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Abstract: Prepayment of Dutch mortgage loans is restricted to a fixed amount per calendar year. Due to path dependence the valuation of these mortgage loans is more complicated than the valuation of unrestricted prepayment options. In this paper we derive an optimal and efficient strategy to price interest-only mortgages with restricted prepayments. For tax reasons an interest-only mortgage is the most popular mortgage type in the Netherlands. The optimal strategy also provides a rational explanation why empirical prepayment models find that prepayments peak in December. Our results indicate that a restricted prepayment option still has significant value.

Keywords: mortgage valuation, partial prepayments, binomial trees.

JEL codes: C61, C63, G21

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*The views expressed in this paper are those of the authors and do not necessarily reflect the views of ABP.

1 Introduction

The Dutch mortgage market is one of the largest in Europe. Charlier and Van Bussel (2003) report that the Netherlands is ranked second among the countries of the European Union in terms of both the outstanding mortgage debt as a percentage of GDP and the outstanding mortgage debt per capita.

The Dutch market differs in two important institutional aspects from the US market. First, mortgage interest payments are fully tax-deductible in the Netherlands. For maximum advantage of tax-deductibility, interest-only (IO) mortgages have become increasingly popular. These mortgages do not have scheduled amortization of the principal before maturity of the loan, such that interest payments remain high. Second, Dutch mortgage loans have restricted prepayment options. Whereas in the US a mortgage loan can be fully prepaid at the discretion of the borrower, a Dutch contract has limits imposed on the maximum prepayment per year. Typically only 10 to 20 percent of the initial loan can be prepaid per calendar year. The only exceptions are sale of the house and default or death of the borrower.

Empirical prepayment behavior for Dutch mortgages has been documented extensively.¹ Optimal exercise of the Dutch prepayment option has not been considered so far, contrary to the large literature on optimal prepayment for US mortgages.² The methods developed for valuation and optimal prepayment of US mortgages do not apply, however, in the Dutch situation. The technical problem is path dependence. The prepayment option is an American style option that is usually valued by backward induction. With partial prepayments, backward induction is not directly applicable, since at any given time the future optimal prepayment policy depends on how often prepayment has taken place in earlier years. Typically, a non-recombining tree is required to value mortgage loans with restricted prepayments. Since in such a tree the number of nodes increases exponentially with time, efficiently determining the optimal prepayment strategy is very difficult. The main contribution of this paper is to derive the optimal prepayment strategy for Dutch interest-only mortgages, by applying an efficient binomial lattice approach.

Both Hayre (2003) and Charlier and Van Bussel (2003) find that prepayments peak in December of each calendar year. They attribute the strong seasonality to holiday and tax effects. In our optimal prepayment model, the December effect arises

¹ See Charlier and Van Bussel (2003), Hayre (2003), Van Bussel (1998) and Alink (2002).

² See McConnell and Singh (1994), Kau et al. (1990, 1998) and textbooks like Hull (2002).

endogenously as an implication of the partial prepayment option. Due to the restriction of a maximum prepayment per calendar year, rational borrowers have an incentive to prepay more often in December than in other months, since each December an option to prepay expires.

The rest of the paper is organised as follows. Section 2 defines the mortgage contract. Section 3 derives the optimal prepayment strategy and provides a valuation algorithm within a lattice framework. Section 4 provides an empirical analysis of the costs of the Dutch prepayment option relative the full prepayment and the option induced spread of Dutch mortgages.

2 Mortgage Contract

We consider the following contract specifications. The contract has a maturity of L years. The contract interest rate ("mortgage rate") is Y percent per year. Contractual payments have the form of a coupon bond. Each period a fraction $y = Y/K$ of the annual coupon is paid, with K the number of periods per year. No regular amortization takes place. The remaining principal, if any, is repaid at maturity. Before maturity, the principal can be repaid according to the following two conditions:

- The borrower has the right of full prepayment when the mortgage rate is adjusted. The mortgage rate is fixed for $M \leq L$ years, after which it is adjusted to prevailing market rates. No caps or floors apply to the interest rate adjustment.
- In each calendar year the borrower has the option to prepay a fraction $1/N$ of the principal. The total loan can thus be prepaid over a period of N years.

Full prepayment is also allowed when the house is sold. We will not include the latter effect, assuming that moving costs are sufficiently high that this is a non-strategic event.

We consider optimal call policies that minimize the present value of the cash flows paid by the borrower. The optimal call policy provides a lower bound on the loan value to the lender. For several reasons borrowers might not all follow the optimal prepayment policy. Foremost is the tax incentive. Interest rate payments on a mortgage on a main residence are fully tax deductible, whereas earned interest is not taxed in many cases (for example when household wealth is below a threshold). Another reason is the link between a mortgage loan and a life insurance contract, present in part of the outstanding mortgage contracts.

Since the interest rate is adjusted at the end of year M to the new market rate, and full prepayment is allowed at that moment, the value of the contract at the end of year M will be equal to the principal. Hence, for valuation we do not need to look beyond year M . The contract is in effect a coupon bond with maturity M that is callable in N steps. The term of the loan therefore does not matter for the analysis as long as $M \leq L$. Actual mortgages in the Netherlands have a lifetime of 30 years, while most often interest rates are fixed for periods between 5 and 15 years.

3 Valuation

The valuation problem is complicated by the path dependence of the partial prepayment option. As an example, consider a 3-year bond that allows prepayment of 50% of the principal each calendar year. For an American option we would like to apply backward induction within an interest rate lattice. But at any node of the lattice in year 3 the value of the contract will differ depending on whether prepayment has taken place in years 1 and 2, or not. The terminal conditions of year 3 are path dependent, as they depend on the number of prepayments in previous years.

Path dependence can often be solved by introducing an additional state variable, which keeps track of all possible values of a function of the state variable at each node. This is the technique applied to price American lookback options presented in Hull (2002). The present problem is different. The path dependence is not through a function of the state variable but through endogenous decisions of the borrower. In that sense the partial prepayment problem is related to shout options (see Cheuk and Vorst (1997)).

Path dependence does not occur for all partial prepayment mortgages. When $M < N$, the loan cannot be fully prepaid until maturity. We can decompose this loan as a portfolio of a non-callable coupon bond and a bond with maturity M that is callable in exactly M annual steps. In symbols,

$$V(M, N) = \frac{M}{N}V(M, M) + (1 - \frac{M}{N})P(M), \quad M < N, \quad (1)$$

where $V(M, N)$ is the value of a mortgage contract with maturity M years that can be called in N annual steps and $P(M)$ is the value of a non-callable coupon bond with the same maturity. To keep the notation as light as possible, we suppress the dependence on the contract rate Y . The mortgage loan can be replicated by investing a fraction M/N in a bond of which a fraction $1/M$ can be called in each calendar year.

The remainder $1 - M/N$ is invested in the coupon bond. This part of the loan cannot be prepaid before maturity. Valuation of a non-callable coupon bond is trivial. The callable bond, with value $V(M, M)$, can be further decomposed as a portfolio of M simple callable bonds,

$$V(M, M) = \frac{1}{M} \sum_{\ell=1}^M C(\ell), \quad (2)$$

where $C(\ell)$ is the value of a coupon bond with maturity M that is only callable in year ℓ and not in any other year.

We conclude that options having $M \leq N$ are simple American style options, without path dependence, that can be easily valued by a lattice method. The more complicated contracts are the ones with $M > N$. These are contracts that can be fully prepaid before the contract rate adjustment date. For such contracts we have to decide how to assign N prepayment options to $M > N$ calendar years. We thus concentrate on pricing mortgage contracts with $M > N$.

The idea for the valuation of a partially callable mortgage is the construction of two lattices. The first lattice has annual time steps and describes the evolution of the prepayments. Figure 1 depicts the lattice for a mortgage with a fixed rate period of 7 years and an annual prepayment restricted to 20% of the principal ($N = 5$). The original mortgage contract is indicated as the root node (7,5) in the figure. If the borrower has not exercised the prepayment option in the first year, the contract will become a mortgage with maturity of six years of which 20% can be prepaid annually. In the lattice this is node (6,5). The alternative is that the borrower will have exercised the first prepayment option. This is equivalent to paying 20% of the principal and converting the remainder 80% to a contract with maturity $M = 6$ that can be prepaid in $N = 4$ steps; this is node (6,4).

Terminal nodes can be reached in two ways. First, whenever we are at a node (m, m) we have a contract that we can value without path dependence. To reach these nodes, we need to expand the annual lattice. If, starting at node (6,5), the borrower again does not prepay in year 2, we arrive at node (5,5). This is a contract without path dependence for which (2) provides the value. We can compute that value in the lattice by starting at the end of year 7, and discounting back to the end of year 2 each of the five constituent callable bonds.

Second, if we reach a node $(m, 1)$ we have a fully callable bond, which can be called in one step. Such a contract can also be easily priced in a spot rate lattice. Therefore we can obtain terminal values of all the contracts at the end of a branch of the lattice

in figure 1.

Nodes in the prepayment conversion lattice recombine: node (5,4) is reached by prepaying in year 1 and not in year 2 and also by prepaying in year 2 and not in year 1. Formally, the nodes of the prepayment lattice have transitions

$$\begin{array}{r}
 \nearrow (m-1, n-1) \quad \text{if } n > 1 \\
 (m, n) \\
 \searrow (m-1, n) \quad \text{if } n < m.
 \end{array} \tag{3}$$

Once we have obtained values for all the path independent contracts, we can work backwards to price the original mortgage.

For the actual valuation we use a different lattice with much finer time steps within a year. Nodes in this lattice refer to spot interest rates and are denoted (i, t) . Each node (i, t) has two successors, $(i+1, t+1)$ and $(i, t+1)$. Both successor nodes are reached with probability one half. Each node is associated with a spot rate r_{it} and discount factor $d_{it} = (1 + r_{it})^{-1}$ ($i = 0, \dots, t$).³ Of special importance are the end-of-year nodes (i, T_ℓ) at times $t = T_\ell$ ($\ell = 1, \dots, M$). Figure 2 illustrates the procedure. It shows a snapshot of the lattice within year 2 of the example. Nodes at the end of year 2 are denoted (i, T_2) and nodes at the end of year 1 by (i, T_1) . The year is subdivided in $T_2 - T_1$ time steps.

Suppose that in figure 2 we are at the end of year 2 and have obtained values of the (5,5), (5,4) and (5,3) contracts at each node at time $t = T_2$. We perform two backward recursions through year 2 to obtain the values $V(6,4)$ and $V(6,5)$ at the end of year 1. The first recursion determines $V(6,4)$, the second $V(6,5)$. Both are required to perform the final valuation of the of the (7,5) contract in figure 1.

At time $t = T_2$ the values $V_{i,T_2}(5,5)$, $V_{i,T_2}(5,4)$ and $V_{i,T_2}(5,3)$ are given. At this point we initialise the values of the two contracts, $V(6,5)$ and $V(6,4)$, at each node (i, T_2) by

$$\begin{aligned}
 V_{i,T_2}(6,5) &= \min \left(V_{i,T_2}(5,5), \frac{1}{5} + \frac{4}{5} V_{i,T_2}(5,4) \right) \\
 V_{i,T_2}(6,4) &= \min \left(V_{i,T_2}(5,4), \frac{1}{4} + \frac{3}{4} V_{i,T_2}(5,3) \right)
 \end{aligned}, \quad i = 0, \dots, T_2. \tag{4}$$

Knowing the terminal values, discounting back to $t = T_1$, taking into account early exercise, is done by backward induction. For example, for the (6,4) contract the recursion

³ Definitions of the spot rates could come from any term structure model that can be implemented in a lattice framework. We describe a binomial lattice for spot rates, but a multinomial structure would work as well.

follows

$$V_{it}(6, 4) = \min \left(V'_{it}(6, 4), \frac{1}{4} + \frac{3}{4}V_{it}(5, 3) \right), \quad (5)$$

for $i = 0, \dots, t$ and $t = T_1, \dots, T_2 - 1$, with

$$V_{it}(5, 3) = d_{it} \left(y + \frac{1}{2}V_{i,t+1}(5, 3) + \frac{1}{2}V_{i+1,t+1}(5, 3) \right) \quad (6)$$

$$V'_{it}(6, 4) = d_{it} \left(y + \frac{1}{2}V_{i,t+1}(6, 4) + \frac{1}{2}V_{i+1,t+1}(6, 4) \right), \quad (7)$$

where y is the periodic coupon. The prepayment condition (5) compares the continuation value $V'_{it}(6, 4)$ with the conversion value $\frac{1}{4} + \frac{3}{4}V_{it}(5, 3)$, and sets $V_{it}(6, 4)$ to the minimum of the two. The discount recursions (6) and (7) keep track of the value of the two alternatives at each time during year 2. An analogous recursion provides the value of $V_{it}(6, 5)$.

The general form of the recursions, including partial prepayment decision, is given by

$$V_{it}(m, n) = \min \left(V'_{it}(m, n), \frac{1}{n} + \frac{n-1}{n}V_{it}(m-1, n-1) \right) \quad (8)$$

with

$$V'_{it}(m, n) = d_{it} \left(y + \frac{1}{2}V_{i,t+1}(m, n) + \frac{1}{2}V_{i+1,t+1}(m, n) \right) \quad (9)$$

$$V_{it}(m-1, n-1) = d_{it} \left(y + \frac{1}{2}V_{i,t+1}(m-1, n-1) + \frac{1}{2}V_{i+1,t+1}(m-1, n-1) \right) \quad (10)$$

At each node a prepayment decision must be made. Without prepayment the contract is characterized by m calendar years and n partial prepayment options left. No prepayment means continuation of the same contract (m, n) . The value is obtained by discounting the expected contract value plus interest payment. Prepayment implies that the remaining contract has $m-1$ calendar years and $n-1$ opportunities to prepay, with value $V_{it}(m-1, n-1)$. The fraction $\frac{1}{n}$ of the contract is repaid (at the nominal price of 1) and the remaining fraction $\frac{n-1}{n}$ is obtained in the contract $(m-1, n-1)$. The value of this contract is also determined by backward recursion. The mortgage value $V_{it}(m, n)$ is the minimum of the continuation value and the portfolio of prepayment value and remaining contract value. During year ℓ we evaluate ℓ different contracts $V(m, n)$, for $m = M - \ell$ and $n = N - \ell, \dots, N$, all in the same spot rate lattice.

At the end of the year a node (m, n) in the prepayment conversion lattice leads to either $(m-1, n)$ or $(m-1, n-1)$, whereas in other periods (m, n) either remains (m, n) or changes to $(m-1, n-1)$. The general form of the end-of-year contract conversion is given by

$$V_{i,T_\ell}(m, n) = \min \left(V_{i,T_\ell}(m-1, n), \frac{1}{n} + \frac{n-1}{n}V_{i,T_\ell}(m-1, n-1) \right) \quad (11)$$

At the year-end nodes the maturity m is increased from $m - 1$ to m (going backwards).

The end-of-year initialisation equations induce additional prepayments. The end-of-year conversion condition (11) may trigger a partial prepayment even if the normal prepayment condition (8) does not indicate prepayment. Each prepayment decreases the future principal, and hence future interest payments. A partial prepayment can be optimal even if an immediate loss is faced, as long as the expected decrease of future interest payments offsets this loss. ‘December’ prepayments can be optimal, because a prepayment option might expire if not exercised in December. A partial prepayment can thus be optimal in nodes where full prepayment is not.

4 Empirical Results

What is the value of the Dutch prepayment options relative to unrestricted prepayment and relative to no prepayment at opportunities at all? For a comparison of different contracts we look at fair rates. The fair rate of a mortgage is the contract rate at which the present value of all payments equals the nominal value. If a mortgage rate is fair, the value of the mortgage at initiation is equal to the loan principal. For the empirical example we calibrate an interest rate lattice to the observed swap curve and swaption volatilities at the beginning of the years 2002 to 2005. The calibration uses a Black, Derman and Toy (1990) model with spot rates at nodes (i, t) given by

$$\ln r_{it} = a(t) + b(t)i, \quad i = 0, \dots, t, \quad (12)$$

with $a(t)$ determined by the yield curve and $b(t)$ by the swaption volatilities.⁴ The BDT model is used for illustrative purposes, since the methodology does not depend on the details of the term structure model as long as it can be represented within a lattice.

Fair rates for interest-only mortgage contracts with a varying number of prepayment opportunities N and a varying fixed rate period of M years are presented in table 1. N takes on the values 1 (corresponding to a fully callable mortgage), 5, 10 or infinity (non-callable mortgage). The fixed rate period M equals 5 or 10 years.

The fair rate spread of a mortgage is defined as the difference between the fair rate of the mortgage itself and the fair rate of a non-callable mortgage with otherwise

⁴ As not all swaption volatilities of swap maturity m and option expiration k can be fitted, the calibration is based on the function $b(t)$ that minimises the sum of squared errors subject to smoothness conditions.

the same conditions. Table 2 shows that a partial prepayment option has significant value, although a prepayment of only a small loan amount is allowed once per calendar year. For an interest-only mortgage with a ten-year fixed rate period, the average full prepayment spread (in terms of fair rates) equals 73 basis points. A partial prepayment option, embedded in an interest-only mortgage with a ten-year fixed rate period and $N = 5$, implying a 20% penalty free prepayment each calendar year, has an average premium of almost 30 basis points. Therefore, a 20% prepayment option is worth more than 40% of a full prepayment option. The premium for a 10% prepayment option ($N = 10$) is half the premium of a 20% prepayment option: on average 15 basis points or 20% of a full prepayment option.⁵

For shorter fixed rate periods, the prepayment option is less valuable, since the cumulative prepayment during the fixed rate period is more restricted. When considering a fixed rate period of 5 years, a full prepayment option is worth 60 basis points. The premium of a 20% (10%) prepayment option is about 27% (12%) of the premium of a full prepayment option, corresponding to 17 (7) basis points.

Figure 3 shows the effect of changing the number of prepayment opportunities on the mortgage price. The underlying interest rate lattice is calibrated at January 1, 2005. The fixed rate period equals 10 years ($M = 10$). The contract rate is either 4% or 4.5%. The mortgage principal has been scaled to 1. For the higher contract rate the mortgage value is more sensitive to prepayment options, as the price difference between a fully callable and a non-callable mortgage is larger. Prices increase faster for increasing prepayment opportunities if N is low. For N approaching infinity, the price asymptotically approximates the non-callable mortgage value. Since the fair rate for a fully callable mortgage (see table 1) is just below 4.5%, the value of this mortgage is almost equal to one at $N = 1$. Adding partial prepayments restrictions ($N > 1$) raises the value above one.

Finally we consider the December effect. In the lattice prepayment occurs if the spot rate at time t goes below a critical level r_{it}^* . The time path of the critical rates r_{it}^* defines the prepayment boundary. Figure 4 shows the prepayment boundary for the January 1, 2005 instance for a 10-year fixed rate period and a 20% prepayment option. The figure clearly shows the prepayment peaks present in December, at expiration of the prepayment options. Prepayment in December occurs at higher interest

⁵ In practice some contracts also have commission costs, which are typically a fixed upfront expense of 1% of the loan. Including 1% commission costs the relative value of the partial prepayment option increases to 56% of an unrestricted prepayment option for the same instance.

rates than in the rest of the year. The figure also shows a general increase in the prepayment boundary over time. In the first few years a prepayment option is only exercised at very low interest rates, as it is optimal to wait until interest rates have decreased substantially. On average, over all four instances and all December nodes, the prepayment boundary in December is about 30 basis points above the boundary in all other months.

5 Conclusion

We have developed an efficient algorithm for the valuation of interest-only mortgages with partial prepayment options. The algorithm decomposes an interest-only mortgage contract into several callable bonds and a non-callable coupon bond. Valuation takes place by simultaneously feeding a series of related securities through a recombining tree and considering the relevant conversion options granted to the borrower.

Prepayment of a partially callable mortgage may occur earlier than a full prepayment, because at the end of a calendar year a prepayment option expires. A partial prepayment in December, just before the annual option expiration, can be optimal, even if a full prepayment is not, thereby decreasing the remaining loan and future interest payments.

In terms of fair rates, a 20% prepayment option is worth 30 basis points on average, when considering a ten year fixed rate period. This is 40% of the value of an unrestricted prepayment option.

In this paper we focus on interest-only mortgages. The proposed valuation method does not apply to mortgage contracts with scheduled amortization during the term of the loan. For mortgages with a regular, periodical redemption schedule, the conversion tree in figure 1 is no longer recombining. This means that the number of different contracts that must be kept track of in the backward recursion grows exponentially. For general mortgage contracts Kuijpers and Schotman (2005) provide an alternative approach based on upper and lower bounds in a linear programming formulation.

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Table 1: Fair rates with partial prepayment options

The table reports fair rates (in annualized percentages) for interest-only mortgages with fixed-rate periods equal to $M = 5$ or $M = 10$ years, which can be prepaid in $N = 1$, $N = 5$, $N = 10$ or $N \rightarrow \infty$ annual steps of a fraction $1/N$ of the principal. The case $N \rightarrow \infty$ corresponds to a non-callable mortgage, a mortgage is fully callable if $N = 1$. All mortgages mature after 30 years. The interest payments are based on an annual contract rate following $y = Y/K$ with $K = 12$. Underlying term structures on four different dates are considered. The interest rate lattice uses monthly time steps.

Date	N	$M = 10$				$M = 5$			
		∞	10	5	1	∞	10	5	1
Jan 1, 2002		5.123	5.289	5.442	5.905	4.572	4.646	4.747	5.217
Jan 1, 2003		4.310	4.440	4.565	5.007	3.610	3.673	3.759	4.234
Jan 1, 2004		4.305	4.447	4.582	4.998	3.619	3.684	3.771	4.178
Jan 1, 2005		3.688	3.851	4.010	4.423	3.116	3.189	3.293	3.680

Table 2: Fair rate spreads

The table reports fair rate spreads (in annualized percentages) for interest-only mortgages. Entries report the difference of the fair rates of mortgage with prepayment options relative the benchmark of a non-callable mortgage.

Date	N	$M = 10$			$M = 5$		
		10	5	1	10	5	1
Jan 1, 2002		0.166	0.319	0.782	0.074	0.175	0.645
Jan 1, 2003		0.130	0.255	0.697	0.063	0.149	0.624
Jan 1, 2004		0.142	0.277	0.693	0.065	0.152	0.559
Jan 1, 2005		0.163	0.322	0.735	0.073	0.177	0.564
average		0.150	0.293	0.727	0.069	0.173	0.598

Figure 1: Prepayment conversion lattice

The figure shows the evolution over time (in calendar years) with respect to the number of years M and the number of prepayment options N remaining. Nodes are labelled (M, N) . Leaves have either the possibility of prepayment in each calendar year ($M = N$), or unrestricted prepayment ($N = 1$).

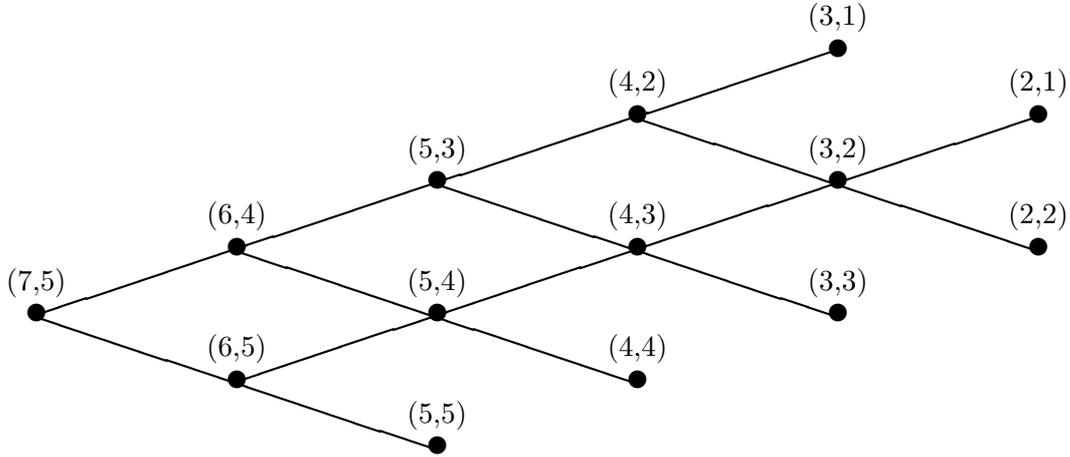


Figure 2: Valuation lattices

The figure illustrates the valuation within a given calendar year. The example refers to the valuation of the terminal nodes of the (6,4) and (6,5) contracts at time $t = T_1$ in figure 1 given the values of the (5,3), (5,4) and (5,5) contracts at time $t = T_2$. The example assumes that calendar year 2, between times T_1 and T_2 , is divided in 4 subperiods.

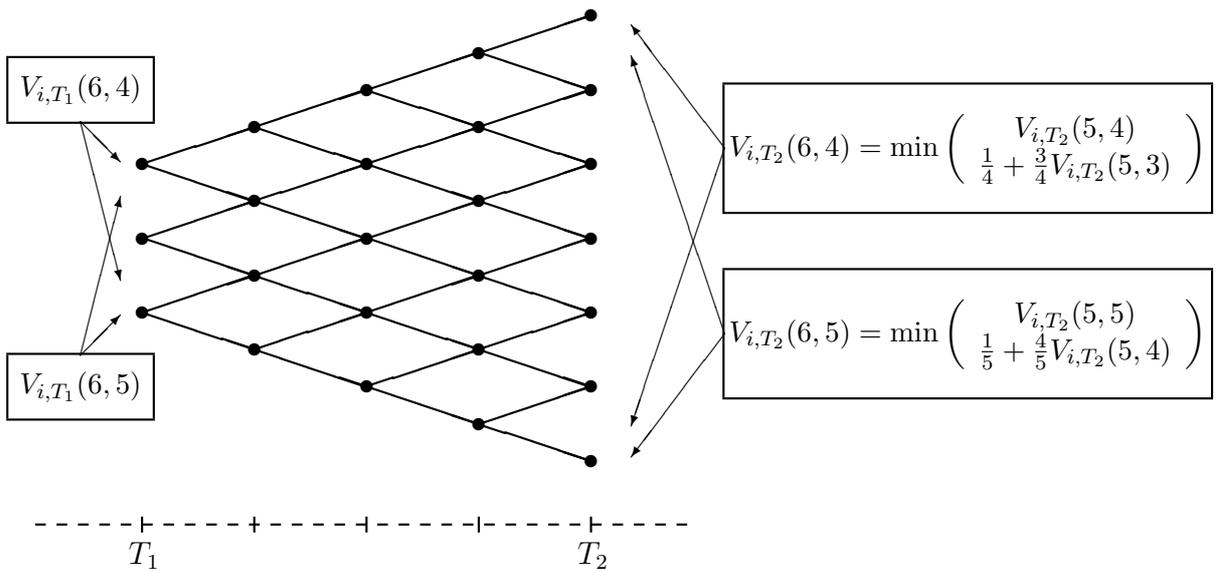


Figure 3: Mortgage price for varying N

This figure displays mortgage prices for a varying number of prepayment opportunities N . The January 1, 2005 instance is used with $M = 10$. Results are based on two different contract rates. Both curves asymptotically approach the corresponding non-callable mortgage value ($N \rightarrow \infty$), represented by the horizontal dashed lines. The nominal loan value equals 1.

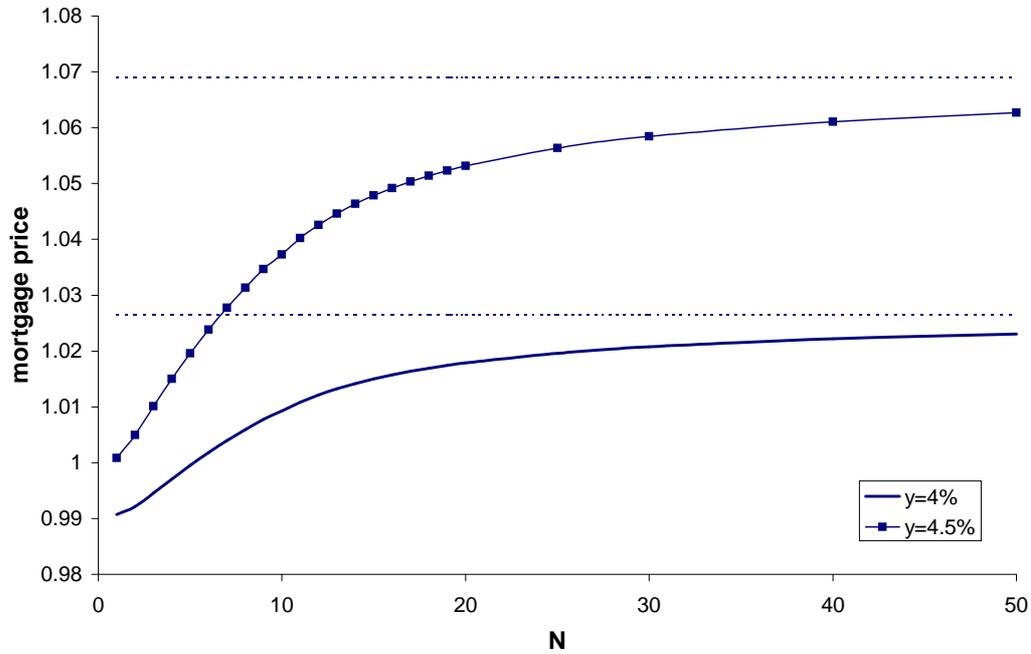


Figure 4: Prepayment boundary

The figure displays the discretised prepayment boundary based on the fair contract rate of the January 1, 2005 instance, with $M = 10$ and $N = 5$. For interest rates in the dark area a partial prepayment is optimal, for interest rates above the light area prepayment does not occur. The true prepayment boundary is contained in the light area, which is based on two subsequent interest rates in the interest rate lattice.

