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Portfolio Choice over the Life Cycle

A Sector Allocation Approach

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PORTFOLIO CHOICE OVER THE LIFE CYCLE:
A SECTOR ALLOCATION APPROACH

MSc Thesis Economics and Finance of Aging

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ABSTRACT

In a changing pension landscape, asset accumulation and allocation over the life-cycle are increasingly important topics. In this thesis, I refine the basic life-cycle model of portfolio choice by determining the allocation to one of the core asset classes, equity, along sector lines. Using a wide range of sectors, I show that the total allocation to equity is much higher at all ages in a sector allocation model than in a standard specification in which the equity component only consists of a broad stock market index. I find that this is mostly due to better diversification in the sector model. Also differences in correlation between the various equity sectors and labor income growth affect the optimal sector allocations but this hedging effect is small. The higher allocation to equity in the sector model leads to a much higher expected return on financial wealth. As a consequence, both consumption per unit of labor income and wealth per unit of labor income are much higher in the sector model than in the market index model specification.

1. Introduction

Gradual but sustained increases in longevity and considerable changes in the average career path have both had a significant impact on pensions in recent years. They have led to many changes in the provision of pensions and decision-making about retirement-related issues. A general trend observable throughout the wide range of aspects is that it becomes more important for individuals to make retirement choices themselves. Among others, this involves decisions on how much to save during working life and retirement and how to allocate the accumulated assets. An often used theory to help individuals make these choices is the life-cycle portfolio choice framework. In its most basic form, life-cycle portfolio theory assumes away labor income risk and describes a financial market in which only equity and bonds are available. Furthermore, equity is generally taken to be a broad stock market index and bonds are assumed to be risk free. As discussed in more detail below, several authors have altered these highly restrictive assumptions (examples include: Heaton and Lucas, 2000a,b; Viceira, 2001; Cocco, 2005, and, Munk and Sørensen, 2010). However, the changes and extensions made in the existing literature are mostly in the riskiness of labor income and in the number of asset classes available for investment. In this thesis, I expand on the general life-cycle framework by refining one of the core asset classes, equity. A refinement in one of the basic asset classes could have important implications for life-cycle portfolio choice. This clearly holds for equity as within this asset class different subclasses generally behave differently over the business cycle. These differences in cyclicalities can be exploited to create a more diversified portfolio than would be possible when investing only in a broad stock market index. Further, since a working individual also has an exposure to the business cycle via labor income, allocating a portfolio to a set of various equity sectors instead of a broad market index could lead to improved investment decisions via the hedging of labor income risk. This point is closely related to the contemporaneous correlation of returns on equity and human capital which is one of the central issues in life-cycle investment theory. Empirical estimates of this correlation are often small (Heaton and Lucas, 2000b; Cocco, 2005, and Eiling, 2013). However, Heaton and Lucas (2000b) and Eiling (2013) find that once labor income growth is lagged one period, the correlation with stock returns increases sharply. These findings suggest that there is scope for improvement in the life-cycle model by including equity sectors that have different properties. Such a sector allocation approach makes it possible to reduce exposure to sectors that have a relatively high correlation with human capital when young and increase the exposure to these sectors when older. This means that an investor is better able to fine-tune the asset allocation over the life cycle as not only the total allocation to equity can be varied but also within the equity domain shifts in the allocation can occur.

My research adds to the vast literature on life-cycle portfolio choice that has emerged after the seminal contribution by Merton (1969). From that point onwards, the life-cycle literature has developed into several directions. My work fits into the strand of research that concentrates on the effects of labor income for portfolio choice. Particularly in the early work on this topic, labor income is taken to be just a component of total wealth (see, for example, Merton, 1971). Bodie et al. (1992) use the same approach in a more generalized framework. Their work highlights the importance of the riskiness of labor income and the correlation with returns on risky assets. In the baseline riskless labor income case, the ‘investment’ in this risk-free asset increases the allocation to risky assets early in life when the discounted sum of future labor income is large relative to financial wealth. As the investor ages, human capital wealth declines and, to compensate for that, the investor reduces the investment in the risky asset (relative to financial wealth). Bodie et al. (1992) also show that in the other extreme case in which labor income is perfectly positively

correlated with the return on the risky asset, the portfolio allocation to the single risky asset is lower than in the first case in which the correlation is zero. Jagannathan and Kocherlakota (1996) reach similar conclusions on the optimal allocation to stocks and bonds over the life cycle. In addition, they intuitively add the more realistic intermediate case in which labor income is correlated with stock returns but only imperfectly. Viceira (2001) formally incorporates this intuition into the life-cycle model of portfolio choice. He also emphasizes the influence of labor income risk. This latter issue is also raised by several more detailed studies on the influence of different background risks. Heaton and Lucas (2000a,b) are prime examples of this part of the literature. The authors show that households facing higher background risk such as more risky entrepreneurial income should and also do hold less of their wealth in stocks. Cocco et al. (2005) reach a similar conclusion in a study of employment types with different career and layoff risks. More recently, Benzoni et al. (2007) develop a framework in which the optimal allocation to risky assets first increases and then decreases again as the investor ages. This hump-shaped pattern is driven by the cointegration between labor income and dividends. The authors argue that when the investor is young, this cointegration has a long time to act and labor income is thus more risky. However, when the investor becomes older, labor income becomes more ‘bond like’ and the allocation is determined as in the models with low-risk labor.

Although they may differ in their conclusions on the optimal portfolio choice over the life-cycle, the models above have in common that the correlation between returns to labor and a risky asset is one of the crucial elements. However, since there is only one risky asset available in these models, they are not able to take into account the different risk and return properties of different risky assets. Later research has expanded on this by including other risky assets into the basic framework. One of such additional investment classes is housing wealth. An example of a study that incorporates this into the analysis is Cocco (2005) who shows that house price risk reduces equity holdings. In addition to housing, several authors have expanded the set of available asset classes with annuity products. Examples of this strand of the literature are Horneff et al. (2008) and Chai et al. (2009) who also endogenize labor supply and retirement. My research is closely related to these studies but deviates from them in a crucial respect. The studies discussed above incorporate a broader investment set by including more asset classes into the model. I keep the asset classes limited to bonds and equity but split up the latter class into sectors. Also Davis and Willen (2000) examine, among others, the effect of including industry portfolios in the life-cycle model. Nevertheless, a fundamental difference with their work is that I analyze the effect of including equity sectors for an individual not working in a specific sector but with average labor income characteristics. Davis and Willen (2000) examine the effect of including shares of firms from a particular industry in a portfolio of a person working in that same industry. Munk and Sørensen (2010) expand the investment universe by refining an asset class as well. However, they focus on the fixed income side of the portfolio which they refine by incorporating long-term government bonds and stochastic interest rates. For the equity side, they use a broad market index. My thesis complements the existing extensions of the basic life-cycle model by allocating the equity side of the portfolio along sector lines. Such an approach takes into account that there are potential gains from recognizing that not all equity sectors are alike. There are many differences between them in terms of risk, return and cyclicity. The general classification of an equity index as a uniform risky asset does not allow for these differences and makes it therefore impossible to exploit them.

I find that there are large quantitative differences between a life-cycle portfolio model specification in which the equity segment is allocated along sector lines and a specification in which the equity part only

consists of a broad stock market index. In the sector specification, the allocation to equity is much higher than in the market index specification. This is mainly due to more opportunities for diversification in the former specification. The role of hedging of labor income risk is minor. The higher returns on financial wealth in the sector model lead to a much higher consumption per unit of labor income. These findings are robust to changes in labor income measure, growth rate and riskiness of this growth rate.

The remainder of this thesis is structured as follows. Section 2 outlines the life-cycle portfolio choice model used in the calculations while section 3 describes the data and parameter choices. This is followed by the results in section 4. Section 5 concludes.

2. The model

As reviewed in the introduction, depending on the focus of the analysis, there are various specifications of the life-cycle model of portfolio choice in the presence of labor income. The model I use is a multi-period, dynamic model in which consumption and asset allocation are determined simultaneously. In order to fully concentrate on the financial asset side of the model, the labor side of the problem is simplified. I closely follow Viceira (2001) and Campbell and Viceira (2002:182-194) in terms of set-up of the model. The main difference with their model is the higher level of detail in the equity domain.

Following Viceira (2001) and Campbell and Viceira (2002:182-194), I assume that the expected growth rate of labor income is constant over the life cycle and that all shocks to labor income are permanent. The investor is either working or retired. Once retired, the investor does not return to the labor market. The probability that the investor is still employed in the next period depends on her age and is denoted by π_t^y . Analogously, the probability of retirement is $\pi_t^r = (1 - \pi_t^y)$. Both probabilities are strictly positive. In retirement, the investor only receives income from financial wealth. When employed, she receives income both from labor and from financial wealth. The labor income process is described as follows:

$$Y_{t+1} = Y_t e^{g + \xi_{t+1}} \quad (1)$$

Where, Y_t denotes labor income in period t , g is the expected growth rate of labor income and ξ_{t+1} is a normally distributed shock with mean zero and variance σ_ξ^2 .

On the financial asset side, I assume there is only one fixed-income security available which is a risk-free asset. The log (ex-post) real return on this risk-free asset is constant and denoted by r_f . Further, the investor can invest in several equity sectors which are jointly lognormally distributed. The one-period log (ex-post) real return on equity sector i is $r_{i,t+1}$. The excess log return of equity sector i is then

$$x_{i,t+1} = r_{i,t+1} - r_f \quad (2)$$

The vector x_{t+1} contains all sector excess returns. That is,

$$x_{t+1} = \begin{pmatrix} x_{1,t+1} \\ \vdots \\ x_{n,t+1} \end{pmatrix} \quad (3)$$

The expected excess returns on the equity sectors are constant with,

$$E_t[x_{t+1}] = \mu \quad (4)$$

Where μ is a vector of expected excess log returns on the equity sectors.

For simplicity, I assume the variance of the excess log returns on the equity sectors is constant. The covariance matrix of the sector returns is then defined as

$$Var_t[x_{t+1}] = \Sigma_{ee} \quad (5)$$

The excess log return on the equity sectors may also be correlated with the growth rate of labor income. That is,

$$Cov_t(x_{t+1}, \Delta y_{t+1}) = \sigma_{\xi e} \quad (6)$$

Where, Δy_{t+1} is log labor income growth and $\sigma_{\xi e}$ is a vector of the covariances of the different equity sectors with labor income growth.

I assume that the investor's preferences are captured by a utility function of the CRRA type. More specifically,

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad (7)$$

Where, C_t is consumption in period t and γ is the coefficient of relative risk aversion which is strictly positive.

Let $\alpha_{i,t}$ be the fraction of (financial) wealth invested in equity sector i in period t . The vector α_t contains the portfolio weights of all equity sectors. By definition, the remainder of financial wealth is invested in the risk-free asset. Following Viceira (2001) and Campbell and Viceira (2002:182-194), I assume that the investor has an infinite investment horizon. However, as these authors also show, a finite horizon can be incorporated by adjusting the discount factor. Let the investor's discount factor during working life be denoted by δ_y . During retirement, the discount factor changes to:

$$\delta_r = (1 - \pi^d)\delta_y \quad (8)$$

Where, π^d is the inverse of the expected lifetime after retirement.

Using all of the above, the optimization problem of the investor can thus be written as follows:

$$\max_{C_t, \alpha_t} E_t \sum_{t=0}^{\infty} \delta_s^t \frac{C_t^{1-\gamma}}{1-\gamma} \quad (9)$$

Subject to

$$W_{t+1} = (W_t + Y_t - C_t)(1 + R_{w,t+1}) \quad (10)$$

Where, $s = y, r$, W_t is financial wealth at the start of period t and $R_{w,t+1}$ is the one-period discrete net return on financial wealth in period t :

$$R_{w,t+1} = \alpha'_t X_{t+1} + R_f \quad (11)$$

Where, X_{t+1} and R_f are the discrete equivalents of the previously defined variables x_{t+1} and r_f .

The investor's optimization problem as outlined above can be solved analytically only when she is retired and receives no labor income. When the investor is still working, the presence of labor income as an alternative source of income in addition to income from financial wealth complicates the optimization problem. Since labor income growth is risky but has different properties than the asset returns, the optimization problem depends on the realization of labor income growth and is thus not analytically solvable. To obtain an approximate solution in case the investor is not yet retired, I use the log-linearization method developed by Viceira (2001) and Campbell and Viceira (2002). For convenience, I use this method as well for the optimal portfolio allocation of the retired investor. In the model description as in the actual optimization itself, I start with the optimization problem of the retired investor. The main reason for this is that there are feedback effects from the retirement stage to the employment stage as a consequence of the strictly positive probability of retirement in all periods of life. The optimal allocation of the employed investor thus depends on the choices made as a retired investor and hence can be determined only after a solution has been found for the optimization problem in retirement.

As shown by Viceira (2001) and Campbell and Viceira (2001), the budget constraint (8) can be log-linearized. For the retired investor, it is then written as:

$$w_{t+1} - w_t \approx k^r - \rho_c^r (c_t - w_t) + r_{w,t+1} \quad (12)$$

Where, w_t is log financial wealth at the start of period t , k^r and ρ_c^r are constants and $r_{w,t+1}$ is the log return on financial wealth.

A similar procedure can be followed for the budget constraint when the investor receives labor income. The budget constraint in that case reads as (Viceira, 2001, and Campbell and Viceira, 2001),

$$w_{t+1} - y_{t+1} \approx k^y - \rho_w^y (c_t - w_t) - \rho_c^y (c_t - y_t) - \Delta y_{t+1} + r_{w,t+1} \quad (13)$$

Where, y_{t+1} is the log of income in period $t+1$, k^y , ρ_w^y and ρ_c^y are endogenously determined constants.

As shown by Campbell and Viceira (2002:29), $r_{w,t+1}$ can be approximated by:

$$r_{w,t+1} \approx \alpha'_t (x_{t+1} + \frac{1}{2} \sigma_e^2) + r_f - \frac{1}{2} \alpha'_t \Sigma_{ee} \alpha_t \quad (14)$$

Where, σ_e^2 is a vector that contains the variances of the sector excess log returns.

Combining the approximate log return on financial wealth with log-linear approximations of the budget constraints in both the employed and retired states and the utility specification results in optimal consumption and portfolio allocation rules.

For the retired investor, the optimal rules are relatively simple because of the assumption that she receives no labor income. In the retirement stage of life, optimal log consumption, c_t , can be written as a linear function of wealth (Viceira, 2001):

$$c_t = b_0^r + w_t \quad (15)$$

Where, b_0^r is a constant.

In retirement, the optimal asset allocation only consists of a speculative component which captures the risk-return tradeoff of the equity sectors. This implies that from the retirement date onwards, the allocation to equity sectors is fixed. That is,

$$\alpha_t = \alpha = \frac{1}{\gamma} \Sigma_{ee}^{-1} \left(\mu + \frac{1}{2} \sigma_e^2 \right) \quad (16)$$

The optimal rules for the employed investor are more complex. In this case, optimal log consumption is expressed relative to log labor income according to the following equation (Viceira, 2001):

$$c_t - y_t = b_0^y + b_1^y (w_t - y_t) \quad (17)$$

Where, b_0^y is a constant and $b_1^y \in (0,1)$ is the elasticity of consumption to financial wealth for the employed investor. It follows from the parameters π^y , ρ_w^y and ρ_c^y (Viceira, 2001). $(1 - b_1^y)$ is the elasticity of consumption to labor income for the employed investor.

In the asset allocation part of the problem, the presence of labor income means that the optimal share of financial wealth invested in each of the equity sectors is determined as the sum of two components:

$$\alpha_t = \frac{1}{\gamma \bar{b}_1} \Sigma_{ee}^{-1} \left(\mu + \frac{1}{2} \sigma_e^2 \right) - \left(\frac{\pi_t^y (1 - b_1^y)}{\bar{b}_1} \right) \Sigma_{ee}^{-1} \sigma_{\xi e} \quad (18)$$

Where, $\bar{b}_1 = \pi_t^y b_1^y + \pi_t^r$.¹

The first term on the right-hand side of equation (18) is a speculative component. This component captures the tradeoff between risk and return of the equity sectors. It is similar to the optimal asset allocation rule of the retired investor. The difference between the two is that in the employment stage of life, the speculative component is levered up due to the inclusion of an alternative income source next to income from financial wealth. The magnitude of this effect is determined by the retirement probability and hence age of the investor. The second term on the right-hand side of equation (18) is a hedging component. It is an adjustment factor to the speculative component which takes into account the co-movement in labor income growth and sector returns. If an equity sector is positively (negatively)

¹ \bar{b}_1 is a probability weighed average of the elasticity of consumption to financial wealth during employment and retirement. However, during retirement, this elasticity is equal to 1 by construction of the model.

correlated with labor income growth, this term adjusts the optimal allocation downwards (upwards). However, note that, as the portfolio contains multiple sectors, in addition to the correlation between a sector and labor income growth, also the correlation structure of the sectors themselves influences the hedging component. Next to these correlations, the hedging component is also related to the retirement probability of the investor. Just as the speculative component, also the hedging component is thus directly related to the investor's age. Therefore, in contrast to the optimal asset allocation of the retired investor, the optimal allocation to the equity sectors of the employed investor varies across age.

3. Data and parameters

I choose the parameters such that the model fits a U.S.-based investor who only invests domestically and receives the average labor income. I set most of the parameters on the basis of historical data. These data and the resulting parameter choices are described in more detail below. Based on data availability, the sample period starts in January 1973 and ends in December 2013. All data are at a monthly frequency and unless otherwise indicated they are from Datastream. In addition to exogenous parameters, the model also contains several endogenous parameters, i.e., the log-linearization constants. I solve for these endogenous parameters by setting their initial values and then recalculating the model outcome until the parameters have converged.

3.1 Data

As a proxy for the real risk-free rate, I take the 3-month U.S. Treasury-bill rate. I convert this to an ex-post real interest rate by subtracting the CPI inflation rate from the nominal T-bill rate.² For consistency with the equity indices, I compute the interest rate on the basis of a total return index. As sector indices, I select 10 level-1 sector indices covering the following wide range of sectors: Basic materials, consumer goods, consumer services, financials, health care, industrials, oil & gas, technology, telecom, and utilities. To compare the refined life-cycle model to the usual specification with only one broad equity index I use a general total market index. All equity indices are from Thomson Reuters Datastream as these indices have the longest data history.

The final core input of the model is labor income. As discussed in more detail below, there are various measures for labor income. Mainly because of the high frequency (monthly), I use data on labor income from the National Income and Products Accounts (NIPA) Tables of the Bureau of Economic Analysis (BEA).³ The wage data are reported on an aggregate basis. I convert them into per capita income by dividing them by the population as reported in the NIPA tables as well. To get a measure of real income per capita, I deflate the income by the CPI. The (log) rate of real labor income growth is then defined as the log of real income in period $t+1$ divided by the log of income in period t .

There are several possible measures for labor income on the basis of the data in the NIPA tables. I take a narrow definition of labor income in the baseline specification. That is, I only include salaries and wages of employees. This income measure fits best in the setup of the life-cycle model I use. Part of the model assumptions is that the investor only receives income from labor when working and receives no income

² I calculate the CPI inflation rate on the basis of the CPI index for all urban consumers, covers all items and is seasonally adjusted.

³ The data on wages, salaries and the population are part of table 2.6 and are retrieved from: <http://www.bea.gov/iTable/iTable.cfm?ReqID=9&step=1#reqid=9&step=3&isuri=1&910=x&911=0&903=76&904=1973&905=2013&906=q>

when retired. Including other (labor-related) components of compensation such as transfers or social security contributions by employers would thus not be consistent with the assumptions made. In the baseline, I also do not include proprietors' income since this may have different dynamics than wage income which could influence the results (Heaton and Lucas, 2000b).

Table 1 presents summary statistics of the variables.

Table 1: Summary Statistics

	<i>Mean</i>	<i>Standard deviation</i>	<i>Min</i>	<i>Max</i>
<i>Real interest rate</i>	1.09	1.10	-1.08	1.80
<i>CPI</i>	4.26	1.19	-1.79	1.79
<i>Basic Mats</i>	6.94	21.86	-32.42	20.03
<i>Cons Gds</i>	5.03	19.13	-30.67	18.38
<i>Cons Svs</i>	6.06	19.10	-32.79	17.51
<i>Financials</i>	7.10	20.10	-24.48	23.47
<i>Health</i>	7.55	15.90	-20.10	23.73
<i>Industrials</i>	7.53	19.23	-30.86	17.42
<i>Oil & Gas</i>	8.21	19.22	-21.36	18.37
<i>Technology</i>	6.74	24.31	-32.71	19.40
<i>Telecom</i>	6.04	17.81	-17.69	27.36
<i>Utilities</i>	5.52	14.88	-15.49	17.62
<i>Market index</i>	6.15	15.77	-23.78	15.53
<i>Labor income</i>	0.66	2.47	-5.27	6.15

All statistics are expressed in percentage terms. They relate to the excess returns for the equity sectors and to the growth rate for labor income. For convenience, all means are converted to their annual discrete equivalents. Also the standard deviations are annualized.

3.2 Baseline parameters

For the investor's discount factor during employment, I follow the convention in the life-cycle literature and set δ_y equal to 0.96 (see, for example, Campbell and Viceira, 2002: 187; Cocco et al., 2005 and Gomes and Michaelides, 2005). I assume the retirement age is 65 and I fix the expected lifetime after retirement at 19 years. This is the average life expectancy at age 65 in the U.S. according to the latest available data from the National Centre of Health Statistics.⁴ I vary the risk aversion parameter γ between the values 5 and 10. These values give an indication of the results for moderate and high risk-aversion levels.

⁴ Data are retrieved from: <http://www.cdc.gov/nchs/data/hus/hus12.pdf#018>

Most of the asset parameters, such as expected returns, variances and covariances follow directly from the data described above. To obtain an estimate of the hedging capacity of the different sectors ($\Sigma_{ee}^{-1}\sigma_{\xi e}$), I run the following regression:

$$\Delta y_{t+1} = \alpha + \beta' x_{t+1} + \varepsilon_{t+1} \quad (19)$$

Where, ε_{t+1} is an error term with mean 0 and variance σ_{ε}^2 .

Results of the regression are the first column of table 2. All estimated betas are surprisingly low and in some cases even negative. With the exception of industrials they are all insignificantly different from zero. Also the explanatory power of the regression is negligible. Part of the insignificance could be driven by multicollinearity due to the inclusion of a broad range of sectors. However, also jointly the sectors (other than industrials) are not significant which suggests that multicollinearity is not an issue. As a robustness check, I test a specification with a broad definition of labor income in which transfers, social security contributions and proprietors' income are included. Results of this are in the second column of table 2. Although the precise estimates differ somewhat, the results are qualitatively unaltered by using a broader definition of labor income. Again, only the industrials sector enters significantly positively in the regression. The point estimates of the betas are somewhat smaller (in absolute terms) for most of the sectors. The exceptions to this are consumer goods and utilities for which the coefficients increase sharply.

For comparison purposes, columns (3) and (4) report the results of a regression with the total market index. Although the results are not directly comparable to previous research due to differences in the definition of labor income and sample period, they are broadly in line with previous findings (see, for example, Cocco, 2005, Munk and Sørensen, 2010, and Eiling, 2013). The regression results of the market index are similar to those of the sector indices in terms of explanatory power. However, the estimated beta of the broad index is more affected by the choice of labor income. In the baseline regression, the index does not enter significantly while in the robustness check with a broad labor income definition it is highly significant. This change in significance is potentially related to the point made by Heaton and Lucas (2000b). They show that proprietary income has a much higher (positive) correlation with equity. This is included in the broad labor income specification but not in the baseline, which could explain (part of) the change in the significance of the index. Still, some caution is needed as, just as for the baseline labor income definition, explanatory power of the regression is very low in the broad labor income specification.

In contrast to the market index, as mentioned above, despite the inclusion of, among others, proprietors' income, most of the sectors remain insignificant in the broad specification. A possible explanation for this is that in the sector model the correlations of the sectors with each other affects the coefficients. The positive correlation between the sector returns makes the regression coefficients of the individual sectors smaller and hence could potentially lead to insignificance of the individual sectors. However, this is only a partial explanation as also jointly the sectors are not significant. Another part of the explanation is that, with the exception of industrials all sector correlations with labor income growth are lower than the correlation of the market index with labor income growth.⁵ This highlights that the market index is only a particular combination of sectors and not necessarily the optimal one from a life-cycle perspective.

⁵ This holds for both the baseline and the broad specifications. The only exception is the telecom sector which has a higher correlation with labor income growth than the market index in the baseline definition but not in the broad definition of labor income.

Table 2: Regression results

	<i>Sectors</i>		<i>Market index</i>	
	<i>Baseline definition (1)</i>	<i>Alternative definition (2)</i>	<i>Baseline definition (3)</i>	<i>Alternative definition (4)</i>
<i>Basic Mats</i>	-0.01526 (0.01039)	-0.00625 (0.00629)		
<i>Cons Gds</i>	0.00363 (0.01047)	0.00610 (0.00805)		
<i>Cons Svs</i>	-0.01296 (0.01505)	-0.00930 (0.01156)		
<i>Financials</i>	0.00464 (0.01114)	0.00401 (0.00856)		
<i>Health</i>	-0.01478 (0.01164)	-0.01186 (0.00894)		
<i>Industrials</i>	0.03962*** (0.01531)	0.02259* (0.01177)		
<i>Oil & Gas</i>	-0.00354 (0.00818)	0.00296 (0.00629)		
<i>Technology</i>	-0.00626 (0.00768)	-0.00049 (0.00590)		
<i>Telecom</i>	0.01215 (0.00817)	0.00422 (0.00628)		
<i>Utilities</i>	0.00074 (0.01022)	0.00336 (0.00785)		
<i>Market index</i>			0.00941 (0.00707)	0.01568*** (0.00541)
<i>Constant</i>	0.00048 (0.00032)	0.00084*** (0.00024)	0.00049 (0.00032)	0.00084*** (0.00025)
<i>Obs.</i>	491	491	491	491
<i>Adjusted R²</i>	0.0053	0.0110	0.0016	0.0149

The results are based on the monthly labor income growth figures computed on the basis of the NIPA labor income data. Standard errors are in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level respectively.

The final exogenous parameters that need to be set are the baseline labor income growth and risk parameters. In principle, the expected growth rate and variance can be easily derived from the NIPA labor income data. However, this yields estimates for both parameters that are much lower than estimates reported in previous work on the basis of data on individuals.⁶ For example, a conventional estimate of the (average) growth rate of labor income is 3 per cent per year (see, for example, Carroll and Samwick, 1997, and Munk and Sorensen, 2010).⁷ A further issue is that labor income growth tends to vary over the life cycle (see, for example, Cocco et al., 2005, Gomes and Michaelides, 2005, and Benzoni et al., 2007). Although this feature cannot be incorporated in the model directly, it can indirectly by computing the effects of varying the growth rate of labor income (Viceira, 2001). To incorporate both issues into the analysis, I test the model outcome for different growth rates of labor income: 0, 3 and 6 per cent on an annual basis. In the baseline specification I set the growth rate at the mid-point value and conventional

⁶ These estimates are mostly based on the Panel Study of Income Dynamics (PSID) of the University of Michigan.

⁷ Usually the definitions of labor income are somewhat broader than the baseline definition I use.

estimate of 3 per cent per year. For the riskiness of labor income growth, I follow a similar procedure. In the baseline calculations, I use a standard deviation of labor income growth of 10 per cent per year (following Viceira, 2001, and Campbell and Viceira, 2002:182-194). In a robustness check, I examine the effect on the model outcome of a lower risk of the growth rate of labor income.

4. Results

4.1 Baseline results

The baseline results are presented in table 3. For comparison purposes, the equivalent results of a model specification with only a broad market index are included in the table as well. The general pattern of the allocation to equity is similar in both specifications. The allocation to equity is highest when young and gradually declines as the investor ages. This is a standard result in the life-cycle portfolio choice literature (see, among others, Bodie et al. 1992, Viceira, 2001, Campbell and Viceira, 2002, and Cocco et al., 2005). As already indicated before, my work is most closely related to the research of Viceira (2001) and Campbell and Viceira (2002:182-194). Therefore, a direct comparison between my results on the index specification and the findings of these authors is useful.⁸ There are some notable differences between the two sets of results. For a risk aversion level of 5, the equity portfolio shares presented here are considerably higher. In the high risk aversion case, they are still higher than those of Campbell and Viceira (2002:182-194) at all ages but lower than those of Viceira (2001) up to the age of 50. These differences in the optimal allocations are a consequence of the differences in the choice of the parameters on the investor's discount rate, lifetime after retirement, and risk and expected return of the market index in excess of the risk-free rate.

The main difference between the sector and market index specifications is the much higher allocation to equity in the model with sectors rather than an index. This holds also during retirement which indicates that it is caused to a considerable extent by better diversification opportunities when investing in different equity sectors rather than a broad index. Another, albeit smaller part of the large differences in allocation to equity between the sector model and the index model is explained by the different correlations with labor income. This is most clearly illustrated in table 4 which shows the hedging component for a risk-aversion level of 5. In the sector allocation model, some of the hedging components are positive while others are negative.⁹ For the market index all hedging parts are negative. The hedging components of the sector model indicate that it is optimal to adjust the allocation upwards to some sectors and downwards to others. As this is not possible when investing in a broad market index, these results suggest that a somewhat better asset mix can be achieved by investing in equity sectors rather than only in a market index with fixed sector weights. However, the adjustment and hence potential improvement of the asset allocation due to better hedging of labor income risk is small for most of the sectors. Surprisingly, there is also no clear pattern in terms of allocation adjustments to more or less cyclical sectors. While the very cyclical basic materials and technology sectors have a positive hedging component, the portfolio weight of industrials is quite strongly adjusted downwards. On the more defensive side, utilities and telecoms both

⁸ I compare my results to their results for the case in which the correlation between stock returns and labor income growth is 0. Given the low estimated regression coefficient for the baseline labor income definition, these results are most comparable to mine.

⁹ Note that in the case of a short position a positive value indicates that the short position would have been larger in the absence of a hedging part.

have a negative hedging component whereas the optimal allocation to oil and gas and health care is higher than in the case in which there would be no hedging component. These hedging components directly correspond to the sector betas from the regression in section 3. A positive (negative) regression coefficient there translates into a negative (positive) hedging component here. This direct relation between the hedging components and regression results implies that the hedging components of the sectors are mainly driven by a combination of two elements: the correlation between a sector and labor income growth and the correlations of the sector with the other sectors. However, what ultimately determines these two elements is beyond the scope of the research here. A third element driving the hedging components is a scaling factor that depends on the age of the investor. However, as this varies only across time, it is the same for all sectors at a certain point in time.

A final point worth discussing is the large positive and negative hedging components for consumer goods and services. These can be explained by the relatively small speculative allocations to these sectors. As discussed in the model description of section 2, in the employment stage, the optimal allocation to the sectors consists of a speculative and a hedging part. For the consumer goods and services sectors, the speculative component of the allocation is quite small relative to the allocation to the other sectors. The hedging part is similar to that of other sectors. Taken together, the small speculative component combined with the moderate hedging part imply that the hedging component has a relatively large weight in the optimal allocation. As a percentage of the portfolio share (in absolute value), the adjustment in the allocation to these sectors due to the correlation with labor income growth is thus large.

A second noteworthy difference between the model specification with the sector allocation and the specification with a market index is the speed of convergence towards the retirement solution. In both specifications, the speed of convergence is increasing over the life cycle. That is, the investor draws down the equity share of the portfolio in an accelerating pace. However, in the low risk aversion case, the equity share of the portfolio decreases much more rapidly in the market index specification than in the sector model. In the high risk aversion case, the opposite holds. This difference in convergence speed between the two specifications is, again, to a large extent explicable by diversification. A higher risk aversion level leads to a lower optimal allocation to equity in both model specifications. Nevertheless, the decline in the equity share is considerably smaller in the sector specification than in the broad index case. This is directly related to the risk-return properties of the optimal portfolios (not shown). For the low level of risk aversion the optimal portfolios of the sector model are more risky than those of the market index model at all ages. In the case risk aversion is high, the sector model starts to yield less risky optimal portfolios than the market model specification from around the age of 35. However, this lower level of risk is achieved for a total equity share that is about double that of the market model. This again indicates that particularly in a life-cycle setting, the optimal (total) asset allocation is strongly dependent on the allocation choices made within the equity segment.

As a final illustration of the large differences between the two model specifications, table 5 presents the expected consumption-to-labor income and wealth-to-labor income ratios. Just as the optimal asset allocations, the optimal consumption-to-labor income and wealth-to-income ratios differ strongly between the two model specifications. Both ratios are much larger in the sector model than in the market specification due to the much higher returns on financial wealth generated under the former specification. These higher returns allow for a much higher consumption and wealth accumulation per unit of labor income. Nevertheless, in comparison with the findings of previous, related work such as Viceira (2001) and Campbell and Viceira (2002:182-194) also the wealth-to-labor income ratios of the market index

specifications are very high. This is mainly caused by the different choices for the discount factor, expected lifetime after retirement and, to a lesser extent, risk and return properties of the market index.

Table 3: Baseline results

γ		Age								
		25	30	35	40	45	50	55	60	Retired
5	<i>Basic Mats</i>	-46.12	-45.30	-44.37	-43.22	-41.89	-40.31	-38.46	-36.41	-34.86
	<i>Cons Gds</i>	-0.74	-0.71	-0.69	-0.66	-0.63	-0.59	-0.55	-0.50	-0.46
	<i>Cons Svs</i>	-3.90	-3.85	-3.79	-3.73	-3.65	-3.56	-3.45	-3.33	-3.24
	<i>Financials</i>	-15.29	-15.00	-14.67	-14.26	-13.79	-13.24	-12.58	-11.86	-11.31
	<i>Health</i>	64.66	63.45	62.07	60.36	58.38	56.05	53.29	50.26	47.96
	<i>Industrials</i>	60.41	59.37	58.19	56.73	55.05	53.05	50.70	48.12	46.15
	<i>Oil & Gas</i>	49.30	48.39	47.36	46.07	44.59	42.84	40.77	38.50	36.76
	<i>Technology</i>	-29.51	-28.97	-28.37	-27.63	-26.76	-25.74	-24.54	-23.22	-22.21
	<i>Telecom</i>	23.26	22.85	22.39	21.83	21.17	20.39	19.47	18.46	17.69
	<i>Utilities</i>	5.09	5.00	4.90	4.77	4.62	4.44	4.23	4.00	3.83
	Total equity	107.18	105.22	103.00	100.26	97.08	93.32	88.89	84.02	80.31
	<i>Market index</i>	84.01	78.77	73.77	69.01	64.46	60.11	55.90	51.75	48.47
10	<i>Basic Mats</i>	-28.34	-27.01	-25.67	-24.32	-22.94	-21.52	-20.07	-18.59	-17.43
	<i>Cons Gds</i>	-0.63	-0.58	-0.53	-0.48	-0.43	-0.38	-0.33	-0.27	-0.23
	<i>Cons Svs</i>	-1.84	-1.81	-1.79	-1.76	-1.73	-1.70	-1.67	-1.64	-1.62
	<i>Financials</i>	-9.85	-9.34	-8.83	-8.30	-7.77	-7.23	-6.67	-6.10	-5.66
	<i>Health</i>	41.45	39.32	37.18	35.01	32.80	30.53	28.21	25.84	23.98
	<i>Industrials</i>	36.19	34.59	32.98	31.36	29.70	28.00	26.25	24.47	23.08
	<i>Oil & Gas</i>	31.24	29.67	28.09	26.50	24.87	23.20	21.50	19.75	18.38
	<i>Technology</i>	-18.30	-17.42	-16.54	-15.65	-14.74	-13.80	-12.85	-11.87	-11.11
	<i>Telecom</i>	14.08	13.44	12.80	12.15	11.49	10.81	10.11	9.40	8.85
	<i>Utilities</i>	3.18	3.02	2.87	2.71	2.55	2.39	2.22	2.05	1.91
	Total equity	67.17	63.87	60.56	57.21	53.79	50.29	46.70	43.04	40.15
	<i>Market index</i>	29.94	29.16	28.41	27.71	27.02	26.34	25.64	24.90	24.24

Allocation of financial wealth to the different sectors and the total allocation to equity (bold numbers) at different ages and for different risk aversion levels. The final row contains the allocation to equity in case the investor invests a broad equity market index rather than sector indices. All are in percentage terms. For all the calculations labor income growth is set to 3 per cent per year in discrete terms, the standard deviation of labor income growth is set to 10 per cent annually.

Table 4: Baseline Results, hedging components

γ		Age							
		25	30	35	40	45	50	55	60
5	<i>Basic Mats</i>	1.1209	1.0597	0.9840	0.8880	0.7703	0.6201	0.4295	0.1957
	<i>Cons Gds</i>	-16.5470	-15.7946	-14.8428	-13.6033	-12.0291	-9.9299	-7.1061	-3.3760
	<i>Cons Svs</i>	11.2736	10.6003	9.7769	8.7492	7.5118	5.9695	4.0677	1.8177
	<i>Financials</i>	-1.0167	-0.9623	-0.8948	-0.8090	-0.7033	-0.5678	-0.3947	-0.1807
	<i>Health</i>	0.7733	0.7319	0.6804	0.6151	0.5346	0.4315	0.2998	0.1371
	<i>Industrials</i>	-2.2150	-2.0929	-1.9419	-1.7509	-1.5171	-1.2196	-0.8431	-0.3833
	<i>Oil & Gas</i>	0.2398	0.2269	0.2109	0.1906	0.1655	0.1335	0.927	0.423
	<i>Technology</i>	0.7214	0.6822	0.6336	0.5720	0.4964	0.3998	0.2771	0.1264
	<i>Telecom</i>	-1.7722	-1.6749	-1.5546	-1.4022	-1.2155	-0.9777	-0.6764	-0.3078
	<i>Utilities</i>	-0.4643	-0.4391	-0.4079	-0.3683	-0.3197	-0.2576	-0.1786	-0.0815
	<i>Market index</i>	-0.8365	-0.7606	-0.6781	-0.5885	-0.4903	-0.3829	-0.2626	-0.1252

Table shows the adjustment to the allocation at a certain age compared to the situation in which the hedging part as described in section 3 would have been zero. The hedging components are expressed relative to the absolute value of the actual allocation (α_{it}) and are in percentage terms.

Table 5: Baseline Results, consumption and wealth ratios

<i>C/Y</i>		Age							
		25	30	35	40	45	50	55	60
	<i>Sectors</i>	2.2879	2.3094	2.3460	2.4062	2.5043	2.6720	2.9846	3.6637
	<i>Market index</i>	1.2048	1.2283	1.2569	1.2904	1.3315	1.3834	1.4522	1.5562
<i>W/Y</i>	<i>Sectors</i>	48.4567	48.2374	48.3320	48.9041	50.2342	52.9338	58.4580	71.0771
	<i>Market index</i>	24.7834	27.2979	30.1506	33.3349	36.9086	40.9386	45.5594	51.2754

Table shows the expected consumption-to-labor income ratios and wealth-to-income ratios (all in discrete terms) at different ages for the sector specification of the model and the market index specification. For all the calculations labor income growth is set to 3 per cent per year in discrete terms, the standard deviation of labor income growth is set to 10 per cent annually. Risk aversion is at a level of 5.

4.2 Robustness checks

As a first check on the effects of different parameters, I continue the robustness check with a different labor income definition that I performed earlier in the regression analysis as well. For comparability of the results, I assume the growth rate of labor income and the volatility of this rate are the same as in the baseline definition. The only difference between the two definitions is thus in the correlation of the returns with labor income. Results of the model with the broad labor income definition are shown in table 6. As expected on the basis of the regression results, this changes the model outcome somewhat but these

changes are very small both quantitatively and qualitatively. In the broad labor income specification, the beta estimates are smaller for most of the sectors but larger for some of them. The portfolio allocations reflect these differences in the estimated betas. As their beta estimates are smaller for the broad labor income definition, the portfolio weights of, for example, industrials, health care and telecoms are higher than in the baseline. On the contrary, the allocation to basic materials, consumer goods and utilities is lower. The total equity share of the portfolio is about equal to that of the baseline in all periods. The effect of the different income measure on the market index model is slightly stronger. The allocation to equity in that model is unambiguously lower than under the baseline income definition. However, quantitatively, the differences are quite small.

Table 6: Robustness Check, broad labor income definition

	Age								
	25	30	35	40	45	50	55	60	Retired
<i>Basic Mats</i>	-46.56	-45.70	-44.72	-43.53	-42.13	-40.48	-38.56	-36.46	-34.86
<i>Cons Gds</i>	-0.82	-0.80	-0.77	-0.73	-0.69	-0.63	-0.57	-0.51	-0.46
<i>Cons Svs</i>	-4.03	-3.97	-3.91	-3.83	-3.73	-3.62	-3.49	-3.35	-3.24
<i>Financials</i>	-15.31	-15.02	-14.68	-14.28	-13.80	-13.23	-12.58	-11.86	-11.31
<i>Health</i>	64.74	63.51	62.11	60.41	58.39	56.03	53.27	50.25	47.96
<i>Industrials</i>	61.14	60.04	58.79	57.27	55.47	53.36	50.90	48.20	46.15
<i>Oil & Gas</i>	49.22	48.31	47.26	46.00	44.51	42.75	40.71	38.47	36.76
<i>Technology</i>	-29.78	-29.23	-28.60	-27.83	-26.92	-25.85	-24.61	-23.25	-22.21
<i>Telecom</i>	23.59	23.16	22.67	22.07	21.36	20.53	19.56	18.50	17.69
<i>Utilities</i>	5.02	4.93	4.83	4.71	4.57	4.40	4.20	3.99	3.83
Total equity	107.21	105.24	102.99	100.26	97.03	93.24	88.82	83.98	80.31
<i>Market index</i>	83.33	78.19	73.32	68.64	64.18	59.92	55.77	51.71	48.47

Allocation of financial wealth to the different sectors and the total allocation to equity (bold numbers) at different ages. All are in percentage terms. For all the calculations the coefficient of relative risk aversion is set at 5. The percentage annual discrete growth rate of labor income is 3 per cent per year in discrete terms, the standard deviation of labor income growth is set to 10 per cent annually.

As outlined in section 3, in addition to the baseline value of 3 per cent, I examine what the implications for the model outcome are of different labor income growth rates. Table 7 shows the model outcomes for a real labor income growth rate of 0 and 6 per cent on an annual basis. In the no-growth case, the portfolio allocation is essentially unchanged over the life-cycle. The presence of labor income gives rise to a somewhat higher allocation to the sectors in earlier years but the allocation remains very close to the retirement solution. The absence of growth in labor income makes the investor more reliant on income from financial wealth which implies the optimal asset allocation is closer to the retirement solution at all ages. This stands in sharp contrast to the high labor income growth case. In this situation, the allocation to the sectors is levered up strongly. Although the exact long and short positions are admittedly extreme, the results indicate that a higher labor income growth makes it optimal for the investor to take considerably larger equity positions (in absolute terms) when working. The younger the investor is, the larger the total

equity share in the portfolio. The market index specification displays the same patterns. The difference between the two models is again in the magnitude of the numbers. That is, a higher growth rate of labor income increases the optimal allocation to equity more in the sector model than in the market index specification.

Table 7: Robustness Check, labor income growth

Growth rate		Age								
		25	30	35	40	45	50	55	60	Retired
0	<i>Basic Mats</i>	-34.93	-34.92	-34.90	-34.89	-34.89	-34.88	-34.87	-34.86	-34.86
	<i>Cons Gds</i>	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46
	<i>Cons Svs</i>	-3.24	-3.24	-3.24	-3.24	-3.24	-3.24	-3.24	-3.24	-3.24
	<i>Financials</i>	-11.33	-11.33	-11.33	-11.32	-11.32	-11.32	-11.31	-11.31	-11.31
	<i>Health</i>	48.06	48.04	48.02	48.01	48.00	47.99	47.97	47.97	47.96
	<i>Industrials</i>	46.24	46.22	46.21	46.19	46.18	46.17	46.16	46.16	46.15
	<i>Oil & Gas</i>	36.84	36.83	36.81	36.80	36.79	36.79	36.78	36.77	36.76
	<i>Technology</i>	-22.26	-22.25	-22.24	-22.24	-22.23	-22.23	-22.22	-22.22	-22.21
	<i>Telecom</i>	17.73	17.72	17.72	17.71	17.71	17.70	17.70	17.70	17.69
	<i>Utilities</i>	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83
	Total equity	80.47	80.44	80.42	80.39	80.37	80.35	80.33	80.32	80.31
<i>Market index</i>	48.54	48.53	48.52	48.51	48.51	48.50	48.49	48.48	48.47	
6	<i>Basic Mats</i>	-83.49	-76.55	-69.89	-63.50	-57.35	-51.43	-45.63	-39.82	-34.86
	<i>Cons Gds</i>	-1.65	-1.48	-1.32	-1.16	-1.01	-0.87	-0.72	-0.58	-0.46
	<i>Cons Svs</i>	-6.07	-5.67	-5.28	-4.91	-4.55	-4.21	-3.87	-3.53	-3.24
	<i>Financials</i>	-28.48	-26.04	-23.68	-21.43	-19.25	-17.16	-15.12	-13.06	-11.31
	<i>Health</i>	120.09	109.80	99.92	90.44	81.32	72.54	63.94	55.31	47.96
	<i>Industrials</i>	107.71	98.93	90.50	82.41	74.62	67.13	59.79	52.42	46.15
	<i>Oil & Gas</i>	90.92	83.19	75.78	68.66	61.81	55.22	48.76	42.28	36.76
	<i>Technology</i>	-53.70	-49.21	-44.90	-40.76	-36.78	-32.95	-29.19	-25.42	-22.21
	<i>Telecom</i>	41.73	38.30	35.01	31.85	28.81	25.88	23.02	20.14	17.69
	<i>Utilities</i>	9.30	8.52	7.77	7.05	6.36	5.69	5.04	4.38	3.83
	Total equity	196.34	179.79	163.90	148.65	133.98	119.85	106.01	92.13	80.31
<i>Market index</i>	139.98	123.39	108.44	95.12	83.42	73.19	64.21	55.85	48.47	

Allocation of financial wealth to the different sectors and the total allocation to equity (bold numbers) at different ages. All are in percentage terms. For all the calculations the coefficient of relative risk aversion is set at 5. The percentage annual discrete growth rate of labor income is stated in the first column, the standard deviation of labor income growth is set to 10 per cent annually.

As a final change to the baseline parameter values, I rerun the model for a lower level of labor income risk. Table 8 presents the outcome for a standard deviation of 2.47 per cent per year. This is the standard deviation of labor income growth of the baseline NIPA labor income data. I consider this to be a reasonable lower bound. Furthermore, such a large decrease in the labor income risk parameter gives a clear picture on the general effects of this risk on the optimal portfolio choice. The other parameters are at their baseline levels and as in the other robustness checks, I set risk aversion at a level of 5. Particularly when young, the much lower labor income risk puts upward pressure on the holdings of the equity sectors. Both long and short positions are larger in absolute terms. This suggests that a lower riskiness of labor income growth is compensated for by an increase of risk taking in financial wealth. However, interestingly, the response to the parameter change is not the same for all the sector allocations. Again, in particular the consumer goods and services sectors differ from the others in sensitivity to the change. Whereas the effect on the allocation to the consumer goods sector is much larger than the average, that on the allocation to consumer services is much smaller. These effects can be explained by the hedging components. As discussed above, the effect of a decline in income risk is more risk taking in the allocation of financial wealth. Technically, this works through a decrease in the elasticity to financial wealth, b_1^y . A lower elasticity implies that the hedging components are a more important component of the portfolio weights. The sectors most affected by this are those with the largest negative and positive hedging components, which are the consumer goods and services sectors.

Table 8: Robustness Check, labor income risk

	Age								
	25	30	35	40	45	50	55	60	Retired
<i>Basic Mats</i>	-53.96	-51.92	-49.77	-47.54	-45.17	-42.64	-39.94	-37.07	-34.86
<i>Cons Gds</i>	-0.93	-0.88	-0.82	-0.77	-0.71	-0.65	-0.58	-0.51	-0.46
<i>Cons Svs</i>	-4.35	-4.23	-4.11	-3.98	-3.84	-3.69	-3.54	-3.37	-3.24
<i>Financials</i>	-18.06	-17.34	-16.58	-15.79	-14.95	-14.06	-13.10	-12.09	-11.31
<i>Health</i>	76.29	73.27	70.08	66.76	63.25	59.51	55.49	51.23	47.96
<i>Industrials</i>	70.33	67.75	65.03	62.20	59.20	56.01	52.58	48.95	46.15
<i>Oil & Gas</i>	58.04	55.76	53.37	50.88	48.25	45.43	42.42	39.22	36.76
<i>Technology</i>	-34.58	-33.26	-31.87	-30.42	-28.89	-27.26	-25.50	-23.64	-22.21
<i>Telecom</i>	27.14	26.13	25.06	23.96	22.79	21.54	20.20	18.78	17.69
<i>Utilities</i>	5.98	5.75	5.51	5.25	4.99	4.70	4.40	4.07	3.83
Total equity	125.89	121.02	115.89	110.55	104.91	98.89	92.42	85.58	80.31
<i>Market index</i>	95.16	87.57	80.55	74.07	68.11	62.61	57.46	52.50	48.47

Allocation of financial wealth to the different sectors and the total allocation to equity (bold numbers) at different ages. All are in percentage terms. For all the calculations the coefficient of relative risk aversion is set at 5. The percentage annual discrete growth rate of labor income is 3 per cent per year in discrete terms, the standard deviation of labor income growth is set to the standard deviation labor income growth of the baseline income definition which is 2.47 per cent annually.

5. Conclusions

The research in this thesis has added to the life-cycle portfolio choice literature by refining the equity side of the portfolio. In most existing models, the equity component consists of a broad stock market index. I have presented a model in which funds are allocated over various equity sectors instead. The results of this research can be directly applied to portfolio allocation decisions. In addition, the findings here can serve as a starting point for further research on portfolio choice over the life cycle with a higher level of detail of the asset allocation within the equity segment or within other asset classes.

I have shown that the differences in outcome between a sector model and a market index model are considerable. In the sector model, the allocation to equity is much higher than in the general market index model in all periods of life. This result is independent of the choice of key parameters. As a consequence, for almost equivalent risk, the return on financial wealth is much higher in the sector model. This, in turn, leads to a considerably higher consumption per unit of labor income.

There are several reasons for the different optimal portfolio allocations in the two model specifications. The main explanation is that the sector model better exploits diversification possibilities within the equity segment. This results in optimal portfolios with a much higher return for only slightly more or even less risk. Another advantage of investing on a sector basis is that it is possible to adjust the allocation of a certain sector up-or downwards depending on its correlation with labor income growth and the other sector returns. I have found that it is also optimal to do so. However, as the relationship between labor income growth and a set of equity sectors is quite weak, hedging does not materially alter the optimal allocation for most of the sectors. The same holds for the allocation to stocks in a model with only a broad market index. Another similarity between the two models is that the presence of labor income increases the allocation to equity. The allocation to equity is thus largest when young. Over the life cycle, the equity share of financial wealth gradually declines. This is a standard result in the life-cycle literature. However, the results of the research here indicate that the magnitude of the allocation and decline depend strongly on risk aversion, labor income growth and model specification. For some parameter combinations, the sector model indicates the investor's portfolio should be largely allocated to equity while the market specification suggests that it is optimal to invest most of the portfolio in low-risk bonds. This highlights that it is important to carefully determine the investment set. Different investment sets, even within the same asset class, might lead to considerably different outcomes.

Although there is already a long literature on portfolio choice over the life cycle, the results here indicate there are still interesting areas for future research left. Whereas I used different domestic equity sectors as portfolio components, other diversification approaches are potentially insightful as well. A key one of these would be international investment. This could add to life-cycle models by offering opportunities for diversification across countries. It could potentially also be useful from a hedging perspective as different countries may well be at different points in the economic cycle at the same point in time. In addition, the applications of a multiple asset model as presented here need not be limited to the allocation to different classifications of equity. It can also be used for including a set of other asset classes and subclasses in a life-cycle framework. Important in this respect is that the point made in the introduction on the varying properties of equity sectors easily carries over to other asset classes and their subclasses. Also these subclasses generally differ in risk and return properties which is not well captured by the classification of the main class into risky or low risk. Given the results here, it would thus be interesting to also incorporate other asset classes and their subclasses into the life-cycle portfolio choice model.

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