Modeling Human Capital in Life-Cycle Portfolio Choice: Riskless or Risky?

By

Ingmar Minderhoud (148656)

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Faculty of Economics and Business Administration
Tilburg University

Supervisors

Dr. R.J. Mahieu[1]
Prof. Dr. Th.E. Nijman[1]
Drs. R.D.J. Molenaar[2]
Prof. Dr. E.H.M. Ponds[12]

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[1] Tilburg University
[2] APG

Abstract

We study the impact of risky human capital in life-cycle portfolio choice and survey the academic literature on the optimal asset allocation over the individual's life-cycle, where we emphasize the nature of human capital. A distinction is made between the *riskless* conception of human capital as having bond-like characteristics, and the *risky* conception of seeing this future income stream as having stock-like properties. In particular, attention will be paid to the models presented in Cocco, Gomes, and Maenhout (2005) and Benzoni, Collin-Dufresne, and Goldstein (2007). We use the idea of Benzoni *et al.* to study the welfare implications of portfolio choice when labor income and dividends are co-integrated. This dynamic portfolio choice problem is analyzed for two sectors, public and construction, and for the Netherlands as a whole. The results indicate that hump-shaped asset allocations are welfare improving for all three groups. Further, we show that similar conclusions are obtained using a simple vector autocorrection model.

Keywords: human capital, life-cycle investment, co-integration, dynamic portfolio choice, wage profiles

Acknowledgments

The standard explanation for econometrics is that the econometrician is capable of translating (economic) problems into mathematical models, and tries to solve these. However, in the recent years when I studied Econometrics and during my internship at APG, I have learned that two important aspects are missing in this story which are interpretation and common sense. Can we explain what we are doing, can we explain the outcome? Is it reasonable what we are doing?

Another important lesson I have learned during my internship is that whenever you are facing a problem, do not hesitate to ask colleagues or other people for helping you out since they are more than happy to. In this respect I would like to thank my first supervisor from Tilburg University, Ronald Mahieu, for guiding me throughout the internship with useful comments about theory and modeling. Next to that I would like to thank my supervisors at APG, Roderick Molenaar and Eduard Ponds, for providing me the opportunity to do my research at APG. They proved to be an excellent supervising combination since they provided me with the important, balanced mix of mathematical and economical insights, needed to bring this internship to a success. Further, I am thankful to my second university supervisor, Theo Nijman, for ensuring this research to keep on the right track, and to Peter Vlaar for providing clear views. Finally, I would like to thank my family and everyone else who supported me during this research.

Contents

1 Introduction

The problem of optimal life-cycle investment has received substantial attention in the academic literature. In the theory of life-cycle investment it is still widely assumed that human capital, which is defined as the discounted value of future labor income, can be seen as a position in the risk-free asset. As a consequence, the optimal portfolio allocation over the life-cycle should be high in stocks in the beginning of the agent's career and declining afterwards. Since the agent has implicit holdings of human capital in his portfolio he should tilt his financial portfolio towards stocks so that his total dollar holdings of each asset equal the optimal holdings. The economic intuition is that early in life the fraction of human capital is high compared to the fraction of financial wealth. Young agents are less dependent on financial wealth for consumption since they have labor income as alternative income source. It is therefore affordable for them to take more risk with financial wealth then elderly agents who almost entirely depend on this type of wealth for their consumption. A common advice financial planners give to their clients is to invest in stocks according to the *100 minus age* rule.

Nevertheless, in the past years several papers appeared in the academic literature which assume that human capital can be risky, and hence has more stock-like properties. Empirical evidence shows that risky asset holdings over the life-cycle typically are 'hump-shaped': young agents hold very little stocks, progressively increase these holdings as they age, and decrease their exposure when retirement is approached (e.g. Campbell (2006)). This pattern contrasts with the common knowledge that young agents should place most of their savings in stocks and switch their holdings to bonds as they age. These recent studies raise doubts about the way of handling human capital as an implicit investment in the riskless asset. They tend to treat the risk profile of human capital as having stock-like properties. For example, in Benzoni *et al.* (2007) a young agent will find himself overexposed to market risk in which case it will even be optimal for him to take a short position in the market portfolio. Hence, in the academic literature we can roughly classify the way of thinking about the nature of human capital in two groups. The group where it is assumed that human capital is riskless will be denoted by the *riskless view* on human capital. The group where this assumption is challenged will be denoted by the *risky view* on human capital.

Pension funds study age-differentiation in investment and indexation policies (see Molenaar and Ponds (2009)). This issue has become more important because of the 2008 credit crisis. The crisis has led to a drop of funding ratios below the required minimum for many Dutch pension funds. In this respect life-cycle theory is used in order to study the impact of such new policies. An important assumption underlying this theory is labor income's risk profile. We conjecture that the risk profiles of different types of employees will have different influence on the optimal asset allocation over the life-cycle[3]. The question we consider is which asset allocation approximates the optimal investment decision over the agent's life cycle. Furthermore, we examine whether different asset allocations hold for different types of labor income risk profiles. An interesting observation which we touch upon concerns the difference in wage profiles between the Netherlands and the US. In the Netherlands we observe increasing wage profiles while in the US these

[3]With different we mean more risky in the sense that the employment instability for the construction sector is likely to be higher then for the public and educational sectors. In the remainder we will denote the public and educational sectors by 'public sector'.

profiles are more hump-shaped. As a consequence, in the Netherlands human capital declines at a lower rate during the final years before retirement. This means that the agent's implicit bond holding will be higher during those years, which might causes the investor to hold larger fractions in the risky asset compared to the Benzoni results.

In order to study the welfare implications of different asset allocations over the life-cycle we model labor income following Benzoni *et al.* (2007), in which labor income and dividends are co-integrated.

We find that the optimal asset allocation will not be decreasing as the individual ages, but rather shows a hump-shaped pattern as described earlier in the introduction. Further, we do not find differences in results between the different sectors. However, this could be due to our way of approximating the optimal allocation. If we would optimize the dynamic portfolio choice problem, we might find a different asset allocation for each sector. It appears that our results are less controversial than the results presented in Benzoni *et al.* (2007) in the sense that we obtain positive asset holdings in the beginning of the agent's career. This might be caused by the difference in wage profiles. Finally, we find similar results using a vector autocorrection model, without the co-integrating relation.

The remainder of this paper is organized as follows. In Section 2 we survey the academic literature on life-cycle investment in which we emphasize the underlying assumptions about the characteristics of human capital. In Section 3 we specify the life-cycle portfolio choice model. The details of estimation and simulation are explained in Section 4. Finally, Section 5 concludes.

2 The Theory of Life Cycle Investment

In this Section we will survey the academic literature on life-cycle theory, particularly focussing on the role of human capital when solving the dynamic portfolio choice problem.

Life-cycle saving and investing is a matter which concerns people all around the world. It is about how much to consume now and how much to safe for retirement. The next question then is how to optimally allocate savings between stocks and bonds. The first economists who developed models in order to solve dynamic portfolio choice problems were Samuelson (1969) and Merton (1969). They show that if investment opportunities are constant the agent should optimally maintain a constant fraction of wealth invested in stocks over the life-cycle. However, an important and unrealistic assumption made in these models is that the agent has no labor income. If we assume the agent receives labor income until retirement, the allocation to the risky asset will be related to the life-cycle.

An important study on the impact of labor income on the investment strategy is done in Bodie, Merton, and Samuelson (1992). Their main result states that the fraction of the agent's financial wealth invested in the risky asset should decline with age. The first reason is that human capital is usually seen as riskless asset and the value of human capital declines as the investor ages. A second argument relates to the flexibility young investors have to alter their labor supply. This allows them to invest more aggressively in stocks compared to older agents. However, the opposite is also possible. Consider the often given example from Samuelson (1969). If the agent works as a businessman or stock analyst, his labor income is highly correlated with stock markets. This results in a very low fraction of financial wealth invested in stocks when the agent is young, which increases as one ages.

Roughly speaking there are two views about human capital. The more *riskless view* as we will call it here means that human capital acts like a risk free asset and hence can be treated as though the agent has an implicit holding in this asset[4]. Papers which study this kind of human capital are Bodie *et al.* (1992), Merton (1971), Heaton and Lucas (1997), Jaganathan and Kocherlacota (1998), Campbell and Viceira (2002), and Viceira (2009). A survey of recent academic literature on financial planning over the life-cycle can be found in Bovenberg, Koijen, Nijman, and Teulings (2007).

The main conclusion of the riskless view is that the optimal portfolio holdings in the risky asset will generally be high early in the agent's working life and declines when the agent ages (see Figure 2).

The more recent developed view about the risk profile of human capital, which we will denote by *risky view*, challenges the assumption about human capital being riskless. Instead, different ways of modeling are presented in which the risky nature of human capital is reflected. Papers which study the effect of labor income risk on portfolio choice are Viceira (2001), Cocco *et al.* (2005), and Benzoni *et al.* (2007). The main conclusion of the risky view is that modeling human capital as having stock-like properties results in a lower or even negative fraction of financial wealth invested in stocks early in working life. As the agent ages this fraction will increase until he is around the age of 55. As he approaches retirement his fraction in stocks will decline to the level it was before (see Figures 6 and 7).

An important determinant in modeling household portfolio holdings, which

[4]We assume the existence of a riskless asset and in our analysis we will take the bond for that purpose.

we will not consider here, is the influence of housing on portfolio holdings. Papers which study these effects are Cocco (2004), Hu (2005), and Yao and Zhang (2004). The main result is that home-ownership crowds out stock market participation. Investment in risky housing substitutes for risky stocks, thereby partially helping to resolve the limited stock market participation puzzle.[5]

In Sections 2.1 and 2.2 we will explain the theory behind the *riskless* and the *risky* view, respectively, in more detail.

2.1 'Riskless' view on human capital

Let us first consider the classic portfolio choice problem of investing total wealth into two assets, a risky asset and a riskless asset. We assume that asset returns are lognormal. The riskless asset has a constant return denoted by R_f. The risky asset has a return denoted by R_t and expected excess return equals $E_t R_{t+1} - R_f = \mu$. The agent will invest a fraction α_t in the risky asset and $(1-\alpha_t)$ in the riskless asset. The portfolio return then equals:

$$R_{p,t+1} = \alpha R_{t+1} + (1-\alpha_t)R_f = R_f + \alpha_t(R_{t+1} - R_f) \tag{1}$$

The mean portfolio return is $E_t R_{p,t+1} = R_f + \alpha_t(ER_{t+1} - R_f)$ and the variance equals $\sigma^2_{p,t} = \alpha_t^2 \sigma_t^2$. Following Campbell and Viceira (2002) we maximize power utility which gives

$$\max_{\alpha_t}(E_t R_{p,t+1} + \frac{1}{2}(1-\gamma)\sigma^2_{p,t}) \tag{2}$$

[5] Another non-tradable asset which might have similar risk characteristics is privately owned business.

Where γ denotes the agent's risk aversion parameter.

To come to a solution we have to relate the log portfolio returns to the log returns of the individual assets. We do this by using a Taylor approximation, which gives the following result (See Campbell and Viceira (2001b)):

$$R_{p,t+1} - R_f = \alpha_t(R_{t+1} - R_{f,t+1}) + \frac{1}{2}\alpha_t(1-\alpha_t)\sigma_t^2 \tag{3}$$

Substituting equation 3 into equation 2, the problem becomes:

$$\max_{\alpha_t} \alpha_t(E_t R_{t+1} - R_{f,t+1}) + \frac{1}{2}\alpha_t(1-\alpha_t)\sigma_t^2 + \frac{1}{2}(1-\gamma)\alpha_t^2\sigma_t^2 \tag{4}$$

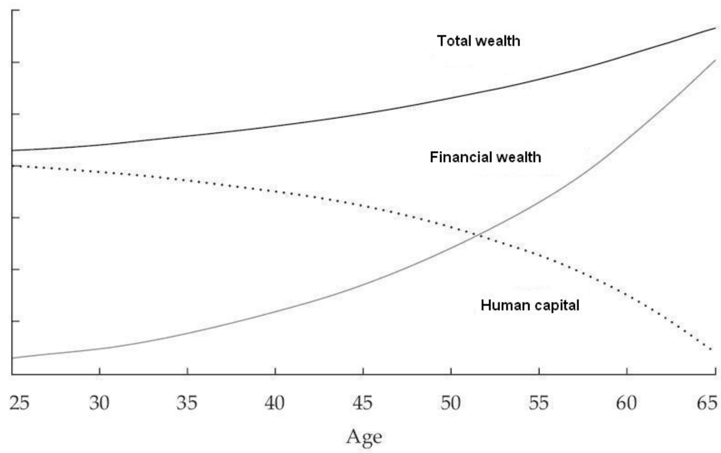
Solving for α_t we obtain:

$$\alpha_t = \frac{E_t R_{t+1} - R_{f,t+1} + \frac{\sigma_t^2}{2}}{\gamma\sigma_t^2} \tag{5}$$

The fraction α_t of total wealth invested in the stock is independent of the time horizon. Suppose we extent the one-period investment to a t-period investment. The expected excess return then equals $t\hat{\mu}$ and the variance of the excess return equals $\frac{1}{t}\alpha_t^2\sigma^2 t^2 = \alpha_t^2\sigma^2 t$. Both vary proportionally with time, hence the optimal fraction invested in stocks does not depend on the length of the investment horizon.

However, as said earlier human capital can be seen as an implicit holding in the riskless asset. This implicit holding pays out dividends in the form of labor income. Hence, the agent's wealth consists of tradable financial assets and non-tradable human capital. Figure 1 displays the development of human capital and financial wealth over the life-cycle.

Figure 1: Development of wealth components over the life-cycle.
Source: Ibbotson *et al.*(2007)



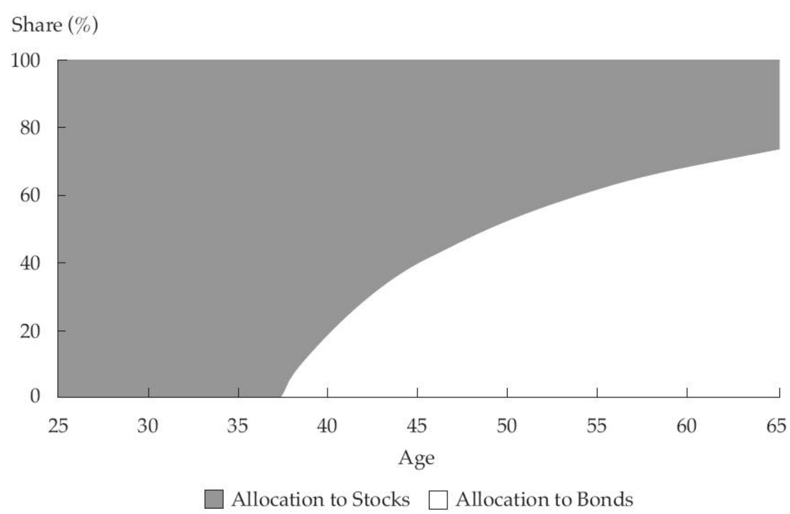
The argument for human capital being non-tradable is a moral hazard kind of problem. If human capital were tradable, the agent could sell claims against future labor income. Afterwards, he no longer has an incentive to work anymore and the sold claims become worthless. The question is how the optimal asset allocation under equation 5 changes when we take this non-tradable human capital into account. The agent should adjust his portfolio in such a way that his total holdings of each asset equal the optimal holdings under equation 5. This can be done by investing $\alpha_t(FW_t+HC_t)$ in the risky asset and $(1-\alpha_t)(FW_t+HC_t)-HC_t$ in the riskless asset.[6] Hence, the total fraction of risky capital should be equal to α (i.e., $\hat{\alpha}\frac{FW_t}{FW_t+HC_t} = \alpha$). The optimal fraction of financial wealth invested in stocks then equals:

$$\hat{\alpha} = \frac{\alpha(FW_t + HC_t)}{FW_t} = \frac{\mu + \frac{\sigma^2}{2}}{\gamma\sigma^2}(1 + \frac{HC_t}{FW_t}) \tag{6}$$

[6]This means that we multiply $(1 - \alpha_t)$ with total wealth and substract the *implicit bond holding* from it. What remains is invested in the riskless asset.

The 'folk wisdom' that young workers should hold most of their financial wealth in stocks is justified by the above equation. The fraction of financial wealth invested in stocks, $\hat{\alpha}$, tends to decrease over the life-cycle for two reasons. First, as the agent ages, he consumes part of his human capital so that HC_t declines. Second, his financial wealth increases as he saves part of his human capital, hence FW_t increases. This results in a high fraction of financial wealth invested in stocks for young workers, and a lower fraction for older workers. Figure 2 displays the portfolio holdings over the life-cycle.

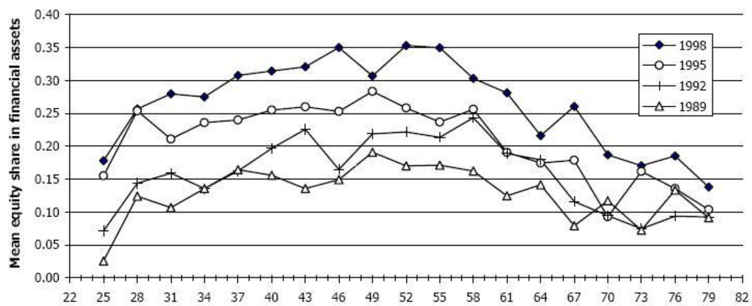
Figure 2: Fraction of financial wealth invested in stocks and bonds over the life-cycle.
Source: Ibbotson *et al.* (2007)



2.2 'Risky' view on human capital

Empirical evidence shows that life-cycle theory as described in Section 2.1 does not match our observations. Observations show that risky asset holdings typically are low for young agents, increase when the agent ages, and finally decrease when retirement is approached. Figure 3 shows such typical hump-shaped patterns.[7] In this section we review life-cycle theory in which

Figure 3: Fraction of financial wealth invested in risky asset.
Source: Ameriks and Zeldes (2001)



the more realistic assumption of wage income being stochastic is modeled. Most literature generally agrees that human capital risk can be decomposed into two components. First, an aggregate stochastic component which captures the effect of economy-wide shocks on all individuals together. Second, an idiosyncratic stochastic component which is subject to individual-specific shocks. In theory the idiosyncratic risk is uncorrelated to other risks in the economy and is therefore of less importance to large diversified pension

[7] These hump-shaped patterns are, among others, reported by Heaton and Lucas (2000), Ameriks and Zeldes (2001), and Campbell (2006).

funds.

Whereas bond-like human capital increases the demand for stocks, stock-like human capital reduces it. Since human capital acquires more stock-like properties, it becomes less important compared to financial wealth. Hence, the fraction of financial wealth invested in stocks in order to obtain the desired risk exposure of the overall portfolio, will decline over the entire life-cycle.

This way of modeling human capital can be seen as a possible explanation of why young agents hardly participate in the equity market (Figure 3). Since Cocco *et al.* (2005), and Benzoni *et al.* (2007) produce rather interesting results, we will discuss the life-cycle problems used in these papers in more detail.[8]

2.2.1 Wage profiles

Before discussing the 'Benzoni' and 'Cocco' models in more detail, we stress that the results as stated in Cocco *et al.* (2005) and Benzoni *et al.* (2007) are obtained using hump-shaped wage profiles, where these profiles decline after age 50. In the Netherlands, however, we observe increasing wage profiles which do not decline as retirement approaches. These profiles are displayed in Figures 4 and 5[9]. Because we will use data from the Netherlands to estimate our model, this could influence our results. Due to the increasing pattern for the Netherlands, human capital will decline at a lower rate which might lead to different results compared to Benzoni *et al.* (2007). This means that although we use the same way of modeling labor income risk,

[8]In the remainder of this paper we will denote the model as decribed in Cocco *et al.* (2005) and Benzoni *et al.* (2007), by the 'Cocco' model and the 'Benzoni' model, respectively.

[9]See Euwals, De Mooij, and Van Vuuren (2009) for more details.

the results for the Netherlands could be different from the US only because the wage profile for Dutch employees behaves differently over the life-cycle. Looking at the figures these results probably also hold for other European countries such as Germany and France (see Figure 5), whereas for example the analysis for the UK probably finds results similar to that of the US.

Figure 4: Wage profile by age for US, UK, Australia and Canada.
Source: Euwals, De Mooij, and Van Vuuren (2009)

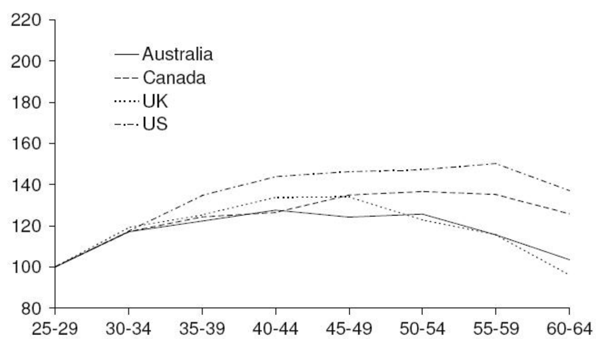
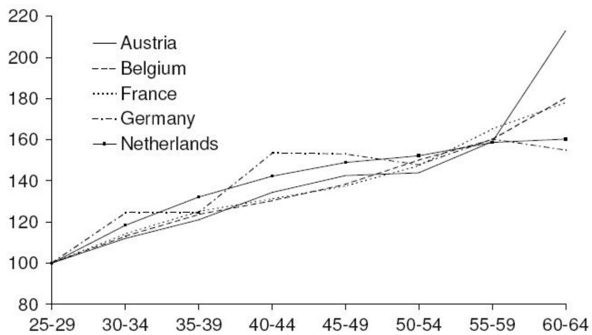


Figure 5: Wage profile by age for the Netherlands, Germany, France, Belgium and Austria.
Source: Euwals, De Mooij, and Van Vuuren (2009)



2.2.2 Allowing for disastrous labor income shocks

As in most literature on life-cycle theory Cocco *et al.* (2005) specify agent i's preferences by a time-separable power utility function. Before retirement (i.e., $t \leq 65$), labor income Y_{it} is given by:[10]

$$\log(Y_{it}) = f(t, Z_{it}) + \nu_{it} + \epsilon_{it} \tag{7}$$

Where $f(t, Z_{it})$ is a deterministic function of age and of a vector of other individual characteristics Z_{it}. ν_{it} is given by

$$\nu_{i,t-1} + u_{it} \tag{8}$$

[10] They define the labor income process after retirement as a constant fraction λ of permanent income in the last working year: $\log(Y_{it}) = \log(\lambda) + f(K, Z_{iK}) + \nu_{iK}$, for $t > K$, where K is the retirement age.

where $u_{it} \sim N(0, \sigma_u^2)$, denotes a permanent shock. ϵ_{it} is an idiosyncratic temporary shock distributed as $N(0, \sigma_\epsilon^2)$, and is uncorrelated with u_{it}.
Further, u_{it} is decomposed into an aggregate component ξ_t and an idiosyncratic component ω_{it}:

$$u_{it} = \xi_t + \omega_{it} \tag{9}$$

It is assumed that there are two assets in which the agent can invest, a risky (S_t) and a riskless asset (B_t). The riskless asset has a constant return $\bar{R}_f$. The risky asset has return R_t and its excess return is given by

$$R_{t+1} - \bar{R}_f = \mu + \eta_{t+1} \tag{10}$$

where $\eta_{t+1} \sim N(0, \sigma_\eta^2)$ i.i.d.
Cash-on-hand in period t is defined by $X_{it} = W_{it} + Y_{it}$, where W_{it} denotes agent i's wealth in period t. The agent then has to decide how much to consume and how to invest the remaining cash-on-hand between stocks and bonds. Next period wealth is then given by:

$$W_{i,t+1} = R^p_{i,t+1}(W_{it} + Y_{it} - C_{it}) \tag{11}$$

where R^p_{t+1} is the portfolio return:

$$R_{i,t+1} \equiv \alpha_{it} R_{t+1} + (1 - \alpha_{it})\bar{R}_f \tag{12}$$

where α_t is the fraction of financial wealth invested in the risky asset. Together with borrowing constraints $B_{it} \geq 0$, and $S_{it} \geq 0$, which imply that $\alpha_{it} \in [0, 1]$ and wealth is non-negative, the optimization problem reads as follows:

$$V_{it}(X_{it}) = \max_{C_{it}\geq 0, 0\leq\alpha_{it}\leq 1}[U(C_{it}) + \delta p_t E_t V_{i,t+1}(X_{i,t+1})] \tag{13}$$

where

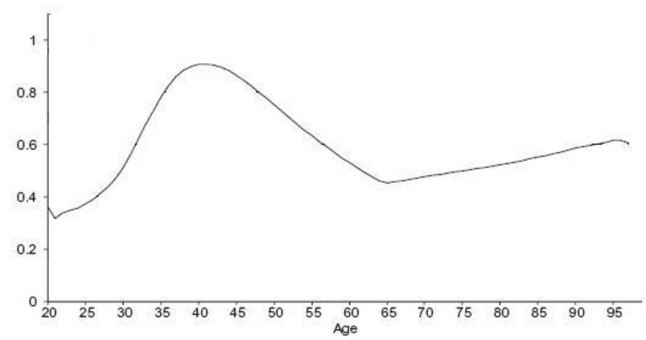
$$X_{i,t+1} = Y_{i,t+1} + (X_{it} - C_{it})(\alpha_{it}R_{t+1} + (1-\alpha_{it})\bar{R}_f) \tag{14}$$

δ is the discount factor, and p_t denotes the probability that the agent is alive at date $t+1$.

The results demonstrate that labor income risk actually has a minor effect on portfolio holdings, while the empirical evidence on the value of the correlation between labor income innovations and stock returns is mixed. Allowing for disastrous labor income shocks, however, substantially lowers the average allocation to risky assets[11]. In fact, in that case they find a hump-shaped pattern which looks like the pattern from empirical observations. Incorporating disastrous labor income shocks when modeling human capital therefore seems to be quite important in explaining data. Figure 6 displays the portfolio holding in the risky asset when incorporating disastrous income shocks. Of all the extensions they investigated, the empirically calibrated probability of a disastrous labor income shock seems to work best. However, as a zero labor income draw actually never occurs in the Netherlands, future research has to point out whether the results are robust against other levels of income shocks.

[11] With disastrous labor income shocks Cocco *et al.* (2005) mean that the agent receives zero labor income with positive probability.

Figure 6: Fraction of financial wealth invested in stocks with a 0.5% probability of a zero-income realization.
Source: Cocco *et al.* (2005)



2.2.3 When the stock and labor markets are co-integrated

Previous studies on stochastic labor income show that only unrealistic high contemporaneous correlations between labor income shocks and stock returns, or including the possibility of a disastrous labor income shock can explain the level of risky asset holdings for young agents.[12] These models also specify long-run correlations between stock market returns and human capital to be low or zero. This is a point of debate since it seems plausible to conjecture that a long period of high economic growth will be reflected by a strong stock and labor market performance in the long-run. Along these lines Benzoni *et al.* (2007) find evidence that aggregate labor income and dividends are co-integrated.[13] Their specification agrees with the empirical observation of low contemporaneous correlations between market returns

[12]See e.g. Viceira (2001) or Cocco, Gomez, and Meanhout (2005).

[13]Other studies which model along these lines are Baxter and Jerman (1997), Santos and Veronezi (2006), and Lettau and Ludvigson (2001a).

and changes to aggregate labor income, but allows for a significantly higher long horizon correlation between human capital returns and dividends. For the full model specification we refer to Benzoni *et al.* (2007) and Section 3.1.

The dividend process $D(t)$ of the risky asset follows a geometric brownian motion, and using standard arguments, the stock price is the discounted value of all future dividends.

The log-gain process (capital gain plus dividends), defined as $s(t) = \log S(t)$, follows[14]:

$$ds = (\mu - \frac{\sigma^2}{2})dt + \sigma dz_3 \tag{15}$$

The labor income process is specified as follows. Define $l(t) = \log L(t)$ where $L(t)$ is the product of two elements. In line with Section 2, $L_1(t)$ denotes aggregate income, and $L_2(t)$ denotes idiosyncratic shocks. Consequently, log-labor flow is specified as:

$$l(t) = l_1(t) + l_2(t) \tag{16}$$

Benzoni *et al.*(2007) specify $l_1(t)$ such that it captures two features. First, contemporaneous correlations between market returns and aggregate labor income shocks have to be low. Second, aggregate labor income and aggregate dividends have to be co-integrated. In order to capture these two features,

[14]Note that the stock return volatility equals the volatility of the dividend growth rate. In reality the stock return volatility is much higher. The authors emphasize, however, that only the stock return volatility is relevant for the portfolio choice problem. Yet, in De Jong (2007) the stock price dynamics are modeled to have a larger volatility. The portfolio implications are much less severe. Stock demand is reduced, but remains positive for all age groups and is declining with age.

they define the difference between the log of $L_1(t)$ and the log of $D(t)$ as:

$$y(t) = l_1(t) - \hat{d}(t) - \bar{ld} \tag{17}$$

where $\bar{ld}$ should be interpreted as the longrun log-ratio of aggregate labor income to dividends. To capture the co-integration relation between labor income and dividends, they assume $y(t)$ to be a mean-reverting process:

$$dy(t) = -\kappa y(t)dt + \nu_1 dz_1(t) - \nu_3 dz_3(t) \tag{18}$$

where z_1 is a standard brownian motion independent of z_3. The dynamics of the logarithm of the idiosyncratic shocks are modeled as arithmetic brownian motion. The time dependence in the drift term of the idiosyncratic labor income process is modeled as:

$$\alpha(t) = \alpha_0 + \alpha_1 t \tag{19}$$

Where α_0 and α_1 are calibrated to capture the observed hump shape of earnings over the life-cycle.

From equation 16 and 17 we obtain:

$$l(t) = y(t) + \hat{d}(t) + \bar{ld} + l_2(t) \tag{20}$$

Finally, using Ito's lemma we obtain the following dynamics for the labor income process:

$$\frac{dL}{L} = (-\kappa y(t) + g_D - \frac{\sigma^2}{2} + \alpha(t) + \frac{\nu_1^2}{2} + \frac{(\sigma - \nu_3)^2}{2})dt + \nu_1 dz_1(t) + \nu_2 dz_{2,i}(t) + (\sigma - \nu_3)dz_3(t) \tag{21}$$

In contrast to common thinking, the results show that it is optimal for young agents to take a substantial short position in the risky asset, or at least do not participate in the stock market. In fact, the results display a hump-shaped pattern which is similar to the results when allowing for disastrous labor income shocks.

The authors' interpretation can be summarized as follows. Due to the co-integration between human capital and dividends, there exists long-horizon correlation between human capital returns and market returns. The level of exposure is controlled for by the mean-reversion coefficient κ. A large value indicates a high rate of mean-reversion, which means that there exists a strong relation between the two variables. This strong relation indicates a high level of long-term correlation. If the agent's remaining employment is larger then $\frac{1}{\kappa}$, in other words, if the agent is young his human capital is highly correlated with market returns, i.e., human capital has stock-like properties. Moreover, a young agent's total wealth mainly consists of human capital. Consequently, due to this long-run labor income risk the agent implicitly holds a large position in the risky asset. To offset his exposure to the risky asset, he will place (a large fraction of) his financial wealth in the riskless bond.

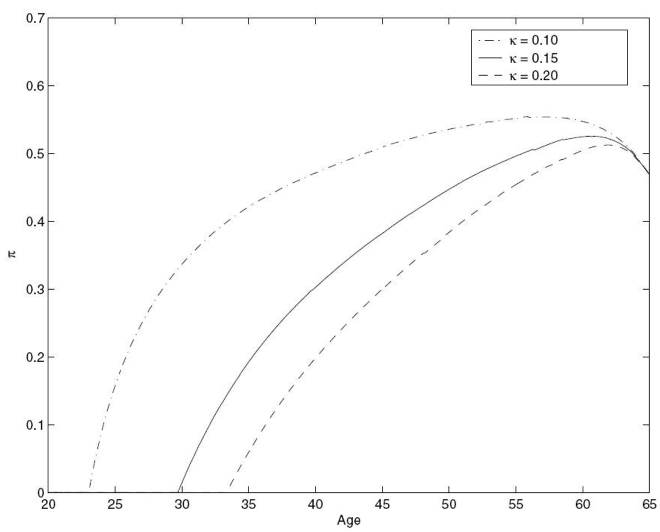
However, as the agent ages the process of co-integration (i.e., the *long-run* labor income risk) has less time to act, which means that labor income becomes less risky, and acquires more bond-like properties. Therefore, the fraction of financial wealth invested in the risky asset will increase to offset the larger implicit holding in the bond.

Finally, as the agent approaches retirement two offsetting effects are at work. First, we have that the process of co-integration has less time to act due to the agent getting older, as explained above. Second, because the agent reaches retirement his amount of human capital reaches zero. This means that the implicit bond position in his portfolio declines. This second effect will eventually become more important which causes the agent to reduce his holding in the risky asset to buy more bonds.

In other words, co-integration makes human capital a close substitute for stocks, especially for younger agents which have long investment horizons. Hence, young agents invest less in stocks then older agents do.

Figure 7 shows the results for different values of κ. It is seen that the

Figure 7: Life-cycle profile of stock holdings.
Source: Benzoni *et al.* (2007)



inverse of κ indeed acts as time-scale for the agent. Larger values of κ increase the agent's exposure to the risky asset, and thus reduces his stock holding. Furthermore, based on κ alone we can estimate at which age the agent's allocation to the risky asset reaches it's maximum. For $\kappa = 0.15$ this maximum is reached around age 58. The results are roughly consistent with previous empirical studies such as Campbell (2006), and Ameriks and

Zeldes (2001). When there is no co-integration (i.e., when $\kappa \rightarrow 0$), $\Leftrightarrow \frac{1}{\kappa} \rightarrow \infty$ and the time point t for which the allocation to risky stocks will attain its maximum will be at the very beginning of the agent's life-cycle. Hence the 'Benzoni' model specification reduces to the 'riskless' situation.

2.2.4 A Comparison between the Cocco and Benzoni model

In this Section, which is written in line with Benzoni (2008), we make a comparison between the 'Cocco' model and 'Benzoni' model. A difference we have to bridge is that the 'Cocco' model is specified in discrete time, while the 'Benzoni' model is specified in continuous time.
Cocco *et al.* (2005) specify the labor income process to be:

$$\log(Y_{it}) = f(t, Z_{it}) + \nu_{it} + \epsilon_{it} \tag{22}$$

as described in Section 2.2.1. while Benzoni *et al.* (2007) specify the log labor dynamics as decribed in equation 21.
In the 'Cocco' model the excess return on the risky asset is given by:

$$R_{t+1} - \bar{R_f} = \mu + \eta_{t+1} \tag{23}$$

like equation 10, while in the 'Benzoni' model the return dynamics are defined in a similar way by equation 15[15]. The difference in labor income

[15]In the 'Cocco' model the returns are normally distributed, while in the 'Benzoni' model they are lognormally distributed.

between date t and date $t+1$ can be written as follows:

$$\begin{aligned}\log(Y_{i,t+\triangle t}) - \log(Y_i,t) &= f(t, Z_{i,t+\triangle t}) - f(t, Z_{i,t}) + \nu_{i,t+\triangle t} - \nu_{i,t} + \epsilon_{i,t+\triangle t} - \epsilon_{i,t} \\ &= f(t, Z_{i,t+\triangle t}) - f(t, Z_{i,t}) + u_{i,t+\triangle t} + \epsilon_{i,t+\triangle t} - \epsilon_{i,t} \\ &= [f(t, Z_{i,t+\triangle t}) - f(t, Z_{i,t})] + \omega_{i,t+\triangle t} + \xi_{t+\triangle t} + [\epsilon_{i,t+\triangle t} - \epsilon_{i,t}]\end{aligned} \tag{24}$$

where the second and third equality follow from equations 8, and 9, respectively. This labor income specification already closely matches the Benzoni specification in equation 21. Equation 24 also consists of a deterministic drift term and three stochastic parts.
In the remainder the temporary shock $[\epsilon_{i,t+\triangle t} - \epsilon_{i,t}]$ is ignored due to the finding of, among others, Benzoni *et al.* (2007) that this term has insignificant effect on optimal asset allocations.

Following Benzoni (2008) we relabel $\log(Y_{i,t+\triangle t}) - \log(Y_{i,t}) \equiv \triangle l(t)$, $\omega_{i,t+\triangle t} \equiv \nu_2 \triangle z_{2,i}(t)$, and $[f(t, Z_{i,t+\triangle t}) - f(t, Z_{i,t})] \equiv (g_D - \frac{\sigma^2}{2} + \alpha(t) - \frac{\nu_2^2}{2})$.

Since the 'Cocco' model allows aggregate labor income shocks ξ to correlate with market return innovations η, ξ is decomposed into two terms $\xi_\perp \equiv \nu_1 \triangle z_1$ and $\xi_\| \equiv (\sigma - \nu_3)\triangle z_3$. We can now rewrite the change in labor income of the 'Cocco' model in terms of the 'Benzoni' model as follows:

$$\triangle l^{Cocco}(t) = [g_D - \frac{\sigma^2}{2} + \alpha(t) - \frac{\nu_2^2}{2}]\triangle t + \nu_1 \triangle z_1 + \nu_2 \triangle z_{2,i}(t) + (\sigma - \nu_3)\triangle z_3. \tag{25}$$

These two models are equal in the limit where $\kappa \to 0$, and there is no co-integration. This means that for cases when κ is small, they will be difficult to distinguish given only a few years of data. For example, if $\kappa = 0.05$ the co-integration effects are expected over a timespan of 20 years. With 60 years of data we then only have 3 independent data points. However, although Benzoni *et al.* (2007) find small values for the mean-reversion coefficient, the results for $\kappa = 0$ and $\kappa = 0.05$ differ substantially. Apparently, the results are very sensitive to the mean-reversion parameter.

3 Model specification

As already mentioned in Section 2.1 several ways of modeling are proposed in the academic literature, most of them generating results which do not differ significantly from what we call the *riskless* view in this paper. Models which generate interesting results that do differ significantly from this view are the 'Cocco' model and the 'Benzoni' model. Econometrically, these two models do not differ much (see Section 2.2.4). The results, however, indicate opposite conclusions. This suggests that the 'Benzoni' model is very sensitive to the size of the mean-reversion parameter. Therefore in this study, where we use the 'Benzoni' model as guideline, we will perform a sensitivity analysis to check the robustness of our simulation results[16]. Furthermore, a side issue concerning the riskless view is that those results do not explain empirical evidence. The 'Benzoni' and 'Cocco' model generate results which resemble these observations. In this Section we present the model we will use for our analysis. We follow the idea used in Benzoni *et al.*(2007). However, in this paper we specify the co-integration relation $y(t)$ by means of a vector error correction model instead of the mean-reversion way of modeling in the Benzoni paper. Our approach is statistically more founded since we first test for co-integration before actually estimating the VECM. In contrast, in the Benzoni paper they capture the co-integration relation by assuming a priori that $y(t)$ is a mean-reverting process. In our approach we model according to a VECM. If it turns out that there is no co-integration, the adjustment coefficients will be insignificant and a standard VAR model remains. With

[16]In the 'Cocco' model they find that when allowing for disastrous labor income shocks as specified in Section 2.2.1, comparable hump-shaped results are obtained. In a further research we could expand our analysis by extending the 'Benzoni' model for allowing disastrous labor income shocks and hence make it more realistic.

our approach we circumvent the doubtful approach taken in Benzoni *et al.* (2007) where the stock return volatility is taken equal to the volatility of the dividend growth rate.

3.1 Co-integration between dividends and labor income

3.1.1 Financial market

To keep the analysis simple we assume that the financial market contains only two assets in which the agent can invest, a risky stock and a riskless bond. The riskless bond has a constant return $\bar{R}_f$. The risky stock has return R_t, and its excess return, defined as $R^e_{t+1} = R_{t+1} - \bar{R}_f$, is given by:

$$R^e_{t+1} = \mu + \eta_{t+1} \tag{26}$$

where $\eta_{t+1} \sim N(0, \sigma^2_\eta)$ i.i.d.

3.1.2 Labor income process

We specify the labor income process as follows. Define $l(t) = \log L(t)$. Often, like in Benzoni *et al.* (2007), $L(t)$ denotes the product of two elements, $L_1(t)$ which denotes aggregate labor income, and $L_2(t)$ which denotes the idiosyncratic part. In our approach, we do not specify labor income process a priori to consist of two components. We let the data speak for itself so to say and do not make a distinction between different components. What is important here is the way labor income is modeled. Furthermore, from the pension fund's perspective the idiosyncratic labor income component will be of less interest, since such individual shocks may be diversified away in collective schemes[17].

In order to capture the feature that contemporaneous correlations between market returns and aggregate labor income shocks are low, but that long-term correlations between these variables might be significantly higher, i.e., that labor income and dividends might be co-integrated, we model these variables using a vector autocorrection mechanism. An elaboration on co-integration and such mechanisms is given in Section 3.2.

3.1.3 Agent's preferences

At each date $t \in [0, 40]$ the agent has to decide how much to consume (c_t) and how to invest (x_t) the remaining wealth between stocks and bonds. Next period wealth is then given by[18]:

$$W_{t+1} = L_{t+1} + (1 - c_t)W_t(x_t R^e_{t+1} + R^f) \tag{27}$$

The agent derives utility from consumption during his life-cycle. Hence, the following intertemporal optimization problem can be formalized[19]:

$$\begin{aligned} V(t, T, W_t, L_t) &= \max_{(x_s, c_s)_{s=t}^T} E_t[\sum_{s=t}^{T} \beta^{s-t} u(c_s W_s)] \\ &= \max_{x_t, c_t} [u(c_t W_t) + \beta E_t[V_{t+1}(W_{t+1})] \end{aligned} \tag{28}$$

[17] Actually, when we model labor income for a representative agent there is no idiosyncratic risk component.

[18] Note that we follow Benzoni *et al.* (2007) in which we analyse a 40-year period, starting at age 25.

[19] The optimization problem can be extended for example by adding mortality probabilities (See Section 2.2.2).

where β denotes the discount factor and $u(\cdot)$ denotes the time-separable constant relative risk aversion (CRRA) utility function, which summarizes the agent's preferences[20].

3.2 VECM specification

In time series analysis, the finding that a linear combination of two non-stationary variables is itself a stationary variable is denoted by *co-integration*. In our analysis it seems plausible to conjecture that at long horizons, labor income and stock markets are likely to move together. When we have a sustained period of high economic growth, this will be reflected by a strong stock and labor market performance in the long-run. Hence, $l(t)$ and $d(t)$ might be co-integrated. The stationary linear combination, $l(t) - \beta d(t)$, is called the *co-integrating equation*, which means that the two variables share a common trend. This can be interpreted as a long-run equilibrium relationship among those variables.

In order to model the co-integrating relation $y(t) = l(t) - d(t)$ we construct a vector error correction model (VECM), which in fact is a restricted VAR model. The VECM can be used for series which themselves are non-stationary but are known to be co-integrated. The main characteristic of a VEC model is the notion of an equilibrium long-run relationship and the introduction of past disequilibria as explanatory variables. In other words, such a model describes how the two series behave in the short-run consistent with a long-run co-integrating relationship. Therefore, the first step

[20]The power utility function is defined by $u(W) = \frac{W^{1-\gamma}}{1-\gamma}$, where $u(W) = \ln(W)$ for $\gamma = 1$.

is testing whether the two variables are non-stationary[21]. This is done by performing an Augmented Dickey Fuller (ADF) unit root test on both series separately.

The second step is testing whether the two series are co-integrated. This can be done by estimating the regression $l(t) = a + bd(t) + \epsilon(t)$ by OLS and see whether the residual series is stationary. This is known as the Engle-Granger approach[22]. Important in this approach is that we distinguish between co-integration and what is called *spurious regression*. Suppose that we estimate the co-integrating regression and test whether the error term is stationary by testing for the presence of a unit root in the OLS residuals. We have to be aware of the additional complication in testing for unit roots in OLS residuals. Ordinary least squares chooses a and b such that the sample variance of the residuals is minimized. This means that also in cases where there is no co-integration the OLS estimator will make the residuals look as stationary as possible (Verbeek(2004)). As a consequence, if we use the standard ADF test, we may reject the null hypothesis (presence of unit root) too often. Hence, when performing these tests we should use critical values which are more conservative, calculated by Davidson and MacKinnon[23].

An alternative test which we will use is the Johansen co-integration test. This test does not suffer from the drawback mentioned above. The Jo-

[21]A variable is non-stationary if it is (at least) integrated of order 1, which is denoted by $I(1)$. If we difference this variable once we obtain a stationary variable, which is denoted by $I(0)$.

[22]Note that if there are more than two variables involved, the co-integrating vector generalizes to a co-integrating space. If we have a number of k $I(1)$ variables, there exist at most $k-1$ independent linear combinations that are $I(0)$. If we would have k co-integrating relations, this would imply that there exist k independent linear combinations that are $I(0)$, such that all endogenous variables would be $I(0)$. In order to determine the number of co-integrating relations we can use the Johansen co-integration test.

[23]see Table 9.2 in Verbeek(2004)

hansen test actually consists of two tests, the trace test and the maximum eigenvalue test.

The third step is estimating the error-correction model using the estimated co-integrating vector from the previous step[24]. Assuming that there exists a co-integrating equation, the VEC model is given by:

$$\begin{aligned}\triangle l(t) &= \alpha_1(l(t-1) - \hat{a} - \hat{b}d(t-1)) + \beta_{11}\triangle l(t-1) + \beta_{12}\triangle d(t-1) + \epsilon_1(t)\\ \triangle d(t) &= \alpha_2(l(t-1) - \hat{a} - \hat{b}d(t-1)) + \beta_{21}\triangle l(t-1) + \beta_{22}\triangle d(t-1) + \epsilon_2(t)\end{aligned} \tag{29}$$

The first term on the right-hand side denotes the error correction term which ensures that if $l(t)$ and $d(t)$ deviate from the long-run equilibrium, this term will correct the series through a number of partial adjustments. The coefficient α_i is the adjustment parameter, measuring the speed of adjustment towards the equilibrium. The second and third terms are the usual VAR terms (in first differences).

[24]The two-step estimation procedure as described in step two and three is carried out as follows: The first step is to determine the number of co-integrating relations using the Johansen co-integration test. This information has to be provided in the next step, where the error correction terms are constructed from the co-integrating equations, and a VAR model is estimated including these terms as regressors. Note that since the explanatory variables are the same for each equation and the error terms are assumed to be independent of the history of $l_1(t)$ and $d(t)$, OLS gives consistent estimates. The error terms, however, are correlated.

4 Model estimation and simulation

4.1 Data

We study yearly dividends and returns on the MSCI Netherlands from 1969 through 2008. The data is taken from datastream. To construct a proxy for aggregate labor income, $l(t)$, we use Dutch, public sector, and construction sector price-index rates, taken from CBS[25]. Since CBS data for the public sector ends in 2002, we use APG data as explanatory variable in a regression in order to predict the remaining years until 2008[26]. Table 1 presents descriptive statistics.

Table 1: Summary statistics. In this table the summary statistics on growth rate are given for the variables, which are denoted in first differences. MSCI data is in log excess returns. l^N denotes Dutch labor income growth rate, l^P denotes public sector labor income growth rate, l^C denotes construction sector labor income growth rate, and d denotes dividends growth rate.

variable	l^N	l^P	l^C	d	MSCI
mean	0.041268	0.032942	0.048396	0.059655	0.047541
std.Dev.	0.036730	0.041259	0.040230	0.065139	0.223982

4.2 Estimating VECM with stochastic trend

Testing for Co-integration

The first step in constructing our VEC model is testing whether $l(t)$ and $d(t)$ are non-stationary. We do this by performing ADF unit root tests. Table 2 provides the p-values for each series. l^N, l^P, and d are non-stationary at the one, five and ten percent significance level. l^C has a p-value of 0.0630 which means that this series appears to be non-stationary at the one and five percent level, but at the ten percent level the null hypothesis is rejected.

[25]CBS denotes the statistics bureau for the Netherlands.

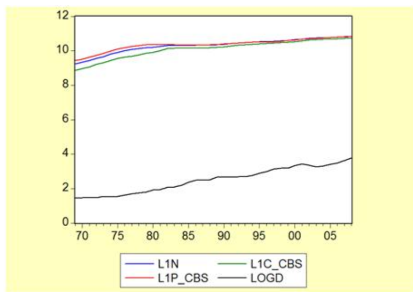
[26]We used APG data as explanatory variable in a regression where we regressed CBS data against APG data. The remaining years until 2008 in the CBS data are estimated by inserting the APG data.

Table 2: ADF test results

variable	l^N	l^P	l^C	d
p-value	0.1764	0.1781	0.0630	0.9914

Figure 8 displays the relation between labor income and dividends where the series are in logs. From this it appears that in the long-run the series might

Figure 8: Relation between labor income and dividend series.



have positive co-movements.

Since all series appear to be non-stationary the next step is testing for co-integration. Since we only have two endogenous variables a simple approach we might use is the Engle-Granger approach of regressing $l(t)$ on a constant and $d(t)$ and testing for a unit root in the residuals. This can be done by running an ADF test on the OLS residuals and applying the critical values of Table 9.2 in Verbeek (2004). Another method we will apply is the Johansen test.

A method for testing the OLS residuals on having a unit root is performing an ADF test. Table 3 presents p-values for each test. For the Dutch market and the public sector we can reject the null of no co-integration at the five percent significance level, but for the construction sector the OLS residuals appear to be non-stationary since we cannot reject the null at the five or ten percent level. Next, we perform the Johansen co-integration test. To

Table 3: ADF test results for OLS residuals.

variable	l^N	l^P	l^C
p-value	0.0341	0.0178	0.1198

compute these tests we need to specify the maximum lag length p in the VAR model. Choosing p to small will invalidate the test and choosing p to large results in a loss of power (Verbeek(2004)). The tests show that there is some sensitivity with respect to the choice of maximum lag length in the vector autoregressions. The results for $p = 2$ are presented in Figure 12. When estimating the VECM it follows from the AIC that a VAR(2) is the most appropriate model, which is explained in more detail on the next page. For the Dutch market and the public sector the null hypothesis of no co-integrating relations is rejected at the 1% level, so we can assume the existence of one co-integration relation for these groups. For the construction it seems that there is no evidence in favor of co-integration. However, if we specify the lag length to be 2 in differences, the test does indicate the existence of a co-integrating relation as is shown in Figure 13.

In sum, the empirical evidence seems to suggest the existence of co-integration between Dutch labor income and dividends, and public sector income and dividends. For the construction sector the statistical evidence seems to reject the existence of co-integration between labor income and dividends.[27] As our period is relatively short, and tests for co-integration usually have lack of power which means that the null hypothesis of no co-integration can

[27] Another, more suggestive, test is based on the Durbin Watson statistic. This test indicates we cannot reject the null hypothesis of no co-integration. However, we will proceed on the basis of the Johansen test since this test is known to be more reliable.

be rejected too often. Nevertheless, economic intuition provides us an incentive to believe in co-integration between these variables. Hence, we will proceed under the assumption of co-integration.

Estimating VECM

We now consider the vector error correction model for this system. In order to determine the appropriate lag length we estimate a VECM with no lagged differences, one lagged difference, and two lagged differences. To conclude which lag length seems most appropriate we compare the AIC's. The model with the lowest AIC is typically preferred which, in this case, is the VEC model with one lagged difference corresponding to a VAR(2) model. Estimation results are presented in Figure 14. It seems that the short-run adjustments to the long-run equilibrium are mostly done by the labor income variable, since the adjustment coefficients for dividend is insignificant in all three models.

4.3 Estimating VECM with deterministic trend

Note that in the previous part we estimated VECM assuming that all trends in the data are stochastic. However, Arpaia, Prez, and Pichelmann (2009) show that labor income share has declined from 1975 to 2005 for many European countries including the Netherlands, which seems evidence in favor of a negative trend coefficient in the co-integrating equation. Therefore, we will repeat the analysis by testing for a deterministic time trend and, if necessary, include such a trend in our VECM specification.

Testing for a deterministic time trend in data

Again we first test whether all series are non-stationary, but this time we include a deterministic time trend in our ADF test. Table 4 below presents the ADF test results and the estimation results for allowing a trend in the specification. The results indicate that we can reject the null of having a unit root for the variables l^N and l^P at reasonable significance levels. It seems that the non-stationarity is caused by the presence of a deterministic time trend rather than a unit root. This is an indication for these series to be trend-stationary. The variables l^C and d, however, do not seem to be appropriately described by a trend-stationary process, although the dividend series appears to be almost trend stationary at the five percent level.

Table 4: ADF test results. We see that for the Dutch market as well as for the public sector we can assume that these series are trend-stationary.

	l^N	l^P	l^C	d
p-value (test)	0.0031	0.0006	0.1442	0.0543

The next step is testing whether the series are co-integrated using the information from Table 4. This means that when we test for co-integration by means of the Johansen test, we incorporate the series to have a deterministic trend. Output can be found in Figure 15. Although the ADF test indicates that the construction sector labor income series may not appropriately be characterized by the presence of a deterministic trend we still tested for co-integration including a trend since the statistical evidence regarding this result is weak. The test for the construction sector now indicates one integration relation for the construction sector at the 5% significance level. However, the trend coefficient appears to be negative which is not in line with the development of labor income share, since this means that the trend coefficient will be positive in the co-integrating equation. In contrast, Arpaia, Prez, and Pichelmann (2009) show that labor income share has declined from 1975 to 2005 for many European countries including the Netherlands, which seems evidence in favor of a negative trend coefficient in the co-integrating equation. This is the case for the Dutch market and the public sector, where the trend coefficient appears to be positive (negative in the cointegrating equation). In conclusion, in the following we assume that l^N and l^P are trend-stationary series.

Estimating VECM

After comparing the AIC's for different lag lengths it again seems that a VECM(1) model characterizes our system in the most appropriate way. Estimation results are presented in Figure 16. Including a trend in the VECM for the construction sector seems to produce odd results as stated above. This could be due to the fact that the trend did not appear to be significant in the ADF test and therefore does not provide an appropriate characterization. In addition, labor income in the construction sector tends to behave more cyclical over the years. A third argument might be that the MSCI Netherlands index was mostly build up by financial companies and Shell. Compared to the Dutch market and the public sector, the construction sector will be least related to this index. Hence, we will not simulate construction according to a VECM with deterministic trend. The results for the public sector changed significantly. Apparently, including a time trend is of significant importance. This could be due the fact that the deterministic trend helps explaining the decrease in labor shares from the past years. The differences with the results without a trend are quite large. The adjustment coefficients measure the speed of adjustment to the long-run equilibrium.

Apparently, the speed of adjustment for public sector labor income and dividends towards their long-run equilibrium occurs rather slow. This might be justified by saying that public sector labor income probably behaves more bond-like in the sense that in the short run it varies less than for example construction sector labor income, and hence needs fewer corrections to the long-term equilibrium. However, we have to be aware of the fact that our data set only consists of four decades which forces us to treat the results with care, since co-integration is a long-term feature. Nonetheless, guided through economic intuition it seems plausible to assume the existence of co-integration. However, since the co-integration results seem to advocate modeling the relation between construction labor income and dividends as VAR instead of VECM we will also include simulation based on this type of model.

4.4 Estimating the VAR model

Before continuing to our simulation model, we are wondering how big the impact of modeling life-cycle investment according to co-integration really is. Do our results solely depend on the co-integrating relation within the VEC model, or do we obtain similar results by modeling labor income dynamics according to an unrestricted vector autoregression model (VAR). Further, co-integration tests for construction labor income make us doubt whether there really exists such a long-term relation between these two variables. To answer this question we estimate the same model without the co-integrating relation[28]:

$$\triangle l(t) = \beta_{11} \triangle l(t-1) + \beta_{12} \triangle d(t-1) + \epsilon_1(t)$$
$$\triangle d(t) = \beta_{21} \triangle l(t-1) + \beta_{22} \triangle d(t-1) + \epsilon_2(t) \tag{30}$$

The results are presented in Figure 17.

A VAR model describes the dynamic evolution of $l(t)$ given the joint history of $l(t)$ and $d(t)$. Since the coefficient for the lagged difference of $d(t)$ in the equation for $l(t)$ is insignificant it seems that the dividend history has little influence on explaining labor income. However, looking at the coefficients of the co-integrating equations in the regression results of the VECM results, we see that there seems to exist a significant relation between labor income and dividends in levels. We conclude that these variables in some way do influence each other's dynamics via their joint history.

4.5 Simulation results

Using the results of Sections 4.2.1 and 4.2.2 we set up different simulation models. For both the Netherlands and the public sector we simulate using VECM with and without deterministic trend. For the construction sector we simulate according VECM but only without deterministic trend since incorporating such a trend in our VECM estimation produces unrealistic results. Finally, we set up a simulation model using VAR estimation results. In sum we find that for benchmark case the hump shapes are preferred above the other allocations.

[28]Note that we estimate the VAR model in differences, since the variables are non-stationary at the level. If we do not impose stationarity the variance of the process will be infinite over time. This will produce inconsistent estimates since statistical inference is based on the assumption of stationarity.

Benchmark case

In our simulation models we use the following assumptions concerning the financial market: Using data on the MSCI Netherlands we estimated the excess return to be 4%. In our simulation we will vary this parameter in order to analyze its impact on the results.

The key parameter in the power utility function is the risk aversion coefficient γ. Following Benzoni *et al.*(2007) we use $\gamma = 5$ for the benchmark case. We include several coefficients for this parameter in our analysis to study the impact it has on the outcomes.

Finally, in order to study the impact of parameter uncertainty concerning the adjustment coefficients we construct confidence intervals using the estimated standard errors, and simulate again using the upper- and lower bounds as input.

simulating scenarios

We solved the problem via simulation. Firstly, the VECM estimates are used as input for simulating forward labor income and dividends by using equation 28. A Cholesky decomposition is used in order to model the multivariate normal distribution of the error terms. The Cholesky decomposition is commonly used in Monte Carlo simulation for systems with multiple correlated variables. The simulation produces a matrix containing 100.000 sample paths each with a length of 40 years. Next, we simulate excess returns by using equation 26. In order to simulate wealth dynamics we first set the consumption level at a constant 50% per year[29]. Since we want to compare different asset allocations we can simply keep consumption constant in all

[29] Other levels of consumption will give similar results

simulations.

Firstly, we simulate using the following allocations: The allocation which results from the traditional life-cycle theory, where we used the '100 minus age' rule of thumb. Next an allocation which is contrary to this allocation (see Basu *et al.*(2009). Further the allocation which most pension funds use as default allocation with a constant 60% allocation to stocks, and two rather extreme allocations with respectively 20% and 90% allocated to stocks. These allocations are depicted in Figure 9. We will also use approximations of the allocations which were found in Cocco *et al.* (2005) and Benzoni *et al.* (2009) to see whether these allocations improve our results. The 'hump' allocation is in between the 'cocco' and the 'benzoni' allocation. The 'opposite' allocation is a 'hump-shaped' allocation which actually applies some kind of opposite strategy compared to the other 'hump-shaped' allocations. This idea is taken from Basu *et al.* (2009). These allocations are depicted in Figure 10. Table 5 explains the accompanying legends. Wealth dynamics are then simulated by using equation 27. Thus, each strategy has 100.000 wealth sample paths resulting in 100.000 wealth outcomes at the end of the 40-year horizon.

Table 5: Definition of allocations. the first five allocations are depicted in Figure 9, the other four are depicted in Figure 10.

const90	Constant allocation 90% in risky asset
const20	Constant allocation 20% in risky asset
const60	Constant allocation 60% in risky asset
lifecycle	Linearly declining allocation
basu	Linearly increasing allocation
hump	Linearly increasing till t=60, linearly decreasing afterwards
cocco	Hump shaped allocation
benzoni	Approximated Benzoni hump shaped allocation
opposite	Hump shaped allocation with maximum early in working life

4.5.1 Results

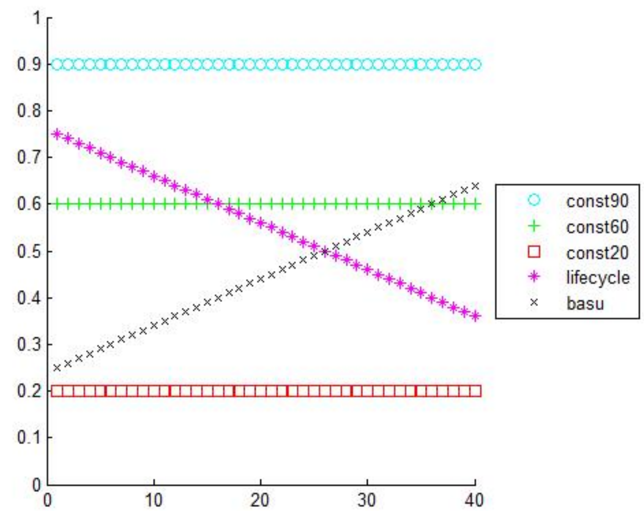
In this Section we present the simulation results using a VECM model with stochastic trend. Since the results with respect to the preference order of allocations is the same for each model we will only present results for the Netherlands. The results for the public sector and the construction sector, and using the VECM model with deterministic trend and the VAR model, can be found in the appendix.

In table 6 we present results for the lifecycle allocation, the basu allocation, and the three constant fraction allocations (see Figure 9).

Table 6: Simulation results for Netherlands, using VECM with stochastic trend. Allocation 'lifecycle' is normalized to one. The results show that only for the benchmark case it holds that the declining allocation, which follows from the life-cycle theory where human capital is assumed to be risk-free, performs best. For the more 'extreme' market conditions, where each time one condition is changed from the benchmark case, we see that the boundary allocation 'const20', or 'const90' performs best. Note that the 'const60' allocation, which most pension funds use, does not give the best performance in any of the situations.

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
lifecycle	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
basu	0.9928	0.8839	1.0050	1.0548	0.9645	0.8988	1.0381
const60	0.9956	0.9023	1.0050	1.0528	0.9695	0.9137	1.0351
const20	0.9886	1.0153	0.9951	0.9445	1.0104	1.0258	0.9715
const90	0.9612	0.6797	1.0100	1.0981	0.9025	0.7130	1.0842

Figure 9: Allocation to the risky asset.



Next we present results for all allocations together to see whether the hump-shaped allocations found in Cocco *et al.* (2005) and Benzoni *et al.* (2007) are welfare improving. The allocations are depicted in Figure 10, the results are presented in Table 7.

The fact that it is still widely assumed that human capital can be seen as a position in the risk-free asset is reflected in the advice which financial planners commonly give to their clients: Invest according to the '100 minus age' rule. In table 6 we see that this allocation indeed seems to work well. Nevertheless, in the recent years papers appeared which challenged the assumption of human capital being riskless. As a consequence these papers find other optimal asset allocations. Looking at table 7 we see that our results seem to underline these findings. For the benchmark case it holds that the 'cocco' allocation performs best. Note that also the 'benzoni'

Figure 10: Allocation to the risky asset.

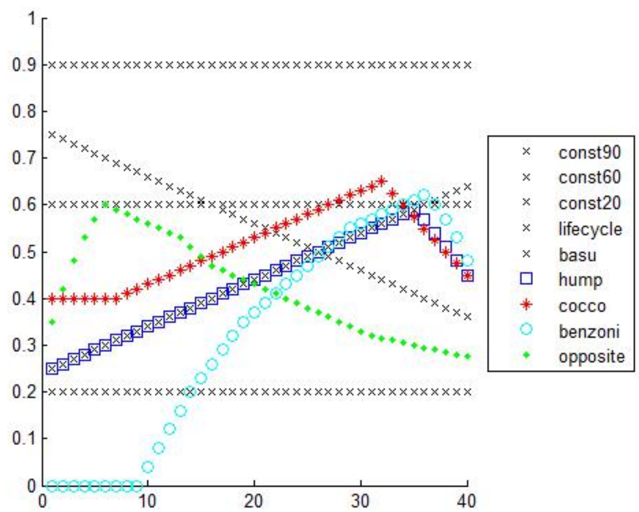


Table 7: Simulation results for Netherlands, using VECM with stochastic trend. Allocation 'cocco' is normalized to one. The results show that for the benchmark case it holds that the lifecycle allocation does not give the best performance anymore. The hump shaped allocation found in Cocco *et al.*(2005) now performs best. Note that although allocation 'hump' has the same performance after normalizing, in utility levels this allocations gives a lower utility than allocation 'cocco'. For the more 'extreme' market conditions we see that the boundary allocation 'const20', or 'const90' again performs best. Note that the 'cocco' allocation does not perform worse in any of the situations, and actually can be seen as a hedge allocation to extreme market conditions.

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
lifecycle	0.9973	1.0330	0.9967	0.9668	1.0122	1.0340	0.9811
basu	0.9908	0.9166	1.0020	1.0206	0.9762	0.9291	1.0183
const60	0.9935	0.9351	1.0018	1.0185	0.9812	0.9445	1.0154
const20	0.9856	1.0467	0.9911	0.9128	1.0231	1.0608	0.9533
const90	0.9598	0.7093	1.0061	1.0633	0.9138	0.7225	1.0633
hump	1.0000	0.9972	1.0002	1.0017	0.9991	0.9974	1.0011
benzoni	0.9986	0.9724	1.0013	1.0136	0.9914	0.9753	1.0093
opposite	0.9925	1.0467	0.9939	0.9391	1.0193	1.0526	0.9664

and 'hump' allocation, which are slightly different allocations, perform better in comparison with the 'lifecycle' portfolio. In the other environments, which can be seen as rather extreme market conditions, we observe that the 'const20', or 'const90' allocations give the best performance, in line with our expectations. For example, if a person is very risk averse, which is the case for $\gamma = 10$, the 'const20' allocation performs best, since this portfolio has the smallest allocation to the risky asset. If market conditions are good and the mean excess return is 0.1, the 'const90' allocation performs best since this is the allocation with the largest position to the risky asset.

The results seem to confirm the findings of Benzoni *et al.* (2007), namely that the co-integrating relation between labor income and dividends makes labor income, or human capital, having stock-like properties. Especially in the beginning of the agent's career when his human capital is large this means that the agent has a large position in the risky asset. To adjust his overall portfolio to the right risk level he has to compensate for this large risky position by putting a large fraction of his financial wealth into the risk-less asset. As the agent ages the co-integrating relation, i.e., long-run labor income risk, has less time to act and his human capital will require more bond-like properties. Hence he should compensate for this by enlarging his position to the risky asset. As the agent approaches retirement two offsetting forces are at work. Firstly the co-integrating relation has less time to act. Secondly, his human capital is declining at a fast rate which declines the implicit risk-free asset position. The latter effect eventually will dominate, which causes the agent to reduce his position in the risky asset to buy more risk-free assets. Note that our results are less extreme than those of Benzoni *et al.* (2007), where it is even optimal to take a short position in the risky asset. As stated in the introduction, maybe this is due to the fact that wage profiles for the Netherlands are different from wage profiles for the US. The results are similar for each sector and for the Netherlands as a whole. Apparently, modeling the life-cycle problem in this way outweighs the fact that labor income risk for the public sector generally is less than for the construction sector. However, if we would optimize we might see differences in asset allocations for the different sectors.

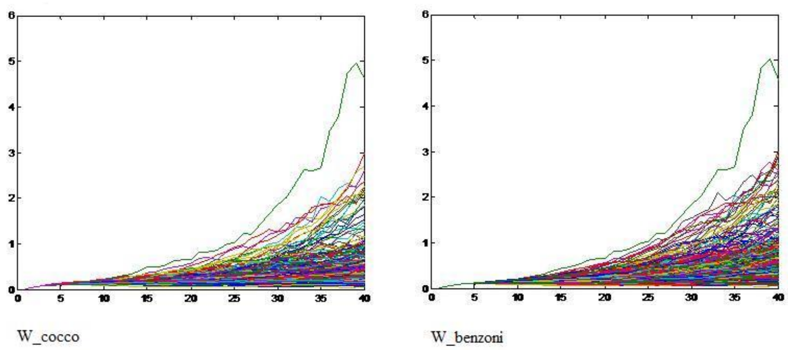
Looking at the 'cocco' allocation's performance we see that this allocation never performs really good in these situations but also not really bad. This means that we can see this allocation as some kind of hedge allocation[30]. Although the 'cocco' portfolio only performs best in the benchmark case, using this allocation prevents us from performing very bad in extreme market situations. Note that the 'const60' allocation which most pensions use as default, performs worse than the 'lifecycle' allocation. Only in situations where market conditions are good we see that this allocation performs well, since in those cases this allocation takes advantage of it's relatively high position in the risky asset.

We observe that in good market conditions the ordering between the three hump shaped allocations; 'cocco', 'hump', and 'benzoni' is reversed. In those situations the 'benzoni' allocation performs best out of these three. This result is in line with Basu and Drew (2009), where they find that portfolios which are contrary to common life-cycle strategies result in a much higher expected (or median) final wealth. They explain this by looking at the accumulation paths of wealth over the simulation period and argue that since these paths steepen as they move along the horizon, potential for fast

[30]To be precise, hedge is not the right definition for this allocation since we are not really investing in offsetting positions whatsoever, to reduce risk. A definition which might suite better is *minimum-regret portfolio*: Choose the allocation that the agent would regret the least.

growth of wealth comes only in the later years. Looking at Figure 11 we see that the accumulation paths for the 'cocco' allocation in the final years lag a little behind those of the 'benzoni' allocation. Since the sensitivity of growth in accumulated wealth to the asset allocation becomes more and more significant in the final years before retirement when the portfolio size is larger than at the beginning of the horizon. This explains why the 'benzoni' allocation performs better than the 'hump' and 'cocco' allocation. Although the allocation to the risky asset is smaller during a large part of the horizon, the fact that the 'benzoni' allocation reaches it's maximum at a later stage in the simulation period results in a higher accumulated expected final wealth. Nevertheless, we have to be careful using this explanation. If we encounter successive years of bad returns at the end of the agent's working life, this will produce severe results for the agent who employs a high equity allocation (See The Ambachtsheer Letter (august 2009)).
Note that the VAR model gives the same results concerning the preference ordering of the allocations. This seems to suggest that the co-integrating relation between labor income and dividends is not a necessary condition for these results. Apparently, if we model labor income and dividends according to their joint history it might well be the case that the interdependencies between the variables caused by each others history and their correlated error terms result in the outcomes being the similar to the co-integration case. However, we tend to prefer the VECM since this kind of modeling produces less variation between simulated scenarios as is seen in figure 20.

Figure 11: Wealth accumulation paths. We observe that the accumulation paths for the 'cocco allocation lag behind those of the 'benzoni' allocation. Since the sensitivity of growth in accumulated wealth to the asset allocation becomes more significant in the final years of the horizon, when portfolio size is larger than at the beginning of the horizon, the expected accumulated final wealth for the 'benzoni' allocation will be higher. For a better examination of the paths we only plotted 300 simulations.



Conclusions simulation results

- Cocco allocation performs best in benchmark case.
- Benzoni and hump allocation also perform better in benchmark case compared to other allocations.
- Cocco allocation serves as a hedge portfolio against rather extreme market conditions.
- VAR gives same results in terms of preference ordering between the allocations as the VECM.

4.6 Sensitivity analysis

This Section examines the sensitivity of the results with respect to average final labor income to changes in parameter settings. Since the models for the Netherlands, the public sector, and the construction sector are very similar for each model specification we will mainly perform the analysis for the Netherlands. In the following we will examine the sensitivity of the parameter estimates to a change of 1 x standard error. For the adjustment coefficients it holds for both stochastic and deterministic VECM that the results are quite robust against parameter estimates. For the coefficients of the dynamics, which we will denote by B, there is more discrepancy in sensitivity results. The models for public labor income appear to be more sensitive to parameter estimates than the models for the Netherlands and construction labor income.

4.6.1 Speed of adjustment

VECM with stochastic trend

Since tests on co-integration usually have lack of power, which means that the null hypothesis of no co-integration can be rejected too often, we might handle our estimation results with care. In this respect we examine the robustness of our results to the value of the adjustment coefficients.

From simulations we can extract 100.000 sample paths of the correction terms (α_1 x co-integrating equation). After summing up the forty corrections over each path we obtain for each path a sum of corrections. A histogram, which is shown in Figure 18, shows the distribution of these 100.000 sums of corrections. We observe that roughly 80% lies below zero. Hence, on average a more negative value of α_1 will result in a decrease in labor income.

Table 8: Sensitivity to the adjustment coefficient. In this Table sensitivity results with respect to average final labor income are presented using VECM with stochastic trend, where the results of a decrease in α_1 are denoted with (). The public sector appears to be the least sensitive to changes in adjustment coefficients. The construction sector is the most sensitive. We do not present results concerning α_2 since this parameter appears to be insignificant in the estimation results.

	α_1
Netherlands	+1.97% (−1.24%)
Public	+0.12% (−0.02%)
Construction	+5.23% (−3.90%)

We observe that a change in adjustment parameter has little impact on the public sector, while it has the most impact on the construction sector. Note that a change of 1 x standard error is more significant for the construction sector than for the public sector. However, this change is even less significant for the Netherlands, while results for the Netherlands are higher than for the public sector. Hence, this does not completely explain the results. In general, construction sector labor income is more risky than public sector labor income. Perhaps this is the reason why construction sector labor income is more sensitive to the results. However, we have seen that statistical evidence regarding co-integration between construction sector labor income and dividends is rather weak. As a consequence, estimation results could be unreliable, which causes this parameter to be more sensitive.

The results regarding the preference order of the asset allocations do not change.

VECM with deterministic trend

The conclusions with respect to asset allocations for this robustness analysis are in accordance with those made for the VECM with stochastic trend. In this model, α_2 is significant to. Hence, a change in parameter estimate probably has an impact on our results. Again, the results concerning the the preference order of asset allocations remains unaffected. See Table 9.

Table 9: Sensitivity to the adjustment coefficient. In this Table sensitivity results with respect to average final labor income are presented using VECM with deterministic trend, where the results of a decrease in α are denoted with (). Now also α_2 appears to be significant. The sensitivity results for both sectors are quite similar. Only the sign is reversed.

	α_1	α_2
Netherlands	+0.87% (−0.80%)	+0.72% (−1.24%)
Public	−0.90% (+0.65%)	−0.61% (+0.90%)

4.6.2 Dynamics

Stochastic trend

From Figure 14 it appears that for every model only parameter estimate $B(1,1)$ is significant. We expect that changing parameter settings for the other coefficients will not significantly affect our results. As a check we also present results for $B(1,2)$.

Table 10: Sensitivity results for dynamics. In this table sensitivity results with respect to average final labor income are presented using VECM with stochastic trend, where the results of a decrease in the estimates are denoted with (). For $B(1,2)$ we only present sensitivity results concerning an increase in the estimate. Note that for this parameter we analyze a change of 10% instead of 1 x standard error, which is approximately equal to a change of 1 x standard error for the significant coefficients. The sensitivity results for the public sector are extremely high compared to the other sectors.

	$B(1,1)$	$B(1,2)$
Netherlands	+5.72% (−4.98%)	+0.36%
Public	+39.02% (−19.93%)	+1.31%
Construction	+2.97% (−2.67%)	+0.09%

The $B(1,1)$ parameter is more sensitive to changing parameter settings compared to the adjustment coefficients. As expected an increase in $B(1,2)$ has only little impact on results.[31] The results can be explained by looking at the lagged difference $(l(s-1)-l(s-2))$ which is a positive increment, hence a decrease in $B(1,1)$ will result in a decrease of this lagged difference and thus in a decrease of labor income. The results with respect to asset allocations remain unchanged.

Deterministic trend

For the VECM with deterministic trend we analyze $B(1,1)$ since this parameter is significant in all models. From the remaining parameters $B(2,2)$

[31] Note that an increase of 1 x standard error for the $B(1,2)$ parameter denotes a 100% change, while a change of 1 x standard error for the significant variables denotes a change of approximately 10%.

appears to be least insignificant, hence we include this parameter in the analysis. We see that the sensitivity results for the Netherlands are larger

Table 11: Sensitivity results for dynamics. In this table sensitivity results with respect to average final labor income are presented using VECM with deterministic trend, where the results of a decrease in the estimates are denoted with (). For $B(2,2)$ we only present sensitivity results concerning an increase in the estimate. Note that for this parameter we analyze a change of 10% instead of 1 x standard error, which is approximately equal to a change of 1 x standard error for the significant coefficients.

	$B(1,1)$	$B(2,2)$
Netherlands	$+22.69\%$ (-14.61%)	$+4,67\%$
Public	$+28.33\%$ (-15.72%)	$+5.62\%$

as for the VECM with stochastic trend. However, for the public sector the sensitivity results actually became smaller.

The important results concerning allocation preferences remain unaffected.

Var model

In the VAR model we see that $B(1,1)$ again is significant in all models. $B(2,1)$ is nearly significant for the Netherlands, and is significant for the public sector. Thus, we include this parameter in our analysis.

Table 12: Sensitivity results for dynamics. In this table sensitivity results with respect to average final labor income are presented using the VAR model, where the results of a decrease in the estimates are denoted with (). For $B(2,1)$ we only present sensitivity results concerning an increase in the estimate. Note that although $B(2,1)$ is significant for the public sector we still analyze a change of 10% instead of 1 x standard error, since the parameter is not highly significant. A change of 1 x standard error denotes approximately 50%.

	$B(1,1)$	$B(1,2)$
Netherlands	$+163.51\%$ (-33.26%)	0.99%
Public	$+130.01\%$ (-27.79%)	0.34%
Construction	$+137.05\%$ (-32.64%)	0.34%

We observe that the results are quite similar. Compared to the two VEC models the VAR model is far more sensitive to the parameter estimates. These results tend to speak in favor of the vector error correction models.

4.6.3 Trend

In this Section we examine the sensitivity results with respect to the trend parameter in the VECM with deterministic trend. We analyze an increase (decrease) of 10%, which is approximately equal to the 1 x standard error changes for the other parameters.[32]

Table 13: Sensitivity results for trend. In this table sensitivity results with respect to average final labor income are presented using VECM with deterministic trend, where the results of a decrease in the estimates are denoted with (). Note that for this parameter we analyze a change of 10%, instead of 1 x standard error, which is approximately equal to a change of 1 x standard error for the significant coefficients.

	trend
Netherlands	−16.56% (+21.24%)
Public	−22.55% (+29.26%)

We see that the results are quite similar, although the public sector shows to be slightly more sensitive to a change in parameter estimate.

In sum, the results suggest that we should use one of the vector error correction models since the VAR model is far more sensitive to the parameter estimates. Another reason for preferring a VECM above the VAR model can be found by comparing the variation in scenarios. Because of the possible adjustments the VECM will make if the variables deviate to far from the long-run equilibrium, it will produce more accurate simulations than the VAR model. This is nicely illustrated by plotting 100.000 simulations from both models, see Figure 20. Obviously, there is more variation in simulated scenarios for the VAR model than for the VECM.

Choosing between the two VEC models is less evident. The results are in favor of the stochastic VECM, although the deterministic VECM has a nice

[32] Note that for the estimate of the trend parameter a change of 1 x standard error equals 46% for the Netherlands and 32% for the public sector.

interpretation regarding the trend estimate. For the construction sector we should use the stochastic VECM anyway, since statistical evidence shows that a deterministic VECM is inappropriate in describing the co-integration between labor income and dividends. For the public sector we tend to choose for the deterministic VECM, since sensitivity results are less severe. For the Netherlands it again appears that the stochastic VECM is the most appropriate characterization.

5 Conclusions

Nowadays financial planners often recommend individuals to use a simple rule of thumb when investing over the life-cycle: 100 - age percent should be invested in the risky asset. This rule is justified by the life-cycle theory in which the key assumption about human capital is that the expected present value of future labor income is risk-free. Hence, in the beginning of his career the agent has a large implicit position in the risk-free asset. To compensate for this, he should place a large fraction of his financial wealth in the risky asset. As the agent ages his human capital reduces which also reduces the implicit bond position. As a consequence he should reduce his position to the risky asset and buy more bonds.

In that sense our results are quite controversial. We showed that it is not the lifecycle allocation which performs best, nor does the allocation most pension funds use as default, which consists of 60% stocks. Our results show that it is the hump-shaped allocation found in Cocco *et al.* (2005), and the hump-shape found in Benzoni *et al.* (2007) which give the best performance. Even in extreme market conditions these allocations perform reasonably well, which demonstrate that we can actually see them as some kind of hedge allocation. Under normal conditions the hedge performs best, however under extreme market conditions we are hedged against large shortfalls.

Our results are in line with those in Cocco *et al.* (2005) and Benzoni *et al.* (2007). However, if we simulate using a VAR model, so without the co-integrating relation between labor income and dividends, the conclusions are similar. Apparently, the co-integration is not a key concept for obtaining these rather contradictory results. Looking at our VAR estimation results it seems that only the lagged difference for labor income in the labor income equation is significant. The coefficient is nearly significant for the dividend equation in the model for the Netherlands, and is significant in the model for public sector economics. Although just few coefficients are significant, it seems that modeling labor income and dividends in this way might cause the interdependencies between the variables, through each others history and their correlated error terms, to lead to results similar to the co-integration models.

Does this mean we can throw away our co-integration models? Although the VAR model gives similar results, it is far more sensitive to parameter estimates compared to the error correction models which makes the VEC models more reliable. Further, as already shown in Figure 20 we see that the VECM produces far less variation between its simulations. Choosing between the two VEC models, however, is more difficult. The deterministic VECM shows plausible results concerning the negative trend in the co-integrating equation, which is in line with the finding that labor shares have decreased over the last years. Though, sensitivity results show that this model is more sensitive to estimation results than the stochastic VECM, except for the public sector. Furthermore, statistical evidence seems to reject the existence of co-integration between construction sector labor income and dividends, which is reflected in the non-interpretable estimation results. This leaves us with a deterministic VECM for the Netherlands and the public sector, and a stochastic VECM for the construction sector. In sum, it seems appropriate to characterize the Netherlands with a deterministic VECM although this model is more sensitive to parameter estimates than the stochastic VECM. For the construction sector a stochastic VECM seems appropriate, and for the public sector we again prefer the deterministic VECM.

It seems important for pension funds to take this information into account in designing more optimal contracts for participants. However, it is difficult to translate our findings into useful pension policies, and how this must be done is a key question for future research. In the future pension funds have to find the right balance between collective and individual pension plans, in which the risk sharing benefit from collective schemes should be combined with individual characteristic from individual schemes.
To clarify the overall picture we summarize the main conclusions:

- The 'hump-shaped' allocations perform better compared to more traditional allocations such as the '100 minus age' allocation.
- The 'hump-shaped' allocations serve as a hedge portfolio against extreme market conditions.
- Although the VAR model produces similar results, the VECM is to be preferred since this model is less sensitive to parameter estimates and produces less variation between scenarios.

Our results show similarities with those of Benzoni *et al.* (2007). However, discrepancies between our most preferred allocation and Benzoni's allocation appear at the beginning of the agent's career. We show that it is better for the agent to start with a 40% allocation to the risky asset in the first years, while benzoni's allocation states that it is optimal to start with a 0% allocation to the risky asset in the first years. An interesting question to explore would be if this difference is related to the country's wage profile. In Section 2.2.1. for the Netherlands we observed increasing wage profiles which do not decline as retirement approaches. For the US we observe hump-shaped wage profiles which decline after age 50. This means that in the Netherlands human capital declines at a lower rate. This causes the agent's implicit bond holding to be higher during his final years before retirement, which might explain why the agent should hold larger fractions in the risky asset over his life-cycle compared to the Benzoni results[33]. In a further research it would be interesting to find out how large the impact of different wage profiles is on the optimal asset allocation. Can we attribute the discrepancies between our results and those of Benzoni *et al.* (2007) to the difference in wage profiles?

We model the life-cycle problem using co-integration between labor income and dividends. Cocco *et al.* (2005) modeled labor income where they allow for disastrous income shocks. Interestingly, the results are quite similar. This raises the question of what would be the outcome if we extend our model by allowing such disastrous labor income shocks to occur. What is even more interesting is that we produce similar results using a VAR model, in which almost none of the coefficients are significant. In future research it would be worth investigating what causes this result.

We a priori specified certain allocations found in the academic literature, and which are used by financial planners or pension funds. After simulation we compared the allocations using utility levels. To be complete, in an extension of this research we could actually solve the dynamic programming optimization to find the exact optimal allocation. In this way we might find differences in asset allocations between the public and construction sector.

Finally, we find that in good market conditions (i.e. for the cases where $\gamma = 2$, $R_e = 0.1$, and $\sigma = 0.05$) the ordering between the three hump shaped allocations 'cocco', 'benzoni', and 'hump' is reversed compared to

[33]Note that during the final years before retirement the agent's human capital acquires more bond-like properties since the process of co-integration has less time to act.

the benchmark case. We explain this by referring to Basu and Drew (2009). Potential for fast growth comes only in the later years. From Figure 11 we see that the wealth accumulation paths for the 'cocco' allocation lag behind those of 'benzoni'. However, the difference in allocations in the later years is very small. We leave the question of why such small differences in allocations lead to these results for future research.

Below we summarize our main questions for further research:

- What is the impact of using different kind of wage profiles on the optimal asset allocation?
- What causes the VAR model to produce similar results?
- Do we find differences in asset allocations between the public and construction sector if we solve the dynamic optimization problem?
- Can Basu and Drew (2009) explain why the ordering between the three hump shapes is reversed in good market conditions, even if the differences in allocations in the later years is small?

It is clear that this area of research has to be exploited in several directions in the coming years. The world of pensions is a dynamic system which has to be adapted constantly in order to fulfil the desires of the plans' participants and to ensure a financially stable system at the same time.

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7 Appendix: Figures and Tables

Figure 12: Johansen co-integration test results. The first table denotes the results for Netherlands, the second for the public sector, and the third denotes results for the construction sector. As is seen from the first two tables, according to the Johansen test there exists a co-integrating relation between labor income and dividends for the Netherlands and the public sector. From the third table we observe that there is no evidence in favor of a co-integrating relation between labor income and dividends for the construction sector.

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.416714	21.96242	15.41	20.04
At most 1	0.038134	1.477437	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 1 cointegrating equation(s) at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.416714	20.48498	14.07	18.63
At most 1	0.038134	1.477437	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at both 5% and 1% level:

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.384147	19.47190	15.41	20.04
At most 1	0.027293	1.051538	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 1 cointegrating equation(s) at the 5% level
Trace test indicates no cointegration at the 1% level

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.384147	18.42036	14.07	18.63
At most 1	0.027293	1.051538	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at the 5% level

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.262292	14.23219	15.41	20.04
At most 1	0.067908	2.672322	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates no cointegration at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.262292	11.55986	14.07	18.63
At most 1	0.067908	2.672322	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates no cointegration at both 5% and 1% levels

Figure 13: Johansen co-integration test results for construction sector using 2 lagged differences. If we test for co-integration between labor income and dividends using two lagged differences the test results do indicate a co-integrating relation.

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.337746	18.62215	15.41	20.04
At most 1	0.087161	3.374244	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 1 cointegrating equation(s) at the 5% level
Trace test indicates no cointegration at the 1% level

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.337746	15.24790	14.07	18.63
At most 1	0.087161	3.374244	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at the 5% level
Max-eigenvalue test indicates no cointegration at the 1% level

Figure 14: Vector error correction estimates. The estimation results are more or less the same for all three models. The adjustment coefficient in the labor income equation is significant, while the adjustment coefficient in the equation for dividends is not. Furthermore, only the lagged difference for labor income in the equation for labor income appears to be significant. Standard errors in ( ) and t-statistics in [ ].

Cointegrating Eq:	CointEq1	
L1N(-1)	1.000000	
LOGD(-1)	-0.405069 (0.05178) [-7.82351]	
C	-9.305368	
Error Correction:	**D(L1N)**	**D(LOGD)**
CointEq1	-0.074076 (0.01612) [-4.59628]	0.077039 (0.07909) [0.97407]
D(L1N(-1))	0.694265 (0.07333) [9.46831]	-0.296173 (0.35983) [-0.82309]
D(LOGD(-1))	0.022796 (0.03403) [0.66992]	0.208845 (0.16699) [1.25065]
C	0.009845 (0.00452) [2.17994]	0.061136 (0.02216) [2.75844]

Cointegrating Eq:	CointEq1	
LOGPUBLICK(-1)	1.000000	
LOGD(-1)	-0.322478 (0.04912) [-6.56461]	
C	-9.618591	
Error Correction:	**D(LOGPUB...**	**D(LOGD)**
CointEq1	-0.085824 (0.02024) [-4.23942]	0.055564 (0.07842) [0.70855]
D(LOGPUBLICK(...	0.705122 (0.07740) [9.10978]	-0.431097 (0.29983) [-1.43780]
D(LOGD(-1))	0.023587 (0.04255) [0.55437]	0.195497 (0.16481) [1.18620]
C	0.007350 (0.00484) [1.51901]	0.064692 (0.01874) [3.45153]

Cointegrating Eq:	CointEq1	
L1C_CBS(-1)	1.000000	
LOGD(-1)	-0.393428 (0.09959) [-3.95061]	
C	-9.141201	
Error Correction:	**D(L1C_CBS)**	**D(LOGD)**
CointEq1	-0.071243 (0.02122) [-3.35701]	0.054245 (0.06215) [0.87287]
D(L1C_CBS(-1))	0.467405 (0.12596) [3.71062]	-0.148562 (0.36887) [-0.40275]
D(LOGD(-1))	0.002585 (0.05724) [0.04516]	0.230995 (0.16762) [1.37806]
C	0.023383 (0.00830) [2.81818]	0.054881 (0.02430) [2.25877]

Figure 15: Johansen test results for VECM with deterministic trend. The conclusions regarding co-integration remain the same.

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.422083	38.61505	25.32	30.45
At most 1 **	0.373659	17.77872	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 2 cointegrating equation(s) at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.422083	20.83633	18.96	23.65
At most 1 **	0.373659	17.77872	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 2 cointegrating equation(s) at the 5% level
Max-eigenvalue test indicates no cointegration at the 1% level

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.395005	35.54682	25.32	30.45
At most 1 **	0.351379	16.45046	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 2 cointegrating equation(s) at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.395005	19.09636	18.96	23.65
At most 1 **	0.351379	16.45046	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 2 cointegrating equation(s) at the 5% level
Max-eigenvalue test indicates no cointegration at the 1% level

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None *	0.362835	28.48996	25.32	30.45
At most 1	0.258447	11.36232	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 1 cointegrating equation(s) at the 5% level
Trace test indicates no cointegration at the 1% level

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.362835	17.12763	18.96	23.65
At most 1	0.258447	11.36232	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates no cointegration at both 5% and 1% levels

Figure 16: Vector error correction estimates including deterministic time trend.
Including a determinstic time trend has significant impact on the results. The results for the construction sector does not seem to be correct as the trend coefficient appears to be negative, which means that it is positive in the co-integrating equation. According to Arpaia *et al.*(2009) labor income share is declining over the past years. This results are in line with the ADF results where it seems that including a deterministic trend does not appear to be an appropriate characterization. Furthermore, now both adjustment coefficients are significant. Standard errors in ( ) and t-statistics in [ ].

Cointegrating Eq:	CointEq1	
L1N(-1)	1.000000	
LOGD(-1)	-1.423414 (0.46041) [-3.09162]	
@TREND(69)	0.062006 (0.02886) [2.14864]	
C	-8.037378	
Error Correction:	**D(L1N)**	**D(LOGD)**
CointEq1	-0.041272 (0.01121) [-3.68185]	0.107036 (0.04844) [2.20970]
D(L1N(-1))	0.799719 (0.06842) [11.6877]	-0.256014 (0.29568) [-0.86586]
D(LOGD(-1))	0.009825 (0.03693) [0.26604]	0.249629 (0.15959) [1.56423]
C	0.006214 (0.00463) [1.34333]	0.057136 (0.01999) [2.85825]

Cointegrating Eq:	CointEq1	
LOGPUBLICK(-1)	1.000000	
LOGD(-1)	-2.712146 (0.75713) [-3.58212]	
@TREND(69)	0.147827 (0.04797) [3.08186]	
C	-6.690722	
Error Correction:	**D(LOGPUB...**	**D(LOGD)**
CointEq1	-0.026104 (0.00939) [-2.77878]	0.076740 (0.03009) [2.55025]
D(LOGPUBLICK(...	0.812467 (0.07765) [10.4626]	-0.375060 (0.24875) [-1.50781]
D(LOGD(-1))	-0.000801 (0.04848) [-0.01652]	0.273755 (0.15530) [1.76274]
C	0.004992 (0.00533) [0.93719]	0.058253 (0.01706) [3.41432]

Cointegrating Eq:	CointEq1	
L1C_CBS(-1)	1.000000	
LOGD(-1)	2.677591 (0.63653) [4.20652]	
@TREND(69)	-0.187687 (0.03935) [-4.76990]	
C	-12.95079	
Error Correction:	**D(L1C_CBS)**	**D(LOGD)**
CointEq1	-0.026847 (0.01637) [-1.64008]	-0.132083 (0.03729) [-3.54168]
D(L1C_CBS(-1))	0.610494 (0.13620) [4.48222]	-1.111420 (0.31031) [-3.58159]
D(LOGD(-1))	0.029595 (0.06592) [0.44893]	0.372762 (0.15020) [2.48181]
C	0.014809 (0.00865) [1.71270]	0.094057 (0.01970) [4.77448]

Figure 17: VAR estimation results. The lagged difference for labor income is significant in the equation for labor income in all three models. Note that for the public sector it holds that the lagged difference for labor income is also significant in the equation for dividends. Standard errors in ( ) and t-statistics in [ ].

	D(L1N)	D(LOGD)
D(L1N(-1))	0.896169	-0.506152
	(0.07368)	(0.28790)
	[12.1627]	[-1.75808]
D(LOGD(-1))	0.027395	0.204062
	(0.04269)	(0.16679)
	[0.64177]	[1.22343]
C	0.001208	0.070118
	(0.00515)	(0.02014)
	[0.23445]	[3.48172]
R-squared	0.819499	0.157949
Adj. R-squared	0.809185	0.109832
Sum sq. resids	0.008785	0.134121
S.E. equation	0.015843	0.061903
F-statistic	79.45257	3.282599
Log likelihood	105.1545	53.36571
Akaike AIC	-5.376552	-2.650827
Schwarz SC	-5.247269	-2.521544
Mean dependent	0.039944	0.060805
S.D. dependent	0.036268	0.065611

	D(LOGPUB...	D(LOGD)
D(LOGPUBLICK(...	0.867228	-0.536047
	(0.08201)	(0.25883)
	[10.5749]	[-2.07105]
D(LOGD(-1))	0.026450	0.193644
	(0.05184)	(0.16361)
	[0.51022]	[1.18355]
C	0.001517	0.068468
	(0.00565)	(0.01784)
	[0.26838]	[3.83769]
R-squared	0.773670	0.183633
Adj. R-squared	0.760736	0.136984
Sum sq. resids	0.013054	0.130030
S.E. equation	0.019312	0.060952
F-statistic	59.82060	3.936447
Log likelihood	97.62941	53.95427
Akaike AIC	-4.980495	-2.681804
Schwarz SC	-4.851212	-2.552521
Mean dependent	0.033358	0.060805
S.D. dependent	0.039482	0.065611

	D(L1C_CBS)	D(LOGD)
D(L1C_CBS(-1))	0.760795	-0.371953
	(0.10317)	(0.26473)
	[7.37444]	[-1.40501]
D(LOGD(-1))	0.001022	0.232185
	(0.06510)	(0.16705)
	[0.01571]	[1.38994]
C	0.009063	0.065784
	(0.00809)	(0.02077)
	[1.11974]	[3.16747]
R-squared	0.625959	0.132515
Adj. R-squared	0.604585	0.082945
Sum sq. resids	0.020984	0.138172
S.E. equation	0.024485	0.062831
F-statistic	29.28630	2.673267
Log likelihood	88.61079	52.80032
Akaike AIC	-4.505831	-2.621069
Schwarz SC	-4.376548	-2.491786
Mean dependent	0.046487	0.060805
S.D. dependent	0.038938	0.065611

Table 14: Simulation results for public sector, using VECM with stochastic trend. Results for the public sector give the same conclusions about the preference order of the allocations, as for the Netherlands. For the benchmark case the 'cocco' allocation performs best. The 'const60' allocation most pension funds use actually performs even worse than the 'lifecycle' allocation. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-6.9096	-3.8870	-0.0405	-6.0905	-7.3497	-7.5263	-6.6349
hump	-6.9098	-3.8984	-0.0405	-6.0801	-7.3565	-7.5456	-6.6272
benzoni	-6.9187	-4.0017	-0.0404	-6.0088	-7.4131	-7.7127	-6.5736
lifecycle	-6.9287	-3.7556	-0.0406	-6.2996	-7.2610	-7.2841	-6.7630
const60	-6.9542	-4.1678	-0.0404	-5.9803	-7.4894	-7.9593	-6.5337
opposite	-6.9632	-3.7007	-0.0407	-6.4858	-7.2109	-7.1596	-6.8666
basu	-6.9732	-4.2551	-0.0404	-5.9688	-7.5282	-8.0888	-6.5147
const20	-7.0163	-3.6817	-0.0408	-6.6729	-7.1851	-7.1053	-6.9617
const90	-7.1981	-5.5325	-0.0402	-5.7307	-8.0402	-10.1390	-6.2375

Figure 18: These histograms shows 100000 sums of correction terms α_1 (upper plot) and α_2 (lower plot) over their sample paths, for the VECM with stochastic trend.

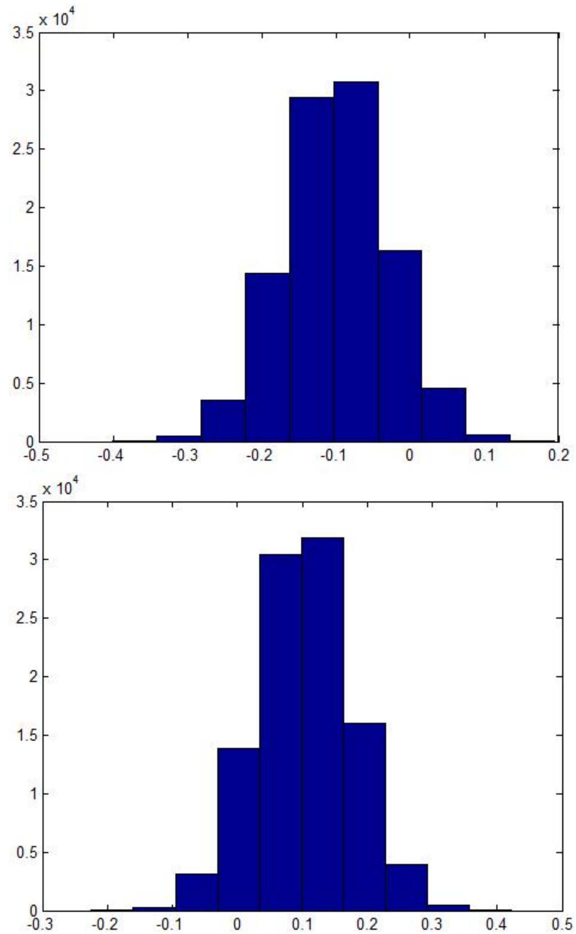

Figure 19: These histograms shows 100000 sums of correction terms α_1 (upper plot) and α_2 (lower plot) over their sample paths, for the VECM with deterministic trend.

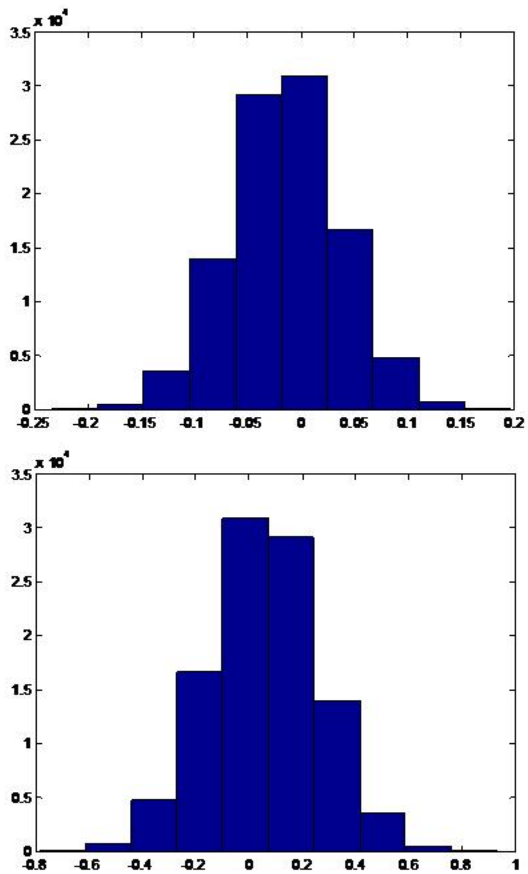

Figure 20: The left figure shows 100.000 simulated scenarios of possible labor income paths for the VECM while the other figure shows these for the VAR model. Obviously, there is more variation in scenarios for the VAR model.

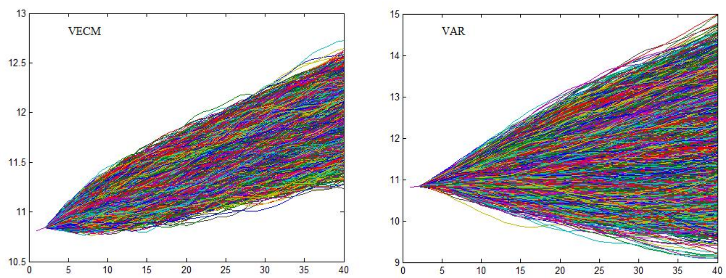

Table 15: Simulation results for construction sector, using VECM with stochastic trend. Again conclusions are the same as for the Netherlands. The 'cocco' allocation is most welfare improving. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-7.0939	-4.0464	-0.0407	-6.2520	-7.5456	-7.7396	-6.8126
hump	-7.0942	-4.0581	-0.0407	-6.2413	-7.5525	-7.7595	-6.8047
benzoni	-7.1045	-4.1615	-0.0407	-6.1682	-7.6108	-7.9327	-6.7498
lifecycle	-7.1117	-3.9166	-0.0409	-6.4664	-7.4543	-7.4876	-6.9440
const60	-7.1431	-4.3259	-0.0407	-6.1387	-7.6896	-8.1874	-6.7088
opposite	-7.1456	-3.8648	-0.0410	-6.6573	-7.4028	-7.3574	-7.0502
basu	-7.1636	-4.4129	-0.0406	-6.1268	-7.7297	-8.3213	-6.6893
const20	-7.1951	-3.8647	-0.0411	-6.8493	-7.3762	-7.3019	-7.1471
const90	-7.4043	-5.6984	-0.0405	-5.8818	-8.2584	-10.4750	-6.4050

Table 16: Simulation results for the Netherlands, using VECM with deterministic trend. Although these results are obtained using a VECM with deterministic time trend, conclusions are similar, which states that our findings are quite robust against different model specifications. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-6.6632	-3.5859	-0.0401	-5.8883	-7.0850	-7.2829	-6.4039
hump	-6.6633	-3.5964	-0.0401	-5.8784	-7.0915	-7.3019	-6.3965
benzoni	-6.6715	-3.6910	-0.0400	-5.8108	-7.1454	-7.4667	-6.3450
lifecycle	-6.6824	-3.4664	-0.0402	-6.0876	-7.0007	-7.0430	-6.5271
const60	-6.7059	-3.8435	-0.0400	-5.7844	-7.2182	-7.7105	-6.3067
opposite	-6.7160	-3.4172	-0.0403	-6.2651	-6.9534	-6.9188	-6.6267
basu	-6.7243	-3.9238	-0.0400	-5.7738	-7.2553	-7.8385	-6.2885
const20	-6.7637	-3.4145	-0.0405	-6.4441	-6.9293	-6.8656	-6.7176
const90	-6.9372	-5.1051	-0.0399	-5.5509	-7.7457	-10.9170	-6.0219

Table 17: Simulation results for the public sector, using VECM with deterministic trend. Conclusions remain unchanged. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-6.9503	-3.8631	-0.0406	-6.1343	-7.3999	-7.6110	-6.6790
hump	-6.9506	-3.8742	-0.0406	-6.1240	-7.4068	-7.6312	-6.6713
benzoni	-6.9597	-3.9736	-0.0405	-6.0528	-7.4643	-7.8059	-6.6172
lifecycle	-6.9695	-3.7381	-0.0407	-6.3435	-7.3099	-7.3561	-6.8082
const60	-6.9962	-4.1327	-0.0405	-6.0240	-7.5420	-8.0631	-6.5770
opposite	-7.0040	-3.6874	-0.0408	-6.5298	-7.2589	-7.2234	-6.9126
basu	-7.0156	-4.2166	-0.0405	-6.0124	-7.5815	-8.1981	-6.5578
const20	-7.0536	-3.6862	-0.0409	-6.7174	-7.2324	-7.1656	-7.0079
const90	-7.2437	-5.4394	-0.0404	-5.7751	-8.1027	-10.4230	-6.2784

Table 18: Simulation results for Netherlands, using VAR. Although we now obtain results using a VAR model, where we have not made use of the co-integrating relation between labor income and dividends, the conclusions remain similar. this suggests that the co-integration is not a necessary ingredient for obtaining these controversial outcomes. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}.

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-6.8931	-4.1348	-0.0403	-6.0919	-7.3593	-7.5280	-6.6370
hump	-6.8934	-4.1469	-0.0403	-6.0817	-7.3661	-7.5471	-6.6293
benzoni	-6.9028	-4.2558	-0.0403	-6.0109	-7.4231	-7.7135	-6.5757
lifecycle	-6.9115	-3.9963	-0.0405	-6.2997	-7.2699	-7.2870	-6.7655
const60	-6.9394	-4.4306	-0.0403	-5.9824	-7.5002	-7.9600	-6.5358
opposite	-6.9451	-3.9386	-0.0406	-6.4847	-7.2193	-7.1634	-6.8693
basu	-6.9589	-4.5226	-0.0403	-5.9708	-7.5394	-8.0891	-6.5167
const20	-6.9935	-3.9343	-0.0407	-6.6711	-7.1932	-7.1123	-6.9640
const90	-7.1854	-5.8675	-0.0401	-5.7343	-8.0572	-10.1030	-6.2390

Table 19: Simulation results for the public sector, using VAR. Just as before, also for the public sector conclusions are similar. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-7.7369	-5.3454	-0.0414	-6.8119	-8.2306	-8.4394	-7.4306
hump	-7.7372	-5.3609	-0.0414	-6.8003	-8.2383	-8.4613	-7.4221
benzoni	-7.7473	-5.5003	-0.0414	-6.7208	-8.3026	-8.6522	-7.3623
lifecycle	-7.7581	-5.1704	-0.0416	-7.0457	-8.1293	-8.1623	-7.5739
const60	-7.7876	-5.7250	-0.0414	-6.6894	-8.3898	-8.9358	-7.3178
opposite	-7.7961	-5.0997	-0.0417	-7.2537	-8.0717	-8.0192	-7.6897
basu	-7.8090	-5.8434	-0.0413	-6.6768	-8.4340	-9.0848	-7.2966
const20	-7.8506	-5.0983	-0.0418	-7.4631	-8.0416	-7.9584	-7.7953
const90	-8.0627	-7.5342	-0.0412	-6.3907	-8.9893	-11.4850	-6.9870

Table 20: Simulation results for the construction sector, using VAR. It does not matter which sector we use for simulation, in this case the construction sector. Outcomes are in accordance with those of the Netherlands and the public sector. Results are times 10^{-7}, except for $\gamma = 10$ where the results are times 10^{-14}

	benchmark	$\gamma = 10$	$\gamma = 2$	$R_e = 0.1$	$R_e = 0.01$	$\sigma = 0.4$	$\sigma = 0.05$
cocco	-6.5317	-3.5287	-0.0398	-5.7456	-6.9309	-7.1154	-6.2608
hump	-6.5320	-3.5389	-0.0398	-5.7359	-6.9372	-7.1337	-6.2535
benzoni	-6.5413	-3.6310	-0.0397	-5.6687	-6.9893	-7.2922	-6.2030
lifecycle	-6.5438	-3.4123	-0.0399	-5.9425	-6.8498	-6.8848	-6.3816
const60	-6.5762	-3.7792	-0.0397	-5.6410	-7.0592	-7.5254	-6.1653
opposite	-6.5783	-3.3645	-0.0400	-6.1177	-6.8045	-6.7656	-6.4793
basu	-6.5947	-3.8573	-0.0397	-5.6298	-7.0949	-7.6474	-6.1474
const20	-6.6229	-3.3621	-0.0401	-6.2940	-6.7817	-6.7151	-6.5685
const90	-6.8102	-5.0035	-0.0396	-5.4037	-7.5681	-9.5247	-5.8859