03971	0.03988	0.03966	0.03914	0.03854	0.03795	0.03741	–	–	–
0.4000	0.03971	0.03989	0.04000	0.03998	0.03958	0.03908	0.03859	–	–	–	–
0.5000	0.03989	0.04000	0.04009	0.04014	0.03985	0.03942	–	–	–	–	–
0.6000	0.04000	0.04009	0.04015	0.04018	0.04006	–	–	–	–	–	–
0.7000	0.04009	0.04015	0.04018	0.04021	–	–	–	–	–	–	–
0.8000	0.04015	0.04018	0.04021	–	–	–	–	–	–	–	–
0.9000	0.04018	0.04021	–	–	–	–	–	–	–	–	–
1.0000	0.04021	–	–	–	–	–	–	–	–	–	–

The table presents the minimum rates of return for different variants of the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. Note that a customer with a strong form of habit formation is considered, i.e., $\alpha = 0.2, \beta = 0.3$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 15. Certainty equivalent surplus consumption values for different variants of the indirect payment method

ρ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1000	2.2408	2.2921	2.3092	2.3089	2.3029	2.2957	2.2889	2.2824	2.2758	2.2684	–
0.2000	2.2921	2.3118	2.3206	2.3230	2.3206	2.3172	2.3131	2.3097	2.3057	–	–
0.3000	2.3118	2.3194	2.3233	2.3267	2.3274	2.3259	2.3236	2.3211	–	–	–
0.4000	2.3194	2.3231	2.3251	2.3275	2.3295	2.3293	2.3282	–	–	–	–
0.5000	2.3231	2.3251	2.3266	2.3273	2.3295	2.3302	–	–	–	–	–
0.6000	2.3251	2.3266	2.3275	2.3278	2.3288	–	–	–	–	–	–
0.7000	2.3266	2.3275	2.3278	2.3281	–	–	–	–	–	–	–
0.8000	2.3275	2.3278	2.3281	–	–	–	–	–	–	–	–
0.9000	2.3278	2.3281	–	–	–	–	–	–	–	–	–
1.0000	2.3281	–	–	–	–	–	–	–	–	–	–

The table presents the certainty equivalent surplus consumption for different variants of the indirect payment method and the minimum rates of return presented in table 14. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a strong form of habit formation is considered, i.e., $\alpha = 0.2, \beta = 0.3$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 15 shows the CESC-values for different contracts using the indirect payment method corresponding to the minimum rates presented by table 14. The worst contract for the customer is the one with the parameters $\varphi = 0.0$ and $\rho = 0.1$, which is the same for the three different customers that we discussed. The most valued contract for a customer with a stronger form of habit formation is the contract with $\varphi =$

0.5 and $\rho = 0.5$. Note that the difference in value between the worst contract and the best contract is relatively small for a customer with stronger habit formation. For the customer with stronger habit formation, the best contract leads to a 3.99% higher CESC compared to the worst contract, while this is 4.98% for the customer with weaker habit formation, and 5.40% for the customer without habit formation. The changes in the CESC for different contracts, are similar to the other cases.

The optimal consumption choice changes in the same way as with the other type of customers. Figure 15 shows this optimal consumption choice for the three following cases: The worst case (i.e. $\varphi = 0.0$ and $\rho = 0.1$), the best case (i.e. $\varphi = 0.5$ and $\rho = 0.5$), and the case without the guarantee. The consumption paths change in the same way as they did for the other customers: The consumption paths are smoother, with the consumption path in the worst case below the consumption path in the best case, and a spike in the last period. Theses spikes in the last period are relatively smaller for this customer than for the other two customers. Notice that the consumption paths are steeper than in the other two cases as was previously shown in figure 2.

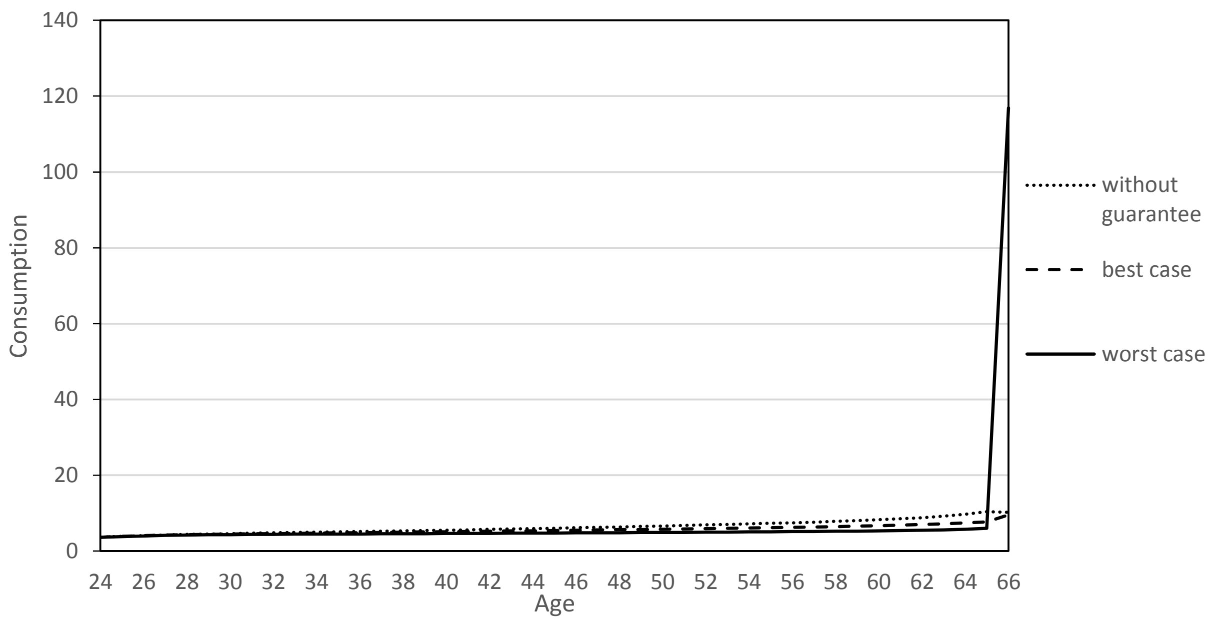

Figure 15. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer with a stronger form habit formation and the indirect payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0.2$, $\beta = 0.3$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\xi = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\rho = 0.1$. The best case is the contract with parameters $\varphi = 0.5$ and $\rho = 0.5$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

In the worst-case scenario, the customer has a CESC of 2.2408, which is higher than in the case without the guarantee, where the CESC is 2.1675. So, in the worst-case scenario, the CESC increases with 3.38%, which is much better than the worst cases for the other customers, where the CESC decreases if the customer agrees to the worst contract. The best contract increases the CESC with 7.51%, from 2.1675 to 2.3302. So, for the indirect payment method, the minimum rate guarantee is most favorable for the customer with the strong form of habit formation.

Now we consider the direct payment case for the customer with the strong form of habit formation. Table 16 present the minimum rates that are offered by the company for different direct payment contracts. The changes in the minimum rates for the different contracts are similar to the changes for the other customers. Note that the minimum returns are slightly lower for this case of stronger habit formation, which is for the same reason as explained before.

Table 16. Values for g for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	0.03876	0.03945	0.03961	0.03936	0.03896	0.03856	0.03818	0.03785	0.03757	0.03733	0.03710
0.0050	0.04395	0.04410	0.04421	0.04399	0.04366	0.04332	0.04297	0.04270	0.04246	0.04224	0.04205
0.0075	0.04740	0.04742	0.04747	0.04733	0.04708	0.04684	0.04660	0.04638	0.04619	0.04603	0.04590
0.0100	0.05025	0.05025	0.05025	0.05017	0.05000	0.04981	0.04963	0.04948	0.04933	0.04921	0.04908
0.0125	0.05286	0.05286	0.05286	0.05282	0.05273	0.05261	0.05248	0.05236	0.05224	0.05213	0.05203
0.0150	0.05538	0.05538	0.05538	0.05538	0.05534	0.05527	0.05519	0.05510	0.05501	0.05493	0.05486
0.0175	0.05788	0.05788	0.05788	0.05788	0.05787	0.05785	0.05780	0.05774	0.05767	0.05761	0.05755
0.0200	0.06038	0.06038	0.06038	0.06038	0.06038	0.06038	0.06036	0.06034	0.06029	0.06024	0.06019
0.0225	0.06288	0.06288	0.06288	0.06288	0.06288	0.06288	0.06288	0.06287	0.06286	0.06283	0.06279
0.0250	0.06538	0.06538	0.06538	0.06538	0.06538	0.06538	0.06538	0.06538	0.06537	0.06537	0.06536

The table presents the minimum rates of return for different variants of the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a strong form of habit formation is considered, i.e., $\alpha = 0.2, \beta = 0.3$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

Table 17 presents the CESC-values for the different contracts for the minimum rates presented in table 16. Again, the worst contract in the direct payment case, $\varphi = 0.0$ and $\xi = 0.0025$, is slightly worse than the worst contract in the indirect payment case, while the best contract in the direct payment case, $\varphi = 0.3$ and $\xi = 0.0100$, is better than the best contract in the indirect payment case. Note that the difference between the best contract in the direct payment case and the indirect payment case is considerably lower for this customer than for the other two, the same holds for the worst contracts.

Table 17. Certainty equivalent surplus consumption values for different variants of the direct payment method

ξ	φ										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0025	2.2301	2.2611	2.2827	2.2915	2.2943	2.2955	2.2950	2.2945	2.2939	2.2935	2.2932
0.0050	2.2968	2.3049	2.3171	2.3210	2.3214	2.3211	2.3197	2.3188	2.3179	2.3170	2.3162
0.0075	2.3185	2.3208	2.3274	2.3301	2.3303	2.3297	2.3290	2.3286	2.3280	2.3276	2.3274
0.0100	2.3254	2.3261	2.3294	2.3316	2.3315	2.3313	2.3305	2.3302	2.3298	2.3298	2.3296
0.0125	2.3267	2.3270	2.3288	2.3305	2.3314	2.3315	2.3313	2.3310	2.3305	2.3304	2.3303
0.0150	2.3261	2.3262	2.3272	2.3290	2.3302	2.3307	2.3310	2.3309	2.3308	2.3306	2.3307
0.0175	2.3254	2.3255	2.3259	2.3270	2.3283	2.3295	2.3299	2.3302	2.3300	2.3300	2.3299
0.0200	2.3250	2.3251	2.3252	2.3258	2.3268	2.3280	2.3287	2.3294	2.3296	2.3296	2.3294
0.0225	2.3248	2.3248	2.3249	2.3252	2.3257	2.3265	2.3274	2.3281	2.3286	2.3288	2.3288
0.0250	2.3248	2.3248	2.3248	2.3250	2.3252	2.3256	2.3261	2.3267	2.3271	2.3277	2.3280

The table presents the certainty equivalent surplus consumption for different variants of the direct payment method and the minimum rates of return presented in table 16. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0$, $\beta = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. Note that a customer with a strong form of habit formation is considered, i.e., $\alpha = 0.2, \beta = 0.3$. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

As in the previous cases, the optimal investment choice is not affected by the minimum rate of return guarantee and the consumption choice changes in a similar fashion. The consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee are presented in figure 16. As mentioned, the changes are similar to the other cases. In the worst case, the spike in the last period is relatively smaller for this customer. In the best case, the spike is smaller for this customer than for the customer without habit formation both in absolute and relative terms, while it is larger than for the customer with a weak form of habit formation.

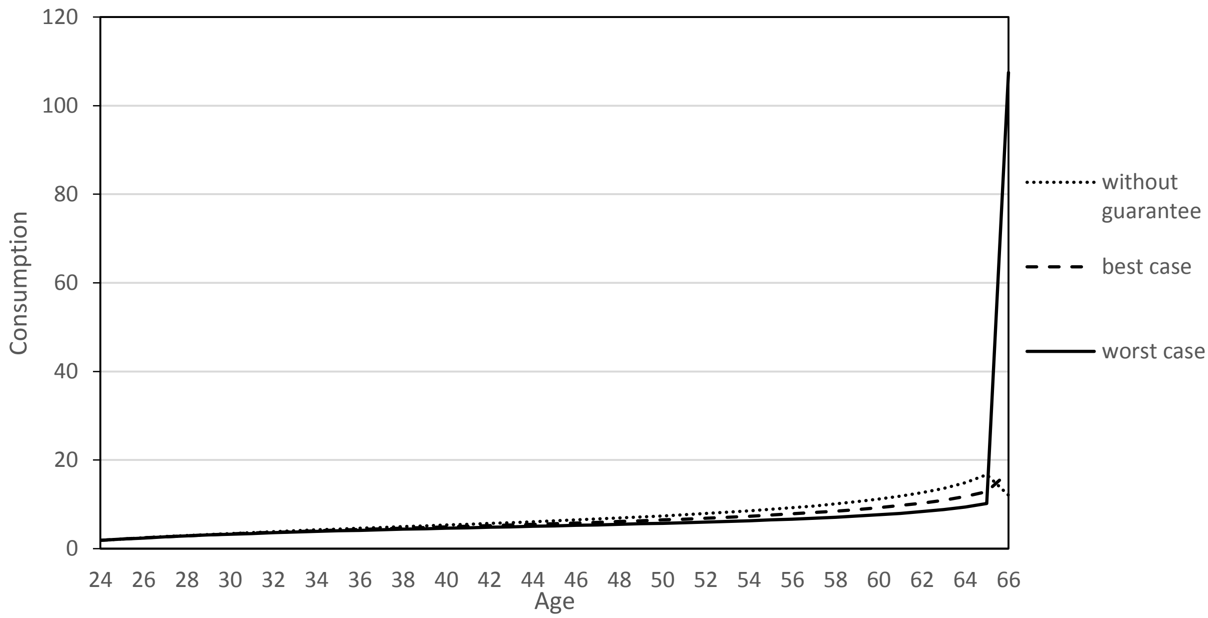

Figure 16. Optimal consumption paths for the worst-case scenario, the best-case scenario, and the case without the guarantee for a customer with a stronger form of habit formation and the direct payment method. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\gamma = 5$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $\alpha = 0.2$, $\beta = 0.3$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, $\rho = 0$, and $r = 0.03$. The worst case is the contract with parameters $\varphi = 0.0$ and $\xi = 0.0025$. The best case is the contract with parameters $\varphi = 0.3$ and $\xi = 0.0100$. Note that the spike in consumption is caused by the payout of the bonus reserve in the last period. Furthermore, note that the results presented in this figure are averages over the total number of simulations (1000).

As in the case of an indirect payment, the CESC is higher in both the worst case and the best case. In the worst case, the CESC increases with 2.89%, while the CESC increases with 7.57% in the best case. So, introducing a minimum rate of return guarantee is relatively most favorable for the customer with a stronger form of habit formation when compared to customers with either a weak form of habit formation or no habit formation.

Thus, for the basic customers with different habit parameters α and β, introducing a minimum rate guarantee leads to an increase in the certainty equivalent surplus consumption, except for the worst contracts for the customer without habit formation and the customer with the weak form of habit formation. Furthermore, note that the worst contracts are the same for all three types of customers, while the best contracts differ in terms.

5.2 Optimal strategies with a minimum rate of return guarantee for alternative parameters

Now that we know that a minimum rate of return guarantee is favorable in almost all cases for the basic customers with the different habit parameters, it is time to consider some alternative customers with the same habit parameters. As before, the following parameters are changed: The risk aversion coefficient γ, the constant risk-free interest rate r, the Sharpe-ratio λ, and the initial habit level h_0. The tables below will present the minimum guaranteed rate and the corresponding CESC for both the worst and the best case, the direct and the indirect method, and for the three different types of customer considered earlier in this section. The optimal investment choice will not be discussed since the introduction of the minimum rate guarantee does not affect this choice and therefore it will not alter the results when compared to the case without the guarantee. Also, the consumption choices will not be discussed since the changes will be similar to the changes discussed in section 4.2.1-4.2.4, but then for the case with the minimum guarantee. In order to determine if the minimum guarantee is valued, a comparison will be made between the minimum rates that are offered and the corresponding CESC-values.

Below, the parameters in the worst case are the parameters that on average lead to the lowest CESC for the three basic customers discussed above. Thus, it might be that the worst case as discussed below is not actually the worst case; however, it is extremely likely that the considered worst case is actually the worst case. The best case is determined in a similar fashion and is therefore also not necessarily equal to the actual best case. The worst and the best case are chosen in this way to avoid superfluous results.

The remaining part of this section is structured in the following way. Section 5.2.1 presents the results for alternative values of the risk aversion coefficient, section 5.2.2 considers changes in the risk-free interest rate, section 5.2.3 provides the results for alternative Sharpe-ratios, and section 5.2.4 shows the outcomes for different initial habit levels. These results are compared to the results in section 4.2.1-4.2.4 in order to determine the value of the minimum rate of return guarantee.

5.2.1 Alternative risk aversion levels

This section considers alterations in the risk aversion coefficient and studies the effects of these alterations. We find that a lower risk aversion coefficient leads on average to a higher certainty equivalent surplus consumption, while a higher risk aversion level lowers the CESC. If the risk aversion coefficient decreases to $\gamma = 2$, the best-case scenario increases the CESC most for a customer without habit formation (15.71%), while the CESC increases least for a customer with strong habit formation (6.55%). If the risk aversion level increases to $\gamma = 10$, the best-case scenario increases the CESC most for a customer with strong habit formation (5.54%), while it increases least for the customer with weak habit formation (0.51%). Below, the results will be discussed in more detail.

Two alterations are considered: a drop in the risk aversion parameter to $\gamma = 2$ and an increase in the risk aversion parameter to $\gamma = 10$. Table 18 presents values for the minimum rate of return and the CESC for the alternative risk aversion levels and both the direct and indirect payment method for the three types of customers considered earlier in this section.

If the risk aversion level drops to $\gamma = 2$, we see that in the case of an indirect payment to the company, the CESC is higher in both the worst case and the best case for all three types of customers when compared to the CESC in the case without the guarantee, except for the customer with strong habit formation if the worst contract terms are offered. This means that it is optimal in almost all cases to invest in such a guarantee if there is an indirect payment to the company. Furthermore, it is striking that the minimum rate guaranteed in the worst case is now lower than the minimum rate offered in the best case since it is the other way around when $\gamma = 5$. In addition, the minimum rates are now much higher due to the increased optimal risky asset weight, which increases the expected return on investments, and thus the payment to the company, allowing them to offer a higher rate. In the case of a direct payment, we see that the minimum rates have increased and that the difference in the minimum rates between the worst and the best case is now larger. Moreover, we see that the worst contract in the direct payment case is much worse than the worst contract in the indirect payment case. On top of this, the worst contract leads to a considerably lower CESC than in the case without the guarantee, making it unfavorable for the customer to invest in the guarantee. On the other hand, in the best-case scenario, the CESC is only slightly lower than in the indirect payment case for all three customers. Since we know that the worst case is far worse than all other contracts, it is very likely that the customer should buy the guarantee in all cases except in the worst case when the direct payment method is considered. In the best-case scenario, the CESC increases by 15.71% for the customer without habit formation, by 13.29% for the customer with the weak form of habit formation, and by 6.55% for the customer with the strong form of habit formation. The guarantee is thus relatively most valuable to the customer without habit formation.

Table 18. Values for g and CESC for alternative risk aversion levels and different contracts.

		γ=2		γ=10	
α	β	Worst case	Best case	Worst case	Best case
Indirect payment		ρ=0.1 & φ=0	ρ=0.5 & φ=0.5	ρ=0.1 & φ=0	ρ=0.5 & φ=0.5
0	0	0.06002	0.05859	0.03347	0.03576
		6.6592	7.0216	4.3233	4.4961
0.1	0.5	0.05764	0.05654	0.03301	0.03535
		5.5501	5.8858	3.6820	3.7978
0.2	0.3	0.05104	0.05088	0.03187	0.03427
		3.0094	3.2255	1.5709	1.8023
Direct payment		ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4	ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4
0	0	0.04435	0.06822	0.03717	0.04870
		5.0210	6.9551	4.4080	4.5072
0.1	0.5	0.04226	0.06619	0.03686	0.04831
		4.2050	5.8434	3.7460	3.8088
0.2	0.3	0.04021	0.06114	0.03625	0.04725
		2.4492	3.2185	1.6480	1.6264

The table presents the minimum rate of return and the certainty equivalent consumption level for different contracts and different risk aversion levels. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\sigma = 0.2$, $\lambda = 0.3$, $h_0 = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, $X_0 = 100$, and $r = 0.03$. Note that the results are presented for three different types of customers. Further, note that the worst-case scenario is defined as the average worst case for the three customers with the basic assumptions. This means that by definition, the worst case does not have to be the absolute worst case; however, this will be true in most cases. The best case is determined in a similar way. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

If the risk aversion level increases to $\gamma = 10$, we see that for the indirect payment case, the minimum rates of return decrease. This is caused by a lower optimal weight in risky assets, which lowers the expected return on investments and therefore lowering the payment to the company. When looking at the CESC, we see that the worst case lowers the CESC for all three customers, while the best case increases the CESC. For the direct payment case, the worst case still leads to a lower CESC than for the case without the guarantee, making it unfavorable to invest in the guarantee. However, in the best-case scenario, the CESC is higher, making the guarantee interesting for the customers with weak habit formation and without habit formation. For the customer with strong habit formation, the guarantee lowers the CESC in both the worst and the best case, making the guarantee unfavorable when there is a direct payment. Furthermore, note that the worst-case scenario actually leads to a higher CESC than the best case, which is due to the definition of these cases as explained before. So, when the risk aversion level increases, the guarantee becomes less attractive; however, in the best-case scenario the guarantee might still improve the CESC. In the best case, the CESC is increased by 0.72% for the customer without habit formation, by 0.51% for the customer with weak habit formation, and by 5.54% for the customer with the strong form of habit formation. Therefore, the guarantee is relatively most valuable to the customer with the strong form of habit formation.

5.2.2 Alternative risk-free interest rates

This section considers alterations in the risk-free interest rate and studies the effects of these changes. The results show that a decrease in the interest rate leads to a lower certainty equivalent surplus consumption, while an increase in the risk-free rate leads to an increase in the CESC. If the risk-free rate is lowered to $r = 0.01$, the best-case scenario increases the CESC most for a customer with strong habit formation (4.85%), while the CESC increases least for a customer with weak habit formation (2.96%). If the interest rate increases to $r = 0.05$, the best-case scenario increases the CESC most for a customer without habit formation (6.38%), while it increases least for the customer with strong habit formation (3.19%). The remainder of this section will discuss the results in more detail.

We consider a drop in the risk-free rate to $r = 0.01$ and an increase in the interest rate to $r = 0.05$. Table 19 presents values for the minimum rate of return and the CESC for the alternative risk-free rates for three different types of customers and both the direct and indirect payment method.

Table 19. Values for g and CESC for risk-free rats and different contracts.

		r=0,01		r=0,05	
α	β	Worst case	Best case	Worst case	Best case
Indirect payment		ρ=0.1 & φ=0	ρ=0.5 & φ=0.5	ρ=0.1 & φ=0	ρ=0.5 & φ=0.5
0	0	0.01900	0.01877	0.06153	0.06483
		3.3810	3.5222	6.5649	6.9147
0.1	0.5	0.01833	0.01814	0.06036	0.06470
		2.8534	2.9659	5.5709	5.8433
0.2	0.3	0.01644	0.01672	0.05716	0.06164
		1.5079	1.5455	3.1942	3.3246
Direct payment		ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4	ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4
0	0	0.01778	0.03307	0.06377	0.07861
		3.1523	3.5330	6.5431	6.9376
0.1	0.5	0.01738	0.03246	0.06268	0.07750
		2.6888	2.9733	5.5591	5.8615
0.2	0.3	0.01698	0.03083	0.06068	0.07444
		1.4575	1.5400	3.2228	3.3315

The table presents the minimum rate of return and the certainty equivalent consumption level for different contracts and different risk-free interest rates. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\sigma = 0.2$, $\lambda = 0.3$, $\gamma = 5$, $h_0 = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, and $X_0 = 100$. Note that the results are presented for three different types of customers. Further, note that the worst-case scenario is defined as the average worst case for the three customers with the basic assumptions. This means that by definition, the worst case does not have to be the absolute worst case; however, this will be true in most cases. The best case is determined in a similar way. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

If the risk-free interest rate drops to 1%, the return on the investment portfolio will decrease, lowering both the minimum rates and the CESC levels. As table 19 shows, in the case of an indirect payment, the guarantee is valuable in the best-case scenario for both the customer without habit formation and the customer with the weak form of habit formation, while it increases the CESC in both the worst and best case for the customer with strong habit formation. In the case of a direct payment, the guarantee is valuable to all three types of customers in the best-case scenario, while it lowers the CESC in the worst cases. In the best-case scenario, the indirect payment method is favored by the customer with strong habit formation, while the other two customers favor the direct payment method. In the worst-case scenario, the indirect payment method leads to a higher CESC for all customers when compared to the worst-case scenario for the direct payment method. In the best case, the CESC increases by 3.93% for the customer without habit formation, by 2.96% for the customer with the weak form of habit formation, and by 4.85% for the customer with the strong form of habit formation. The guarantee is therefore most valued by the customer with the strong form of habit formation.

If the risk-free interest rate increases to 5%, the total expected return increases, leading to higher minimum rates and CESC levels. When there is an indirect payment to the company, the guarantee is favorable in both the worst and best case for the customer without habit formation. For the other two customers, the guarantee only increases the CESC in the best case. If we look at the direct payment case, we see that the results are similar: The guarantee is valuable in both cases for the customer without habit formation, while it is only valuable to the other two customers in the best case. In the best case, the CESC increases by 6.38% for the customer without habit formation, by 5.11% for the customer with the weak form of habit formation, and by 3.19% for the customer with strong habit formation. The guarantee is thus most valuable to the customer without habit formation in this case. Note that the indirect payment method is favored when the best case is considered.

5.2.3 Alternative Sharpe-ratios

This section considers changes in the Sharpe-ratio and studies the effects of these alterations. It is found that a decrease in the Sharpe-ratio leads to a lower CESC, while an increase in the Sharpe-ratio leads to a higher CESC. If the Sharpe-ratio decreases to $\lambda = 0.2$, the best-case scenario increases the CESC most for a customer without habit formation (4.73%), while the CESC increases the least for a customer with strong habit formation (3.39%). If the Sharpe-ratio increases to $\lambda = 0.4$, the best-case scenario increases the CESC most for a customer with strong habit formation (5.54%), while it increases the least for the customer with weak habit formation (0.51%). The results are explained in more detail below.

The two following alterations to the Sharpe-ratio are considered: A drop in the Sharpe-ratio to $\lambda = 0.2$ corresponding to an expected excess return of 4% and an increase in the Sharpe-ratio to $\lambda = 0.4$ corresponding to an expected excess return of 8%. Table 20 presents the minimum rates of return and the corresponding CESC values for the three different customers for both the indirect and direct payment method.

If the Sharpe-ratio drops to $\lambda = 0.2$, the expected excess return decreases, which lowers the total return on investments. This causes the CESC levels and the minimum rates to be lower. For the indirect payment case, the guarantee is valuable for all three customers when the best contract is offered. The worst contract is also valuable for the customer without habit formation and the customer with strong habit formation. In the case of a direct payment, the guarantee is valuable in both the best and worst case for all three customers. Note that the direct payment case is favored in both cases for all three customers, except for the customer with strong habit formation, who favors the indirect payment method in the best-case scenario. In the best-case scenario, the CESC increases by 4.73% for the customer without habit formation, by 3.87% for the customer with the weak form of habit formation, and by 3.39% for the customer with strong habit formation. So, a drop in the Sharpe-ratio causes the guarantee to be most valuable to the customer without habit formation.

Table 20. Values for g and CESC for alternative levels of the Sharpe-ratio and different contracts.

		λ=0,2		λ=0,4	
α	β	Worst case	Best case	Worst case	Best case
Indirect payment		ρ=0.1 & φ=0	ρ=0.5 & φ=0.5	ρ=0.1 & φ=0	ρ=0.5 & φ=0.5
0	0	0.03511	0.03773	0.04456	0.04618
		4.4492	4.6420	5.1616	5.5073
0.1	0.5	0.03453	0.03718	0.04339	0.04509
		3.7761	3.9257	4.3749	4.6460
0.2	0.3	0.03295	0.03575	0.03988	0.04212
		2.0920	2.1607	2.4100	2.5280
Direct payment		ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4	ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4
0	0	0.03826	0.05074	0.04351	0.05924
		4.4921	4.6561	4.8676	5.5296
0.1	0.5	0.03778	0.05020	0.04265	0.05818
		3.8147	3.9379	4.1673	4.6612
0.2	0.3	0.03698	0.04876	0.04106	0.05530
		2.1232	2.1597	2.3686	2.5266

The table presents the minimum rate of return and the certainty equivalent consumption level for different contracts and different Sharpe-ratios. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\sigma = 0.2$, $\gamma = 5$, $h_0 = 0$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, and $X_0 = 100$. Note that the results are presented for three different types of customers. Further, note that the worst-case scenario is defined as the average worst case for the three customers with the basic assumptions. This means that by definition, the worst case does not have to be the absolute worst case; however, this will be true in most cases. The best case is determined in a similar way. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

If the Sharpe-ratio increases to $\lambda = 0.4$, the total return increases, thereby increasing the minimum rates that can be offered and the CESC levels. In the indirect payment case, the guarantee is valuable to the customers in the best-case scenario; however, the customers are not interested in the guarantee if the worst contract is offered. The same holds for the direct payment case. The direct payment case is better for the customer without habit formation and the customer with the weak form of habit formation if the best contract is offered, while the indirect payment method is favored in all other cases. In the best case, the CESC increases by 2.46% for the customer without habit formation, by 1.53% for the customer with the weak form of habit formation, and by 3.22% for the customer with strong habit formation. The guarantee is thus most valuable to the customer with the strong form of habit formation.

5.2.4 Alternative initial habit levels

This section considers alternative initial habit levels and studies the effects of increases in this initial habit level. It is found that an increase in the initial habit level lowers the CESC for all habit customers. A larger increase in the initial habit level leads to the same effects with a larger magnitude. If the initial habit level increases to $h_0 = 1.5$, the best-case scenario increases the CESC most for a customer with strong habit formation (5.53%), while the CESC increases the least for a customer with weak habit formation (4.22%). If the initial habit level increases further to $h_0 = 3$, the best-case scenario increases the CESC most for a customer without habit formation (5.37%), while it increases the least for the customer with weak habit formation (4.22%). Below, the results are explained in more detail.

First, we assume that the habit level increases to $h_0 = 1.5$. Then, we assume that the habit level increases further to $h_0 = 3$. Table 21 presents the minimum rates and the CESC levels for the worst and the best contract, the direct and indirect payment method, and for the three different customers.

If the initial habit level increases to $h_0 = 1.5$, the habit customers will alter their consumption paths, while this does not affect the customer without habit formation. As we can see, the minimum rates and the

CESC-values do not change for the customer without habit formation. In the indirect payment case, the guarantee is valuable to both habit customers in the best-case scenario, while the contract is only valuable to the customer with strong habit formation in the worst-case scenario. As before, for the customer without habit formation, the guarantee is only valuable in the best case. The same holds for the direct payment method. Note that the direct payment method is favored by the customers in the best case, while the indirect payment method is favorable in the worst case. In the best case, the CESC increases by 5.37% for the customer without habit formation, by 4.22% for the customer with the weak form of habit formation, and by 5.53% for the customer with strong habit formation. The guarantee is relatively most valuable to the customer with the strong form of habit formation, but the difference with the customer without habit formation is only marginal.

Table 21. Values for g and CESC for alternative levels of the Sharpe-ratio and different contracts.

		h=1,5		h=3	
α	β	Worst case	Best case	Worst case	Best case
Indirect payment		ρ=0.1 & φ=0	ρ=0.5 & φ=0.5	ρ=0.1 & φ=0	ρ=0.5 & φ=0.5
0	0	0.03990	0.04249	0.03990	0.04249
		4.8030	5.0618	4.8030	5.0618
0.1	0.5	0.03903	0.04168	0.03907	0.04171
		3.9355	4.1319	3.8067	3.9965
0.2	0.3	0.03638	0.03927	0.03624	0.03913
		2.0397	2.1203	1.8378	1.9093
Direct payment		ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4	ξ=0.0025 & φ=0	ξ=0.0125 & φ=0.4
0	0	0.04047	0.05589	0.04047	0.05589
		4.6602	5.0827	4.6602	5.0827
0.1	0.5	0.03988	0.05508	0.03993	0.05511
		3.8440	4.1469	3.7184	4.0111
0.2	0.3	0.03879	0.05257	0.03881	0.05238
		2.0336	2.1214	1.8358	1.9103

The table presents the minimum rate of return and the certainty equivalent consumption level for different contracts and different Sharpe-ratios. The parameter values are as follows: $T = 42$, $\delta = 0.02$, $\sigma = 0.2$, $\lambda = 0.3$, $\gamma = 5$, $A_0 = 100$, $B_0 = 0$, $C_0 = 0$, and $X_0 = 100$. Note that the results are presented for three different types of customers. Further, note that the worst-case scenario is defined as the average worst case for the three customers with the basic assumptions. This means that by definition, the worst case does not have to be the absolute worst case; however this will be true in most cases. The best case is determined in a similar way. Furthermore, note that the results presented in this table are averages over the total number of simulations (1000).

If the initial habit level increases to $h_0 = 3$, the effects are similar to the previous case; however, the magnitude is larger. Again, the customer without habit formation is not affected by this change. In the indirect payment case, the guarantee is valuable to all customer in the best case, while the customer with the strong form of habit formation is the only customer that is interested in the worst case. In the direct payment case, the results are similar. As before, the indirect payment method is favored in the worst cases, while the direct payment method is favored in the best cases. In the best-case scenario, the CESC increases by 5.37% for the customer without habit formation, by 4.22% for the customer with the weak form of habit formation, and by 4.82% for the customer with strong habit formation. The guarantee is thus relatively most valuable to the customer without habit formation. As the initial habit level increases, the guarantee becomes less valuable to the habit customers. This effect is stronger for customers with a stronger form of habit formation.

6. Concluding remarks

This is the concluding section of this paper and consists of the following subsections. Section 6.1 will first summarize the methods and goals of this paper. Thereafter, this section sums up the most important conclusions and gives a recommendation about the minimum rate of return guarantee. Then, section 6.2 will discuss some interesting topics of future research.

6.1 Conclusion

This paper aimed at answering whether a minimum rate of return guarantee would be welfare improving for a customer whose preferences reflect habit formation. To answer this question, I used the model by Munk (2008) to determine the optimal consumption and investment policies for habit customers when investment opportunities are constant. The optimal policies derived from this model helped us to attach values to the consumption paths of the different customers. Using these optimal polices, I valued the optimal consumption paths for different settings and for customers with different strengths of habit formation. In order to value a minimum rate of return guarantee, I derived another model by combining the model by Munk (2008) and the model by Hansen and Miltersen (2002). This model derives the optimal consumption and investment choices for different habit customers when investment opportunities are constant and a minimum rate of return guarantee is introduced. This model determines the minimum guaranteed rate using the condition that the issuer of the guarantee has zero profits in a risk neutral world. I consider a guarantee in which the customer has an option on the final bonus reserve, which is paid out to the customer if it is positive at maturity. The customer pays for the guarantee by paying an annual fee to the issuer. Furthermore, the customer provides an initial deposit and is allowed to consume out of the customer's account. The first model allows us to calculate the value of the optimal consumption path in the case that a minimum rate of return guarantee is not available. The second model offers us a way to calculate the value of the optimal consumption path when the customer invests in a minimum rate of return guarantee. By comparing the simulation results of the two models, it is possible to determine whether a minimum rate of return guarantee is an interesting investment strategy for habit customers.

By simulating the model in which a minimum rate of return guarantee is unavailable, we find that habit customers are more conservative investors, i.e., habit customers allocate a smaller share to risky assets. This is caused by the fact that habit customers reserve some wealth in order to ensure that future consumption will meet the habit level. Furthermore, habit customers have a steeper optimal consumption path when compared to customers without habit formation. The results of the simulation are compared for three different customers: A customer without habit formation ($\alpha = 0, \beta = 0$), a customer with a weaker form of habit formation ($\alpha = 0.1, \beta = 0.5$), and a customer with a stronger form of habit formation ($\alpha = 0.2, \beta = 0.3$). I have checked whether the minimum rate of return guarantee was valuable for these three customers for a broad range of different settings. When the basic setting is considered, we find that a minimum rate of return guarantee is an interesting investment for all three customers as long as the issuer does not offer the worst contract terms. The minimum return guarantee offers a welfare increase of more than 4% for all three customers, with a welfare increase of 7.57% for a customer with a stronger form of habit formation. Furthermore, I consider different settings by changing one of the following four parameters: The risk aversion coefficient, the constant risk-free interest rate, the Sharpe-ratio, and the initial habit level. These settings lead to the conclusion that a minimum rate of return guarantee is always interesting for all three customers if the issuer offers the best contract, while the customers would not be interested in the guarantee if the worst contract terms are offered. The results are thus robust against changes in the four variables mentioned above.

As the results show, the minimum rate of return guarantee is an interesting investment strategy to consider for both customers with and without habit formation. However, the customer must pay attention to the contract that is offered since the worst contract terms lead to a welfare loss in most cases. Note that I have made a number of assumptions that might not reflect the real world. Therefore, if some assumptions are relaxed, it may be the case that the results change and that the guarantee will become more or less valuable to the different customers. However, in the settings that I have presented in this paper, investing in a minimum rate of return guarantee is a recommended strategy.

6.2 Topics of future research

This paper showed that investing in a minimum rate of return guarantee could increase the welfare for both customers with and without habit formation. However, there are some assumptions that might not be realistic. The next step would be to make the model more realistic by changing these assumptions. A possible extension to the model could be the introduction of mean reversion in stock returns. The empirical evidence on the subject of mean reversion in stock returns is mixed; however, some recent papers have found evidence in favor of the existence of mean reversion[8]. Munk (2008) provides a model that includes mean reversion in stock prices and relaxes the assumption of constant investment opportunities at the same time. In a slightly adapted form, this model could be used to add the mean reversion assumption to a model that includes a minimum rate of return guarantee.

Another possible extension to the model could be the relaxation of the assumption that the risk-free interest rate is constant, i.e., the model could include stochastic interest rates. Stochastic interest rates are widely used in the academic literature[9] and are more realistic. Munk and Sørensen (2010) argue that it is important to include stochastic interest rates since it allows an investor to distinguish between risk-free assets with different maturities. As in the case of mean reversion, Munk (2008) provides a model that includes stochastic interest rates. Furthermore, Munk (2008) derives a model that includes both mean reversion in stock returns and stochastic interest rates.

The model presented in this paper uses a so-called “difference-habit model’’, where the utility depends on the difference between the consumption and the habit level. It would be interesting to test whether a minimum rate of return guarantee is valuable if another habit model is used. A possibility would be the so-called “ratio habit model”, where the instantaneous utility depends on the ratio of consumption to the habit level. The instantaneous utility function would then be as follows: $u_t = (c_t/h_t)^{1-\gamma}/(1-\gamma)$. However, De Jong and Zhou (2014) argue that the ratio habit utility function affects the investment choice only marginally.

At last, the model presented in this paper does not include income and assumes that the customer is only endowed with initial wealth. A model that includes income would be somewhat more realistic. The inclusion of income in this model would lead to an unrealistic consumption path, which is why the model should be adapted in order to include income and lead to realistic results.

[8] Fama and French (1988); Poterba and Summers (1988); Balvers, Yangru, and Gilliland (2000); Gropp (2004); and Mukherji (2011) provide empirical evidence in favor of mean reversion, while Richardson and Stock (1989); Richardson (1993); Kim, Nelson, and Startz (1991); and McQueen (1992) doubt the existence of mean reversion in stock returns.

[9] See for example Sørensen, 1999; Brennan and Xia, 2000 and 2002; Munk, 2008; and Munk and Sørensen, 2010.

Appendix A: Calculation of the risk-neutral discounted profits

This appendix explains in more detail how the condition in equation (23) leads to the minimum rates calculated in the results section. First, it is important to know how $\varepsilon_{t+\Delta t}$ is distributed in the normal setting, which is as follows

$$\varepsilon_{t+\Delta t} = \begin{cases} +1 & with\,probability & P \\ -1 & with\,probability & 1-P \end{cases} = \begin{cases} +1 & with\,probability & \frac{1}{2} \\ -1 & with\,probability & \frac{1}{2} \end{cases},$$

where $\varepsilon_{t+\Delta t}$ is distributed such that it has mean 0 and variance 1. In order to calculate the profits of the issuer in a risk-neutral setting, $\varepsilon_{t+\Delta t}$ must be distributed according to risk-neutral probabilities. This means that

$$\varepsilon^{Q}_{t+\Delta t} = \begin{cases} +1 & with\,probability & Q \\ -1 & with\,probability & 1-Q \end{cases},$$

where Q is the risk-neutral probability of the stock earning the high return and $1-Q$ is the risk-neutral probability of the stock earning the low return. The risk-neutral probability is calculated using the following formula

$$Q = \frac{e^{r\Delta t} - d}{u - d}.$$

where d is the price of the stock when the return is low (i.e. $\varepsilon_{t+\Delta t} = -1$), u is the price of stock when the return is high (i.e. $\varepsilon_{t+\Delta t} = 1$), and it is assumed that the stock price equals 1 at the start of the period. u and d are then calculated as

$$u = 1 + \ (r\,\mathbf{1} + \sigma\lambda)\,\Delta t + \sigma\sqrt{\Delta t}$$

and

$$d = 1 + \ (r\,\mathbf{1} + \sigma\lambda)\,\Delta - \sigma\sqrt{\Delta t}.$$

Once we know the risk-neutral probabilities, it is possible to simulate the amounts in the accounts B_T and C_T in a risk-neutral setting using the following formulas

$$(A_t + C_t) = \left((A_{t-\Delta t} + C_{t-\Delta t}) - c_{t-\Delta t}\right)e^{\max\left\{g,\ln\left(1+(\varphi+\rho)\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t}+C_{t-\Delta t}}-\theta\right)\right)\right\}},$$

$$A_t = (A_{t-\Delta t} - c_{t-\Delta t})e^{\max\left\{g,\ln\left(1+\varphi\left(\frac{B_{t-\Delta t}}{A_{t-\Delta t}+C_{t-\Delta t}}-\theta\right)\right)\right\}-\xi},$$

$$C_t = (A_t + C_t) - A_t,$$

and

$$B_t = B_{t-\Delta t} + X_t - X_{t-\Delta t} - (A_t + C_t) + (A_{t-\Delta t} + C_{t-\Delta t}).$$

Note that these formulas have already been described in section 3.2. Once these values are simulated, the amount in account B at maturity can be written as

$$B_T = B_T^+ - B_T^-,$$

where B_T^+ is the amount in the bonus reserve at maturity if this amount is positive and B_T^- is the value of the bonus reserve deficit at maturity. Note that the value of B_T^- is a positive amount and that B_T^+ and B_T^- can never be positive at the same time. Once we know the value of C_T and B_T^-, it is easy to determine the profit of the company at maturity, which is

$$V_T = E^Q[C_T - B_T^-].$$

This value can be discounted at the risk-free rate since we are in a risk-neutral setting. The discounted value of the profits equals

$$V_0 = e^{r(T-1)} E^Q[C_T - B_T^-].$$

The discounted profits of the company can be averaged over all simulations. In this paper, the number of simulations used is 1000. The value of the minimum rate of return g is then determined by trial-and-error. The trial-and-error process is continued until we find the minimum rate with 5 decimals that leads to the abnormal profit closest to zero. So the minimum rate of return is determined such that

$$V_0 = e^{r(T-1)} E^Q[C_T - B_T^-] = 0.$$ [10]

The values for the minimum rate of return are in turn used to calculate the optimal strategies and the certainty equivalent surplus consumption for the different customers.

[10] Note that this value will not be exactly equal to zero.

References

Abel, A. B. (1990). Asset prices under habit formation and catching up with the Joneses. *The American Economic Review*, 38-42.

Abel, A. B. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, *43*(1), 3-33.

Bacinello, A. R. (2001). Fair pricing of life insurance participating policies with a minimum interest rate guaranteed. *Astin Bulletin*, *31*(02), 275-297.

Backus, D. K., Gregory, A. W., & Telmer, C. I. (1993). Accounting for forward rates in markets for foreign currency. *The Journal of Finance*, *48*(5), 1887-1908.

Balvers, R., Wu, Y., & Gilliland, E. (2000). Mean reversion across national stock markets and parametric contrarian investment strategies. *The Journal of Finance*, *55*(2), 745-772.

Boldrin, M., Christiano, L. J., & Fisher, J. D. (2001). Habit persistence, asset returns, and the business cycle. *American Economic Review*, 149-166.

Braun, P. A., Constantinides, G. M., & Ferson, W. E. (1993). Time nonseparability in aggregate consumption: International evidence. *European Economic Review*, *37*(5), 897-920.

Brennan, M. J., & Xia, Y. (2000). Stochastic interest rates and the bond-stock mix. *European Finance Review*, 4(2), 197-210.

Brennan, M. J., & Xia, Y. (2002). Dynamic asset allocation under inflation. *The journal of finance*, 57(3), 1201-1238.

Campbell, J. Y., & Cochrane, J. H. (1995). *By force of habit: A consumption-based explanation of aggregate stock market behavior* (No. w4995). National Bureau of Economic Research.

Campbell, J. Y., & Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of financial studies*, *1*(3), 195-228.

Constantinides, G. M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of political Economy*, 519-543.

De Jong, F. C. J. M., & Zhou, Y. (2014). Habit Formation: Implications for Pension Plans. *NETSPAR Design Papers*, *32*.

Dunn, K. B., & Singleton, K. J. (1986). Modeling the term structure of interest rates under non-separable utility and durability of goods. *Journal of Financial Economics*, *17*(1), 27-55.

Dynan, K. E. (2000). Habit formation in consumer preferences: Evidence from panel data. *American Economic Review*, 391-406.

Eichenbaum, M., & Hansen, L. P. (1990). Estimating models with intertemporal substitution using aggregate time series data. *Journal of Business & Economic Statistics*, *8*(1), 53-69.

Eichenbaum, M. S., Hansen, L. P., & Singleton, K. J. (1986). A Time Series Analysis of Representative Agent Models of Consumption andLeisure Choice Under Uncertainty.

Fama, E. F., & French, K. R. (1988). Dividend yields and expected stock returns. *Journal of financial economics*, *22*(1), 3-25.

Ferson, W. E., & Constantinides, G. M. (1991). Habit persistence and durability in aggregate consumption: Empirical tests. *Journal of Financial Economics*, *29*(2), 199-240.

Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, 367-390.

Gomes, F., & Michaelides, A. (2003). Portfolio choice with internal habit formation: A life-cycle model with uninsurable labor income risk. *Review of Economic Dynamics*, *6*(4), 729-766.

Grishchenko, O. V. (2010). Internal vs. external habit formation: the relative importance for asset pricing. *Journal of Economics and Business*, *62*(3), 176-194.

Gropp, J. (2004). Mean reversion of industry stock returns in the US, 1926–1998. *Journal of Empirical Finance*, *11*(4), 537-551.

Grosen, A., & Jørgensen, P. L. (2000). Fair valuation of life insurance liabilities: the impact of interest rate guarantees, surrender options, and bonus policies. *Insurance: Mathematics and Economics*, *26*(1), 37-57.

Hansen, M., & Miltersen, K. R. (2002). Minimum rate of return guarantees: the Danish case. *Scandinavian Actuarial Journal*, *2002*(4), 280-318.

Heaton, J. (1993). The interaction between time-nonseparable preferences and time aggregation. *Econometrica: Journal of the Econometric Society*, 353-385.

Heien, D., & Durham, C. (1991). A test of the habit formation hypothesis using household data. *The Review of Economics and Statistics*, 189-199.

Jensen, B., Jørgensen, P. L., & Grosen, A. (2001). A finite difference approach to the valuation of path dependent life insurance liabilities. *The Geneva Papers on Risk and Insurance Theory*, *26*(1), 57-84.

Kim, M. J., Nelson, C. R., & Startz, R. (1991). Mean reversion in stock prices? A reappraisal of the empirical evidence. *The Review of Economic Studies*, *58*(3), 515-528.

Korniotis, G. M. (2010). Estimating panel models with internal and external habit formation. *Journal of Business & Economic Statistics*, *28*(1), 145-158.

McQueen, G. (1992). Long-horizon mean-reverting stock prices revisited. *Journal of Financial and Quantitative Analysis*, *27*(1), 1-18.

Meghir, C., & Weber, G. (1996). Intertemporal nonseparability or borrowing restrictions? A disaggregate analysis using a US consumption panel. *Econometrica: Journal of the Econometric Society*, 1151-1181.

Miltersen, K. R., & Persson, S. A. (2003). Guaranteed investment contracts: distributed and undistributed excess return. *Scandinavian Actuarial Journal*, *2003*(4), 257-279.

Muellbauer, J. (1988). Habits, rationality and myopia in the life cycle consumption function. *Annales d'Economie et de Statistique*, 47-70.

Mukherji, S. (2011). Are stock returns still mean-reverting?. *Review of Financial Economics*, *20*(1), 22-27.

Munk, C. (2008). Portfolio and consumption choice with stochastic investment opportunities and habit formation in preferences. *Journal of Economic Dynamics and Control*, *32*(11), 3560-3589.

Munk, C., & Sørensen, C. (2010). Dynamic asset allocation with stochastic income and interest rates. *Journal of Financial Economics*, *96*(3), 433-462.

Polkovnichenko, V. (2007). Life-cycle portfolio choice with additive habit formation preferences and uninsurable labor income risk. *Review of Financial Studies*, *20*(1), 83-124.

Poterba, J. M., & Summers, L. H. (1988). Mean reversion in stock prices: Evidence and implications. *Journal of financial economics*, *22*(1), 27-59.

Richardson, M. (1993). Temporary components of stock prices: A skeptic's view. *Journal of Business & Economic Statistics*, *11*(2), 199-207.

Richardson, M., & Stock, J. H. (1989). Drawing inferences from statistics based on multiyear asset returns. *Journal of Financial Economics*, *25*(2), 323-348.

Shore, S. H., & White, J. S. (2003). External habit formation and the home bias puzzle. Retrieved from Social Science Research Network website: http://ssrn.com/abstract=334943

Sørensen, C. (1999). Dynamic asset allocation and fixed income management. *Journal of financial and quantitative analysis, 34*(04), 513-531.

Sundaresan, S. M. (1989). Intertemporally dependent preferences and the volatility of consumption and wealth. *Review of financial Studies*, *2*(1), 73-89.

Verdelhan, A. (2010). A Habit-Based Explanation of the Exchange Rate Risk Premium. *The Journal of Finance*, *65*(1), 123-146.

Wachter, J. A. (2006). A consumption-based model of the term structure of interest rates. *Journal of Financial economics*, *79*(2), 365-399.

Winder, C. C., & Palm, F. C. (1989). Intertemporal consumer behaviour under structural changes in income. *Econometric reviews*, *8*(1), 1-87.