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Valuation of the Sponsor Support Option

A Practical Tool for Risk Management

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VALUATION OF THE SPONSOR SUPPORT OPTION

A practical tool for Risk Management

by

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Abstract

This thesis proposes a valuation method for the sponsor support option in defined benefit pension contracts, including an extensive implementation of sponsor's default probabilities. The value of sponsor support option gives a clear view on the economic conditions and explicitly values the real economic risks of both the pension fund and the underlying company. The results show that risky investment strategies and the sponsor's credit rating both have significant impact on the value of the sponsor support option, and vice versa. Besides that, not only the local characteristics of the pension fund and its underlying company themselves, but also broad market conditions affect the value of the sponsor support option. The current European debate about the introduction of real economic risks on pension fund's holistic balance sheets confirms the importance of sponsor support option valuation for pension fund's risk management. For regulatory purposes, this thesis concludes that the valuation of the sponsor support option may work counterproductive. Regulation based on false guarantees creates a downwards spiral that in the end will harm the pension's participants.

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1. Introduction

1.1 Problem Description

Occupational corporate defined benefit (DB) pension schemes provide guarantees to their participants. As the name DB indicates, employees know in advance what pension benefits to expect in the future while contributions to the pension fund can vary over time. Regulators impose minimum required funding ratios, so that the solvency of pension funds is secured and guarantees can be carried out. A funding ratio represents the current pension fund's asset value over the present value of its outstanding liabilities to participants. In case of underfunding (i.e. a funding ratio below the regulatory required ratio) one or more of the stakeholders in the pension deal need to intervene.

In the United Kingdom, the only stakeholder who is bearing the risk of underfunding is the employer, as it is obliged by the law (The Pension Regulator, 2004). Employers have to make an immediate remedial contribution in case of a deficit of their pension fund. This legally imposed guarantee is often referred to as parent guarantee, sponsor covenant or sponsor support. The frequency and size of the sponsor support can affect the financial conditions and continuation of sponsor.

From a financial perspective, the sponsor support can be interpreted as a put option. The option gives the owner (the pension fund) the right to exchange the underlying (the value of the assets in the fund) with the writer of the option (the employer) at a specific strike price (the value of the liabilities the pension fund has). Since the pension fund does not actually pay a premium for the option, the option is called "embedded" (Kocken 2008). The sponsor support option implicitly allows the pension fund to take additional risks, entirely borne by the employer, without actually paying a premium for it. Without the sponsor support option, the pension fund would have probably increased the employer's contribution rate (since the scheme is DB). With the sponsor support option, the premium the employer should have received for writing the option is implicitly received and virtually invested in the pension fund as a buffer.

Knowing the price of the sponsor support option is useful for various stakeholders in the pension deal. Firstly, since the price of the option is virtually invested in the pension fund as a buffer, it strengthens the solvability position of the fund. Current pension funds should not only be worried about regulatory funding requirements. Real economic risks, both now and in the future, should have the full attention of pension fund's boards. The value of the sponsor support option will be a useful instrument in managing these risks. A proposal for revision of the European pension regulation (IORP) introduces the holistic balance sheet (European Insurance and Occupational Pensions Authority [EIOPA], 2012). On this holistic balance sheet, several security mechanisms (such as the sponsor support option) are added to the assets and liabilities from the traditional balance sheet. The holistic balance sheet gives a better view on the economic conditions and explicitly values the economic risks of the pension fund. For that reason it could be useful in pension fund's risk management.

Secondly, where the sponsor support option can be interpreted as a buffer on the pension fund's balance sheet, it is an outstanding liability from the perspective of the employer. As UK law imposes, the risk of underfunding of the pension fund is entirely borne by the employer. However, a changing pension environment from defined benefit (DB) to defined contribution[1] (DC) schemes shows that employers do not want to bear this risk anymore (Broadbent, Palumbo & Woodman, 2006). In that case, the employer could decide to eliminate the guarantee it has given to the pension fund by means of a buy off. Understanding the value of the sponsor support option is helpful in the negotiations of the buy off price.

As the holistic balance sheet approach advocates for security mechanisms on the balance sheet of the pension fund, one could argue that the value of the sponsor support option should also be present on the balance sheet of the employer. Because of the fact that it is interpreted as an implicit liability, it would in fact affect the market value of the company. The sponsor support option value reflects the real economic risks the company faces and for that reason, decisions about mergers and acquisitions should take this value into account.

The third stakeholder that makes use of the sponsor support option value is the regulator (The Pension Regulator in the United Kingdom). Implementation of security mechanisms (such as the sponsor support option) on the pension fund's holistic balance sheet enables the regulator to adapt its regulatory policy to the real economic risks of the pension fund. However, regulation based on stakeholder's guarantees requires proper supervision on the governance of the pension fund, which otherwise could harm the pension's participants.

The applications show that a proper valuation method for the sponsor support option would make life much easier for some stakeholders in the pension deal. Despite the fact that several recent studies cover valuation methods to price the sponsor support option (Kocken, 2008; De Haan, Janssen & Ponds, 2012; Barrie & Hibbert, 2013) no leading methodology exists among European pension funds yet. In June 2013, EIOPA announced that, within the new European pension regulation, it will postpone the holistic balance sheet approach (Investments and Pension Europe [IPE], 2013). One of the main reasons for this is that no uniform methodology for the valuation of the sponsor support option is found yet, due to variety in pension contracts and haziness about sponsor's default probabilities.

[1] In defined contribution schemes the amount of the contribution is specified. Benefits are based on the investment earnings from these contributions.

The goal of this thesis is to develop a clear and understandable method for the valuation of the sponsor support option. Besides that, applications for the pension fund, the employer and the pension regulator will be studied. Sensitivity to uncertain parameters and assumptions will be tested to understand how the sponsor support option value is affected by policy decisions and vice versa.

1.2 Research Description

The focus of this thesis will be on pension schemes in the United Kingdom, since in the UK the guarantees the employer has to its pension fund are clear and firm. In other countries, for example the Netherlands, not only the employer is bearing the risk of underfunding. Possibilities to cut benefits and lower indexation in case of shortage of capital make the pension participants a bearer of the risk of underfunding too.

Many pension funds find a stochastic valuation approach (i.e. using Monte Carlo simulations) to value the sponsor support option complex and time consuming (EIOPA, 2013) and therefore use a deterministic approach. Stochastic and deterministic approaches are probabilistic counterparts: The first describes a process with many directions (randomness) and the second describes a process which can only evolve in one way (no randomness). This thesis will explain and apply the stochastic approach, since it is assumed to be the most general approach to calculate the market value of the sponsor support option (Kocken, 2008).

Another reason why the sponsor support option is still hard to value is credit risk. In traditional option pricing (Black & Scholes, 1973) it is assumed that both parties are credible and will meet the obligations if necessary. In reality however, the employer can default on the remedial contribution, depending on its own credit state and the broad market conditions. In a bad economic environment, the probability of underfunding of the pension fund will be higher and the economic condition of the employer will be weaker[2]. These two effects will results in an even higher default probability of the sponsor. In this thesis, default probabilities are calculated from historical default rates (related to credit rating) and credit default swap spreads.

Besides explaining a general approach to the valuation of the sponsor support option, this thesis will pay most attention to the applications for the pension fund board, the employer's (or corporate) board and the pension regulator.

Since the sponsor support option is interpreted as an implicit buffer for the pension fund, the pension fund board will adjust its strategy that results in the highest possible sponsor support option value. Increasing

[2] It is assumed that the company's economic condition is positively correlated with the broad market conditions.

the riskiness of the investment portfolio is such a strategy. This is due to the fact that deficits are more likely to occur when more risk is taken. The employer on the other hand, aims for the lowest possible sponsor support option, since it reflects the value of its guarantee which is an implicit liability. Bargaining about the investment strategy between the pension fund board and the corporate board will decide the eventual policy.

An employer who has a higher probability of default would only be able to give a smaller guarantee to the pension fund than more stable companies. The results show that for lower rated companies, the value of the sponsor support option significantly falls. Companies that are more (positive) correlated with the broad market show higher option values in good economic environments and lower option values in bad economic environments.

Whereas the value of the sponsor support option turns out to be a useful tool in risk management for the pension fund and the underlying company, implementation of the sponsor support option value on the holistic balance sheet does not benefit the pension regulator. Higher funding ratios under the holistic balance sheet approach (including sponsor support option) than under the traditional balance sheet approach (excluding sponsor support option) will result in softer funding ratio requirements. In that case, it will amplify the behaviour of the pension fund to increase the riskiness of its investment portfolio, lower pension premiums (decrease assets) or raise pension benefits (increase liabilities) up till a level that the pension fund " holistic" funding ratio (traditional balance sheet plus the sponsor support option in this case) exactly meets the minimum required funding ratio.

Then, the increased sponsor support option value can cause the sponsor to be downgraded, since it reflects an implicit liability of the sponsor. This downgrading decreases the sponsor support option value, so that there occurs a gap in the pension fund's balance sheet. A remedial contribution from the sponsor will be required, which can again downgrade the sponsor. Moreover, the previous downgrading could indicate that the sponsor had financial insufficiencies in the first place. The new remedial contribution can further downgrade the sponsor which will again result in a decrease in sponsor support option value. This situation can be interpreted as a downwards spiral up till a moment in time where the company's guarantee appears to be not credible anymore. Then, the softened regulation turns out to be a counterproductive instrument for regulatory purposes.

It should be mentioned that the holistic balance sheet approach as proposed by EIOPA is not based on a system with infinite guarantees provided by the employer and infinite decision power for the pension fund board. Proper governance of the pension fund is therefore necessary. Besides that, pension fund boards should be aware of the fact that policy decisions based on uncertain guarantees will not benefit the continuation of the pension fund.

1.3 Research Structure

In order to perform an ex ante valuation of the sponsor support option a stylised pension fund is introduced. The characteristics of this stylised pension fund that is studied need to be defined. Chapter 2 contains a general description of DB pension funds in the United Kingdom and defines the stylised version that will be used in this thesis. Initial characteristics of the studied pension fund are formed in this chapter. The actual value of the sponsor support option depends on the values of assets and liabilities at each specific moment. Chapter 3 introduces a cash flow model that technically illustrates how assets and liabilities evolve over time. If assets and liabilities are valued for each period, the value of sponsor support option can be calculated. Chapter 4 explains the methodology that is used to do these calculations. It will include the sponsor's credit state and sponsor default probabilities. Due to the uncertainty of variables, a pure analytical solution is hard to find. For that reason the Monte Carlo simulation approach will be used to make the calculations. Chapter 5 will calibrate stochastic models for the instantaneous short term interest rate, the interest rate term structure and equity returns which are calculated in Chapter 5 can thereafter be inserted in the cash flow model from Chapter 3 and the option pricing model from Chapter 4. Chapter 6 will show the results that come out of the simulations. Sensitivity analyses on investment strategy, credit ratings, correlation with the stock market index and market conditions will be performed in this chapter as well. In the end, traditional and holistic balance sheets are compared and regulatory usefulness is discussed. Besides studying a stylized fund, all methods (chapter 3 to 5) can be applied to an actual existing pension fund. Chapter 7 will show the valuation of the sponsor support option for British Airways Pension Fund[3].

2. Pension Fund Characteristics

This thesis makes calculations for both a stylised fictitious United Kingdom pension fund and an actual existing UK pension fund. Firstly, pension regulation in the UK will be explained in general. Secondly, the assumptions made for the stylised contract as studied in this thesis will be summed up. All necessary simplifications will be explained.

[3] The choice of British Airways Pension Fund was done based on publicly available data on the pension fund itself and the sponsor. The valuation methods can be used as a blueprint for most corporate DB pension funds.

2.1 UK Pensions

In the UK, employer guarantees are firm, which means that in the case of underfunding of the pension fund the employer is obliged to make a remedial contribution to the fund. Underfunding means that the market value of assets is less than the present value of the liabilities, or technical provisions.

The remedial contribution is calculated by means of a Recovery Plan, which is a contract between the pension fund and the underlying employer. The Recovery Plan has to be approved by the Pension Regulator thereafter. It is a plan that has a negotiated maturity and quotes for remedial contributions that the employer has to make to the fund. Other elements of the Recovery Plan can be indexation agreements or temporarily cuts in pension benefits. The aim of the plan is to eliminate the pension fund's deficit within the maturity of the plan. Most Recovery Plans impose the entire risk of underfunding on the employer.

The focus of this thesis is on the remedial contribution as the most important factor in eliminating the pension fund's deficit. Two extremes of a remedial contribution strategy can be distinguished: i) the softest strategy: a remedial contribution in case of inability to pay the immediate necessary pension benefits only; ii) the toughest strategy: a remedial contribution equal to the deficit, starting immediately in case of underfunding. All other strategies lie between these two extremes. The contribution guarantee is firm, which implies that a bankruptcy of the firm would be the only way to default on this obligation. In reality however, it will be desirable to avoid such a situation in time.

Instead, three possible scenarios can be drawn if the employer would not be able to close the gap. Firstly, underfunded pension funds can be placed under the Pension Protection Fund (PPF). This fund can be seen as a mutual insurance fund from all UK pension funds together. Secondly, the pension fund can negotiate with the Pension Regulator to temporarily lower the remedial contribution under strict supervision. This would lengthen the time period in which the gap should be closed. Thirdly, the pension fund can decide to take over the underlying company.

A recent trend in the UK and other European countries is that stakeholders want to eliminate the risk of underfunding. This results in a shift from defined benefit (DB) to defined contribution (DC) schemes. Most traditional DB pension schemes in the UK are therefore closed. Two types of closure exist.

1) Closed for new members. Existing participants continue to contribute and accrue benefits after the closure. New member will enter a new scheme.
2) Closed for contributions and accruals. Existing and new members will contribute and accrue benefits in a new scheme. This type of closure can be interpreted as a liquidation of the pension scheme.

Where the first type is more common among UK pension funds, a clear shift can be seen to the second type in recent years and in the near future (Pension Protection Fund & The Pensions Regulator, 2012). From a practical perspective a closure of a pension fund makes the principle of sponsor support option valuation logical to understand. One of the applications of the valuation of the sponsor support option could be a buy off of the guarantee by the employer. Substandard pension fund portfolio returns (in for example low interest rate environments) could make the employer willing to eliminate the risk of underfunding. The value of the sponsor support option after the closure could then be useful in the negotiation process to find a price for the deal.

Pension funds in the UK differ from each other in terms of their asset allocation. In 2012, the average asset allocation was 43.7% in stocks, 36.1% in inflation linked UK Government Bonds (Gilts) and fixed interest and the remaining 20.2% in real estate, cash, insurance policies and other risky assets (Pension Protection Fund & The Pensions Regulator, 2012).

Liabilities of DB pension funds in the UK are by law discounted with the Gilt rate plus a certain percentage which can vary up to 2% (The National Association of Pension Funds Limited [NPAF], 2013). Indexation is performed against the Limited Price Indexation (LPI). The LPI is the Retail Prices Index (RPI) capped at a certain percentage. Since the implementation of the Pension Act 2004 (The Pension Regulator, 2004), the LPI is the RPI capped at 2.5%. This means that indexation of pension benefits is linked to the price inflation up to a maximum of 2.5%.

2.2 Stylised Pension Fund

The pension fund that is studied in this thesis is a simplistic version of an average UK DB pension fund as explained before. The next paragraphs will sum up the characteristics in terms of both liabilities and assets.

2.2.1 Liabilities

- The studied pension fund refers to one company only.
- The initial amount of participants in the fund is 100,000. Exact demographics of the initial pension fund population can be found in Appendix A.1.
- It is assumed that the fund is a representation of the total UK population. Late entries and early exits are neglected and all participants are expected to start working at age 25 and retire at age 65.
- It is assumed that people live at maximum 99 years. Survival probabilities can be found in Appendix A.2 and are assumed to be constant for the entire horizon of the fund. If someone deceases before age 99 the remaining accrued benefits stay within the fund, so no bequests are given.

- Benefits are accrued at a constant rate per working year of 1/60 of final wages. The maximum accrued benefit is therefore 40/60 of the final wage.
- Average final wages in 2013 are assumed to be 40,000 UK£ per year.
- Pension benefits are indexed against wage inflation which is capped at 2.5% (LPI).
- In case of overfunding no contribution holidays are given.
- Liabilities are discounted with the Gilt rate (UK Government bonds) plus 1%, as this is the average rate in the United Kingdom (NAPF, 2013).
- The pension fund is assumed to be fictitiously closed at the initial time period according to the first type of closure. This means that no new participants can enter the fund. Initial participants continue to contribute and accrue benefits afterwards.

2.2.2 Assets

- The initial asset value is equal to the initial present value of the liabilities (technical provisions), so the initial funding ratio is 100%.
- Contribution rate (employer + employee) is 20% of average wages.
- Assets are invested in an investment portfolio which consists of two types of assets only: stocks/equity (60%) and (risk-free) inflation linked bonds/Gilts (40%). Inflation linked bonds are hold to hedge against inflation risk. Stocks are hold to achieve additional return from investments.
- Rebalancing of the portfolio is performed at a yearly frequency.
- The return from stocks/equity is based on the return from the FTSE 100 index.
- The return from inflation linked bonds is based on short rates from the same yield curve of Gilts that is used for discounting the liabilities.
- The minimum required funding ratio is 1 (or 100%). If the asset value is lower than the value of discounted liabilities at a certain moment in time the fund is underfunded.
- The risk of underfunding is totally borne by the employer. The toughest possible contribution strategy is adopted, which means that in case of underfunding of the pension fund the employer has to make a direct remedial contribution equal to the deficit.
- Valuation is performed at annual frequency, at the end of each year.

3. Pension Fund Cashflow model

Sponsor guarantees or sponsor support options in pension contract arise from probable deficits in the pension funds. Such deficits depend on the value of assets and liabilities at a specific moment. This chapter describes the model that is used to calculate the values and evolution of assets and liabilities through time. With the information from this model as input, Chapter 4 will explain the option pricing method used to value the sponsor support option.

Asset and Liability Management (ALM) for pension funds is a standard modelling approach which takes into account assets, liabilities and different policies the board of the pension fund can apply. A traditional balance sheet of a pension fund consists of assets A_t^s and liabilities in present value (technical provisions) L_t^s. The difference between assets and liabilities is either a surplus or a deficit and the ratio of assets over liabilities is the funding ratio $F_t^s = A_t^s / L_t^s$.

Assets are composed of contributions made by the active participants minus benefits paid to retired participants and are invested in an investment portfolio with a risk-return profile determined by its composition. The initial liabilities are calculated as the present value of the total accrued benefits at time $t = 0$. A planning horizon of T with annual observations is assumed in the ALM modelling and this annual time steps are denoted by subscript t. Time $t = 0$ is the current time (2013) and $t = T$ is the length of the horizon: $t \in T \coloneqq \{0, \dots, T\}$. The uncertainty of the stochastic scenario sets (or evolutionary uncertainty) is modelled through a large but finite number of stochastic scenarios S which is denoted by a superscript $s \in S \coloneqq \{1, \dots, S\}$. Chapter 5 will elaborate more on the calibration of these stochastic scenarios.

3.1 Liabilities

By using age (number of worked years) the accrued benefits per worker can be calculated. Accrued benefits should be updated with wage inflation each year and the present value of liabilities is found by discounting these accruals with a discount factor. In order to model the liabilities a simple cash flow model is constructed. This cash flow model is based on the paper by De Haan, Janssen and Ponds (2012), who apply a similar study, and modified for the purpose of this thesis.

It is assumed that an employee works from age 25 until age 65 and dies at latest at an age of 99. The fund is closed for new member so no participants are younger than 25. The number of employees in cohort $Pop_{x,t}$ with age x at time t is calculated recursively by multiplying $Pop_{x-1,t-1}$ by the one year survival probability $p_{x-1,t-1}$ as denoted in equation (1).

$$Pop_{x,t} = Pop_{x-1,t-1} * p_{x-1,t-1} \,, \tag{1}$$

where $25 \leq x \leq 99$ and $t = 0, \dots, T$. Survival probabilities are held constant under all scenarios and contain rates for both males and females. The results from equation (1) can be shown in a *74 x T* population matrix $\boldsymbol{Pop_{x,t}}$:

$$\boldsymbol{Pop_{x,t}} = \begin{bmatrix} Pop_{25,0} & \dots & Pop_{25,T} \\ \vdots & \ddots & \vdots \\ Pop_{99,0} & \dots & Pop_{99,T} \end{bmatrix}$$

In this matrix each column represents an annual time step and each row represents an age cohort. Benefits are accrued at a rate of 1/60 of final wage per working year so the maximum accruals that can be built up per participant are 40/60 of the average final wage. The matrix for accrued benefits $\boldsymbol{B_{x,t}}$ for each age cohort from the age of 25 until the age of 65 for each time period t is illustrated in equation (2).

$$\boldsymbol{B_{x,t}} = \boldsymbol{Pop_{x,t}} * \left[\frac{40}{60} - \frac{(|65-x|)}{60}\right] = \begin{bmatrix} B_{25,0} & \dots & B_{25,T} \\ \vdots & \ddots & \vdots \\ B_{99,0} & \dots & B_{99,T} \end{bmatrix}, \tag{2}$$

where $25 \leq x \leq 99$. It is assumed that average wages increase with the indexation rate only. Wage increases due to career improvements are neglected in this model. As explained in the previous chapter, the indexation rate is linked to wage inflation capped at 2.5% in the United Kingdom. The accrued benefits matrix $\boldsymbol{B_{x,t}}$ will be multiplied by 1.025 each year. For simplicity, accrual rates and indexation rates are assumed to be fixed and constant. They do not change per scenario and therefore no superscripts of scenarios s are used yet.

To determine the present value at time t of the accrued benefits, matrix $\boldsymbol{B_{x,t}}$ is discounted with a discount factor *D* consisting of the term structure of real interest rates (inflation linked Gilts) plus an additional percentage ζ. This percentage differs per pension contract and would normally be below 2%. Equation (3) shows the discount factor for a certain age cohort x at a specific moment in time t. The parameter uncertainty is modelled through a large but finite number of stochastic scenarios S which is denoted by a superscript s. Instantaneous short Interest rates IR_t^s are calibrated in Chapter 5 for each year and for each scenario. From the instantaneous short rates, the interest rate term structures for each of the scenarios can be found. The yields $y_{m,t}^s$ from this term structure are used to discount liabilities with maturities m. The first discounting period will be m periods from the current time $t = 0$ and the last discounting period will be at age 99. For the age cohorts that are retired ($x \geq 65$) the first discounting period will be the current time $t = 0$.

$$D_{x,t}^s = \sum_{\mathrm{i=max}(m,0)}^{99-x} \frac{1}{(1+\zeta+y_{m,t}^s)^m} \tag{3}$$

The intuition behind equation (3) can be illustrated in an example: suppose an age cohort $x = 45$ at period $t = 15$. Accrued benefits for this age cohort is the quantity of this age cohort $Pop_{45,15}$ times the rate for accrued benefits: $Pop_{45,15} * \left[\frac{40}{60} - \frac{(|65-x|)}{60}\right] = \frac{20}{60}$. The first pension benefit will be paid at the age of 65 in year 35 and the second pension benefit will be paid at the age of 66 in year 36. These payments continue until all participants have deceased. Discounting to present value is done by dividing the first benefit by the discount factor at time $t = 35$, which is $(1 + \zeta + y^s_{20,15})^{20}$. The second paid benefit is divided by the discount factor at time $t = 36$, which is $(1 + \zeta + y^s_{21,15})^{21}$. This calculation continues for all age cohorts until accruals for oldest age cohort with the age of 99 are valued. The discount factor from equation (3) is a *S x 74* discount matrix for a certain age cohort x at time t under scenario s:

$$\boldsymbol{D^s_{x,t}} = \begin{bmatrix} D^1_{25,t} & \dots & D^1_{99,t} \\ \cdots & \ddots & \cdots \\ D^S_{25,t} & \dots & D^S_{99,t} \end{bmatrix}.$$

In this matrix, each column represents an age cohort and each row represents a scenario. Multiplying the discount matrix by the matrix with accrued benefits $\boldsymbol{B^s_{x,t}}$ results in a *S x 74* matrix (4) with accrued benefits in present value $\boldsymbol{L^s_{x,t}}$. Note that the multiplication is done elementwise or entrywise. This multiplication is sometimes referred to as a Hadamard[4] product, denoted by $D \circ B$.

$$\boldsymbol{L^s_{x,t}} = \begin{bmatrix} L^1_{25,t} & \cdots & L^1_{99,t} \\ \vdots & \ddots & \vdots \\ L^S_{25,t} & \cdots & L^S_{99,t} \end{bmatrix} = \boldsymbol{D^s_{x,t}} \circ \boldsymbol{B^s_{x,t}} = \begin{bmatrix} D^1_{25,t} & \dots & D^1_{99,t} \\ \cdots & \ddots & \cdots \\ D^S_{25,t} & \dots & D^S_{99,t} \end{bmatrix} \circ \begin{bmatrix} B^1_{25,t} & \cdots & B^1_{99,t} \\ \vdots & \ddots & \vdots \\ B^S_{25,t} & \cdots & B^S_{99,t} \end{bmatrix} \qquad (4)$$

In matrix $\boldsymbol{L^s_{x,t}}$ each column represents the liabilities for a certain age cohort at time t and each row represents a scenario. The present value of total liabilities for each scenario at time t can then be found by summing each of the columns: $L^s_{N,t} = \sum_{x=25}^{99} L^s_{x,t}$. The resulting S x 1 vector will be used for the option pricing model from Chapter 4.

3.2 Assets

The initial asset value is set to obtain the targeted initial (at time $t = 0$) funding ratio: $F_0 = A^s_0 / L^s_{N,0}$. $L^s_{N,0}$ reflects the present value of all the accrued liabilities or the technical provisions. Suppose the initial funding ratio of the stylized pension fund is 1 (or 100%). The initial asset value will then be equal to the present values of the pension fund's accrued liabilities. Each year the asset value of the pension fund changes due to the contributions c_t made by working participants minus benefits $B_{x,t}$ paid to retired participants. The assets are invested in an investment portfolio which gives a stochastic return r^s_t. Equation (5) reflects the asset value at a specific moment in time.

[4] A Hadamard product is a binary operation that takes two matrices of the same dimension and produces a new matrix where each element ij is the product of the elements ij of the original two matrices.

$$A_{t+1}^{s} = \left[A_t^s * + \sum_{x=25}^{64} c_t * Pop_{x,t} - \sum_{x=65}^{99} B_{x,t} * Pop_{x,t}\right] * (1 + r_t^s), \tag{5}$$

where $t = 0, \dots, T$ and $s = 0, \dots, S$. The investment portfolio of the pension fund studied in this thesis consists of two assets: return seeking stocks and liability hedging inflation linked bonds. Weight w_t is invested in stocks and $(1 - w_t)$ is invested in inflation linked bonds. The return from these inflation linked bonds (Gilts in the UK) is equal to the instantaneous short term interest rate IR_t^s that was used for discounting the liabilities and the return from stocks is equal to the return from the stock market index (FTSE 100). The total return from the portfolio is shown in equation (6).

$$r_t^s = w_t * r_{stocks,t}^s + (1 - w_t) * IR_t^s \tag{6}$$

The initial investment strategy used in this thesis has 60% invested in stocks and 40% inflation linked bonds and is rebalanced annually. The returns from stocks and inflation linked bonds are generated using a stochastic model, see Chapter 5. With the initial asset value and its evolution in mind the next step is to insert these values into models for option pricing.

4. Sponsor Support Option Valuation

According to the United Kingdom pension law, employers bear the risk underfunding of their pension funds and have the obligation to make remedial contributions in case of underfunding of the fund. In Chapter 1 it was explained that such a guarantee can be interpreted as a put option written by the employer to the pension fund. Chapter 2 and 3 explained the characteristics of the pension fund that is studied in this thesis. This chapter will show the methodology behind the valuation of the sponsor support put option. First, a naïve approach will explain the general concept of pricing of the sponsor support option, assuming a risk free sponsor and constant equity return and interest rates. After that, the extended approach will develop a method that can be applied in a more realistic world.

4.1 Naïve Approach

The employer is providing a remedial contribution to the pension fund in case of underfunding, valued at an annual frequency. The amount the employer is forced to pay can be calculated as the sponsor payoff with the use of equation (7):

$$\text{Sponsor Payoff}_t = \max\left\{0; L_{N,t+T} * \exp(-D_T T) - A_t\right\} \tag{7}$$

In equation (7) $L_{N,t+T}$ reflects the value of the liabilities the pension fund has to its participants N at time $t + T$ that was earlier calculated in paragraph 3.1. This liabilities value determines the required asset value in T years from t to avoid underfunding. D_T is the current discount rate as explained in the previous chapter, T is the time to maturity and A_t is the current asset value. The subscript t reflects the moment of valuation, which is at the end of each year. $L_{N,t+T} * \exp(-D_T T)$ can be interpreted as the present value (at time $t = 0$) of the total accrued liabilities, which is referred to as technical provisions and reflects the assets that are currently required in order to have sufficient capital to cover liabilities in the future.

The sponsor payoff is equal to the difference between the required technical provisions and the current asset value. If the fund is overfunded (i.e. a current asset value that is larger than the present value of the accrued liabilities), the sponsor payoff is zero. Equation (7) looks like a typical formula for a short position in a forward contract, written by the employer to the pension fund. The value of the technical provisions ($L_{N,t+T} * \exp(-D_T T)$) is the strike price and the current value of the assets (A_t) is the underlying price in the forward contract.

The sponsor guarantee can also be interpreted as a put option written by the employer to the pension fund: the sponsor support option. This option gives the owner (the pension fund) the right to exchange the underlying (the value of the assets) with the writer (the employer) for a specific strike price (the value of the liabilities times the required funding ratio) at a specific moment in time. In financial literature such an option is called a European option because the exchange will only take place at the end of the contract (i.e. at the end of each year).

The traditional Black-Scholes (Black & Scholes, 1973) option pricing formula (8) can be used to calculate the value of the sponsor support option. It gives a theoretical estimate of European options under the following assumptions: i) there is no arbitrage opportunity; ii) the risk-free interest rate is constant and; iii) stock prices follow a Geometric Brownian Motion[5] with constant drift and volatility (Hull, 2012).

$$\text{Sponsor Support Option}_t = L_{N,t+T} * \exp(-D_T T) * \phi[-d_2] - A_t * \phi[-d_1] \tag{8}$$

with $\begin{cases} d_1 = \dfrac{\ln \frac{A_t}{L_{N,t+T}} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 = \dfrac{\ln \frac{A_t}{L_{N,t+T}} + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \end{cases}$

r is the return from the portfolio in which the pension fund has invested its assets and σ is the volatility of the asset portfolio return. The function $\phi[x]$ is a cumulative probability distributions function for a standard normal distribution $N(0,1)$. A simple numerical example can be used to explain the concepts of the sponsor support option valuation.

[5] A further explanation about the concept of the Geometric Brownian Motion is given in Chapter 5.

4.2 Numerical Example – Naïve Approach

A simplified situation is assumed, with only one participant in the pension fund who is 45 years of age and worked at the company for 20 year. The pension will be received as a lump sum at the age of 65. Per working year the participant accrued 1/60 of his final wage. On top of that he will be compensated for price inflation with a constant indexation rate of 2.5% per year. Assuming that his current wage is 40,000€ per year, the value of the lump sum pension payment in 20 years from now will be $\left(\frac{20}{60} * 40{,}000\right) e^{0{,}025*20} = 21{,}982.95$€. Discounting is performed with the Gilt rate plus 1% (as explained in the Chapter 2), continuously compounding. With a 20-years Gilt rate of 2% the resulting value is $21{,}982.95e^{-0.03*20} = 12{,}064.50$€. It is assumed that the initial funding ratio of the pension fund is 1 (or 100%). This means that the current asset value is equal to the present value of accrued liabilities. The assets are invested in a portfolio with an annual return of 4% and a volatility of 15%. If 20 years from now the pension fund does not have sufficient money to do the pension payment, the underlying company will pay the difference.

In this example the sponsor option can be calculated using equation (8) which results in a sponsor support option value of:

$$12{,}064.50 * \phi[-0.8572] - 12{,}064.50 * \phi[-1.5280] = \mathbf{1{,}597.58}$$

This value is $\mathbf{13.2}\%$ of the current asset value of the fund. This percentage reflects the market valuation of the guarantee the company has given to its pension fund as an embedded option.

For a more complex pension contract in which pension payments are spread over several years the same approach can be used. Each payment is then considered as a separate contract and option prices of these contracts are added together. Two core assumptions of this naïve approach are not realistic: a risk free sponsor and constant expected equity return and interest rates.

4.3 Extended Approach

4.3.1 Default Probabilities

An employer is not always able to pay a remedial contribution in case of underfunding of the pension fund. As assumed in the contract specifications, the employer is obliged by the United Kingdom law to back the deficit of the pension fund. In 2012, the liabilities of the FTSE 350 pension schemes have reached 35% of their sponsoring companies' combined market capitalization (Aon Hewitt, 2012). Small changes in the pension schemes therefore have disproportionate effects on the sponsor's finances. In case of underfunding of the pension fund the remedial contribution by the sponsor company would severely damage the continuation of sponsor. For that reason it is not impossible that a company defaults on its obligations at some point in time.

To make the naïve model more realistic the sponsor will have credit-risk allowing for a non-zero default probability. It is assumed that the company will only default on its pension obligations if it is practically unable to make the necessary extra contribution. This is the same as a company unable to fulfil the obligations to its regular debt holders. The pension fund can be viewed as a debt holder of the underlying company. If the pension fund faces a deficit, the sponsor needs to fill this gap. In this thesis it is assumed that no priority is given to neither a regular debt holders nor the pension fund when fulfilling obligations. If this is the case, credit risk can be evaluated for firms that trade corporate debt.

Since sponsors' default probabilities cannot directly be derived from market data, the default probabilities applied in this study are implied default probabilities, based on financial products that indirectly reflect default occurrences. Furthermore, a risk-neutral measure is used to perform valuations in this study which makes the results market consistent. This measure is also known as the equivalent martingale measure or $\mathbb{Q}$-measure. Risk-neutrality is a theoretical measure of probability derived from the combined assumptions that the current value of financial assets is equal to their expected payoffs in the future discounted with the risk-free rate and there is an absence of arbitrage. This measure makes it possible to calculate probabilities without knowing the actual market's degree of risk, which is in many cases an uncertain and unobservable variable. Investors are assumed to be indifferent between choices with equal expected payoffs even if one choice is riskier. It does not mean that calculations are wrong if investors would be risk-averse or risk-seeking but it is a straightforward method to perform valuations and it allows the risk-free interest rate curves within the set to be used for discounting all cash flows. Two methods will be explained that model implied risk-neutral default probabilities

Company Specific Credit Default Swaps

Implied risk-neutral default probabilities can be retrieved from observable market prices of credit default swaps which are a form of insurance against default occurrences. A credit default swap (CDS) is a derivative that gives the owner the possibility to get compensation in the event of a financial default of the underlying company. The receiver pays a certain premium for such a CDS. If the implied default probability of the underlying company increases (due to for example a falling credit rating), the premium of the CDS will increase in value and vice versa. On the other hand, market prices of these CDS's imply that risk premiums are included. These risk premiums are not only driven by the credit worthiness of the company but also by the economic conditions of the market as a whole. While the credit rating of a company goes up, falling stock prices could still lead to higher implied default probabilities.

Next to credit default swaps, the Recovery Rate (RR)[6] should be introduced when implied default probabilities are calculated. The Recovery Rate, normally used in the bonds market, is the percentage of an investment that will be paid back to the counterparty, notwithstanding the default. Since more empirical information can be found on the RR it is easier to make assumptions on this rate. Empirical data finds that typical Recovery Rates are 35%-40% of the promised amount when unsecured corporate debt defaults (Moody's Investor Services, 2012). In this thesis a RR of 0.4 is assumed. Further research could calibrate the Recovery Rate by fitting the equations below to financial products that have the same characteristics as default occurrences.

The Loss Given Default (*1-RR*) is embedded in the market pricing of CDS's. This is explained as follows: first, the price of a zero-coupon bond (*ZCB*) can be calculated by 1 minus the probability of default times the loss given default (*LGD*). This leads to equation (9) for the implied probability of default.

$$\text{Implied Probability of Default} = \frac{(1-ZCB)}{(1-RR)} \tag{9}$$

Second, the zero-coupon bond price can be calculated straightforward from the credit spread ($spread_T$) from credit default swaps. From the equation (10) for the implied cumulative probability of default for maturity T can be constructed.

$$\text{Implied Cumulative Probability of Default}_T = \frac{(1-e^{spread_T * T})}{(1-RR)} \tag{10}$$

In order to calculate implied default probabilities for an actual existing company, data on traded credit default swaps is needed. These credit default swaps are provided by Bloomberg (2013) and are priced in terms of their credit spread over the risk-free interest rate. A distinction in terms of risk-free interest rate can be made between government bond prices and interest rate swap rates. Both financial products can be used to value the risk-free interest rate. This section uses swap rates for the risk-free interest rate, as this is consistent with the methods used in Chapter 5 of this thesis.

Since the longest available maturity for a credit default swap is 10 years credit spreads are extrapolated by using final spot extrapolating (CEIOPS, 2010). This extrapolating technique uses the spread rate of the credit default swap with the largest maturity available on the market and takes this spread rate as the rate for all longer maturities: $R_t^{(T_{end}+T)} = R_t^{(T_{end})}$. So for $T \geq 10$ the credit spread is assumed to be constant. The problem is however that implementing this extrapolation in equation (10) will results in implied

[6] The equivalent of the Recovery Rate (1-RR) is the Loss Given Default (LGD). The LGD is the loss that debt holders (participants in the pension plan) experience when an actual default occurs. The Loss Given Default is a common parameter in risk models for financial institutions. It refers to a percentage of loss over the total exposure when a bank's counterparty goes bankrupt.

default probabilities of over 100%, as can be seen in Figure 4.1. This is not a convenient result, so a different extrapolating technique should be adopted for sake of comparison. To remedy the problem, the following constraint is added to equation (10):

$$\begin{cases} \dfrac{(1-e^{spread_T * T})}{(1-RR)} = \text{Implied Cumulative Probability of Default}_T \leq 1 \\ \qquad\qquad\qquad\qquad spread_T \leq -\dfrac{1}{T}\log(RR) \end{cases}$$

Figure 4.1 shows the calculated implied cumulative default probabilities for some companies. The left hand side graph illustrates the results without the credit spread constraint and the right hand side graph has incorporated the constraint. The complete results can be found in Appendix B.1.

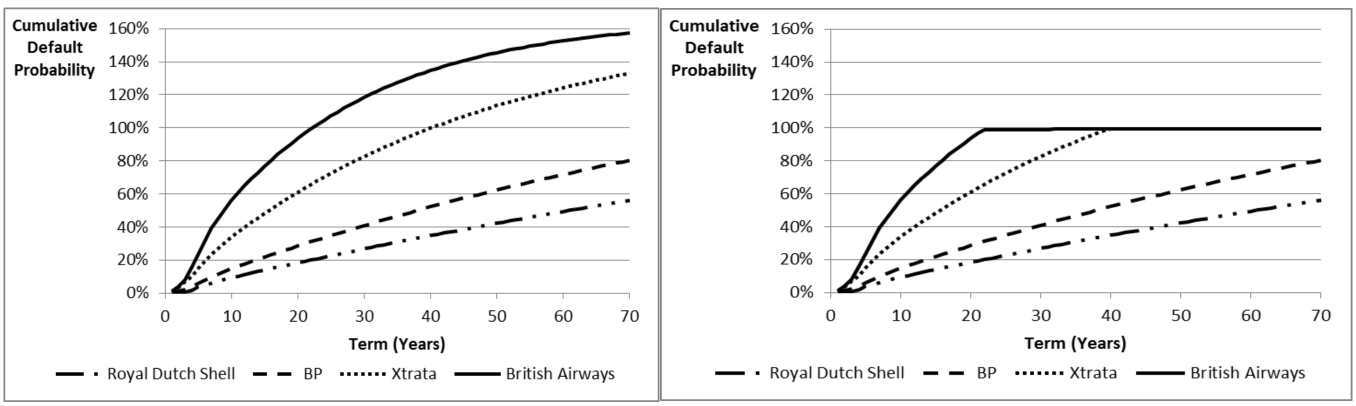

Figure 4.1: Implied Risk-neutral Cumulative Default Probabilities without (L) and with (R) constraint on credit spread

Since the default probability rates are risk-neutral, no direct economic conclusions can be drawn from the graph. For example, the market price of 30 year credit default risk from Royal Dutch Shell can be derived from the joint assumption that the 30 year default rate is 26.90% and that corporate debt-holders do not require a risk premium from bearing credit risk. Note that Royal Dutch Shell is a dual listed company, which means that it is listed on both the AEX and the FTSE. The observed rates can be used in market-consistent cash flow valuations for the studied company and its pension fund.

Royal Dutch Shell is AA-rated, BP is A-rated, Xstrata is BBB rated and British Airways is BB rated. Note that at the moment of writing this thesis there was no FTSE 100 company with AAA-rating that traded credit default swaps. From the graph it can be seen that higher rated companies have lower cumulative default probabilities. Credit default swap prices can differ between companies with the same credit rating and default probabilities cannot be assumed to be equal per credit rating.

Barrie and Hibbert (2012) argue that in order to study default rates of firms that do not trade corporate debt (mostly smaller firms) one should look at other firms (that do issue corporate debt) with comparable credit worthiness (i.e. credit rating). The Pension Regulator in the United Kingdom assumes that unrated sponsors will have the same probability of insolvency as a BBB rated company (EIOPA, 2013).

Historical Default Rates Linked to Credit Ratings

A second method to calculate default probabilities of companies can be to study historical default rates. Standard & Poor’s (Standard & Poor’s [S&P], 2013) mapped the hypothetical 1-year default rate of companies’ based historically observed default rates. For Standard & Poor’s, the AAA-rating is the highest “prime” score. AA is the high grade, A is the upper medium grade, BBB is the lower medium grade and BB is the non-investment (speculative) grade. Lower ratings are typified as higher speculative and more risky. According to S&P (2013), companies that are BB-rated and lower have a 1-year default rate of 100%, which means that they will always default. Note that these rates are defined as hypothetical and may differ from what would be expected in reality.

With the use of the 1-year data, the default probabilities for longer maturities can be calculated by assuming that the 1-year default probability will remain constant. For example, a 1-year default probability of 2% for AA-rated companies implies that after one year 98% of the AA-rated companies have not defaulted. Next year 2% of 98% will default, so that after 2 years 96.04% will remain as not having defaulted. Figure 4.2 shows the results for all credit ratings by applying this method. The complete results can be found in Appendix B.1. According to Standard & Poor’s (S&P, 2013) the 1-year default rates of BB, B, CCC and CCC- are 100%. This means that these companies will default in any case. A triple A-rated company is expected to have a zero default probability for all horizons, so S&P (2013) assumes that this is the risk-free sponsor. As Figure 4.2 shows, the implied cumulative default probabilities of BBB- and A-rated companies seem to be overestimated. This method would probably suit the higher rated companies better than the lower rated companies. Chapter 6 will test this statement. Besides that, the 1-year default rates as provided by S&P do not necessarily imply that companies with the same credit rating always have the same default probabilities. They can still differ in terms of default probabilities and might even migrate between credit ratings. In order to calculate company specific default probabilities, the credit default swap method as shown in Figure 4.1 might result in more realistic default probabilities.

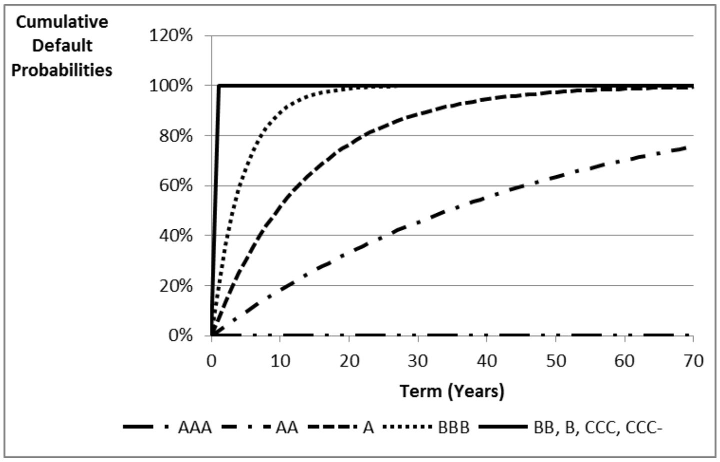

Figure 4.2: Hypothetical Cumulative Default Probabilities based on historical rates (S&P, 2013)

With the cumulative default probabilities from Figure 4.1 and 4.2 derived by the two methods, a "default threshold" can be constructed: $\phi^{-1}(CPD_T)$. $\phi^{-1}(x)$ is the inverse of a cumulative normal distribution with mean 0 and standard deviation 1 and CPD is the implied risk-neutral cumulative default probability from one of the two methods as illustrated in Figure 4.1 and 4.2. The cumulative default probability that is eventually used is the one that has the same maturity as the average maturity of the liabilities of the pension fund.

If the credit state of the sponsor is below the default threshold the sponsor will default on its obligations to the pension fund and other regular debt holders. This means that it will not make the required remedial contribution. Instead, the sponsor will only be able to pay a contribution equal to the Recovery Rate (RR) as explained earlier. How the credit state of the sponsor can be determined is explained below.

4.3.2 Sponsor's Credit State

The credit worthiness of a company is not only dependent on the default probabilities of the company. Market prices of credit default swaps contain risk premiums. These risk premiums are not only driven by the credit rating of the company but also by the economic conditions of the market as a whole. Suppose, for example, a fall in stock prices. This results in a decrease in value of assets for both the pension fund and the underlying company, assuming a positive correlation between the sponsor's asset value and the broader stock market. The risk adjusted default probability would therefore be even higher than in case there would not be a fall in stock prices. This would increase the credit risk of the sponsor support option (guarantee) and reduces the value of the payoff and the put option. Second, if the pension fund is increasing the riskiness of its investment portfolio in order to generate extra return to close the gap, pension fund members will be even more negatively affected by falling stock prices. The sponsor support option will lose much of its value.

For valuation of the credit state of the company, the correlation between the sponsor default and the return from the broad stock market is needed. The correlation between a pension fund's default probability (credit state) and its own equity return is expected to be negative (positive) strong on average, since a higher credit state will result in higher market valuation. So the correlation between the pension fund's stock return and its credit state could be equivalently considered as the correlation between the pension fund's equity return and the equity market return index. "A typical market implied correlation between a firm's equity return and the broader equity market index is +0.5" (Barrie & Hibbert, 2012, p. 7). The credit state of the sponsor at time t is calculated by equation (11).

$$\text{Credit State}_t = \rho * r_{index,t} + \sqrt{(1-\rho^2)}\varepsilon_t \tag{11}$$

This equation shows the dependency between sponsor credit state and assets in both the pension fund itself as the broader market. The error ε is a random variable with mean 0 and standard deviation 1: $\varepsilon \in N(0,1)$ under the risk-neutral measure. ρ is the correlation between the company's equity return and the broader equity market return. This can be equivalently considered as the correlation between the credit state of the sponsor and the broader equity market return. A high correlation means that more defaults will occur when the asset market index falls in value and vice versa. The return from the equity market index is retrieved from the FTSE 100 index and will mostly have a relatively small value compared to a standard normal variable with mean 0 and standard deviation 1. To avoid that the error term weights too heavily in the credit state, the variable $r_{index,t}$ is a normalised variable of the stock index return from equation (12).

$$r_{index,t} = \frac{\mu(r_{stocks,t}) - r_{stocks,t}}{\sigma(r_{stocks,t})} \tag{12}$$

In equation (12) $\mu(r_{stocks,t})$ is the mean stock index return, $r_{stocks,t}$ is the expected stock return and $\sigma(r_{stocks,t})$ is the standard deviation of the stock index return at period t. The sponsor will default if $\text{Credit State}_t < \phi^{-1}(CPD)$. In that case the sponsor will only pay the amount equal to the Recovery Rate (RR), which was assumed to be 0.40. Using the default threshold and the credit state, a default indicator is implemented in the naïve valuation approach. This indicator is 1 if $\text{Credit State}_t < \phi^{-1}(CPD)$ and 0 otherwise. $\phi^{-1}(x)$ is the inverse of a cumulative normal distribution with mean 0 and standard deviation 1 and CPD is the implied risk-neutral cumulative default probability from one of the two methods as illustrated in Figure 4.1 and 4.2. The cumulative default probability that is used that has the same maturity as the average maturity of the liabilities of the pension fund.

4.3.3 Simulation Based Approach

Since it is unknown beforehand whether the default will take place within the studied horizon T, equations (7) and (8) need to be recursively updated. Pension funds' assets grow with the rate of return of its asset portfolio (13), which is a function of the portfolio composition.

$$A_{t+T}^{S} = A_t * \exp\left[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} * \epsilon_t\right] \tag{13}$$

$\epsilon \in N(0,1)$ under the risk neutral measure. Note that this is not the same error term as in equation (11). r is the return from the asset portfolio and σ^2 is the volatility of the asset portfolio return. For now both portfolio return and volatility are assumed to be constant and are the same variables as in equation (8). Updating equation (7) with equation (13) and including the default probability indicator leads to equations (14).

$$\text{Sponsor Payoff}_t = \left[\max\left\{0; L_{N,t+T} - A_t * \exp[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} * \epsilon_t]\right\} * \left\{1 - (1 - RR)I_{def}\right\}\right] \tag{14}$$

I_{def} is the default indicator and is 1 if $\text{Credit State}_t < \phi^{-1}(CPD)$ and 0 otherwise. The Black-Scholes equation (8) for option pricing assumed constant risk-free interest rates. Equation (14) contains stochastic variables which makes the outcome uncertain. For that reason the Black-Scholes approach cannot be used with the default indicator anymore. Since there is a correlation between the credit state of the company and the growth of the portfolio of the pension fund, a purely analytical value of the sponsor support option is difficult to calculate. Therefore, the Monte Carlo (MC) approach will be adopted that runs 5,000 simulations of equation (14). Since risk-neutral valuations are applied, the expected credit state of the company can be found with the risk-neutral measure $\mathbb{Q}$ as showed in equation (15).

$$\frac{\text{Sponsor Support Option}_t}{N_t} = E^{\mathbb{Q}}\left[\sum_{i=0}^{T}\left(\frac{\text{Sponsor Payoff}_{t+i}}{N_{t+i}}\right)\right] \tag{15}$$

Equation (15) shows that each simulated sponsor payoff for each year is discounted with N_t. Then, all sponsor payoffs are added and the expected value under risk-neutral measure $\mathbb{Q}$ is taken, which is the average. The sponsor support option is the expected sponsor payoff and is in itself not a stochastic variable. N_t is a numéraire, which is a basic standard to compute values under the risk-neutral measure. The risk-free interest rate is used in this case as the numéraire. This was earlier explained as the discount factor so it can be stated that $N_t = \exp(-D_{T,t}T)$. A simple numerical example can be used to explain the concepts of the extended approach to the sponsor support option valuation.

4.4 Numerical Example – Extended Approach

The numerical example from paragraph 4.2 is extended in this paragraph. The participant (45 years of age) will now receive two lump sum payments of its pension, at age 65 and 75. Applying the same indexation rules as in paragraph 4.2 the pensioner will receive 21,982.95€ and 28,226.67€ at the ages of 65 and 75 respectively. Discounting these values to the present period results in: $21{,}982.95e^{-0.03*20} + 28{,}226.67e^{-0.03*30} = 12{,}065.50€ + 11{,}476.11€ = 23{,}540.61€$. Since the initial funding ratio is 1 (or 100%), the initial asset value is equal to the present value of the accrued liabilities. The pension fund will currently hold $12{,}065.50€$ for the first pension payment and $11{,}476.11€$ for the second pension payment made to the participants. The assets are invested in a portfolio with an annual return of 4% and a volatility of 15%.

The average duration of the liabilities is 24 years. The company underlying the pension fund has a AA credit rating (S&P, 2013). The 24-years risk-neutral implied cumulative default probability for a AA rated company from Figure 4.2 is 38.42%. The default threshold is then -0.29447. The correlation between the companies stock return and the stock market index return is +0.5. If the credit state of the company falls below the threshold, the company will only pay the Recovery Rate (0.4).

Now 5,000 Monte Carlo simulation of the sponsor payoff equation (14) will lead to a sponsor support option (SSO) value as shown below in Table 4.1. The results are illustrated relatively to the current asset value. A comparison is made with a AAA-rated company. AAA rated companies are expected to have a default probability of zero, so this comparison could illustrate the difference between the naïve and the extended option pricing approach.

	SSO	σ	$\sigma/\sqrt{N}$	95%-Confidence Interval	
AA	0.1606	0.0487	0,0007	0,1592	0,1619
AAA	0.2087	0.0462	0,0006	0,2074	0,2099
% Change	-23.1%				

Table 4.1: Results Sponsor Support Option Value: Numerical Example – Extended Approach

The first column in Table 4.1 shows the sponsor support option value as it is the average of the 5,000 MC simulations. The second and third column show the volatilities of the sample and the average sponsor support option respectively. With the use of the third column, a 95%-Confidence interval as illustrated in the fourth column can be constructed.

Table 4.1 shows that by introducing the extended approach (i.e. including default probabilities and credit state) results in a decrease of the sponsor support option value of 23.1%. This result is significant against the 0.1% significance level. For companies with credit ratings lower than AA, the difference between the naïve and the extended approach will be even higher. This effect seems logical to understand since the option price reflects the value of the guarantee the company has given to its pension fund. Obviously a guarantee from a lower rated company is less worth than the same guarantee from a higher rated company.

5. Stochastic Variables

In Chapter 4 it was assumed that both the portfolio return (r) and the discount rate (D_T) are fixed and constant. This is not a realistic assumption and in this chapter we relax this assumption. Short term interest rates, interest rate term structures and equity return will follow a stochastic process, which describes a process with many directions (randomness).

5.1 Approach

This chapter introduces a market consistent economic scenario generator (ESG). An ESG can be used to calibrate stochastic variables that are input for the cashflow (ALM) model described in Chapter 3 and the valuation model from Chapter 4. The ESG can either produce real world scenarios or market consistent scenarios in econometric models.

The first type reflects the observed real world and can be used to project the expected future evolution of the economy. Risk premiums are included and the calibrations (estimations) are based on historical data. The second type is used for market consistent valuations and generates market prices. Calibration/Estimation is performed by using current market data. Market consistent scenarios do not include risk premiums (i.e. they are risk-neutral) and can therefore be used to value derivatives. The latter type of ESG will be used in this thesis since the aim of this thesis is to value the derivative sponsor support option. This approach is in line with international regulation (Solvency II) which places some requirements for a market-consistent ESG. Firstly, the market data used to perform the calibrations should be from products that are available at deep, liquid and transparent financial markets. Secondly, there should be no opportunity for arbitrage. Under this condition all assets are priced appropriately and outperforming the market is not possible without lowering risk aversion.

The purpose behind a market consistent valuation is to reproduce market prices of assets which have similar characteristics as the variables that need to be calibrated. Liabilities and guarantees in pension contracts have the characteristics of options. An ESG calibrated to option prices would therefore give a good fit to the market price of options for the variables needed in this thesis. The first variable that is calibrated in this chapter is the instantaneous nominal short term interest rate. Part of the pension fund's portfolio is invested in inflation linked government bonds. The return from these bonds can be derived from the instantaneous nominal short term interest rates. The second variable that is calibrated is the interest rate term structure. This curve is used to discount the liabilities the pension fund has to its participants. The calibrated instantaneous nominal short term interest rate will serve as the basis of this curve. The third variable that is calibrated is equity return. The part from the pension fund's portfolio that is not invested in inflation linked government bonds is invested in equity. Equity prices are calibrated to European call options and by using a constant equity return volatility and correlation between the earlier calibrated instantaneous nominal short term interest rate and equity prices.

For all three variables a model is compared with market data. By adjusting parameters in these models (calibrating) a curve can be found that best fits the market data. These adjustments are made by minimizing the sum of squared differences between market prices and models prices. Based on the calibrated parameters, the ESG generates a set of simulations that will be inserted in the cashflow (ALM) model from Chapter 3 and the valuation model from Chapter 4.

Both interest rates and equity return are modelled as stochastic processes which satisfy the stochastic differential equation (SDE) as shown in equation (16). This equation follows a Geometric Brownian Motion (GBM) (Hull, 2012). Some of the arguments for using GBM are: i) expected relative returns are independent of the value of the process (stock price), which agrees with what we would expect in reality; ii) only positive values are assumed, just like real stock prices; iii) the same kind of 'roughness' in its paths as are seen in real stock prices are present. Using GBM is relatively easy for calculating stock prices. A disadvantage is that volatility is assumed to be constant over time, which is not the case in reality. This assumption will hold sufficient for the research topic in this thesis.

$$dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t), \tag{16}$$

where $X(0) = x_0$ and $\mu\big(t, X(t)\big)$ is the drift term. This drift term is the mean change per unit time for a stochastic process. $\sigma\big(t, X(t)\big)$ is the diffusion coefficient. The diffusion coefficient is the variance per unit time. $W(t)$ is a Wiener process (Hull, 2012), which is a continuous time stochastic process with independent increments. The properties of this Wiener process are: if $s < t \leq u < v$ then $W(v) - W(u)$ and $W(t) - W(s)$ are independent stochastic variables, and $W(t) - W(s)$ follows a normal distribution $N(0, \sqrt{t - s})$. The volatility for each model is determined from option prices on the market.

5.2 Interest Rates

5.2.1 Model

In the modelling of the interest rates differences can be made between short rate models, forward rate models and LIBOR/swap market models. In this paper a short rate model is applied since these types of models are well known and popular among financial engineers for pricing interest rate derivatives. The 3-month rate is used as the short interest rate (Baldvinsdóttir & Palmborg, 2011). Within short rate models the stochastic process for the future behaviour of the short term interest rate uniquely defines the arbitrage free prices today of risk-free bonds of any term. Therefore, the specification of the stochastic process for the short term interest rate also implies a bond pricing formula for the valuation of a risk free cash flow of any duration.

Within short rate models differences can be made between equilibrium/endogenous models and no-arbitrage models. Whereas the first type of model uses today's interest rates as output, the second type of model uses today's interest rates as input. Furthermore, in an equilibrium model the drift of the short rate is not a function of time and in a no-arbitrage model the drift is in general dependent on time. In order to get to a market consistent calibration of interest rates, the current term structure of interest rates should be used as input. For that reason it is assumed that a no-arbitrage model suits this study best. Equilibrium models can be converted into no-arbitrage models by including a function of time in the drift and the short rate. Examples of equilibrium models are: Vasicek (1977), Cox-Ingersoll-Ross (1985) and Rendleman-Bartter (1980). Examples of no arbitrage models are: Ho-Lee (1986), Hull-White (1990, 1994), Black-Derman-Toy (1990) and Black-Karasinski (1991). In order to use a model that is consistent with today's term structure of interest the Hull-White (1990) model is chosen in this thesis. This is an extension of the original Vasicek (1977), which models the risk-neutral process by equation (17) (Hull, 2012).

$$dr = a(b - r)dt + \sigma dz, \tag{17}$$

where $a,\ b$ and σ are constants. This model is mean reverting. The essence of mean reversion is that both a stock's high and low prices are temporary and that a stock's price will tend to move to the average price over time. This is likely to lead to the fact that actual risk events (both positive and negative) neglected in expectations, since the mean reversion parameter a slows down the volatility. In equation (17) the short rate is reverting towards the average level b at rate a. Hull and White (1990) proposed an extension of the Vasicek model in which some or all of the parameters are not constant anymore but can vary with time. Hereby, the model can match the current term structure of interest rates. They proposed the following extended Vasicek equation (Hull, 2012):

$$dr = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t),$$

where $r(0) = r_0$, $a(t)$, $\sigma(t)$ and $\theta(t)$ are deterministic functions of time and $W(t)$ is a Wiener process. $a(t)$ is the rate at which $r(t)$ reverts to its mean $\theta(t)$. Since theta varies over time it cannot really be denominated as a mean, but it is named "mean" here according to the mean reversion principle. $\sigma(t)$ is the volatility of the instantaneous short rate. If we set a and σ as (positive) constants equation (18) can be constructed.

$$dr = [\theta(t) - ar(t)]dt + \sigma dW(t), \tag{18}$$

where $r(0) = r_0$ and $\theta(t)$ can be found that matches the current term structure of interest rates best. Equation (19) reflects the deterministic function for $\theta(t)$.

$$\theta(t) = \frac{\partial F(0,t)}{\partial T} + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}), \tag{19}$$

where $F(0,t)$ is the market instantaneous forward rate at time 0 for maturity T: $-\frac{\partial \ln P(0,T)}{\partial T}$. $P(0,T)$ is the zero-coupon price for maturity T. The price of a bond at a time t with maturity T is given by equation (20).

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{20}$$

where $\begin{cases} B(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right] \\ \ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F_t(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - e^{-at}) \end{cases}$

Equation (20) can be used to calculate zero-coupon bond prices at a future time in terms of the current short rate and current bond prices. Current bond prices can be calculated from the current term structure. The explicit formula (20) for zero-coupon bond prices is very useful since it leads to an equation for prices for European options on bonds (21). This analytical formula for the price of a European put at time t with maturity T on a zero-coupon bond with maturity S and strike K is shown in (21) (Brigo & Mercurio, 2005).

$$ZBP(t,T,S,K) = KP(t,T) * N\left(-h + \sigma_p\right) - P(t,S) * N(-h) \tag{21}$$

where $\begin{cases} \sigma_p = \sigma\sqrt{\frac{1-e^{-2a(T-t)}}{2a}}B(T,S) \\ h = \frac{1}{\sigma_p}\ln\frac{P(t,S)}{P(t,T)K} + \frac{\sigma_p}{2} \end{cases}.$

In order to calibrate the interest rate term structure by using current market data as input, it is necessary to find a product from the financial market that best "describes" interest rates. Such a product should characterize interest rates and should be available at deep and liquid markets. For that reason interest rate swaption rates are used. An interest rate swaption is an option that gives the owner the right to enter into an underlying swap. A (interest rate) swap is a popular and highly liquid financial derivative in which two parties agree to exchange interest rate cash flows. A European swaption can be seen as an option on a coupon bearing bond. Baldvinsdóttir and Palmborg (2011) proposed formula (22) for swaption prices. The authors made use of Jamshidian's[7] decomposition to get to this equation.

$$PS(t,T,t_1,\dots,t_n,N,K) = N\sum_{i=1}^{n} c_i\,[ZBP(t,T,t_i,K_i)], \tag{22}$$

where PS is the price of a payer swaption with strike K, maturity T and nominal value N. It gives the holder the right to enter at time T an interest rate swap with payment times $\{t_1,\dots,t_n\}, t_1 > T$ where he pays at the fixed rate K and receives LIBOR. τ_i is the year fraction from t_{t-1} to $t_i, i = 1,\dots,n, c_i = K\tau_i$ for $i = 1,\dots,n-1$ and $c_n = 1 + K\tau_n$. $K_i = A(T,t_i)\exp(-B(T,t_i)r^*$ and r^* is the value of the spot rate at time T for which: $\sum_{i=1}^{n} c_i A(T,t_{i\_}\exp(-B(T,t_i)r^* = 1$ (Baldvinsdóttir & Palmborg, 2011).

5.2.2 Calibration

The previous paragraph derived the model that is used to perform the calibration for interest rates. This calibration is needed to perform a simulation study based on valuation methods described in Chapter 3 and 4. Eventually the calibration will result in constant values for a and σ_r. These two variables reflect the reversion rate and the interest rate volatility respectively. The market consistent calibration of the Hull-White interest rate model is performed for data at May 27th 2013. At that date, prices of at-the-money interest rate swaptions traded in the UK (UK swaptions) on the OTC market are retrieved from Bloomberg. Swaptions are quoted as implied volatilities and from these volatilities the actual prices can be calculated. Appendix B.2 shows the implied volatility matrix for at-the-money interest rate swaption at May 27th 2013. Maturity reflects the maturity of the swaption and tenor reflects the maturity of the underlying bond. Figure 5.1 illustrates these implied volatilities graphically. Implied volatility is a decreasing function of the maturity (years) of the swaption. For the maturity of the underlying bond, implied volatility is increasing up to a tenor of 4 years. Thereafter, the implied volatility decreases.

[7] Jamshidian (1989) showed that the wealth obtained by restriction to a continuous time model of the type similar to those given in Cover's universal portfolio, lies in explicit formulae for the wealth of constant rebalanced portfolios.

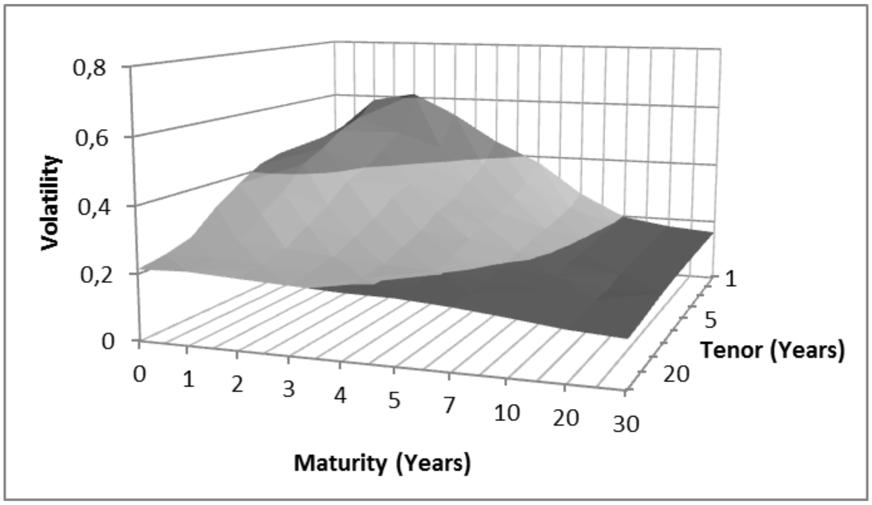

Figure 5.1: UK Interest Rate Swaptions Implied Volatilities, 27-5-2013. Source: Bloomberg 2013

From the implied volatilities the prices of the UK swaptions can be calculated by using the Black-formula (Black, 1976). A software package applied to Microsoft Excel available at Cardano Risk Management is used to perform the calibrations. This package calibrates the Hull-White parameters directly from swaption implied volatilities. Besides swaptions prices, the Hull-White model also needs to exactly fit the yield curve at the same date. This risk-free interest rate term structure is based on the UK interest rate swaps curve retrieved from Bloomberg. Figure 5.2 shows the yield curve for UK government bonds at May 27th, 2013. The complete results can be found in Appendix B.2.

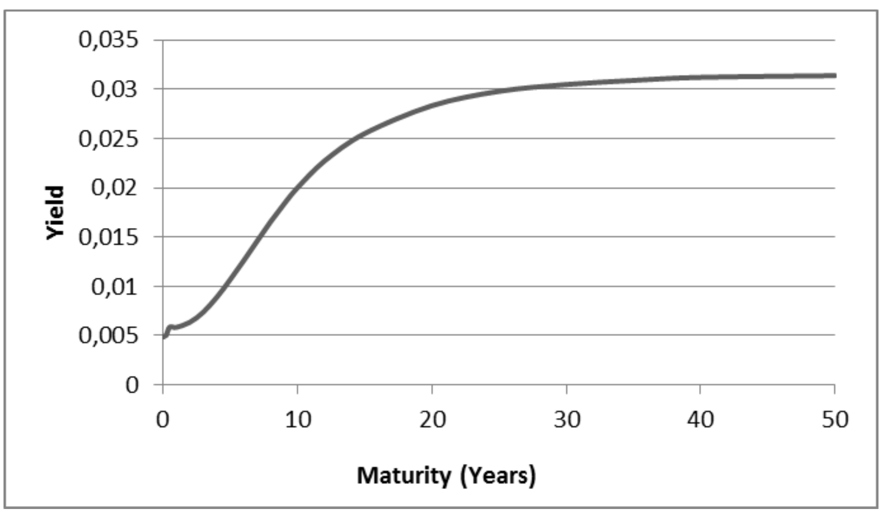

Figure 5.2: Interest Rate Term Structure based on UK swap rate curve, 27-5-2013. Source: Bloomberg 2013

The deterministic function for $\theta(t)$ as shown in equation (19) needs to be calibrated so that the model exactly matches the current term structure of interest in Figure 5.2. Thereafter, the parameters a and σ_r are calibrated so that they give the best fit of the models to market swaption prices. The calibration to both curves is performed by the Levenberg-Marquardt algorithm[8]. The results are shown in Table 5.1.

	$\boldsymbol{a}$	$\boldsymbol{\sigma_r}$
Hull-White Results	0.005904311	0.007469748

Table 5.1: Hull White Parameters calibrated to swaption prices

The reversion rate a towards the mean interest rate is 0.0059. This "mean" interest rate is the deterministic function $\theta(t)$ as showed in Equation (19). Since theta varies over time it cannot really be denominated as a mean. Table 5.1 shows that the calibrated one year interest rate volatility is 0.75%. This is in line with historical data, which shows that in the United Kingdom the one year interest rate volatilities fluctuated between 0.8% and 1% in the past 20 years (Bloomberg, 2013). Previous research on the estimation of single-factor Hull-White parameters (Moody's Analytics, 2013) shows similar results: $a = 0.008$ and $\sigma_r = 0.01$. Note that these are results for December 2010, but could be assumed as comparable due to the same (low) interest rate environment as in May 2013.

5.2.3 Simulations

From the two Hull-White parameters in Table 5.1 the deterministic function $\theta(t)$ can be constructed with equation (19). Monte-Carlo simulations are used to predict future instantaneous nominal short term interest rates. Then, from these future instantaneous short rates the interest rate term structure for each of the future years within the horizon of the fund can be constructed. With the term structures, linked to inflation, the predicted future liabilities are discounted. The instantaneous short rates are used to calculate how portfolio investments in inflation linked government bonds will evolve over time.

To simulate the returns of a specific asset the stochastic differential equation has to be discretized. The most straightforward method to do this discretization is the Euler approximation. Equation (18) is discretized with equation (23) (Rouah, 2005).

$$r(t + \Delta t) = r(t) + [\theta(t) - ar(t)]\Delta t + \sigma \Delta W(t)\sqrt{\Delta t}, \tag{23}$$

where $\Delta W \sim N(0, \sqrt{\Delta t})$. The initial short rate $r(0)$ is assumed to be the 3-month zero rate at May 27th 2013, retrieved from the data in Figure 5.2. From equation (23) 5,000 Monte Carlo Simulations are produced for the instantaneous short interest rate. A horizon of 70 years is applied. The resulting average rate is 0.024.

[8] The Levenberg-Marquardt method provides a numerical solution to non-linear minization problems, especially in least squares curve fitting (Levenberg, 1944).

From the instantaneous short interest rates the interest rate term structures for each year within the horizon can be calculated. The deterministic equation (19) is used to match the current term structure of interest rates: $\theta(t) = \frac{\partial F(0,t)}{\partial T} + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})$. 5,000 Monte Carlo simulations for the yield curves can be constructed for each period t. These simulations are used to discount the liabilities the pension fund has to its participants for each year within the horizon of the fund by adding a percentage according to the pension contract.

5.3 Equity Return

5.3.1 Model

A commonly accepted model for equity returns is the Black-Scholes model (Black & Scholes, 1973). The original Black-Scholes model assumes that interest rates are deterministic. However, the effect of stochastic interest rates when pricing options with long maturities is much more significant and should therefore not be neglected in the modelling of equity return. Both the interest rate volatility σ_r and reversion rate a from the Hull-White model in previous paragraphs are implemented in the Black-Scholes model.

Equity option prices can be used to calibrate the model. An equity call or put option gives the holder the right (voluntary) to buy or sell the underlying stock at the maturity date of the option. Distinction can be made between European and American options, where the first gives the holder the right to exercise the option at the expiration date and the latter gives the holder the right to exercise the option at any time before the expiration date. No general formulas exist for valuing American options. European options are typically valued by the Black-Scholes model (1973). For that reason the European style options are used. The Hull-White one factor short rate model with a given mean reversion factor a and volatility σ_r is applied as described in the previous section. We implement this in the traditional Black-Scholes model. The price of the European call option at time t on asset S with maturity T, strike K then equals equation (24).

$$c(t,T,K) = S(t)N(d_1) - KP(t,T)N(d_2), \tag{24}$$

with $\begin{cases} d_1 = \dfrac{\ln\frac{S(t)}{KP(t,T)} + \frac{1}{2}v^2(t,T)}{v(t,T)} \\ d_2 = \dfrac{\ln\frac{S(t)}{KP(t,T)} - \frac{1}{2}v^2(t,T)}{v(t,T)} \end{cases}$

And $\begin{cases} v^2(t,T) = V(t,T) + \sigma_s(T-t) + 2\rho_{r,s}\frac{\sigma_r\sigma_s}{a}[T-t-\frac{1}{a}(1-e^{-a(T-t)})] \\ V(t,T) = \frac{\sigma_r^2}{a^2}[T-t+\frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}] \end{cases}$

Note that $\rho_{r,s}$ is the instantaneous correlation parameter that describes the degree of dependence between changes in the interest rate model and the equity model. The correlation is regarded as an endogenous parameter that is obtained by calibration to market prices. σ_s is the instantaneous volatility of stock prices and is also regarded as an endogenous parameter obtained by calibration.

5.3.2 Calibration

From the calibration of the model the parameters σ_s and $\rho_{r,s}$ are set in such a way that the equation (24) best fits the market price of European call options. Appendix B.3 shows option prices, volatilities and strike prices for FTSE 100 index (UKX index) call options retrieved from Bloomberg at May 27th 2013. Option prices are shown for differences in both moneyness and maturity. Moneyness refers to the relative price of the underlying asset with respect to the strike price. A moneyness of 100% means that the option is at the money; the strike price is equal to the spot price. A moneyness below 100% means that the option is out of the money; the strike price is below the spot price. A moneyness above 100% means that the option is in the money; the strike price is above the spot price.

Equation (24) is compared with the market data from Appendix B.3. Options with maturities of 1 month to 2 years are available on the market and only the longest three maturities are used. This is because of the fact that short term option can fluctuate heavily (high volatility) due to short term shocks. By minimizing the sum of squared differences between the market prices and the model prices, parameter values for σ_s and $\rho_{r,s}$ can be found. The modeling is done in Microsoft Excel and the sum of squared differences is found by the Solver Tool. Table 5.2 shows the results that come from the calibrated model that best fits this market data.

	$\boldsymbol{\sigma_s}$	$\boldsymbol{\rho_{r,s}}$
Black-Scholes/Hull-White Results	0.182295	-0.20044

Table 5.2: Black Scholes Parameters calibrated to swaption prices at May 27th 2013

First, from the results it can be found that the instantaneous correlation between interest rates and equity return (-0.20) is negative. This is in line with historical experience, which shows that for the past 30 years the correlation between various index returns and yields on government bonds was between -0.8 and 0 (Choi, Richardson & Whitelaw, 2012). These correlations are calculated for market prices. Second, it can be noticed that the calibrated 1-year stock volatility is 0.18. This is in line with historical observations, which shows that for the past 25 years the yearly index volatility was around 0.2 (Bloomberg, 2013). Baldvinsdóttir and Palmborg (2011) performed the same calibration study for December 31st 2009, which resulted in similar parameter values: 0.267 and -0.224 for $\boldsymbol{\sigma_s}$ and $\boldsymbol{\rho_{r,s}}$ respectively.

5.3.3 Simulations

From the calibration results in Table 5.2 the same simulation method can be used as performed for the instantaneous short term interest rate. To simulate the returns of a specific asset the stochastic differential equation has to be discretized. The most straight forward method to do this discretization is the Euler approximation as explained in paragraph 5.3 (Rouah, 2005) and shown in equation (25). The only difference is now that a correlation is included between instantaneous nominal short term interest rate as found before and the equity return.

$$S(t+\Delta t) = S(t) + S(t)\left[r(t)\Delta t + \rho_{r,s}\sigma_s W(t) + \sqrt{1-\rho_{r,s}^2}\sigma_s Z(t)\right], \tag{25}$$

where, $W \sim N(0, \sqrt{\Delta t})$ and $Z \sim N(0, \sqrt{\Delta t})$ are independent variables. From equation (25) 5,000 Monte Carlo simulations are produced for equity prices. The equity return can be calculated from these prices. The average equity return from all year within the horizon is 0.056.

6. Valuation Results

With the valuation framework completed in Chapter 3, 4 and 5, the actual valuation of the stylized fund sketched in Chapter 2 can be performed. The sponsor's default probabilities are inserted by replacing the stylized AAA rated company by lower rated companies. Besides that, the applications for the pension fund board, the corporate board and the pension regulator can be studied and discussed. At the end of this chapter, the sensitivity of the sponsor support option to market conditions is tested and the model uncertainty is studied.

6.1 Stylized Pension Fund

The fund is assumed to be closed to new members but not for contributions and accruals from current members. By combining the fund demographics and survival probabilities, the remaining time horizon of the pension fund can be calculated. Regardless of the initial number of participants in the fund, the last pensioner will decease at time $t = 70$. All necessary curves need to be extrapolated to at least a maturity of 70. The initial valuation is performed at time $t = 0$. At this moment, the demographics of the pension fund are considered as in appendix A.1. The survival probabilities from appendix A.2 are assumed to be constant for the entire horizon of the pension fund.

Figure 6.1 shows the outgoing liability cash flows from the pension fund. Both nominal and real (nominal plus indexation) cash flows are shown.

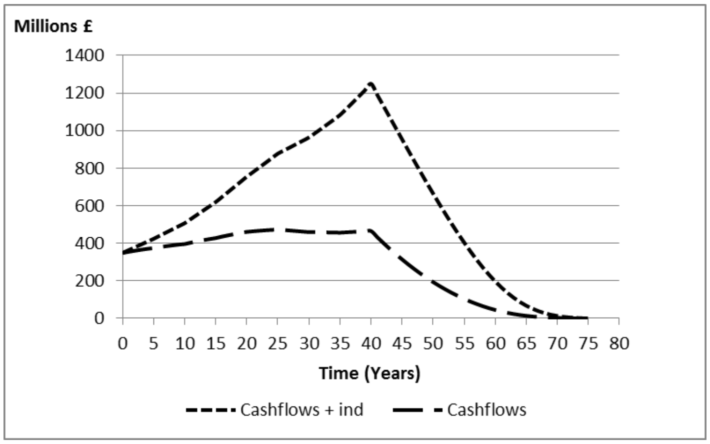

Figure 6.1: Stylized UK Pension Fund Cashflows (t=0 is 2013)

It is assumed that the average final wage in the UK is 40,000£ at time $t = 0$. Figure 6.1 shows that the value of the pension payments increases in the first 40 years. This is because of the fact that until time $t = 40$, each year more people will retire. After 40 years, all people are retired and will decease at some moment. Equation (2) from Chapter 3 is used to calculate the accrued benefits matrix $\boldsymbol{B_{x,t}}$. The columns of this matrix are summed up to get to the total number of accruals per year for the stylized pension fund. By using equation (3) from Chapter 3 the accrued benefits are discounted to each year $t + T$. An additional percentage ζ of 1% on top of the interest rate term structure from nominal Gilts is adopted in this case (which is close to what the average the UK among pension funds applies). Note that the interest rate term structure is derived from the calibrated instantaneous short term interest rate in Chapter 5. By using equation (3), 5,000 different present values for the accrued liabilities at time t were calculated. The expected values were calculated under the risk-neutral measure $\mathbb{Q}$ as explained in Chapter 4. The results are illustrated in Figure 6.2.

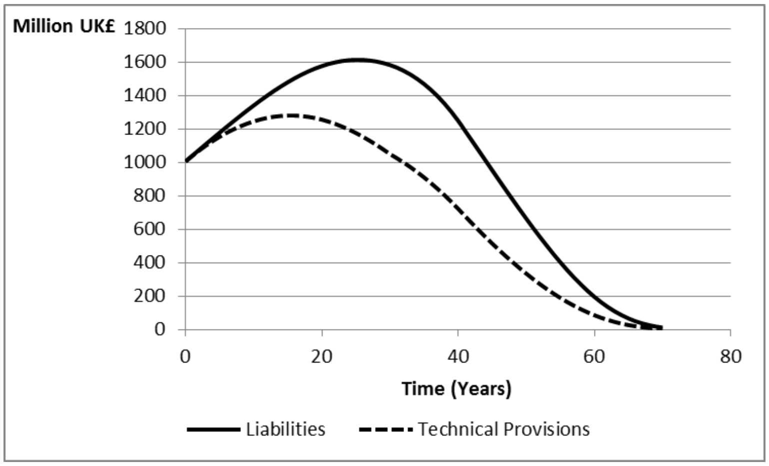

Figure 6.2: Accrued Liabilities in Present Value for the entire horizon

Figure 6.2 represents the liabilities (solid line) of pension fund for each of the years within the horizon of the fund. This line represents the values the assets should have in each year to avoid underfunding. The dotted line represents the liabilities from the solid line discounted to time $t = 0$. Taking $t = 35$ as an example for further explanation: at year $t = 35$ (2045) the pension fund will have accrued liabilities to its participants of 1,400 million UK£. This means that in order to have a sufficient funding ratio (≥ 1) in 2045, the pension fund should hold a certain value in assets at time $t = 0$ (2013). This required asset value is illustrated by the dotted line and is 900 million UK£. The same holds for all of the 70 years. By summing up all the points on the dotted line the total present value of accrued liabilities at time $t = 0$ is calculated. The initial funding ratio was assumed to be 1 (or 100%) so the initial asset value is equal to the initial present value of these liabilities.

Now that all the necessary starting assumptions are known the actual calculation of the sponsor support option value at time $t = 0$ can be performed by applying equation (14):

$$\text{Sponsor Payoff}_t^s = \left[\max\left\{0; L_{N,t+T} - A_t * \exp[\left(r_t^s - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} * \epsilon_t]\right\} * \{1 - (1 - RR)I_{def}\}\right]$$

Portfolio return r has become stochastic and is not constant anymore:

$$r_t^s = w_t * r_{stocks,t}^s + (1 - w_t) * IR_{bonds,t}^s,$$

where $r_{stocks,t}^s$ and $IR_{bonds,t}^s$ are the simulations from Chapter 5 and the portfolio is 60% invested in stocks and 40% invested in inflation linked bonds. The portfolio return volatility changes to:

$$\sigma_t = \sqrt{{w_t}^2{\sigma_s}^2 + (1 - w_t)^2{\sigma_r}^2 + 2\rho_{s,r}\sigma_s\sigma_r w_t(1 - w_t)},$$

where σ_s, σ_r and $\rho_{s,r}$ are the calibration parameters from Chapter 5. The 5,000 Monte Carlo simulations for equation (14) will be performed and the expected sponsor support option value is calculated by the risk neutral valuation method as explained in Chapter 4. The results for the stylized pension fund are shown in Table 6.1. The complete results can be found in Appendix C.1.

	SSO	σ	$\sigma/\sqrt{N}$	**[95%-Confidence Interval]**	
Stylised Fund	0.2543	0.0489	0.0006	0.2529	0.2556

Table 6.1: Sponsor Support Option results for the stylised pension fund

The first column in Table 6.1 and Appendix C.1 shows the sponsor support option value as it is the average of the 5,000 MC simulations. The second and third column show the volatilities of the sample and the average sponsor support option respectively. With the use of the third column, a 95%-Confidence interval as illustrated in the fourth column can be constructed.

The average sponsor support option value is 25.4% of the current assets in the pension fund. This value shows the price of the guarantee the sponsor has given to its pension fund. It could be denoted as the risk premium for bearing the risk of underfunding. Since the calculation does not include sponsor's default probabilities, no clear interpretation in terms of application of the value can be given yet.

6.2 Credit Risk

The assumption that the sponsor will always be able to make the necessary remedial contribution is in reality not plausible. Companies can be credit risky and can default on their obligations. For that reason paragraph in 4.3 an extended approach to the sponsor support option valuation was introduced. Credit risk is dependent on the sponsor's specific default probability and the size of the potential deficit of the pension fund. The latter is dependent on the sponsor's stock return and the broader stock market return. A correlation between sponsor's stock return and broader stock market return of +0.5, a Recovery Rate of 0.40 and a 60/40 asset allocation as in the previous paragraph are assumed in this section. The average maturity of the pension fund's liabilities is 25 years, which is calculated from the solid line in Figure 6.2. Figure 6.3 shows the results of the calculations using the historical default rates according to credit ratings. The complete results can be found in Appendix C.2.

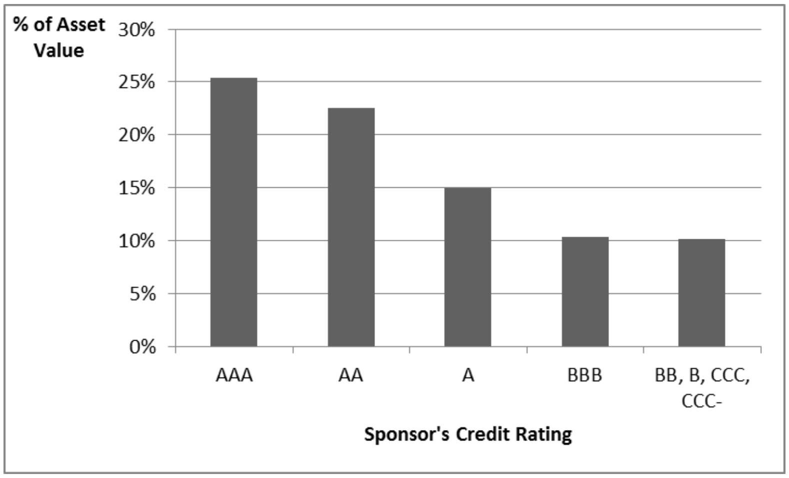

Figure 6.3: Sponsor Support Option Value - Sponsor Credit Rating

As the sponsor becomes more risky (i.e. lower credit rating), the sponsor support option decreases in value. The difference between a risk-free sponsor (AAA-rated) and a AA-rated sponsor is 11.4% (3 percentage points) in sponsor support option value. A single A-rated company has a 41.1% (10.4 percentage points) lower option value than the AAA-rated company. For the BBB- and BB-rated companies the values are 59.3% and 59.9% (15.1 and 15.3 percentage points) respectively lower than the AAA-rated sponsor. A chi-square test results in a p-value smaller than 0.001 ($p < 0.001$) for all of the calculations, which means that the null hypothesis is rejected at a 0.1% significance level. This null hypothesis states that there is no relationship between the two measured phenomena (credit rating and sponsor support option). In other words, the results show statistical significance that there is a relationship between credit rating and sponsor support option.

Companies with credit rating BB or lower are expected to be pure defaulting sponsors as was the result in Chapter 4. The positive relation between companies' credit rating and sponsor support option value can be explained by the fact that if credit risky sponsors are introduced, the sponsor has in fact the possibility to default on its guarantee. Lower rated companies are expected to have a higher default probability than higher rated companies. The guarantee given by lowly rated companies to their pension fund has a lower value than the same guarantee for high rated company. This is reflected in a lower sponsor support option value. A default will only occur if the sponsor is actually not able to make the necessary remedial contribution. In that case it will pay only the Recovery Rate, which is assumed to be 0.40.

Besides using historical default rates according to credit ratings, Chapter 4 introduced the calculation of implied risk-neutral cumulative default probabilities with the use of credit default swap spreads. Figure 6.4 illustrates the sponsor support option values for the companies studied in Chapter 4, compared with the values according to their credit ratings. Note that the characteristics of the stylised pension fund are used, not company specific pension fund's characteristics.

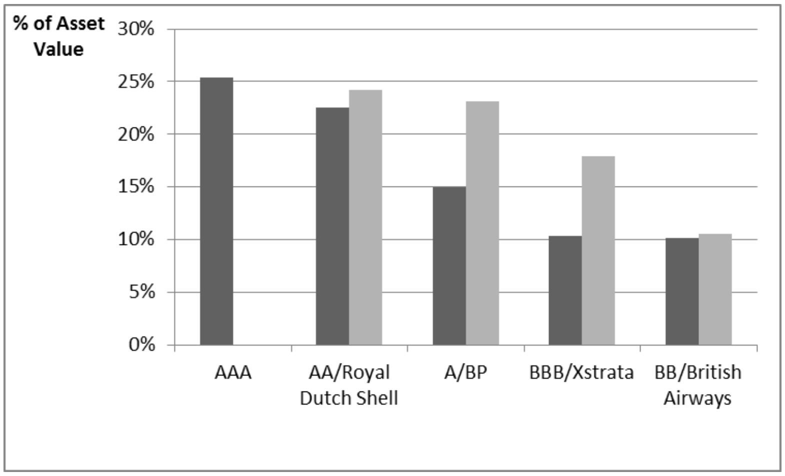

Figure 6.4: Mean Sponsor Support Option Value – Company specific Default Probabilities

As argued in Chapter 4, the historical 1-year default rates would probably result in overestimated cumulative default probabilities for the lower rated companies (A and BBB). The findings in Figure 6.4 confirm this. For Royal Dutch Shell, the results from credit default swaps and historical rates seem to be more or less similar. For BP and Xstrata, the use of credit default swap spread instead of historical default rates according to credit ratings results in significant ($p < 0.001$) higher sponsor support option values. For British Airways, which is assumed to be a company close to a pure defaulting company, the option values under both methods are more or less similar. Calculations for other FTSE 100 companies also show that A and BBB rated cumulative default probabilities are overestimated under the historical rates approach. The positive relationship between credit rating and sponsor support option value that was visible in Figure 6.3, is also shown in Figure 6.4.

6.3 Applications for Pension Fund Board and Corporate Board: Investment Strategy

The stylized pension fund invested 60% in stocks and 40% in inflation linked bonds. It is interesting to study how dependent the resulting sponsor support option is on this assumption. By changing the asset mix in terms of percentage invested in stocks, a sensitivity analysis can be given. Figure 6.5 shows the sponsor support option values for all different credit rated companies, under different pension fund's asset mixes. Despite the fact that previous section proved that under the historical rates approach some of the default probabilities are overestimated, the sensitivity analysis is performed by using this historical rates approach. The reason for this is that not the actual value of the sponsor support option, but the differences between the credit ratings are interesting to study. The complete results can be found in Appendix C.1 and C.2.

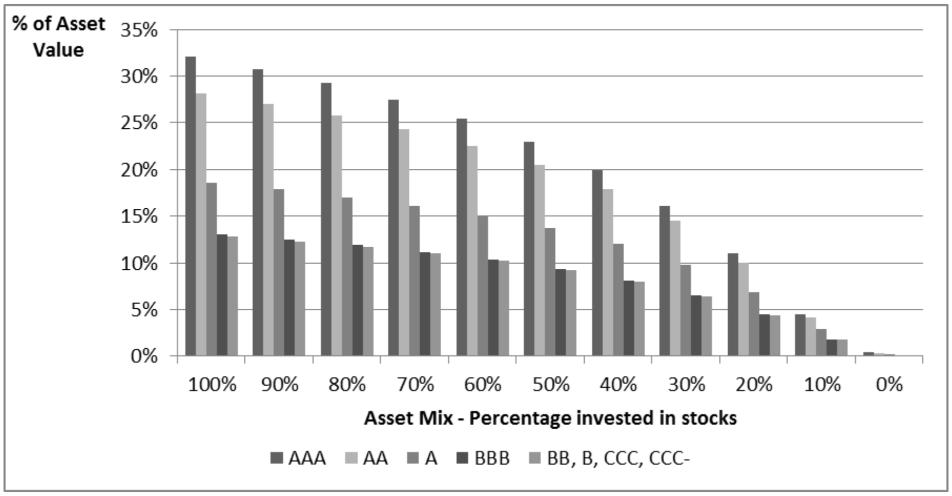

Figure 6.5: Mean Sponsor Support Option Value

The figure shows that a portfolio invested 100% in risky equity holds the highest sponsor support option value for all of the credit ratings. A portfolio with only stocks is more risky than a diverse portfolio with inflation linked bonds (Gilts), which are (quasi) risk-free assets. For a pension fund holding a risky asset portfolio, the probability of a deficit is higher than for a less risky portfolio. Because of that, the guarantee the sponsor has given to the pension fund has a higher value. The sponsor support option value would therefore be expected to decrease if more Gilts are part of the portfolio (i.e. the portfolio becomes less risky). From Figure 6.5 it can be seen that this is exactly the case.

The sponsor support option value is regressed on the investment strategy in terms of percentage invested in stocks, ceteris paribus. The resulting coefficient is 0.3150. This means that for each extra percentage invested in stocks, the sponsor support option would be expected to increase with 0.3150 percentage points of the current asset value. The p-value is extremely small ($p < 0.001$), which means that there is significant evidence (at the 0.1% significance level) against the null hypothesis that the sponsor support option and the investment mix are not related. The R-squared value in this regression is 0.9388, which means that 93.9% of the change in the sponsor support option value is explained by movements in the investment mix.

A pension fund board wants to have the highest possible sponsor support option value (i.e. the highest possible guarantee from the sponsor). This would allow the board to take additional investment risk without actually paying a premium for it. On the other hand, the employer wants to have the lowest possible sponsor support option, since it reflects an implicit liability.

A clear conflict of interest between the pension fund board and the corporate board occurs. It depends on the pension contract or on regulation which party has what decision power in setting the investment strategy. In the United Kingdom, independent pension fund boards have had the highest decision power in the past. New regulation reduces the possibility for the pension fund to game the system and its sponsor (The Pension Regulator, 2013).

Suppose that stock market prices fall. Then, the value of both the pension fund's assets and the sponsor's stock price will fall, assuming that the company's stock price is positively correlated with stock market prices. Under these two conditions the probability that the sponsor will default or even goes bankrupt will be higher, which makes both parties losing. Besides that, a higher sponsor support option value could eventually lead to a downgrading of the sponsor (due to an increase in implicit liabilities), which on itself increases the probability of default. A too risky investment strategy would thus not be optimal from the perspective of the pension fund board.

6.4 Applications for Regulator: Holistic Balance Sheet

The holistic balance sheet (hbs) is discussed in Chapter 1. The current proposal to revise the European pension regulation (IORP) includes the holistic balance sheet (EIOPA, 2012). This balance sheet differs from the traditional balance sheet by adding several security mechanisms to the balance sheet, as for example the sponsor support option. The holistic balance sheet would then better illustrate the economic conditions of the pension fund and take into account real economic risks.

The mean sponsor support option value for the stylised pension fund was calculated in paragraph 6.1 at 25.4% of current asset value. Since the initial funding ratio (under the traditional approach) was assumed to be 100%, the holistic balance sheet approach would result in a funding ratio of 125.4%. Note that the proposed European pension regulation (IORP) includes more security mechanisms (depending on the pension contract) than only the sponsor support option, which can influence the funding ratio under the HBS approach both negatively and positively.

The initial funding ratio (under the traditional approach) can differ from the assumed 100%. Figure 6.6 shows the funding ratios under the traditional approach (black) and the holistic balance sheet approach (black + grey) for different initial funding ratios under the traditional approach. Other security mechanisms that are normally part of the HBS are not taken into account in the graph. The complete results can be found in Appendix C.2.

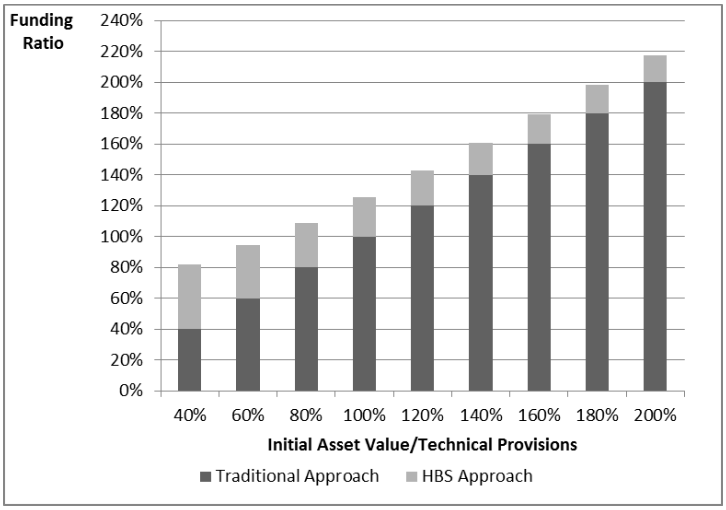

Figure 6.6: Funding Ratios for AAA rated company, 60% invested in stocks

On the horizontal axis the initial funding ratio under the traditional approach is given. It represents the current value of assets divided by the technical provisions at the same moment in time. For example: If asset value is 80% of technical provisions, the sponsor support option value is 29%, which makes the funding ratio under the hbs approach 109%.

In some cases the traditional valuation approach would result in underfunding while the holistic balance sheet approach would result in overfunding. Figure 6.6 shows by how much the hbs funding ratio will increase if the traditional balance sheet is lowered. Solving a simple optimization problem results in a minimum asset value of 67.8% of the technical provisions in order to achieve a funding ratio (under the hbs approach) above 100%.

If the ratio of pension fund's initial asset value over technical provisions increases, the sponsor support option value decreases. This can be explained by the fact that an initially underfunded pension fund will probably need more remedial contributions in the future than others. This results in a higher sponsor support option value. Lower rated companies than AAA show similar results.

Assuming that the regulator's goal is to protect steady pension incomes for pension's participants, the holistic balance sheet is not useful as a regulatory instrument, which is illustrated in Figure 6.7. Higher funding ratios under the holistic balance sheet approach (including sponsor support option) than under the traditional balance sheet approach (excluding sponsor support option) will result in softer funding ratio requirements. In that case, it will amplify the behaviour of the pension fund to increase the riskiness of its investment portfolio, lower pension premiums (decrease assets) or raise pension benefits (increase liabilities) up till a level that the pension fund " holistic" funding ratio (traditional balance sheet plus the sponsor support option in this case) exactly meets the minimum required funding ratio. As calculated above, the minimum asset value that should be hold by the stylised pension fund is 67.8% of the technical provisions.

The increased sponsor support option value can cause the sponsor to be downgraded, since it reflects an implicit liability of the sponsor. This downgrading decreases the sponsor support option value, so that there occurs a gap in the pension fund's balance sheet which was exactly matched with the technical provisions. A remedial contribution from the sponsor will be required, which can again downgrade the sponsor. Moreover, the previous downgrading could indicate that the sponsor had financial insufficiencies in the first place. The new remedial contribution can further downgrade the sponsor which will again result in a decrease in sponsor support option value. This situation can be interpreted as a downwards spiral up till a moment in time where the company's guarantee appears to be not credible anymore. Then, the softened regulation turns out to be a counterproductive instrument for regulatory purposes.

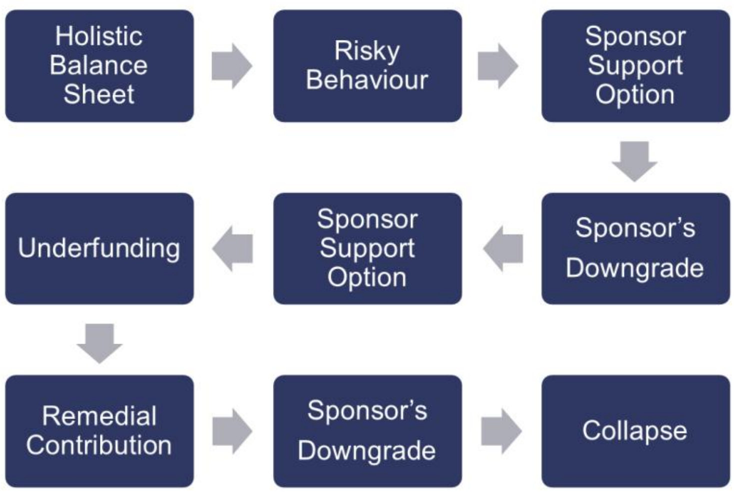

Figure 6.7: The sponsor support option value as regulatory instrument

It should be mentioned that the holistic balance sheet approach as proposed by EIOPA is not based on a system with infinite guarantees provided by the employer and infinite decision power for the pension fund board. Proper governance of the pension fund is therefore necessary. Besides that, pension fund boards should be aware of the fact that policy decisions based on uncertain guarantees will not benefit the continuation of the pension fund.

The only way to save the employees in this situation is to have well-funded pension scheme (without the sponsor option included on the balance sheet) and an investment mix that will not negatively affect the funding status. The sponsor option does negatively affect the traditional funding status and encourages irresponsible behaviour of the pension fund, which should disqualify the sponsor support option as a regulatory tool.

6.5 Sensitivity to Market Conditions

Calibration to market prices of UK interest rate swaptions and FTSE 100 index European call options makes the valuation results extremely dependent on the moment of calibration. The calibrations in Chapter 5 are all based on market prices at May 27th 2013. At this moment, the UK economy still experienced the aftermath of the 2007 financial crisis and the 2010 European sovereign debt crisis. A comparison between market conditions from 2013 and historical market conditions from the past 40 could place the results from paragraph 6.1 in a different perspective. Mean parameter values for the two different market conditions are shown in Table 6.2.

	Market Conditions 1 (based on May 27th 2013)	**Market Conditions 2 (1970-2013)**
Nominal Short term Interest Rate	2.4%	7.3%
Inflation	2.8%	4.0%
30Y Nominal Interest Rate	3.7%	5.6%
Equity Return (Annual FTSE 100)	5.6%	10.1%

Table 6.2: Mean Parameter Values for the two Market Conditions

A comparison between the two Market Conditions results in the findings as shown in Table 6.3. The parameter values based on historical data from Table 6.2 are inserted in equation (14) from Chapter 4. The volatilities of equity return and interest rate return are assumed to be constant and similar to the stylised calculations. The stylised fund characteristics described in Chapter 2 and Chapter 3 are used: the portfolio is invested 60% in equity and 40% in inflation linked bonds and the underlying company has a AAA credit rating. The results are compared with the results from paragraph 6.1.

	$\boldsymbol{SSO}$	$\boldsymbol{\sigma}$	$\boldsymbol{\sigma/\sqrt{N}}$	**[95%-Confidence Interval]**	
Market Conditions 1	0.2543	0.0489	0.0006	0.2529	0.2556
Market Conditions 2	0.1477	0.0473	0.0007	0.1463	0.1490
% Change	-41.7%				

Table 6.3: Comparison between current market conditions (2013) and historical market conditions (1970-2013)

Table 6.3 shows that the sponsor support option value under historical market data is 41.7% (10.7 percentage points) lower than the value under the current market conditions. This result is significant at the 0.1% significance level. Under Market Conditions 2 the returns from inflation linked bonds and equity are higher than under Market Conditions 1. This results in less occurrences of underfunding within the horizon of the pension fund, and thus less remedial contributions. This makes the guarantee the sponsor has to its pension fund (i.e. the sponsor support option) lower in value.

It can be tested what impact the different market variables have on the simulated payoff (equation (14)) values that lead to the sponsor support option. Regressing the payoff values on the instantaneous short term nominal interest rate (ceteris paribus) results in a coefficient of -0.01873. This means that for every percentage point extra return inflation linked bond return, the simulated payoff value will decrease with 1.8 percentage point. A chi-square test results in a p-value below 0.02, which means that there is significant evidence (2% significance level) that there is a relation between the return from inflation linked bonds and the simulated payoff value. The R-squared value is 0.053. This means that only 5.3% of the change in simulated payoff value is explained by movements in the inflation linked bond return.

The same regression is performed for the equity return, which results in a coefficient of -0.28, a p-value of smaller than 0.001 and an R-squared value of 0.596 The fact that the effect from equity return on the simulated payoff value is larger than the effect from inflation linked bond return on the simulated payoff value can be explained by the higher volatility of equity return than the volatility of inflation linked bonds return (0.2 versus 0.075). Larger volatilities result in more risk and thus more volatility in the eventual deficit, which causes the sponsor support option price to increase.

6.6 Parameter Uncertainty

The model that is used to value the sponsor support option in this thesis makes use of some crude assumptions. It can be studied to what degree these assumptions have an impact on the eventual results, which is often referred to as model uncertainty. The two parameters that are studied in this section are the correlation parameter (between stock market price and the sponsor's stock price), which was assumed to be +0.5, and the recovery rate, which was assumed to be 0.4. The sponsor support option values that are calculated in this thesis are expected values derived from simulated payoff value according to equation (14) in Chapter 4. For that reason, the parameter uncertainty is tested on the results from the simulated payoff values.

Correlation Parameter

Studying the sensitivity of the stock market index on the simulated payoff values can be tested by implementing different values for the correlation between sponsor's stock return and stock market index return, which was previously assumed to be +0.5. For the AAA- and BB-rated companies a change in the correlation parameter does not result in a change on the simulated payoff values, since the implied cumulative default probabilities are 0% (never default) and 100% (always default) respectively. For AA-, A- and BBB-rated companies a change in the correlation parameter can cause a change in the simulated payoff values. Figure 6.8 shows the mean results (the expected sponsor support option under risk-neutral measure $\mathbb{Q}$) for a pension fund 60% invested in stocks. The complete results can be found in Appendix C.2.

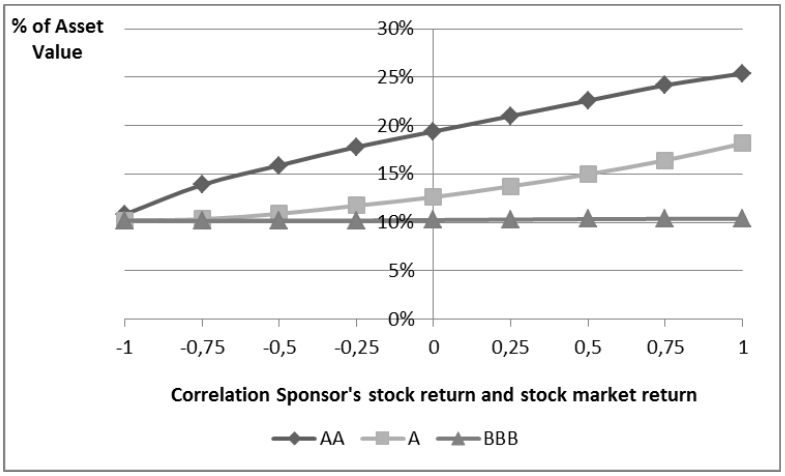

Figure 6.8: Sponsor Support Option Value – Correlation Parameter

The simulated payoff values are regressed on the correlation between stock market return and the sponsor's stock return, ceteris paribus. The resulting coefficients are 0.0705, 0.0402 and 0.0012 for AA-, A- and BBB-rated companies respectively. The complete results can be found in Appendix C.1. The p-values are extremely small (<0.001), which means that there is very strong evidence against the null hypothesis that the simulated payoff values and the correlation parameter are not related (at a 0.1% significance level). The R-squared values in these regressions are all above 0.9, which means that for all of the credit rated companies, at least 90% of the change in the simulated payoff values is explained by movements in the correlation parameter. These results show that the assumption about the correlation parameter is in a large extent affecting the valuation results and therefore needs accurate examination.

A higher correlation with stock market prices implies that the credit state of the company becomes more linked to the market, so more volatile. This high volatility means that the credit state of the company is higher when stock prices increase and lower when stock prices fall, compared to a lower correlation (i.e. low volatility). This means that a high correlation results in less defaults when stock prices are high and more defaults when stock prices are low, compared to a lower correlation. So finally a high correlation results in higher simulated payoff values when stock prices are high and lower simulate payoff values when stock prices are low. This explains the positive coefficient from the regression.

Recovery Rate

The same sensitivity analysis as for the correlation parameter can be performed for the recovery rate (RR), which is in the rest of this thesis assumed to be 0.4. For the AAA-rated companies a change in the recovery rate does not result in a change of the simulated payoff values, since the implied cumulative default probabilities is 0%. For AA-, A-, BBB- and BB-rated companies a change in the correlation parameter can cause a change in the simulated payoff values. Regressing the simulated payoff values on the recovery rate results in coefficients for AA-, A-, BBB- and BB-rated companies of 0.046696, 0.173907, 0.251461 and 0.257456 respectively. The complete results can be found in Appendix C.2. All results are significant against the 0.1% significance level and R-squared for all of the regressions are above 99%. As the credit rating of the sponsor decreases, the effect from a change in recovery rate on the simulated payoff values increases compared to higher rated companies. This is due to the fact that lower rated companies are expected to default more often. A high R-squared value shows that 99% of the change in simulated payoff values is explained by movements in the recovery rate. These results show that the assumption about the recovery rate is largely affecting the valuation results and needs accurate examination.

The results from the stylised fund show how the simulated payoff value (and indirectly the sponsor support option value) is dependent on investment decisions, credit risk, market conditions and assumption about the correlation parameter and the recovery rate. It is also concluded that the sponsor support option can be a useful tool for pension fund risk management, but can be counterproductive for regulatory purposes.

7. Case Study: British Airways Pension Fund

The valuation framework for the sponsor support option developed in this thesis is applied to British Airways Pension Fund. The pension fund was chosen based on publicly available data on the pension fund itself and the sponsor. This example can be used as a blueprint for most corporate DB pension funds. The extended Black-Scholes model for option pricing is used. The model needs three variables as input: outgoing liability cashflows and technical provisions for all of the years within the horizon of the fund, information about investment strategies and market returns and default probabilities of the sponsor. The figures below show the outgoing liability cashflows (solid line) and technical provisions (dotted line) for the two pension schemes British Airways Pension Fund consists of: APS and NAPS. Both schemes are closed for new members. The difference between the curves is due to the fact that APS was closed in 1984 and NAPS in 2003. As a consequence APS is a more mature and “aged” pension scheme compared to NAPS. Funding ratios are 93% for APS and 78.6% for NAPS.

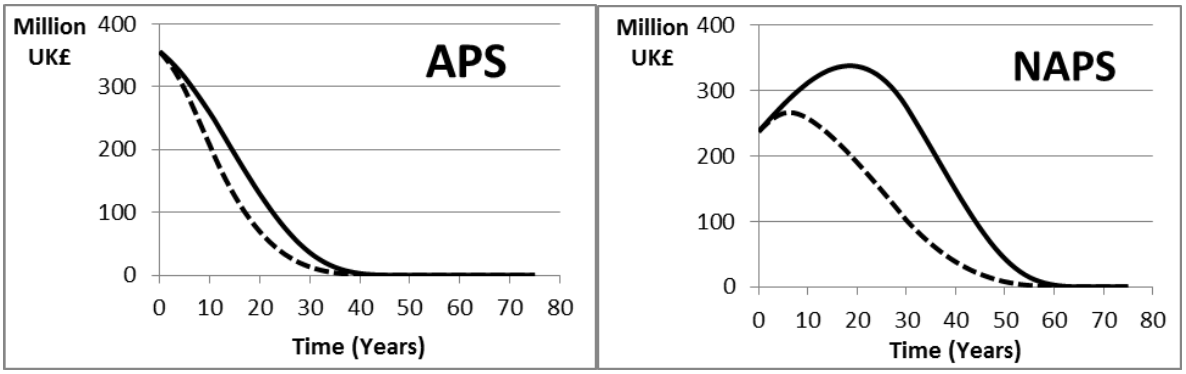

Figure 7.1: Liability Cashflows and Technical Specifications for both British Airways Pensions Schemes

The outgoing liability cashflows are calculated by using the pension payments in 2011, current demographics of the British Airways Pension fund (British Airways Pensions, 2012a, 2012b) and the survival probabilities for average UK population (Appendix A.2). The technical provisions are calculated by using the current interest rates term structure from inflation linked bonds (Gilts) plus 0.3% for pensioners and minus 0.3% for non-pensioners, as this is used by the British Airways Pension scheme (British Airways Pensions, 2012a, 2012b).

APS is 25% invested in equity and 75% invested in bonds and cash. NAPS is 70% invested in equity and 30% invested in bonds and cash. Predictions about returns from equity and return from inflation linked bonds are made by using the calibrations from Chapter 5. The table below shows the calculated sponsor support option values for APS and NAPS relative to their current asset value, excluding the potential default of the sponsor.

Scheme	$\boldsymbol{SSO}$	$\boldsymbol{\sigma}$	$\boldsymbol{\sigma/\sqrt{N}}$	**[95%-Confidence Interval]**	
APS	0.1384	0.0278	0.0004	0.1376	0.1391
NAPS	0.3176	0.0512	0.0007	0.3161	0.3190

Table 7.1: Results British Airways Pensions Sponsor Support Option Valuation – Risk free

The "younger" NAPS scheme has more outstanding liabilities, which results in a higher sponsor support option value. In this table, British Airways is assumed to be a risk free sponsor. This means that it will always be able to make the remedial contribution. This is not a realistic assumption, especially not if one knows that British Airways' credit rating is BB, which is a credit rating that is assumed to be risky (S&P, 2013). The probability that British Airways defaults on the remedial contribution is retrieved from the credit spread from British Airways credit default swaps using the methodology developed in Chapter 4. Average maturities of the liabilities are 8 and 19 years for APS and NAPS respectively. In case of a default British Airways will only pay the Recovery Rate, which is 0.4 of the deficit. The 8-years and 19-

years implied risk neutral cumulative default probabilities are 44.94% and 90.56% respectively. The table below shows the theoretical valuation of the sponsor support option when the sponsor credit risk is taken into account and reflects the difference with Table 7.1.

Scheme	SSO	σ	$\sigma/\sqrt{N}$	**[95%-Confidence Interval]**	
APS	0.1171	0.0235	0.0003	0.1164	0.1177
% Change	-15.4%				
NAPS	0.1613	0.0386	0.0005	0.1602	0.1623
% Change	-49.2%				

Table 7.2: Comparison between naive and extended valuation approaches

Including a default probability results in large and significant (at the 0.1% significance level) decreases in option value for both pension schemes. The higher decrease in the NAPS scheme is due to a longer maturity of liabilities in the NAPS scheme compared to the APS scheme. This results in a higher cumulative default probability.

As showed earlier, the funding ratios for APS and NAPS are 93% and 78.6% respectively. This calculation is performed with the traditional approach (excluding the sponsor support option). One aspect of the earlier explained new European pension regulation (IORP) is the introduction of the so called holistic balance sheet (hbs) approach. On this holistic balance sheet, extra security mechanisms (such as the sponsor support option) can be added next to assets and liabilities from the traditional balance sheet. The proposed approach gives a better insight in the economic conditions of the pension fund. If the sponsor support option value would be added to the traditional balance sheet, the funding ratios of APS and NAPs would be 104.7% (93% + 11.7%) and 94.7% (78.6% + 16.1%) respectively. Where both APS and NAPS are underfunded under the traditional approach, only NAPS is underfunded under the holistic balance sheet approach. Note that under the holistic balance sheet approach more security mechanisms than only the sponsor support option are added, depending on the pension contract. Since the focus of this thesis is on the sponsor support only, these are not included.

7.1 Applications

The results from the sponsor support option valuation for British Airways Pension Fund can be of use for several applications for the different stakeholders in the pension fund. The funding ratios that are given below are all calculated under the holistic balance sheet approach (traditional balance sheet funding ratio plus the sponsor support option value).

Firstly, the *pension fund board* should take the value of the sponsor support option into account when determining the investment strategy. With a contract in which the sponsor bears the entire risk of underfunding of the pension fund, the pension fund board wants to take as much risk as possible in its investment strategy. Even with the probability that the sponsor will default, the pension fund's funding ratio in the APS scheme will be highest with 100% invested in stocks: 114.7%.

On the other hand, the *British Airways board* wants to hold as little guarantees as possible to the pension fund. Its optimal investment strategy would be to invest as safely as possible. Assuming that the regulatory minimum required funding ratio is 100%. From the perspective of British Airways board, the optimal investment strategy in the APS scheme would be 98% in inflation linked Gilts and 2% in stocks, which results in a funding ratio of exactly 100%.

For the NAPS scheme, a change in investment policy alone cannot lead to a funding ratio above 100%. If the pension fund would invest 100% in stocks, the funding ratio would be 96.2%. The sponsor would still be obliged to make a direct remedial contribution of 3.8% of the pension funds asset value.

It depends on the pension contract and regulation which party has how much decision power in the investment policy. The two parties have a severe conflict of interest in terms of the value of the sponsor support option. On the other hand, they share one single interest: If the pension fund would invest too much in stocks and the UK stocks market would fall in the upcoming year, assets from the pension fund and from British Airways would fall. Together with an increase in sponsor support option value which can cause a downgrade of British Airways, this would probably lead to a bankruptcy of British Airways. In that case, both parties would lose. These aspects will play an important role in the negotiations in the "investment game".

In case of a merger between British Airways and another company (as was the case with Iberia in 2011) with a higher (lower) credit rating than British Airways (BB), the value of the sponsor support option will increase (decrease) (assuming that pension guarantees are part of the merger deal). The value of the sponsor guarantee should for that reason be taken into account in M&A negotiations.

Finally, the *UK Pension Regulator* could use the sponsor support option value when setting up Recovery Plans and minimum required funding ratios for pension funds. Higher funding ratios under the holistic balance sheet approach (including sponsor support option) than under the traditional balance sheet approach (excluding sponsor support option) will result in softer capital requirements for the fund. In that case, it will amplify behaviour by British Airways Pensions Fund to reach a funding ratio that exactly meets the minimum required funding ratio. Such a high situation can lead to a downwards spiral as explained in Section 6.4 and illustrated in Figure 6.7. To avoid this situation, proper governance of British Airways Pensions is necessary. Besides that, pension fund boards should be aware of the fact that policy decisions based on uncertain guarantees will not benefit the continuation of the pension fund.

8. Conclusions

Defined benefit pension schemes are based on guarantees provided by the stakeholders in the pension contract. Such guarantees have consequences for policy decisions for the pension fund boards, corporate boards and pension regulators. Especially in the United Kingdom, where only the employers bear the risk of pension fund's underfunding, the value of the sponsor guarantee or sponsor support can be of great importance in contract negotiations and investment decisions. Besides that, current pension funds should not only be worried about regulatory requirements. Real economic risks, both now and in the future, should have the full attention of pension fund's boards. A proper sponsor support option valuation will be a useful instrument in studying these risks.

As the public policy debate shows nowadays, a uniform method for the valuation of the sponsor support option is difficult to find. This is due to the large variety in pension contracts within Europe and due to haziness about sponsor defaults. This thesis developed a simulation based valuation of the sponsor support option for United Kingdom defined benefit pension contracts. By including default probabilities of credit risky sponsors, the sponsor support option value illustrates the real economic value of the guarantee provided by the employer. Previous research derived default probabilities from historical rates based on credit ratings only. However, default probabilities do also depend on broad market conditions at a specific moment in time and the correlation of the company with the broader market. Besides that, default probabilities can vary between companies with the same credit rating. For that reason, default probabilities retrieved from company specific credit default swap spreads and the correlation between the stock market return and the company´s stock return are incorporated in this thesis.

A pension fund board wants to have the highest possible sponsor support option value since it represents an implicit buffer on its balance sheet. On the contrary, the employer is willing to have the lowest possible sponsor support option since it represents an implicit liability.

More risk in the investment portfolio results in higher sponsor support option values. This is because of the fact that risky portfolios have a higher probability of being underfunded than safer portfolios. Lower credit rated companies have lower sponsor support option values since the guarantee they provide to their pension fund are lower in value. Besides that, high correlation between the stock market index prices and the company stock price results in a more volatile sponsor support option value.

From the results it can be concluded that a pension fund board wants to have a risky investment strategy, whereas the corporate board supports a safe investment strategy. However, a too risky investment strategy could (in bad economic conditions) result in a downgrading or even bankruptcy of the company. Then, both parties would lose. A bargaining process determines the eventual decision about the investment strategy.

Since the sponsor support option value reflects the economic conditions of a company, it would be useful when determining the market value of the company in M&A activities. A merger with a company with a different credit rating would then by itself again change the sponsor support option value. In a changing pension environment from DB to DC, the sponsor could decide to eliminate the risk of underfunding and plan a buy off of the guarantee. The support option value can be an instrument in the negotiation of the buy off price.

Finally, the pension regulator can use the sponsor support option value when setting up recovery plans and minimum required funding ratios for pension funds. Higher funding ratios under the holistic balance sheet approach (including sponsor support option) than under the traditional balance sheet approach (excluding sponsor support option) will result in softer funding ratio requirements. In that case, it will amplify the behaviour of the pension fund to increase the riskiness of its investment portfolio, lower pension premiums (decrease assets) or raise pension benefits (increase liabilities) up till a level that the pension fund " holistic" funding ratio (traditional balance sheet plus the sponsor support option in this case) exactly meets the minimum required funding ratio.

The increased sponsor support option value can cause the sponsor to be downgraded, since it reflects an implicit liability of the sponsor. This downgrading decreases the sponsor support option value, so that there occurs a gap in the pension fund's balance sheet which was earlier exactly matched with the technical provisions. A remedial contribution from the sponsor will be required, which can again downgrade the sponsor. Moreover, the previous downgrading could indicate that the sponsor had financial insufficiencies in the first place. The new remedial contribution can further downgrade the sponsor which will again result in a decrease in sponsor support option value. This situation can be interpreted as a downwards spiral up till a moment in time where the company's guarantee appears to be not credible anymore. Then, the softened regulation turns out to be a counterproductive instrument for regulatory purposes.

The only solution to the downwards spiral is proper governance of the pension fund, both by the pension fund board and the corporate board. Besides that, pension fund boards should be aware of the fact that policy decisions based on uncertain guarantees will not benefit the continuation of the pension fund and will harm the pension’s participants.

The only way to save the employees in this situation is to have well-funded pension scheme and an investment mix that will not negatively affect the funding status. The sponsor option does negatively affect the traditional funding status and encourages irresponsible behaviour of the pension fund, which should disqualify the sponsor support option as a regulatory tool on itself.

The results from this thesis show how the sponsor support option value is dependent on investment decisions, credit risk, market conditions and assumption about the correlation parameter and the recovery rate. It is also concluded that the sponsor support option can be a useful tool for pension fund risk management, but can be counterproductive for regulatory purposes without proper governance of the pension fund.

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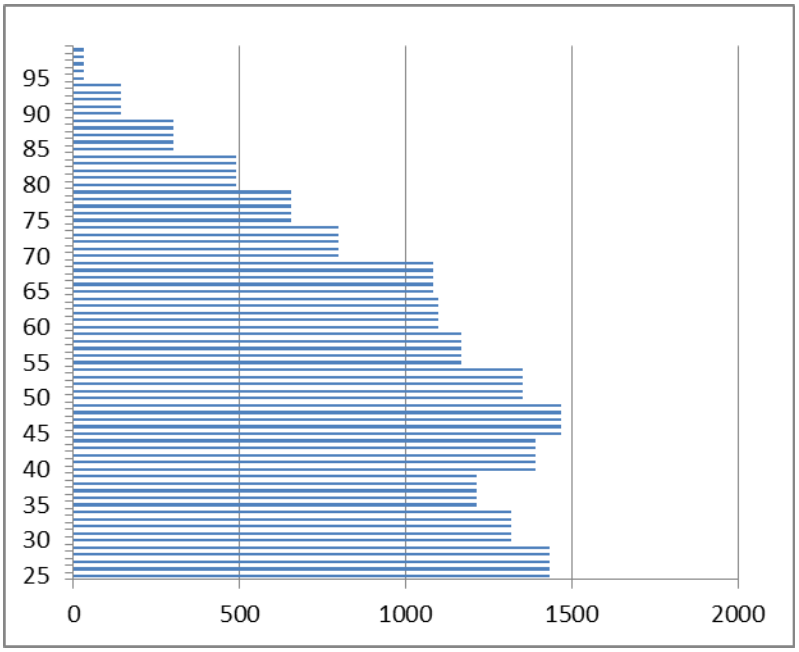
Appendixes

A Demographics

A.1 Initial Age Distribution Stylized Pension Fund 2013



Source: 2010-based National Population Projections, Published 26 October 2011

A.2 Survival Probabilities per age cohort

25	26	27	28	29	30	31	32	33	34
0,999561	0,999549	0,999535	0,999496	0,999483	0,99942	0,999425	0,999358	0,999346	0,999256
35	36	37	38	39	40	41	42	43	44
0,999189	0,999143	0,999037	0,998958	0,998913	0,99877	0,998689	0,99859	0,998493	0,998306
45	46	47	48	49	50	51	52	53	54
0,998181	0,998073	0,998018	0,997773	0,997562	0,997367	0,997064	0,996666	0,996405	0,996065
55	56	57	58	59	60	61	62	63	64
0,995685	0,995152	0,9948	0,994298	0,993813	0,993179	0,992692	0,992214	0,991382	0,990529
65	66	67	68	69	70	71	72	73	74
0,989592	0,988391	0,98751	0,985975	0,984422	0,982602	0,981099	0,979063	0,977144	0,974345
75	76	77	78	79	80	81	82	83	84
0,971889	0,968094	0,964691	0,960322	0,95561	0,949638	0,943636	0,936443	0,928989	0,920326
85	86	87	88	89	90	91	92	93	94
0,910997	0,900899	0,888788	0,876017	0,859035	0,849644	0,837976	0,825813	0,803001	0,7803
95	96	97	98	99					
0,759473	0,741939	0,721352	0,702223	0,687805					

Source: 2010-based National Population Projections, Published 26 October 2011

B Market Data Input

B.1 Cumulative Default Probabilities

Implied Risk-Neutral Cumulative Default Probabilities.
Per company, derived from credit spreads from credit default swaps

Horizon	Royal Dutch Shell	BP	Xstrata	British Airways
0	0,00%	0,00%	0,00%	0,00%
1	0,14%	0,42%	0,91%	1,08%
2	0,41%	1,33%	3,32%	3,94%
3	0,88%	2,44%	6,79%	8,19%
4	1,79%	3,83%	10,47%	15,91%
5	3,78%	6,05%	14,85%	23,67%
7	5,76%	9,75%	23,26%	39,24%
10	9,50%	14,95%	34,00%	56,34%
20	18,45%	28,57%	61,07%	93,64%
30	26,90%	40,96%	82,61%	100,00%
40	34,86%	52,24%	99,76%	100,00%
50	42,37%	62,50%	100,00%	100,00%
60	49,45%	71,85%	100,00%	100,00%
70	56,13%	80,36%	100,00%	100,00%

Hypothetical Cumulative Default Probabilities.
Per credit rating, derived from 1-years historical default rates (S&P, 2013)

Horizon	AAA	AA	A	BBB	BB, B, CCC, CCC-
0	0,00%	0,00%	0,00%	0,00%	0,00%
1	0,00%	2,00%	7,00%	20,00%	100,00%
2	0,00%	3,96%	13,51%	36,00%	100,00%
3	0,00%	5,88%	19,56%	48,80%	100,00%
4	0,00%	7,76%	25,19%	59,04%	100,00%
5	0,00%	9,61%	30,43%	67,23%	100,00%
6	0,00%	11,42%	35,30%	73,79%	100,00%
7	0,00%	13,19%	39,83%	79,03%	100,00%
8	0,00%	14,92%	44,04%	83,22%	100,00%
9	0,00%	16,63%	47,96%	86,58%	100,00%
10	0,00%	18,29%	51,60%	89,26%	100,00%
20	0,00%	33,24%	76,58%	98,85%	100,00%
30	0,00%	45,45%	88,66%	99,88%	100,00%
40	0,00%	55,43%	94,51%	99,99%	100,00%
50	0,00%	63,58%	97,34%	100,00%	100,00%
60	0,00%	70,24%	98,71%	100,00%	100,00%
70	0,00%	75,69%	99,38%	100,00%	100,00%

B.2 Interest Rate Calibration Input

Implied Volatilities from at the money interest rate swaption at May 27th 2013

Maturity	Tenor	1y	2y	3y	4y	5y	7y	10y	15y	20y	30y
1m		0,386	0,388	0,4275	0,425	0,412	0,365	0,31	0,242	0,217	0,203
3m		0,434	0,421	0,4294	0,437	0,418	0,373	0,32	0,253	0,228	0,214
1y		0,586	0,533	0,505	0,46	0,426	0,356	0,3084	0,2491	0,2257	0,219
2y		0,613	0,487	0,439	0,3973	0,376	0,3176	0,2786	0,2364	0,2188	0,2105
3y		0,541	0,4382	0,3917	0,3546	0,329	0,2849	0,2535	0,223	0,2094	0,203
4y		0,4484	0,38	0,343	0,3101	0,293	0,2567	0,2336	0,2108	0,204	0,1949
5y		0,38	0,328	0,301	0,2722	0,2614	0,239	0,2195	0,2033	0,1924	0,192
7y		0,273	0,248	0,236	0,227	0,22	0,211	0,1957	0,1836	0,184	0,181
10y		0,1978	0,197	0,193	0,19	0,187	0,183	0,178	0,173	0,168	0,165
20y		0,174	0,171	0,17	0,168	0,167	0,164	0,1523	0,156	0,15	0,147
30y		0,162	0,163	0,162	0,161	0,16	0,158	0,154	0,148	0,141	0,137

Source: Bloomberg 2013

B.3 Equity Return Calibration Input

European Call Options Prices UKX Index European Call Options at May 27th 2013

Moneyness	Maturity	1M	3M	6M	9M	1Y	2Y
90%	Option Price	646,75	641,50	668,44	701,12	700,80	748,19
	Volatility	0,22	0,19	0,18	0,18	0,18	0,18
	Strike	5924,78	5924,78	5924,78	5924,78	5924,78	5924,78
95%	Option Price	336,57	367,52	421,15	469,12	482,57	564,30
	Volatility	0,17	0,16	0,16	0,17	0,17	0,17
	Strike	6253,94	6253,94	6253,94	6253,94	6253,94	6253,94
100%	Option Price	94,84	157,06	224,05	280,38	307,03	411,66
	Volatility	0,14	0,14	0,15	0,15	0,16	0,17
	Strike	6583,09	6583,09	6583,09	6583,09	6583,09	6583,09
105%	Option Price	9,59	44,61	98,07	147,66	177,85	291,59
	Volatility	0,13	0,13	0,14	0,14	0,15	0,16
	Strike	6912,24	6912,24	6912,24	6912,24	6912,24	6912,24
110%	Option Price	0,70	10,18	36,99	69,92	95,87	203,72
	Volatility	0,14	0,13	0,13	0,14	0,14	0,16
	Strike	7241,40	7241,40	7241,40	7241,40	7241,40	7241,40
115%	Option Price	0,04	2,20	13,21	31,63	49,40	140,65
	Volatility	0,15	0,13	0,13	0,14	0,14	0,15
	Strike	7570,55	7570,55	7570,55	7570,55	7570,55	7570,55

Source: Bloomberg 2013

C Results

C.1 Summary Statistics for AAA rated Company

% in Stocks	SSO	σ	$\sigma/\sqrt{N}$	[95%-Confidence interval]	
1	0,320835	0,056345	0,000797	0,319273	0,322397
0,9	0,307719	0,054906	0,000776	0,306197	0,309241
0,8	0,292268	0,053493	0,000757	0,290785	0,293751
0,7	0,274959	0,051263	0,000725	0,273538	0,27638
0,6	0,254338	0,048984	0,000693	0,25298	0,255696
0,5	0,229321	0,046165	0,000653	0,228041	0,230601
0,4	0,199581	0,041786	0,000591	0,198423	0,200739
0,3	0,161065	0,036081	0,00051	0,160065	0,162065
0,2	0,110219	0,027809	0,000393	0,109448	0,11099
0,1	0,044866	0,015158	0,000214	0,044446	0,045286
0	0,003944	0,003007	4,25E-05	0,003861	0,004027

C.2 Resulting Sponsor Support Option Values

Sensitivity to the investment mix, per credit rating of the sponsor

% in stocks	AAA	AA	A	BBB	BB, B, CCC, CCC-
1	0,320835	0,281551	0,185335	0,130239	0,128554
0,9	0,307719	0,270565	0,178513	0,124907	0,12288
0,8	0,292268	0,257623	0,170438	0,119017	0,11702
0,7	0,274959	0,242904	0,160892	0,111416	0,109877
0,6	0,254338	0,2255	0,149748	0,103459	0,101905
0,5	0,229321	0,204475	0,136806	0,093338	0,092007
0,4	0,199581	0,178453	0,119896	0,08125	0,079889
0,3	0,161065	0,144817	0,098009	0,065793	0,064358
0,2	0,110219	0,100304	0,068278	0,045097	0,044096
0,1	0,044866	0,041193	0,028829	0,018484	0,017925
0	0,003944	0,002992	0,001987	0,001581	0,00159

Comparison between the traditional and the holistic balance sheet

$A_0^s / L_{N,0}^s$	Traditional Approach	$\overline{SSO}$	HBS Approach
40,00%	40,00%	42,31%	82,31%
60,00%	60,00%	34,52%	94,52%
80,00%	80,00%	29,14%	109,14%
100,00%	100,00%	25,41%	125,41%
120,00%	120,00%	22,88%	142,88%
140,00%	140,00%	20,99%	160,99%
160,00%	160,00%	19,57%	179,57%
180,00%	180,00%	18,41%	198,41%
200,00%	200,00%	17,46%	217,46%

Sensitivity to the correlation between sponsor's stock return with the broader market index

Correlation	Rating	AA	A	BBB
	-1	0,108227	0,101783	0,1017
	-0,75	0,1387	0,10323	0,101557
	-0,5	0,158856	0,10888	0,101767
	-0,25	0,177892	0,117213	0,101791
	0	0,193885	0,126047	0,102368
	0,25	0,209898	0,137332	0,102861
	0,5	0,225778	0,149659	0,103353
	0,75	0,241728	0,164139	0,103573
	1	0,253823	0,181377	0,103697

Sensitivity to the Recovery Rate

Recovery Rate	Rating	AA	A	BBB	BB
	0	0,207683	0,080306	0,002805	0,006793
	0,1	0,212194	0,097534	0,027895	0,031591
	0,2	0,216603	0,114919	0,052955	0,05638
	0,3	0,221621	0,131946	0,078271	0,080959
	0,4	0,226174	0,150024	0,103298	0,105535
	0,5	0,231278	0,167661	0,12835	0,130516
	0,6	0,23558	0,184821	0,153692	0,155253
	0,7	0,240422	0,201996	0,17882	0,18001
	0,8	0,244845	0,21909	0,203926	0,204544
	0.9	0,249584	0,236393	0,228971	0,229433
	1	0,254155	0,254331	0,254278	0,25446