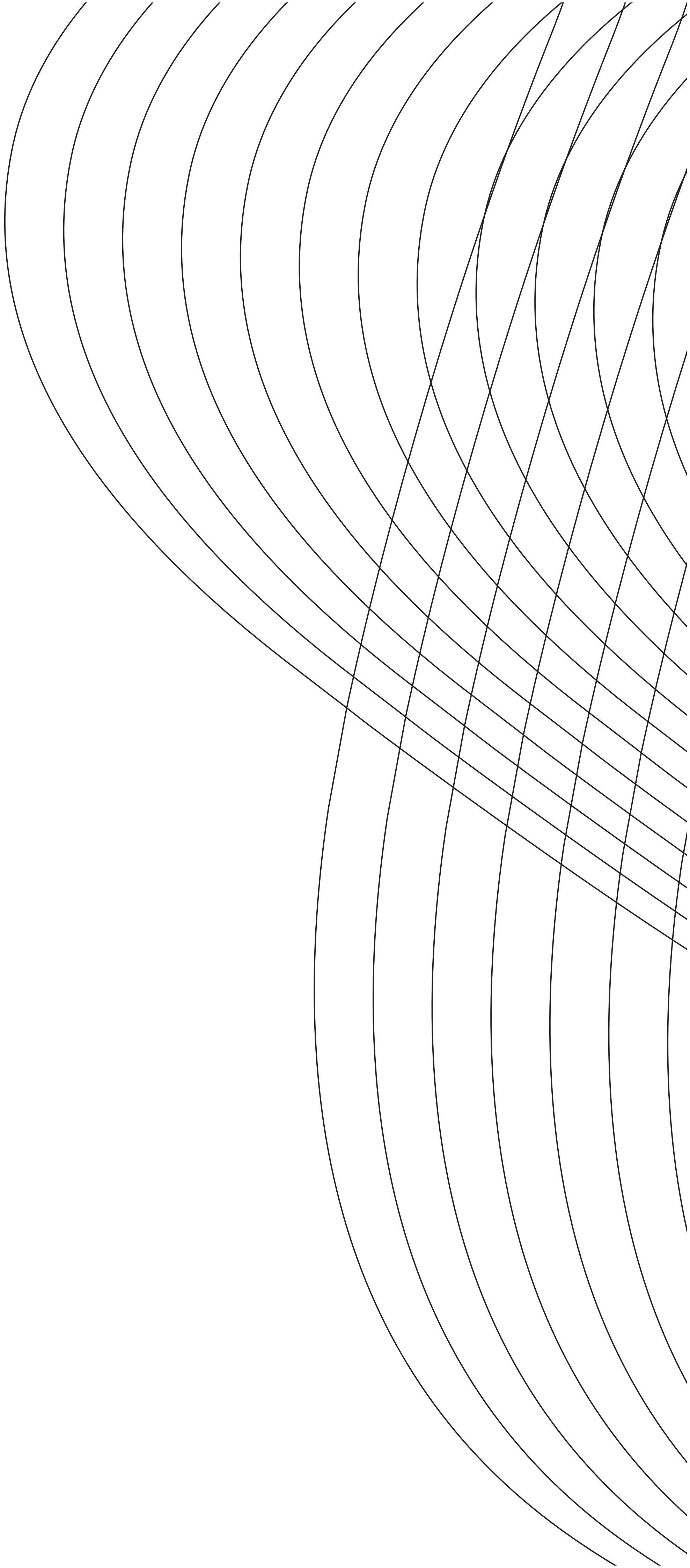
Netspar Panel Papers

Frank de Jong, Peter Schotman and Bas Werker

Strategic Asset Allocation



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PREFACE

Netspar stimulates debate and fundamental research in the field of pensions, aging and retirement. The aging of the population is front-page news, as many baby boomers are now moving into retirement. More generally, people live longer and in better health while at the same time families choose to have fewer children. Although the aging of the population often gets negative attention, with bleak pictures painted of the doubling of the ratio of the number of people aged 65 and older to the number of the working population during the next decades, it must, at the same time, be a boon to society that so many people are living longer and healthier lives. Can the falling number of working young afford to pay the pensions for a growing number of pensioners? Do people have to work a longer working week and postpone retirement? Or should the pensions be cut or the premiums paid by the working population be raised to afford social security for a growing group of pensioners? Should people be encouraged to take more responsibility for their own pension? What is the changing role of employers associations and trade unions in the organization of pensions? Can and are people prepared to undertake investment for their own pension, or are they happy to leave this to the pension funds? Who takes responsibility for the pension funds? How can a transparent and level playing field for pension funds and insurance companies be ensured? How should an acceptable trade-off be struck between social goals such as solidarity between young and old, or rich and poor, and individual freedom? But most important of all: how can the benefits of living longer and healthier be harnessed for a happier and more prosperous society?

The Netspar Panel Papers aim to meet the demand for understanding the ever-expanding academic literature on the consequences of aging populations. They also aim to help give a better scientific underpinning of policy advice. They attempt to provide a survey of the latest and most relevant research, try to explain this in a non-technical manner and outline the implications for policy questions faced by Netspar's partners.

Let there be no mistake. In many ways, formulating such a position paper is a tougher task than writing an academic paper or an op-ed piece. The authors have benefited from the comments of the Editorial Board on various drafts and also from the discussions during the presentation of their paper at a Netspar Panel Meeting.

I hope the result helps reaching Netspar's aim to stimulate social innovation in addressing the challenges and opportunities raised by aging in an efficient and equitable manner and in an international setting.

Henk Don
Chairman of the Netspar Editorial Board

Summary

Most financial managers of pension funds choose to allocate the wealth of their funds to traditional asset classes such as stocks, bonds and short-term depostits (cash). Increasingly, however, other asset classes including commodities, currencies, corporate bonds, derivatives, hedge funds and various illiquid assets (direct real estate, private equity, ...) have become part of the asset allocation problem. The strategic asset allocation decision concerns the choice of investments across these broad investment categories, and is the major determinant of a pension fund's investment risk profile. This decision determines a fund's exposure to market-wide risk factors such as the business cycle, the stock market, interest rates, inflation, etc. In its strategic asset allocation, the fund has several objectives. First, the fund is interested in a good risk-return trade-off; this is similar to the standard textbook investments setting. Second, in contrast to a traditional investment manager, a pension fund has specific liabilities in the form of future pension benefits, which are often indexed to price- or wage growth. Third, the pension fund invests on behalf of its members, who often have large specific investments in human capital and housing. All of these factors have to be balanced in order to reach an optimal strategic asset allocation choice. This paper reviews the recent advances in the literature on these topics.

One of the premises of this panel paper is that pension funds invest on behalf of their members. Individual investors make risk-return trade-offs over the full course of their life. This means that, for all age cohorts, stocks and other risky assets are part of the optimal asset allocation. These exposures depend on the characteristics of the members. For a fund whose members are endowed with risky human capital and are strongly exposed to the business cycle, the equity investments will be lower than for a fund with members that have almost riskless human capital. As long as the equity premium is positive, all investors (whether young or old) prefer a positive equity exposure in their portfolio. An important policy implication of this result is that pension funds (which invest on behalf of their members) should accept mismatch risk. An investment policy that is fully in risk-free assets is suboptimal. The assumption that the pension fund aggregates the asset allocation preferences of its members implies that the pension fund should accept mismatch risk and not invest all of its money in risk-free assets.

One of the open issues in defining the optimal investments of a pension fund concerns the allocation of benefits and of investment risk and returns to the participants. Pension funds pool the assets of all participants, and all participants likewise share the risks and

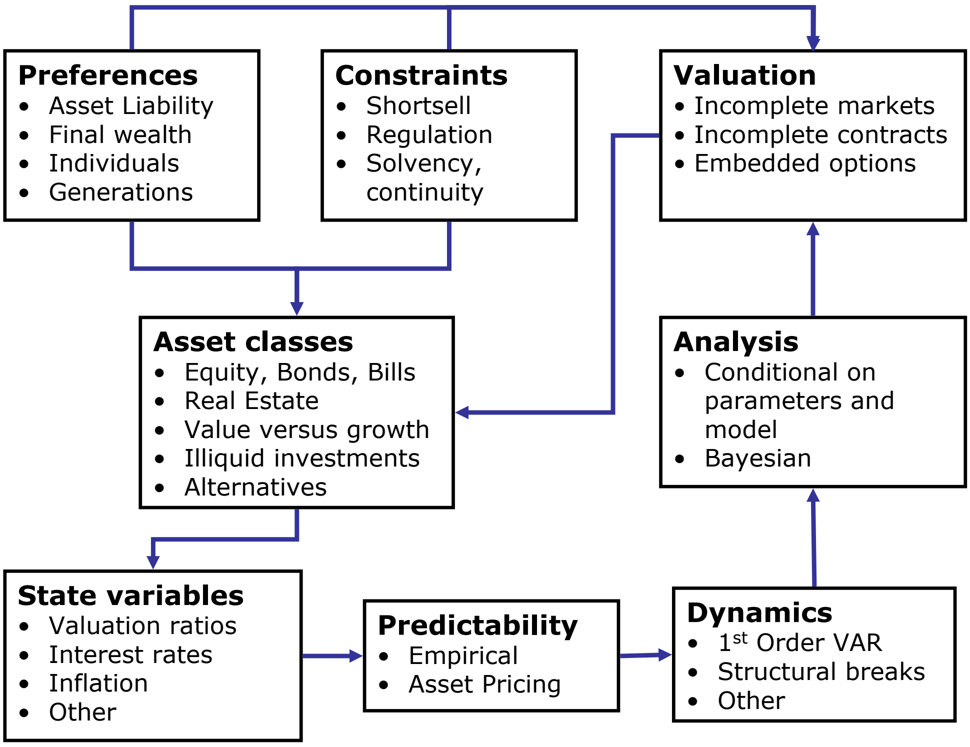
returns. Certain pension fund policies (such as conditional indexation) give various participants different levels of exposure to the investment risk. The pension fund can thus spread risk over different groups while maintaining a single aggregate investment portfolio.

Over the life cycle, young investors prefer larger equity exposures than older investors do, for basically two reasons. First, young investors have a large fraction of their wealth in human capital. This can be used to "leverage up" the equity exposure in the financial portfolio. A second reason why young investors may prefer larger equity is because they have more ways to buffer financial risks (for example, labor supply or consumption smoothing). This implies that pension funds may want to allocate investment risks differently for various age groups.

This paper reviews in detail the recent literature concerning long-term investing, which is of obvious importance for retirement saving. Among other things it is shown that restrictions on individuals with regard to short-sales can be very costly, leading to an additional argument for the existence of institutional investors (like pensions funds). It is also shown that the optimal exposure to different risk-factors varies with an individual's age. This leads to the possibility of differentiating an individual's exposure to such risk-factors while, at the same time, maintaining intergenerational risk-sharing. These concepts are not at odds. The effects of home ownership on the optimal asset allocation are also considered. Similar to human capital, housing has a leverage effect. But housing is also a large, risky position in a very specific asset. This makes investors more cautious in their financial portfolio. Financial contracts that allow older investors to hedge the price risk of selling their house in the future can be valuable here, and instruments such as individual house futures or reverse mortgages could accomplish this.

Finally, the paper reviews the quality of the econometric models of asset returns and the critical parameters for strategic asset allocation. The equity premium and the mean reversion of equity returns are important drivers of large equity positions in the optimal portfolios. Since both of these empirical regularities are also subject to much uncertainty, this paper advocates a Bayesian approach to modelling and decision making.

Figure 1: Strategic Asset Allocation



1 Introduction

Strategic asset allocation concerns the allocation of wealth to broad investment categories. Although institutional investors, such as pension funds, traditionally choose asset classes like stocks, bonds and short-term deposits (cash), they are increasingly considering also alternative asset classes such as commodities, currencies, credits (i.e. corporate bonds), derivatives and hedge funds. The choice of which specific stocks or bonds to buy is part of an investor's tactical asset allocation problem.[1]

The strategic asset allocation decision is the major determinant of a pension fund's investment risk profile, and determines the fund's exposure to market-wide risk factors such as stock market return, interest rate changes, inflation, default risk etc. In its strategic asset allocation, the fund has several objectives. First, the fund is interested in a good risk-return trade-off; this is similar to the standard textbook investments setting. Second, in contrast to a traditional investment manager, a pension fund has specific liabilities in the form of future pension benefits, which are often indexed to prices or to wage growth. Third, the pension fund invests on behalf of its members, who often have large specific investments in human capital and housing. All of these factors must be balanced in order to reach an optimal strategic asset allocation choice. This paper reviews the recent advances in the literature on these topics.

The starting point in this paper is Campbell and Viceira's book (2002) *Strategic Asset Allocation*. Their analysis is based on the preferences of an individual investor. Although the pension fund invests on behalf of its individual members, the objectives of the pension fund are somewhat different in a defined-benefit scheme. Section 2 of this paper therefore reviews pension fund objectives. To limit the scope of the paper, we take the optimal pension plan design and intergenerational risk sharing as given revealed preferences. These were discussed extensively in the NETSPAR panel paper Bovenberg, Koijen, Nijman and Teulings (2007).

Pension funds invest on behalf of their members. The literature has convincingly shown that individual investors make risk-return trade-offs over the full course of their life.[2] This means that stocks and other risky assets are part of the optimal asset allocation for all age cohorts. Young investors, however, prefer larger equity exposures than older investors do, for two reasons. First, young investors have a large fraction of their

[1] This is outside the scope of this paper. What exactly defines an asset class is not completely clear, however. For example, we treat equity as one asset class. Some recent studies like Jurek and Viceira (2006), however, distinguish between value and growth stocks as separate asset classes.

[2] See, for example, Bovenberg, Koijen, Nijman and Teulings (2007).

wealth in human capital. This can be used to "leverage up" the equity exposure in the financial portfolio. A second reason why young investors may prefer larger equity is because they have more ways to buffer financial risks (for example, through labor supply (Bodie, Merton, Samuelson) or consumption smoothing (Gollier)). Although this second channel will not be considered in this paper, we will look at the impact of human capital and other investments. The assumption that the pension fund aggregates the asset allocation preferences of its members implies that the pension fund should accept mismatch risk (i.e. it should not invest all of its money in a risk-free asset).

Section 3 reviews three models that we consider to be representative for the general results on strategic asset allocation. De Jong (2008a) presents a stylised model of asset returns in which the effects of different institutional arrangements (such as conditional indexation) can be analysed. The model for the economic environment and for asset returns is stylised. The asset menu is restricted to three asset classes: stocks, bonds and bills. The equity premium is constant, and real and nominal bond returns are endogenously determined through a simple term-structure model. The model focuses on inflation and interest rate risk. Campbell, Chan and Viceira (2003) consider a much more general economic environment represented as an unrestricted vector autoregression. On top of inflation and interest rate risk, the model also features time-varying bond- and equity premia. Strategic asset allocation is studied from an individual perspective. Individual preferences include separate parameters for risk attitudes, time preferences and intertemporal substitution of consumption. The risk attitudes affect the choice of asset class, while the intertemporal substitution determines overall savings. Hoevenaars, Molenaar, Schotman and Steenkamp (2008) expand the number of asset classes to include commodities, hedge funds, corporate bonds and real estate. They also explicitly analyse the difference between the asset-only- and asset-liability perspectives.

Section 4 considers two characteristics of non-financial wealth with relevance for strategic asset allocation. With regard to human capital, we look more closely at the relation between wages and financial returns. Benzoni, Collin-Dufresne and Goldstein (2007) argue that the long-run correlation between dividends and labor income implies a much lower optimal equity exposure for young people. The second source of non-financial wealth is housing. This paper uses the results in De Jong, Driessen and Van Hemert (2007) to address the interaction of asset allocation, house price risk and mortgage choice.

Section 5 addresses estimation issues. It reviews the econometric evidence on the size of the equity premium and the predictability of returns. The paper concludes with a series of recommendations.

2 Pension fund objectives

This paper focuses on pension funds that execute a pension plan on behalf of a group of firms (e.g. an industry). The funds receive contributions from participants, invest assets and pay out retirement benefits. The focus here is on the asset allocation strategy, taking contributions and pay-out policies as given.

Industry pension funds differ from corporate pension funds. For corporate pension plans, pension obligations are liabilities of the sponsoring firm. The classic corporate finance perspective maintains that pension liabilities are part of the capital structure of the firm.[3] When a corporation and a pension fund are consolidated, the problem of optimal investment policy is the same as determining the optimal capital structure of the firm. For industry pension funds, the balance sheet is separated from the sponsoring firms, which do not have an obligation to cover for underfunding of the pension plan. The pension fund invests on behalf of all its members, and participants do not have individual accounts at the fund. Since participants differ with regard to age, human capital, health, wealth, job tenure and many other characteristics, each participant might wish for a specific investment portfolio. Yet, the fund has only a single aggregate portfolio. If the pension deal is a complete contract, then each participant has predefined claims on the aggregate wealth in the fund. For a defined-benefit plan the claims are a function of wage and job tenure. For a defined-contribution plan the benefits are a function of the contributions and the investment returns. Most pension funds in the Netherlands have incomplete pension contracts, which are a hybrid mix of DB and DC.

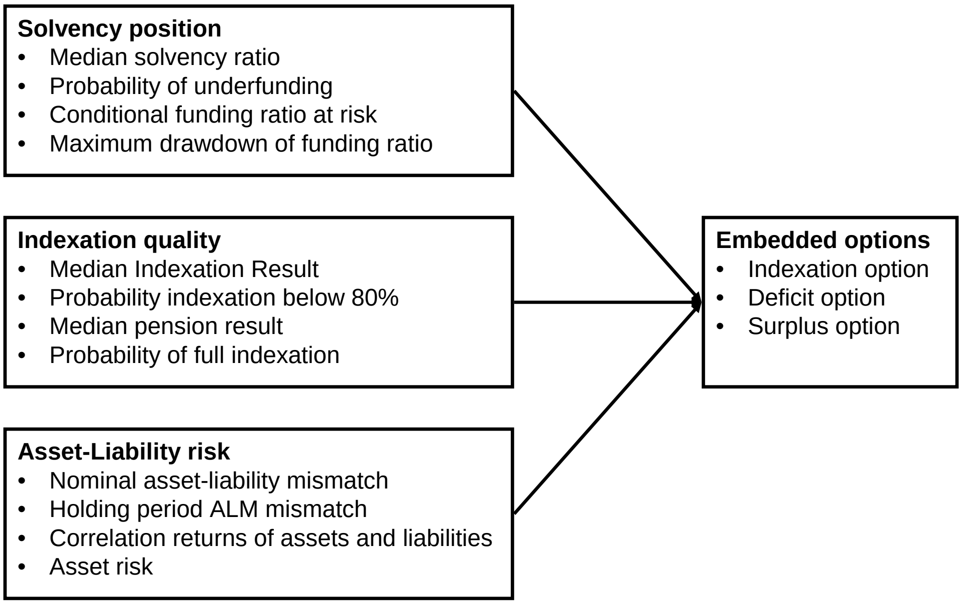
With hybrid and incomplete contracts, definition of the pension fund's preferences is somewhat fuzzy. Stated preferences often refer to providing a safe pension at low costs. A safe pension is seen as a DB pension that is fully indexed. Investing in long-term index-linked bonds would be the safest way to guarantee benefits. This option is usually rejected by pension funds as being too costly. Costs consist of the premiums paid by the participants. The pension fund (and thus implicitly its members) is willing to accept funding risk in order to benefit from the higher expected returns of stocks and other risky assets. Higher expected returns enable lower average contributions. Conditional indexation and limited adjustments in the contribution rate are the resulting risks for the participants in different age cohorts. This is the primary risk-return trade-off faced by the typical pension fund.[4]

[3] A non-Dutch example: in the Italian system, workers leave about 7% of their salary in the firm. For the firms this is a form of cheap debt on which it pays a low interest rate. Workers receive their claim to the TFR (*Trattamento di Fine Rapporto*) funds when they leave the firm.

[4] Although these are the stated preferences of many pension funds, it is not sure if these preferences

In practice, the process of decision-making in a multiple member and multiple objective pension plan depends on the pension fund governance, the financial position of the fund and risk attitudes. Stakeholders want more than merely a healthy financial position; they also prefer low contribution rates and high indexation and they demand insights into the intergenerational value transfers in collective schemes. Furthermore, there is not just one single investment horizon, but there are many. Pension payments, inflation compensation and solvency positions are considered at many points in time. Regulatory frameworks also impose requirements on the short-term solvency and the long-term continuity of the pension fund.

Figure 2: Evaluation Scorecard



Source: Hoevenaars (2008).

A scorecard approach can help in evaluating an asset allocation strategy from various perspectives. Boender (1997) and Hoevenaars, Molenaar and Steenkamp (2003) describe the implementation of this approach. For each policy alternative, many scenarios should be stimulated for asset returns and the macroeconomic environment. The consequences of each policy are presented to the pension fund board in the form of a wide range of statistics regarding different aspects of the policy. Hoevenaars (2008) presents an example

are shared by participants. Participants may have a larger preference for guaranteed indexed benefits the closer they get to retirement.

of a scorecard. His evaluation criteria (see figure 2) are considered to be representative of an average collective hybrid DB/DC pension scheme.

The measures evaluate the investment strategies in the short run (one year) and in the long run (ten years). The simulated distributions are characterised by median, standard deviation and extreme downside risk. Important criteria for the solvency position are the median funding ratio, the probability of underfunding in any particular year and the overall probability that the funding ratio falls at least once below 100% within a period of ten years. The conditional funding ratio at risk represents tail risk as the expected funding ratio in the 2.5% worst-case outcomes.

From the participant's perspective, the indexation quality is summarized by the probability of full indexation, the indexation result and pension result. The latter incorporates both nominal liabilities and indexation. A pension result of 90% means that participants who have 40 years of service receive 90% of the 80% average wage promise as a pension payment (therefore 72% of the average wage). The indexation result excludes the nominal liabilities and focuses solely on the appointed indexation. The holding period mismatch risk is the volatility of geometric funding-ratio returns under the policy settings. The maximum drawdown measures the largest fall of the nominal funding ratio in the next year in percentage points.

Embedded options put prices on the various risks. Instead of producing pure statistical measures, the embedded options attach an economic value to risks. Kocken (2006) identifies the employer guarantee, the option to default and the inflation indexation option. The indexation option is the present value of the conditional inflation indexation over the next one- to ten years. The evolution of indexation rights are determined along each scenario path and valued using a stochastic discount factor. The stochastic discount factor reflects the price of each scenario. Kortleve and Ponds (2006) and Hoevenaars and Ponds (2006) value the surplus and deficit of the fund for different age cohorts among the participants in order to gain insight into the value transfers between current and future generations. For instance, a high deficit option value ten years from now and a high indexation option for the coming years indicates that current beneficiaries receive high indexation benefits at the expense of future beneficiaries, who face large downside risks.

With incomplete markets, the valuation of the embedded options requires a utility function for the part of the options that cannot be replicated by traded financial instruments. This generates a circular reasoning, when the scorecard is used to elicit preferences from the board by showing them the outcomes. De Jong (2008b) provides a detailed analysis.

Although the scorecard contains many evaluation criteria, none of them explicitly relates to the preferences of the participants. It is assumed that participants have homogeneous preferences. Furthermore, the embedded options distinguish between age cohorts, but do not look at other differences between individuals. Heterogeneity in preferences is difficult to cope with for a collective fund that invests on behalf of all of its participants. One option is a flexible retirement age, which is already part of many pension contracts. Individuals or households with a strong preference for leisure could retire early by accepting actuarially fair lower benefits. The average preference for early or delayed retirement could affect the optimal investment portfolio of the pension plan, as it determines the effective duration of the investment horizon.

For tractable quantitative derivation of optimal investment plans of a pension fund, the preferences are usually simplified to a unidimensional criterium function. The two most popular choices are utility of wealth or utility as a function of the funding ratio,

$$u_F = \mathsf{E}_t\left[\frac{(A/L)_T^{1-\gamma}}{1-\gamma}\right], \tag{1}$$

$$u_W = \mathsf{E}_t\left[\frac{(A-L)_T^{1-\gamma}}{1-\gamma}\right], \tag{2}$$

where $\gamma \geq 1$ is a risk-aversion parameter, A are the assets and L the liabilities of the fund. Both formulas have the obvious property that the fund prefers high assets compared to liabilities; both also assign more weight to the downside than to the upside. The second formulation does not accept any underfunding at all, as the utility function is not defined for negative net wealth $A - F < 0$.[5]

A drawback of both formulations is the absence of the pension contributions in the objective functions. As argued before, the reason that pension funds accept mismatch risk is that taking this risk allows them to offer the same benefits at lower expected costs. A slightly more involved formulation is adopted by Van Binsbergen and Brandt (2007), who explicitly account for the requests for additional contributions from sponsor or members,

$$U_{ALM} = \mathsf{E}_t\left[\beta^T \frac{(A/L)_T^{1-\gamma}}{1-\gamma} - \lambda \sum_{t=1}^{T} \beta^t c_t\right], \tag{3}$$

where c_t are the additional contributions above the base level that the fund requires.

Simple formulations with a single risk-aversion parameter such as (1) have more drawbacks in intertemporal portfolio problems. The emphasis on the final wealth or funding ratio ignores intermediate payouts. In the context of individual investors, intermediate consumption may affect optimal portfolio choice. Most of the effects occur when preferences

[5] The functional form of the utility is Constant Relative Risk Aversion (CRRA).

from risk, measured by γ, are separated from preferences of intertemporal substitution of consumption. Campbell, Chan and Viceira (2003), among various other studies, therefore specify Epstein-Zin preferences. This two-parameter specification of preferences gives separate control over the decisions on how much to save and consume, and on what the allocation of wealth will be over asset classes. Since this paper concentrates on the asset allocation decision, and abstains from studying overall consumption smoothing and the level of pensions, we will not elaborate on the intertemporal rate of substitution.

Most preference functions relate to the absolute level of wealth or consumption. Other preference functions consider the relative wealth or consumption of one individual (or group) relative to another individual (or group). An example is the hypothesis of external habit formation, used in Campbell and Cochrane (1999), which evaluates a person's consumption relative to that of someone else. The idea is that bad outcomes are not so bad if we are collectively poor. The hypothesis has been characterised as "catching up with the Joneses". This paper ignores such relative evaluations of pension results for the collective and large pension funds under consideration here.

A functional form with constant relative risk aversion is convenient for analytical tractability and also ensures scalability and time-series comparability, in the sense that the optimal portfolio shares do not depend on the trend growth in wealth or GDP. The CRRA form also does not depend on the size of the pension fund. It is thus equally applicable for small and large funds and in times with high or low aggregate wealth in the economy.

The utility function weights the objectives of the individual investor or the pension fund in a particular way. This is a simplification, but it is difficult to build a full-fledged objective function for a pension fund with heterogeneous members and a sponsor. The advantage of the utility function approach is the tractability, which allows in many cases for insightful analytical solutions to the optimal portfolio problem. Another limitation is that the paper abstracts from regulatory constraints (such as solvency- or for the Dutch case FTK rules). Again, this buys us more tractability of the optimal investment problem.

3 Long-term investments theory

This section of the paper reviews the long-term investments literature, as it was presented in the book by Campbell and Viceira (2002), and as it has developed since then. Investors saving for retirement have a long investment horizon. For a young individual, this may even exceed 60 years. A typical pension fund has liabilities with a duration (that is, the value-weighted average time to payout) of 15 to 20 years. Moreover, this duration is often (at least partly) in real terms (that is, the liabilities are indexed or, equivalently, form a short position in a real fixed-income portfolio). For long-term investors, several risks that are negligible in the short run become very important in the long run. The most important long-run concerns are interest rate risk (Section 3.1) and inflation risk (Section 3.2). However, the long-term investor may also be able to profit from predictability in stock returns and to exploit that predictability to get a better risk-return trade-off (Section 3.3).

3.1 Interest rate risk

Traditional investment theory, along with "modern portfolio choice" of Markowitz, considers static models with, generally, investment horizons of up to one year at most. In such models, short-term deposits ("cash" or treasury bills) act as an essentially risk-free asset. It is important to realize that -also in this case- the investment horizon need not necessarily equal the maturity of the cash or bills that are available for investment. If the investment horizon is larger, then the bills would have to be rolled over, which induces a risk. We first analyze the amount of risk involved in such fixed-income roll-over strategies.

For the traditional investment horizons, rolling over the position does induce some (although limited) risk. This is due to the empirical fact that short-term interest rates are fairly persistent (meaning that short-term rates are not expected to change too much over the course of, say, one year). At longer horizons, however, risks increase significantly. This can be illustrated (and formalized) using a simple (one-factor) interest rate model. Consider one-month treasury bills. Denote the (annualized) interest paid on these bills in month t by r_t. In order to allow for some mean reversion in the interest rate, but acknowledging its high persistence, we model r_t as an autoregressive process of order 1, with a high (that is close to unity) autoregressive parameter. In financial parlance, this boils down to the use of the Vasicek term-structure model. Formally, we have

$$r_t = \mu + \rho \left(r_t - \mu \right) + \sigma \varepsilon_t. \tag{4}$$

In order to illustrate our ideas, we use some typical parameter values. We take a long-run average short-term rate of $\mu = 4.0\%$. A typical mean-reversion parameter (at the monthly frequency we use) is found to be $\rho = 97.5\%$. Finally, interest rates move in our model with a volatility of $\sigma = 0.30\%$, implying that they move with at most (using a 95% level) 60 basis points (two times 30) per month. For illustrative purposes, we also assume that the current short-term rate is relatively low and equals $r_0 = 2.5\%$.

Figure 3: Confidence interval short-term interest rate

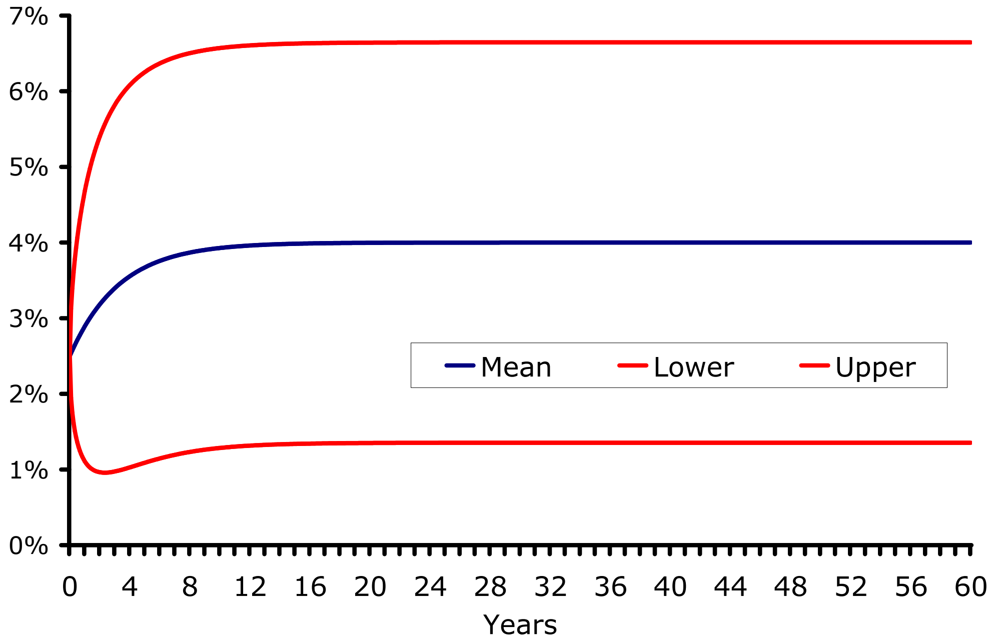
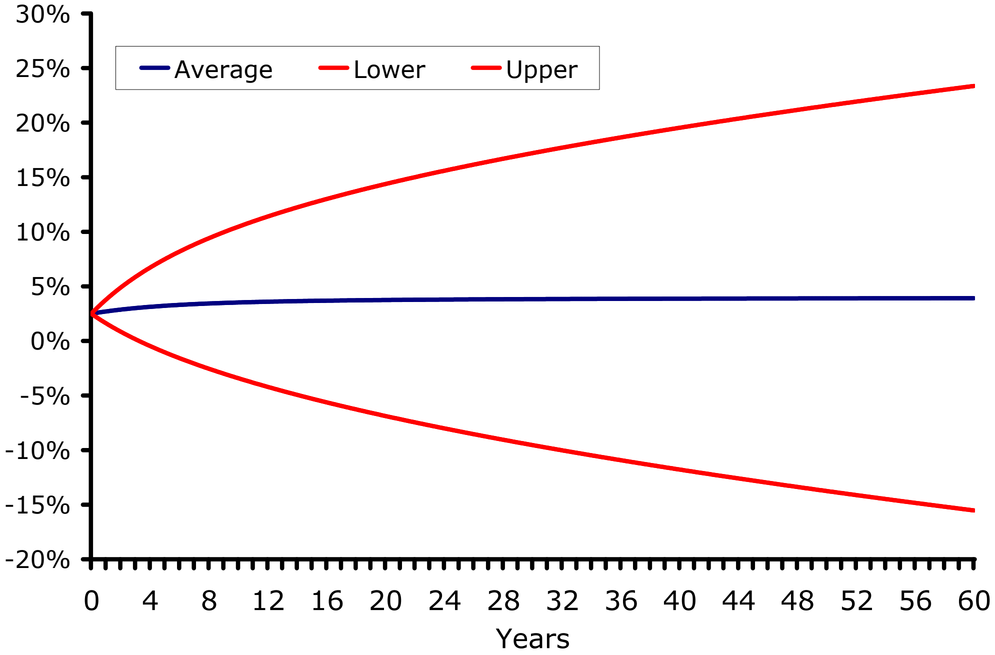


Figure 3 shows confidence intervals for the evolution of *interest rates* over time.[6] First of all, note that the confidence interval's lower bound is not monotonically decreasing. This is a result of the low initial short rate. The mean reversion in the model induces a positive drift (as can also be seen from the evolution of the mean short-term rate over time). Moreover, note that, as of a forecasting horizon of about 12 years, the process has converged to its so-called "stationary distribution". After this amount of time, the predicted mean and volatility remain constant. Phrased differently, the current value of the short-term rate is uninformative of its value over such long horizons. The mean reversion in the model thus induces some serial dependence between short-term rates at small horizons, but at long horizons the short-term rates behave essentially independently. We will see below that the amount of time needed to get essentially independent interest rate behavior is of crucial importance for portfolio decisions.

More important from a risk-management perspective is the risk-return behavior of the roll-over strategy. In such a strategy, money is invested each month against the then-prevailing one-month treasury bill rate. Such an investment is risky, as future interest rates are not known. Figure 4 shows the (annualized) expected return and the (annualized)

[6] The Excel file with the necessary calculations is available from the authors.

Figure 4: Roll-over risk



The figure shows the annualized expected return and volatility of a monthly roll-over strategy in treasury bills for typical parameter values.

volatility of such a strategy.

Note that the expected return converges fairly quickly to the long-term average short-term interest rate of 4% in this example. Given the low initial interest rates, the mean reversion induces a strong upward drift in the interest rates, which leads to a steep increase in the expected return at small horizons. However, while over short horizons the risk is fairly small, it builds up quickly over long horizons. With an horizon of about 30 years, the roll-over strategy has an annualized volatility of about 14% not far from stock index holdings.

For a long-horizon investor, an investment in a long-term bond (with exactly the same maturity as his investment horizon) is (obviously) the appropriate risk-free asset, as it guarantees a fixed payoff (in nominal terms). However, an important fact of life for such long-term investors is inflation risk, which is the subject of Section 3.2.

3.2 Inflation risk

Investors are, of course, interested less in the nominal (dollar or euro) value of their investment, and more in the purchasing power of their wealth. For a short investment horizon (say a year), inflation is largely predictable and the real payoff is just the nominal payoff minus a given (which is known at the beginning of the investment period) expected inflation. In the long run, however, inflation uncertainty accumulates to levels that cause it to become an important risk factor. The main reason for this is the fact that inflation shocks are very persistent: an unexpected increase in inflation this year implies high inflation levels in subsequent years as well. This cumulative effect makes inflation a much more important source of risk in the long run than it is in the short run. It also makes long-term nominal bonds a poor investment vehicle: although they do hedge interest rate risk, the exposure to inflation is too large. Clearly, a solution to this is the use of real long-term bonds (to the extend that they are available on the market or can be reconstructed using other investment classes). The issue of reconstructing real bonds from nominal ones will be explored below.

As mentioned in the introduction, investment classes exhibit different exposures to various (financial of macroeconomic) risk factors. The investor aims to optimally choose exposures (both for myopic risk/return reasons and to hedge given background risk), and then to select the investment portfolio accordingly. This two-step approach is also key in the underlying mathematics of dynamic portfolio choice (the so-called martingale method): first obtain the optimally desired exposures to risk factors and then construct an investment strategy that realizes these exposures. If many asset classes are available, it's possible that the desired exposures can be obtained in many ways, which opens the possibility of secondary arguments (like liquidity) playing a role.

Brennan and Xia (2002), BX02 from now on, was one of the first papers to study dynamic asset allocation while taking inflation risk explicitly into account. Modelling inflation (and thus distinguishing between nominal and real interest rates) and observing that (as mentioned above) investors care about real purchasing power, shows that all investors have an implicit exposure to inflation risk that they would like to hedge. For short-term investors, once more, inflation is largely predictable and thus inflation *risk* is negligible. These investors can safely ignore this risk (but need to account for the current inflation level when optimizing their portfolio). To illustrate this literature, we will discuss the BX02 model in more detail.

BX02 considers a two-factor term structure[7]. The relevant risk factors are (expected) inflation and the real interest rate. Nominal *short* interest rates follow endogenously from the model, and are, as we shall see, in the simplest version of the model actually equal to the real short-term rate plus expected inflation (this is not true for longer maturity yields).

[7] A version of this model is available in the Tilburg Finance Tool, which is available from the last author's website: center.uvt.nl/staff/werker.

Inflation is modelled as a price index Π_t evolving according to

$$\mathsf{d}\Pi_t = \pi_t \Pi_t \mathsf{d}t + \sigma_\Pi \mathsf{d}W_{\Pi t}, \tag{5}$$

$$\mathsf{d}\pi_t = -\alpha_\pi (\pi_t - \bar{\pi}) \mathsf{d}t + \sigma_\pi \mathsf{d}W_{\pi t}. \tag{6}$$

The price index thus has a drift that is itself time varying. This drift is called the expected inflation. The model thus allows for periods of increasing inflation rates and periods of decreasing inflation rates. These movements generally take place at lower frequencies (that is, they are generated by persistent processes, with a small mean-reversion coefficient α_π). This will be discussed in more detail below.

The second factor in the BX02 model is the real short-term interest rate, which is denoted r_t and assumed to be described by

$$\mathsf{d}r_t = -\alpha_r (r_t - \bar{r}) \mathsf{d}t + \sigma_r \mathsf{d}W_{rt}. \tag{7}$$

Note that an implication of these processes (called Ornstein-Uhlenbeck processes) is that real interest rates and expected inflation levels are normally distributed. This implies that, theoretically, these models allow for negative interest rates. As long as the parameters are such that the probability of this happening is small, the models may still be used in practice. Also, due to this normality property, the models are analyzed easily and are part of the so-called affine term-structure models.

BX02 show formally that the short-term nominal interest rate is given by

$$r_{nom,t} = r_t + \pi_t - \lambda_\Pi, \tag{8}$$

where λ_Π is a risk premium for unexpected inflation shocks. If the Fisher equation would hold, we would have $\lambda_\Pi = 0$, which is essentially imposed in much of the literature. An equation like (8) also holds for longer yields if two adaptations are made. First, the coefficients of the real short-term rate and expected inflation are no longer equal to unity and, second, the risk premium is no longer zero even if the Fisher hypothesis holds for short-term rates. Thus, in order to complete the model, we need to describe how the market compensates for the risks involved (expected inflation risk, realized inflation risk, and real interest rate risk). This compensation follows from the risk-aversion of the representative agent. As always, the agent is only compensated for risks that cannot be hedged away. No data is necessary on real bonds in order to derive the market risk-premium on expected inflation risk. The reason for this is simple: standard nominal bonds are sensitive to changes in both the real interest rate and expected inflation.

Various observations may be made concerning the empirical evidence in the BX02 model. First of all, the literature finds invariantly that expected inflation is much more persistent than the real interest rate. To be precise, BX02 estimates $\alpha_\pi = 0.027$, while $\alpha_r = 0.631$.[8] As a result (compare the discussion in Section 3.1), shocks to the real interest rate have only a short-term effect (as they die out relatively quickly, bringing the real interest rate back to its stationary distribution). Long-term investors should thus be (primarily) concerned with expected inflation risk. Clearly, this is hampered by the fact that it may not always be easy to hedge this risk completely. We discuss this in more detail below, after concluding our discussion with the other empirical findings.

The long-run real interest rate level is found to be $\bar{r} = 1.2\%$, while the long-run expected inflation is estimated at $\bar{\pi} = 5.4\%$. This latter number seems, currently, to be very high. Recall, however, that the estimates that were obtained include the Volcker period (with extremely high inflation rates). A general problem of these models is that estimates are fairly sensitive to the data period used. The volatility of the short-term real interest rate is estimated at $\sigma_r = 2.6\%$, while the expected inflation is less volatile, with $\sigma_\pi = 1.4\%$. Nevertheless, these levels induce quite some risk in the long run.

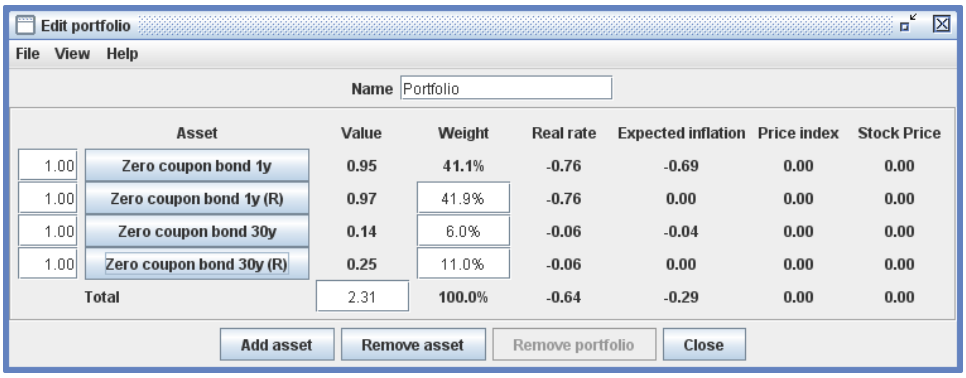
Two important parameters are the prices of risk involved with both risk factors. As is well known, bearing (systematic) risk in any asset class is compensated by a higher expected return on that asset class. While stock indices earn a positive risk premium, the price of risk for the real interest rate and expected inflation is negative. Since bond returns are inversely related to interest rate changes, note that this still implies positive risk premiums on bonds, both nominal and real. The reason for the negative prices of risk on the real-term rate of inflation and expected inflation is that people are averse towards increases in both, while, concerning stocks, people are averse towards decreases in the index level. This aversion leads people to pay a risk premium to insure them against these bad outcomes; this is reflected in a higher discount rate. BX02 estimates the real-term rate price of risk at $\lambda_r = -21\%$ and for expected inflation $\lambda_\pi = -11\%$. Although it's rather difficult to interpret these numbers directly, using the Tilburg Finance Tool makes it easy to get an idea of the effect on the term structure (both nominal and real) of given prices of risk.

Tilburg Finance Tool Example

This section discusses in greater detail the effect of both the real interest rate and expected inflation levels on real and nominal yields. In a two-factor term structure model (as described above), all yields are known (affine, linear) functions of the underlying factors. As a result, as soon as two yields are observed, the prevailing real interest rate and

[8] We report their estimates obtained over the period 1970-1995.

Figure 5: The sensitivity of (real and nominal) bonds to the real rate and expected inflation. (Source: Tilburg Finance Tool)



Edit portfolio

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Name Portfolio

	Asset	Value	Weight	Real rate	Expected inflation	Price index	Stock Price
1.00	Zero coupon bond 1y	0.95	41.1%	-0.76	-0.69	0.00	0.00
1.00	Zero coupon bond 1y (R)	0.97	41.9%	-0.76	0.00	0.00	0.00
1.00	Zero coupon bond 30y	0.14	6.0%	-0.06	-0.04	0.00	0.00
1.00	Zero coupon bond 30y (R)	0.25	11.0%	-0.06	0.00	0.00	0.00
	Total	2.31	100.0%	-0.64	-0.29	0.00	0.00

Add asset Remove asset Remove portfolio Close

expected inflation level are observed as well. Reality is, of course, different as no pair of yields is perfectly dependent on any other pair of yields. However, such a model still provides a useful framework that can be used to consider hedging relevant risks. Another consequence of the above observation is that, as soon as yields on two traded bonds are available, any exposure combination to both risk factors can, theoretically, be obtained. Given the above-mentioned difference in the persistence of the real interest rate and expected inflation, it seems reasonable to use a short- and a long-maturity nominal bond as they differ most in their individual exposures to the risk factors. This is illustrated in Figure 5.

The figure shows that all bond prices have negative exposures to both the real interest rate and the expected inflation factor. Clearly, real bonds have no exposure to the expected inflation factor: changes in (expected) inflation are one-for-one compensated in the payoff of the bond and hence do not affect prices. It's also clear that bond prices are more sensitive to changes in the real interest rate than in the expected inflation level although for short bonds this effect is more pronounced than for long bonds. For long bonds, the sensitivity to expected inflation is about two-thirds of that to the real interest rate, while for short bonds it's about 90%. It's exactly this difference in sensitivities for long- and short bonds that enables a portfolio strategy that combines positions in both to concurrently provide the required real interest rate exposure and the (expected) inflation risk exposure.

While this latter idea is fairly straightforward, it gives rise to an important practical issue. In standard life-cycle models, the desired exposures may necessitate shorting bonds. This can be costly, or even impossible, for individuals or pension funds. These issues are discussed in detail in Koijen, Nijman and Werker (2007a). One way out would be to allow for investment in interest rate derivatives, which may lead to sizeable factor exposures with relatively small investments.

3.3 Predictability

The discussion above that long-horizon investors face additional risks (e.g., in terms of inflation). However, there are also opportunities. Several studies maintain that stock returns are predictable (for example, from the dividend-price ratio), and have argued that this predictability implies a better risk-return trade-off in the long run than in the short run. The interested reader is referred to, among others, Campbell and Viceira (2005) and Campbell and Viceira (1999).

During the eighties, the most popular model for stock prices was the random-walk model. In this model, by construction, stock returns are not predictable in any way. At that time there was no way to reject this hypothesis in favor of predictability. The reasons for this are twofold. First, at the time, the available data series covered a relatively short period and/or were of poor quality. Secondly, formal time series analysis had not developed the rich set of techniques that exists nowadays, especially concerning very persistent explanatory variables.

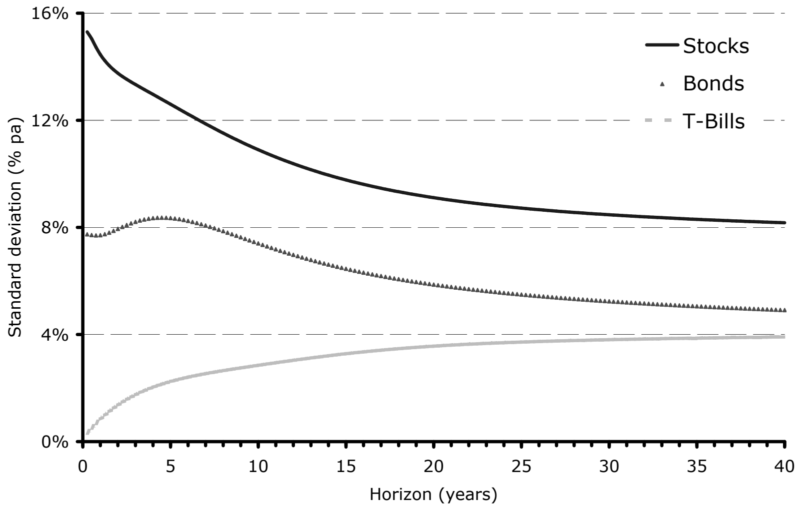
Currently, the view is different. A number of papers document some predictability of stock returns. Predictability has two effects. First, most forms of predictability imply long-term mean reversion, which allows for time diversification. This occurs if higher order autocorrelations of stock returns are mostly negative. In this case, the variance of cumulative returns increases less than linearly with the investment horizon, making stocks less risky for long-term investors and making them attractive for hedging. The second effect of predictability is that the investment opportunities change over time, which prompts the investor to make frequent portfolio adjustments in the speculative portfolio.

Mean reversion has been well documented, and disputed, over the last twenty years.[9] Campbell and Viceira (2005) summarise the mean reversion effects in a single compelling figure. Figure 6, based on our own computations, reproduces their results for slightly different data and model specification. It shows the per annum conditional standard deviation

$$V(k) = \sqrt{\frac{1}{k}\,\mathsf{Var}\left[x_{t+k}^{(k)}\right]} \tag{9}$$

[9] A standard textbook reference for the early literature includes chapters 2, 7 and 8 in Campbell, Lo and MacKinlay (1997).

Figure 6: Term Structure of Risk



The figure shows the term structure of risk implied by a 2^{nd} vector autoregression for stock returns, bond returns, real T-Bills, nominal T-Bills, dividend yield, term spread and credit spread. Data are quarterly for the US. Details of data, estimation and the definition of the term structure are in Schotman, Tschernig and Budek (2008).

of stocks, bonds and bills as implied by an estimated vector autoregression. From the empirical VAR models it appears that the long-term risk of equity is only half the short-term risk. Hence for long-run investors stocks are a good hedge against their own risk.

It is important to realize that the amount of predictability is often relatively small in terms of regression R-squares. Typical values range from 3–8%. Although such values are of academic importance, their practical relevance is debatable. Moreover, predictability of stock returns is often based on economic variables that are very persistent. Often-used predictors include price-dividend ratios, interest rates and inflation. This persistence leads to a low variation in the predictive variable and, as a result, imprecisely estimated regression coefficients. We return to these issues in Section 5.

3.3.1 Time-varying bond risk premiums

While enjoying the predictability in stock returns, long-horizon investors can also profit from the predictability in bond returns, or more precisely, in bond risk premiums. The Brennan and Xia (2002) model discussed in section 3.2 is, by now, a standard model with which to study the optimal life-cycle investment of an individual. In the absence of intergenerational risk-sharing, this also solves the optimal portfolio (or, once more, exposure) allocation of pension funds. Key in these models is the imposed preference structure for individual consumers. Also, to obtain realistic settings, the models should allow for human capital depletion over the life-cycle and housing needs. Both of these lead to additional risk, but also to additional flexibility in subsuming financial market shocks as long as labour force participation can be adapted and the housing market is sufficiently flexible. These issues are discussed in detail at the end of this paper.

The key risk factor here is inflation risk. Standard life-cycle models use the expected utility framework to describe the preferences of individuals. Although many shortcomings of this framework have been revealed (leading to alternative preference specification including, for instance, habit formation and peer pressure), it's still the standard financial paradigm. In the expected utility framework, the individual maximizes the expected utility of retirement wealth. Several observations are important here. First, the utility index function should be chosen so as to allow for empirically observed levels of risk aversion. A standard CRRA utility function is often used for ease of analysis. Such a choice leads to mathematical homogeneity properties that facilitate the analysis significantly.[10] Second, it is often not so much retirement wealth that is important, but the associated annuity income. In many countries retirement wealth is replaced, either partly or fully, by an annuity income. This is advantageous, as it allows the individual to hedge the risk of mortality. This effect is discussed in detail in Koijen, Nijman and Werker (2007c). However, note that one reason not to convert all retirement wealth into annuities is precautionary savings to hedge (costly) health shocks. Finally, and most importantly for this section, the investor is interested in his or her wealth in real terms, rather than in nominal terms. As such, the BX02 indeed derives a hedging component with which to hedge inflation risk. This component leads positively on inflation, exchanging extra returns when inflation is high (when you need more money in nominal terms to achieve the same wealth level in real terms) in return for smaller returns when inflation is low. Clearly, as "everybody" (or at least the representative investor) follows a similar strategy, such a hedging strategy is costly. The reduced risk, however, outweighs the costs. Both BX02 and Campbell and Viceira (2002) find that the size of this hedging portfolio is not overly affected by the individual's risk aversion. Risk aversion plays a role primarily in the myopic component of the demand for risk exposures. Clearly, very risk-averse agents will not favor a great deal of exposure in this myopic term, whereas they still desire risk exposures in their asset menu to offset their inflation risk. In this very simple setting, extremely risk-averse agents would like to hold real bonds only, hedging away any possible inflation risk (but keeping

[10] Dynamic life-cycle problems are extremely computer intensive as soon as a realistic number of state variables is used.

real interest rate risk, which is not punished in such utility functions).

3.4 Optimal Portfolios

The long-run risks discussed in this chapter have a profound impact on the strategic asset allocation. Brennan and Xia (2002) and De Jong (2008a) consider a setting with interest rate risk and inflation risk, but without predictability. The objective of the long-term investor is to maximize the expected utility of real wealth at a date T

$$\begin{aligned} V &= \max \mathsf{E}\left[U(W_T/\Pi_T)\right], \\ U(x) &= \frac{x^{1-\gamma}}{1-\gamma}, \end{aligned} \tag{10}$$

where W_T is nominal wealth and Π_T is the price level. The time T can be interpreted, for example, as the retirement date. The optimal portfolio of a long-term investor is then

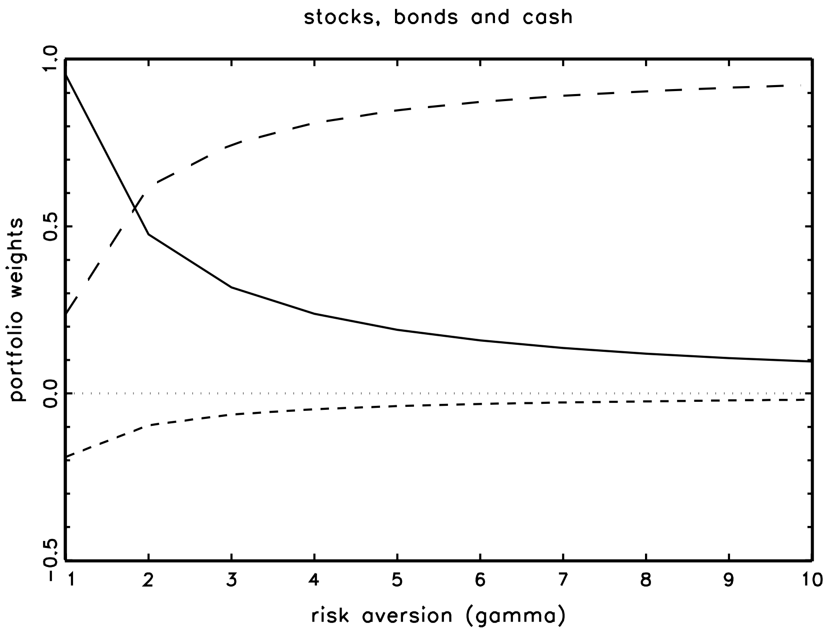
$$w_t = \frac{1}{\gamma}\Sigma^{-1}\mathsf{E}[R_t - R_t^f] + \left(1 - \frac{1}{\gamma}\right)\Sigma^{-1}\mathrm{Cov}(R_t,\, R_t^{ILB_T}), \tag{11}$$

where Σ is the covariance matrix of the asset returns, $\mathsf{E}[R_t - R_t^f]$ is the vector of (instantaneous) risk premiums (expected asset returns in excess of the risk-free rate), and $\mathrm{Cov}(R_t,\, R_t^{ILB_T})$ is the covariance between the asset returns and the returns on an index-linked bond with maturity T. The optimal portfolio for a long-term investor has two components: a *speculative* component and a *hedge* component. The speculative portfolio is the usual Markowitz' mean-variance optimal portfolio. This portfolio exploits the risk premiums on all assets and optimizes the risk-return trade-off. The hedge portfolio is a portfolio of assets that hedges optimally against the long-term risks. An index-linked bond (ILB) that matures at time T would be the ideal hedge instrument. Hence, if such a bond is available, the hedge portfolio would consist solely of index-linked bonds. If an ILB that matures at time T is not available, the hedge portfolio tries to replicate the return on the (hypothetical) ILB as best it can. The hedge portfolio composition $\Sigma^{-1}\mathrm{Cov}(R_t,\, R_t^{ILB_T})$ can be viewed as the slope coefficients β in a multiple regression of the ILB return on the asset returns:

$$R_t^{ILB_T} = \alpha + \beta' R_t + \varepsilon_t \tag{12}$$

The vector β is multiplied by $1 - \frac{1}{\gamma}$ to obtain the hedge portfolio weights. The weights given to the speculative portfolio and the hedge portfolio reflect the preferences of the investor. For the pension fund that wishes to provide a safe pension, all wealth would be allocated to the ILB. The higher the risk tolerance, the more the investment moves towards the optimal mean-variance portfolio.

Figure 7: Optimal Portfolio Weights



The figure plots the optimal portfolio weights as a function of risk aversion. The solid line is the weight in stocks, the line with long dashes the weight in bonds, and the line with short dashes the weight in cash.

The long-term optimal portfolio has the property that the degree of risk aversion determines how much the investor allocates to stocks, bonds and the risk-free asset. A typical graph is Figure 7. The left-hand side of this graph shows the portfolio allocation for an investor with $\gamma = 1$, who has only the speculative asset demand and no hedge demand. This investor's portfolio is dominated by equity and only a small fraction of bonds. The investor also borrows money (holds a short position in cash) to finance his speculative investments. For risk aversion larger than one, the hedge portfolio starts to get a greater weight. The example is constructed such that the hedge portfolio consists entirely of (index linked) bonds. For greater risk aversions, the bond allocation increases and crowds out the equity investments. For example, with $\gamma = 5$ (which is a value often chosen in this literature for long-term investors), the equity weight in the portfolio is 25% and the bond weight is 80%, with a small short position in cash.

Campbell, Chan and Viceira (CCV, 2003) have a more general and flexible setup. They allow for interim consumption and the more general Epstein-Zin utility function. They also allow for time-varying interest rates and very general predictability of asset returns. They use a discrete time first-order Vector AutoRegressive (VAR) model for the dynamics of asset returns and predictive variables. Their model is

$$z_{t+1} = Az_t + u_{t+1}, \tag{13}$$

where z_t is the state vector

$$z_t = \begin{pmatrix} r_t \\ x_t \\ s_t \end{pmatrix}, \tag{14}$$

with r_t the short-term interest rate; x_t the vector of logarithmic asset returns in excess of the risk-free rate; and s_t a vector of variables that are used to predict the asset returns. CCV assume that the optimal portfolio is linear in the state variables

$$x_t = A_0 + A_1 z_t. \tag{15}$$

As in the Brennan and Xia model, the optimal portfolio can be seen as a weighted average of a speculative portfolio and a hedge portfolio

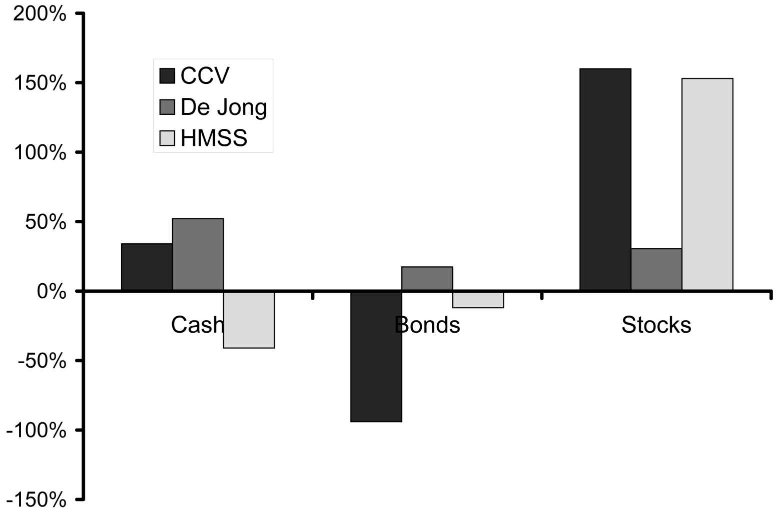
$$w_t = \frac{1}{\gamma}\Sigma^{-1}(\mathsf{E}_t[x_{t+1}] + \tfrac{1}{2}\mathrm{Var}(x_{t+1})) + \left(1 - \frac{1}{\gamma}\right)\Sigma^{-1}\mathrm{Cov}(x_{t+1}, c_{t+1} - w_{t+1}), \tag{16}$$

where $c_{t+1} - w_{t+1}$ is the logarithm of the consumption-wealth ratio. This solution is only approximately analytic, since it relies on a log-linear approximation of the investor's budget equation and log-normal approximations to expected returns.

Adding more asset classes to the investment menu may result in additional diversification opportunities for the speculative portfolio, and additional correlations with the long-term risks (especially inflation) that are important for the hedge portfolio. Hoevenaars, Molenaar, Schotman and Steenkamp (HMSS, 2008) add four alternative asset classes to the VAR of CCV: corporate bonds, listed real estate, commodities and hedge funds. An ILB is not available in their asset menu. From the long-run covariance properties of the returns they conclude that commodities are a valuable alternative asset class in the hedge portfolio. Commodities provide risk reduction through diversification, since they have positive correlation with inflation and low correlation with stocks and bonds at any investment horizon. Of negligible value is listed real estate, due to its high correlation with the basic asset classes of bills, bonds and equity. Hedge funds also do not help in hedging the liabilities risk, since their returns are volatile and hardly correlated with interest rates.

HMSS also consider the effect of the objective function. They compare the portfolios from the asset-only perspective of De Jong (2008a) and CCV (2003) with an asset-liability perspective based on the funding ratio metric in (1). They use an approximate mean-variance solution to decompose the portfolio for the asset-liability investor in a speculative

Figure 8: Portfolio weights in three different models



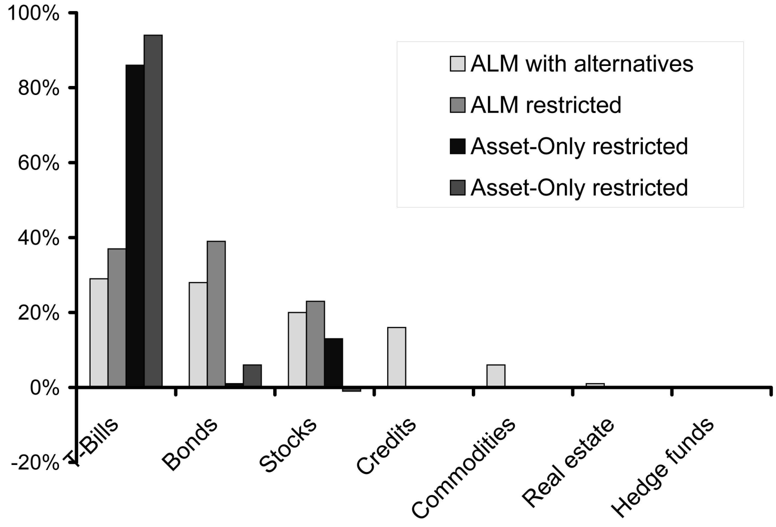
Source: Campbell, Chan and Viceira (CCV, 2003), De Jong (2008) and Hoevenaars et al. (HMSS, 2008)

portfolio and a hedge portfolio with weights $\frac{1}{\gamma}$ and $1 - \frac{1}{\gamma}$. The hedge portfolio is based on the covariance of the asset returns with the return on the liabilities. Since HMSS assume that the pension fund offers full indexation, the liabilities are subject to both inflation risk and interest rate risk.

Figure 8 compares the implications of the three models. Results in the figures are for $\gamma = 5$ and an asset menu excluding index-linked bonds. The available assets for all models are restricted to equity, T-Bills and long-term nominal bonds. In figure 8 the portfolio weights are from the perspective of an asset-only investor. For HMSS the utility function is the same as in De Jong (2008a). The investment horizon in both is 20 years.

The portfolio of CCV is dominated by equity, with an average portfolio weight of more than 100%. Equity is attractive in this model, due to the large equity premium (7.7%) in historical data and the mean reversion in equity returns. The equity premium induces speculative demand for equity, while the mean reversion makes equity attractive for hedging. Nominal long-term bonds have a negative portfolio weight, as the bond premium is very small (1.1%). Since stocks and bonds are positively correlated, the model implies that bonds are a good hedge for the equity risk in the portfolio. As shown, the portfolio weights in the figure are unrestricted. Adding short-sell constraints will certainly reduce the extreme positions in stocks and bonds and lead to a portfolio that is close to 100% in equity.

Figure 9: Alternative asset classes and the ALM perspective



Source: Hoevenaars et al. (HMSS, 2008)

With the restricted asset menu and from the asset-only perspective, the HMSS model is very similar to that of CCV, since the underlying vector autoregression is estimated on similar time-series data. It therefore also has a large equity premium and strong mean reversion. The weights implied by the model of De Jong (2008a) are far less extreme, since the equity premium in his model is much lower (3.2%) and stocks are more risky due to the absence of mean reversion.

Figure 9 highlights the effect of the alternatives and the ALM perspective in the HMSS model. The figure shows the optimal hedge portfolio ($\gamma \to \infty$) for the asset-only investor and the ALM investor. Portfolio weights for both investors are with and without the alternative asset classes. From the alternatives, the only two asset classes in the hedge portfolio are commodities and credits. Commodities are included because they have low correlation with stocks and bonds, but a positive correlation with inflation. Corporate bonds are a partial substitute for long-term government bonds. The asset-only portfolio is very different, as it is dominated by the real T-Bill. From the ALM perspective, both stocks and bonds are present in the liability hedge portfolio.

4 Recent advances

This section reviews some recent developments in the long-term investments literature and discusses the relevance for pension funds. The first section focuses on the impact of human capital, the second section on the ownership of a house.

4.1 Human capital

4.1.1 Human capital and portfolio choice

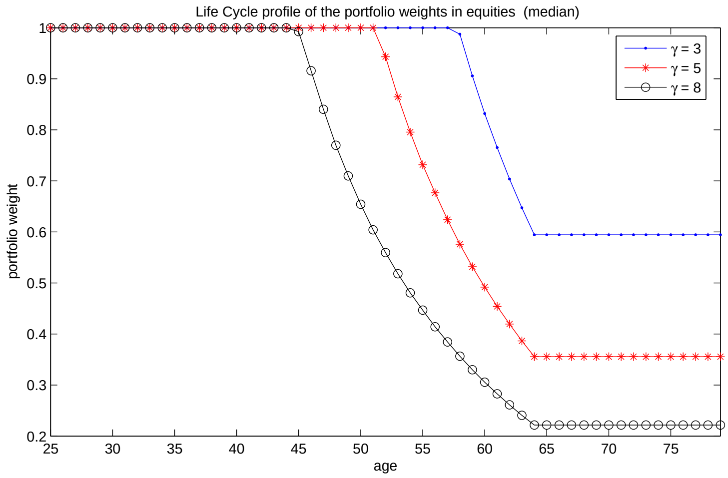
This section explores the impact of human capital on financial portfolio choice. At first, we will assume that labor income is riskless. Human capital then is the present value of future labor income, discounted at the risk-free rate. The only effect of human capital in this simple setting is a leverage effect, multiplying all of the financial portfolio weights with the ratio of total wealth over financial wealth. This keeps the risk exposure on the total wealth exactly as desired. For the sake of simplicity, this section assumes a simple asset menu with only stocks and a risk-free asset. In this setting, the optimal portfolio weight for stocks is

$$w^* = \frac{1}{1-h} \frac{\mu - R^f}{\gamma \sigma^2}. \tag{17}$$

In this formula, h denotes the fraction of human capital in total wealth; the leverage ratio $W/F = 1/(1-h)$, where F is financial wealth, H is human wealth, and $W = F + H$ is total wealth. The notation here is as follows: $\mu - R^f$ is the expected excess return (or risk premium) on the stock; σ is the volatility of the stock return; and γ is the coefficient of relative risk aversion of the investor. In a more complex setting with stocks, bonds and cash, human capital reduces the hedge demand for bonds because it provides an automatic hedge against real interest rate fluctuations.

For young investors, the leveraged portfolio weights often exceed 100%. Typically, the investor faces portfolio constraints, in the sense that the weights on all assets must be between zero and one. The optimal portfolio strategy has to take these constraints into account. An important observation is that the possibility of hitting the portfolio constraints in the future affects the consumption and portfolio weight decisions today. The problem with constraints therefore cannot be solved analytically. Instead, numerical methods for dynamic programming are needed to solve the optimal consumption/asset allocation problem. Figure 10 shows the optimal portfolio weights for the different levels of investor risk aversion over the life cycle. Notice that these are averages over many scenarios, as the fraction of human wealth to total wealth depends on the stochastic

Figure 10: Portfolio Choice with Human Capital



The figure plots the optimal portfolio choice for investors with constant labor income and different degrees of risk aversion. Source: Cui, De Jong, and Ponds (2007)

outcomes of the financial portfolio investments, and therefore h varies over time. The figure shows that the optimal portfolio weight declines with age. Young investors almost always invest all of their wealth in stocks. For middle-aged people, the portfolio weight gradually is reduced. After retirement, there is no longer any human capital; portfolio weights are thus fixed and equal to $(\mu - R^f)/\gamma\sigma^2$.

The argument has thus far assumed that labor income is riskless in real terms. In practice, labor income is somewhat correlated with stock returns. When human capital is not riskless, the results are somewhat different, although not dramatically so. The leverage effect is not changed at all, and the hedge demands can be adjusted easily, as follows. Let β_H be the regression coefficients of a regression of human capital returns (i.e. labor income growth rates) on stock returns. Then, the demand for stocks is

$$w^* = \frac{1}{1-h}\left[\frac{\mu - R^f}{\gamma\sigma^2} - h\beta_H\right]. \tag{18}$$

The second part of this expression is a hedge term of the correlation between labor income and stock returns. With a higher correlation, the demand for stocks is reduced. Empirical estimates on quarterly data for the US and the Netherlands in De Jong (2008a) show that the correlation between wages and stock returns is positive but rather small, with an estimated human capital beta $\beta_H = 0.18$.

The overall implication of this analysis is that young investors (with a high human-capital to wealth ratio) should have much more equity exposure in their financial portfolio than older investors have. The portfolio weight equations are only exact if labor income can be capitalized; labor income must not have any idiosyncratic risk or this risk must be insurable. When there is idiosyncratic labor income risk, the portfolio weights are somewhat smaller, due to the presence of background risk; see Grossman and Laroque (1990) and Gollier and Pratt (1996). Later on, we show that ignoring the idiosyncratic labor income risk actually has only negligible effects on utility.

4.1.2 Life-cycle model of labor income, consumption and portfolio choice

Cocco, Gomes and Maenhout (CGM, 2005) investigate the role of human capital in a life-cycle model of consumption and portfolio choice. They assume that investors are not able to borrow against future labor income, and therefore face borrowing and portfolio constraints: they cannot take short positions in the risk-free asset in order to leverage up the financial portfolio. The asset menu is quite simple: a real riskless asset and a stock with a constant real risk premium. Labor income is exogenous and follows a random walk with age-dependent drift, plus an independent error term. After retirement, households receive a defined-benefit pension, which equals a fraction of final income.[11] In the baseline calibrations, labor income is not correlated with the stock return. CGM solve the optimal life-cycle consumption and investment policies by numerical methods (dynamic programming). The outcomes are as follows. First, young people save little or not at all. Only around the age of 40 do households start saving a significant part of their income. Second, the portfolio weight of stocks is declining with age. On average (in their simulations), up to age 40, whatever is saved is invested fully in stocks. After age 40, the portfolio weight gradually declines to around 60%. This is somewhat higher than the results in Figure 10, but the pension system in CGM is quite generous and provides a high risk-free income, so that there is substantial leverage in the portfolio even after retirement.

CGM calculate the welfare losses of several sub-optimal policies. First, they consider a portfolio rule that completely ignores leverage (i.e. $x^* = (\mu - R^f)/\gamma\sigma^2$ truncated between zero and one). Second, they consider the optimal rule with leverage derived without the life-cycle borrowing constraints and ignoring the income risk, equation (17), with the weight truncated between zero and one. Finally, they consider two simple rules: no stocks at all, and the "100-age" strategy (which makes the portfolio weight of stocks equal to (100-age) percent). Table 1 shows the welfare losses of these suboptimal strategies

[11] The replacement rate in the CGM study is quite high, and varies from 68% for highschool graduates to 94% for college graduates (see their Table 2).

Table 1: Welfare loss of suboptimal life-cycle investment strategies

no stock investments	–2.541 %
ignoring leverage	–1.561 %
ignoring labor income risk	–0.135 %
100-age rule	–0.986 %

This table presents the welfare losses of sub-optimal investment strategies in the presence of risky labor income, for an investor with relative risk aversion coefficient $\gamma = 5$. Source: Cocco, Gomes and Maenhout (2005).

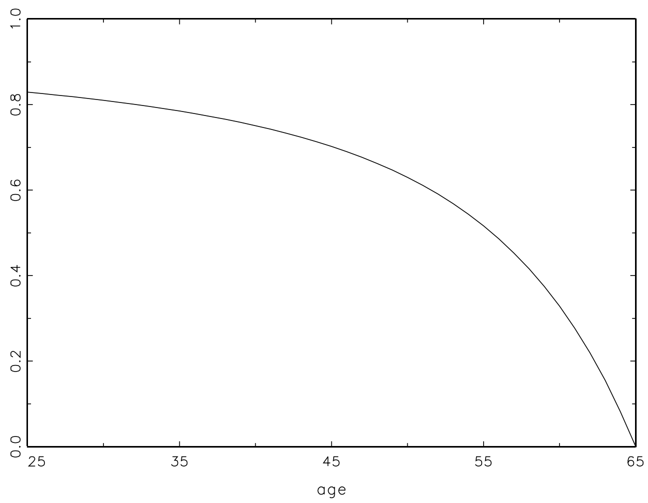
compared to the optimal life-cycle strategy for an investor with a relative risk-aversion coefficient $\gamma = 5$. The welfare loss is expressed as the percentage reduction in certainty-equivalent consumption over the life cycle. Ignoring the leverage effect appears to be harmful, as is not investing in stocks at all. The optimal rule derived by ignoring the riskiness of labor income does quite well, and has only a very small welfare loss. The 100-age rule falls somewhere in between. The conclusion is that taking into account the leverage provided by human capital is essential for the optimal investment policy; the riskiness of labor income and the portfolio constraints are less important.

CGM show that their results still hold when looking at larger income risk, although the investors the become more careful. Some larger changes are observed when allowing for positive correlation between labor income and stock returns (GGM study correlations of 0.2 and 0.4). For young people, this correlation reduces the portfolio weight in stocks, which is consistent with the hedge effect discussed above. But for middle-aged people, the stock-labor correlation actually increases the portfolio weight. This somewhat counterintuitive result can be explained from the reduction in overall savings caused by this correlation, which increase the leverage effect for this age category. But all in all, the optimal portfolio weights still exhibit a declining pattern with age.

4.1.3 Cointegration

In a recent paper, Benzoni, Collin-Dufresne and Goldstein (2007) challenge the conclusions of the previous analyses that optimal portfolio weights are declining with age. They point out that in the long run, the shares of capital income and labor income in the economy are fairly stable. This implies that dividends and labor income are cointegrated. Even though dividends and wages are only weakly correlated in the short run (in quarterly data, for example), cointegration implies that the long-run correlation between dividends and

Figure 11: Human capital beta



The figure plots the beta of human capital in the Benzoni et al. (2007) model.

wages is much higher. As stocks are just the present value of future dividends, and human capital the present value of future labor income, the cointegration also implies that stock returns and human capital returns are much more strongly correlated in the long run. This correlation reduces dramatically the demand for stocks by young investors. Benzoni et al. calibrate the model and show that the optimal position in stocks for young investors may even be negative. The life-cycle pattern of optimal stock investments is hump shaped, with young- and retired investors having low equity exposure, and middle-aged investors the highest exposures.

Suppose that only stocks and a risk-free asset are available, and stock returns and labor income are contemporaneously uncorrelated. Then, the optimal portfolio weight for stocks is

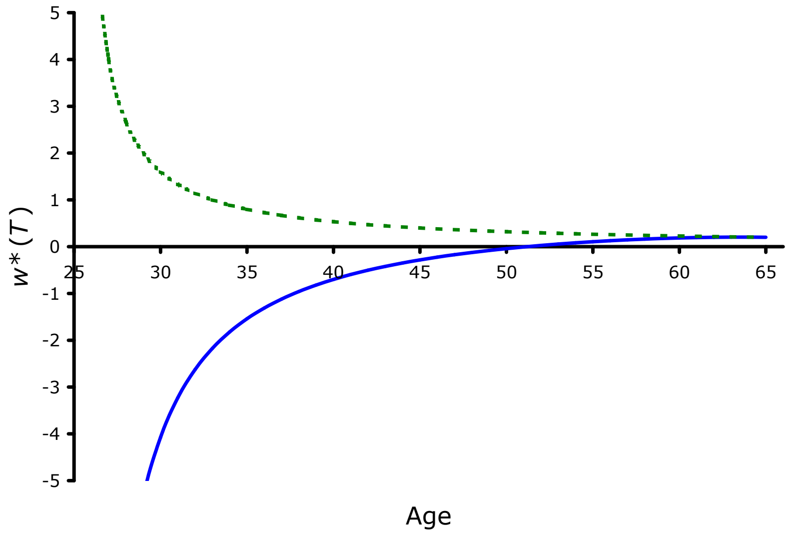
$$w^* = \frac{1}{1-h}\left[\frac{\mu}{\gamma\sigma^2} - h\beta_H(T)\right]. \tag{19}$$

The first term in brackets is the usual mean-variance optimal portfolio. The second term is the human capital hedge term. Notice that the absolute value of this term is smaller than one, as both $h < 1$ and $\beta_H(T) < 1$. Both terms are leveraged up with the inverse of the share of financial wealth to total wealth, $1/(1-h)$. The human capital beta in this model equals

$$\beta_H(T) = 1 - \exp(-\kappa T),$$

with κ the mean reversion coefficient of the error-correction term (equal to the feedback of the error correction term in the wage growth equation). Benzoni et al. estimate $\kappa = 0.18$,

Figure 12: Portfolio weights with and without cointegration



The figure plots the portfolio weight of stocks in the Benzoni et al. (2007) model (solid line) and the model without cointegration (dashed line).

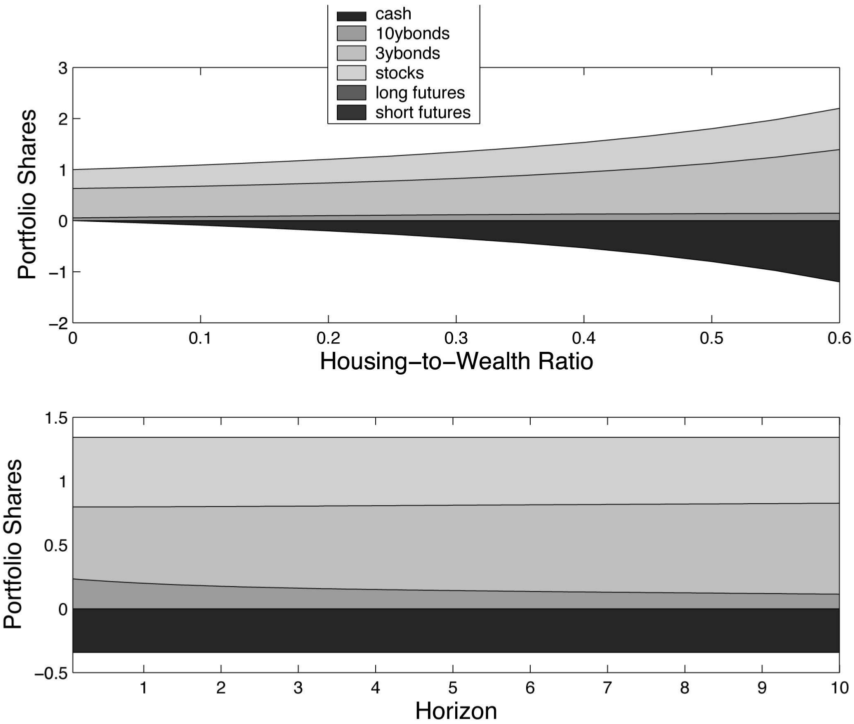
which yields the picture in Figure 11.

To gauge the impact of the human capital hedge on the asset allocation, we take the same model parameters and make some additional assumptions. We assume an investor who retires at age 65. The ratio of his human capital over total wealth decreases linearly with age from $h = 1$ at age 25 to $h = 0$ at age 65. Furthermore, we assume that the (unleveraged) speculative demand for stocks $\mu/\gamma\sigma^2 = 0.20$, which is fairly conservative. Figure 12 shows the demand for stocks, with (solid line) and without (dashed line) the human capital hedge effect. Taking into account the human capital hedge reduces strongly the demand for stocks. For many investors (except those close to retirement), the demand for stocks actually becomes negative.

4.2 Housing

Perhaps the single-most important asset for individual households (apart from their human capital) is housing. Most people certainly most of those who are close to retirement own the house in which they live. The value of the house often far exceeds the value of the household's financial investments. This section discusses the implications of owning a house for financial portfolio choice. We also study the choice for a mortgage, which is perhaps the largest financial decision that a household faces.

Figure 13: Portfolio Choice with Housing



The figure plots the optimal portfolio choice for a $\gamma = 5$ investor. Source: De Jong, Driessen and Van Hemert (2007)

4.2.1 Optimal portfolios with housing

The impact of the house on financial portfolio choice is in many respects similar to the effect of human capital. The wealth embedded in the house leads to a leverage effect and to hedging demands against the correlations of house prices with financial market returns. However, houses have some specific aspects: (i) below-market expected returns due to convenience yield of living in the house, (ii) a large unhedgable idiosyncratic (house-specific) price risk,[12] and (iii) the ability to use the house as collateral for a mortgage loan. De Jong, Driessen and Van Hemert (DDV, 2007) show that these three effects are all important for the financial portfolio choice.

DDV study an investor with a horizon of ten years, at which point the house is sold and the investor consumes all of his wealth. This setting is obviously a simplification of the life-cycle planing problem, but Van Hemert (2007) shows that it gives a fairly good approximation for households that are close to retirement. Since household has a net

[12] This aspect is also relevant for other non-tradeables, such as human capital and labor income (see Munk, 2000, and Munk and Sørensen, 2007).

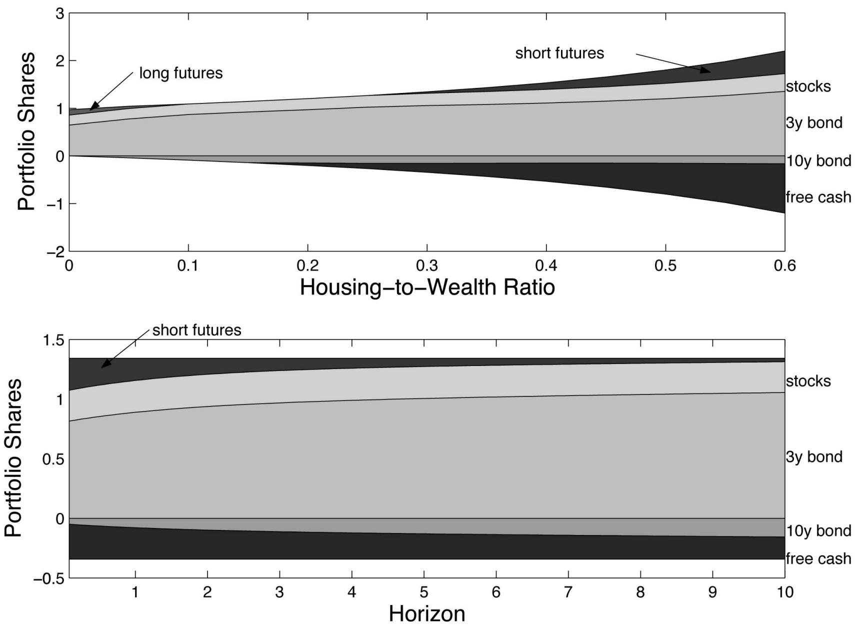
worth that is larger than the value of the house, the housing-to-total-wealth ratio h is between zero and one. This may be a reasonable assumption if we include the capitalized value of social security and pension rights in the financial wealth. DDV assume an asset menu with stocks, medium-term bonds (duration three years), long-term bonds (ten-year duration, which is approximately the duration of a thirty-year mortgage) and nominally risk-free cash. DDV impose portfolio constraints, but allow for short positions in bonds or cash up to 80% of the value of the house. The house serves as collateral for this short position. This is a simple way to include a mortgage in the analysis. DDV analyze the optimal portfolios for investors with $\gamma = 5$. Investors differ in their housing-to-wealth ratio h, which runs from 0 to 0.6 in the figures. The results show that all investors have a substantial portfolio weight in stocks, mainly driven by the speculative demand. The composition of the bond portfolio is more interesting. The investor invests mainly in stocks and medium-term bonds, and to some extent in long-term bonds. These bond investments are speculative and capture the bond risk premiums (which are in turn caused by real interest rate and inflation risk premiums). The investor takes advantage of the borrowing allowed by the house, by assuming a mortgage loan up to the full amount allowed by the constraint. This loan is exclusively in risk-free cash (i.e. an adjustable rate mortgage).

DDV also include housing futures in the asset menu, but it turns out that the demand for these for a $\gamma = 5$ investor is nil and does not affect the allocation to other risky assets. The $\gamma = 10$ investor, however, likes to include some futures in his portfolio. Figure 14 shows the optimal demands for all assets including futures. Investors with a large house (relative to their total wealth) have an interest in shorting housing futures. This hedges them against the risk of house price fluctuations. In contrast, investors with no house or a very small house want to take a long position, in order to profit from the risk premium earned on the housing futures. The $\gamma = 10$ investor also prefers a mixture of long bonds and cash in the mortgage loan. This observation will be discussed in greater detail in the section discussing mortgage choice.

4.2.2 Housing futures

A recent financial innovation is in the area of housing futures. These are contracts based on house price indices for several metropolitan areas in the US, or the US as a whole. These futures are potentially very useful as a hedge against the price fluctuations of houses. Indeed, Van Hemert (2007) shows that a young investor who plans to buy a house, or move to a bigger house, can profit from such contracts. However, the contracts turn out

Figure 14: Portfolio Choice with Housing Futures



The figure plots the optimal portfolio choice for a $\gamma = 10$ investor. Source: De Jong, Driessen and Van Hemert (2007)

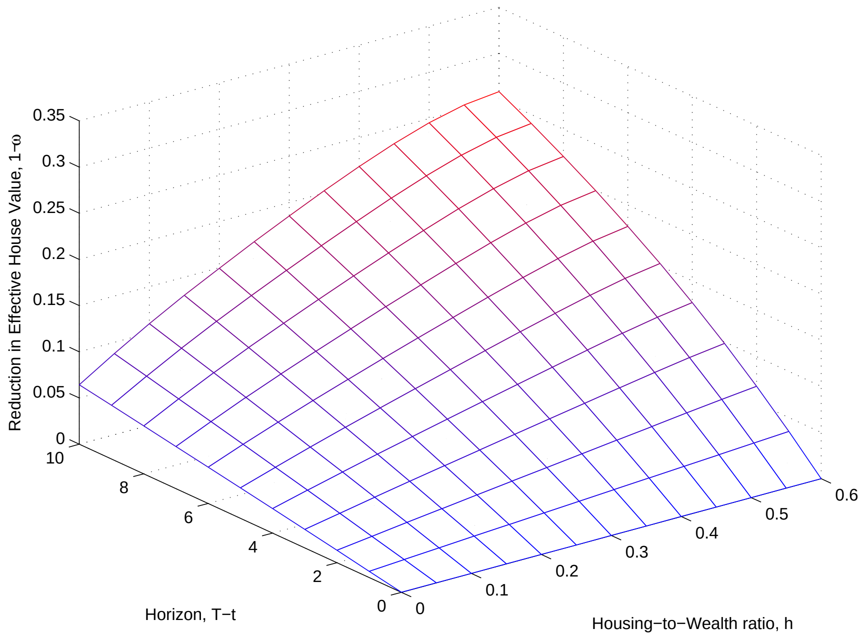
to be much less useful for older investors who plan to sell their house. The reason is that most risk on the house is idiosyncratic (i.e. house-specific) risk. The hedge demand for the house is substantially reduced because of this risk (see Figure 15 for the discount applied to the house with idiosyncratic risk). Another reason for the lack of demand for futures is that the (long) speculative demand cancels out the (short) hedging demand, except for investors with a large housing-to-wealth ratio. DDV show that a contract that can hedge the house-specific risk is much more valuable, and may yield up to a 4% welfare gain for an investor with $T = 10$ and $\gamma = 5$, and even higher for more risk-averse investors.

4.2.3 Mortgage choice

Campbell and Cocco (2003) study the choice between an Adjustable Rate Mortgage (ARM) and a Fixed Rate Mortgage (FRM). With an ARM, the interest payments are adjusted every year inline with the market interest rates. In contrast, the FRM has fixed interest payments for the full length of the mortgage contract, typically 30 years in the US.[13] The ARM has the advantage that it is cheaper: the term structure is, on average,

[13] In the US, an FRM typically has a pre-payment option embedded, which allows the mortgagee to repay his mortgage whenever he likes and to enter into a new mortgage contract at no cost or for relatively small transaction costs. This option is valuable when interest rates for new mortgages fall below the

Figure 15: Idiosyncratic Risk Discount



The figure plots the reduction in effective housing wealth due to idiosyncratic risk for a $\gamma = 5$ investor. Source: De Jong, Driessen and Van Hemert (2007).

upward sloping, so that short-term interest rates are below long-term interest rates.

More carefully, even starting with above- or below average interest rates, we can invoke the expectations hypothesis. This assumes that long rates are an average of expected future short rates, plus a constant term premium. Campbell and Cocco (CC, 2003) assume a two-factor interest rate model, where the short rate is the sum of the real rate and expected inflation. The real rate is constant plus an IID error, whereas inflation is a persistent AR(1) process.[14] CC estimate the term premium for the long mortgage rate to be 1% per year. A household entering an ARM thus saves 1% times the value of the house per year on mortgage interest payments. Suppose the mortgage is four times the households annual income; a 1% lower interest rate then implies a 4% lower expenditure share to housing and hence a 4% higher consumption on other goods. In order to carry out a full welfare analysis, however, we also need to take into account the risk features of the different mortgage types.

The disadvantage of the FRM (apart from the higher cost) is the wealth risk: in times

interest rate paid on the existing mortgage. In the Netherlands, costless prepayment is typically permitted only when moving house. Otherwise, a penalty equal to the present value of the interest rate differential between the old- and the new mortgage has to be paid, making the prepayment option worthless. Even for the US-style contracts, CC show that the value of the prepayment option is limited.

[14] There is no unexpected inflation in their model, so that inflation equals expected inflation.

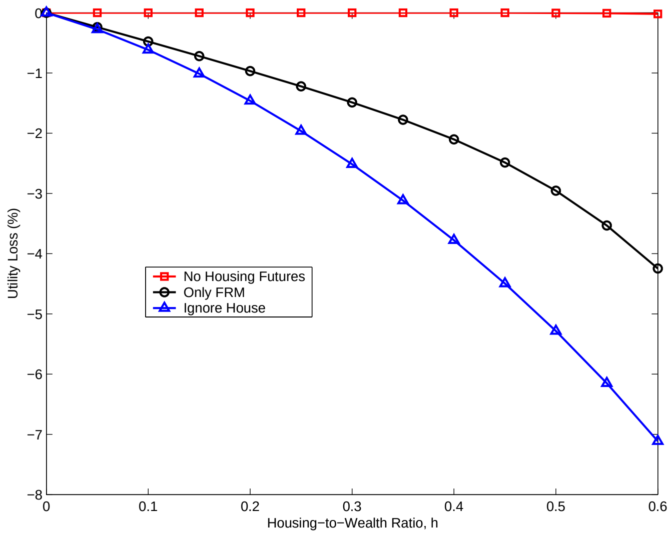
of lower inflation the FRM payments are very high, whereas in times of high inflation the FRM payments are cheap. The ARM, in contrast, yields payments that (in real terms) are not sensitive to inflation shocks. On the other hand, the ARM has the risk of increases in the *real* interest payments. If interest rates rise, then consumers who are borrowing-constrained run the risk that the monthly interest payments come close to or exceed their available income. In that case, these consumers may be forced into very costly default. An FRM does not have this risk because its payments are fixed. Default with an FRM happens only when inflation is extremely low and wages fall in nominal terms, but that happens only rarely.

So, a risk-averse investor needs to balance the real interest rate risk of the ARM with the wealth risk and the higher average cost of the FRM. The analysis of CC shows that for most realistic situations, the lower cost of the ARM outweighs the real interest rate risk. Their conclusion is that the ARM is the best mortgage contract. Only for people with a very high mortgage relative to their wage income, or very high income volatility, does an FRM become interesting.

The analysis in CC did not consider the possibility of a combination of ARM and FRM contracts, or an FRM with a shortened period of fixed interest rates, as is common in the Netherlands (where most mortgages have interest rates reset after five or ten years). Such combinations of ARM and FRM might be interesting in an attempt to balance the conflicting objectives. Also, CC probably underestimate the real interest rate risk by assuming that real interest rate shocks are purely transitory. In practice, they are persistent, although less persistent than inflation shocks (see Section 3.2).

De Jong, Driessen and Van Hemert (2007) address these issues. They specify a model for an investor who owns a house and invests in stocks and bonds of various maturities. A mortgage is modeled as a short position in a short-term bond (ARM), a long-term bond (FRM), or a combination of both. This model does not include income risk, but does feature persistent real interest rate shocks. The analysis again shows that most investors prefer an ARM. Only very risk-averse investors prefer a mixture of ARM and FRM (see, for example, Figure 14 for the $\gamma = 10$ investor). Having only FRMs available leads to large welfare losses for all investors considered. Figure 16 depicts the welfare loss for investors as a function of the ratio of housing wealth to total wealth, h. The loss associated with having only an FRM may rise to 4%, whereas the loss of completely ignoring the house (and its leverage effect) may reach 7%.

Figure 16: Welfare Loss of Suboptimal Portfolio Choice



The figure plots suboptimal portfolio choice for a $\gamma = 5$ investor. Source: De Jong, Driessen and Van Hemert (2007).

5 Parameter Estimates and Estimation Error

In the empirical models of section 3.4 the optimal portfolios are heavily tilted towards equity for two reasons: the equity premium promises high returns, while mean reversion provides these returns at low long-term risks. The equity premium and the predictability of stock returns are thus crucial parameters.

5.1 Equity Premium

In the typical VAR model for asset returns and macroeconomic variables, the equity premium is usually large. The estimate reflects the historical equity premium of around 8% per year in most economies in the Western world.[15]

In a forward-looking model, the equity premium comes out much lower. The forward-looking equity premium is the discount rate that sets the current stock price equal to the present value of future earnings or dividends. Using data on US stock prices and the forecasts of analysts regarding earnings in the 1980's and 1990's, Claus and Thomas (2001) find that the average forward-looking premium was about 3.4% much lower than average realised stock returns.

A similar low equity premium is reported in a series of surveys among finance professors

[15] See Dimson, Marsh and Staunton (2002, 2007) for the historical equity premium around the world.

conducted by Ivo Welch (2001, 2008). Surveyed in August 2001, these professors provided a median estimate of the long-run equity premium of 5.5%. With a interquartile range of 4 to 7%, the opinions of the academics show considerable dispersion. Repeating the survey in December 2007, Welch (2008) reports more consensus among the finance professors on a long-run equity premium of 5%. A third reason why the historical equity premium overestimates the expected future return is selectivity bias. Jorion and Goetzmann (1999) look at all the major stock markets existing around 1900 (including markets such as Russia and Argentina). An investor who would have held a diversified portfolio of all of the international stock markets at the time would have realised a much lower return than the realised return on a portfolio composed of the US market and a few of the other better performing Western stock markets. Ex-post, the US market realised a large equity premium, but ex-ante it would have been impossible to predict the difference between Russia and the US.

From a theoretical perspective, the equity premium is also mostly very low. The equity premium puzzle posed by Mehra and Prescott (2005) related the requited return to the covariance between the stock market and real consumption. In the simplest models with constant relative risk-aversion utility the (arithmetic) equity premium is given by

$$\mathsf{E}[R - R_f] = \gamma \mathsf{Cov}(R, \Delta \ln C), \tag{20}$$

where $\mathsf{E}[R - R_f]$ is the equity premium, C is consumption and γ the relative risk aversion. To get a feel for the numbers, suppose that $\gamma = 5$, the correlation between consumption growth and the stock market is at maximum unity, the standard deviation of consumption is about 3% and the standard deviation of stock returns is not larger than 20%. Substituting in (20) gives an equity premium of 3%, far below the historical equity premium. It has proven to be difficult to design a theoretical model that can produce a substantially higher equity premium without running into other conflicts with the stylised facts on real growth, stock returns and the interest rate. The most successful models all have very different preferences for individual consumers. Campbell and Cochrane (1999) introduce an external habit, meaning that people measure their consumption utility relative to the consumption of other people. Benartzi and Thaler (1995) introduce the concept of myopic loss aversion, which is based on the psychological concept that individuals are more sensitive to losses than to gains relative to their current wealth. If they also frequently (at least annually) review their returns, then they will consider stocks to be a very risky asset class.[16]

[16] The discussion in this section is kept intentionally brief, as there are many excellent existing surveys

Equation (20) was obtained by assuming that returns are approximately log-normally distributed. It is possible to obtain a high equity premium with standard preferences if we allow for extreme negative returns (such as a stock market crisis). Since negative returns are deemed highly undesirable by investors, a small probability of a huge loss has a great impact on the expected utility. The more negatively skewed the return distribution, the larger the risk of the asset. Barro (2006) argues that the deviation from normality in stock returns can explain a large part of the equity premium puzzle.

Summarising, the optimal allocation to equity should, for speculative reasons, be less than that which is implied by the standard models. This may be due to the fact that the historical average excess return overestimates the future equity premium. But it may also follow from theories that can reconcile a high equity premium. If the high equity premium is an equilibrium result attributable to fancy preferences, how can we be sure that the pension fund (or its participants) do not have similar preferences that are fundamentally different from constant relative risk aversion? If the large equity premium is related to crisis risk, then the models in section 3 should also explicitly use non-normal return distributions in the scenario generators of an ALM study.

5.2 Predictability

Mean reversion in stock returns has been well documented, and disputed, over the last twenty years. Critical for the mean reversion effect is the proper kind of predictability of equity returns. Stock returns must be predictable by variables for which the contemporaneous correlation is opposite from the effect on future expected returns. In the simple VAR model,

$$x_{t+1} = a + bz_t + \epsilon_{t+1} \tag{21}$$

$$z_{t+1} = \mu + \rho(z_t - \mu) + \eta_{t+1} \tag{22}$$

$$\mathrm{Cov}(\eta_t, \epsilon_t) = \psi, \tag{23}$$

b and ψ must have opposite signs, and ρ must be large, but not too close to unity.[17] The most common predictor is the dividend yield (or another valuation ratio such as earnings-to-price). An increase in the dividend yield raises the expected future return on stocks. At the same time, the positive shock to the dividend yield is very strongly

in the equity premium literature. See, for example, Campbell, Lo and MacKinlay (1997, ch 8+9), Dimson, Marsh and Staunton (2002), Cochrane (2005, ch 22) or Kocherlakota (1996).

[17] The simple model (21)-(22) has been analysed in great detail to study the long-run time-series properties of equity returns and their implications for strategic asset allocations. Cochrane (2005, Chapter 20) has a textbook treatment of the properties and empirical relevance of this model.

negatively correlated with stock returns. In CCV03 and HMSS08 the correlation is -0.98 for quarterly data. A shock to the dividend yield is bad news for current stock returns, but it also signals better investment opportunities in the future. The future benefits are stronger the more persistent the shock. The combined effect implies mean reversion.

Other predictor variables with potential mean reversion properties are inflation, interest rates and volatility. The hedge potential of inflation was explored in Schotman and Schweitzer (2000), who, using the framework of (21)-(22), find that stocks are a hedge against long-term inflation. Stock returns are negatively correlated with inflation news in the short run ($\psi < 0$), but long-term cumulative stock returns are positively correlated with stock returns ($b > 0$ and $\rho \approx 1$).

Estimation of b is difficult. It can be estimated reliably only for very long time series. The problem is threefold. First, in efficient markets there cannot be too much predictability; we thus expect a low R^2 of the predictive regression (21). Second, but related, the predictive regression is very unbalanced. On the left-hand side is a return series that is mostly unpredictable noise. The predictive variables are all slowly moving near-unit-root time series. This creates a danger of spurious regressions. Third, in small samples the OLS estimate of b will be biased if $\mathrm{Cov}(\eta_t, \epsilon_t)$ is non-zero. As shown by Stambaugh (1999), the size and direction of the bias depend on ψ. This is unfortunate, since the very variables that have potentially strong mean reversion effects are also the ones with a strong correlation between ϵ_t and η_t (-0.98 for the dividend yield and stock returns).[18]

A skeptical view on the predictability of stock returns is presented by Goyal and Welch (2003, 2007). They perform an out-of-sample prediction test for a wide range of prediction variables. Their main conclusion is that none of the models has predictive power; a model with a constant equity premium is best out-of-sample. Significant in-sample coefficients are often the result of early historical episodes in which prediction worked. For the dividend yield (and other price ratios) the in-sample predictive power comes from the period until the oil shock in the mid 1970s. After that, and especially since the 1990s, the dividend yield does not have predictive power. The coefficient b in (21) could thus well be zero.

Goyal and Welch (2007) attribute the predictive failure to instability of the models. Addressing this issue, various studies have considered time-varying parameters, structural breaks and regime-switching models.[19] Model instability, and its effect on strategic asset allocation, continues to be an active research area. More complicated econometric models

[18] Recent detailed econometric analyses can be found in Ang and Bekaert (2007), Campbell and Yogo (2006) and Cochrane (2007).

[19] An example is Guidolin and Timmermann (2007). Elliott and Timmermann (2008) provide a general survey of alternative prediction methods and evaluation criteria including examples in the field of portfolio choice.

require very computationally intensive numerical methods for solving the asset demands for long-term investors.

Even though predictability is weak, it could still be meaningful enough to exploit. Small changes in the predicted returns can generate substantial long-run mean reversion and have large effects on the optimal portfolio composition. In all models this leads to the well-known decomposition of the optimal portfolio in a myopic portfolio and a hedge portfolio. The total portfolio often implies an extremely aggressive investment strategy with optimal weights that often range between minus and plus 300 percent. Such large swings very likely overstate the benefits from active strategies. The situation is closely related to the problems in static mean-variance analysis.

What remains is evaluation of the benefits of the more complicated portfolio rules. Benefits could be small. Starting with Lucas (1987) it has generally been found that big changes in economic policy rules often have negligible effects on measures of welfare. For strategic asset pricing this is still an open issue. Some studies report large gains in certainty equivalents, while others report only modest gains. The reported gains are usually for in-sample estimates, and therefore open to the same criticism of error-maximisation as they are in the mean-variance portfolio choice literature.

The main problem with the dynamic prediction model is that it tends to generate large fluctuations in optimal asset allocations. Bayesian methods offer a promising way forward, either formal of informal, An example of an informal Bayesian prior is to use the economic insight that the ex-ante equity premium can not be negative. Every time model (21) predicts a negative equity premium (it is replaced by zero). Campbell and Thompson (2007) find that this improves the predictive power. Another implicit Bayesian prior is to rule out extreme portfolio weights, as these can never be an equilibrium solution. This would be the strategic asset allocation counterpart of the Black and Litterman (1992) approach in static mean-variance portfolio choice.

Formal Bayesian models can add subjective prior views to support the weak sample evidence in estimating b. Various kinds of prior information have been considered. Cremers (2002) adds views on the expected R^2 in (21) and on the number of predictive variables. Hoevenaars, Molenaar, Schotman and Steenkamp (2007) add prior views on the long-term means of all variables in the vector autoregression. Wachter and Warusawitharana (2007) also consider the expected R^2, which they relate to the autocorrelation in the dividend yield. Although not much evidence is available yet, the Bayesian procedures seem to improve portfolio performance because they shrink the amount of predictability and avoid overly aggressive strategies.

The lesson for practical applications would be to use predictions cautiously by combining estimation results with subjective views of the investment opportunities. Predictability should not overstated, and models with high R^2 should be mistrusted.

5.3 Bayesian decision theory

As is clear from the previous discussion, the process that generates returns is surrounded by uncertainty. Parameters in the return model are subject to substantial estimation error, and even the model itself is uncertain. Estimation error was ignored until a few years ago, and the point estimates of the parameters were merely plugged into the portfolio formulas. This clearly understates the risk of using the wrong parameters. Bayesian decision theory acknowledges that parameters are not known with complete certainty. It provides a methodology to quantify the parameter uncertainty and integrate it into the decision making. A Bayesian optimises

$$\mathsf{E}_t \left[\int u(W_T|\theta)\ p(\theta|\mathsf{data})\ \mathsf{d}\theta \right]. \tag{24}$$

For a given strategy, the Bayesian averages the expected utility over all possible values of the unknown parameters weighted by their posterior probability. Bayesian decision theory has now been fully incorporated in the portfolio choice literature. Barberis (2000) highlights the conceptual differences and quantitative effects on strategic asset allocation.

In general, parameter uncertainty introduces additional risk. The following example illustrates the effect. Suppose an investor can choose between a risk-free bond and a risky equity with expected return μ and standard deviation σ. We assume that returns are uncorrelated over time. In that case, for given parameters the portfolio weight of equity does not depend on the investment horizon. We will assume, however, that μ is unknown and must be estimated, maintaining for simplicity that σ is known. We will show that for a Bayesian decision maker the investment in equity will decrease with the investment horizon.[20]

Since μ is unknown, the investor needs an estimate. The simplest estimator is the historical mean

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} x_t, \tag{25}$$

based on a sample of T observations. Conditional on $\hat{\mu}$, the optimal plug-in portfolio is

$$w_{MV} = \frac{1}{\gamma}\ \frac{\hat{\mu}}{\sigma^2}. \tag{26}$$

[20] The example is a simplified version of the analysis in Barberis (2000).

With an historical excess return $\hat{\mu} = 8\%$, a standard deviation $\sigma = 20\%$ and $\gamma = 5$, the optimal weight for equity is thus $w_{MV} = 40\%$. The portfolio weight does not depend on time, since both expected returns and the variance increase linearly with the investment horizon. For a horizon of k years the optimal portfolio is still $w_{MV} = 40\%$ under the plug-in method.

A Bayesian with an uninformative prior will infer that, conditional on $\hat{\mu}$, the true parameter μ is

$$\mu = \hat{\mu} + \xi, \tag{27}$$

with ξ the estimation error with mean zero and variance ω^2. Integrating out the parameter uncertainty leads to the predictive distribution of future returns

$$x_{t+1} = \mu + \epsilon_{t+1} = \hat{\mu} + \epsilon_{t+1} + \xi, \tag{28}$$

which has expected value $\hat{\mu}$ and variance $\sigma^2 + \omega^2$. The Bayesian will thus choose the portfolio

$$w_B = \frac{1}{\gamma} \frac{\hat{\mu}}{\sigma^2 + \omega^2}. \tag{29}$$

Since the estimation error variance ω^2 is always positive, the Bayesian portfolio will always contain less equity than the plug-in portfolio. For the single-period example, the effect will be small, since $\omega^2 \ll \sigma^2$. If returns are truly identically and independently distributed, then the variance of the sample mean (25) will be

$$\omega^2 = \frac{1}{T}\, \sigma^2, \tag{30}$$

and the Bayesian portfolio becomes

$$w_B = \frac{1}{1 + 1/T} w_{MV} \tag{31}$$

For a long historical sample of 60 years, the proportion of equity is reduced by a negligible amount, from 40% to 39%.[21]

Over longer horizons, the effect of parameter uncertainty is much more pronounced. Aggregating (28) gives the k-period return

$$x_{t+k}^{(k)} = k\hat{\mu} + k\xi + \sum_{i=1}^{k} \epsilon_{t+i}, \tag{32}$$

[21] Technically the effect of parameter uncertainty is of the order N/T, with N the number of assets. Therefore, parameter uncertainty matters a lot if N is relatively large as in typical static allocation problems. This is a very different literature that is not discussed here. See Kan and Zhou (2006) for the technical details and possible solutions.

which has mean and variance

$$\mathsf{E}_t[x_{t+k}^{(k)}|\hat{\mu}] = k\hat{\mu} \quad (33)$$

$$\mathsf{Var}_t[x_{t+k}^{(k)}|\hat{\mu}] = k\sigma^2 + k^2\omega^2. \quad (34)$$

A Bayesian long-run buy-and-hold portfolio therefore has an equity weight of

$$w_B^{(k)} = \frac{1}{\gamma}\,\frac{\hat{\mu}}{\sigma^2 + k\omega^2} = \frac{T}{k+T} w_B^{(1)}, \quad (35)$$

and will decrease with the investment horizon k. The reason is the persistence of the estimation error η. Getting the average return wrong is a small mistake. But getting it wrong in the same way every period adds up to a potentially big mistake. For the long-term investor, the effect of parameter uncertainty becomes quantitatively important: risk at a 20-year horizon increases by 33%.

For an ALM study based on scenarios, applying Bayesian analysis is no more complicated than using the plug-in method. An ALM study usually starts by generating scenarios for returns and macroeconomic variables that are relevant for the decision making. In the plug-in method, the scenarios are generated for a single parameter vector. In the Bayesian method, every scenario is based on a draw from the posterior for the parameters and conditional on the parameter with which a scenario path is generated. Much more involved is the construction of optimal portfolios from (24). This has only recently become feasible after advances in computational methods developed by Brandt, Goyal, Santa-Clara and Stroud (2005), and subsequent refinements to these methods.

Hoevenaars, Molenaar, Schotman and Steenkamp (2007) apply the Bayesian scenario generator to extend the example to a full vector autoregression. They specify an informative prior on the long-term means of equity, bonds, bills, inflation, term spread and dividend yield. Parameter uncertainty leads to an increase in the term structure of risk. Relative to the plug-in method, the annualised volatility for stocks increases by 4%-6% at the 15-year horizon, depending on the prior precision and the specific long-run view. In many cases, mean reversion disappears completely at longer horizons and the term structures of risk become upward sloping. In these cases, the uncertainty of the expected returns (the a and b parameters in (21)) dominate the risk in the return scenarios.

Hoevenaars et al. (2007) find that all asset classes become much more risky. Since correlations are hardly affected by the parameter uncertainty, the composition of long-term portfolios does not change much.

5.4 Robustness

Application of Bayesian methods requires a prior view on the relevant parameters in order to quantify the importance of alternative parameters. But sometimes consensus about a prior can not be reached. Also, some risks are not quantifiable, as Lowenstein (2000, p 62-63) explains in analysing the rise and fall of Long-Term Capital Management:

> "We may deduce that if we buy a share of IBM at $80, we have a greater chance of making a profit than if we buy at $90, just as your child is at greater risk if two of his buddies are sick, rather than only one. But *how much* greater? We don't have enough facts to quantify either the risk of market loss or the risk of contagion.
>
> Notice that there is a key difference between a share of IBM (or an infectious disease) and a pair of dice. With dice, there is *risk* — you could, after all, roll snake eyes — but there is no *uncertainty*, because you know (for certain) the chances of getting a 7 and every other result. Investing confronts us with both risk and uncertainty. There is a risk that the price of IBM will fall, and there is uncertainty about how likely it is to do so. So many variables — political, economic, managerial, competitive factors — can affect the results that the uncertainty all but overwhelms us."

Other risks have not been thought of yet, but may affect decision making. Former president of the Federal Reserve Board Alan Greenspan (2007, p. 196) recalls how the LTCM crisis made him aware of model uncertainty:

> "The way the Fed responded to the Russian crisis reflected a gradually evolving departure from the policymaking textbook. Instead of putting all our energy into achieving the single best forecast and then betting everything on that, we based our policy response on a range of possible scenarios. When Russia defaulted, the Fed's mathematical models showed that it was highly likely that the U.S. economy would continue expanding at a healthy pace in spite of Russia's problems and with no action by the Fed. Yet we opted to ease interest rates all the same because of a small but real risk that the default might disrupt global financial markets enough to severely affect the United States. That was a new kind of trade-off for us: we judged this unlikely but potentially greatly destabilizing event to be a greater threat to economic prosperity than the higher inflation that easier money might cause. I suspect there had been many such decisions in the Fed's past, but the underlying decision-making process had never been made systematic or explicit.

> Weighing costs and benefits systematically in this way gradually came to dominate our policymaking approach. I liked it because it generalized from a number of ad hoc decisions we'd made in years past. It let us reach beyond econometric models to factor in broader, though less mathematically precise, hypotheses about how the world works."

The last quote points at directions that might be taken in order to cope with the more fundamental uncertainty. Thus, in addition to plug-in methods or Bayesian decision making, we see a third option of dealing with uncertainty: the application of a worst-case scenario, also called robust portfolio optimisation. The robust investor maximises

$$\min_{\theta \in \Theta} \mathsf{E}_t \left[u(W_T|\theta) \right], \tag{36}$$

where Θ is the set of possible values of θ. In this case, the investor either does not trust the prior of the Bayesian or is subject to ambiguity aversion and strongly dislikes model uncertainty. In a robust analysis the decisionmaker does not know which model is best and has no way to quantify how exactly the actual model will deviate from the estimated econometric model. The approach is treated in detail in the Hansen and Sargent (2008) textbook.

Continuing the example in section 5.3 on estimation error in the unconditional equity premium, consider a robust investor who decides on the worst case out of a wide confidence set. Using the same historical sample mean, such an investor will infer that with 95% confidence the expected return is in the interval $(\hat{\mu} - 2\omega, \hat{\mu} + 2\omega)$, implying the worst-case optimal portfolio

$$w_R = \frac{1}{\gamma} \frac{\hat{\mu} - 2\omega}{\sigma^2}, \tag{37}$$

which would reduce the fraction of equity to 20%.

For strategic asset allocation, Lutgens and Schotman (2007) re-analyse the Barberis (2000) model (i.e. (21)-(22)) to find the critical parameters that define the worst case for a long-run investor. These would be the parameters most deserving attention for improved estimation. In the application, the worst case depends on the predictability parameter b together with the long-run mean of the dividend yield.

6 Policy implications

What can be learned from this panel paper? The state of the long-term investment literature about six year ago was nicely summarized in Campbell and Viceira (2002). One of their main lessons was that the optimal portfolio of long-term investors is a weighted average of a speculative portfolio (based on the myopic mean-variance optimal portfolio) and a hedge component (in the setting of CV02, the hedge portfolio is the best hedge against interest rate- and inflation risks). The more risk averse the investor, the higher the weight on the hedge portfolio. All investors therefore prefer a positive equity exposure in their portfolio. An important policy implication of this result is that pension funds, which invest on behalf of their members, should accept mismatch risk: an investment policy fully in risk-free assets is suboptimal.

The recent literature has studied many extensions of the original CV02 setting for example, with predictability in stock- and bond risk premiums and background risks. In all of these extensions the basic portfolio structure survives: the optimal portfolio is a weighted average of a speculative portfolio and a portfolio that hedges against long-term risks. We now discuss a few specific extensions and their main lessons.

First, stock returns are somewhat predictable, and it turns out that the long-run volatility of stocks is smaller than the short-run volatility. This allows investors with a long investment horizon to assign a higher portfolio weight to stock investments. Bond risk premiums are also somewhat predictable, allowing investors to 'time' the bond market by appropriately adjusting their investments in short-, medium- and long-term bonds. However, the predictability of stock- and bond risk premiums is heavily debated. There are several econometric problems with the predictability regressions. Recently, several papers have included predictability and estimation error in a unified framework for portfolio choice. We find the Bayesian approaches the most promising.

Second, the asset allocation of a pension fund should depend on the individual characteristics of its members. The equity exposure or the risk allocation should depend on human capital, the riskiness of labor income and the correlation of income with the stock market. The investment policy should therefore be a function of individual characteristics such as age, education and occupation. Financial contracts that allow older investors to hedge the price risk of selling their house in the future may be invaluable. Instruments such as individual house futures or reverse mortgages could do this. Notice that these optimal portfolio solutions could be implemented by collective entities such as pension funds. These are able to provide cost-effective investment services and also (if desired) facilitate risk sharing between generations.

An open question that remains is how we can find practical and easily implementable default asset allocations. Gomes, Kotlikoff and Viceira (2007) study asset allocations for individual pension plans and conclude that a simple balanced life-cycle fund, with stock- and bond investments, and the weight on stocks declining with age, comes close to the optimal investment strategy for most investors (in terms of welfare gains), and is substantially better than investing only in risk-free assets.

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SUMMARY OF DISCUSSION

By Zhen Shi

Strategic Asset Allocation
By Frank de Jong, Peter Schotman and Bas Werker

Chairman: Henk Don (Erasmus University Rotterdam)
Discussants: Hens Steehouwer (Ortec) and Jaap van Dam (PGGM)

Netspar Panel: April 23, 2008

Hens Steenhouwer (Ortec) opens the discussion by emphasizing the importance of the strategic asset allocation for resulting investment returns. His first comment is about a pension fund's preference versus its participants' preferences. The pension fund invests on behalf of its members. Therefore, Schotman, de Jong and Werker (2008) argue that the investment strategy of a pension fund should depend on the preferences of its members. However, since the members differ in many characteristics, for example, age, income, health condition and job, it is difficult to aggregate all the participants' preferences. Hence, it is natural to ask how far the integration of individual and pension fund decision making should be pursued. Steenhouwer's suggestion is to separate the collective (pension fund investment management) and individual investment decision making (private financial management). **Theo Nijman** (Tilburg University) argues that the utility function of individuals is better defined. He suggests a pension fund should have different utility functions for different groups of individuals.

Steenhouwer's second comment is about multi-period decision making. Fundamentally, the asset allocation decision of a pension fund is a long-term issue. However, the investment return in the short-term is also important due to the requirements of the regulators (e.g., FTK and IFRS) and of the pension fund itself in order to check whether the fund is on the right track to meet the long-term goal. Steenhouwer wonders how an optimal multi-period asset allocation can satisfy these short-term and long-term targets at the same time.

Steenhouwer also points out a few methodological issues. Strategic asset allocation decisions depend heavily on the outcomes of financial models. However, the model outcomes are very sensitive to the underlying assumptions. **Peter Schotman** (Maastricht University) suggests a robust way to deal with the uncertainty, namely, pooling scenarios from different models and taking more scenarios from preferred models. The second methodological issue Steenhouwer points out is about the length of the data period. The fundamental information about long-term behavior of financial variables, for example, nominal interest rates, lies in long-term data. But how long is 'long-term'? Steenhouwer shows that the mean of nominal interest rates from 1970 until now does not equal the mean derived from the data between 1810 and now. **Theo Nijman** suggests that theories may be helpful. He also points out that researchers should use data from other countries.

The first comment of the second discussant, **Jaap van Dam** (PGGM), is also about the objective of a pension fund. He agrees with Steenhouwer that a pension fund's investment should be separated from individual financial management. His second comment is about incomplete contracts. The contracts between a pension fund and its participants are neither clear nor complete. He believes that, despite the contract incompleteness, the utility of participating in a pension fund should be larger than the utility obtained from DIY (do it yourself) solutions where individuals make investment decisions by themselves. The main benefits of participating in a pension fund are, among others, the solidarity and convenience. Van Dam's third comment is about the relationship between strategic asset allocation and the pension fund's current solvency condition. He argues that the strategic asset allocation of a pension fund should take into account the current solvency condition of the fund.

In the panel paper, de Jong, Schotman and Werker summarize a paper by Benzoni et al (Journal of Finance, 2007). Benzoni et al (2007) claim that young people may wish for negative equity exposure due to the strong positive long-term correlation between returns in human capital and stock returns. **Lans Bovenberg** (Tilburg University) argues that introducing different preference functions for different ages might generate a different result.

PUBLICATIONS IN THE PANEL PAPERS SERIES

1. *Saving and investing over the life cycle and the role of collective pension funds*
   Lans bovenberg , Ralph Koijen, Theo Nijman and Coen Teulings
2. *What does behavioural economics mean for policy? Challenges to savings and health policies in the Netherlands*
   Peter Kooreman and Henriëtte Prast
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   Johan Mackenbach, Wilma Nusselder, Suzanne Polinder and Anton Kunst
8. *Strategic asset allocation* (2008)
   Frank de Jong, Peter Schotman and Bas Werker
9. *Pension Systems, Aging and the Stability and Growth Pact* (2008) Revised version
   Roel Beetsma and Heikki Oksanen

STRATEGIC ASSET ALLOCATION

This paper reviews the recent advances in the literature on strategic asset allocation decision. In its strategic asset allocation, a pension fund has several objectives, such as the interest in a good risk-return trade-off. A pension fund has specific liabilities in the form of future pension benefits, which are often indexed to prices or to wage growth. Furthermore a pension fund invests on behalf of its members, who often have large specific investments in human capital and housing. The strategic asset allocation decision concerns the choice of investments across these broad investment categories, and is the major determinant of a pension fund's investment risk profile. The researchers limit the scope of the paper by taking the optimal plan design and intergenerational risk sharing as given revealed preferences. The paper also reviews the quality of the econometric models of assets returns and the critical parameters for strategic asset allocation.

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