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Frans de Roon

Jinqiang Guo

Jenke ter Horst

## **Being Locked Up Hurts**

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# Being Locked Up Hurts

Frans A. de Roon, Jinqiang Guo, and Jenke R. ter Horst<sup>\*</sup>

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## ABSTRACT

This paper examines multi-period asset allocation when portfolio rebalancing is difficult or impossible for some assets due to the existence of a lockup period. Our empirical analysis shows that both the unconditional strategy and conditional strategy benefit from adding hedge funds. More importantly, both the unconditional strategy and conditional strategy are hurt by the presence of a hedge fund lockup period. In an unconditional setting, we find a Sharpe ratio of 1.23 for a three-month lockup period for hedge funds and monthly rebalancing of stocks and bonds. For the same portfolio, but without a lockup, we find a significantly higher Sharpe ratio of 1.53. The certainty equivalents using the two-asset portfolio as the benchmark are 3.4% and 7.6% for the portfolio with a lockup and the one without a lockup, respectively. Therefore, the economic significance of a lockup period is also evident. Investors compensate for the lockup period of hedge funds by making adjustments to their equity and bond holdings. Adding hedge funds to the portfolio of stocks and bonds reduces the allocation to stocks and increases the allocation to bonds in each month. Finally, the effect of a lockup period on portfolio performance is less pronounced when investing in funds of hedge funds relative to investing in individual hedge funds when the investment horizon is short, suggesting that funds of funds are able to suppress the effect of a lockup period.

*JEL classification:* G11; G12

*Keywords:* Multi-period asset allocation; return predictability; hedge funds; lockup period.

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<sup>\*</sup> De Roon, Guo and ter Horst are from Tilburg University (Department of Finance and CentER for Economic and Business Research) and Netspar. Postal Address: Department of Finance, Tilburg University, P.O.Box 90153, 5000 LE, Tilburg, the Netherlands. E-mail address: [F.A.deRoon@uvt.nl](mailto:F.A.deRoon@uvt.nl) (F.A. de Roon), [J.Guo@uvt.nl](mailto:J.Guo@uvt.nl) (J.Guo), [J.R.terHorst@uvt.nl](mailto:J.R.terHorst@uvt.nl) (J.R. ter Horst). The authors thank Ralph Koijen for insightful comments.

## I. Introduction

An important question for both practitioners and academics in portfolio analysis is the multiperiod investment problem. The question is how to rebalance the portfolio before the investment horizon, which is often complicated by restrictions such as the inability to go short and the fact that some positions in illiquid assets cannot be rebalanced easily. For instance, investments in hedge funds are often accompanied by a lockup period, during which investors cannot withdraw their money. There are other restrictions such as a redemption notice period and redemption frequency that make it difficult for an investor to get his money out of hedge funds. The implication of a hedge fund lockup period is best illustrated by the experience of hedge fund investors during the recent financial crisis beginning in July, 2007. Investors would like to liquidate their investments in hedge funds to avoid further losses or to meet liquidity need elsewhere. But there is no escape if the hedge fund lockup period has not yet expired.

Institutional investors from many countries show an increasing allocation (in terms of both absolute dollar amounts and portfolio weights) to hedge funds, private equity and venture capital (Source: The 2007-2008 Russell Investments Survey on Alternative Investing). In this paper, we study the asset allocation problem for a multi-period investor when some of the assets such as hedge funds have a lockup period. The framework in this paper can be modified to take into account the redemption notice period and redemption frequency constraints. The analysis extends Brandt and Santa-Clara (2006) where the multi-period investment portfolio is solved in a static Markowitz-framework. We take the perspective of a multi-period investor who periodically rebalances his portfolio that consists of liquid assets and illiquid assets during the investment period. We show that a hedge fund lockup period can be incorporated into the multi-period asset allocation decision by an investor who periodically re-adjusts his portfolio. In addition, we find that the lockup considered in this paper is empirically highly relevant. However, our empirical analysis shows that even with a hedge fund lockup, investing in hedge funds can improve portfolio outcomes under both the unconditional strategy and conditional strategy.

We also contribute to the hedge fund literature by evaluating the economic value of hedge fund investments from a portfolio perspective. The evaluation of hedge fund performance has been studied in several papers, including Agarwal and Naik (2004), Fung, Hsieh, Naik and Ramadorai (2008), Kosowski, Naik, and Teo (2005), Malkiel and Saha (2005). In these studies,

the performance of individual hedge funds or groups of hedge funds is typically evaluated on the basis of a factor model or a benchmark model. We take a portfolio perspective and compute the optimal allocation to different asset classes in a portfolio. Asset classes such as hedge funds are often considered attractive investments because of their superior risk-return profile and low correlation with stocks and bonds. However, investments in hedge funds often face more restrictions than investments in stocks and bonds. For instance, many hedge funds impose a lockup period, ranging from a few months up to several years. As the lockup periods lengthen, the negative effect of having such illiquid assets grows and may lead to the situation that a portfolio of liquid assets and illiquid assets is dominated by a portfolio of only liquid assets.

As in Brandt and Santa-Clara (2006), we solve a multi-period portfolio problem that consists of a set of timing portfolios and conditional portfolios. In a multi-period setting, a timing portfolio for a risky asset is a strategy that invests in the risky asset in one period only and in the risk-free asset in all remaining periods. Therefore, the multi-period asset allocation can be derived by solving the static Markowitz problem on the basis of timing portfolios and scaled returns or conditional portfolios. We incorporate the constraint of a lockup period for hedge funds to the asset allocation problem. If we assume that the investment horizon is equal to the length of the lockup period, there is no timing portfolio for hedge funds, because once an investment in hedge funds is made, the investor has to hold on to it until the lockup restriction expires. A portfolio of stocks, bonds and hedge funds with a lockup will certainly behave differently from a portfolio of the same assets without a lockup, in terms of allocations to different assets over time, as well as the portfolio performance.

The paper uses broad market indexes as the proxy for stocks, bonds and hedge funds/funds of funds in the empirical analysis. Indeed, we find that a lockup period induces additional hedge demand for stocks in order to obtain the desired intertemporal equity exposure that cannot be obtained by hedge funds due to the lockup. For the portfolio of stocks, bonds and funds of funds (using the HFRIFoF composite as the proxy) with a three-month lockup period under the unconditional strategy, the Markowitz demands for stocks are 60%, 58% and 43% in Month 1, Month 2 and Month 3. The inclusion of the HFRIFoF composite generates hedge demands of -41% , -27%, and -16% over the three months. As a result, the total demands for stocks in the three-asset portfolio are lower than the Markowitz demand, and display an inverted U-shape over time. The hedge demands for bonds are positive in all months, so the total demands for bonds are higher than the Markowitz demands for bonds. Both the total

demands and the Markowitz demands for bonds are increasing over time, suggesting that an investor allocates more to less risky assets as the investment horizon approaches.

Our empirical analysis shows that both the unconditional strategy and conditional strategy can be improved upon when adding hedge funds to the stock/bond portfolio, but the portfolio performance is hurt when there is a hedge fund lockup period. For instance, the annualized Sharpe ratio for the unconditional strategy with stocks, bonds and funds of hedge funds (HFRIFoF composite) with a three-month lockup period is 1.23, which is significantly higher, both economically and statistically, than the Sharpe ratio of 0.91 for the unconditional strategy of stocks and bonds only. But if there is no hedge fund lockup period, the portfolio Sharpe ratio with the three asset classes is 1.53, which is significantly different from the reported Sharpe ratio of 1.23 for the conditional strategy with a three-month lockup period. The effect of a lockup period is stronger when the HFRI composite index and the HFRI strategy indexes are considered than for the fund of funds indexes (i.e. HFRIFoF composite and strategy indexes), especially at the short investment horizon. This suggests that fund of funds managers are able to structure their funds in such a way that their clients are hurt less by a lockup period.

The rest of the paper is organized as follows. Section II explains the methodology to derive the optimal asset allocation for a mean-variance investor facing lockup periods for hedge funds. Section III describes the data. Section IV shows the results of a lockup period under the unconditional strategy, while Section V presents the results in the conditional framework. The bootstrap results are shown in Section VI. Finally, Section VII concludes.

## **II. Asset Allocation with a Lockup Period**

We consider the allocation problem for a risk-averse investor. The portfolio consists of liquid assets and illiquid assets. Liquid assets include stocks, bonds, money market instruments, etc., while illiquid assets can be hedge funds, private equity and venture capital investment. We restrict our attention to stocks, bonds, Treasury bills and hedge funds in our empirical analysis, but the model here holds for all asset classes with similar features of illiquidity. The investor can change the allocation to liquid assets every period, but adjusting allocations to illiquid assets is difficult if not impossible. The form of illiquidity in this paper is restricted to the situation in which a lockup period is imposed for investments in hedge funds.

## A. Multi-period Asset Allocation with Lockup Constraints

We first illustrate the two-period asset allocation problem with lockup constraints, and generalize the method to the longer period setting. There are  $K_1$  liquid risky assets and  $K_2$  illiquid risky assets with a lockup period equal to  $L$ . For simplicity, the investment horizon has the same length as the lockup period. Consider the two-period quadratic utility optimization problem for an investor:

$$\max E_t \left[ r_{t \rightarrow t+2}^p - \frac{\gamma}{2} (r_{t \rightarrow t+2}^p)^2 \right], \quad (1)$$

where  $r_{t \rightarrow t+2}^p$  is the excess portfolio return over two periods and  $\gamma$  is the coefficient of risk aversion. Denote portfolio weights on liquid assets and illiquid assets at time  $t$  by  $w_{z,t}$  and  $w_{x,t}$ , respectively. In addition, denote the one-period gross return on the risk-free asset at time  $t$  by  $R_t^f$ , and gross returns of illiquid assets by  $R_{t+1}^x$ . The vector  $r_{t+1}$  contains one-period excess returns of liquid risky assets. The two-period excess return of the portfolio with only liquid assets is:

$$\begin{aligned} r_{t \rightarrow t+2}^p &= (R_t^f + w'_t r_{t+1}) (R_{t+1}^f + w'_{t+1} r_{t+2}) - R_t^f R_{t+1}^f \\ &= w'_t (R_{t+1}^f r_{t+1}) + w'_{t+1} (R_t^f r_{t+2}) + (w'_t r_{t+1}) (w'_{t+1} r_{t+2}) \\ &\approx w'_t (R_{t+1}^f r_{t+1}) + w'_{t+1} (R_t^f r_{t+2}). \end{aligned} \quad (2)$$

Because  $r_{t+1}$  and  $r_{t+2}$  are excess returns, the product  $(w'_t r_{t+1})(w'_{t+1} r_{t+2})$  is very small at short horizons, so the excess portfolio return over two periods is approximately the sum of  $w'_t (R_{t+1}^f r_{t+1})$  and  $w'_{t+1} (R_t^f r_{t+2})$ .

Brandt and Santa-Clara (2006) interpret  $w'_{z,t} (R_{t+1}^f r_{t+1})$  and  $w'_{z,t+1} (R_t^f r_{t+2})$  as “timing portfolios”. First,  $w'_{z,t} (R_{t+1}^f r_{t+1})$  is the two-period excess return from investing in risky assets at time  $t$  and then investing in the risk-free asset. Second,  $w'_{z,t+1} (R_t^f r_{t+2})$  is the two-period excess return from investing in the risk-free asset at time  $t$  and then investing in risky assets.

When the portfolio includes assets with a two-period lockup, the two-period portfolio excess return takes the form of the following:

$$r_{t \rightarrow t+2}^p = (R_t^f + w'_{z,t} r_{t+1}) (R_{t+1}^f + w'_{z,t+1} r_{t+2}) - R_t^f R_{t+1}^f + w'_{x,t} r_{t \rightarrow t+2}^x$$

$$\approx w'_{z,t} (R_{t+1}^f r_{t+1}) + w'_{z,t+1} (R_t^f r_{t+2}) + w'_{x,t} r_{t \rightarrow t+2}^x. \quad (3)$$

where  $r_{t \rightarrow t+2}^x$  is the  $K_2$  dimensional vector of excess returns of illiquid assets, and for each illiquid asset,  $r_{i,t \rightarrow t+2}^x = R_{i,t+1}^x R_{i,t+2}^x - R_t^f R_{t+1}^f$  for  $i=1,2,\dots,K_2$ . For investment in illiquid assets, one dollar will grow by  $R_{i,t+1}^x R_{i,t+2}^x$  and after paying back the risk-free loan, the two-period excess return on illiquid assets is  $R_{i,t+1}^x R_{i,t+2}^x - R_t^f R_{t+1}^f$ . There is no “timing” portfolio for illiquid assets since they are locked up over two periods.

The  $S$  dimensional vector of  $z_t$  is a set of state variables at time  $t$ . The portfolio weights are assumed to be linear in state variables. For liquid risky assets,

$$w_{z,t} = \beta_1 z_t \text{ and } w_{z,t+1} = \beta_2 z_{t+1}, \quad (4)$$

where the matrices  $\beta_1$  and  $\beta_2$  both have a dimension of  $K_1 \times S$ . For illiquid assets, we have

$$w_{x,t} = \beta_x z_t, \quad (5)$$

where  $\beta_x$  is a  $K_2 \times S$  matrix. Throughout this paper, the portfolio strategy using the constant as the only state variable is called the “unconditional strategy”. If state variables include time-varying instruments, then the portfolio strategy is called the “conditional strategy”. For the optimal portfolio under the conditional strategy, an investor can simply maximize the utility (1) after inserting (3) into the utility function. It is obvious that the unconditional strategy is the special case of the conditional strategy with constants being the only state variable. The derivation of the optimal conditional strategy is more complicated, as we continue to show the steps below.

The equations (4) and (5) express portfolio weights as linear combinations of state variables, and the two-period portfolio excess return in (3) becomes

$$r_{t \rightarrow t+2}^p = (\beta_1 z_t)' (R_{t+1}^f r_{t+1}) + (\beta_2 z_{t+1})' (R_t^f r_{t+2}) + (\beta_x z_t)' r_{t \rightarrow t+2}^x. \quad (6)$$

Using some linear algebra, we find

$$(\beta_1 z_t)' (R_{t+1}^f r_{t+1}) = \text{vec}(\beta_1)' R_{t+1}^f (z_t \otimes r_{t+1}), \quad (7)$$

$$(\beta_2 z_{t+1})' (R_t^f r_{t+2}) = \text{vec}(\beta_2)' R_t^f (z_{t+1} \otimes r_{t+2}), \quad (8)$$

$$(\beta_x z_t)' r_{t \rightarrow t+2}^x = \text{vec}(\beta_x)' (z_t \otimes r_{t \rightarrow t+2}^x). \quad (9)$$

where  $\text{vec}(\beta_j)$  is a vector that stacks the columns of the matrix  $\beta_j$ ,  $j=1,2,x$ , and  $\otimes$  is the Kronecker product. The investment menu becomes a set of scaled returns or expanded asset

return space,  $\tilde{r}_{t+1} = z_t \otimes r_{t+1}$ ,  $\tilde{r}_{t+2} = z_{t+1} \otimes r_{t+2}$  and  $\tilde{r}_{t \rightarrow t+2}^x = z_t \otimes r_{t \rightarrow t+2}^x$ . The investor's problem is to choose a set of unconditional weights to maximize the multi-period mean-variance utility:

$$\max_{\tilde{w}} E_t \left[ \tilde{w}' \tilde{r}_{t \rightarrow t+2} - \frac{\gamma}{2} \tilde{w}' \tilde{r}_{t \rightarrow t+2} \tilde{r}'_{t \rightarrow t+2} \tilde{w} \right], \quad (10)$$

where the unconditional weights are  $\tilde{w}' = (\text{vec}(\beta_1)' \text{vec}(\beta_2)' \text{vec}(\beta_x)')$ , and  $\tilde{r}'_{t \rightarrow t+2} = \left( (R_{t+1}^f \tilde{r}_{t+1})' \quad (R_t^f \tilde{r}_{t+2})' \quad (\tilde{r}_{t \rightarrow t+2}^x)' \right)$ . The portfolio weights  $\tilde{w}$  that maximize the

conditional expected utility at all dates  $t$  should also maximize the unconditional expected utility. The optimization still makes use of the static Markowitz approach on the basis of the unconditional moments of scaled returns. The optimal static or unconditional weights are:

$$\tilde{w} = \frac{1}{\gamma} E[\tilde{r}_{t \rightarrow t+2} \tilde{r}'_{t \rightarrow t+2}]^{-1} E[\tilde{r}_{t \rightarrow t+2}]. \quad (11)$$

The sample analogue of the population moments in equation (11) leads to a consistent estimate of the unconditional weights  $\tilde{w}$ . It is a vector of length  $(2K_1S + K_2S)$ , and we can recover the optimal portfolio weights on  $K_1$  risky assets at time  $t$  and  $t+1$ ,  $w_{z,t}$  and  $w_{z,t+1}$  as

$$w_{z,t}^i = (\tilde{w}_{(i)} \quad \tilde{w}_{(i+K_1)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_1)}) z_t, \quad i = 1, 2, \dots, K_1. \quad (12)$$

$$w_{z,t+1}^i = (\tilde{w}_{(i+K_1S)} \quad \tilde{w}_{(i+K_1+K_1S)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_1+K_1S)}) z_{t+1}, \quad i = 1, 2, \dots, K_1. \quad (13)$$

For illiquid assets, the portfolio weights at time  $t$  can be derived in the same way as those of liquid risky assets.

$$w_{x,t}^i = (\tilde{w}_{(i+2K_1S)} \quad \tilde{w}_{(i+K_2+2K_1S)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_2+2K_1S)}) z_t, \quad i = 1, 2, \dots, K_2. \quad (14)$$

However, the static optimal portfolio weights in (11) do not give direct solutions to the portfolio weights of illiquid assets at time  $t+1$ . We can normalize the initial portfolio value to one and the portfolio weight of illiquid asset  $i$  is the ratio of its value to the portfolio value at the beginning of time  $t+1$ .

We can generalize the method above to the L-period asset allocation problem with lockup constraints on certain risky assets. The optimal static portfolio weights are

$$\tilde{w} = \frac{1}{\gamma} E[\tilde{r}_{t \rightarrow t+L} \tilde{r}'_{t \rightarrow t+L}]^{-1} E[\tilde{r}_{t \rightarrow t+L}], \quad (15)$$

where  $\tilde{r}_{t \rightarrow t+L}$  is a set of timing portfolios with scaled returns of liquid assets and L-period excess returns of illiquid assets scaled by the information set  $z_t$ .

The solution in (15) may produce negative weights for illiquid assets with a lockup period at time  $t$ . In reality, while shorting stocks and bonds is relatively easy, shorting illiquid assets is either too costly or impossible. For instance, investors cannot short hedge funds or transfer their stakes in hedge funds to other investors. In this case, investors should add a nonnegative constraint on portfolio weights of illiquid assets to the analysis.

### *B. Econometric Issues*

We estimate the set of portfolio weights in (15) by sample analogue. In addition, we can test whether the portfolio weights of each asset are equal across the investment horizon by a Wald test or F test. The construction of the estimated covariance matrix of  $\tilde{w}$  and the test procedure follow the method by Britten-Jones (1999).

Given a time-series sample of asset returns, the estimation of  $\tilde{w}$  can be sensitive to the choice of starting dates of the sample. Specifically, for a lockup period of  $L$ , we have  $L$  choices of starting dates, and the resulting  $L$  sets of the estimated  $\tilde{w}$  are all consistent asymptotically. Following Jegadeesh and Titman (1993), and Rouwenhorst (1998), we consider  $L$  strategies that contribute equally to a composite portfolio. Specifically, at the start of each period, the composite portfolio consists of  $L$  sub-portfolios. Each sub-portfolio invests optimally according to one set of estimated  $\tilde{w}$  on the basis of an estimation window. For example, suppose the lockup period is two months and the sample data consists of ten-year monthly asset returns. We can estimate  $\tilde{w}$  using two different windows: one starting one month later than another in the data. The composite portfolio invests one half according to the first set of estimated  $\tilde{w}$  and one half according to the second set of estimated  $\tilde{w}$ . The method is comparable to that in Jegadeesh and Titman (1993), and Rouwenhorst (1998). In those two papers, they report the monthly average return of  $K$  strategies for a  $K$ -month holding period in order to evaluate the relative strength portfolios.

## **III. Data**

For hedge funds, we obtain various hedge fund indexes and fund of funds indexes from Hedge Fund Research, Inc. (HFR, Inc.). A fund of funds or fund of hedge funds is a hedge fund that invests with multiple managers of hedge funds or managed accounts. Since a fund of funds

holds a diversified portfolio of hedge funds, it lowers the risk of investing with an individual hedge fund manager and gives access to hedge funds that are closed to new money (Nicholas (2004)). The length of the lockup period depends on the liquidity of the underlying individual hedge funds in the fund of funds. Some funds of funds require no lockup periods, but a lockup period of 3 months up to 2 years is not uncommon. An individual U.S. hedge fund typically requires a one-year lockup period plus a notice period ranging from 1 month to 3 months. In contrast, less than 40 percent of funds of funds require a lockup period, and among those funds of funds that do, about two third of them set a lockup period of 6 months or longer (Nicholas (2004)). The HFRI Fund of Funds Composite Index (HFRIFoF) is an equally-weighted index that includes over 800 funds of hedge funds with at least USD 50 Million under management. Monthly returns are net of all fees. HFR, Inc. also provides four equally-weighted sub-indexes according to the classification of fund of funds strategies: Conservative, Diversified, Market Defensive, and Strategic. A fund of funds is classified as “Conservative” if it tends to invest in funds with conservative strategies such as Equity Market Neutral, Fixed Income Arbitrage, etc. that exhibit low historical volatilities. A fund of funds is “Diversified” if it invests with various strategies/managers and exhibits performance close to that of the HFRIFoF composite index. A “Market Defensive” fund of funds invests in funds with short-biased strategies and exhibits a low or negative correlation with the equity market benchmark. Finally, a “Strategic” fund of funds tends to invest in hedge funds with more opportunistic strategies and exhibits greater volatility relative to the HFRIFoF composite index. For the composite index based on individual hedge funds, we use the HFRI Fund Weighted Composite Index (HFRI), which is an equally-weighted index based on more than 2000 individual hedge funds. The HFRI index excludes funds of funds to prevent double counting of performance figures. In addition, HFR, Inc. classifies individual hedge funds into four primary strategies: Equity Hedge, Event-Driven, Macro, and Relative Value. Each primary strategy includes several sub-strategies. HFR, Inc. provides detailed descriptions of primary and sub-strategies in its products and website. From CRSP, we obtain the value-weighted NYSE index as the proxy for stocks, the 1-month Treasury bill as the proxy for the risk-free asset, and the Fama Bond Portfolio (Treasuries) with maturities greater than 10 years as the proxy for bonds. We construct quarterly returns from monthly index returns of stocks, bonds, and hedge funds. The relatively short sample period for the hedge fund data limits the empirical analysis to the sample period from December 1989 through December 2007. Table 1 gives summary statistics of stocks, bonds and hedge funds.

Over the sample period, the average return and volatility of stocks are 11.4% and 12.6%, respectively. Bonds have an average return of 8.5% and volatility of 7.9%, but the Sharpe ratio of bonds is only slightly lower than that of stocks. The HFRIFoF composite index has a lower average return (9.7%) and volatility (5.5%) compared to stocks, and a Sharpe ratio of 1.03, which is almost twice as large as the Sharpe ratio of stocks or bonds. The HFRIFoF Conservative index has the lowest volatility among all fund of funds indexes, consistent with the style classification. The HFRIFoF Diversified index shows a similar average return and volatility compared to the composite index. Although average returns and volatilities differ among four HFRIFoF strategy indexes, their Sharpe ratios are not too far away from each other. In contrast, the HFRI Relative Value shows a Sharpe ratio that is higher than the other three HFRI strategy indexes and the HFRI composite index, mainly due to its low volatility. But given the nature of the Relative Value strategy, the returns of this strategy show fat tails. The fourth moment of the Relative Value returns is 10.43, much larger than what it would be under the normal distribution. In this case, comparisons on the basis of means and volatilities should be made with caution. The average returns of the HFRI composite index and the HFRI strategy indexes are quite high compared to stocks, bonds and fund of funds indexes. The average return of the HFRI composite index is 13.2%, which is 3.5% higher than the average return of the HFRIFoF composite index, while the volatility of the HFRI composite index is about 6.6%, only 1.1% higher than that of the HFRIFoF composite index. The difference in Sharpe ratios of the two composite indexes is 0.35, so it seems that funds of funds offer lower risk-adjusted returns relative to the aggregate individual hedge funds. The double fee structure of fund of funds investments may account for some of the difference in risk-adjusted returns, but some studies argue that the greater survivorship bias underlying individual hedge funds may cause the reported under-performance of funds of funds (e.g. Fung and Hsieh (2000)).

We obtain the state variables from CRSP<sup>1</sup>. We include the market dividend-price ratio that is known to predict asset returns.<sup>2</sup> The market dividend-price ratio is based on the value-weighted NYSE equity index, calculated as the ratio of sum of dividends over past twelve months to the NYSE index level. Figure 1 plots the time series of the market dividend-price ratio from

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<sup>1</sup> This paper includes the market dividend-price ratio as the state variable. Results based on other state variables such as the short-term interest rate, term spread and default spread are available on request.

<sup>2</sup> See Campbell (1987), Campbell and Shiller (1988a), (1988b), Campbell and Viceira, (1999), (2002), Cochrane (2007), Fama and French (1988), (1989), Keim and Stambaugh (1986), Hodrick (1992), and Lettau and Ludvigson (2005). Goyal and Welch (2007) and Campbell and Thompson (2007) include a comprehensive list of these variables along with some others as predictors used in predictability studies.

December 1989 to December 2007. The market dividend-price ratio is closely linked to the ups and downs of the U.S. stock market, so the long bull market in 1990s results in a downward trend of the market dividend-price ratio during this period. Table 2 gives the correlation matrix for risky asset returns and the lagged state variable. For most hedge fund indexes, their correlations to the market dividend-price ratio are stronger than the correlations of stocks and bonds to the market dividend-price ratio. The correlations of hedge fund returns to stock returns are moderate for most hedge fund indexes, except for FoF Market Defensive. Stock returns and bond returns are weakly correlated as expected. Most hedge fund index returns have a low correlation to bond returns, with the exception of HFRI Macro. Notice that the correlation of stock returns to the HFRIFoF composite index returns is high (0.43), but lower than the correlation of stock returns to the HFRI composite index returns (0.69). This implies that the HFRIFoF is a better diversifier than the HFRI does. On the other hand, a high correlation of stock returns to hedge fund returns indicates that hedge funds and funds of funds have large equity exposures.

#### **IV. Unconditional Strategy with a Three-Month Lockup**

Section A starts by reporting the portfolio weights of the unconditional strategy with a three-month hedge fund lockup period. We are interested in the difference in the allocations to stocks and bonds when hedge funds are added to the portfolio, as well as the changes in investment patterns over the three-month investment horizon. We decompose the total demand for stocks and bonds in the portfolio of stocks, bonds and hedge funds into the Markowitz or speculative demand and the hedge demand. In addition, we show how a hedge fund lockup period affects the Markowitz demand, hedge demand and thus the total demand for stocks and bonds. Section B compares the performance of the unconditional strategy with a lockup period and without a lockup period. In addition, we test whether adding hedge funds improves the Sharpe ratio of the portfolio.

##### *A. Portfolio Weights and Decomposition of Stock and Bond Demands*

Table 3 reports the results for the unconditional strategy with a three-month hedge fund lockup period. The estimated parameters, portfolio performance and test statistics are the

averages of three-month rolling windows. We can think of this as the result of a strategy that always invests  $1/L$  of wealth for three months (i.e.  $L = 3$  in Section IV), starting every month, just as in Jegadeesh and Titman (1993) and Rouwenhorst (1998) (see Section II.B. Econometric Issues). The t-statistics for the portfolio weights are based on Britten-Jones (1999). The degree of risk aversion of the investor is 10 for all analyses<sup>3</sup>.

Results for the unconditional strategy in Table 3 show that portfolio weights vary in a systematic way over the investment horizon. The variation in portfolio weights is caused by the presence of timing portfolios. To start out, in the portfolio of stocks and bonds only, allocations to stocks and bonds display distinct patterns over the investment horizon. Over the three months, allocations to stocks decrease monotonically from 60% to 43% while allocations to bonds increase monotonically from 55% to 98%. Thus, an investor starts with a relatively risky portfolio and gradually adjusts his portfolio holdings in order to obtain a less risky portfolio by the end of the investment horizon. This is in line with the idea of life-cycle funds where equity exposure decreases over time, due to the autocorrelation in stock returns. We test the restriction that allocations to stocks or bonds are equal across three months, and the p-values indicate that the null hypothesis cannot be rejected for both stocks and bonds.

Adding hedge funds to the portfolio of stocks and bonds reduces the allocation to stocks and increases the allocation to bonds for each month, irrespective of whether or not a hedge fund lockup period exists. This reflects the fact that investing in hedge funds leads to bigger equity exposure relative to bond exposure. Adding hedge funds to the portfolio changes the pattern of portfolio weights of stocks over the investment horizon, while the pattern of portfolio weights of bonds remain monotonically increasing. For example, inclusion of the HFRI composite with a three-month lockup period will reverse the pattern of investments in stocks from being monotonically decreasing to be monotonically increasing from  $-57\%$  in Month 1 to  $-31\%$  in Month 3. However, inclusion of the HFRIFoF composite with a three-month lockup period to the portfolio of stocks and bonds will change the pattern of investment in stocks over the three-month period from being monotonically decreasing to being an inverted U-shape, as allocations to stocks increase from 19% in Month 1 to 30% in Month 2, and then decrease to 27% in

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<sup>3</sup> The degree of investor's risk aversion in the empirical analysis is based on the mean-variance utility function for computational convenience only. Note that there is a corresponding relation between the mean-variance portfolio and the mean-second moment portfolio (Britten-Jones (1999)) by rescaling the degree of investor's risk aversion. For the same optimal portfolio weights, the degree of investor's risk aversion is lower under the mean-second moment utility.

Month 3. Nevertheless, the difference of allocations to stocks across the three months is statistically insignificant.

To further investigate these changes in the pattern of the portfolio weights of stocks and bonds, we calculate the Markowitz (or pure speculative) demands and the hedge demands for stocks and bonds in the three-asset portfolio with a hedge fund lockup period. Table 4 shows the optimal demand for stocks and bonds as the combination of the Markowitz demand<sup>4</sup> and the hedge demand, using either the HFRI or the HFRIFoF composite index as the proxy for hedge funds in the three-asset portfolio with the lockup restriction. The hedge demand is the product of two determinants: the optimal demand for hedge funds at time  $t$ , denoted by  $w_{x,t}^*$ , and the slope coefficients from the regression of three-month excess returns of hedge funds on a constant and returns of the timing portfolios of stocks and bonds:

$$r_{t \rightarrow t+3}^x = \alpha + b'_{s,1}(R_{t+1}^f R_{t+2}^f r_{t+1}^s) + b'_{s,2}(R_t^f R_{t+2}^f r_{t+2}^s) + b'_{s,3}(R_t^f R_{t+1}^f r_{t+3}^s) \\ + b'_{b,1}(R_{t+1}^f R_{t+2}^f r_{t+1}^b) + b'_{b,2}(R_t^f R_{t+2}^f r_{t+2}^b) + b'_{b,3}(R_t^f R_{t+1}^f r_{t+3}^b) + \varepsilon_t, \quad (16)$$

For instance, the hedge demand for stocks in the first period is  $-(w_{x,t}^*)' \cdot b_{s,1}$ . The hedge demands in other periods and for bonds follow the same logic.

From Table 4, the restriction that hedge demands (as well as optimal demands) are equal across three months cannot be rejected by the Wald test for all cases. We find that for each month, the hedge demand is negative for stocks and positive for bonds. Furthermore, the hedge demand for stocks is most negative in the beginning and increases over time, which results in a pattern of the optimal demands different from the Markowitz demands for stocks. For instance, adding the HFRIFoF to the portfolio gives rise to a small allocation to stocks relative to the Markowitz demand in the first month (19% vs. 60%). The Markowitz demand decreases to 58% in the second month, while the total demand increases to 30% due to an increase in the hedge demand. In the third month, the total demand for stocks decreases to 27%, as the increase in the hedge demand is more than offset by the decrease in the Markowitz demand. This is the reason that the total demand for stocks exhibits an inverted U-shape. For bond investments in the three-asset portfolio, changes in portfolio weights are dominated by the changes in the Markowitz demands. The hedge demands for bonds are relatively small; the

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<sup>4</sup> The Markowitz demands for stocks and bonds in the three-asset allocation are the optimal allocations to stocks and bonds in the two-asset allocation, i.e. the portfolio weights of stocks and bonds in column 2 of Table 3. The difference between the total demands for stocks and bonds in the three-asset allocation (column 4, 6, 8, or 10 of Table 3) and the total demands for stocks and bonds in the two-asset allocation is the hedge demand.

changes in hedge demands over the three-month horizon are not large enough to reverse the pattern of total investments in bonds.

We can explain the difference in the patterns and magnitudes of the hedge demands for stocks and those for bonds by examining the second determinant of the hedge demands, which is the set of slope coefficients from the regression (16). We look at the correlations between hedge funds and stocks or bonds in Table 2 to get a rough estimation of the magnitude and direction of the hedge demand<sup>5</sup>. From Table 2, the correlation between stocks and the HFRI composite is 0.69, while the correlation between bonds and the HFRI composite is near zero. In other words, hedge funds look more like stocks. To hedge the changes in the value of hedge funds, an investor can simply go short on stocks, and the hedge demand for stocks is  $-(w_{x,t}^*)' \cdot b_{s,1}$  for the first month. As bonds and hedge funds are weakly correlated, the hedge demand for bonds is relatively small.

Investing in hedge funds when there is a lockup period, basically leads to an exogenously given exposure to hedge funds after the first period, which induces additional hedge demand for stocks and bonds. The optimal investment in stocks and bonds in the three-asset portfolio is the sum of the Markowitz demands and the hedge demand. The Markowitz demand is the optimal portfolio weights of stocks and bonds when the investment menu includes stocks and bonds only. The hedge demand arises because the investor wants to hedge the changes in the value of hedge fund investment, which is locked up for three months. A negative hedge demand for stocks implies that the overall allocation to stocks will be lower than it would be in the portfolio consisting of only stocks and bonds.

The patterns of investments in stocks differ when different hedge fund indexes are used as a proxy. We can explain the difference by examining the difference in the two determinants of hedge demands for stocks. The first determinant  $w_{x,t}^*$  is larger when the HFRI composite is included in the portfolio, relative to the allocation to hedge funds in the portfolio of stocks, bonds and the HFRIFoF composite. In fact, adding the HFRI composite to the portfolio results in the allocation to hedge funds being almost twice as large as the allocation to hedge funds when the proxy for hedge funds is the HFRIFoF. In addition, the correlation between stock returns and the HFRI composite returns is 0.69, higher than the correlation between stock

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<sup>5</sup> The estimation is not precise because the dependent variable is the three-month excess return of the hedge fund and the independent variables are returns on timing portfolios of stocks and bonds. The correlation matrix in Table 2, however, is based on monthly series of excess returns on hedge funds, stocks and bonds.

returns and the HFRIFoF composite, 0.43. In other words, the second determinant,  $b_{s,1}$ , is more likely to be larger when the HFRI composite is the hedge fund proxy. Both determinants work in the same direction such that the hedge demands are larger (in absolute value or magnitude) in the portfolio of stocks, bonds and HFRI than those in the portfolio of stocks, bonds and the HFRIFoF composite. In fact, the hedge demands are so much larger than the Markowitz demands for stocks when the HFRI composite is included in the portfolio that they lead to negative total demand for stocks.

The total allocations to bonds in the three-asset portfolio are similar to the Markowitz demands when either the HFRI or the HFRIFoF composite is the proxy. Adding either hedge fund composite would not change the trend of investments in bonds. In all cases, total allocations to bonds increases monotonically over the three-month period. The hedge demands for bonds are larger in the portfolio of stocks, bonds and the HFRI composite than those in the portfolio of stocks, bonds and the HFRIFoF composite, mostly due to a large portfolio weight of the HFRI composite (i.e. a large  $w_{x,t}^*$ , the first determinant of the hedge demand for bonds). Nevertheless, the hedge demands for bonds are small relative to the Markowitz demands, and the variation (month-to-month difference) in the hedge demands is not large enough to make a difference in the trend of total investments in bonds. The Markowitz demand for bonds is 55%, 74% and 98% in the first, second and third month. The corresponding hedge demands for bonds in the portfolio of stocks, bonds and the HFRI composite are 24%, 13% and 27% (11%, 7% and 9% in the portfolio of stocks, bonds and the HFRIFoF composite). The variation in the Markowitz demands is 20% from month 1 to month 2, and 24% from month 2 to month 3. In contrast, the variation in the hedge demands is less than 14% (4% in the portfolio of stocks, bonds and the HFRIFoF composite).

### *B. Portfolio Efficiency and Certainty Equivalent*

The above analysis has shown that taking into account lockup periods for hedge funds has important portfolio implications. A question of considerable importance is now whether hedge funds offer diversification benefits when they are added to a portfolio of stocks and bonds only. Table 5 reports the performance of various portfolios of stocks, bonds and hedge funds under the unconditional strategy. The p-values, as they appear in the table, are calculated based on the averaged test statistics over the three overlapping samples. In each case, a different hedge fund

index is used as the proxy. The mean excess return and volatility of the two-asset portfolio are 8.1% and 8.9%, respectively. The three-asset portfolios with or without a lockup period have noticeably higher mean excess returns and volatilities. Moreover, the Sharpe ratios of the three-asset portfolios are much higher than the two-asset portfolio. The difference in mean returns, volatilities and Sharpe ratios of the three-asset portfolios is large for all hedge fund indexes. For instance, the portfolio of stocks, bonds and the HFRIFoF composite with a lockup has a mean excess return of 14.8% with a volatility of 12.1%, compared to a mean excess return of 23.7% and a volatility of 15.3% for the portfolio of stocks, bonds and the HFRI composite. The Sharpe ratio of the first portfolio above is 1.23, lower than the Sharpe ratio of 1.55 of the second portfolio.

The test of portfolio efficiency follows Jobson and Korkie (1982) and De Roon and Nijman (2001). Denote the sample Sharpe ratio for the benchmark portfolio  $r^p$  by  $\hat{\theta}_p$ , and the sample Sharpe ratio for the portfolio of test assets  $r$  and benchmark assets  $r^p$ , by  $\hat{\theta}$ . The Wald statistic of the Sharpe ratio test is:

$$\xi_w = T \left( \frac{\hat{\theta}^2 - \hat{\theta}_p^2}{1 + \hat{\theta}_p^2} \right) \sim \chi_K^2 \quad (17)$$

where  $T$  is the sample size and  $K$  is the degrees of freedom. The degrees of freedom are the difference in the number of parameters between the two portfolios. From the p-values of the Sharpe ratio test in Table 5, the difference in Sharpe ratios between the two-asset portfolio and every three-asset portfolio is statistically significant at the 1% significance level, suggesting that the two-asset portfolio can be significantly improved upon by adding hedge funds.

An investor who ignores the existence of a hedge fund lockup period will get a wrong estimate of portfolio performance. From Table 3 and Table 4, we know that the existence of a three-month lockup period for hedge funds makes a difference in the allocations to stocks, bonds and hedge funds over the investment horizon. If there would be no hedge fund lockup period, a portfolio of stocks, bonds and hedge funds would have a higher mean excess return and volatility, as well as a higher Sharpe ratio, relative to a portfolio of stocks, bonds and hedge funds with a lockup period of three months, regardless of the choice of the hedge fund proxy. As shown in Table 5, the difference in Sharpe ratios between the three-asset portfolio with a hedge fund lockup period and the three-asset portfolio without a hedge fund lockup is large and statistically significant (except for the case when the HFRI Relative Value as the hedge fund

proxy). For instance, the portfolio of stocks, bonds and the HFRIFoF composite with a lockup has the Sharpe ratio of 1.23, but the Sharpe ratio is 1.53 if there is no lockup period. The difference is statistically significant at the 1% significance level. Similarly, for the portfolio of stocks, bonds and the HFRI composite, the difference in Sharpe ratios is 0.23 (1.55 vs. 1.78). Hence, overlooking the existence of a hedge fund lockup period may overstate the performance of three-asset portfolios.

We calculate the certainty equivalent as the difference in utilities (utility function is given by the expression (1) in Section II.A) using the two-asset portfolio as the benchmark. The certainty equivalent can be considered as the fee an investor is willing to pay in order to have a portfolio of stocks, bonds and hedge funds. The portfolio of stocks, bonds and the HFRI composite has a certainty equivalent of 7.9% with a three-month lockup period, and 11.8% if there is no lockup. The certainty equivalent of the portfolio of stocks, bonds and the HFRIFoF composite is 3.4% if there is lockup, and 7.6% if there is no lockup.

## **V. Conditional Strategy with a Three-Month Lockup**

This section reports the portfolio weights and performance of various portfolios under the conditional strategy. We consider asset allocations conditional on one state variable, i.e. the market dividend-price ratio. We analyze the (average) total demand for stocks and bonds in the conditional portfolio of stocks, bonds and hedge funds, as a combination of the speculative demand (Markowitz demand) and the hedge demand due to investments in hedge funds with a three-month lockup period, similar to the previous section. We test the difference in Sharpe ratios of the three-asset portfolio with a lockup period and the portfolio without a lockup period. Furthermore, we test whether using conditional strategy improves the efficiency of the unconditional strategy.

### *A. Portfolio Decision Conditional on the Market Dividend-Price ratio*

Table 6 reports the results of asset allocations under the conditional strategy. The state variable is standardized to have a zero mean and a volatility of one, so the intercepts are the average allocations over the sample period. The average allocations to stocks and bonds change with the passage of time. For the two-asset allocation, we cannot reject the null hypothesis that

average allocations to stocks as well as to bonds are equal across time. The average allocations to stocks are not too different across the three sub-periods (69%, 77% and 66%), while the average allocation to bonds is 62% in the first month and increases from 70% in month 2 to 86% in month 3. This implies that bonds become relatively important in the portfolio as the investment horizon approaches. In addition, for all three-asset portfolios, the average portfolio weights of bonds appear to be increasing over time, a similar pattern to what we found for the two-asset portfolio.

Changes in the state variable lead to changes in portfolio weights of the conditional strategy. The sign of coefficients on the market dividend-price ratio in determining portfolio weights of stocks changes over time. The investor's responses to changes in the state variable will depend on whether hedge funds are added to the portfolio, which hedge fund index is used as the proxy, and which month the rebalancing decision is made. For instance, in the two-asset allocation, the change in the market dividend-price ratio is positively related to the allocations to stocks in the second and third month, but not in the first month. The change in the market dividend-price ratio is always negatively associated with the change in the allocations to bonds. Moreover, at a given month, the sign and magnitude of the slope coefficients on the state variable are different across different portfolios. This stresses the importance for the investor to take into account the lockup period in his conditional strategy. We test the null hypothesis that the slope coefficients of dividend-price ratio are equal across three months. We can't reject the null hypothesis for coefficients related to stocks or bonds in any case.

The average allocations to stocks and bonds in the three-asset portfolios with a three-month hedge fund lockup period increase monotonically. Table 7 shows the decomposition of the total demand for stocks and bonds as the combination of the Markowitz demand and the hedge demand. The Markowitz demands for stocks are 69%, 77%, and 66% in the first, second and third month, respectively. Adding the HFRI composite to the portfolio induces an increasing average hedge demand for stocks in the first, second and third month of -139%, -116% and -78%, resulting in a total demand for stocks of -70%, -40%, and -12%, accordingly. The presence of large, negative hedge demands for stocks is not surprising. Since the investment in hedge funds is locked up for three months, an offsetting position in stocks provides the hedge against possible declining value of hedge funds over the investment horizon. Because hedge funds and stocks are alike, the magnitude of the hedge demands for stocks is large. In this particular case, the inverted U-shape of the Markowitz demands for stocks is overwhelmed by

the increasing hedge demands, such that the total demands for stocks increase over time. Nevertheless, there is no evidence that the total demands differ across the three months at any conventional significance level. Inclusion of the HFRI composite with a three-month lockup period generates positive average hedge demands for bonds. The average hedge demands do not differ too much from month to month. Since the average Markowitz demands for bonds are positive and monotonically increasing over time, the total demands for bonds are also positive and monotonically increasing.

When the HFRIFoF composite is chosen as the hedge fund proxy, the size of hedge demands for stocks becomes smaller. The hedge demand for stocks is  $-35\%$  in the first month, and does not differ from zero at the 10% significance level in the second and third month. Average hedge demands for bonds are negative. In contrast, we have shown that the average demands for bonds are positive when the HFRI composite is the hedge fund proxy. Since the investments in hedge funds are large and positive in both cases, the opposite signs of hedge demands for bonds can only be the result of difference in the covariances between bond returns and returns of the two hedge fund composite indexes.

### *B. Conditional Strategy vs. Unconditional Strategy*

A comparison of the conditional strategy and unconditional strategy reveals some interesting results<sup>6</sup>. One similarity is the patterns of investments in bonds: allocations to bonds increase monotonically over time in all portfolios under both the unconditional strategy and conditional strategy. With respect to portfolio weights of bonds, difference due to the use of the conditional strategy is not too large for the two-asset allocations, as well as the three-asset allocations with the HFRI composite. The conditional strategy seems to reduce allocations to bonds in the portfolios of stocks, bonds and the HFRIFoF composite, compared to the unconditional strategy. The difference is caused by the hedge demands for bonds under the two strategies: there are positive average hedge demands under the unconditional strategy, and negative hedge demands under the conditional strategy. With similar Markowitz demands under both strategies, the difference in hedge demands is passed through to total demands.

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<sup>6</sup> All comparisons of results in Subsection V.B. are made between the conditional strategy and the unconditional strategy.

The conditional strategy on average allocates more to stocks in every period in the two-asset portfolio and the three-asset portfolios with the HFRIFoF composite (with the HFRI composite, the allocations to stocks under the conditional strategy are larger in the second and third month compared to the unconditional strategy, but not in the first month). This reflects the possibility of portfolio rebalancing in response to changing market conditions. It appears that the ability to adjust portfolio weights according to changes in the state variable induces the investor to allocate more aggressively to stocks (and less to bonds in the portfolios of stocks, bonds and the HFRIFoF composite). Larger total demands for stocks in the portfolio of stocks, bonds and the HFRIFoF composite under the conditional strategy are due to both larger Markowitz demands and larger hedge demands. When the HFRI composite is added to the portfolio (with a lockup), even though the average Markowitz demands for stocks are higher under the conditional strategy, the average hedge demand for stocks in each period is lower under the conditional strategy (-139%, -116%, and -78% vs. -117%, -103% and -74% under the unconditional strategy). The net effect on the total demand depends on whether the difference in Markowitz demands (in the absolute term) is larger or smaller than the difference in hedge demands. The conditional strategy leads to lower average allocations to stocks in the first month, as the difference in the hedge demands (-22%) dominate the difference in the Markowitz demands (9%). The effect is reversed in the remaining two periods, as the difference in the Markowitz demands dominates such that the total demands for stocks are higher due to the higher Markowitz demands under the conditional strategy.

Finally, when the conditional strategy includes hedge funds with a three-month lockup period, the average allocations to hedge funds become larger compared to the allocations to hedge funds under the unconditional strategy. Together with our results related to stocks above, we can conclude that investors are in general more aggressive under the conditional strategy.

### *C. Portfolio Efficiency*

Table 8 shows the performance of the conditional strategy using different hedge fund indexes as a proxy for investments in hedge funds. It also reports the certainty equivalents for three-asset portfolios. Three questions arise. First of all, is the two-asset portfolio under the conditional strategy mean-variance efficient or does adding hedge funds to the portfolio improve the portfolio efficiency? Second, are portfolios under the unconditional strategy mean-

variance efficient? Third, what difference does a three-month hedge fund lockup period make in terms of the portfolio performance?

We can use the Sharpe ratio test to determine the portfolio efficiency of the two-asset portfolio under the conditional strategy, and various portfolios under the unconditional strategy against the portfolios under the conditional strategy. For each case, we have p-values from four Sharpe ratio tests. For instance, using the HFRIFoF composite index as the proxy, the Sharpe ratios of the three-asset portfolio under the conditional strategy with a lockup and without a lockup are 1.57 and 1.77, respectively. The p-value (0.000) to the right of the Sharpe ratio of the three-asset portfolio with a lockup period is based on the Sharpe ratio test in which the two-asset allocation as the benchmark portfolio. The p-value (0.128) next to the Sharpe ratio of the three-asset portfolio without a lockup period is based on the Sharpe ratio test of the difference in Sharpe ratios of two three-asset portfolios, i.e. the portfolio with a lockup period vs. the portfolio without a lockup period. The p-values (0.079) and (0.463) below the Sharpe ratios of the three-asset portfolio with a lockup and without a lockup are based on the Sharpe ratio test of the unconditional strategy vs. conditional strategy.

For all cases, the difference in Sharpe ratios of the three-asset portfolio with a three-month lockup period and the two-asset portfolio is significant at the 1% significance level. Hence, we conclude that the portfolio of stocks and bonds is not mean-variance efficient under the conditional strategy. The investor should add hedge funds to the portfolio even though there is a lockup period of three months.

The two p-values below the Sharpe ratios of three-asset portfolios come from the Sharpe ratio test of the unconditional strategy vs. conditional strategy. For the three-asset portfolios with a lockup period and using any hedge fund index, the Sharpe ratios of the portfolios under the conditional strategy do not differ from those of the portfolios under the unconditional strategy at the 10% significance level. Therefore, even if the market dividend yield predicts returns of stocks, bonds and hedge funds, and generates allocations different from those under the unconditional strategy, a mean-variance investor does not benefit from using the conditional strategy. However, in terms of the certainty equivalent, in some cases, the difference between the unconditional strategy and the conditional strategy is quite large. For instance, the certainty equivalent is 3.4% for the three-asset portfolio of stocks, bonds and the HFRIFoF composite with a lockup under the unconditional strategy, and 5.8% under the

conditional strategy. The difference is the largest (17.6% vs. 23.9%) when the hedge fund proxy is the HFRI Relative Value strategy index.

To answer the last question, we perform the Sharpe ratio test of the three-asset portfolio with a three-month lockup period vs. the three-asset portfolio without a lockup period, in order to assess the effect of a three-month hedge fund lockup on the portfolio performance. When the HFRIFoF composite and four HFRIFoF strategy indexes are considered as the proxy for hedge funds, the difference in the Sharpe ratios of the three-asset portfolios with a lockup period and the three-asset portfolios without a lockup period is not significant for 4 cases. The difference is significant at the 10% level only if the HFRIFoF Conservative index is used as the hedge fund proxy in the three-asset portfolio. When the HFRI composite index or the HFRI strategy indexes are used as the proxy for hedge funds, the difference is much larger and significant at the 10% significance level for all cases. Therefore, having a three-month lockup period implies a significant lower Sharpe ratio of the three-asset portfolio of stocks, bonds, and the HFRI composite (and HFRI strategy indexes). We can also compare the certainty equivalents for portfolios with or without a lockup. The difference is in the range of 4% to 5% when the HFRI composite is the hedge fund proxy (the difference is in the range of 2% to 4% when the HFRIFoF composite is the hedge fund proxy).

As the results of the Sharpe ratio tests indicate, a three-month lockup period has smaller negative effect on the performance of the three-asset portfolios of stocks, bonds and funds of funds. Funds of funds seem to be better able to suppress the effect of lockup periods on the portfolio performance of the conditional strategy than individual hedge funds do. Possible explanations which require further research include: a fund of funds typically has more frequent subscriptions and can use new money to pay off redemption requests. Moreover, a fund of funds manager can actively manage the lockup periods of the underlying individual hedge funds, such that each fund has a different lockup expiration date. In this way, a fund of funds can still invest in many individual hedge funds with long lockup periods, while imposing a shorter lockup period for fund of funds investors. From the perspective of an investor following the conditional strategy, when he decides to add funds of funds to the portfolio of stocks and bonds, a three-month hedge fund lockup period should not cause great concerns. In contrast, the investor should not overlook the effect of a three-month lockup period on the portfolio performance when individual hedge funds are considered. He would get a wrong

impression of the incremental benefits of investing in individual hedge funds if he ignores the existence of a lockup period.

## **VI. Bootstrap Samples with a One-Year Lockup Period**

Three-month hedge fund lockup periods are plausible for many funds of funds, but some funds of funds and individual hedge funds have longer lockup periods. The estimation is problematic with longer lockup periods since the history of hedge fund indexes is relatively short. For instance, for the one-year horizon, we have only 18 non-overlapping samples to estimate parameters of interest whose number can be more than 70. Using quarterly returns or fewer state variables will reduce the number of parameters, without decreasing the sample size. We use the bootstrap method to obtain a larger sample size in order to examine the effect of a long lockup period.

We follow the stationary bootstrap method by Politis and Romano (1993) and Sullivan, Timmermann and White (1999) to obtain 5000 bootstrap samples of quarterly data. The smoothing parameter is chosen to be 0.2, so the mean block length is 5 quarters. The choice of the smoothing parameter affects the portfolio weights and performance, but the results of the Sharpe ratio tests are not too sensitive to the smoothing parameter.

Table 9 gives the results of the portfolio performance under the unconditional strategy, using various hedge fund indexes as a proxy for hedge funds. The significantly higher Sharpe ratios for the three-asset portfolios justify the inclusion of hedge funds into an investors' portfolio. Nevertheless, a one-year lockup period seems to make little impact on the performance of the three-asset portfolios of stocks, bonds and the HFRIFoF (or HFRIFoF strategy indexes), as the difference in Sharpe ratios of the portfolios with or without a lockup period is not significant. Adding the HFRI composite or HFRI strategy indexes to the portfolio also increases the Sharpe ratio significantly. However, having a one-year lockup period causes the significant difference in the Sharpe ratios of the three-asset portfolios when the HFRI Event-Driven or the HFRI Relative Value is used as the hedge fund proxy.

Table 10 reports the analysis of the portfolio performance under the conditional strategy. The Sharpe ratios are significantly higher under the conditional strategy than those under the unconditional strategy in all cases. Therefore, an investor can benefit from using conditional information in the portfolio decision with one-year investment horizon. Relative to the two-

asset portfolio, adding hedge funds to the portfolios improves the portfolio payoff in terms of Sharpe ratios under the conditional strategy. However, a portfolio investor would overestimate the portfolio performance when he ignores the presence of a one-year hedge fund lockup period. A one-year lockup period has a significant impact on the portfolio performance whichever hedge fund index is chosen as the hedge fund proxy. It seems that if the lockup period is long, an investor should be concerned with the effect of a lockup period on the performance of his portfolio under the conditional strategy, for investments in funds of funds as well as individual hedge funds.

## **VII. Conclusion**

A lockup period is a realistic feature of investments in hedge funds, private equities and venture capital. This paper considers the impact of a hedge fund lockup period on the asset allocation decisions of a mean-variance investor who re-adjusts the portfolio weights periodically. Due to the presence of a hedge fund lockup period, the investor can only adjust the allocation of stocks and bonds. The mean-variance framework in this paper serves to illustrate the effect of hedge fund lockup periods on multi-period asset allocation, with the potential to extend to other asset classes with similar lockup or illiquid constraints. The empirical analysis indicates that the investor is better off by investing in portfolios of stocks, bonds and hedge funds, relative to a portfolio of stocks and bonds only. In addition, the three-asset portfolios under the unconditional strategy seem to be mean-variance efficient with a three-month horizon and monthly frequency. The conditional strategy can achieve better outcomes in terms of Sharpe ratios than the unconditional strategy only with a longer horizon and quarterly frequency. Most importantly, the presence of a lockup period is not trivial, especially when investing in individual hedge funds. An investor may overstate the benefit from adding individual hedge funds to a portfolio when he overlooks the existence of a hedge fund lockup period. Nevertheless, funds of funds seem to be able to suppress the effect of a short lockup period on the portfolio performance under the conditional strategy and the effect of a long lockup period on the portfolio performance of the unconditional strategy.

**Table 1**  
**Descriptive Statistics of Returns of Stocks, Bonds and Hedge Funds**

This table gives summary statistics of risky assets from January 1990 to December 2007. The value-weighted NYSE index is proxy for stocks, and Fama Bond Portfolio (Treasuries) with maturities greater than 10 years is proxy for bonds. For hedge funds, various indexes are considered: HFRI Fund of Funds composite index (HFRIFoF), HFRIFoF sub-strategy indexes, HFRI Fund Weighted Composite Index (HFRI) and HFRI sub-strategy indexes. Means, standard deviations, maximums and minimums are expressed in percentages. We annualize means, standard deviations and Sharpe ratios, while the remaining statistics are on a monthly basis.

	Mean	Std	Sharpe	Max	Min	Skew	Kurtosis
Stocks	11.4%	12.6%	0.581	10.7%	-14.7%	-0.531	1.347
Bonds	8.5%	7.9%	0.563	7.2%	-8.3%	-0.430	0.845
HFRIFoF	9.7%	5.5%	1.033	6.9%	-7.5%	-0.284	4.049
-Conservative	8.3%	3.2%	1.332	4.0%	-3.9%	-0.506	3.144
-Diversified	9.1%	5.8%	0.870	7.7%	-7.8%	-0.134	4.182
-Market Defensive	9.4%	5.8%	0.939	7.4%	-5.4%	0.148	1.234
-Strategic	12.7%	8.6%	1.010	9.5%	-12.1%	-0.389	3.827
HFRI	13.2%	6.6%	1.383	7.7%	-8.7%	-0.590	2.940
-Equity Hedge	15.7%	8.5%	1.376	10.9%	-7.7%	0.193	1.551
-Event-Driven	13.5%	6.4%	1.475	5.1%	-8.9%	-1.251	4.630
-Macro	14.3%	8.0%	1.295	7.9%	-6.4%	0.394	0.784
-Relative Value	11.2%	3.5%	2.079	5.7%	-5.8%	-0.804	10.433

**Table 2**  
**Correlation Matrix of the State Variable and Asset Returns**

This table displays the correlation matrix of the lagged state variable and risky asset returns from January 1990 to December 2007. The data frequency is monthly. State variables include the market dividend-price ratio.

	Dividend price ratio	Stocks	Bonds	HFRIFoF composite	FoF Conservative	FoF Diversified	FoF Market defensive	FoF Strategic	HFRI composite	Equity Hedge	Event-Driven	Macro	Relative Value
Dividend price ratio	1												
Stocks	0.08	1											
Bonds	0.05	0.05	1										
HFRIFoF composite	0.12	0.43	0.02	1									
FoF Conservative	0.13	0.44	0.05	0.89	1								
FoF Diversified	0.10	0.43	0.00	0.97	0.84	1							
FoF Market Defensive	0.07	0.04	0.11	0.69	0.62	0.63	1						
FoF Strategic	0.19	0.48	0.01	0.93	0.82	0.87	0.55	1					
HFRI composite	0.14	0.69	-0.01	0.83	0.74	0.81	0.35	0.85	1				
Equity Hedge	0.13	0.64	0.00	0.77	0.69	0.75	0.35	0.80	0.93	1			
Event-Driven	0.08	0.67	-0.03	0.67	0.62	0.65	0.26	0.70	0.88	0.78	1		
Macro	0.21	0.40	0.28	0.72	0.64	0.71	0.52	0.68	0.69	0.61	0.56	1	
Relative Value	0.20	0.39	-0.03	0.53	0.54	0.51	0.27	0.53	0.63	0.55	0.65	0.41	1

**Table 3**  
**Asset Allocation under the Unconditional Strategy**  
**[Lockup: Three-month]**

This table reports the results of asset allocations under the unconditional strategy for the degree of risk aversion of the investor,  $\hat{\gamma} = 10$ . The data frequency is monthly. Column 2 to 7 show optimal unconditional weights for various portfolios at each month using the HFRI composite index as the proxy for hedge funds. Column 8 to 11 show optimal unconditional weights for various portfolios at each month using the HFRIFoF composite index as the proxy for hedge funds. Absolute values of t-statistics for the portfolio weights are in square brackets. For each portfolio, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

Period	HFRI as Hedge Fund Proxy						HFRIFoF as Hedge Fund Proxy			
	Two-Asset		Three-Asset with a Lockup		Three-Asset No Lockup		Three-Asset with a Lockup		Three-Asset No Lockup	
Column	2	3	4	5	6	7	8	9	10	11
<b>Stocks</b>										
Month 1	0.601	[1.504]	-0.572	[0.904]	-1.071	[1.330]	0.194	[0.410]	0.017	[0.401]
Month 2	0.578	[1.404]	-0.449	[1.061]	-0.104	[1.134]	0.304	[0.607]	0.504	[1.280]
Month 3	0.426	[1.013]	-0.311	[0.672]	0.259	[1.446]	0.271	[0.675]	0.418	[1.279]
<b>Wald Test</b>		(0.682)		(0.546)		(0.067)		(0.635)		(0.162)
<b>Bonds</b>										
Month 1	0.547	[0.835]	0.781	[0.931]	0.831	[0.904]	0.653	[0.878]	0.771	[0.905]
Month 2	0.743	[1.186]	0.875	[1.107]	0.981	[1.122]	0.817	[1.166]	0.894	[1.147]
Month 3	0.981	[1.535]	1.252	[1.541]	1.392	[1.535]	1.072	[1.498]	1.121	[1.384]
<b>Wald Test</b>		(0.834)		(0.813)		(0.813)		(0.740)		(0.541)
<b>Hedge Funds</b>										
Month 1			2.292	[3.912]	3.950	[2.396]	1.378	[2.686]	2.436	[1.786]
Month 2					2.142	[1.316]			1.157	[1.616]
Month 3					1.173	[1.563]			0.715	[1.611]
<b>Wald Test</b>						(0.022)				(0.011)

**Table 4**  
**Hedge Demands for Stocks and Bonds under the**  
**Unconditional Strategy**  
**[Lockup: Three-month]**

This table displays, for the three-asset portfolio under the unconditional strategy with a three-month hedge fund lockup period, the decomposition of portfolio weights of stocks and bonds at each month into two parts: a Markowitz demand or speculative demand and a hedge demand. Absolute values of t-statistics for those portfolio weights are in square brackets. For each asset, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

	Markowitz Demand [M]		Hedge Demand [H]		Optimal Demand = M + H	
<b>HFRI as Hedge Fund proxy:</b>						
<b>Stocks</b>						
Month 1	0.601	[1.504]	-1.174	[5.355]	-0.572	[0.904]
Month 2	0.578	[1.404]	-1.026	[4.702]	-0.449	[1.061]
Month 3	0.426	[1.013]	-0.737	[3.319]	-0.311	[0.672]
<b>Wald Test</b>		(0.682)		(0.172)		(0.546)
<b>Bonds</b>						
Month 1	0.547	[0.835]	0.235	[0.643]	0.781	[0.931]
Month 2	0.743	[1.186]	0.132	[0.681]	0.875	[1.107]
Month 3	0.981	[1.535]	0.271	[0.785]	1.252	[1.541]
<b>Wald Test</b>		(0.834)		(0.755)		(0.813)
<b>HFRIFoF as Hedge Fund proxy:</b>						
<b>Stocks</b>						
Month 1	0.601	[1.504]	-0.408	[2.912]	0.194	[0.410]
Month 2	0.578	[1.404]	-0.273	[1.943]	0.304	[0.607]
Month 3	0.426	[1.013]	-0.155	[1.063]	0.271	[0.675]
<b>Wald Test</b>		(0.682)		(0.258)		(0.635)
<b>Bonds</b>						
Month 1	0.547	[0.835]	0.106	[0.636]	0.653	[0.878]
Month 2	0.743	[1.186]	0.074	[0.875]	0.817	[1.166]
Month 3	0.981	[1.535]	0.091	[0.432]	1.072	[1.498]
<b>Wald Test</b>		(0.834)		(0.411)		(0.740)

**Table 5**  
**Performance of the Unconditional Strategy**  
**[Lockup: Three-month]**

This table reports performance of portfolios under the unconditional strategy. There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. The benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. The data frequency is monthly. Certainty equivalent or equalization fee is calculated as the difference in the utilities of the three-asset allocation with or without a lockup and the two-asset allocation.

	Two-Asset	Three-Asset with a Lockup	Three-Asset No Lockup	Three-Asset with a Lockup	Three-Asset No Lockup
<b>Hedge Fund Proxy:</b>		<b>HFRI Composite</b>		<b>HFRIFoF Composite</b>	
Mean excess returns	8.1%	23.7%	31.4%	14.8%	23.3%
Std. excess returns	8.9%	15.3%	17.6%	12.1%	15.2%
Sharpe ratio	0.907	1.549 (0.000)	1.782 (0.012)	1.225 (0.001)	1.533 (0.004)
Certainty equivalent		7.9%	11.8%	3.4%	7.6%
<b>Hedge Fund Proxy:</b>		<b>HFRI Equity Hedge</b>		<b>HFRIFoF Conservative</b>	
Mean excess returns	8.1%	25.0%	32.5%	19.0%	26.1%
Std. excess returns	8.9%	15.7%	17.9%	13.7%	16.1%
Sharpe ratio	0.907	1.591 (0.000)	1.814 (0.015)	1.389 (0.000)	1.622 (0.014)
Certainty equivalent		8.6%	12.3%	5.5%	9.0%
<b>Hedge Fund Proxy:</b>		<b>HFRI Event-Driven</b>		<b>HFRIFoF Diversified</b>	
Mean excess returns	8.1%	28.5%	35.6%	13.1%	19.0%
Std. excess returns	8.9%	16.8%	18.7%	11.4%	13.7%
Sharpe ratio	0.907	1.694 (0.000)	1.893 (0.023)	1.153 (0.006)	1.384 (0.019)
Certainty equivalent		10.3%	13.9%	2.5%	5.5%
<b>Hedge Fund Proxy:</b>		<b>HFRI Macro</b>		<b>HFRIFoF Market Defensive</b>	
Mean excess returns	8.1%	17.1%	21.7%	17.1%	23.5%
Std. excess returns	8.9%	13.0%	14.6%	13.0%	15.2%
Sharpe ratio	0.907	1.314 (0.000)	1.481 (0.054)	1.317 (0.000)	1.540 (0.018)
Certainty equivalent		4.6%	6.9%	4.6%	7.7%
<b>Hedge Fund Proxy:</b>		<b>HFRI Relative Value</b>		<b>HFRIFoF Strategic</b>	
Mean excess returns	8.1%	39.3%	42.9%	14.2%	20.3%
Std. excess returns	8.9%	19.7%	20.6%	11.8%	14.1%
Sharpe ratio	0.907	1.994 (0.000)	2.083 (0.195)	1.201 (0.002)	1.434 (0.017)
Certainty equivalent		15.8%	17.6%	3.1%	6.2%

**Table 6**  
**Asset Allocation under the Conditional Strategy**  
**[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]**

This table reports the results of asset allocations under the conditional strategy for the degree of risk aversion of the investor,  $\hat{\gamma} = 10$ . The data frequency is monthly. Column 3 to 8 show intercepts and coefficients of state variables by which optimal conditional weights for various portfolios are determined using the HFRI composite index as the proxy for hedge funds. Column 9 to 12 show intercepts and coefficients of state variables by which optimal conditional weights for various portfolios are determined using the HFRIFoF composite index as the proxy for hedge funds. Absolute values of t-statistics for the intercepts and coefficients are in square brackets. For each portfolio, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

Period	State Variables	HFRI as Hedge Fund Proxy						HFRIFoF as the Hedge Fund Proxy			
		Two-Asset		Three-Asset with a Lockup		Three-Asset No Lockup		Three-Asset with a Lockup		Three-Asset No Lockup	
Column	2	3	4	5	6	7	8	9	10	11	12
<b>Stocks</b>											
Month 1	Constant	0.691	[1.374]	-0.698	[0.882]	-1.117	[1.078]	0.338	[0.508]	0.087	[0.757]
	DP ratio	-0.084	[1.000]	-0.623	[0.868]	-0.071	[0.529]	0.099	[0.765]	0.190	[0.611]
Month 2	Constant	0.773	[1.494]	-0.391	[0.652]	0.340	[1.258]	0.665	[0.991]	1.008	[1.339]
	DP ratio	0.206	[1.306]	-0.207	[0.681]	-0.145	[0.566]	0.599	[0.996]	0.365	[0.502]
Month 3	Constant	0.658	[1.202]	-0.117	[0.620]	0.277	[1.180]	0.693	[1.034]	0.835	[1.274]
	DP ratio	0.264	[1.160]	0.144	[0.623]	-0.034	[0.119]	0.671	[1.133]	0.496	[0.718]
<b>Wald Test of Constant</b>			(0.298)		(0.483)		(0.115)		(0.476)		(0.149)
<b>Wald Test of DP ratio</b>			(0.145)		(0.315)		(0.800)		(0.304)		(0.690)
<b>Bonds</b>											
Month 1	Constant	0.623	[0.767]	0.882	[0.843]	0.784	[0.643]	0.291	[0.295]	0.323	[0.279]
	DP ratio	-0.335	[1.285]	-0.464	[0.400]	-0.685	[0.736]	-0.981	[0.788]	-0.745	[0.538]
Month 2	Constant	0.695	[0.860]	0.906	[0.882]	1.062	[0.916]	0.519	[0.556]	0.701	[0.665]
	DP ratio	-0.314	[1.401]	-0.590	[0.591]	-0.938	[0.810]	-0.683	[0.505]	-0.881	[0.595]
Month 3	Constant	0.860	[1.088]	1.192	[1.171]	1.542	[1.326]	0.599	[0.616]	0.779	[0.715]
	DP ratio	-0.387	[1.110]	-0.589	[0.692]	-0.553	[0.552]	-0.977	[0.768]	-1.004	[0.703]
<b>Wald Test of Constant</b>			(0.592)		(0.922)		(0.793)		(0.893)		(0.817)
<b>Wald Test of DP ratio</b>			(0.194)		(0.744)		(0.650)		(0.974)		(0.922)
<b>Hedge Funds</b>											
Month 1	Constant			2.562	[3.672]	4.377	[2.116]	2.096	[2.986]	3.633	[1.940]
	DP ratio			0.526	[0.805]	-0.429	[0.290]	1.187	[1.677]	0.985	[0.479]
Month 2	Constant					1.788	[1.128]			1.199	[1.151]
	DP ratio					0.121	[0.342]			0.920	[0.419]
Month 3	Constant					2.056	[1.172]			1.740	[1.389]
	DP ratio					0.862	[0.556]			1.658	[0.767]
<b>Wald Test of Constant</b>							(0.220)				(0.159)
<b>Wald Test of DP ratio</b>							(0.897)				(0.721)

**Table 7**  
**Average Hedge Demands for Stocks and Bonds under the**  
**Conditional Strategy**  
**[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]**

This table displays, for the three-asset portfolio under the conditional strategy with a three-month hedge fund lockup period, the decomposition of average portfolio weights of stocks and bonds at each month into two parts: a Markowitz demand or speculative demand and a hedge demand. Absolute values of t-statistics for those portfolio weights are in square brackets. For each asset, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

	Markowitz Demand [M]		Hedge Demand [H]		Optimal Demand = M + H	
<b>HFRI as Hedge Fund proxy:</b>						
<b>Stocks</b>						
Month 1	0.691	[1.374]	-1.389	[4.513]	-0.698	[0.882]
Month 2	0.773	[1.494]	-1.164	[3.913]	-0.391	[0.652]
Month 3	0.658	[1.202]	-0.775	[2.696]	-0.117	[0.620]
<b>Wald Test</b>		(0.298)		(0.252)		(0.483)
<b>Bonds</b>						
Month 1	0.623	[0.767]	0.258	[0.830]	0.882	[0.843]
Month 2	0.695	[0.860]	0.211	[0.980]	0.906	[0.882]
Month 3	0.860	[1.088]	0.332	[0.748]	1.192	[1.171]
<b>Wald Test</b>		(0.194)		(0.598)		(0.744)
<b>HFRIFoF as Hedge Fund proxy:</b>						
<b>Stocks</b>						
Month 1	0.691	[1.374]	-0.353	[2.258]	0.338	[0.508]
Month 2	0.773	[1.494]	-0.108	[1.414]	0.665	[0.991]
Month 3	0.658	[1.202]	0.035	[0.635]	0.693	[1.034]
<b>Wald Test</b>		(0.298)		(0.229)		(0.476)
<b>Bonds</b>						
Month 1	0.623	[0.767]	-0.333	[0.548]	0.291	[0.295]
Month 2	0.695	[0.860]	-0.176	[0.715]	0.519	[0.556]
Month 3	0.860	[1.088]	-0.261	[0.394]	0.599	[0.616]
<b>Wald Test</b>		(0.194)		(0.537)		(0.974)

**Table 8**  
**Performance of the Conditional Strategy**  
**[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]**

This table reports performance of portfolios under the conditional strategy. There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. For each three-asset portfolio, two p-values are reported using two benchmark portfolios. In the ‘Sharpe ratio’ row, the benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. In the ‘Unconditional vs. Conditional’ row, the benchmark portfolio for each three-asset portfolio is the corresponding unconditional portfolio. Certainty equivalent or equalization fee is calculated as the difference in the utilities of the three-asset allocation with or without a lockup and the two-asset allocation. The data frequency is monthly.

	Two-Asset	Three-Asset with a Lockup	Three-Asset No Lockup	Three-Asset with a Lockup	Three-Asset No Lockup	
<b>Hedge Fund Proxy:</b>		<b>HFRI Composite</b>			<b>HFRIFoF Composite</b>	
Mean excess returns	13.1%	29.9%	38.2%	24.5%	30.9%	
Std. excess returns	11.4%	17.2%	19.5%	15.5%	17.5%	
Sharpe ratio	1.149	1.740 (0.000)	1.960 (0.078)	1.572 (0.000)	1.765 (0.128)	
Unconditional vs. Conditional		(0.421)	(0.667)	(0.079)	(0.463)	
Certainty equivalent		8.5%	12.6%	5.8%	9.0%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Equity Hedge</b>			<b>HFRIFoF Conservative</b>	
Mean excess returns	13.1%	29.9%	40.0%	28.8%	36.4%	
Std. excess returns	11.4%	17.2%	19.9%	16.9%	19.0%	
Sharpe ratio	1.149	1.737 (0.000)	2.011 (0.031)	1.704 (0.000)	1.910 (0.098)	
Unconditional vs. Conditional		(0.613)	(0.589)	(0.104)	(0.270)	
Certainty equivalent		8.5%	13.6%	8.0%	11.7%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Event-Driven</b>			<b>HFRIFoF Diversified</b>	
Mean excess returns	13.1%	35.5%	46.2%	21.2%	26.7%	
Std. excess returns	11.4%	18.8%	21.3%	14.5%	16.2%	
Sharpe ratio	1.149	1.891 (0.000)	2.155 (0.035)	1.462 (0.004)	1.643 (0.159)	
Unconditional vs. Conditional		(0.391)	(0.343)	(0.142)	(0.384)	
Certainty equivalent		11.3%	16.9%	4.1%	7.0%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Macro</b>			<b>HFRIFoF Market Defensive</b>	
Mean excess returns	13.1%	22.3%	31.0%	25.5%	30.8%	
Std. excess returns	11.4%	14.8%	17.5%	15.9%	17.4%	
Sharpe ratio	1.149	1.502 (0.002)	1.769 (0.040)	1.607 (0.000)	1.766 (0.210)	
Unconditional vs. Conditional		(0.467)	(0.283)	(0.153)	(0.479)	
Certainty equivalent		4.7%	9.1%	6.3%	9.0%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Relative Value</b>			<b>HFRIFoF Strategic</b>	
Mean excess returns	13.1%	50.3%	60.6%	23.3%	28.1%	
Std. excess returns	11.4%	22.3%	24.5%	15.2%	16.6%	
Sharpe ratio	1.149	2.255 (0.000)	2.472 (0.085)	1.535 (0.001)	1.685 (0.241)	
Unconditional vs. Conditional		(0.186)	(0.082)	(0.097)	(0.408)	
Certainty equivalent		18.9%	23.9%	5.2%	7.6%	

**Table 9**  
**Performance of the Unconditional Strategy**  
**[Lockup: One-year; Bootstrap Samples]**

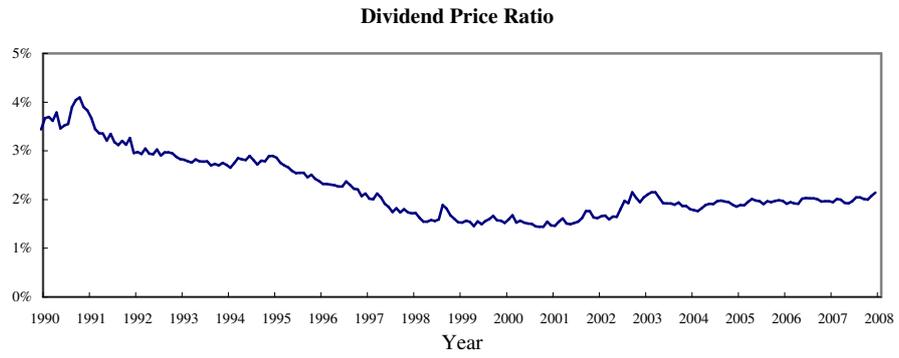
This table reports performance of portfolios under the unconditional strategy. There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. The benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. The data frequency is quarterly. Certainty equivalent or equalization fee is calculated as the difference in the utilities of the three-asset allocation with or without a lockup and the two-asset allocation. The number of bootstrap samples is 5000.

	Two-Asset	Three-Asset with a Lockup	Three-Asset No Lockup	Three-Asset with a Lockup	Three-Asset No Lockup
<b>Hedge Fund Proxy:</b>		<b>HFRI Composite</b>		<b>HFRIFoF Composite</b>	
Mean excess returns	11.0%	22.0%	25.7%	15.4%	18.0%
Std. excess returns	13.5%	14.7%	15.9%	12.3%	13.3%
Sharpe ratio	0.824	1.469 (0.000)	1.588 (0.102)	1.225 (0.000)	1.324 (0.138)
Certainty equivalent		9.2%	11.1%	5.9%	7.3%
<b>Hedge Fund Proxy:</b>		<b>HFRI Equity Hedge</b>		<b>HFRIFoF Conservative</b>	
Mean excess returns	11.0%	20.8%	23.9%	17.0%	19.7%
Std. excess returns	13.5%	14.3%	15.3%	12.9%	13.9%
Sharpe ratio	0.824	1.428 (0.000)	1.531 (0.143)	1.287 (0.000)	1.385 (0.148)
Certainty equivalent		8.6%	10.2%	6.7%	8.1%
<b>Hedge Fund Proxy:</b>		<b>HFRI Event-Driven</b>		<b>HFRIFoF Diversified</b>	
Mean excess returns	11.0%	28.1%	34.1%	14.7%	17.3%
Std. excess returns	13.5%	16.6%	18.3%	12.0%	13.0%
Sharpe ratio	0.824	1.661 (0.000)	1.828 (0.038)	1.195 (0.000)	1.295 (0.133)
Certainty equivalent		12.2%	15.3%	5.5%	6.9%
<b>Hedge Fund Proxy:</b>		<b>HFRI Macro</b>		<b>HFRIFoF Market Defensive</b>	
Mean excess returns	11.0%	15.2%	18.1%	22.2%	25.8%
Std. excess returns	13.5%	12.2%	13.3%	14.8%	15.9%
Sharpe ratio	0.824	1.222 (0.000)	1.331 (0.108)	1.475 (0.000)	1.587 (0.115)
Certainty equivalent		5.7%	7.2%	9.3%	11.2%
<b>Hedge Fund Proxy:</b>		<b>HFRI Relative Value</b>		<b>HFRIFoF Strategic</b>	
Mean excess returns	11.0%	30.3%	36.8%	13.9%	16.2%
Std. excess returns	13.5%	17.3%	19.1%	11.6%	12.6%
Sharpe ratio	0.824	1.727 (0.000)	1.900 (0.034)	1.164 (0.000)	1.255 (0.164)
Certainty equivalent		13.4%	16.7%	5.1%	6.3%

**Table 10**  
**Performance of the Conditional Strategy**  
**[Lockup: One-year; State Variable: Market Dividend-Price Ratio; Bootstrap Samples]**

This table reports performance of portfolios under the conditional strategy. There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values of Sharpe ratio tests. For each three-asset portfolio, two p-values are reported using two benchmark portfolios. In the ‘Sharpe ratio’ row, the benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. In the ‘Unconditional vs. Conditional’ row, the benchmark portfolio for each three-asset portfolio is the corresponding unconditional portfolio. Certainty equivalent or equalization fee is calculated as the difference in the utilities of the three-asset allocation with or without a lockup and the two-asset allocation. The data frequency is quarterly. The number of bootstrap samples is 5000.

	Two-Asset	Three-Asset with a Lockup	Three-Asset No Lockup	Three-Asset with a Lockup	Three-Asset No Lockup	
<b>Hedge Fund Proxy:</b>		<b>HFRI Composite</b>			<b>HFRIFoF Composite</b>	
Mean excess returns	21.8%	58.6%	82.9%	41.0%	60.5%	
Std. excess returns	20.9%	24.0%	28.6%	20.1%	24.4%	
Sharpe ratio	1.084	2.384 (0.000)	2.820 (0.005)	1.998 (0.000)	2.412 (0.002)	
Unconditional vs. Conditional		(0.000)	(0.000)	(0.000)	(0.000)	
Certainty equivalent		29.7%	42.0%	20.8%	30.8%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Equity Hedge</b>			<b>HFRIFoF Conservative</b>	
Mean excess returns	21.8%	47.6%	68.5%	40.1%	57.9%	
Std. excess returns	20.9%	21.7%	26.0%	19.9%	23.8%	
Sharpe ratio	1.084	2.155 (0.000)	2.574 (0.003)	1.977 (0.000)	2.362 (0.004)	
Unconditional vs. Conditional		(0.000)	(0.000)	(0.000)	(0.000)	
Certainty equivalent		24.1%	34.7%	20.3%	29.4%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Event-Driven</b>			<b>HFRIFoF Diversified</b>	
Mean excess returns	21.8%	61.0%	88.0%	39.4%	58.4%	
Std. excess returns	20.9%	24.5%	29.5%	19.7%	23.9%	
Sharpe ratio	1.084	2.437 (0.000)	2.908 (0.003)	1.956 (0.000)	2.370 (0.002)	
Unconditional vs. Conditional		(0.000)	(0.000)	(0.000)	(0.000)	
Certainty equivalent		30.9%	44.6%	20.0%	29.7%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Macro</b>			<b>HFRIFoF Market Defensive</b>	
Mean excess returns	21.8%	39.6%	56.0%	41.8%	60.5%	
Std. excess returns	20.9%	19.7%	23.4%	20.3%	24.4%	
Sharpe ratio	1.084	1.966 (0.000)	2.325 (0.007)	2.019 (0.000)	2.415 (0.004)	
Unconditional vs. Conditional		(0.000)	(0.000)	(0.000)	(0.000)	
Certainty equivalent		20.1%	28.5%	21.2%	30.8%	
<b>Hedge Fund Proxy:</b>		<b>HFRI Relative Value</b>			<b>HFRIFoF Strategic</b>	
Mean excess returns	21.8%	70.7%	106.8%	39.4%	58.4%	
Std. excess returns	20.9%	26.4%	32.5%	19.7%	23.9%	
Sharpe ratio	1.084	2.620 (0.000)	3.196 (0.001)	1.959 (0.000)	2.372 (0.002)	
Unconditional vs. Conditional		(0.000)	(0.000)	(0.000)	(0.000)	
Certainty equivalent		35.7%	53.9%	20.0%	29.7%	



**Figure 1. Evolution of the Market Dividend-Price Ratio**

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