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Jiajia Cui

DC Pension Plan Defaults and Individual Welfare

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Abstract

A growing body of research shows that most DC scheme participants simply follow the given defaults, although they have the freedom to choose. As a result, default designs have a dramatic impact on individuals' behavior with regard to retirement saving. Given the fact that the default design matters, this paper aims to evaluate and design better default options to help DC plan participants to save and invest wisely in terms of welfare improvement. The proposed defaults can be characterized by simple age-dependent contribution and investment rules. We find large economic welfare gains above the current standard default design by following the simple age-dependent contribution and investment defaults. Furthermore, we find that the optimal contribution choice plays a more important role than the optimal portfolio choice does in improving welfare. With regard to modeling and methodology, we solve a realistic life-cycle model with taxable and tax-deferred accounts for the optimal defaults, while taking into account housing and medical expenditures.

Keywords: Tax-Deferred Account, DC plan defaults, retirement savings, life cycle model, DC contribution rate, life cycle fund, liquidity constraints

JEL codes: E21, G11

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†Twente University and APG, the Netherlands; Postal Address: Department of Finance and Accounting, Twente University, PO Box 217, 7500 AE, Enschede, the Netherlands. Phone: +31-53-4893485. E-mail: j.cui@utwente.nl.

1 Introduction

Individual DC pension schemes offer each participant the freedom to choose and to implement the optimal consumption and investment strategies according to their own needs. Life-cycle theory has shown how to determine the optimal saving and investing strategies. In reality, however, early experiences with DC schemes have shown that most people choose highly suboptimal saving and investment strategies. A growing body of research shows that most people simply follow the given defaults[1] (Choi, Laibson, Madrian and Metrick (2004); Beshear, Choi, Laibson and Madrian (2004); Lusardi and Mitchell (2006); Benartzi, Peleg and Thaler (2007)). These studies show that default design has a significant impact on participation, contribution and investment outcomes. Choi et al. (2004) reported that, until late 1990s, non-participation was the standard enrollment default (i.e., individuals are not enrolled unless they opt in). Participation rates were low under such a default, ranging from 26-43% six months after date of hire, and 57-69% three years after date of hire. Automatic enrollment began to be implemented in DC plans in the late 1990s. Consequently, the reported participation rates exceeded 85%, regardless of the tenure of the employee under the automatic enrollment regime. Furthermore, 65-87% of the participants adopted the default contribution rate of 3% or 4% of income, and the default investment in money market accounts. About 45% of the participants still stuck with these defaults three years later. Clearly, default design has a significant impact on retirement saving behavior.

Given the dramatic impact of defaults, it is desirable that they are as good as possible. Is the current popular default design the best possible, in welfare terms? If not, would it be possible to design a better default that might achieve a nearly optimal welfare outcome? This study therefore aims to find the optimal age-dependent contribution and investment rules, and to evaluate to what extent these default rules help to improve individual welfare. We find potentially large economic welfare gains by following the age-dependent defaults above the current standard default design. According to PSCA's *Automatic Enrollment 2001 Survey*[2] in the United States, the most common default contribution rate is 3% or 4% of pay (present in more than 60% of 401(k) plans). The most common investment default is stable value fund and money market fund (present in 67% of plans). In this paper, the current default design refers to automatic enrollment with a flat contribution rate of 4% of income, together with a risk-free investment vehicle.

Life-cycle theory is a useful framework for such analysis, and provides us many insights[3]. There

[1] If individuals do not take action to choose from the available options, then the default settings will be applied automatically. The defaults specify whether or not an individual contributes to the tax-deferred DC account, the level of the contribution rate, and the investment funds in which the contributions will be invested.

[2] www.psca.org

[3] Life-cycle theory has a long history in the finance literature. Particularly, Merton (1969, 1971) emphasizes the role of human capital. More recent papers include Carroll (1992, 1994, 1997) on precautionary saving, Gourinchas and Parker (2002) on life-cycle consumption, Viceira (2001), Gomes and Michaelides (2005) and Cocco, Gomes and

are two controls in life-cycle planning problems: optimal consumption and portfolio choices. Both strategies may depend on an individual's age, income, wealth accumulation and other economic state variables, and hence may differ with age and economic circumstances. A better default design should thus have two aspects: a better saving-rate default and a better investment default. Several studies propose life-cycle funds as the portfolio allocation default (Bodie, McLeavey and Siegel (2007); Viceira (2007)). The idea behind life-cycle funds is to mimic theoretical life-cycle portfolio strategies using an age-dependent portfolio-rebalancing rule. In such life-cycle funds (also known as target maturity funds), portfolio allocation to stock mutual funds declines as an individual ages, and is replaced gradually by safer assets such as bonds and cash. In fact, life-cycle funds implement a simplified version of the optimal portfolio strategy. Life-cycle funds recently started to be implemented as the default by many DC scheme providers, and are expected to have a great impact on the asset allocation outcome of most DC contributors in the future.

However, changing the portfolio default alone does not help much if people do not contribute, or fail to contribute enough. To address this pressing issue, the United States recently passed the Pension Protection Act of 2006, which encourages the adoption of several 'auto-save' features in the DC plans. These 'auto-save' features include automatic enrollment, employer contribution, contribution escalation, and qualified investment default (see Beshears, Choi, Laibson, Madrian and Weller (2008)). The contribution escalation is based on an interesting idea developed by Thaler and Benartzi (2004) called Save More Tomorrow, which dramatically stimulates participants to save more. In such a scheme, participants agree to automatically increase their saving rate whenever they receive a raise. However, as the authors claim, this design is based solely on behavior motivations, rather than financial or economic considerations. Are increasing saving rates optimal or nearly optimal over the life cycle? These questions are not addressed in the literature.

Although life-cycle funds have tackled the asset-allocation aspect of life-cycle theory, they have not addressed the consumption aspect. The next step forward is therefore to extend the fixed-contribution rate to include some age-dependent features. The main idea in this paper is to design simple default rules for DC contribution and investment that will mimic optimal consumption during an individual's life cycle. In addition, this paper evaluates to what extent these age-dependent default contribution- and investment rules are beneficial to the participants and can be recommended as default options for individual pension plans.

The main findings of this paper are as follows. First, substantial economic welfare gains are found by following the smart but simple age-dependent contribution rule above the current standard default design. Compared to current defaults, age-dependent defaults lead to a 7.2% increase in

Maenhout (2005) on life-cycle portfolio choice, Benzoni, Collin-Dufresne, and Goldstein (2007) on life-cycle strategies with cointegrated labor income with market returns, Cocco (2005) on portfolio choice in the presence of housing risk, and Gomes, Kotlikoff and Viceira (2008) on life-cycle investing with flexible labor supply.

certainty equivalent consumption per year. Over 60 years (in adulthood), the welfare gain amounts to 2.78 times first-year labor income. Using the fully optimal strategies as a welfare benchmark, the standard default design delivers maximally 92.7% of welfare relative to the optimal welfare level. The age-dependent contribution and investment default design, however, delivers more than 99%, relative to the optimal welfare level. The simple age-dependent contribution and investment rules can therefore achieve a nearly optimal welfare level.

Second, this paper finds that the contribution (or saving) choice has a greater impact on welfare than portfolio choice does. As compared to the current defaults (with a flat contribution rate of 4% and fully risk-free investment), improving contribution policy alone increases welfare from 92.7% to 97.1% of the optimal welfare level. However, improving asset allocation alone increases welfare only from 92.7% to 95.2%. This paper shows that setting the contribution (or saving) correctly is more important in welfare terms than focusing on portfolio choices, which have been the main area of interest for the life-cycle literature. Our paper finds that the contribution rule plays a more important role in improving welfare.

Where does this welfare improvement originate? Our analyses reveal that it comes from a better trade-off between liquidity constraints and tax advantages. Early in life, individuals face liquidity constraints, because wage earnings are on average upward sloping over the lifetime. During this time, however, individuals cannot borrow against their future labor income to boost their consumption. In addition, in data as well in our model setup, housing expenditure is relatively high for the young compared to the elderly, which makes the liquidity constraint more binding for the young. On top of these things, borrowing or early withdrawal from a DC pension scheme is not allowed, unless under extreme circumstances, and then subject to high penalty costs (about 10% reduction). The DC scheme is therefore (nearly) illiquid during the entire working period[4], which makes the liquidity constraint more severe, especially early in life. Retirement saving through a DC scheme is, however, entitled to tax benefits (and often employer matches), which are higher with age. The age-dependent DC contribution rule thus avoids the early periods in which an individual's liquidity constraint is tight and tax benefit is low, but makes the best use of later periods in which liquidity is abundant and the tax benefit is high.

The paper also stresses the idea of integrated retirement-saving strategies, in which retirement provision is considered jointly with important expenditures such as housing and Medicare. The dynamics and uncertainties of labor income, housing and medical expenditures are therefore carefully modeled in the paper, in order to realistically quantify and evaluate the default designs. The paper demonstrates that the decreasing life-cycle pattern of housing expenditures leads to a postponement of DC savings in the beginning, but that increasing medical expenditures towards

[4] In the baseline model of this paper, the assumption is made that neither borrowing nor early withdrawal from DC plan savings are allowed.

the end of life promote retirement savings.

A closely related paper by Gomes, Michaelides and Polkovnichenko (2006) studies optimal portfolio choices and optimal saving strategies for rational individuals with a taxable account and a tax-deferred DC account.[5] They focus on matching the calibrated life-cycle model with the empirical patterns in portfolio choice, wealth accumulation and stock market participation in a two-account setting. This paper adopts a similar modeling setup as Gomes et al. (2006), but with a focus on the optimal age-dependent contribution and investment rules. Although Gomes, Kotlikoff and Viceira (2008) also study the welfare comparisons of simple defaults, they consider defaults only for portfolio choice.

Classical life-cycle models with a single account are inadequate for studying contribution rules. The two-account setting is therefore particularly important for capturing the liquidity constraint when designing contribution rules. This paper explicitly models the liquid taxable account and illiquid tax-deferred DC plans. Under the two-account setting, the paper first illustrates optimal contribution and investment policies. Various default designs, including constant and age-dependent features, are then presented. The study uses dynamic programming and the Endogenous-Grid Method (Carroll, 2006, 2007) to solve the extended life-cycle model with two accounts.

The paper is organized as follows. Section 2 explains the model setup and the economic environment. Section 3 solves the life-cycle model in order to describe optimal life-cycle saving and investment strategies in an ideal world. We show several age-related patterns regarding asset allocation and consumption strategies over the lifetime. Section 4 explores age-dependent default designs for passive participants in individual-based DC schemes. This paper focuses on the effect of age-dependent designs of contribution and investment defaults, attempting to see how much closer welfare can be pushed to the optimal strategies by using these age-dependent defaults. Section 5 presents the welfare comparisons and policy implications, and Section 6 concludes.

2 Model Setup

2.1 Individual's preferences

This paper assumes that all individuals start working at age 25 ($t = 0$) and retire at 65 ($R = 40$). For simplicity's sake, we assume that the individuals die at age 85 ($T = 60$). During the working period ($1 \leq t \leq R$) the individuals earn stochastic labor income, denoted by Y_t. During the retirement period ($R \leq t \leq T$), the individuals receive no labor income but consume their accumulated

[5] Dammon, Spatt and Zhang (2004) also study optimal portfolio choices with taxable and tax-deferred accounts. They focus on portfolio choices in taxable and tax-deferred accounts due to different tax treatment on dividends, capital gains and interest.

wealth, denoted by W_t. Individuals derive utility over single consumption goods (normalized by price inflation). An individual's preference is captured by the constant relative risk aversion utility function as

$$E\left[\int_{t=0}^{T}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}dt\right]$$

where γ is the risk aversion parameter and β is the subjective discount factor. In the baseline model, we fix $\gamma = 5$, and $\beta = 0.97$, following the life cycle literature.

2.2 Return dynamics

Two financial assets are traded in the market, one risk free and one risky (both in real terms). The real risk free asset offers a fixed real interest rate r. The real price of the risky stock index, E_t, follows geometric Brownian motion with a constant drift. Dividends are reinvested. The aggregate real wage index G_t is stochastic. The uncertain stock returns are potentially correlated with stochastic aggregate wage growth. This contemporaneous correlation is denoted by ρ, where $dZ_{E,t}$ and $dZ_{g,t}$ denote the independent Brownian incremental of stock returns and aggregate wage growth rates. The stock return and wage growth rate dynamics are as follows:

$$dE_t/E_t = \mu dt + \sigma_E\sqrt{1-\rho^2}dZ_{E,t} + \rho\sigma_E dZ_{g,t} \quad (1)$$

$$dG_t/G_t = \overline{g}dt + \sigma_g dZ_{g,t} \quad (2)$$

where μ and $\overline{g}$ are the instantaneous drifts, and σ_E and σ_g are the volatilities of stock returns and wage growth respectively. In the baseline model, we assume $\mu = 6\%$, $\sigma_E = 15\%$, $\overline{g} = 0.8\%$, and $\sigma_g = 4\%$ annualized.

2.3 Labor income and social security

Let t denotes calendar year and t_0 the year of birth, so that $t - t_0$ is the age of the individual under consideration. Following Benzoni, Collin-Dufresne, and Goldstein (2007), we assume that the individuals' real labor income $Y_{t-t_0} = G_tN_{t-t_0}$ can be decomposed into two component, an aggregate wage component, G_t, and an age-dependent idiosyncratic component N_{t-t_0}. The growth rate of aggregate wage component is determined according to eq(2).[6] While the idiosyncratic wage

[6] The aggregate wage growth and equity returns are cointegrated in Benzoni, Collin-Dufresne and Goldstein (2007); the asset allocation to equities may thus be reduced, due to the cointegration effect. Given the focus in this paper on the age-dependent contribution policy rule, we assume that aggregate wage rates and equity returns are correlated, but not cointegrated.

component, N, has an age-dependent drift $f(t-t_0)$ to generate the hump-shape of earnings and normally distributed permanent shocks. The real labor income process is specified as follows:

$$
\begin{aligned}
Y_{t-t_0} &= G_t N_{t-t_0} && (3) \\
\ln N_{t-t_0} &= \ln N_{t-1-t_0} + f(t-t_0) + \sigma_n \eta_t && (4) \\
&= \ln N_{t-1-t_0} + (a_0 + a_1 (t-t_0)) + \sigma_n \eta_t && (5)
\end{aligned}
$$

where $\eta_t \sim i.i.d.N(0,1)$. The starting annual salary at age 25, N_{25}, is normalized to \$20,000. The parameter a_0 and a_1 are set according to the calibration of Benzoni, Collin-Dufresne, and Goldstein (2007) for the high education group ($a_0 = 0.066$ and a_1= -0.0024), and $\sigma_n = 8\%$. Figure 1 shows the quantile distributions of N and G over time.

The old-age social security benefit at age 65, SS_R, is assumed to be a fraction, s, of the final labor income, i.e., $SS_R = s * Y_{R-1}$. Each year, the social security is indexed with the aggregate wage growth, so that $SS_t = G_t SS_R$, for $t > R$. In the baseline model, we consider $s = 30\%$.[7]

2.4 Housing and medical expenditures

From the perspective of integrated retirement saving, retirement provisions should be considered in combination with important expenditures such as housing and Medicare, because these expenditures affect when and how much an individual should save for retirement provisions. As shown in Section 3, the decreasing life-cycle pattern of housing expenditures postpones DC savings in the beginning, but rising medical expenditures towards the end of life promote retirement savings. This paper carefully models the dynamics of housing and out-of-pocket medical expenditures based on the literature. Housing expenditures exhibit a decreasing age profile (Gomes and Michaelides (2005); Amromin, Huang and Sialm (2007)), and medical expenditures exhibit an increasing age profile (Palumbo (1999); Scholz, Seshadri and Khitatrakun (2006)). Both expenditures are modeled as exogenous shocks to the budget process.

We assume that individuals pay off all their mortgages before age 80. The exogenous housing expenditure represents a fraction of labor income during working period, and a fraction of final income during the retirement period. Based on the estimation of Gomes and Michaelides (2005) using PSID data, the ratio of housing expenditure to income has the following age-dependent mean and variances:

$$H_t / Y_t = h_t \sim N\left(\overline{h}(t), \sigma_h^2(t)\right) \tag{6}$$

[7] $s = 30\%$ might overstate the benefit for the high final salary individuals, and understate the benefit for the low final wage individuals.

where the age-dependent mean $\overline{h}\,(age) = h_0 + h_1 * age + h_2 * age^2 + h_3 * age^3$, with $h_0 = 0.71$, $h_1 = -0.035$, $h_2 = 0.00072$, $h_3 = -0.0000049$. Furthermore, uncertainty in the housing expenditure is captured by $\sigma_h = 2\%$, and the correlation between shocks to h_t and income Y_t is set at -0.5. Figure 2 (left panel) depicts the average profile and one simulated scenario of the housing expenditures over the life cycle. Housing expenditures are relatively high for young and mid-age households, but relatively low for retired households, when mortgages are gradually paid off.

Medical expenditure also represents a fraction of labor income during the working period, and a fraction of final income during the retirement period. The ratio of out-of-pocket medical expenditure to income, following the parameterization and estimation results of Scholz, Seshadri and Khitatrakun (2006), is modeled as follows:

$$\ln(M_t/Y_t) = -7.316 + 0.012 * age + 0.00066 * age^2 + \varepsilon_M \tag{7}$$

with $\varepsilon_M \sim N\left(0, \sigma_M^2\right)$ and $\sigma_M = 20\%$. Figure 2 (right panel) depicts the average profile and one simulated scenario of the medical expenditures over the life cycle. Medical expenditures are more costly at advanced ages.

2.5 Tax-Deferred Account and Taxable Account

The tax-deferred account is actually an individual-based DC scheme that is provided by the employer in many countries. Tax advantages, oftentimes combined with employer matching, are used to stimulate savings in the DC schemes. These individual DC schemes are called tax-deferred accounts (TDA), since income tax and dividends tax are exempted or postponed until retirement. In this paper, DC contributions are exempted from a higher income tax, $\tau^y = 30\%$; withdrawals from the DC account are taxable at a lower income tax rate, $\tau^o = 20\%$, when an individual retires. Sometimes, employers will match the contributions made by employees. Such employer matching is a bonus, if the employee contributes. The tax deferral and employer matching provide certain incentives for individuals to save in the tax-deferred DC account. In order to avoid large-scale tax arbitrage, however, the contribution rate is capped at 20%, so that maximally 20% of gross income can be contributed in the DC plan annually. Individuals may thus keep their retirement savings in the tax-deferred account, and may also hold other private wealth in a taxable account.

3 Optimal life cycle strategies

This section studies the optimal life-cycle planning problem of an individual with a tax-deferred account (TDA) and a taxable account (TA), under the realistic economics settings that were used in Section 2. Section 3.1 describes the life-cycle model for an individual with a tax-deferred DC account and a taxable account. Section 3.2 shows the optimal strategies under the tax-benefit-driven setting (the baseline model). Section 3.3 presents the results with additional employer matching.

3.1 Optimization problem with TDA and TA

The optimization problem with a tax-deferred DC account and a taxable account is as follows: The individual optimizes lifetime utility by optimally choosing consumption, contributions to the DC scheme, and asset allocation in both a tax-deferred DC account and a taxable account. The focus here is on when and how much to contribute to the DC scheme. Let m_t denote the contribution rates into the DC pension plan. We assume that one can not withdraw the DC wealth before retirement (that is, $m_t \geq 0$; the contribution rates must be non-negative). In practice, under extreme circumstances early withdrawal from DC account is allowed (but is subject to a 10% penalty cost). In the baseline model of this paper, we assume that borrowing or early withdrawal from the DC savings is not allowed; the DC account is therefore illiquid during the entire working period. The illiquid DC savings make the liquidity constraint potentially more severe.

Let W_t^τ denote an individual's wealth in the taxable account, and W_t^{DC} be the wealth in the tax-deferred DC account. $\widetilde{R}_{t+1}^e = E_{t+1}/E_t$ denotes the total return on equities and $R^f = \exp(r)$ denotes the real risk free rate. The fractions of assets invested in equities are denoted by α_t^τ (for the liquid saving) and α_t^{DC} (for the DC account) respectively. Given the focus in this paper on consumption and saving decisions, we make the simplifying assumption that dividends and capital gains are not taxed, and we further assume that asset allocation in taxable and tax-deferred DC accounts is identical. The descritized optimization problem with TDA and TA can be formalized as follows:

$$V = \max_{\{C_t, \alpha_t^\tau, \alpha_t^{DC}, m_t\}_{t=1}^T} E\left[\sum_{t=1}^T \beta^{t-1} \frac{C_t^{1-\gamma}}{1-\gamma}\right] \tag{8}$$

subject to, the wealth dynamics, before retirement ($1 \leq t < R$), as

$$\begin{aligned}
W_{t+1}^\tau &= (W_t^\tau - C_t - (1-\tau)m_t Y_t)\left(R^f + \alpha_t^\tau\left(\widetilde{R}_{t+1}^e - R^f\right)\right) \qquad (9)\\
&\quad +(1-\tau^y)Y_{t+1} - H_{t+1} - M_{t+1} \\
W_{t+1}^{DC} &= \left(R^f + \alpha_t^{DC}\left(\widetilde{R}_{t+1}^e - R^f\right)\right) \quad \left[W_t^{DC} + m_t Y_t\right] \qquad (10)\\
\text{with } W_1^\tau &= (1-\tau)Y_1, \text{ and } W_1^{DC} = 0 \qquad (11)
\end{aligned}$$

Since the individuals are borrowing constrained, the balances of the two savings must always be non-negative, as in eq(12).

$$
\begin{aligned}
W_t^\tau &\geq 0,\ W_t^{DC} \geq 0 && (12) \\
20\% &\geq m_t \geq 0 && (13)
\end{aligned}
$$

After retirement ($T \geq t \geq R$), DC savings become liquid and available for consumption. The individual thus combines the two savings after deducting the income tax paid on the DC wealth (i.e., $W_R = W_R^\tau + (1-\tau^o)\, W_R^{DC}$). This combined wealth is invested in the taxable account to finance retirement consumption. Formally, the wealth dynamics of savings during the retirement period can be described as follows:

$$
W_{t+1} = \left[\alpha_t^\tau \widetilde{R}_{t+1}^e + (1-\alpha_t^\tau)\, R_t^f\right] [W_t - C_t] + (1-\tau^o) SS_{t+1} - H_{t+1} - M_{t+1} \tag{14}
$$

Furthermore individuals are short-sales constrained, which implies that

$$
1 \geq \alpha_t^\tau \geq 0 \ ,\ 1 \geq \alpha_t^{DC} \geq 0 \tag{15}
$$

The problem has no analytical solution, due to portfolio constraints. We use the dynamic programming principle, together with the Endogenous-Grid Method (Carroll (2006, 2007)), to solve the extended two-account life-cycle model numerically. Appendix A describes the solution technique.

3.2 The life cycle saving and investing profiles

This subsection shows the optimal life-cycle profiles of individuals with both taxable and tax-deferred DC accounts. The distribution of the life-cycle profiles is characterized by 5%, 50% and 95% quantiles.

Figure 3 (left-hand panel) shows the portfolio allocation in stocks over the lifetime. Recall that we assume that the asset allocation in Taxable and Tax-deferred DC accounts is identical. As explained by Campbell and Viceira (2002), the portfolio allocation to equities generally decreases over time, due to the leverage effect of human capital[8]. Here we confirm this finding under the two-account setting. Figure 3 (right-hand panel) shows the consumption profile, which is slightly increasing over time, due to the no-borrowing constraint and the assumed time preference.

Figure 4 (left-hand panel) shows the contribution rate profile. Strikingly, young individuals make zero contribution to the Taxable DC account for the first ten years of their working lives.

[8] The human capital is the present value of the future incomes.

The contribution rate rapidly increases from mid-age, eventually reaching the ceiling of 20% a few years before retirement. Figure 4 (right-hand panel) shows the wealth accumulation in the TA and TDA accounts. Early in life, young individuals accumulate moderate wealth only in the taxable account as precautionary savings against various background risks (such as income uncertainties and expenditure uncertainties). DC wealth starts to grow rapidly after mid-age, when individuals are not liquidity constrained, their retirement-saving motives become stronger, and tax benefits are much higher. The growth of TDA account is boosted by transferals of wealth from TA to TDA account via higher contributions around mid-age.

3.3 With Employer Matching

This subsection considers a variant of the baseline model, including employer matching. As reported by Gomes, Michaelides and Polkovnichenko (2006), about half of all companies do not provide employer matching; regarding the other half of the companies (which do match employee contributions), employer matching may in practice take various forms. Here, we consider a common practice: the employer matches 100% of the employee's contribution up to a limit of 6%. However, the total contribution should not exceed 20%. Let m_t^{DC} denote the total contribution rates into the DC pension plan.

$$m_t^{DC} = \min\left(m_t + \min\left(m_t,\ 6\%\right),\ 20\%\right) \tag{16}$$

The DC wealth dynamic before retirement ($t < R$) is slightly modified to

$$W_{t+1}^{DC} = \left(R^f + \alpha_t^{DC}\left(\widetilde{R}_{t+1}^e - R^f\right)\right)\left[W_t^{DC} + m_t^{DC} Y_t\right] \tag{17}$$

Figures 5 and 6 show the life-cycle profiles of portfolio choice, contribution rates, consumption and wealth accumulations, in the DC scheme with employer matching. Most of the profiles strongly resemble those in the baseline setup. The main difference is in the contribution rates profile, Figure 6 (left-hand panel), which exhibits a clear ladder shape, increasing over the life cycle. The contribution rate is zero for the first nine years of working life. Young individuals are liquidity constrained because they face relatively low incomes but high housing expenditures. They leave the employer's matching offer on the table. Between the ages of 35 and 45, the contribution rate rises quickly to 6%, earning the maximum amount of employer matching. After age 45, it increases to 14%, so that together with the employer matching the total contribution reaches the 20% ceiling.

3.4 Welfare evaluation

We use the certainty equivalent consumption (CEC) as the welfare measure, which can be easily derived from the following equality:

$$V = \sum_{t=1}^{T} \beta^{t-1} \frac{CEC^{1-\gamma}}{1-\gamma} \tag{18}$$

For easy interpretation, we divide the CEC by the first gross labor income Y_{25}. The welfare obtained by implementing the optimal strategies is as follows. In the baseline model (i.e. no employer matching is provided), the optimal CEC reaches 0.688 (in units of first-year income). In the variant in which employer matching is offered, the CEC is higher, reaching 0.702 units of first-year income. The welfare obtained in this section sets upper limits for the comparison across various defaults in the next section.

4 Default designs

The previous section discussed optimal strategies. This section discusses default designs for tax-deferred DC accounts. Given the dramatic impact of defaults on retirement saving behavior, the default should be as good as possible. Is the current popular default design the best possible design, in welfare terms? If not, can we design a better default that may achieve a nearly optimal welfare outcome? In order to design the optimal default options, we model the cases in which individuals stay with the defaults throughout the life cycle, as if the default DC plan had been mandatory.

We consider four specifications of default designs. The main characteristics of these default designs are summarized in the following table.

	Default #1	Default #2	Default #3	Default #4
portfolio	constant	age-dependent	constant	age-dependent
contribution	constant	constant	age-dependent	age-dependent

Table 1: Overview of the default designs

Under each given default design specification, individuals in the model optimize their objective (8), subject to the budget constraints in TA and TDA (9, 10), and no-borrowing constraint (12) and no-short selling constraint (15). A detailed solution methodology is given in Appendix B. Section 5 compares the welfare costs of different default designs relative to the optimal strategies, and investigates whether the age-dependent default is able to help reduce the welfare cost.

4.1 Default #1: constant contribution rate and constant portfolio

This default features a constant contribution rate and a constant portfolio choice throughout the (working) life. These constant features resemble the current standard default options, which typically fix the contribution rate at 4% and invest in money market accounts, without further adjustment.

Individuals under this default design optimize utility over life time consumption, by choosing the consumption level in each period, and a constant saving rate m and a constant portfolio α at the beginning of their careers. Effectively, the optimal strategies in Section 3 are being replaced here by constants (i.e., $\alpha_t^{DC} = \alpha_t^{\tau} = \alpha$, and $m_t^{DC} = m$ (with $0 \leq m \leq 20\%$)). As before, borrowing and short sales are not allowed, which implies $0 \leq W_t^{\tau}$, and $0 \leq \alpha \leq 1$ respectively. The problem is solved numerically, by first optimizing the welfare level for given levels of contribution rate, and then determining the best contribution rate. Details of the solution procedure are given in Appendix B.

Table 2 shows, for a given flat contribution rate, the corresponding optimal fixed portfolio choice and the welfare level under the specification of Default #1. It shows that if the contribution rate is fixed at 4%, then the best portfolio would involve investing 82.5% in equities throughout the life cycle. This provides an annual certainty equivalent consumption of 0.654 units of first-year labor income; Or, relative to the fully optimal TDA / TA benchmark (i.e., the upper limit) $CEC^{TDATA} = 0.6885$, of the optimal welfare level. Table 2 also shows the results for two additional asset allocations: one with 50% in equities and 50% in risk-free assets, and another case with 100% in risk-free assets (as in the current defaults). The 50%/50% asset mix reduces welfare levels slightly, for each given contribution rate. But the 100% risk-free investment strategy is clearly sub-optimal, resulting in a substantial welfare loss. The current default with a fixed contribution rate of 4% and investment in money market accounts gives maximally 92.7% of the optimal welfare benchmark level.

When comparing the CEC across different contribution rates, we find that (for the assumed amount of tax benefit, and without employer matching) zero contribution is the best outcome. Zero contribution means that the tax benefit for an individual in our baseline model is not large enough to compensate for the liquidity loss when he or she was young. Because the individual cannot borrow from his or her future labor income, the contribution rate for the early working period is too high, so that the liquidity constraint becomes more severe.

Table 3 shows the variant with employer matching. In this case, the employee contributes m percent of income, and the employer matches the contributions with a cap at 6%, as shown in (16). Table 3 shows that for a contribution rate of 4%, the optimal asset mix is 82.5% in equities. This gives an annual certainty equivalent consumption of 0.664 units of first-year labor income;

relative to the fully optimal TDA / TA benchmark $CEC^{TDATA} = 0.702$, it gives 94.6% of the optimal welfare level. When comparing the CEC across different contribution rates, it seems that the employer matching together with the tax deferrals stimulate DC savings. Our model suggests that a small but positive contribution rate of 1% is beneficial for the individual.

4.2 Default #2: constant contribution rate and age-dependent portfolio

In a departure from Default #1, we relax the restriction of a constant portfolio throughout life, replacing it with an age-dependent strategy. One popular rule describes a linear relationship between age (denoted by $t - t_0$) and the share of risky assets as

$$\omega_t = \max[0, \quad \min[f_0 + f_1 (t - t_0), 1]] \tag{19}$$

The individual follows the age-dependent allocation rule, optimizing his or her lifetime utility of consumption by choosing consumption in each period, and choosing the constant saving rate m and the parameter (f_0, f_1) at the beginning of his career. As before, neither borrowing nor short sales are allowed, which implies $0 \leq \omega_t \leq 1$ and $0 \leq m \leq 20\%$.

Figure 7 shows the optimized age-dependent portfolio rule corresponding to a given contribution rate of 4%. This life-cycle fund resembles the age profile of the optimal portfolio choices shown in Figure 3. Figure 8 shows the resulting life-cycle quantile profiles of consumption (left-hand panel) and the wealth accumulation in both TA and TDA accounts (right-hand panel). Compared to the optimal strategies in Section 3, consumption in this default is slightly lower for the young, and slightly higher for retirees, because of the constant contribution rate of 4%. Since the contribution rate is fixed at 4%, the accumulated DC wealth at the end of the working period is substantially smaller than the wealth accumulated in the optimal situation.

4.3 Default #3: age-dependent contribution rate and constant portfolio

Starting from Default #1, we relax the restriction of a constant contribution rate throughout life, but still keep the constant portfolio in both TA and TDA. The TDA is thus characterized by the age-dependent contribution rate and a constant portfolio choice. A simple age-dependent contribution rate may depend linearly on age as follows:[9]

[9] An alternative modeling of the age-dependent contribution rate may include the quadratic term in age. For example

$$m_t = d_0 + d_1 * (t - t_0) + d_2 * (t - t_0)^2 \tag{20}$$

This alternative modeling is able to capture the possible hump shape of the optimal contribution rates as seen in Section 3. This specification will be investigated in the future research.

$$m_t = \max[0, \quad \min[d_0 + d_1(t - t_0), \quad 20\%]] \tag{21}$$

Suppose that the individual follows the age-dependent contribution rule, optimizing his or her lifetime utility of consumption by choosing consumption in each period, and choosing the constant portfolio α and the parameter (d_0, d_1) for the contribution schedule at the beginning of his career. As before, borrowing and short sales are not allowed, which implies $0 \leq \alpha \leq 1$ and $0 \leq m_t \leq 20\%$.

Figure 9 shows the optimized age-dependent contribution rule and portfolio choice (which is 85%) for Default #3. The age-dependent contribution rule resembles the optimal life-cycle contribution profile that was shown in Figure 3 (Section 3). Following this default, individuals do not contribute to the DC plan during the first 12 years of their working lives, because of liquidity constraints. Between the ages of 38 and 60, the contribution rates are non-zero and increase linearly with age, by about 0.8% per year. Between the ages of 60 and 64, individuals are no longer liquidity constrained; the maximum contributions are therefore optimally chosen, driven by the tax benefits. Figure 10 shows the resulting life-cycle quantile profiles of consumption (left-hand panel) and wealth accumulation in both TA and TDA accounts (right-hand panel). We see that DC wealth accumulates rapidly after mid-age.

4.4 Default #4: Age-dependent contribution rate and portfolio policies

Default #4 is a combination of Defaults #2 and #3. It specifies an age-dependent contribution rule and an age-dependent portfolio rule, i.e.,

$$m_t = \max[0, \quad \min[d_0 + d_1(t - t_0), \quad 20\%]] \tag{22}$$

$$\omega_t = \max[0, \quad \min[f_0 + f_1 (t - t_0), 1]] \tag{23}$$

Figure 11 shows the optimized age-dependent contribution rate (left-hand panel) and age-dependent portfolio rule (right-hand panel) of Default #4, assuming no employer matching. There is thus no employer matching provided. The age-dependent contribution rate profile is very close to that obtained under Default #3 in Figure 9. The individual starts contributing to the tax-deferred DC scheme after the age of 35, and increases the contribution rate each year until the age of 55, when the contribution cap is reached. The age-dependent portfolio profile shows a similar decreasing pattern, where equity exposure starts to decline from 100% around age 50, until 60% at the end of life. The resulting consumption and wealth accumulation profiles are very close to the ones shown in Figure 4, as in the optimal benchmark setup.

Figure 12 (left-hand panel) shows the optimized age-dependent contribution rate of Default #4 when employer matching is provided. The solid line depicts the contribution rate made by the employee, and the dashed line depicts the total contribution rate together with the employer matching. The individual starts contributing to the tax-deferred DC scheme after age 35, and increases the contribution rate each year until age 50, when the total contribution rate reaches the ceiling of 20%. The age-dependent portfolio profile shows a similar decreasing pattern, but with slightly higher exposure to equities. The equity exposure starts to decline from 100% around the age of 55, until it reaches 70% at the end of life.

5 Welfare comparisons

5.1 Baseline model

Table 4 summarizes the main findings of this paper, comparing the optimal strategies with the current default design, and the step-by-step improvements to it. The first row ("TDA & TA") reports the certainty equivalent consumption (CEC) of the optimal strategies, which sets an upper limit of the welfare level. This welfare measure is expressed in units of first-year gross income. The second row ("Current Default") reports the CEC obtained under the current default setting of 4% contribution rate and zero equity exposure ($\alpha = 0$). The current default maximally reaches 92.7% of the optimal welfare benchmark.

Starting from the current situation, we first improved the asset allocation to a portfolio with 50% in equities, while keeping the contribution rate at 4%. This step improved welfare from 92.7% to 94.8% of the optimal level, as in the third row. When we further improved the asset allocation to an optimal level of 82.5% in equities, keeping the contribution rate at 4%, welfare improved slightly to 95% of the optimal benchmark, as reported in the fourth row. Then, we improved the asset allocation further by choosing an optimal life-cycle fund, using the specification of Default #2. This step improved welfare to 95.2%, as reported in the fifth row. Note that the completely risk-free asset allocation is clearly sub-optimal for a reasonable risk-averse individual ($\gamma = 5$). Having a naive portfolio (e.g., 50% / 50%) improves welfare by 2%. Having an optimal age-dependent portfolio further enhances welfare only slightly (by another 0.2%).

Now, instead of improving the asset allocation, we replaced the flat contribution rate by an optimal age-dependent contribution rate (as specified in Default #3), with a portfolio investing 85% in equities. This step improved welfare by 4%, up to 99.2% of the optimal welfare level, as reported in the seventh row. Compared to the current default, the total welfare gain amounts to 2.7 times first-year income over an adulthood of 60 years.

In the last row, when both contribution- and portfolio defaults are age-dependent (as in Default #4), welfare comes very close to the optimal level (i.e., reaching 99.4% of the optimal level). Similarly, the welfare improvement obtained from the age-dependent portfolio rule above the optimal fixed mix is small (about 0.2%).

Another way to improve the current defaults would involve first changing the contribution rates to an age-dependent rule, while keeping the asset allocation in risk-free assets, i.e., $\alpha = 0$. The CEC immediately improves to 0.668 from 0.638, as given in the sixth row. Improving the contribution policy alone increased welfare from 92.7% to 97.1% of the optimal level, while improving asset allocation alone increased welfare from 92.7% to 95.2%. We see that contribution policy plays a greater role in improving welfare than portfolio choice does.

Figure 13 shows additional results for the sixth and seventh rows (Default #3). The figure depicts the two age-dependent contribution rate rules that correspond to two different asset allocations (i.e., 100% risk free investment vs. the optimal asset mix of $\alpha = 85\%$, based on the baseline model assumptions ($\gamma = 5$, $\beta = 0.97$). The two contribution defaults are close to each other, which means that the optimal age-dependent contribution rule is robust with respect to portfolio mix. The difference in contribution rates is small, maximally 2%. Figure 13 shows that slightly higher contribution rates are necessary to compensate for the low equity exposure (the lower expected returns). The resulting welfare loss is 2%, as reported in Table 4 (sixth and seventh rows).

The variants of DC schemes with employer matching give very similar welfare results, as reported in Table 5. The findings observed from the baseline model still hold in the setting with employer matching.

Our main findings are as follows. First, the simple age-dependent contribution rule and appropriate investment strategies can achieve nearly optimal welfare level. We find potentially large economic welfare gains by following the simple age-dependent contribution rule above the current standard default design. Using the fully optimal strategies as welfare benchmark, the current default design delivers maximally 92.7% of welfare relative to the optimal welfare level. The proposed age-dependent contribution and investment default design, however, delivers up to 99.4% of the optimal welfare level. Compared to the current defaults, the age-dependent defaults lead to a 7.2% increase of the certainty equivalent consumption per year (which is 0.046 units of first-year labor income). Over 60 years (in adulthood), the welfare gain amounts to 2.78 times first-year labor income.

Second, we find that the contribution (or saving) choice has a larger impact on welfare than the portfolio choice does. Compared to the current default (with $\alpha = 0$, $m = 4\%$), improving the contribution policy alone (keep $\alpha = 0$) increases welfare from 92.7% to 97.1% of the optimal level; while improving the asset allocation alone (with $m = 4\%$) increases welfare from 92.7% to 95.2%.

Another example shows a similar result: Replacing the optimal fixed-asset mix by an optimal life-cycle fund, welfare increased only by 0.2%. The life-cycle literature has focused mainly on the importance of portfolio choices. Our study shows that setting the contribution (or saving) correctly is more important, in welfare terms.

5.2 Sensitivity analyses

Also of interest is the sensitivity of the age-dependent contribution and investment rules with respect to different model assumptions, e.g. risk and time preferences, income profile, life expectancy, etc. If the age-dependent default rules are not very sensitive to the assumptions, then the defaults are applicable for heterogeneous participants. Otherwise, we would do better to tailor the defaults by first characterizing the participants by way of questionnaires.

5.2.1 Risk and time preference

What are the optimal strategies for a $\gamma = 8$ individual? Figure 14 and 15 show the age profiles of the optimal contribution rates, optimal portfolio choices, and the resulting consumption and wealth accumulation. The optimal welfare for the $\gamma = 8$ investor is $\text{CEC}^{TDATA,\gamma=8} = 0.582$. The $\gamma = 8$ individual starts contribution after age 30 (which is 5 years earlier than a $\gamma = 5$ individual does as in Figure 3), and then gradually increases the contribution rate till the maximal level. The portfolio choice is also more conservative since mid-age, where equity exposure is 20-30% lower as compared to $\gamma = 5$.

Figure 16 compares the optimal age-dependent contribution rate for a more risk-averse individual ($\gamma = 8$) with that of the benchmark $\gamma = 5$ individual. The age-dependent contribution rule in Figure 16 (according to Default #3) for the $\gamma = 8$ individual approximates the age profile of the optimal contribution rates in Figure 14. The corresponding asset mix (according to Default #3) for the $\gamma = 8$ individual consists 55% in equities, compared to 85% of equities investments preferred by the $\gamma = 5$ individual. We fix other parameters according to the baseline model. We observe that the contribution rule for the more risk-averse individual is 4% higher than that for the $\gamma = 5$ individual. Also, the optimal strategy suggests that the more risk averse individual should start contributing 5 years earlier than the $\gamma = 5$ individual does.

What happens if a $\gamma = 8$ individual steps into a scheme in which the age-dependent contribution rule is designed for $\gamma = 5$ individuals, but the asset allocation is fixed at the right level (i.e., 55% in equities)? The resulted CEC in this case is 0.571, which is 98.1% of the optimal level. The welfare loss is about 2%, compared to the optimal welfare level. The cumulative welfare loss amounts to 0.6 unit of first-year salary over a lifetime.

With regard to sensitivity to the time preference parameter β, our results indicate that both contribution rate and asset allocation rules are rather insensitive to different values for β, e.g. $\beta = 0.95, 0.97, 0.99$. To save space, the results are not shown here.

To conclude, we find that the age-dependent contribution rate should be higher and that asset allocation should be more conservative for a more risk-averse individual. The welfare cost for a more risk-averse individual to follow a contribution rule that is designed for a less risk-averse individual might be sizable. It would therefore be better to first use a questionnaire to characterize individual risk preferences and then to design the age-dependent defaults for each group.

5.2.2 Wage earning profile

The baseline model assumes an individual with a steeper upward-sloping earning profile, which might be the case for higher educated individuals, for example. The average earning profile for lower educated individuals is flatter. How does the flatter earning profile affect the default designs? We set parameters a_0 and a_1 according to the calibration of Cocco, Gomes and Maenhout (2005) for the low education group, while keeping other parameters in line with those of the baseline setup. Figure 17 shows the optimal contribution and investment strategies for individuals with flat earning profile, as predicted by TA/TDA model. We see that the portfolio strategy is very close to that of a steeper earning individual (in Figure 3). However the contribution strategy starts about five years earlier and increases more gradually than the steeper earning case.

The age-dependent contribution and portfolio default rules mimic the optimal strategies closely, which are shown in Figure 18. We notice that, the age-dependent contribution rule for flat wage earners (e.g. lower educated individuals) can start a few years earlier and gradually increase over time.

5.2.3 Life expectancy

The baseline model has assumed a life span of 85 years, with 40 years of working and 20 years of retirement. What if the life span is increased to 90 years, with 40 years of working and 25 years of retirement? Figure 19 (left panel) shows the optimal contribution strategy for the case with prolonged retirement period. The optimal DC contribution starts around age 35, which is the same as the baseline result, to avoid the liquidity constrained period. However, the contribution rates increase more rapidly afterwards. This is because, the individual has to save more to finance the longer retirement period. Figure 19 (right panel) depicts the age-dependent contribution default rule, which is similar as to the baseline result in Figure 11 (left panel), but now with a steeper slope. Therefore, when designing the age-dependent defaults, it is important to take the updated (population) life expectancy into account. The current uniform default has no concern about life expectancy at all, which is very inappropriate. The optimal portfolio strategy and consumption profiles are very similar to the baseline results, hence the figures are not shown here.

6 Conclusions

Given the dramatic impact of defaults in individual DC schemes, this paper investigates whether or not, and to what extent, we can improve welfare by changing the default design. Potentially large economic welfare gains are found by following the simple age-dependent contribution rule above the current standard default design. Using the fully optimal strategies as a welfare benchmark, the current default design delivers only 92.7% of welfare relative to the optimal welfare level. The age-dependent contribution and investment default design, however, delivers up to 99.4% of the optimal welfare level. In terms of certainty-equivalent consumption, the age-dependent default leads to a 7.2% increase in annual consumption. The simple age-dependent contribution and investment rules can thus achieve a nearly optimal welfare level.

Second, the paper finds that the contribution (or saving) choice has a greater impact on welfare than the portfolio choice does. Compared to the current defaults (with flat contribution rate of 4% and fully risk-free investment), improving the contribution policy alone increases welfare from 92.7% to 97.1% of the optimal welfare level. Improving asset allocation alone, however, increases welfare only from 92.7% to 95.2%. Here we show that setting the contribution (or saving) level appropriately is more important, in welfare terms. The life-cycle literature has focused mainly on portfolio choices. We find, however, that the contribution rule plays a more important role in improving welfare.

Regarding the sensitivity of the defaults and the related implementation issues, the paper shows that it is better to use a questionnaire to categorize risk preferences and then to design age-dependent defaults for each group. Furthermore, wage earning profiles, which often associated with attained education, have an impact on the default design; It is important to take the updated (population) life expectancy into account when designing the age-dependent contribution defaults.

As for policy implications, the age-dependent contribution and investment rules can be recommended as default. Our finding is consistent with the auto-save features encouraged by the Pension Protection Act of 2006. This paper contributes to the life-cycle literature and the DC industry by characterizing the optimal age-dependent contribution and investment rules for DC participants. We show that contribution policy plays a bigger role than portfolio choice does in improving welfare.

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A Solution method of Section 3, with TA and TDA

We use the dynamic programming and the Endogenous-Grid Method (Carroll (2006, 2007)) to solve the extended life cycle model with two accounts. In the appendix, the consumption (C_t), wealth ($W_t^{\tau}, W_t^{DC}, W_t$) and expenditures ($H_t, M_t$) are all normalized by income Y_t. The normalized variables are denoted in small letters throughout. The state variables in this model are time and two wealth processes.

A.1 Solving the retirement period

First, we solve the retirement periods ($R \leq t \leq T$). Upon retirement, the taxable savings and tax-deferred DC savings are combined into one savings, because both are freely accessible for consumption. The combined wealth is denoted by $W_R = W_R^{\tau} + (1-\tau^o)\, W_R^{DC}$. A lower tax rate τ^o is applied to the DC wealth at the retirement date. Formally, the normalized objective function and the normalized budget constraint, for the retirement period $R \leq t \leq T$, are

$$
\begin{aligned}
v\left(w_t\right) &= \max \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t\left[\left(R_{t+1}^G R_{t+1}^N\right)^{1-\gamma} v\left(w_{t+1}\right)\right] & (24)\\
s.t.\ w_{t+1} &= \left(w_t - c_t\right)\left[R^f + \alpha_t\left(\widetilde{R}_{t+1}^e - R^f\right)\right]\left(R_{t+1}^G\right)^{-1} + (1-\tau^o)ss - h_{t+1} - med_{t+1} & (25)\\
w_t^{\tau} &\geq c_t; \quad 0 \leq \alpha_t \leq 1 & (26)
\end{aligned}
$$

The optimal consumption and portfolio choice $\omega_t^*(a)$, $c_t^*(a)$ and the endogenous optimal wealth $w_t^*(a)$ (for $a = \{a_j\}_{j=1}^J$) can be found by following procedure. The procedure starts by defining a new variable, $a_t = w_t - c_t$, as the after-consumption-wealth. Construct a grid of $a_t = \{a_j\}_{j=1}^J$. Now we solve for the optimal consumption and portfolio policies for each given a_j. The first order conditions w.r.t. α_t and c_t are

$$
\begin{aligned}
0 &= \beta E_t\left[v'\left(w_{t+1}\right)\left(\widetilde{R}_{t+1}^e - R^f\right)\left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right] & (27)\\
u'\left(c_t\right) &= \beta E_t\left[v'\left(w_{t+1}\right)\left(R^f + \alpha_t\left(\widetilde{R}_{t+1}^e - R^f\right)\right)\left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right] & (28)
\end{aligned}
$$

The Envelope theorem implies that $u'\left(c_t\right) = v'\left(w_t\right)$, since

$$
v'\left(w_t\right) = \beta E_t\left[v'\left(w_{t+1}\right)\left(R^f + \alpha_t\left(\widetilde{R}_{t+1}^e - R^f\right)\right)\left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right] \quad (29)
$$

Replace $v'\left(w_{t+1}\right)$ by $u'\left(c_{t+1}\right)$ in the two first order conditions, we have

$$0 \quad = \quad \beta E_t \left[u' \left(c^*_{t+1}[w_{t+1}] \right) \left(\widetilde{R}^e_{t+1} - R^f \right) \left(\widetilde{R}^G_{t+1} \right)^{-\gamma} \right] \tag{30}$$

$$c^*_t \left(a_t \right) \quad = \quad I_u \left(\beta E_t \left[u' \left(c^*_{t+1}[w_{t+1}] \right) \left(R^f + \alpha^*_t \left(a_t \right) \left(\widetilde{R}^e_{t+1} - R^f \right) \right) \left(\widetilde{R}^G_{t+1} \right)^{-\gamma} \right] \right) \tag{31}$$

where $c^*_{t+1}[w_{t+1}]$ as the optimal consumption policy at time $t+1$, and $I_u(\centerdot)$ denotes the inverse function of $u'(c_t)$. Using any numerical solver, Eq (30) will give the optimal portfolio weight $\alpha^*_t(a_t)$ for any given amount of investment a_t. Because of the borrowing and short-selling constraints, we then impose the restriction $0 \leq \alpha^*_t \leq 1$. Then, eq (31) will give the corresponding consumption $c^*_t(a_t)$ for any given amount of investment a_t. Finally, the optimal wealth process is endogenously determined by $w^*_t = c^*_t(a_t) + a_t$. The advantage of this method is that the numerical search is only needed once in solving $\alpha^*_t(a_t)$, while $c^*_t(a_t)$ can be directly obtained from (31).

Hence we obtain the corresponding policy function $c^*_t(w_t)$ for $t \geq R$. The value obtained at time R can be decompose into two terms $v(w_R) = u(w_R) K(T-R)$ where $K(t) = \frac{1}{F}(1 - \exp(-Ft))$, and $F = \frac{\delta - r(1-\gamma)}{\gamma} - \frac{(1-\gamma)(\mu - r)^2}{\gamma^2 \sigma^2}$. [10]

Then, we solve the working periods ($1 \leq t < R$). But before moving backward into the working period, we need to map the vector of the single state variable $\{w^*_R(j)\}_{j=1}^J$ into two state variables with $w^\tau_{i,j} = w_R(j)\frac{i}{I}$, $w^{DC}_{i,j} = w_R(j)\frac{I-i}{I}(1-\tau^o)^{-1}$, with $i = 0, 1, ..., I$. This step is because both taxable and DC savings are state variables for the working period optimization problem. In a similar way, we map the vector $\{c^*_R(j)\}_{j=1}^J$ into a matrix with $c^*_{ij} = c^*_R(j)$ for $\forall i = 0, 1, ..., I$. Hence we obtain the corresponding policy function $c^*_{R,i,j}\left(w^\tau_{R,i,j}, w^{DC}_{R,i,j}\right)$ at time $t = R$.

[10] After retirement, since there is no labor income nor social security available for the individual, the model is the classical Merton (1969) model. Without any portfolio constraint, the value function time time R has the following expression $v(w_R) = \frac{w^{1-\gamma}}{1-\gamma} K(T-R)$, as shown in Merton (1969) and Munk (2007). With portfolio constraint (e.g. no-borrowing constraint), Grossman and Vila (1992, proposition 3.2.) show that the value function has the expression $v(w_R) = \frac{w^{1-\gamma}}{1-\gamma} \overline{K}(T-R)$, with $\overline{K}(t) = \exp\left(r + \mu k - A\sigma^2 k^2/2\right)(1-A)t$.

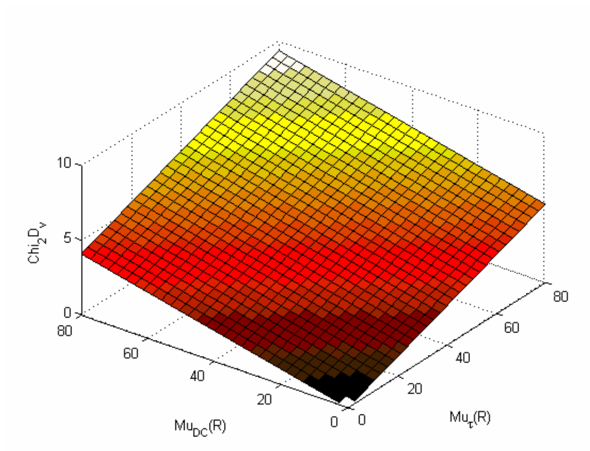

Figure: optimal consumption policy
$c_R^*\left(w_R^\tau, w_R^{DC}\right)$ at time $t = R$.

A.2 At time $t = R - 1$

At time $t = R - 1$, the individual has two accounts, TA and TDA. Therefore the normalized value function has two state variables, $w_{R-1}^\tau, w_{R-1}^{DC}$. The individual has to decide how much to consume, c_{R-1}, out of the TA wealth, and where to locate his savings among the two accounts (by choosing m_{R-1}^{DC}); and finally the individual has to decide the right portfolio's $(\alpha^\tau, \alpha^{DC})$ for both TA and TDA.

Formally, the normalized value function and the normalized budget constraint are

$$\varpi\left(w_{R-1}^\tau, w_{R-1}^{DC}\right) = \max_{c_{R-1},\alpha^\tau,\alpha^{DC},m_{R-1}^{DC}} \frac{c_{R-1}^{1-\gamma}}{1-\gamma} + \beta E_{R-1}\left[\left(R_R^G R_R^N\right)^{1-\gamma} v\left(w_R\right)\right] \tag{32}$$

s.t. the budget constraint

$$w_R = w_R^\tau + (1-\tau^o)\, w_R^{DC} \tag{33}$$

$$w_R^\tau = \left(w_{R-1}^\tau - c_{R-1} - (1-\tau^y) m_{R-1}^{DC}\right)\left[R^f + \alpha_{R-1}^\tau\left(\widetilde{R}_R^e - R^f\right)\right]\left(R_R^G R_R^N\right)^{-1} + (1-\tau^o)ss - h_R \tag{34}$$

$$w_R^{DC} = \left(w_{R-1}^{DC} + m_{R-1}^{DC}\right)\left[R^f + \alpha_{R-1}^{DC}\left(\widetilde{R}_R^e - R^f\right)\right]\left(R_R^G R_R^N\right)^{-1} \tag{35}$$

and the non-negative constraint (i.e. no borrowing) in both accounts

$$w_{R-1}^\tau - c_{R-1} - (1-\tau^y) m_{R-1}^{DC} \geq 0, \text{ and } m_{R-1}^{DC} \geq 0 \tag{36}$$

Notice that the value function is changed from with one state variable, $v(w_R)$, to the value function with two state variables $\varpi\left(w^{\tau}_{R-1}, w^{DC}_{R-1}\right)$.

Follow Carroll's idea, we define two new wealth variables, namely the amount available for investment in the taxable account $a^{\tau}_{R-1} = w^{\tau}_{R-1} - c_{R-1} - (1-\tau^y)m^{DC}_{R-1}$, and the amount available for investment in the DC account $a^{DC}_{R-1} = w^{DC}_{R-1} + m^{DC}_{R-1}$. We then choose a non-negative 1-D grid to discretize $a^{\tau}_{R-1} = \{a^{\tau}_j\}^{J}_{j=1} \geq 0$, and do the same for $a^{DC}_{R-1} = \{a^{DC}_h\}^{H}_{h=1} \geq 0$.

A.2.1 Optimize portfolio's

The first step is to compute the optimal portfolio strategies for both accounts. The first order conditions w.r.t. α^{τ}_{R-1} and α^{DC}_{R-1} are

$$
\begin{aligned}
0 &= \beta E_{R-1}\left[u'\left(w_R\left(a^{\tau}_j, a^{DC}_h\right)\right)\left(\widetilde{R}^e_R - R^f\right)\left(R^G_R R^N_R\right)^{-\gamma}\right] && (37)\\
0 &= \beta E_{R-1}\left[u'\left(w_R\left(a^{\tau}_j, a^{DC}_h\right)\right)\left(\widetilde{R}^e_R - R^f\right)\left(R^G_R R^N_R\right)^{-\gamma}\right](1-\tau^o) && (38)
\end{aligned}
$$

Notice that the portfolio's are function of a^{τ} and a^{DC}. We need to determine α^{τ}_{R-1} and α^{DC}_{R-1} for each combination of $\left(a^{\tau}_j, a^{DC}_h\right)$. One special case is $\omega^{\tau}_{R-1} = \omega^{DC}_{R-1} = \omega_{R-1}$. Assuming $\omega^{\tau}_{R-1} = \omega^{DC}_{R-1} = \omega_{R-1}$ for any combination of $\{a^{\tau}_j, a^{DC}_h\}$. First, for a given vale of investable wealth in TA and TDA, we simulate the next period total wealth for certain portfolio choice α :

$$
\begin{aligned}
\widetilde{w}^{\tau}_R\left(a^{\tau}_j\right) &= a^{\tau}_j\left[R^f + \alpha\left(\widetilde{R}^e_R - R^f\right)\right]\left(R^G_R R^N_R\right)^{-1} + (1-\tau^o)ss - h_R && (39)\\
\widetilde{w}^{DC}_R\left(a^{DC}_h\right) &= a^{DC}_h\left[R^f + \alpha\left(\widetilde{R}^e_R - R^f\right)\right]\left(R^G_R R^N_R\right)^{-1} && (40)
\end{aligned}
$$

The optimal portfolio ω^*_{R-1} is the one that solves the following equation based on the FOC w.r.t α

$$
\begin{aligned}
0 &= \beta E_{R-1}\left[v'\left(\widetilde{w}_R\right)\left(\widetilde{R}^e_R - R^f\right)\left(R^G_R R^N_R\right)^{-\gamma}\right] && (41)\\
&= \beta E_{R-1}\left[u'\left(c_R\left(\widetilde{w}^{\tau}_R, \widetilde{w}^{DC}_R\right)\right)\left(\widetilde{R}^e_R - R^f\right)\left(R^G_R R^N_R\right)^{-\gamma}\right] && (42)
\end{aligned}
$$

where $v'\left(\widetilde{w}_R\right) = u'\left(c_R\left(\widetilde{w}^{\tau}_R, \widetilde{w}^{DC}_R\right)\right)$, and $c_R\left(\widetilde{w}^{\tau}_R, \widetilde{w}^{DC}_R\right)$ is obtained by interpolating the previously constructed policy function $c^*_{R,i,j}\left(w^{\tau}_{R,i,j}, w^{DC}_{R,i,j}\right)$.

A.2.2 Optimize consumption

The second step is to calculate the optimal consumption for each combination of $\{a_j^{\tau}, a_h^{DC}\}$. We know the first order conditions w.r.t. c_{R-1} is

$$\begin{aligned} u'\left(c_{R-1}\right) &= \beta E_{R-1}\left[v'\left(\widetilde{w}_R\right)\left(R^f+\alpha_{R-1}^*\left(\widetilde{R}_R^e-R^f\right)\right)\left(R_R^G R_R^N\right)^{-\gamma}\right] && (43)\\ &= \beta E_{R-1}\left[u'\left(c_R\left(\widetilde{w}_R^{\tau}, \widetilde{w}_R^{DC}\right)\right)\left(R^f+\alpha_{R-1}^*\left(\widetilde{R}_R^e-R^f\right)\right)\left(R_R^G R_R^N\right)^{-\gamma}\right] && (44)\end{aligned}$$

Therefore, we compute c_{R-1}^* directly for each underlying $\left(a_j^{\tau}, a_h^{DC}\right)$ as

$$c_{R-1}^*\left(a_j^{\tau}, a_h^{DC}\right)=\left\{\beta E_{R-1}\left[u'\left(c_R\left(\widetilde{w}_R^{\tau}, \widetilde{w}_R^{DC}\right)\right)\left(R^f+\omega_{R-1}^*\left(\widetilde{R}_R^e-R^f\right)\right)\left(R_R^G R_R^N\right)^{-\gamma}\right]\right\}^{-1/\gamma} \quad (45)$$

Define a new variable called before-consumption wealth $b_{R-1}=a^{\tau}+c_{R-1}^*=w_{R-1}^{\tau}-(1-\tau^y)m_{R-1}^{DC}$. Now we can generate an endogenous 2-D grid for b_{R-1} using

$$b_{R-1}^*\left(a_j^{\tau}, a_h^{DC}\right)=a_j^{\tau}+c_{R-1}^*\left(a_j^{\tau}, a_h^{DC}\right). \quad (46)$$

In addition, we can evaluate the expected utility of the next period, for the given value of $\{a_j^{\tau}, a_h^{DC}\}$, as

$$\begin{aligned} EV_{R-1}\left(a_j^{\tau}, a_h^{DC}\right) &= \beta E_{R-1}\left[\left(R_R^G R_R^N\right)^{1-\gamma} v\left(w_R\right)\right] && (47)\\ &= \beta E_{R-1}\left[\left(R_R^G R_R^N\right)^{1-\gamma} u\left(w_R\right)\right] K\left(T-R\right) && (48)\end{aligned}$$

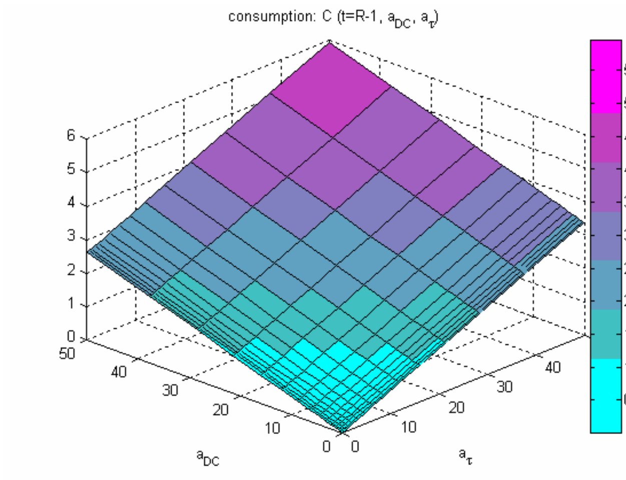

Figure: Consumption as function of investable wealth $c^*_{R-1}\left(a^{\tau}, a^{DC}\right)$ at time $R-1$.

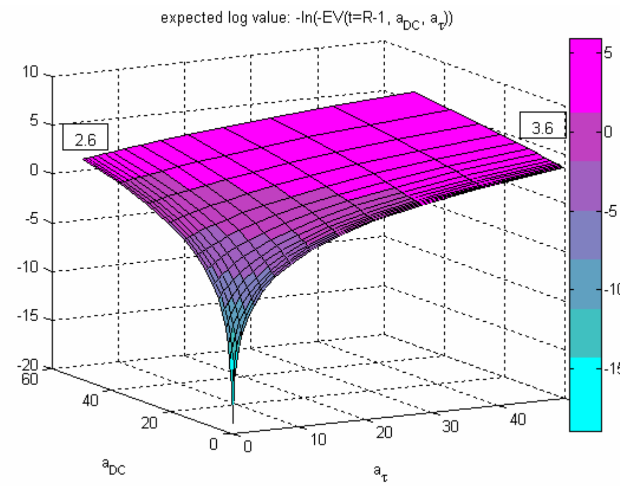

Figure: The expected value at time $R-1$, $EV_{R-1}\left(a^{\tau}, a^{DC}\right)$.

A.2.3 Optimize contribution rate

The third step is to compute the optimal contribution rate *0.2*$\geq m_{R-1} \geq 0$. It sets an upper limit on contribution rate, i.e. the contribution is no larger than 20% of gross salary income.

FOC condition w.r.t. m_{R-1} can not solve the optimal contribution rate.[11] We need to use the value function itself to search the optimal contribution rate numerically. First construct two exogenous grids for possible values of wealth $\widehat{w}^{\tau} = \{W_n\}_{n=1}^{N}$ and $\widehat{w}^{DC} = \{W_n\}_{n=1}^{N}$. Then, we construct a grid of possible contribution rates. We start with the case without employer match. The grid of possible contribution rates is denoted by $\widehat{m}_{R-1} = \{m_i\}_{i=1}^{M} \subset [0, m^{\max}]$, with $m^{\max} = \min\left(1, \widehat{w}^{\tau}/(1-\tau^{y})\right)$. These implies a set of before-consumption wealth $\widehat{b}$ and investable wealth $\widehat{a}^{DC}$ for any given combination of $\{\widehat{w}^{\tau}, \widehat{w}^{DC}, \{m_i\}_{i=1}^{M}\}$, as

[11] FOC w.r.t. m^{DC}_{R-1} is

$$(1-\tau^{y})\beta E_{R-1}\left[\left(R^{G}_{R}R^{N}_{R}\right)^{-\gamma} v'\left(w_R\right)\widetilde{R}^{P,\tau}_{R}\right] = (1-\tau^{o})\,\beta E_{R-1}\left[\left(R^{G}_{R}R^{N}_{R}\right)^{-\gamma} v'\left(w_R\right)\widetilde{R}^{P,DC}_{R}\right]$$

which clearly doesn't hold in general. Since $\tau^{y} > \tau^{o}$, this FOC implies marginal cost of contribution $<$ martinal benefit of contribution, therefore m^{DC}_{R-1} should take some maximum value (if exists). However the danger of doing so is that it implies w^{τ}_{R-1} might be unlimited. Due to this reason, we need to resort to value function to find the optimal m^{DC}_{R-1} for any given level of $\left\{w^{\tau}, w^{DC}\right\}$.

$$\begin{aligned}
\widehat{b}_i &= \widehat{w}^{\tau} - (1-\tau^y)\widehat{m}_i > 0 && (49)\\
\widehat{a}_i^{DC} &= \widehat{w}^{DC} + \widehat{m}_i && (50)
\end{aligned}$$

With employer match, e.g. $m_t^{DC} = \min\left(m_t + \min\left(m_t, 6\%\right), 20\%\right)$, the grid of possible contribution rates remains the same, but the implied $\widehat{b}$ and $\widehat{a}^{DC}$ become

$$\begin{aligned}
\widehat{b}_i &= \widehat{w}^{\tau} - (1-\tau^y)\widehat{m}_i > 0 && (51)\\
\widehat{a}_i^{DC} &= \widehat{w}^{DC} + \widehat{m}_i^{DC} && (52)
\end{aligned}$$

If $\widehat{b}_i$ and $\widehat{a}_i^{DC}$ are known, then with the help of the interpolation relation $b_{R-1}^*\left(a^{\tau}, a^{DC}\right)$ we can back out the corresponding private wealth $\widehat{a}_i^{\tau}$. It then leads to the optimal consumption $\widehat{c}_i^* = \widehat{b}_i - \widehat{a}_i^{\tau}$, for any given set of $\{\widehat{w}^{\tau}, \widehat{w}^{DC}, m_i\}$. We denote the implied consumption as $c_i^*\left(\widehat{w}^{\tau}, \widehat{w}^{DC}, m_i\right)$. Furthermore, we can compute the expected value of the next period $\widehat{EV}_i\left(\widehat{a}_i^{\tau}, \widehat{a}_i^{DC}\right)$ based on relation $EV_{R-1}\left(a^{\tau}, a^{DC}\right)$. Finally, we can evaluate the trade-off between the consumption and the contribution using the recursive objective function as

$$\varpi_{R-1}(\widehat{w}_{R-1}^{\tau}, \widehat{w}_{R-1}^{DC}) \equiv \max_{\{m_i\}_{i=1}^M} \frac{\left(\widehat{c}_i^*\right)^{1-\gamma}}{1-\gamma} + EV_{R-1}\left(\widehat{a}_i^{\tau}, \widehat{a}_i^{DC}\right) \tag{53}$$

The optimal contribution rate is the one that maximize the above expression for given values of $(\widehat{w}_{R-1}^{\tau}, \widehat{w}_{R-1}^{DC})$. Let's denote it as $m^*(\widehat{w}^{\tau}, \widehat{w}^{DC})$. As a by-product, we also get the corresponding consumption policy $c^*(w^{\tau}, w^{DC}) = c^*\left(\widehat{w}^{\tau}, \widehat{w}^{DC}, m^*(\widehat{w}^{\tau}, \widehat{w}^{DC})\right)$.

A.2.4 Euler equation

Finally, the Envelope theorem tells us

$$\begin{aligned}
\varpi_{\tau}'\left(w_{R-1}^{\tau}, w_{R-1}^{DC}\right) &= \beta E_{R-1}\left[\left(R_R^G R_R^N\right)^{-\gamma} v'\left(w_R\right)\left[R^f + \alpha_{R-1}\left(\widetilde{R}_R^e - R^f\right)\right]\right] && (54)\\
&= u'\left(c_{R-1}^*\left(w_{R-1}^{\tau}, w_{R-1}^{DC}\right)\right) && (55)\\
\varpi_{DC}'\left(w_{R-1}^{\tau}, w_{R-1}^{DC}\right) &= \left(1-\tau^o\right)\beta E_{R-1}\left[\left(R_R^G R_R^N\right)^{-\gamma} v'\left(w_R\right)\left[R^f + \alpha_{R-1}\left(\widetilde{R}_R^e - R^f\right)\right]\right] && (56)\\
&= \left(1-\tau^o\right) u'\left(c_{R-1}^*\left(w_{R-1}^{\tau}, w_{R-1}^{DC}\right)\right) && (57)
\end{aligned}$$

where ϖ_{τ}' and ϖ_{DC}' denote the partial differentials.

A.2.5 Optimal policies for $t = R - 1$

For the last year of working, $t = R - 1$, the optimal contribution rate is largely 100% of the labor income, except for when TA wealth (w^{τ}) is very limited, but the TDA wealth (w^{DC}) is abundant. The optimal consumption increases with TA and TDA wealth in general. The special feature for the optimal consumption is that there are kinks due to the liquidity constraint, i.e. individual can not consume more than what they have in the taxable account. Similarly there are also kinks for the value function. Except for this, the value function increases with both wealth accounts.

A.2.6 Repeat the procedure

Now we are able to proceed to $t = R - 2, R - 3, ...1$, by repeating the similar procedure as for $t = R-1$. To generate the average pattern of life cycle portfolio holding, as in Figure 1, we simulate the model from time 1 to T for 10,000 scenario's, and take the average over all simulated scenario's.

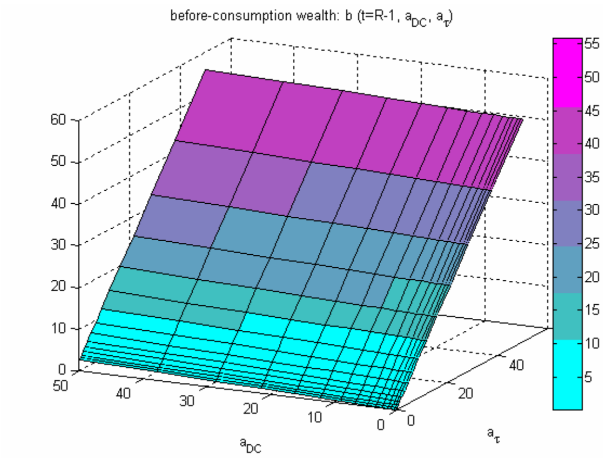

Figure: The implied before-consumption wealth b as function of investable wealth $b_{R-1}\left(a^{\tau}, a^{DC}\right)$ at time $R-1$.

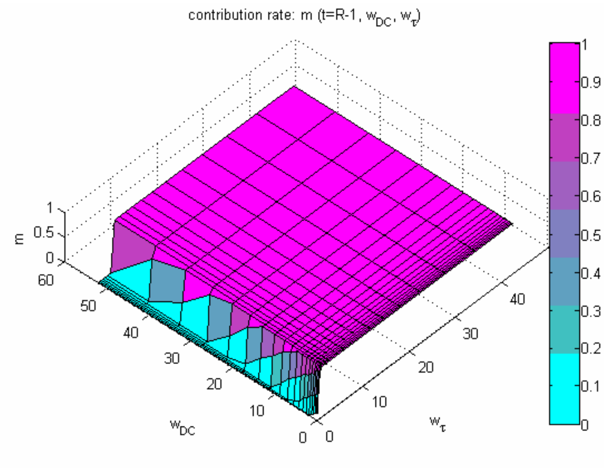

Figure: The optimal contribution rate $m^{*}_{R-1}\left(w^{\tau}, w^{DC}\right)$ at time $R-1$.

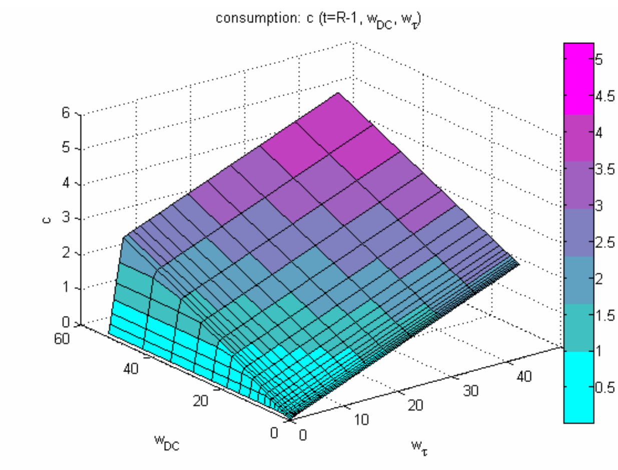

Figure: The optimal consumption policy $c^{*}_{R-1}\left(w^{\tau}, w^{DC}\right)$.

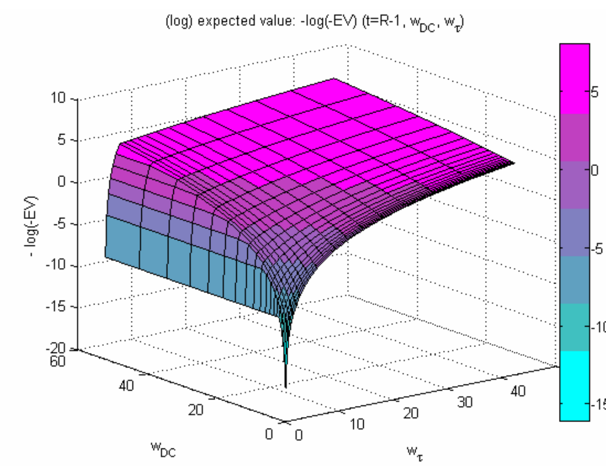

Figure: (log) value function -ln$(-\varpi(w^{\tau}, w^{DC}))$ at time R-1.

B Solution method of Section 4

B.1 Default #1

In Default #1, the contribution rate $m \geq 0$ and portfolio allocations $0.2 \geq \alpha \geq 0$ are chosen at the beginning of the career ($t = 0$), and are fixed throughout life time. Notice that we the portfolio allocation for both accounts are assumed to be the same constant mix through out life time, $\alpha^{\tau} = \alpha^{DC} = \alpha$. Therefore, the value functions, for given the chosen level of m and α, are denoted as $\varpi\left(w_t^{\tau}, w_t^{DC} \mid \alpha, m\right)$ for working period, and $v\left(w_t \mid \alpha, m\right)$ for retirement period. We solve the model numerically using dynamic programming.

For the final period, the optimal consumption policy is to consume everything $c_T^* = w_T$. The corresponding value function is given by $v\left(w_T \mid \alpha, m\right) = \frac{\left(c_T^*\right)^{1-\gamma}}{1-\gamma}$. Then we proceed to time $t = T-1$. During the retirement period ($R \leq t \leq T-1$), the normalized value function (in recursive form) and wealth process are

$$v\left(w_t \mid \alpha, m\right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t\left[\left(R_{t+1}^G\right)^{1-\gamma} v\left(w_{t+1} \mid \omega, m\right)\right] \tag{58}$$

subject to the budget dynamics

$$w_{t+1} = \left(w_t - c_t\right) \widetilde{R}_{t+1}^{P,\omega} \left(R_{t+1}^G\right)^{-1} + \left(1-\tau\right) ss_{t+1} - h_{t+1} - med_{t+1} \tag{59}$$

where we use a shorthand notation $\widetilde{R}_{t+1}^{P,\omega} = R^f + \alpha\left(\widetilde{R}_{t+1}^e - R^f\right)$ for the portfolio returns.

For any given value of m and α, we only need to optimize the consumption choice c_t.

The FOC w.r.t. c_t is

$$u'\left(c_t\right) = \beta E_t\left[v'\left(w_{t+1} \mid \alpha, m\right) \widetilde{R}_{t+1}^{P,\omega} \left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right] \tag{60}$$

The Envelope theorem gives

$$v'\left(w_t \mid \alpha, m\right) = \beta E_t\left[v'\left(w_{t+1} \mid \alpha, m\right) \widetilde{R}_{t+1}^{P,\omega} \left(R_{t+1}^G\right)^{-\gamma}\right] = u'\left(c_t\right) \tag{61}$$

So, pushing one period ahead, we have $v'\left(w_{t+1} \mid \alpha, m\right) = u'\left(c_{t+1}\right)$.

Define a new variable, $a_t = w_t - c_t$, as the after-consumption-wealth. Construct a grid of $a_t = \{a_j\}_{j=1}^J$. Now we solve for the optimal consumption and portfolio policies for each given a_j.

$$\begin{aligned} c_t &= I_u\left(\beta E_t\left[v'\left(w_{t+1} \mid \alpha, m\right) \widetilde{R}_{t+1}^{P,\omega} \left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right]\right) && (62) \\ &= I_u\left(\beta E_t\left[u'\left(c_{t+1}^*[w_{t+1}]\right) \widetilde{R}_{t+1}^{P,\omega} \left(\widetilde{R}_{t+1}^G\right)^{-\gamma}\right]\right) && (63) \end{aligned}$$

Following the EGM, the optimal wealth process is endogenously determined by $w_t^* = c_t^*\left(a_t\right) + a_t$, for a given value of $\{m, \alpha\}$.

At time $t = R$, we split the single wealth variable w_R^* into two wealth variables w_R^τ and w_R^{DC}, and obtain the corresponding policy function $c_R^*\left(w_R^\tau, w_R^{DC}\right)$, as done in Appendix A.

During the working period ($0 \leq t \leq R-1$), the normalized value function (in recursive form) and wealth process are

$$\varpi\left(w_t^\tau, w_t^{DC} \mid \alpha, m\right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t\left[\left(R_{t+1}^G\right)^{1-\gamma} \varpi\left(w_{t+1}^\tau, w_{t+1}^{DC} \mid \omega, m\right)\right]$$

s.t. the wealth dynamics and no borrowing constraint as

$$\begin{aligned} w_{t+1}^\tau &= \left(w_t^\tau - c_t - (1-\tau^y)m\right) \widetilde{R}_{t+1}^{P,\omega}\left(R_{t+1}^G R_{t+1}^N\right)^{-1} + (1-\tau^y) - h_{t+1} - med_{t+1} && (64) \\ w_{t+1}^{DC} &= \left(w_t^{DC} + m^{DC}\right) \widetilde{R}_{t+1}^{P,\omega}\left(R_{t+1}^G R_{t+1}^N\right)^{-1} && (65) \\ 0 &\leq w_t^\tau - c_t - (1-\tau^y)m && (66) \end{aligned}$$

Construct two investable wealth grids $a_t^\tau \equiv (w_t^\tau - c_t - (1-\tau^y)m) = \{a_j^\tau\}_{j=1}^J \geq 0$, and $a_t^{DC} \equiv \left(w_t^{DC} + m^{DC}\right) = \{a_h^{DC}\}_{h=1}^H \geq 0$. Then calculate the optimal consumption for each combination of $\{a_j^\tau, a_h^{DC}\}$. We know the first order conditions w.r.t. c_t is

$$\begin{aligned} u'\left(c_t\right) &= \beta E_t\left[v'\left(\widetilde{w}_{t+1}\right) \widetilde{R}_{t+1}^{P,\omega}\left(R_{t+1}^G R_{t+1}^N\right)^{-\gamma}\right] && (67) \\ &= \beta E_{R-1}\left[u'\left(c_{t+1}\left(\widetilde{w}_{t+1}^\tau, \widetilde{w}_{t+1}^{DC}\right)\right) \widetilde{R}_{t+1}^{P,\omega}\left(R_{t+1}^G R_{t+1}^N\right)^{-\gamma}\right] && (68) \end{aligned}$$

Therefore, we compute c_t^* directly for each underlying $\left(a_j^\tau, a_h^{DC}\right)$ as

$$c_t^*\left(a_j^\tau, a_h^{DC}\right) = \left\{\beta E_t\left[u'\left(c_{t+1}\left(\widetilde{w}_{t+1}^\tau, \widetilde{w}_{t+1}^{DC}\right)\right) \widetilde{R}_{t+1}^{P,\omega}\left(R_{t+1}^G R_{t+1}^N\right)^{-\gamma}\right]\right\}^{-1/\gamma} \quad (69)$$

Following the EGM, the optimal wealth process is endogenously determined by $w_t^{*,\tau} = c_t^*\left(a_t^\tau, a^{DC}\right) + a_t^\tau + (1-\tau^y)m$ and $w_t^{*,DC} = a^{DC} - m^{DC}$, for a given value of $\{m, \alpha\}$.

Repeat the above procedure for all values of $\{m, \alpha\}$. Finally, the optimal $\{\alpha^*, m^*\}$ are the ones that maximize the value function

$$\{\alpha^*, m^*\} = \arg\max \varpi\left(w_0^\tau, w_0^{DC} \mid \alpha, m\right)$$

C Tables and Figures

	Default #1	Default #2	Default #3	Default #4
portfolio	constant	age-dependent	constant	age-dependent
contribution	constant	constant	age-dependent	age-dependent

Table 1: Overview of the default designs

contribution	Optimal portfolio α^*			50% in equities		100% risk free ($\alpha = 0$)	
m	α^*	CEC	% CEC^{TDATA}	CEC	% CEC^{TDATA}	CEC	% CEC^{TDATA}
0%	80%	0.675	98%	0.672	97.6%	0.661	96%
1%	80%	0.671	97.5%	0.67	97.2%	0.6534	95%
2%	80%	0.666	96.8%	0.664	96.5%	0.649	94.2%
3%	82.5%	0.661	96%	0.659	95.6%	0.644	93.5%
4%	**82.5%**	**0.654**	**95%**	**0.653**	**94.8%**	**0.638**	**92.7%**
5%	85%	0.648	94.2%	0.646	94%	0.633	91.9%

Table 2: Default #1, constant contribution rate and constant portfolio, (**no** employer matching). This table presents the welfare (CEC) under given flat contribution rates and several different portfolio weights in equities. ($CEC^{TDATA} = 0.6885$)

contribution	Optimal portfolio α^*			50% in equities		100% risk free ($\alpha = 0$)	
m	α^*	CEC	% CEC^{TDATA}	CEC	% CEC^{TDATA}	CEC	% CEC^{TDATA}
0%	80%	0.675	96.1%	0.672	95.7%	0.661	94.1%
1%	80%	0.676	96.3%	0.674	96%	0.658	93.7%
2%	80%	0.674	96%	0.672	95.7%	0.656	93.5%
3%	82.5%	0.670	95.4%	0.668	95.2%	0.654	93.2%
4%	**82.5%**	**0.664**	**94.6%**	**0.663**	**94.4%**	**0.651**	**92.8%**
5%	85%	0.656	93.5%	0.656	93.4%	0.647	92.1%

Table 3: Default #1, constant contribution rate and constant portfolio, (**with** employer matching). This table presents the welfare (CEC) under given flat contribution rates and several different portfolio weights in equities. ($CEC^{TDATA} = 0.702$).

Designs	default specification		Welfare measure	
	portfolio	contribution	CEC	% to CEC^{TDATA}
Fully Optimal TDA & TA	α_t^*	m_t^*	0.688	100%
Current Default	$\alpha = 0$	$m = 4\%$	0.638	92.7%
Default #1 (with better portfolio)	$\alpha = 50\%$	$m = 4\%$	0.653	94.8%
Default #1 (with best portfolio)	$\alpha = 82.5\%$	$m = 4\%$	0.654	95%
Default #2 (age-dep. portfolio)	α age-dep.	$m = 4\%$	0.655	95.2%
Default #3 (age-dep. contribution)	$\alpha = 0$	m age-dep.	0.668	97.1%
Default #3 (age-dep. contrib., best portf.)	$\alpha^* = 85\%$	m age-dep.	0.683	99.2%
Default #4 (age-dep. contrib, age-dep. portf)	α age-dep.	m age-dep.	0.684	99.4%

Table 4: Welfare comparison across default designs (without employer matching)

Designs	default specification		Welfare measure	
	portfolio	contribution	CEC	% to CEC^{TDATA}
Fully Optimal TDA & TA	α_t^*	m_t^*	0.702	100%
Current Default	$\alpha = 0$	$m = 4\%$	0.651	92.8%
Default #1 (with better portfolio)	$\alpha = 50\%$	$m = 4\%$	0.663	94.4%
Default #1 (with best portfolio)	$\alpha = 82.5\%$	$m = 4\%$	0.664	94.6%
Default #2 (age-dep. portfolio)	α age-dep.	$m = 4\%$	0.668	95.1%
Default #3 (age-dep. contribution)	$\alpha = 0$	m age-dep.	0.681	97%
Default #3 (age-dep. contrib., best portf.)	$\alpha^* = 85\%$	m age-dep.	0.695	99%
Default #4 (age-dep. contrib, age-dep. portf)	α age-dep.	m age-dep.	0.697	99.2%

Table 5: Welfare comparison across default designs (**with** employer matching)

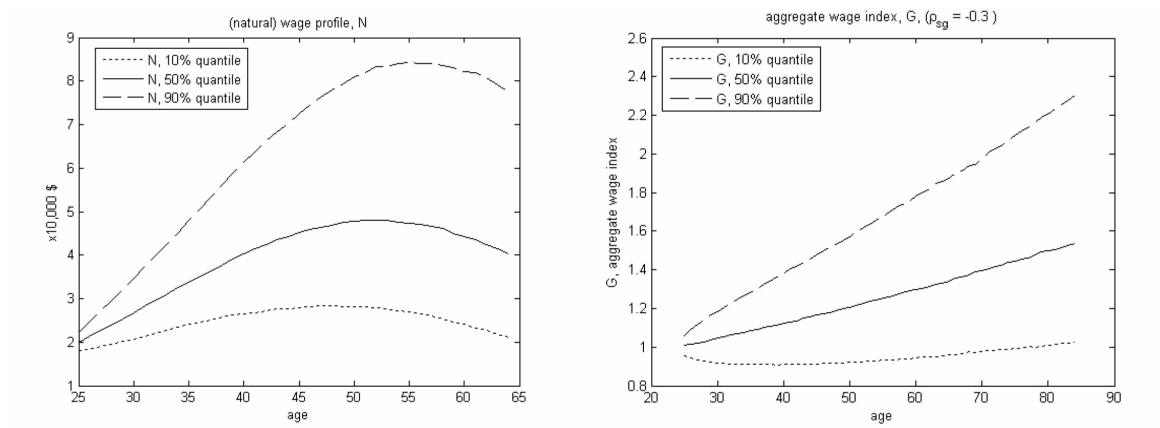

Figure 1: Income profiles. (a) Left panel shows the life cycle profile of the age-dependent idiosyncratic component N_{t-t_0}; (b) Right panel shows the aggregate wage component, G_t.

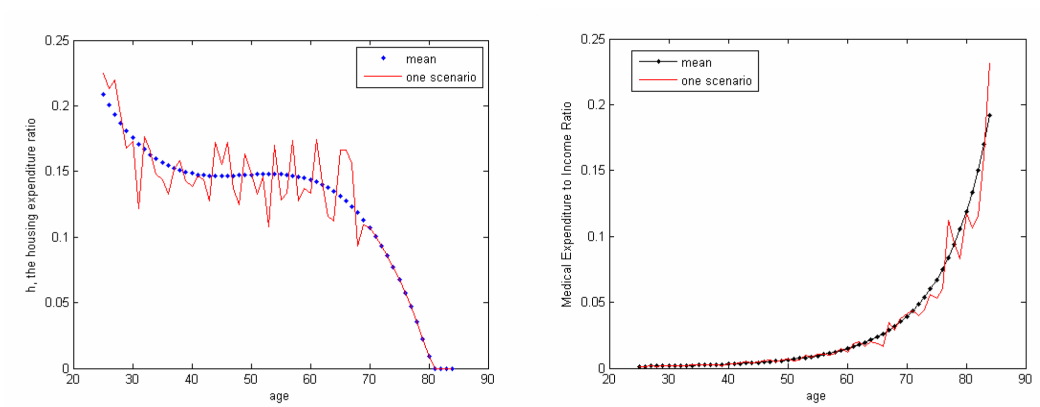

Figure 2: Housing and medical expenditures. (a) Left panel shows the mean and one simulation of the ratio of housing expenditure to income over life cycle; (b) Right panel shows the mean and one simulation of the ratio of out-of-pocket medical expenditure to income over life cycle.

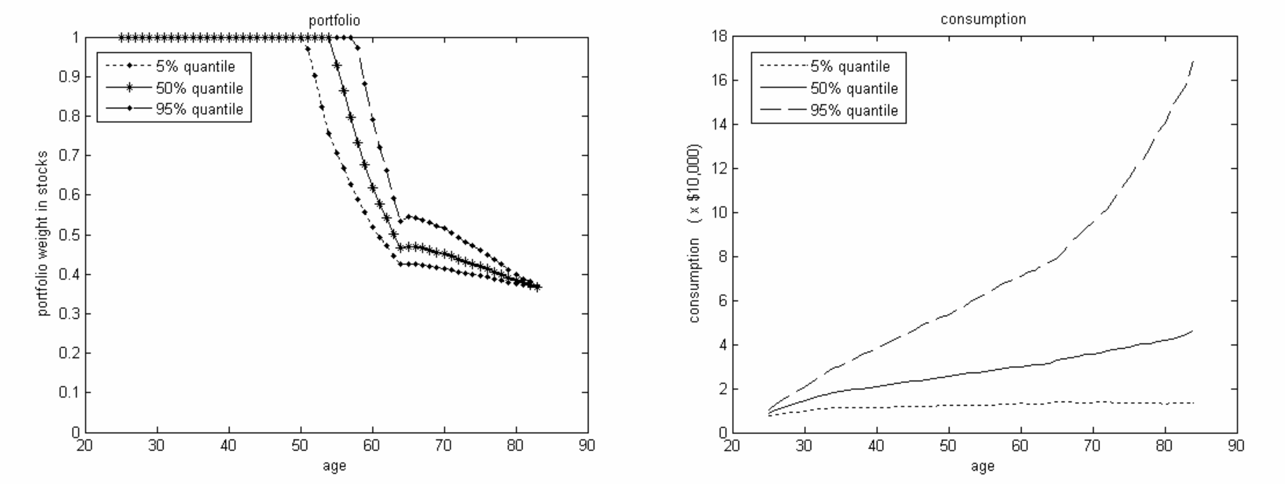

Figure 3: (left panel) Life-cycle portfolio choice assuming the identical asset allocation in Taxable and Tax-deferred DC accounts (5%, 50% and 95% quantiles); (right panel) Life-cycle consumption profile (5%, 50% and 95% quantiles).

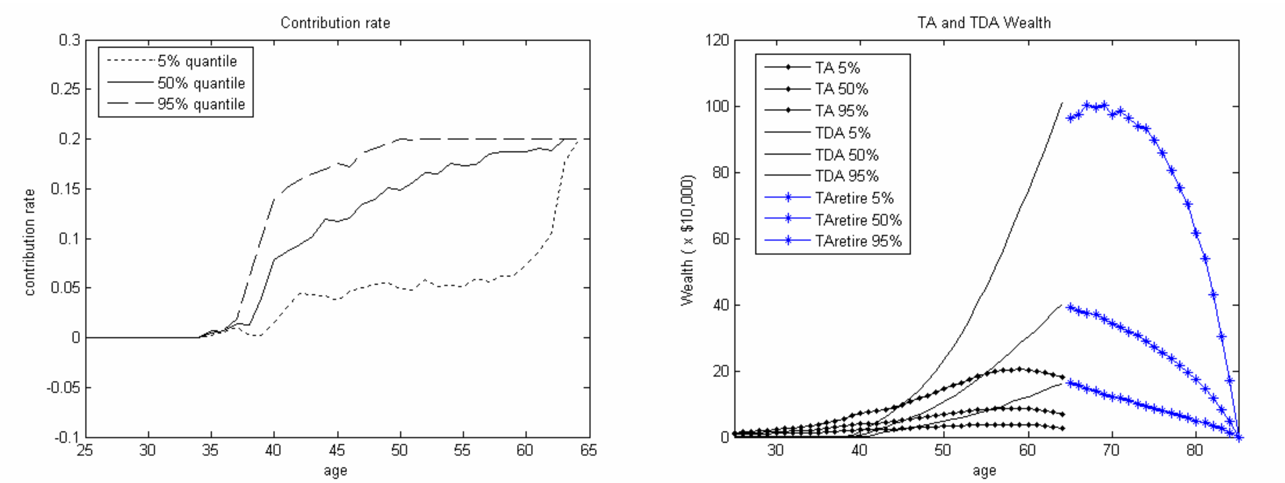

Figure 4: (left panel) life-cycle contribution rate m_t (5%, 50% and 95% quantiles), and (right panel) wealth accumulation in Taxable and Tax-deferred DC accounts (5%, 50% and 95% quantiles).

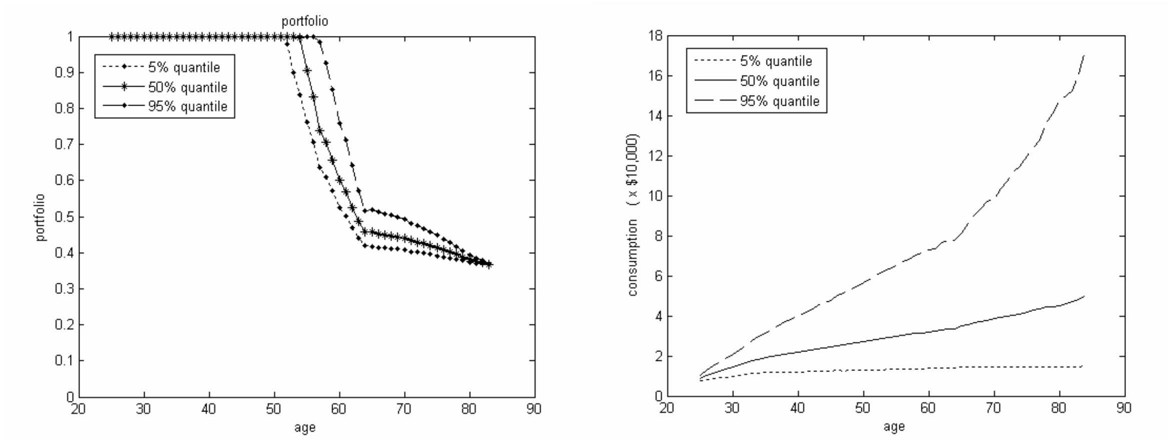

Figure 5: (left panel) Life-cycle portfolio choice assuming the identical asset allocation in Taxable and Tax-deferred DC accounts, (5%, 50% and 95% quantiles); (right panel) Life-cycle consumption **with employer match** (5%, 50% and 95% quantiles).

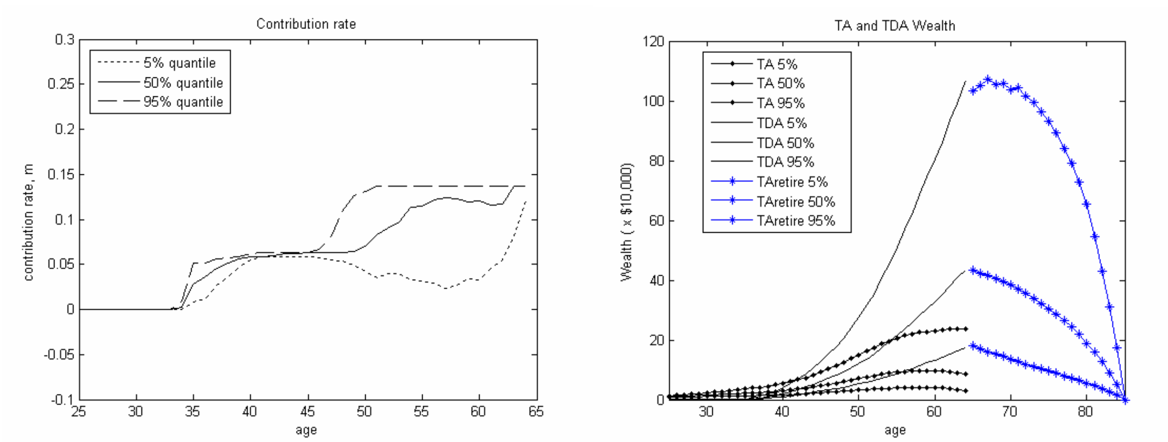

Figure 6: (left panel) life-cycle contribution rate m_t with employer match (5%, 50% and 95% quantiles), and (right) asset accumulation in Taxable and Tax-deferred DC accounts with employer match (5%, 50% and 95% quantiles).

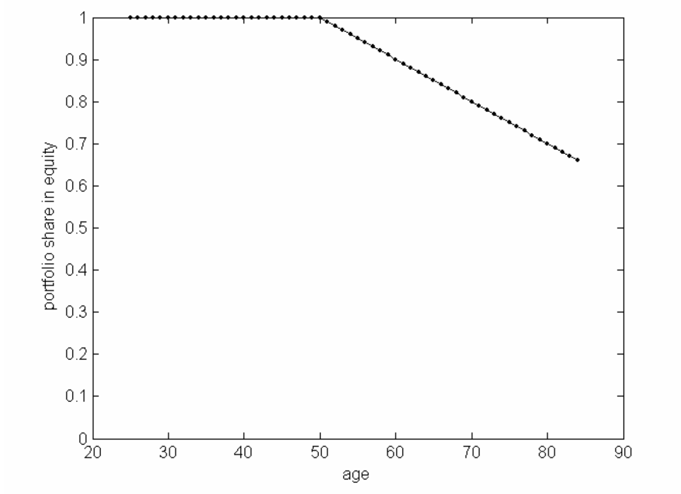

Figure 7: The optimized age-dependent portfolio rule for given contribution rate of 4%, as specified in Default #2.

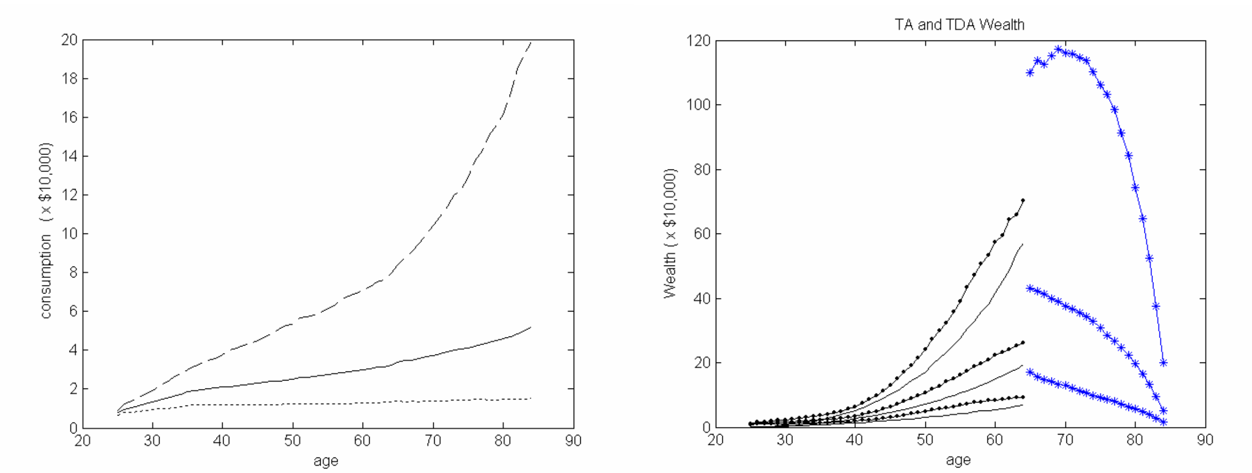

Figure 8: (left) Life-cycle consumption, and (right) asset accumulation in Taxable and Tax-deferred DC accounts, as specified in Default #2 (5%, 50% and 95% quantiles).

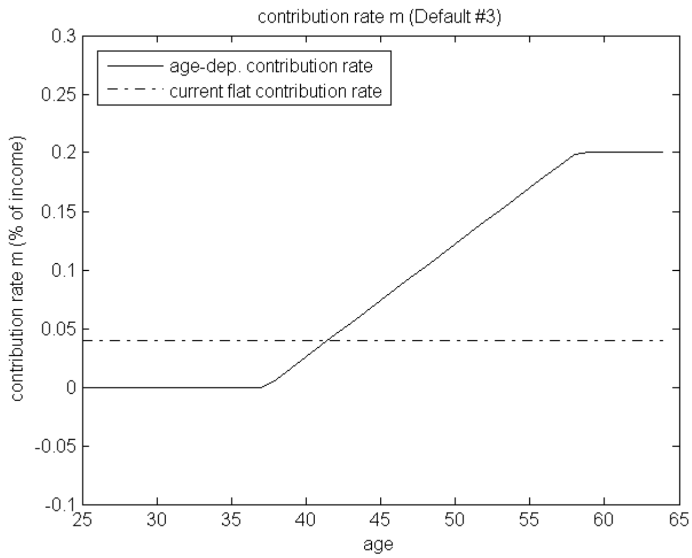

Figure 9: The optimized age-dependent contribution rule for given portfolio choice of 85%, as specified in Default #3.

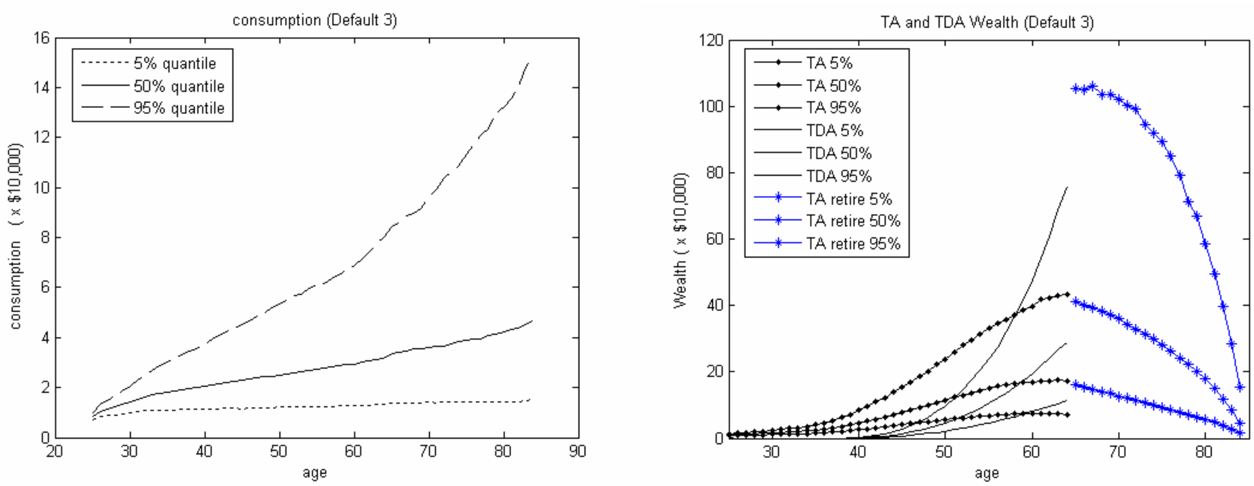

Figure 10: (left) Life-cycle consumption, and (right) asset accumulation in Taxable and Tax-deferred DC accounts, as specified in Default #3 (5%, 50% and 95% quantiles).

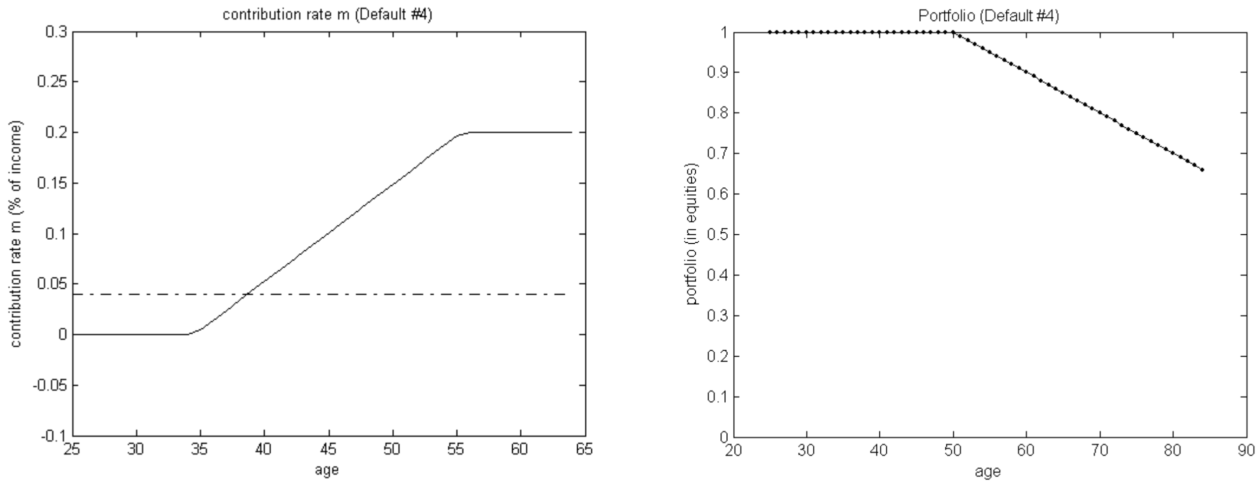

Figure 11: The age-dependent contribution rate (left panel) and age-dependent portfolio rule (right panel) in Default #4 (**no** employer matching).

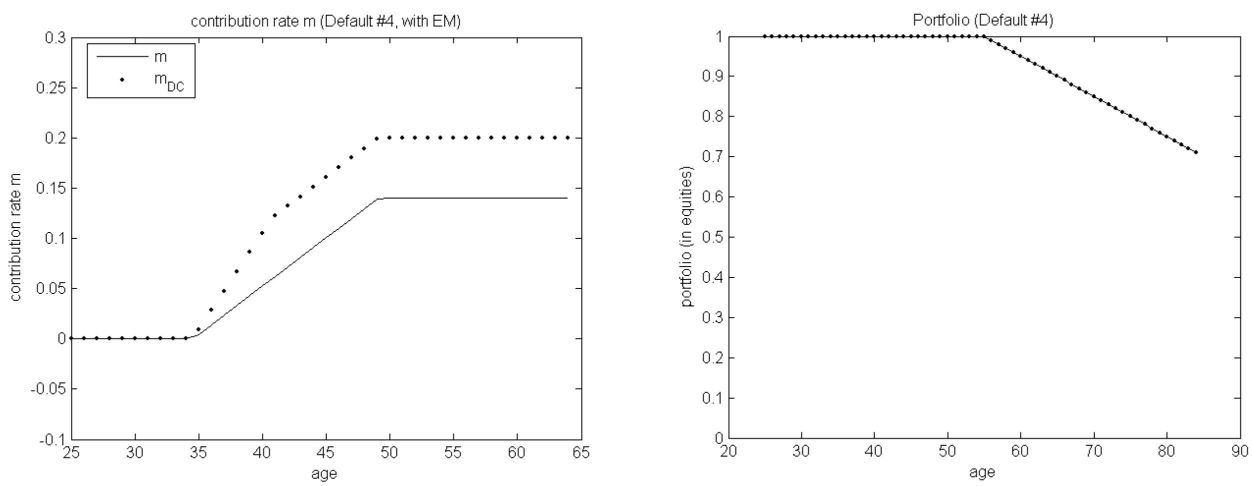

Figure 12: The age-dependent contribution rate (left panel) and age-dependent portfolio rule (right panel) in Default #4 (**with** employer matching).

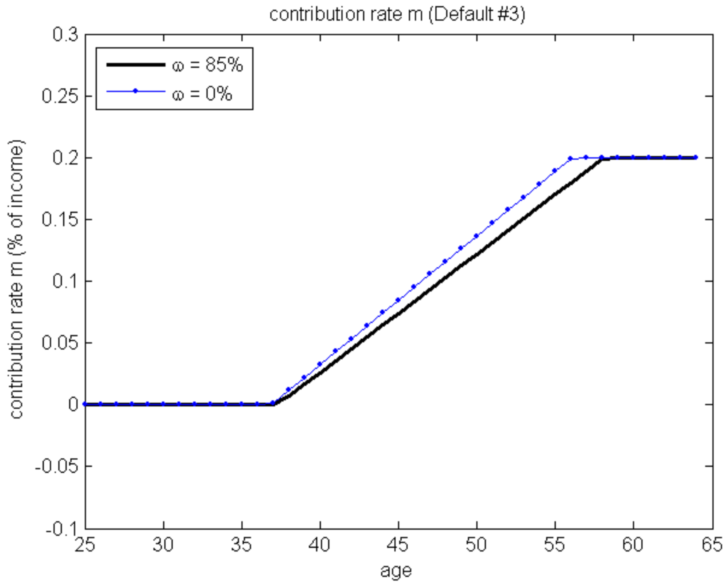

Figure 13: The optimal age-dependent contribution rate rules (default #3) for different asset mix ($\alpha = 0\%$ and 85% respectively), based on the baseline preference assumptions ($\gamma = 5$, $\beta = 0.97$).

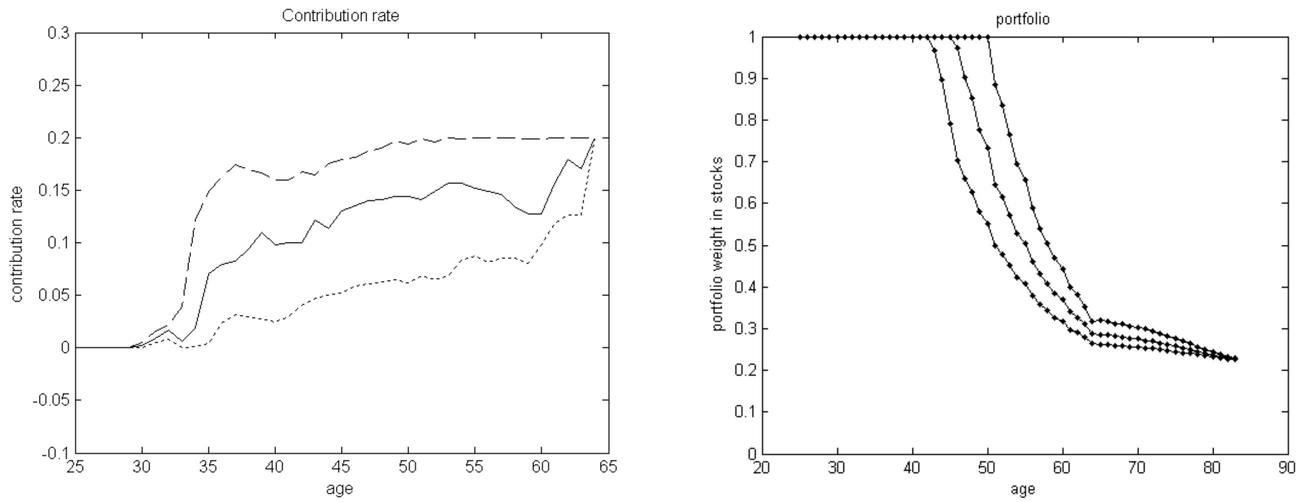

Figure 14: The age profiles of the optimal contribution rates (left panel), optimal portfolio choices (right panel) of a $\gamma = 8$ individual in taxable and tax-deferred DC account, without employer matching (5%, 50% and 95% quantiles)

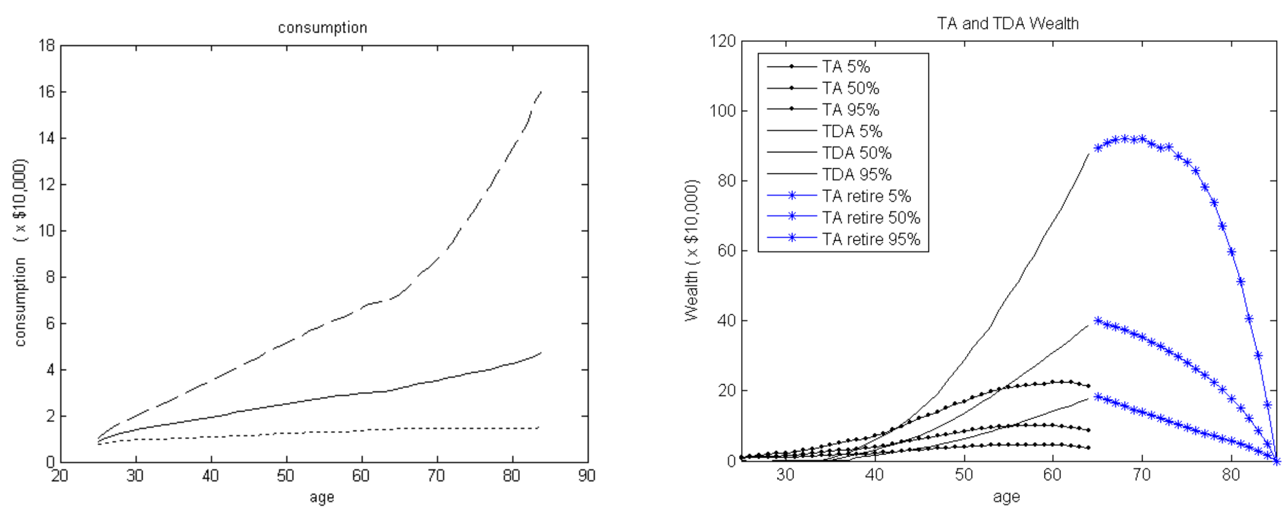

Figure 15: (left panel) The life cycle consumption and (right panel) asset accumulation for a $\gamma = 8$ individual in taxable and tax-deferred DC account, without employer matching (5%, 50% and 95% quantiles)

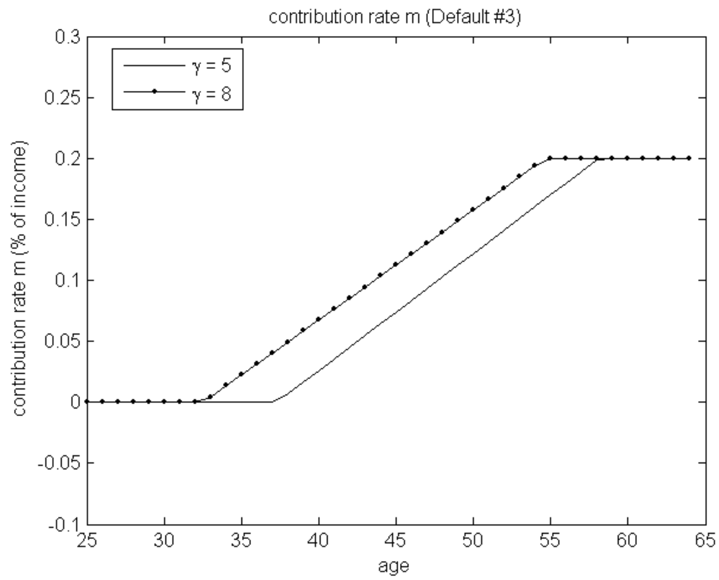

Figure 16: The optimal age-dependent contribution rate rules (default #3) for individuals with different risk aversion ($\gamma = 5$ and 8 respectively).

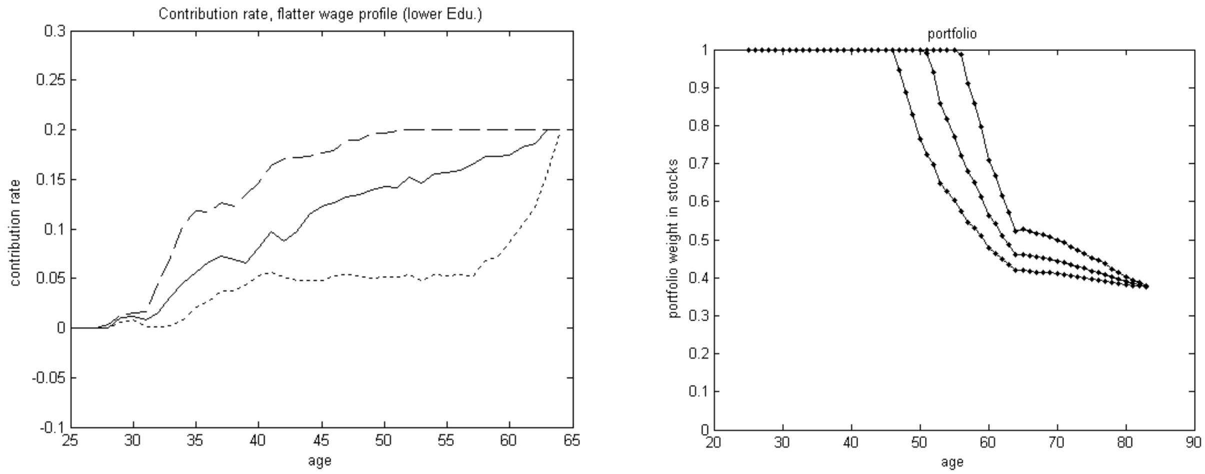

Figure 17: Optimal contribution and portfolio strategies for the case with flat wage earnings. (left panel) Life-cycle contribution rate m_t (5%, 50% and 95% quantiles); (right panel) Life-cycle portfolio choice assuming the identical asset allocation in Taxable and Tax-deferred DC accounts (5%, 50% and 95% quantiles).

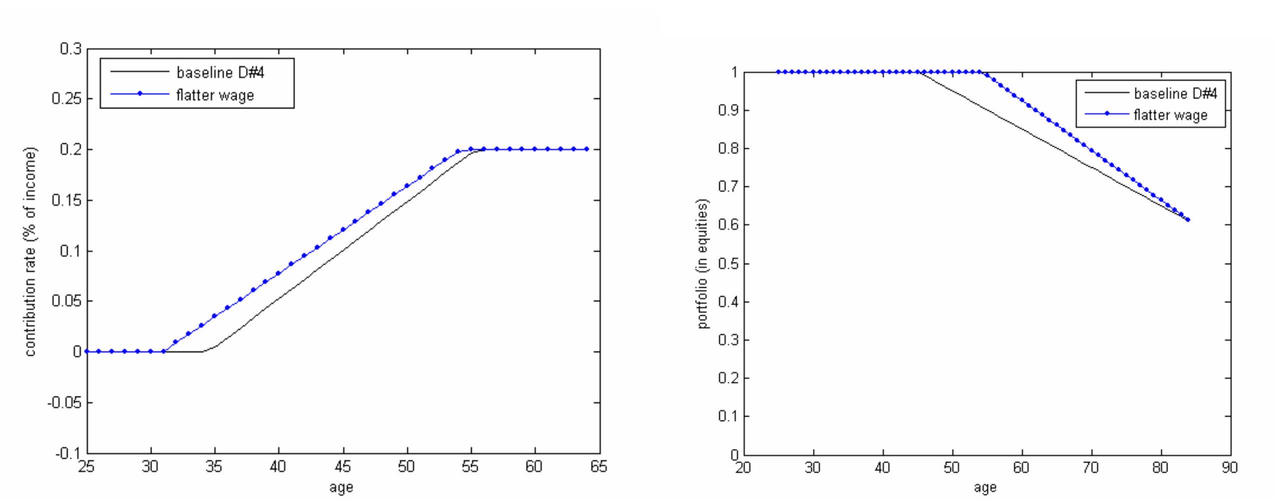

Figure 18: Optimal age-dependent contribution rate and portfolio rules for the case with flatter wage earnings. (left panel) age-dependent contribution rate m_t, and (right panel) age-dependent portfolio rule in Taxable and Tax-deferred DC accounts. The dotted line is the case with flatter wage earnings, and the solid line is the baseline wage earnings.

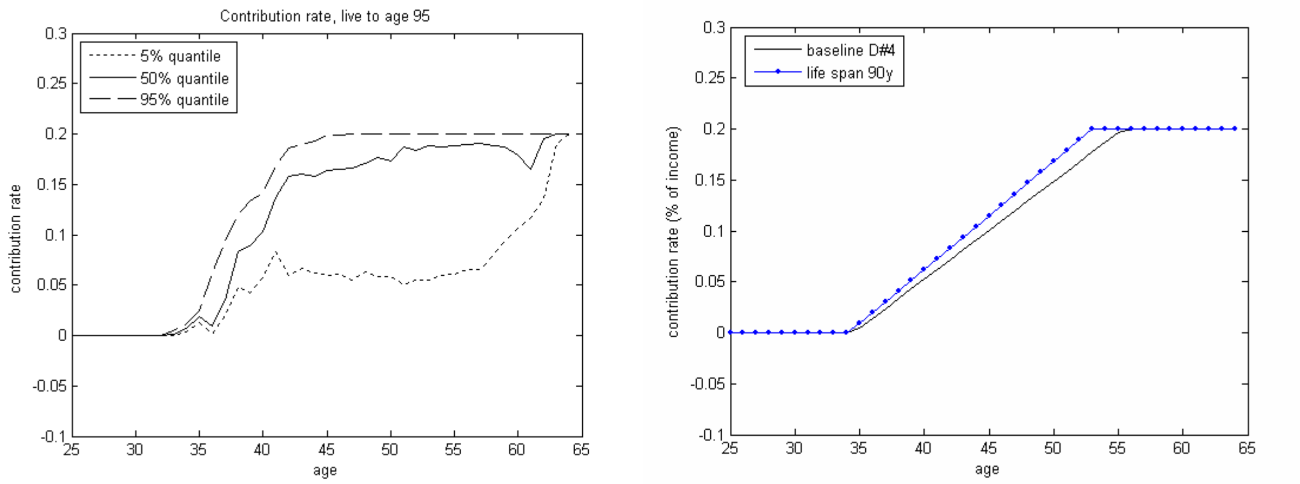

Figure 19: Optimal contribution strategy for the case with longer life span. (left panel) Life-cycle contribution rate m_t (5%, 50% and 95% quantiles); (right panel) optimal age-dependent contribution rate rules. The dotted line is the case with a longer lifespan of 90 years, and the solid line is based on a lifespan of 85 years (baseline).