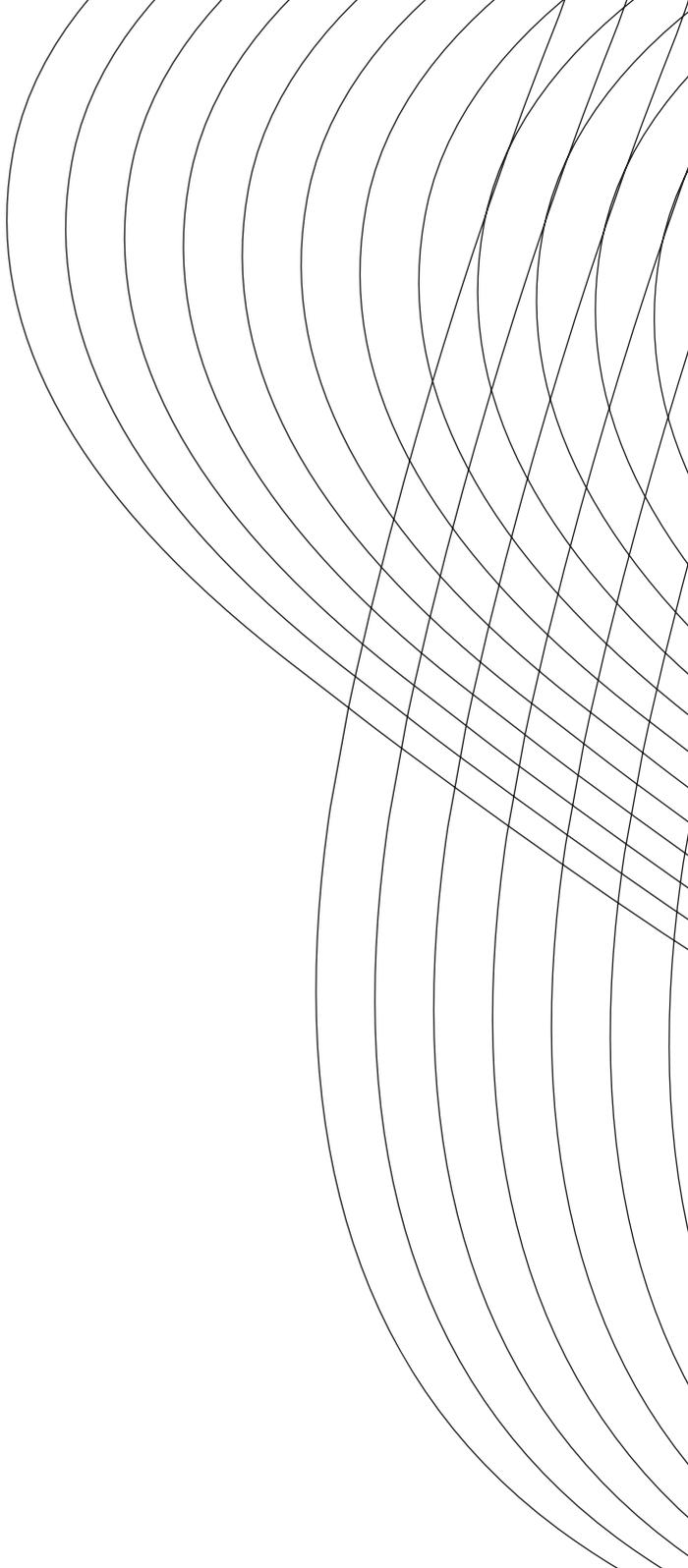




Hans Fehr

Computable Stochastic  
Equilibrium Models and  
their Use in Pension-  
and Ageing Research

Netspar Panel Papers



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Hans Fehr

# Computable Stochastic Equilibrium Models and their Use in Pension- and Ageing Research

PANEL PAPER 13



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# CONTENTS

<i>Preface</i>	7
<i>Abstract</i>	9
<i>Summary</i>	10
1. <i>Introduction</i>	15
2. <i>General Structure of a Stochastic OLG Model</i>	20
3. <i>Idiosyncratic Uncertainty and Intragenerational Risk-Sharing</i>	25
3.1 <i>Long-run Macro and Welfare Effects of Social Security</i>	25
3.2 <i>Does Social Security Privatization Produce Efficiency Gains?</i>	29
3.3 <i>Social Security as a Commitment Device</i>	37
3.4 <i>Family Risk-sharing and Social Security</i>	41
3.5 <i>Marital Risk and the Demand for Life Insurance</i>	47
4. <i>Aggregate Uncertainty and Intergenerational Risk-Sharing</i>	51
4.1 <i>Demographic Uncertainty and Fiscal Sustainability</i>	51
4.2 <i>Portfolio Choice and Financial Risk</i>	55
4.2.1 <i>Investment Risk over the Life Cycle</i>	56
4.2.2 <i>Asset Allocation in Taxable and Tax-Deferred Accounts</i>	60
4.2.3 <i>Annuitized vs. Non-annuitized Assets</i>	64
4.3 <i>Aggregate Risk in Computable General Equilibrium Models</i>	69
5. <i>What Have we Learned and Where Should we Go Next?</i>	73
<i>References</i>	76



## PREFACE

Netspar stimulates debate and fundamental research in the field of pensions, aging and retirement. The aging of the population is front-page news, as many baby boomers are now moving into retirement. More generally, people live longer and in better health while at the same time families choose to have fewer children. Although the aging of the population often gets negative attention, with bleak pictures painted of the doubling of the ratio of the number of people aged 65 and older to the number of the working population during the next decades, it must, at the same time, be a boon to society that so many people are living longer and healthier lives. Can the falling number of working young afford to pay the pensions for a growing number of pensioners? Do people have to work a longer working week and postpone retirement? Or should the pensions be cut or the premiums paid by the working population be raised to afford social security for a growing group of pensioners? Should people be encouraged to take more responsibility for their own pension? What is the changing role of employers associations and trade unions in the organization of pensions? Can and are people prepared to undertake investment for their own pension, or are they happy to leave this to the pension funds? Who takes responsibility for the pension funds? How can a transparent and level playing field for pension funds and insurance companies be ensured? How should an acceptable trade-off be struck between social goals such as solidarity between young and old, or rich and poor, and individual freedom? But most important of all: how can the benefits of living longer and healthier be harnessed for a happier and more prosperous society?

The Netspar Panel Papers aim to meet the demand for understanding the ever-expanding academic literature on the consequences of aging populations. They also aim to help give a better scientific underpinning of policy advice. They attempt to provide a survey of the latest and most relevant research, try to explain this in a non-technical manner and outline the implications for policy questions faced by Netspar's partners. Let there be no mistake. In many ways, formulating such a position paper is a tougher task than writing an academic paper or an op-ed piece. The authors have benefitted from the comments of the Editorial Board on various drafts and also from the discussions during the presentation of their paper at a Netspar Panel Meeting.

I hope the result helps reaching Netspar's aim to stimulate social innovation in addressing the challenges and opportunities raised by aging in an efficient and equitable manner and in an international setting.

*Henk Don*

Chairman of the Netspar Editorial Board

## ABSTRACT

This paper surveys recent advances in the field of computable general and partial equilibrium models that take into account various aspects of uncertainty. After discussing the risk-sharing features of social security in a stochastic environment, I introduce the basic structure of an overlapping-generations model with idiosyncratic labor income and mortality risk. Within this framework I discuss the quantitative results of studies dealing with reform of the social security system. I then move on to models that focus on aggregate factor-productivity risk and/or demographic uncertainty. A distinction is made here between models with uncertain demographics, partial equilibrium models that derive optimal portfolio choice and general equilibrium models quantifying gains from intergenerational risk-sharing.

## SUMMARY

At every stage of the life cycle, agents face substantial amounts of risk and uncertainty. There are numerous forms of idiosyncratic risk – such as mortality, labor income, health, marital or fertility shocks – that are quite dramatic for the individual but do not affect the economy as a whole. On top of that, individuals may be hurt by aggregate shocks such as wars or global financial crises that affect economy-wide wages and capital returns. To the extent that markets provide only incomplete private insurance, households benefit from the informal provision of (at least partial) insurance by families and various government programs. Although welfare gains from such arrangements are well known from theoretical models, it is only recently that advances in computational technology have made it possible to address them quantitatively in applied simulation models. As a consequence, a growing number of numerical models take into account various aspects of uncertainty and quantify the risk-sharing effects of the public pension system and other private insurance institutions. These studies provide results that challenge the existing common wisdom about the costs and benefits of social security.

During the 1990s, most of the quantitative research on social security focused on efficiency losses due to labor market distortions from pay-as-you-go (paygo) financing. Consequently, these deterministic computable general equilibrium (CGE) models either recommended a move towards a more funded system or proposed elimination of intragenerational redistribution by linking benefits to former contributions. Stochastic simulation models, which take into account mortality and individual income risk, shift the focus towards the insurance effects of social security systems. Since the latter implicitly provide an annuity insurance that is not (at least not cheaply) available on the private market, efficiency gains from privatization become much smaller or even negative. In addition, a progressive benefit formula could provide better insurance against income risk than a progressive tax system, since progressivity could be based on lifetime income instead of annual income. Stochastic models also include precautionary savings that are accumulated at the beginning of the life

cycle, so that they can properly account for borrowing constraints. The latter could be quite important in the analysis of social security reforms, since these reforms may increase or reduce existing borrowing constraints significantly.

Furthermore, stochastic models allow the consequences of non-standard preferences to be quantified. As recent studies indicate, privatization of social security may result in significant welfare losses, if the economy is populated by myopic consumers. Again, the idea of governmental interventions motivated by individual myopia and hyperbolic discounting is hardly new, but now we can quantitatively assess the welfare consequences. Finally, stochastic models are able to quantify the interaction between governmentally provided social security and the implicit insurance provision of the family. Since family members also provide (at least an incomplete) insurance against temporary income shocks and longevity, private family insurance may reduce the need for public programs. Therefore, social security privatization is more favorable if it is analyzed within the framework of an explicit family structure. While it is true that the family offers insurance, it is also true that the marital status itself is a source of uncertainty across the life cycle. Declining marriage rates coupled with increased divorce rates have dramatically changed the family structure in almost all industrialized countries in the past. Recent stochastic models introduce changing marital risk and endogenous marriage and divorce decisions into the life-cycle framework. Their results indicate that changes in family structure had a significant positive impact on asset accumulation. Rising marital risk leads to precautionary behavior similar to that resulting from rising income risk. Models with family formation can also capture the welfare effects arising from survivor benefits provided by the public pension system and allow for the introduction of the demand for life insurance.

The discussion until now has considered only idiosyncratic risk at the individual level and the potential welfare gains from intragenerational risk sharing arrangements. Only recently have simulation models that introduce aggregate demographic and productivity risk been developed. Both sources of uncertainty affect life-cycle wages and interest rates of a specific cohort as a whole. They are important for pension and ageing research mainly in two areas. First, the institutional design of the pension system may affect the individual portfolio structure over the life cycle. In order to minimize

welfare losses, we need to know the optimal life-cycle portfolio allocation with respect to the shares of bonds and equities but also with respect to the shares of annuitized and non-annuitized assets. Some recent studies indicate substantial welfare losses due to currently applied regulations of the portfolio structure of pension funds. They also find that complete annuitization may not be optimal in the presence of financial risk. Second, since macroeconomic shocks affect the cohort as a whole, it is important to isolate and quantify the potential intergenerational risk-sharing properties of various governmental programs. Typically, paygo-financed pensions are defined-benefit systems that allow the macroeconomic risk to be spread across generations. However, as results from numerical studies indicate (at least in the long-run), the welfare gains from improved intergenerational risk sharing do not fully compensate for the crowding-out of the capital stock due to unfunded social security. Quantitative research must therefore focus on the design of pension systems that balance intergenerational redistribution and risk-sharing.



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# COMPUTABLE STOCHASTIC EQUILIBRIUM MODELS AND THEIR USE IN PENSION- AND AGEING RESEARCH

## 1. Introduction

This paper provides a broad overview of the recent developments in the rapidly growing field of computable equilibrium models that account for various types of uncertainty. Since the focus here is on pension and ageing issues, I will concentrate on stochastic life-cycle and overlapping-generations models. During the past decade, economists have developed increasingly realistic simulation models that can be used for forecasting and policy analysis. At least to some extent, results from these models have challenged the previously existing quantitative research on the role of government and social security in a market economy.

Although the provision of insurance in an uncertain environment was one of the main motivations for the introduction of social security systems around the world, the academic literature on social security has for some time ignored uncertainty. Based on the seminal contribution by Samuelson (1958), social security was viewed as a "social contrivance" that corrects a fundamental market incompleteness – namely, the impossibility of writing contracts with the unborn. In Samuelson's overlapping-generations (OLG) model, market outcome was not efficient, so that intergenerational transfers from young to old could make everybody better off. For many years, economists focused on these "strange welfare properties of Samuelsonian economies" (Weil 2008). Models were developed that extended Samuelson's simple endowment economy with perishable goods to more realistic settings with less than fully depreciable capital, individual savings and endogenous factor prices. This work included Diamond's (1965) analytical two-period model with government debt as well as Auerbach and Kotlikoff's (1987) 55-period life-cycle simulation model. Individuals in these models were saving for old age by investing in physical capital at an interest rate equal to the marginal product of capital less the rate of depreciation. A central policy question addressed in these deterministic

models was the financing of social security. Should we continue with the paygo system or should we phase it out and switch to a fully funded system of social security?

As it turned out, in a dynamically efficient economy (i.e. where the interest rate exceeds the growth rate of real wages) in which myopia and market failure are absent, the efficiency consequences of social security privatization depend on the changes in labor supply distortions. With fixed labor supply, a Pareto-improving elimination of social security is not possible. Without compensation of transitional cohorts, future cohorts will gain due to higher savings, which increase future capital stocks and real wages. However, these long-run welfare increases are due to income losses of transitional generations. It therefore makes little sense to compare long-run equilibriums with and without social security. If transitional cohorts are fully compensated, then the implicit pension debt and implicit tax rate of the pension system are transformed into explicit public debt and tax figures, and the welfare of future generations remains unaltered (see Breyer 1989, or Sinn 2000). Matters are different when labor supply is endogenous. If social security is progressive so that benefits are flat or only partially linked to former contributions, then privatization reduces existing labor supply distortions. These efficiency gains can be used, at least in principle, to design a strictly Pareto-improving transition path. Homburg (1990) demonstrated this result for the small open-economy case, while Breyer and Straub (1993) extended it to the closed-economy case. If the paygo system is intragenerationally fair (so that benefits are strictly linked to former contributions), then the household considers only a fraction of its contributions as a tax on labor income. This fraction is equal to the difference between the real interest rate and the growth rate of labor income. As Fenge (1995) and Brunner (1996) showed, in such a situation it is not possible to establish a Pareto-improving transition path without resorting to ability-specific lump-sum taxes. Due to the lack of information, such instruments are not available to the government in practice.<sup>1</sup>

However, existing paygo-financed pension systems in many countries include large fractions of progressive elements in which benefits are independent of former contributions. When population ageing began to

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<sup>1</sup> In principle, it would only be possible to apply cohort-specific labor taxes. Conesa and Garriga (2008) provide a quantitative analysis of social security reforms in such a context.

put pressure on these systems to increase contributions and further distort labor supply, social security funding was considered to be a policy option offering the potential of increased efficiency. Numerous deterministic simulation models were then developed that computed the intergenerational and intragenerational redistribution effects of various privatization strategies in different countries.<sup>2</sup> Studies such as Kotlikoff (1998) or Fehr (1999), which fully compensated transitional cohorts, typically computed significant efficiency gains from privatization. This led to the policy conclusion that in the absence of redistributive and paternalistic motivations for establishing a social security program, a paygo system generally makes individuals worse off than a fully funded or privatized system.

There is ample reason to distrust these conclusions, since deterministic models completely ignore the important risk-sharing features provided by social security.<sup>3</sup> When there is uncertainty, the laissez-faire equilibrium of the overlapping-generations model is never optimal, because the unborn cannot take part before their birth in risk-sharing trades with other generations.<sup>4</sup> This Pareto sub optimality of the market economy does not depend on the relation between the interest rate and the growth rate of the economy, and it also holds in the absence of any traditional market failure. While competitive markets inhibit intergenerational risk sharing, they also do not provide efficient intra-generational risk-sharing mechanisms. The seminal work of Rothschild and Stiglitz (1976) established that insurance markets may suffer heavily due to moral hazard and adverse selection problems. This appears to be the key explanation for the fact that markets for annuities, disability pensions, unemployment insurance and health insurance either do not exist at all or are highly undeveloped. When formal market insurance is missing, households can only imperfectly smooth individual income fluctuations – either with self-insurance through precautionary savings or with informal risk-sharing

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2 For a more detailed discussion of different approaches, see Lindbeck and Persson (2003) or Jimeno, Rojas and Puente (2008).

3 In this survey, the terms "risk" and "uncertainty" are used interchangeably. In most of the literature, however, "risk" refers to a situation in which outcomes are drawn from a known probability distribution, whereas "uncertainty" refers to a random outcome with an unknown probability distribution.

4 The literature also distinguishes a weaker concept called "interim" Pareto optimality, which can be achieved in the market economy (see Demange 2002). But it seems to me that usually the stronger concept of "ex-ante" Pareto optimality is applied in the discussion (see Bovenberg and Uhlig 2008, Krueger and Kubler 2006, or Weil 2008).

arrangements among extended family members (as in Kotlikoff and Spivak 1981). Consequently, government intervention is required in order to correct missing markets for intergenerational and inefficient markets for intragenerational risk sharing. Therefore, studies dating back to Eckstein, Eichenbaum and Peled (1986) or Karni and Zilcha (1986) show that social security can in principle correct such market failure or substitute for these missing markets to make all individuals better off. Since then, numerous theoretical papers (such as Shiller 1999, Rangel and Zeckhauser 2001 or Ball and Mankiw 2007) have addressed positive and normative aspects of social security's risk-sharing features. At the same time, due to advances in computer hardware and numerical solution algorithms, stochastic equilibrium life-cycle and overlapping-generations models were developed that quantify whether social security is indeed effective in counteracting market failure. The central problem here is to quantify welfare gains from improved insurance provision relative to the adverse distortionary impact these programs have on labor supply and capital accumulation. In an uncertain world, optimal policy seeks to balance incentive and redistribution effects and the implicit insurance provision.

In order to structure the discussion, I will concentrate on numerical results and focus in the following discussion more on pension research than on that of ageing. Consequently, I will mostly exclude small-scale (i.e. more stylized) equilibrium models with two overlapping cohorts and neglect the normative discussion that compares the first-best command optimum with alternative decentralized market allocations. The next section describes the general structure of this quantitative approach and introduces some central welfare concepts that are important in the interpretation of the numerical results. The third section deals with idiosyncratic uncertainty and intragenerational risk sharing. This focus has both an empirical and a practical motivation. On the empirical side, various studies (such as Pozzi 2007) indicate that idiosyncratic uncertainty is more important for individual welfare than aggregate uncertainty is. With respect to the practical side, one has to admit that the current state of the art in computational economics has a hard time including aggregate uncertainty in large-scale overlapping-generations models. Consequently, section 4 presents three approaches that deal with aggregate risk in life-cycle models. The first two approaches have some severe theoretical drawbacks. Models with uncertain demographics exclude uncertainty from individual decisions. Partial equilibrium dynamic programming models

with aggregate risk exclude feedback effects from factor markets and the government budget constraint. The last approach considers aggregate risk in a full-fledged CGE model with overlapping generations. However, due to technical reasons this line of research is still in its infancy and only some preliminary attempts can be presented. Finally, section 5 provides some policy conclusions and some (more personal) notes on expected future developments.

## 2. General Structure of a Stochastic OLG Model

In order to answer the quantitative questions raised in the introductory section, I will concentrate on overlapping-generations models in the tradition of Auerbach and Kotlikoff (1987).<sup>5</sup> The production side of the model comprises the standard one-sector neoclassical growth model, where capital and labor are employed to produce a single output good, which in turn can be used for both consumption and investment purposes.<sup>6</sup> Capital depreciates at a constant rate and – depending on the household's preference structure – there may be either labor- or time-augmenting technological progress at a constant rate, consistent with the existence of a balanced growth path. On the household side, the original Auerbach and Kotlikoff model was extended in various directions in order to include borrowing constraints and various sources of uncertainty. With respect to social security issues, two sources of uncertainty seem to be especially important.

First, since in reality lifespan is uncertain and actuarially fair private annuity markets are absent, social security provides an implicit insurance against the risk of a household outliving its own resources. Consequently, at each age  $j$  there exists a certain conditional survival probability  $\psi_j$  which decreases to  $\psi_{J+1} = 0$  when the household reaches maximum age  $J$ . Since assets are accumulated in order to reduce longevity risk, unintended bequests are left in the case of premature death.

Second, labor income (or wage) uncertainty is desirable for various reasons. First, it generates precautionary savings, which are empirically important (Gourinchas and Parker 2001) and have a much lower interest elasticity (Cagetti 2001). Consequently, tax-favored saving schemes will have only modest effects on wealth accumulation. Second, earnings uncertainty interacts with the borrowing constraints of low-income

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5 An alternative OLG approach in the tradition of Blanchard (1985) is applied in quantitative studies such as Jaag, Keuschnigg and Keuschnigg (2008). This line of research can be used to study the effects of social security reforms and demographic changes analytically in general equilibrium. However, the concept of economic (or "probabilistic") aging is quite different from the usual idea that age measures time since birth. Consequently, the quantitative results of these models are difficult to interpret and to compare with traditional approaches.

6 I abstract from aggregate risk in this section and describe the basic structure of models discussed in the next section. When I move on to models with aggregate risk, I will explain in detail how this feature changes the structural equations.

households at young ages, and social security reforms may alleviate or exacerbate these borrowing constraints. Third, paygo-financed pensions may provide insurance against lifetime earning uncertainty if benefits are imperfectly linked to former contributions.

Given these idiosyncratic sources of risks and a specific government policy that defines individual net payments, the household has to decide in every period how much to work (in case of variable labor supply), how much to save and how much to consume.<sup>7</sup> Therefore, given the individual state  $z_j = (a_j, \bar{a}_j, e_j)$  consisting of endowment with financial assets  $a_j$  and pension wealth  $\bar{a}_j$  as well as individual productivity  $e_j$ , the household maximizes utility at each age  $j = 1, \dots, J$

$$V(z_j) = \max_{c_j, \ell_j} \left\{ u(c_j, \ell_j)^{1-\frac{1}{\gamma}} + \delta \psi_{j+1} E[V(z_{j+1})^{1-\eta}]^{\frac{1-\frac{1}{\gamma}}{1-\eta}} \right\}^{\frac{1}{1-\frac{1}{\gamma}}}$$

subject to the budget constraint and the accumulation equation of pension wealth

$$\begin{aligned} a_{j+1} &= a_j(1+r) + b_j + q_j - T_j - c_j \\ \bar{a}_{j+1} &= \bar{a}_j + \lambda \Psi(\bar{w}) + (1-\lambda) \Psi(w_j) \end{aligned}$$

as well as the expected future stochastic shock to labor productivity, which follows a Markov chain with transition probabilities

$$\Pi_j(e_{j+1} | e_j) > 0.$$

In the variables  $c_j$  and  $\ell_j$  denote consumption of goods and leisure at age  $j$ , respectively. The parameters  $\delta$  and  $\gamma$  represent the discount factor and the intertemporal elasticity of substitution between consumption in different years. Since lifespan is uncertain, expected utility in future periods is weighted with the survival probability  $\psi_{j+1}$ . Following the approach of Epstein and Zin (1991), it is possible to isolate risk aversion from intertemporal substitution. The expectation operator  $E$  in then indicates that future utility is computed over the distribution of  $e_{j+1}$ , where the parameter  $\eta$  defines the degree of (relative) risk aversion. Note that for the special case  $\eta = \frac{1}{\gamma}$  we are back at the traditional expected utility specification (see Epstein and Zin 1991, p. 266).

The household faces a budget constraint, where income is generated from previously purchased assets  $(1+r)a_j$ , from (accidental) bequests  $b_j$  of households that died prematurely and from labor income  $w_j$  during their

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7 Of course, there may also be other decision dimensions – such as where to save and where to spend the time endowment.

working period (i.e. before they reach retirement age  $j_R$ ) or public pensions  $p_j$  afterwards, i.e.

$$q_j = \begin{cases} w_j = w(h - \ell_j)e_j & \text{if } j < j_R \\ p_j = \Phi(\tilde{a}_{jR}) & \text{if } j \geq j_R \end{cases}$$

While total time endowment  $h$  may change over time due to technical reasons,  $h - \ell_j$  defines working time and  $w$  the wage rate for effective labor. Labor productivity  $e_j$  may comprise a deterministic age component, a fixed effect that reflects the heterogeneity in productivity before labor market entry (i.e. human capital), and a stochastic productivity shock that might depend on age as well (see equation). Pensions are a function  $\Phi(\cdot)$  of accumulated pension wealth at retirement  $\tilde{a}_{jR}$ . Equation defines the accumulation of pension wealth, where the fractions  $\lambda$  and  $1 - \lambda$  define the flat and the contribution-related parts of pension wealth, respectively. The former depends on average income  $\bar{w}$ , while the latter depends on individual income  $w_j$  via the function  $\Psi(\cdot)$ . Households pay income and consumption taxes as well as social security contributions to the government; these are aggregated in  $T_j$ . The tax schedule may include proportional and progressive taxes, while social security contributions could be restricted to specific income thresholds and ceilings. Finally, consumption, labor supply and asset holdings are required to be non-negative (i.e. households face borrowing constraints), and agents are restricted in their leisure consumption by their time endowment (i.e.  $\ell_j \leq h$ ).

The model is able to replicate the tax and social security system in quite some detail by the specification of individual tax payments and pension benefits. In general equilibrium, a continuum of households interacts on labor and capital markets in order to determine real wages and returns to capital. The former equates labor demand by firms to labor supplied by households; the latter equates capital demand by firms and public debt to total desired asset holdings of households. Typically, the government runs an unfunded pension system, so that contributions from workers finance the benefits of pensioners. In addition, the government may also provide a public consumption good that is financed by taxes and public debt. Of course, the public budget has to be balanced intertemporally.

For a fixed policy schedule, a stationary equilibrium is given by a distribution of households by age, wealth and productivity levels that

remains constant over time. This distribution is stationary, although individuals age and change their position in the income and wealth distribution due to idiosyncratic shocks. After a policy reform is announced or enacted, the economy enters a transition path to the new long-run equilibrium. Of course, the explicit calculation of the transition path is a computational challenge.<sup>8</sup> However – as already indicated above – ignoring the transition and simply comparing steady states may result in rather misleading policy conclusions. Figure 1 reproduces the intergenerational welfare effects of a typical simulation run. The horizontal axis shows the different cohorts, while the vertical axis reports the welfare changes (typically computed as equivalent variation and measured in percent of remaining lifetime resources) after the reform. Cohorts to the left of the vertical axis were born before the reform year 0, while cohorts to the right are born either in the reform year or afterwards.

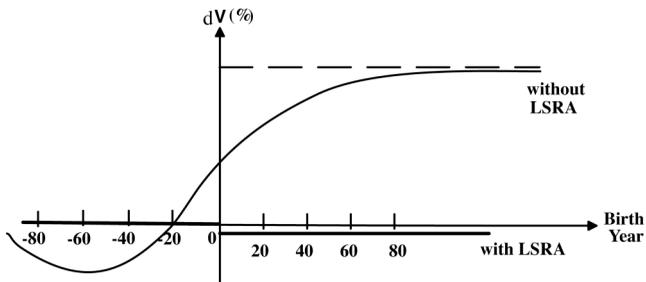


Figure 1: Intergenerational welfare effects of a typical simulation run.

The graph “without LSRA” shows that the considered reform reduces the welfare of currently working and already retired cohorts, whereas cohorts currently younger than 20 years old and future cohorts realize welfare gains. The dotted horizontal line indicates that the considered reform maximizes long-run welfare gains. If the transition is ignored, the welfare effects of the reform would be considered to be very positive. However, most cohorts alive in the reform year are hurt by the reform. This is ignored by a purely long-run analysis, but a comprehensive evaluation of the reform also has to take such intergenerational distribution effects into

8 For the technical details of computing long-run equilibria and the transition path, see Judd (1998), Marimon and Scott (1999) or Heer and Maussner (2009).

account.<sup>9</sup> Once the winners and losers from a reform are identified, the logical next step is to find out whether the winners can compensate the losers. Are the long-run welfare gains in Figure 1 only due to transitional welfare losses or does the reform produce an overall efficiency gain? Auerbach and Kotlikoff (1987) provided a tool that can be used to answer this question. They introduced the so-called "Lump-sum redistribution authority" (LSRA), which compensates existing generations after the reform with lump-sum transfers and taxes so that these generations end up at their pre-reform welfare level. Newborn and future cohorts pay lump-sum taxes to or receive transfers from the LSRA in order to balance the LSRA budget intertemporally. In order to facilitate comparison of efficiency- and welfare effects, LSRA payments are distributed across newborn and future cohorts so that they experience identical welfare gains or losses after compensation. In the example of Figure 1, the LSRA pays lump-sum transfers to cohorts older than 20 years and levies lump-sum taxes on younger cohorts in the reform year. Since taxes on younger cohorts are not sufficient to finance the transfers to older cohorts, the LSRA levies lump-sum taxes on newborn- and all future cohorts. Since LSRA payments dominate welfare gains due to the reform (i.e. without compensation), newborn- and future cohorts realize an identical welfare loss after compensation (see the graph "with LSRA"). This welfare loss reflects the aggregate efficiency effect of the policy reform under consideration. As indicated by the example in Figure 1, the long-run welfare change is generally an ineffective indicator of the aggregate efficiency consequences of the reform.

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9 Of course, one could even disaggregate further within existing cohorts in order to quantify intragenerational distribution effects.

### 3. Idiosyncratic Uncertainty and Intragenerational Risk-Sharing

Hubbard and Judd (1987) extended the original Auerbach and Kotlikoff (1987) model by including lifespan uncertainty and liquidity constraints. Private annuity markets are missing in this model, so that social security provides an insurance against longevity risk. At the same time social security also increases the borrowing constraints of young households. The introduction of social security generates a long-run welfare gain in a small open economy, since insurance benefits compensate rising borrowing constraints. In a closed economy, however, lower individual savings crowd out capital and lead to a reduction of long-run real wages. These additional intergenerational income effects further dampen long-run welfare, which may now even turn negative.

Of course, Hubbard and Judd (1987) make only the first step towards an analysis of insurance and liquidity effects of social security. Their model abstracts from labor income uncertainty, so that precautionary savings are neglected and liquidity constraints become relevant for all young individuals. Hence, a more realistic analysis of the insurance and liquidity effects of social security has to include income risk and intragenerational heterogeneity. Since the mid 1990s, various overlapping-generations general equilibrium models have been developed in this direction. This section reviews the central results of these papers. After first focusing on pension privatization in the standard model, I consider extensions to non-standard preferences and family risk-sharing.

#### *3.1 Long-run Macro and Welfare Effects of Social Security*

The new line of research was initiated by Imrohoroğlu, Imrohoroğlu and Joines (1995, 1999), who introduced uncertainty about labor earnings. The latter is modeled by a binary state variable that either takes a value of unity if the agent is given the opportunity to work in a specific period or is zero otherwise. The employment state follows a two-state Markov process with an age-independent transition-probability matrix. Agents supply labor inelastically when they have a job, and otherwise receive unemployment benefits. After passing the mandatory retirement age, individuals receive pension benefits. Since these benefits are independent of former contributions (i.e.  $\lambda = 1$ ), the pension system also provides insurance against labor income uncertainty. Apart from intergenerational income effects, the welfare consequences of unfunded pensions reflect the

trade-off between the (positive) insurance provision to address income and longevity risk and the (negative) effects of stronger binding liquidity constraints. Since the model abstracts from variable labor supply, the institutional setup already is quite favorable for social security. In addition, Imrohoroglu, et al. (1995) calibrated an initial equilibrium (without social security) that lacks dynamic efficiency – i.e. the growth rate of wages exceeds the marginal product of capital – so that the introduction of social security is Pareto-improving. For the benchmark calibration, the optimal social security replacement rate that maximizes long-run welfare is 30 percent. Due to positive insurance effects, this rate is higher than the one that eliminates dynamic inefficiency.

Since the initial equilibrium represented a highly unrealistic situation, the follow-up study by Imrohoroglu et al. (1999) eliminated dynamic inefficiency of the initial equilibrium (without social security) by incorporating land as a fixed factor of production. In this setting, the introduction of an unfunded pension system has again positive insurance and negative liquidity effects, but now the retired cohorts in the reform year receive a windfall gain at the expense of current young- and future cohorts. The simulations indicate that in the long run, (positive) insurance effects are always dominated by (negative) income effects, so that an economy without social security provides the highest welfare for individuals. In contrast to the authors of this study, however, I do not think that this result promotes the elimination of social security as a superior policy advice. There are no labor supply distortions in this model, so that the introduction of social security more likely has positive efficiency effects after compensating for intergenerational income redistribution. Imrohoroglu et al. (1999) completely ignore this fact, neither considering the transition path nor attempting to compensate intergenerational income effects.

Of course, the same critique applies to all studies that analyze only the long-run consequences of social security reforms. Huggett and Ventura (1999) extend the framework of Imrohoroglu et al. (1999) in various directions. First, their model allows for endogenous labor supply, so that induced labor supply distortions weaken the case for social security. Second, similar to equation, they distinguish between an earnings-related and a flat part of retirement benefits in order to represent the existing U.S. social security system. Third, the labor endowment process is modeled as a mean-reverting stochastic process. The latter allows us to distinguish

agents according to ability levels within a generation, so that intragenerational distributional effects of social security arrangements can be quantified. In this setup, Huggett and Ventura (1999) study the aggregate and distributional consequences of a specific U.S. reform proposal where retirement benefits are set equal to the maximum of a (tax-financed) "floor benefit", and an annuity proportional to the taxes paid up to retirement age, where the proportionality factor is chosen, so that the system is self-financing. Their simulations suggest that both high- and low-ability agents would benefit from a switch to the two-tier system, while median-ability agents would lose. Since most of the agents in the economy are close to median, the considered reform would yield a very robust ex-ante welfare loss.

Despite the innovation compared to the approach in Imrohoroğlu et al. (1995, 1999), Huggett and Ventura (1999) still model a fairly ad-hoc stochastic income process. The very influential study by Storesletten, Telmer and Yaron (1999) overcomes this problem by explicitly introducing three different types of earnings risk: 1) persistent individual differences in ability (or labor "endowment") realized at birth, 2) serially correlated shocks, and 3) transitory shocks to labor supply. The time-varying shocks are designed to provide a "reduced-form" account of factors such as job loss, health problems, child birth, marital transition etc. that may temporarily interrupt an individual labor supply trajectory. Storesletten et al. (1999) used econometric methods to estimate the parameters of the specified stochastic process of labor earnings that matched observed moments of labor earnings in the Panel Study on Income Dynamics (PSID). Their policy reform either replaces the current U.S. pension system by a two-tier system of (funded and unfunded) personal saving accounts (PSA), or by a completely private pension (PP) system. Table 1 reports the long-run welfare effects of both reforms expressed as a consumption equivalent.

Table 1: Welfare Effects of Social Security Privatization (in %)

	PSA	PP
Without SS debt	7.32	0.88
With SS debt	4.03	3.75
Decomposition:		
capital tax distortions	-0.03	-0.72
general equilibrium effects	3.66	6.83
annuity provision	0.68	-0.76
income risk-sharing	-0.29	-1.61

Source: Storesletten, Telmer and Yaron (1999), p. 243.

The first line ("without SS debt") ignores – as does the previous study by Huggett and Ventura (1999) – the initial pension liabilities. Consequently, the phase-down of the existing pension system increases the capital stock and real wages of future households, which therefore realize significant welfare gains. As in Imrohoroglu et al. (1995), the long-run equilibrium in the PP economy without social security is dynamically inefficient. As a consequence, welfare gains are much higher in the PSA economy, where only a fraction of social security is eliminated. Next, the authors try to account for the pension system's obligations to current beneficiaries and contributors by including alternative measures of social security debt in the capital market of their new long-run equilibrium. The second line of Table 1 ("with SS debt") shows that now the welfare effects are rather different. Compared to the previous line, welfare gains in the PSA economy are reduced, since the intergenerational income redistribution is dampened. In the PP economy, welfare gains increase with social security debt, since the previous equilibrium was dynamically inefficient. The difference between the first two lines in Table 1 clearly shows how important it is to account for transitional effects. Therefore, the introduction of social security debt is an enormous improvement compared to previous studies, which ignored this problem completely. In addition, Storesletten et al. (1999) also make a first attempt to disaggregate the overall welfare effects of pension privatization into insurance-, distortion- and intergenerational income components. For that reason, they simulate the considered reforms in alternatively designed economic environments and compare the resulting welfare effects with the benchmark gains of 4.03 and 3.75 percent. The lower part of Table 1 reports

that aggregate welfare effects are dominated by the general equilibrium reaction of factor prices. Changes in tax distortions and the loss of insurance provision play only a minor role. This indicates that social security debt does not neutralize the ongoing intergenerational redistribution. Indeed, as already noted by Imrohoroglu (1999, p. 264), the concept of social security debt that is applied is a fairly arbitrary way to account for transitional cohorts.

### *3.2 Does Social Security Privatization Produce Efficiency Gains?*

All studies discussed thus far only consider the long-run effects of social security and neglect – with the exception of Storesletten et al. (1999) – the consequences for transitional generations. The long-run effects computed there are (at least partly) due to intergenerational income redistribution. In order to improve the assessment of a specific reform, one has to calculate the transition path between steady states and separate intergenerational redistribution from efficiency effects. The first study to address this critique in a stochastic CGE model was Huang, Imrohoroglu and Sargent (1997). In order to simplify computation and aggregation, they abstracted from variable labor supply and applied linear-quadratic preferences that deliver linear decision rules. The study compares two experiments in which the existing unfunded social security system is eliminated and either a private individual account system or a government-run collective-fund system is introduced. In the first experiment, the government suddenly terminates social security and buys out all who were entitled to retirement benefits under the old system by issuing a huge amount of government debt. This debt is fully retired over a 40-year transition period to the new steady state. In the second experiment, social security benefits remain untouched, but the existing system is transformed into a funded one through the build-up of a Trust Fund over a 40-year transition period at a sufficient rate so that all benefits can be paid from interest earnings of the Trust Fund after the transition period. Therefore, funding is financed in both experiments by temporarily raising the labor income tax rate, which hardly affects already retired cohorts. Whereas the consumption of initially working cohorts decreases due to higher labor taxes, it increases for future cohorts, who benefit from reduced labor taxes and the improved capital accumulation. In order to quantify the efficiency consequences of the reforms, Huang et al. (1997) aggregate the wealth-equivalent welfare gains and losses of all cohorts along the transition path and then convert the

aggregate into an annuity value that is expressed as a fraction of the initial equilibrium GDP. Given this measure, both experiments yield a significant aggregate efficiency gain, but the collective fund clearly outperforms the purely private system. The authors do not explain the numerical results in detail. The efficiency gains of the private system, in particular, are somewhat surprising, since households lose the insurance provision of the pension system and may benefit only from reduced intertemporal distortions and lower borrowing constraints. Under the collective fund system, the insurance provision of social security remains fully valid. Therefore, it is hardly surprising that the government-run funding scheme is preferred to privatization. However, there is also a theoretical problem with the applied efficiency measure. The aggregate of individual welfare changes measures efficiency only incompletely, since households are not compensated. The approach is based on an implicit social welfare function that might, in principle, increase (given the appropriate weights) – even if the considered policy is not Pareto-improving.

The follow-up study by De Nardi, Imrohoroglu and Sargent (1999) extended the model by including realistic U.S. demographics, a bequest motive and variable labor supply. The baseline path assumes that the demographic transition is financed purely by higher payroll taxes. The study then introduces various policy reforms (i.e. consumption tax financing, an increase in the retirement age, benefit taxation, an increase in tax-benefit linkage, privatization) and compares the resulting macro and welfare effects of each reform with the respective values from the baseline path. With variable labor supply, consumption taxation typically enhances economic efficiency (see Auerbach and Kotlikoff 1987). Therefore, it is not surprising that the reforms with consumption taxation outperform the respective reforms financed by labor taxation. The study also finds that a stronger tax-benefit linkage has only modest macro effects, but such a policy is strictly Pareto-improving in this framework. This result deserves a comment. Of course, a switch from a defined-benefit to a defined-contribution system reduces labor supply distortions, which in turn increases efficiency. At the same time, however, such a reform reduces the insurance properties of the pension system against income shocks – since now bad outcomes during employment are transferred to the retirement period. It seems that households in this model place little value on such insurance effects, which might be due to the quadratic utility function applied.

Before I return to this issue, I would like to mention a study by Conesa and Krueger (1999) that focuses on political economy aspects of social security reforms.<sup>10</sup> They simulate the transition path of an immediate, a gradual and an announced elimination of social security, and compute the political support for the three proposals in the initial year. Although for all cases considered agents would prefer to be born into the final steady-state, no proposal receives an initial voting majority in the closed-economy case. The political support is declining when intra-cohort heterogeneity is increasing due to the rising insurance gains from flat pensions. While this analysis explains why pension reforms are delayed in democratic systems, it does not include efficiency calculations, which seem to be especially interesting when combined with political economy considerations. One could easily imagine situations in which a policy reform receives no political support, despite the fact that it delivers aggregate efficiency gains and vice versa.

Consequently, even for such political economy considerations, I think it is important to isolate the overall efficiency effects of policy reforms. The first study achieving that in a stochastic CGE model was Nishiyama and Smetters (2007), who simulated a stylized phased-in 50-percent privatization of the U.S. social security system in a model with variable labor supply. The benefits of households aged 65 or older in the reform year are not affected by the reform. Benefits decrease linearly for younger households, so that cohorts aged 25 or younger in the reform year receive only half of their traditional benefits when they turn 65. As before, the considered reform reduces not only labor supply distortions but also insurance provision of the social security system. In order to isolate the overall efficiency effects, the authors introduce LSRA transfers and taxes in a separate simulation in order to compensate initial agents and to distribute the accumulated assets (i.e. efficiency gains) or debt (i.e. efficiency losses) to newborn and future agents (as shown in Figure fig1 above). The reform is first simulated in an economy without income uncertainty. As one would expect, the results show a clear redistribution from the elderly- and middle-aged (i.e. younger than 65 years) cohorts towards younger and future-born cohorts. After compensating the welfare losses of the elderly, the study finds efficiency gains from privatization that

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<sup>10</sup> Other quantitative political economy models of social security reforms include Cooley and Soares (1999) as well as Galasso (1999), among others.

amount to \$18,100 (in 2001 growth-adjusted dollars) per household. Therefore, the loss in annuity provision is overcompensated by reduced labor market distortions and increased liquidity of younger households. Next, the reform is simulated with idiosyncratic labor income uncertainty. Table tab2 reports the welfare changes (i.e. without compensation) and the efficiency effects (i.e. with compensation payments) for different cohorts and income classes.

Table 2: Welfare and Efficiency Effects of Social Security Privatization (in \$1000)

Age in reform year	Welfare effects for selected income classes				Efficiency effects
	Class 1	Class 3	Class 5	Class 8	
79	-0.2	-0.2	-0.3	-0.5	0.0
60	-22.6	-29.7	-37.5	-57.1	0.0
40	-27.6	-46.0	-75.8	-130.7	0.0
20	-5.4	-7.4	-11.2	-22.4	0.0
0	34.3	37.0	38.4	30.7	-2.4
$-\infty$	76.3	84.1	91.3	91.2	-2.4

Source: Nishiyama and Smetters (2007), Table IX, p. 1700.

In the long run, welfare gains from partial privatization range from \$76,300 for low-income households to \$91,200 for top-income households. The higher gains for rich households simply indicate the redistributive features of the U.S. social security system. Similar figures are also reported by the studies discussed above that compute only the long-run consequences of social security reforms. However, these gains are mainly due to income redistribution, since elderly cohorts at the time of the reform realize significant welfare losses due to higher income- or consumption taxes that are required to finance the existing pension claims. Table tab2 shows that 60-year olds at the time of the reform lose between \$22,600 and \$57,100 per household. If the LSRA is introduced and all existing generations at the time of the reform are compensated by transfers, then each household that enters the economy in the reform year or later has to pay \$2,400 to service the debt of the LSRA. In other words, the partial privatization of the U.S. social security system does reduce economic efficiency because the (positive) insurance effects of the U.S. social security system dominate the distortionary effects on labor supply and losses from increased borrowing constraints.

Fehr, Habermann and Kindermann (2008) and Fehr and Habermann (2008) reach a similar conclusion for the German social security system. In

contrast to the U.S. system, benefits in the German system are strongly linked to former contributions; i.e.  $\lambda = 0$  in. While this institutional feature minimizes labor supply distortions, it also reduces insurance provision against income shocks. In addition, the numerical model accounts for significant borrowing constraints. In line with empirical estimates from the German SAVE study, roughly 20 percent of younger individuals are credit constrained in the initial equilibrium. Fehr, Habermann and Kindermann (2008) phase-out the existing paygo-financed German pension system and replace it by private savings. This is accomplished by simply eliminating the further accumulation of pension wealth in equation.

After the reform, existing pension claims are financed by time-invariant payroll taxes, and the intertemporal budget of the general government is balanced by consumption taxes. Consequently, the burden is smoothed across current and future generations with endogenous public debt. The reform reduces payroll taxes by 8 percent and consumption taxes by 3.3 percent, since higher savings for old age boost income-tax revenues. Due to the immediate decrease in consumption taxes, already retired cohorts benefit from the reform. Middle-aged cohorts lose (since the tax share of contributions was below the payroll tax) and future generations win (due to lower payroll taxes). Table 3 reports the aggregate efficiency effect (measured as the percentage of lifetime resources) of this experiment. In order to isolate both the impact of labor market distortions and the insurance and liquidity effects on aggregate efficiency, we simulate the privatization experiment with alternative assumptions about tax progressivity, income and lifetime uncertainty, as well as liquidity constraints. In each simulation we fix the respective interest rate so that the capital-output ratio always remains constant.

Table 3: Efficiency Effects of Social Security Privatization (%)

Scenario	Progressive tax system	Uncertain lifetime	Liquidity constraints	Stochastic income	Deterministic income
(1)	yes	yes	yes	-0.57	-3.48
(2)	no	yes	yes	-0.54	-1.26
(3)	no	no	yes	0.68	-0.03
(4)	no	no	no	0.18	-0.05

Source: Fehr, Habermann and Kindermann (2008), Table 6, p. 898.

Scenario (1) features the benchmark assumptions with a progressive tax system, an uncertain lifespan and binding liquidity constraints for low-income households at young ages. Similar to Nishiyama and Smetters (2007), we find that social security privatization in Germany would reduce economic efficiency. However, in contrast to the previous study, we find that aggregate efficiency deteriorates dramatically with deterministic income. This seemingly contradictory result can be explained straightforwardly. In our reform experiment, privatization induces a shift from consumption taxation towards (progressive) income taxation. As shown in Auerbach and Kotlikoff (1987), such a shift would harm economic efficiency even with proportional taxes. With progressive income taxes, labor supply distortions increase even further. With stochastic income, however, progressive income taxes also provide insurance against income shocks. Consequently, efficiency losses are much smaller in the latter case.

In scenario (2) we assume a proportional income tax of 10 percent in the initial equilibrium, so that both the income insurance effect of the tax system and the labor supply distortions are reduced. With stochastic income, the effects neutralize each other so that aggregate efficiency is hardly affected. With deterministic income, there are no insurance effects – and only labor supply distortions and liquidity effects matter for efficiency calculations. Therefore, aggregate efficiency losses are much lower compared to the respective figure of scenario (1). In scenario (2), the reported aggregate efficiency losses are mainly due to the elimination of longevity insurance, which overcompensates (positive) liquidity effects.

Consequently, in scenario (3) we eliminate the longevity insurance of social security by simulating the model with a certain lifespan of 80 years. Now, reported efficiency gains are solely due to liquidity effects and tax distortions, and the difference between scenarios (3) and (2) roughly captures the provision of longevity insurance by the pension system. Consequently, the (implicit) longevity insurance amounts to 1.1 percent of the remaining resources.<sup>11</sup>

Finally, scenario (4) simulates the model with unrestricted borrowing in the initial equilibrium. The difference between scenarios (4) and (3)

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11 Of course, this rather significant amount is also due to the fact that the model abstracts from a bequest motive. Fehr and Kindermann (2009) include the latter and simulate privatization with and without annuitized accounts in order to quantify the longevity insurance. In this case, the value of the longevity insurance is reduced to roughly 0.5 percent of remaining lifetime resources.

captures the efficiency gains that are due to reduced liquidity constraints after privatization. As already discussed above, liquidity effects are especially important in stochastic economies in which young poor agents expect to climb up to higher income levels in the future. In the benchmark calibration, pension privatization would reduce borrowing constraints, which in turn translates into an efficiency gain of roughly 0.5 percent of aggregate resources. In the deterministic income model, young households do not want to borrow at all initially, so that liquidity effects are absent. Finally, note that the remaining aggregate efficiency effects of simulation (4) are rather small. This could be explained by the fact that the existing German pension system features a tight tax–benefit linkage (i.e.  $\lambda = 0$  in equation). In principle, it would be no problem to go one step further and construct an additional scenario in which the elimination of the paygo pension system is completely neutral.<sup>12</sup> In such a scenario, privatization perfectly converts implicit taxes, savings and debt into their explicit counterparts by a one–time reduction of the contribution rate, but has no impact on the output and welfare of current and future generations (see Fenge 1995).

While a tight tax–benefit linkage is optimal in a deterministic world in order to reduce labor supply distortions, this may be different in a world with stochastic income. A reduced tax–benefit linkage may improve insurance provision. In addition, an exemption level for contributions would improve the liquidity of constrained young households. However, both adjustments increase marginal tax rates, so that labor market distortions rise. In order to analyze the optimal progressivity of the German pension scheme, Fehr and Habermann (2008) introduce a basic allowance for contributions and a flat benefit fraction, and then compute the resulting aggregate efficiency consequences of alternative parameter combinations.

Figure 2 reports the welfare and efficiency consequences when half of the pension consists of flat benefits (i.e.  $\lambda = 0.5$ ) and the basic allowance amounts to 30 percent of average income. Since this reform reduces the tax–benefit linkage and increases marginal contributions, labor supply, employment, consumption and GDP fall after the reform. Aggregate savings and (one period later) the capital stock fall even more strongly, which

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<sup>12</sup> More specifically, such a scenario would require age–specific payroll taxes after the reform, and no capital–income taxation for future interest income from pension debt.

reflects the reduced need for precautionary savings after the reform. Consequently, wages increase initially, but due to the crowding out of capital they fall back even below the initial level during the transition. Due to the basic allowance, the contribution rates have to increase by almost 10 percent. However, since the reform changes only the progressivity of the system, aggregate pension outlays remain almost constant during the transition. Finally, since income tax revenues are reduced and the consumption tax base is smaller, the consumption tax rate has to be increased by 2.6 percent in all transitional periods in order to balance the intertemporal budget. Given the negative long-run macroeconomic consequences, one would expect that at least all future agents are worse off with a progressive pension system. However, this basic intuition is misleading, since it neglects the insurance effects of the reform. As Figure 2 shows, the reform clearly reduces the welfare of most elderly households, while younger and future living households gain.

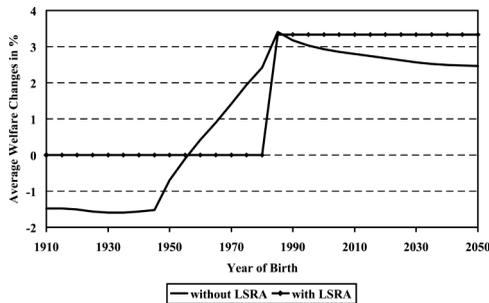


Figure 2: Welfare changes with 50% flat benefits and the basic allowance of 30% of average income. (Source: Fehr and Habermann (2008), p. 434.)

Due to the increase in consumption taxes, the elderly who are already retired (i.e. those who are born before 1940) lose about 1.5 percent of their remaining lifetime resources. Since workers benefit from the improved insurance effects, households that are born after 1960 realize increasing welfare gains, which reach a maximum for those who enter the labor market in the reform year 2005. For future workers, the welfare effects are still positive – although they decrease again due to the long-run fall in wages. After compensating the elderly with lump-sum transfers from the LSRA, young and future households could still experience a welfare increase, which amounts to 3.3 percent of initial resources. Consequently,

the considered progressive pension reform is Pareto-improving! This result contrasts sharply with the introduction of flat benefits in a deterministic model, where such a reform only induces labor market distortions and therefore dramatic efficiency losses; see Fehr (2000).

For the baseline calibration, the above combination of progressivity parameters yields the highest efficiency gain compared to the status quo. The aggregate effect of 3.3 percent of initial resources can be decomposed into a (positive) insurance effect of 3.8 percent, a (positive) liquidity effect of 0.4 percent and a (negative) labor supply effect of -0.9 percent. These qualitative results are quite robust for various parameter choices. Furthermore, it can be shown that the optimal progressivity of the pension system supplements the progressivity of the tax system (i.e. a less progressive tax system requires a more progressive pension system and vice versa).

While I don't think that these results suggest that the German pension system should be radically changed to a more progressive system, I do believe that labor supply distortions are not the only policy target. Numerical results from these studies clearly indicate that social security may provide a quantitatively important insurance role in an uncertain world. As it seems, these (positive) insurance effects may even dominate the (negative) labor supply and liquidity effects of social security for a wide range of parameter values in the standard model. Consequently, I would answer the question raised in the title of this subsection with "Presumably not!" The following two subsections consider extensions of the standard model that might reinforce or weaken the case for social security.

### *3.3 Social Security as a Commitment Device*

Among other reasons, social security was introduced in the first place to protect myopic individuals from their tendency to save inadequate for retirement; see Lindbeck and Persson (2003, p. 77) or Diamond (2004, p. 4). If a sizeable fraction of households really lacks sufficient foresight, one would expect that a paternalistic government could improve the welfare of such short-sighted individuals via the forced-saving element present in paygo-financed social security. In fact, such "paternalistically motivated desire to force savings constitutes an important, and to some the most important, rationale for social security retirement systems." (Kaplou 2008, p. 292)

Therefore, it's hardly surprising that stochastic CGE studies have been applied to analyze the welfare effects of social security funding in models

with short-sighted individuals. Various approaches that rationalize the behavior of individuals who exhibit self-control problems. Imrohoroglu, Imrohoroglu and Joines (2003) consider social security in a model in which consumers apply hyperbolic discounting. While in the standard life-cycle model shown in equation the discount factor  $\delta$  remains constant over time, the discount factor falls in the hyperbolic discounting model as the future draws closer. For example, from the vantage point of today, a person's subjective discount factor between consumption to be received in two years or in three years may be fairly high, so that viewed from today, the individual may plan to postpone a large amount of consumption in year two in order to have more consumption in year three. But after a year has passed, the discount factor between year two and year three is much lower, so that the individual finally decides to save much less than originally planned. Consequently, there is a conflict between the plan formed in the present and the action needed to realize that plan in the future. In other words, people who employ hyperbolic discounting formulate lifetime saving plans that are dynamically inconsistent. Consequently, people will save less than they would actually like to. This prediction is confirmed by individual statements reported, for example, in Choi, Laibson, Madrian and Metrick (2006) that indicate that the actual saving rate falls significantly short of the self-reported target saving rate.

Technically, the decision problem of a hyperbolic consumer is modeled as an intrapersonal game between a sequence of "selves" with conflicting preferences. Taking the strategies of his future selves as given, the current self picks a strategy that is optimal from his own perspective.<sup>13</sup> The time-inconsistency in preferences is evident from the fact that the discount factors upon which individuals base their decisions are different from those that evaluate the decision afterwards.

Alternatively, one could assume that tastes do not change over time (i.e. the discount rate remains constant) but that exercising self-control is costly in psychic terms. Individuals would like to allocate lifetime resources according to the life-cycle model, but they lack the willpower required to delay current consumption that is needed to fulfill the lifetime

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13 The literature distinguishes between so-called "naive" and "sophisticated" hyperbolic consumers; see O'Donoghue and Rabin (1999). The former believe that their future selves will behave in a time-consistent manner, despite the fact that they have consistently violated this belief in the past. The latter correctly foresee that their future selves will also behave in a time-inconsistent way.

consumption plan. As a consequence, individuals act as if they have a kind of dual preference structure: one concerned with the long run, the other with the short run. The act of saving creates an internal conflict between the forward-looking part of the individual and the part that cares only about the present. The individual saving decision then maximizes the long-run goal, while at the same time minimizing psychic costs. Note that in this case preferences are perfectly consistent. Agents can perfectly commit to future actions and do not regret their past actions later on. Kumru and Thanopoulos (2008) apply preferences whereby agents are tempted to consume the entire endowment  $\hat{c}_j$  in every period. With fixed labor supply and standard CRRA utility  $u(c) = \frac{c^{1-\eta}}{1-\eta}$ , the consumer maximizes preferences of the form

$$V(z_j) = \max_{c_j} \left\{ u(c_j) + \delta \psi_{j+1} EV(z_{j+1}) + v(c_j) - v(\hat{c}_j) \right\} \quad \text{with } v' > 0, v'' < 0,$$

subject to the budget constraint, the accumulation equation of pension wealth and the definition for maximum consumption

$\hat{c}_j = (1+r)a_j + b_j + q_j - T_j$ . The utility function  $v(\cdot)$  represents "temptation" (i.e. the individual's urge for current consumption). Consequently, if the consumer would consume  $c_j = \hat{c}_j$ , then the term  $v(c_j) - v(\hat{c}_j)$  would drop out and there would be no cost of self-control. Whenever consumption is less than the entire endowment, the difference  $v(c_j) - v(\hat{c}_j)$  measures the disutility from self-control. Optimal behavior trades off the temptation to consume with the long-run self-interest of the consumer represented by  $u(\cdot) + \delta \psi EV(\cdot)$ .

Individuals who have problems with self-control view social security either as a commitment device or a device that helps to reduce the cost of temptation. Consequently, one would at first expect that social security improves the allocation of resources in an economy with short-sighted individuals (since it dampens losses due to self-control problems). However, Imrohoroğlu et al. (2003) as well as Kumru and Thanopoulos (2008) both find that the introduction of social security still decreases long-run welfare for reasonable discount factors and temptation parameters. Consequently, they conclude that there is little room for social security – even in a world with myopic and/or tempted individuals.

As already explained in the previous section, one should be very careful with policy conclusions from models that compare only long-run equilibriums. Since both studies neglect transitional costs (or benefits), they fail to isolate the exact commitment effect that social security

provides to short-sighted consumers. In order to account for this issue, one must compute the transition and compare the resulting aggregate efficiency effects of social security privatization in models with and without self-control problems. This is the approach of Fehr et al. (2008), which compares the elimination of social security in an economy populated by either rational or hyperbolic consumers. Our long-run macroeconomic and welfare effects are in line with the results in Imrohoroğlu et al. (2003) or Kumru and Thanopoulos (2008), but the most interesting results are the changes in aggregate efficiency effects. Table 3 already showed that the elimination of social security yields an efficiency loss of 0.57 percent of aggregate resources in the baseline calibration of the standard model. In the model with hyperbolic consumers, this efficiency loss increases to 1.78 percent! On average, therefore, each individual loses about 1.2 percent of aggregate resources due to the disappearance of the commitment technology implicitly offered by social security. Again, this is a significant amount and the figure is quite robust for a wide range of parameter values. Consequently, social security is a particularly valuable asset in an economy populated by myopes.

Of course, one could apply the same approach to quantify the value of other commitment devices implicit in government instruments. In a follow-up study, Fehr and Kindermann (2009) compare the introduction of an individual retirement account (IRA) system as a substitute for social security. Even if IRA contributions are voluntary, they represent a commitment device for myopic individuals since the accounts are (almost) illiquid before retirement. In addition, if there is mandatory annuitization, self-control problems are further reduced. Finally, mandatory contributions during employment may reduce self-control problems, but at the same time they strengthen borrowing constraints, so that the overall effect of mandatory accounts is negative.

Summing up, I think that the existing evidence from CGE studies with myopic behavior reinforces the claim of a positive role of social security in Western societies.<sup>14</sup>

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14 In contrast to Kumru and Thanopoulos (2008), a recent study by Bucciol (2009a) also finds a welfare-improving role for social security in a model with temptation. The result is due to a different set of preference parameters, which are in line with estimates from Bucciol (2009b).

### 3.4 Family Risk-sharing and Social Security

Of course, the government doesn't hold a monopoly on providing important risk-sharing institutions and relaxing borrowing constraints when private markets are incomplete or missing due to market failure. Kotlikoff and Spivak (1981) showed that the family can provide insurance against longevity risk – and the same argument can be applied to other forms of economic risk as well. Therefore, a growing body of empirical work (including McGarry and Schoeni 1995, Schoeni 2002, or Attanasio, Low and Sanchez-Marcos 2005) analyzes the responses of family members to changes in the financial situation of other family members. This literature suggests that within-family transfers – either between marriage partners or between parents and children – are an important mechanism that provides insurance against income shocks. However, empirical tests performed by Hayashi, Altonji and Kotlikoff (1996) support the view of incomplete family risk-sharing. As a consequence, and in contrast to the claim of Barro (1974), family responses do not neutralize the effects of governmental interventions in private markets. Nevertheless, when analyzing the redistribution and risk-sharing effects of government policies, it is problematic to abstract from the exchange within families. With respect to social security reform, this means that the previous life-cycle studies (which abstract from family insurance) may have severely exaggerated the impact of social security on the economy. The challenge is therefore to quantify social security reforms in models with multi-person families as decision units.

Fuster (1999) develops such a “family model” in which parents and children are linked by two-sided altruism so that one has to distinguish between cohorts and households of a dynasty. To simplify matters, the model abstracts from lifespan uncertainty (i.e.  $\psi_j = 1$ ), so that each agent lives for  $J$  years. In the middle of his life, at age  $J/2$ , a single child is born. The lifespans of the parent and the child overlap for  $J/2$  periods, which forms the household as a decision unit. Figure 3 shows this structure for the two succeeding cohorts  $i$  and  $i+1$ .

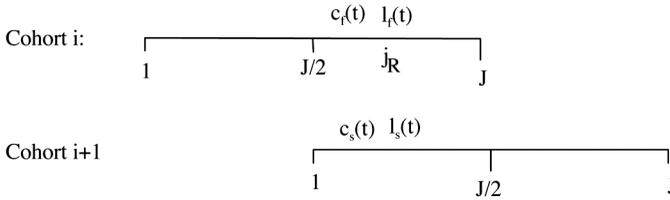


Figure 3: Structure for two succeeding cohorts.

While in the standard life-cycle model each cohort would optimize its own consumption and leisure path separately (i.e. households and cohorts are identical), households in the altruism model consist of two cohorts that optimize their joint behavior during the  $J/2$  periods they overlap. Fuster (1999) allows for variable labor supply, so that the parent (“father”) cohort and the child (“son”) cohort choose consumption  $c_f(t), c_s(t)$  and leisure  $\ell_f(t), \ell_s(t)$  in each period  $t$  in which they overlap. Since agents face neither mortality risk nor period-by-period uncertainty about earnings, social security provides neither longevity nor intragenerational income insurance. However, agents face uncertainty about the “abilities” of their children. Newborn agents may be endowed with either a high or a low lifetime labor productivity, which translates into a specific trajectory for the age-efficiency index  $e_j$ . Thus, each agent knows at birth his complete age-earnings profile as well as the remaining profile of his parent until they retire at age  $j_R$ . However, he/she can form expectations about the productivity of his child born at age  $J/2$  only after the death of the parent, since the abilities of parent and child are assumed to be correlated.

The optimal behavior of the household is derived in a two-stage decision process. In the first stage there is no uncertainty, since initial and terminal assets  $a_1, a_{J/2}$  at the end of the overlap as well as the efficiency indices  $e^i$  and  $e^{i+1}$  of cohorts  $i$  and  $i+1$  are given. The parent-child household maximizes the sum of period utility functions of the parent and child

$$\widehat{V}(a_1, a_{J/2}, e^i, e^{i+1}) = \max_{\substack{c_f(t), \ell_f(t) \\ c_s(t), \ell_s(t)}} \sum_{t=1}^{J/2} \delta^{t-1} [u(c_f(t), \ell_f(t)) + u(c_s(t), \ell_s(t))] ]$$

subject to the constraint that the present value of the goods and leisure consumption of parent and child and the present value of terminal wealth

$a_{j/2}$  are equal to the sum of initial wealth  $(1+r)a_i$ , the present value of full-time income and the present value of social security claims accumulated by both family members. In the second stage, households decide about the asset holdings they will leave to the next household of the dynasty (consisting of cohorts  $i+1$  and  $i+2$ ); i.e.

$$V(a_i, e^i, e^{i+1}) = \max_{a_{j/2}} \left\{ \widehat{V}(a_i, a_{j/2}, e^i, e^{i+1}) + \delta^{j/2} EV(a_{j/2}, e^{i+1}, e^{i+2}) \right\}.$$

Social security benefits are a linear function of an agent's average lifetime earnings, but do not depend on the earnings of his parent or child. The benefit formula is progressive, so that low-ability retirees receive more generous benefits than high-ability retirees. Because agents behave altruistically, uncertainty about the child's ability implies uncertainty about the parent's lifetime resources. A progressive social security system in which benefits are not linked to the earnings of the child distorts labor supply. But it also provides a form of risk-sharing against the risk of having children with low labor productivity that would be difficult to duplicate on private markets.

Fuster (1999) finds that social security lowers the steady-state capital stock much less in the altruism model than in the standard life-cycle model. Whereas in Auerbach and Kotlikoff (1987) old-age savings are replaced by social security, altruistic households would even like to increase their bequest savings in order to compensate future generations. However, social security has a negative effect on labor supply that is even more pronounced than in the standard model. This is the main reason for the reduced long-run capital stock. With respect to distributional issues, the different combinations of parent/child productivities allow us to distinguish four types of households in the long run. Since social security induces wealthy individuals to increase savings in order to leave a larger bequest, wealth inequality increases despite (or better: because of) the progressive social security system.

In the altruism model of Fuster (1999) both intergenerational redistribution and (negative) liquidity effects induced by public policy are (at least partly) neutralized by *intervivos* transfers and bequests. Due to the certain lifespan, however, the analysis abstracts from the implicit provision of longevity insurance. Fuster, Imrohoroğlu and Imrohoroğlu (2003) address this deficiency of the model by introducing uncertain lifespan. They even assume in their benchmark calibration that an agent's life expectancy is determined by his or her labor productivity. Agents with high labor

productivity can expect to stay alive five years longer than those with low productivity. The study also assumes positive population growth. Consequently, the composition of the household changes when either the parent or the  $m$  children die. Since all children have identical mortality rates, there are three types of households. Either the parent has died (Type 1), or the  $m$  children have died (Type 2) or both cohorts are alive (Type 3). The model abstracts from variable labor supply so that consumption of the parent and the children are always identical. This simplifies computation, and the above two-stage process can be comprised into a single stage in which households decide about consumption and savings in each period, taking into account mortality risk and “ability risk” of the new generation.

Of course, the introduction of social security now provides a longevity insurance (which already is at least partially provided by the children), but it also redistributes across different household types. Households with high-ability parents (i.e. who receive benefits longer) and low-ability children (i.e. who pay low contributions) should benefit most from social security, while those with the opposite combination should benefit least. Table 4 confirms this intuition. The type-3 household in which both parent and children are of high ability is denoted by HH. The remaining type-3 households are denoted by HL, LH and LL, where the first letter indicates the ability of the parent and the second the ability of the children.

Table 4: Long-run Welfare Effects of Social Security Privatization

	Type 3				Type 1		Average
	HH	HL	LH	LL	H	L	
CV (%)	-0.49	-1.71	1.39	-0.13	5.62	6.06	0.45
Measure of types	0.147	0.11	0.107	0.53	0.025	0.08	
Expected returns (in %)	3.2	8.6	<0	5.7	-	-	

Source: Fuster, İmrohoroğlu and İmrohoroğlu (2003), Table 2, p. 1258.

The row labeled “CV” reports the compensating variation in consumption that is necessary to make a household indifferent between eliminating social security and keeping a system with a 44% replacement rate. A negative number means that the household is worse off when social security is eliminated. For example, privatization reduces the welfare of the HH household of type 3 by 0.49 percent of consumption. On the other hand, welfare of an L household of type 1 would increase by 6.06 percent of consumption after social security privatization. If the average measure of

consumption compensation is computed using the measure of households reported in the last row, the average household would be better off without social security. In order to understand the differences in welfare effects, Fuster et al. (2003) compare the rate of return on the capital market of 4.6 percent with the (implicit) rate of return of social security. The latter can be computed for different household types and for different cohort types. If expected contributions and benefits are aggregated over the entire lifespan of the cohort, then the rate of return is 3.0 percent for low- and 2.7 percent for high-productivity types. Consequently, in the standard life-cycle model, low-productivity types benefit from the progressive benefit schedule, but their lower life expectancy dampens the redistribution considerably. The rate of return for different households is computed across cohorts. Type-1 households never receive benefits, since the parent dies before reaching retirement. In addition, they face a higher probability to be borrowing constrained when young. Consequently, they are better off in a steady state without social security. For type-3 households, the rate of return increases with the lifespan of the parent (i.e. parent is H) and reductions in the children's contributions (i.e. children are L). Consequently, expected returns for HL and LL households are above the rate of return on the capital market. For the LH household, the rate of return of social security is lower than the after-tax interest rate for all possible mortality histories. Finally, the expected rate of return of the HH household is below the after-tax return on capital.<sup>15</sup>

Fuster et al. (2008) apply the same model in order to analyze individual retirement accounts as a substitute for the existing social security system. Starting from a benchmark that reflects the existing U.S. paygo social security system, they either eliminate the existing system or substitute half of the contributions by mandatory savings in so-called personal saving accounts (PSAs). PSA funds cannot be withdrawn before retirement. They earn the capital market return and are either annuitized or not after retirement. Not surprisingly, all reforms induce an increase in the steady-state capital stock between 6 and 9 percent. As before, type-3 HL households would prefer to be born in an economy with social security, whereas type-1 households benefit the most from complete privatization. Most of the type-3 households prefer the PSA system with mandatory

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<sup>15</sup> However, if the parent survives to age 85, the (implicit) rate of return increases to 5.7 percent above the rate on the capital market.

annuitization, since parents like to hold annuities during retirement. Annuitization improves insurance against longevity risk. The intergenerational income effects are neutralized by changes in inter vivos transfers.<sup>16</sup>

Consequently, Fuster et al. (2003, 2008) demonstrate that social security reforms could affect different household types within a cohort quite differently. Since intergenerational redistribution is dampened in the altruism model, the long-run effects of social security are much less severe compared to the life-cycle model. However, the studies considered so far only compared long-run equilibria and neglected transitional cohorts. For this reason, the analysis of Fuster et al. (2007) includes the transitional path to the new long-run equilibrium. They also account for variable labor supply and idiosyncratic labor income uncertainty during the working phase. The progressive benefit schedule of social security thus provides an additional insurance, but at the same time distorts labor supply. As it turns out, labor supply distortions dominate long-run welfare effects. Consequently, all household types would prefer now to be born into an economy without social security. Fuster et al. (2007) compare the transition paths of different privatization scenarios and compute – similar to Conesa and Krueger (1999) – the political support of the respective policy in the initial equilibrium. Not surprisingly, they find that although there is more support for privatization in the altruism model than in the life-cycle model, only a fully compensated privatization that is financed by consumption taxes finds a political majority in the initial population.

Summing up, I think that the altruism model of Fuster (1999) and Fuster et al. (2003, 2007, 2008) provides an important alternative to the standard life-cycle approach. Due to compensating inter vivos transfers, the redistributive effects of social security are less pronounced and the missing insurance markets are partly provided by the family. Although Fuster et al. (2007) include uncertainty, variable labor supply and transitional dynamics, they neglect the disaggregation between welfare and efficiency effects highlighted in the previous subsection. Consequently, a final evaluation of social security in the altruism model based on efficiency considerations has yet to occur.

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16 This is in contrast to the life-cycle model of Fehr and Habermann (2008b), where annuitization might reduce long-run welfare due to reduced bequests.

### 3.5 Marital Risk and the Demand for Life Insurance

Up to now, the family was considered as a positive institution that might provide insurance against individual longevity and/or labor income risk. However, when people marry, have children, or separate through divorce or widowhood, they may experience dramatic changes in their financial status. Wealth holdings may increase or be split up, spending needs may change due to housing and/or children, and new expectations arise with respect to future income, longevity or bequests. The problem is further intensified because various instruments of the public tax and transfer system are conditioned on marital status. The income tax burden may change significantly when two partners get married or divorced, and the internal rate of return of social security increases after marriage due to widow/widower pensions. Therefore, family transitions constitute an important source of risk that affects labor supply, consumption and saving behavior.

Cubeddu and Ríos-Rull (2003) develop an OLG model with two genders in order to quantify the impact of such "family shocks" on aggregate savings. Compared to the standard model, the individual state vector changes to  $z_j = (g, m_j, a_j, \tilde{a}_j, e_j)$  where  $g \in \{m, f\}$  denotes the agents' gender and  $m_j \in \{s_o, s_w, 1, \dots, I\}$  denotes the marital status, which includes being single without and with dependents and being married where the index  $1, \dots, I$  denotes the characteristic of the spouse such as age or educational background. The marital status is a random characteristic that follows a Markov process with age-specific transition probabilities. In this model, assets vary both because of savings and because of changes in the composition of the household. Once a couple is married, all assets are shared. Therefore, the problem of the single agent is to form expectations about the assets that a prospective spouse would bring into the marriage. The married couple's problem is to maximize

$$V(z_j) = \max_{c_j} \left\{ u(c_j) + \delta \psi_{j+1}^g EV(z_{j+1}) + \delta \psi_{j+1}^{g^*} EV(z_{j+1}^{g^*}) \right\},$$

where the \* denotes the index of the spouse. Couples maximize subject to a budget constraint, which takes into account a division rule of assets in case of divorce. Cubeddu and Ríos-Rull (2003) abstract from uncertain lifespan, income uncertainty and social security issues. They compare alternative steady states with different processes for the marital status as well as different sharing rules and external costs at divorce. Their quantitative results indicate that marital risk has an enormous impact on

aggregate savings and capital accumulation. While the expectation of a marriage and the threat of divorce may increase or reduce precautionary savings, the net contribution of marital risk depends on the subsequent marriage pattern, the splitting rules for assets and the divorce cost.

In the standard life-cycle model, individuals are only concerned about living too long (i.e. they would like to insure against longevity). There exists no rationale for insuring against dying too early. In reality, however, life insurance coverage is very significant and the two-gender model with marital risk allows us to analyze the simultaneous demand for annuities and life insurance. Since social security provides both an annuity and a widow/widower pension in the case of early death, it is important to analyze the role of social security as a substitute for imperfect insurance markets in the model with marital risk. Consequently, Hong and Ríos-Rull (2007) extend the previous model by introducing lifespan uncertainty and a social security system. Table 5 compares alternative long-run equilibriums with and without social security and different combinations of private annuities and life insurance contracts.

Table 5: Long-run Effects of Different Market Structures in the Family Model

	With Social Security			Without Social Security		
	Bench- mark economy	With annui- ties	Without contingent claims	Bench- mark economy	With annui- ties	Without contingent claims
Output	100.0	100.1	106.8	117.8	116.6	123.5
Wealth	236.2	236.9	291.1	380.3	370.55	426.6
Annuity	0.0	127.0	0.0	0.0	226.0	0.0
Life insurance	129.1	122.7	0.0	130.6	125.2	0.0
SS tax (in %)	7.8	7.8	7.8	0.0	0.0	0.0
CV (in %)	–	-0.04	-0.71	12.39	12.25	11.81

Source: Hong and Ríos-Rull (2007), Tables 2 and 3, pp. 132, 136.

In the market structure of the benchmark economy, agents have access to life insurance contracts but not to annuities. Consequently, people with dependents demand life insurance with an aggregate face value of 129 percent of GDP. People without dependents would like to buy annuities, but are restricted from doing so. In the case of their death, the unclaimed assets are collected and rebated as lump-sum transfers to survivors of the same age, sex and marital status. The social security system includes a survivor's benefits program so that widowed singles can choose between

their own benefits and the benefit amount of the deceased. The payroll tax rate amounts to 7.8 percent of output.

When annuities are allowed in the second column, especially married women buy a significant amount with an aggregate face value of 127 percent of GDP. However, output, wealth and life insurance are only negligibly affected. The latter might be due to the fact that already in the benchmark non-annuitized assets of the deceased were distributed in a very similar way as annuities. Welfare measured as a consumption equivalent is slightly negative (i.e. a newborn in the benchmark would give up 0.04 percent of consumption in order to be indifferent with the economy that allows for annuities). Since markets are more complete in the counterfactual equilibrium, the welfare loss seems surprising at first glance. However, long-run cohorts might be hurt due to negative intergenerational income effects – for example, due to lower bequests. The next column shows the steady state, when there are no annuities and no life insurance contracts. Agents now make up for missing life insurance contracts by increasing private savings. As a consequence, wealth and GDP increase significantly compared to the benchmark. However, since people cannot insure themselves against early death, welfare decreases by 0.7 percent of initial consumption.

Since social security provides both a survivor benefit as well as an annuity, the question is how the privatization of social security affects the equilibrium in the different market structures. Of course, there is a negative effect from intergenerational redistribution, but there are also positive insurance effects that have to be taken into consideration. The right part of Table 5 shows that in this calibration of the model, the insurance effects are overwhelmingly overcompensated by intergenerational income effects. Households in all three market structures would like to be born into a steady state without social security. Of course, welfare gains are smaller in an economy without contingent claims, but the difference is hardly significant compared to the welfare effect in the benchmark economy. In contrast to Hong and Ríos-Rull (2007), I do not think that these results are in favor of social security privatization. In my opinion, they simply document that in the present model long-run income effects dominate the insurance effects of social security. In order to derive a better understanding of the consequences of privatization in this model, one has to compute the transition and derive the aggregate efficiency effects. I would expect that the sign of the aggregate efficiency effects of

privatization depends on the market structure. If no private life and longevity insurance is available, then the elimination of social security would probably reduce efficiency due to the loss of insurance. However, if individuals can substitute private insurance, then the relaxation of borrowing constraints after privatization might increase aggregate efficiency.

#### **4. Aggregate Uncertainty and Intergenerational Risk-Sharing**

Up to now the discussion has assumed idiosyncratic uncertainty, so that the aggregate variables are deterministic while at the same time the individual household faces productivity and/or marital risk, which affects individual behavior. Consequently, intragenerational risk-sharing arrangements (such as a social security system) have the potential to improve the wellbeing of individuals. In reality, however, agents are also facing a substantial amount of aggregate uncertainty surrounding economy-wide shocks such as wars, political upheavals or technological innovations. In this case, both household-level and aggregate variables are stochastic, since the shock affects the whole cohort in the same way. Since models with aggregate uncertainty and many overlapping generations are (at least currently) impossible to solve due to the “curse of dimensionality”, various forms of approximations and simplifications are employed in the literature. The following section concentrates on demographic risk and productivity risk and discusses three approaches that deal with this kind of uncertainty. The first approach combines a stochastic population model with a deterministic overlapping-generations model in order to derive probability distributions of future macroeconomic variables. The second approach abstracts from general equilibrium repercussions in order to analyze individual portfolio choice over the life cycle in the presence of aggregate financial risk. The third approach extends the partial equilibrium life-cycle model to a full-fledged CGE model with aggregate risk.

##### *4.1 Demographic Uncertainty and Fiscal Sustainability*

There is no doubt that population ageing will cause the demographic structure to change dramatically in future decades in all developed economies. This development will put severe pressure on public budgets, since increasing numbers of elderly persons are mainly recipients of public transfers while decreasing working cohorts are mainly contributors to public budgets. However, while there is some agreement about the general trend (at least for the next two decades), huge uncertainty exists with respect to the exact figures for future fertility, mortality and immigration rates. Consequently, crucial ratios (such as dependency ratios) become very uncertain already in the medium run. Given the uncertain demographics, it is difficult to assess the sustainability of existing public finance systems.

Traditionally, quantitative analysis that deals with uncertain demographics compares a baseline scenario (which reflects parameter assumptions considered to be most realistic) with some alternative scenarios in order to reveal the sensitivity of the baseline to some salient variables. This approach suffers from some serious shortcomings. Implicitly, the scenarios assume a specific correlation not only between the vital demographic processes of fertility, mortality and migration but also across age and time for each vital process. As a consequence, traditional population forecasts suffer from systematic biases that tend to underestimate the uncertainty problem. In addition, the method is intrinsically unable to assign probabilities to specific ranges for future population and economic variables.

Stochastic population models are able to address this critique by producing a large number of sample paths for demographic variables from which a predictive distribution for the future demographic structure can be derived in a coherent manner.<sup>17</sup> Of course, the results from the stochastic population model can be utilized either for pure accounting models or for traditional (i.e. deterministic) overlapping-generations models for stochastic sustainability analysis. Alho, Jensen and Lassila (2008) discuss the approach in detail and present applications for various European Union members. In principle, such a stochastic analysis proceeds as follows: Given the sample paths for the population, future public expenditures and taxes associated with each of these population paths are computed with the economic model. Next, the simulation results are transformed into sustainability gaps (from the intertemporal budget constraint) or primary gaps (from the periodic budget constraint). Finally, the predictive distributions of these gaps are computed and evaluated.

In the following, I discuss the results from Armstrong, Draper and Westerhout (2008), who combine stochastic population projections for the Netherlands with the computable general equilibrium model GAMMA for the Dutch economy. The population forecasts are based on the demographic projection from Statistics Netherlands and data from the European UPE (Uncertain Population of Europe) project. Stochastic forecasts are derived for 50 years (i.e. until 2056) and then extrapolated forward by fixing the fertility, mortality and net immigration rates to their 2056

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<sup>17</sup> Lee and Tuljapurkar (2001) discuss stochastic population forecasting techniques in detail.

values. The latter ensures that each sample path converges to a constant population structure in the long run which is a technical requirement for the economic model. The left part of Table tab6 reports the mean and standard deviation (s.d.) for the total population and the dependency ratio (individuals aged 65–99 years divided by number of individuals aged 20–64 years) from 359 sample paths. Since the predictive distribution is quite symmetric, the standard deviations around the mean capture roughly 70 percent of the statistically possible outcomes. Consequently, while the baseline forecast predicts a slight increase of the Dutch population over the next fifty years (from currently 16.6 million to 16.9 million in year 2050), the stochastic projections reveal that there is almost an even bet whether the total population will increase or decrease over that time. There is a seventy percent likelihood that the total population will range between 15.2 and 18.6 million people. Similarly, while the mean forecast for the dependency ratio in year 2050 is 42 percent, the seventy percent predictive interval ranges from 35 to 49 percent. These figures clearly indicate the uncertainty associated with typical population projections. Next, these sample paths for the population are combined with the overlapping-generations model GAMMA. Households start their economic life at age twenty and may live up to age 99. The year-to-year changes in both the cohort sizes and the newborn cohorts are obtained from the specific sample of the population model. In contrast to the computable general equilibrium models surveyed in the previous section, GAMMA is deterministic in the sense that agents know their future labor productivity and can perfectly insure themselves against lifespan uncertainty in private annuity markets. Consequently, since individuals face neither income risk nor longevity risk, GAMMA captures only the life-cycle savings motive. The public sector receives revenues from profit, income and consumption taxes as well as social security contributions. Public outlays consist of age-sensitive items (such as health care, education and public pensions) as well as age-independent items (such as defense and public administration). Finally, GAMMA considers the Dutch economy to be small relative to the outside world so that commodity and factor prices are determined by the global market.

Table 6: Demographic Uncertainty and Public Finances in the Netherlands

Year		Population Model		Economic Model			
		Total Population (in 1000)	Dependency ratio (in %)	GDP per capita (in 1000 €)	Public health (in % of GDP)	Public pensions (in % of GDP)	Primary Deficit
2010	mean	16,667	26	32.1	10.5	5.2	-2.3
	s.d.	60	0	0.1	0.2	0.0	0.2
2020	mean	17,009	35	36.8	10.5	6.7	-0.3
	s.d.	325	1	0.4	0.2	0.2	0.5
2030	mean	17,193	43	40.8	12.5	8.5	1.5
	s.d.	720	3	1.2	0.6	0.5	1.3
2040	mean	17,115	47	48.1	13.5	9.0	2.6
	s.d.	1,204	5	2.3	1.2	1.0	2.3
2050	mean	16,876	42	58.2	13.5	8.3	2.7
	s.d.	1,720	7	3.4	1.8	1.3	3.1

Source: Armstrong et al. (2008), Appendix: summary statistics.

Since GAMMA assumes an exogenous and deterministic rate of labor augmenting technological progress of 1.7 percent annually, it is not surprising that the GDP per capita increases significantly over the next forty years. Uncertainty (as measured by the standard deviation) increases as well, but GDP per capita projections are much less uncertain than population projections. Unfortunately, the latter does not hold for public expenditures. The last three columns of Table 6 show that, due to ageing, both public expenditure (health and pensions) and the primary deficit (if the tax system remains as in base year 2007) increase significantly – as does the uncertainty. The low precision of economic forecasts becomes obvious in the deficit measure, where the 70 percent predictive interval ranges from a surplus of 0.4 percent to a deficit of 5.8 percent of GDP in 2050. Of course, the endogenous primary deficit with unchanged public tax and spending behavior is only one of several options that can be used to assess long-run fiscal sustainability. Alternatively, Armstrong et al. (2008) compute the distributions of the reduction in public expenditure as well as the increases in the consumption (or the income) tax rate required to balance public revenues and outlays intertemporally. These distributions can be utilized for specific policy recommendations. For example, if politicians in the Netherlands aim at an 80 percent probability of sustainable national finances, they would have to reduce public consumption by 4.7 percent or increase the consumption tax by 11 percent immediately. If politicians desire a lower probability, then a less radical adjustment of tax rates and/or expenditures is required.

Of course, it is possible to proceed one step further and compute the distribution of welfare effects for different policy reforms. For example,

Fehr and Habermann (2006) compare the cohort-specific expected welfare changes for two German pension-reform proposals aimed to reduce future expenditures and contributions. In order to capture the implied intergenerational risk-sharing consequences, we compute for each generation the (normalized) ratio of the standard deviation of the sample-path-specific utility levels in order to reveal whether the reform has increased or decreased generation-specific risk. Lassila and Valkonen (2008) discuss various other applications of this approach that analyze the sustainability of national finances in Finland.

Despite the fact that stochastic population projections are by now widely applied with computable equilibrium models, the approach suffers from a systematic theoretical inconsistency. While it aims to quantify the economic consequences of demographic uncertainty, individuals do not take into account demographic risk when they decide about savings, labor supply etc. Within each simulation run agents behave as if they have perfect information about future events, so that they do not exhibit any precautionary behavior. Fully rational individuals would take aggregate demographic uncertainty into account when making decisions and would (even without precautionary motives) suffer welfare losses from the forecast errors in their demographic predictions. Consequently, similar to what happens in adaptive expectation models, macroeconomic and welfare effects include systematic biases. Given a specific mortality process, Alho and Määtänen (2008) compare the utilities from the consumption stream derived with either a fully rational or an adaptive behavior model. Their findings indicate that the welfare consequences from ignoring aggregate mortality risk are rather small. However, in my opinion their results do not give an all-clear signal for the approach. Their study captures demographic risk only partially, since the theoretical structure is rather stylized and the analysis is limited to mortality risk. At present, therefore, stochastic population projections combined with CGE models are only of limited use in policy analysis. While they may improve on the traditional scenario-based approach, the derived welfare calculations have to be interpreted with great caution.

#### *4.2 Portfolio Choice and Financial Risk*

All studies and models discussed so far abstract from investment risk. They all assume a single financial asset that yields a certain return that is endogenously determined by the behavior of supply and demand on the

capital market. While this assumption is necessary for technical reasons, it neglects the fact that in reality individuals have to decide where to invest savings, given a continuum of opportunities with specific risk–return characteristics. At the moment it is almost impossible to include aggregate investment risk in a general equilibrium model with many overlapping cohorts.<sup>18</sup> Nevertheless, the optimal portfolio structure over the life cycle is an important issue in almost every discussion of pension reform. It has important implications for the management of retirement funds, the design and effectiveness of saving incentives and the regulation of private pensions.<sup>19</sup> In order to study such issues quantitatively, researchers apply partial equilibrium life–cycle models of optimal portfolio choice that abstract from the repercussions of individual behavior to the public sector and the economy as a whole. This subsection reviews some recent studies in this direction.

#### *4.2.1 Investment Risk over the Life Cycle*

The starting point for any discussion of financial risk is a well–developed normative theory of optimal portfolio choice. Financial planners routinely advise their clients to start saving with a high equity share and then reduce their equity exposure continuously as they get older. However, such investment behavior is optimal only with very specific assumptions about preferences and the risk structure of life–cycle labor income. The basic idea, which was discussed in Jagannathan and Kocherlakota (1996), can be summarized loosely as follows. If a household's preferences exhibit constant relative risk aversion, then it would like to keep its overall risk exposure constant over the life cycle. In each year of the life cycle, the household owns human capital, which represents the present value of the future stream of labor and unfunded pension income. This implicit and nontradable asset can “crowd out” explicit asset holdings in order to balance the household's overall risk exposure over the life cycle. If labor income is riskless, then bond holdings are crowded out and the household would tilt its portfolio strongly towards risky stocks. If the household is constrained from borrowing to finance the risky investment, then a corner solution may result with an optimal portfolio without bonds. If labor income is risky but uncorrelated with risky financial assets, then bond

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<sup>18</sup> Some initial studies in this direction will be discussed in the following subsection.

<sup>19</sup> For an extended discussion, see Bovenberg, Koijen, Nijman and Teulings (2007). This subsection supplements their analysis.

holdings are still crowded out – but less strongly. If labor income is positively correlated with risky financial returns (think of investment fund managers), then risky assets can be crowded out, tilting the portfolio towards bond holdings.

Human capital is high and even increases in the early years of adulthood, but it peaks fairly early and then declines as workers approach retirement. Consequently, if labor income and stock returns are uncorrelated or only weakly correlated, these considerations suggest that households have a strong desire to hold stocks early in life while they reduce their equity position with rising age. Some recent quantitative life-cycle models with portfolio choice have confirmed these theoretical considerations. The basic set-up is developed in Cocco, Gomes and Maenhout (2005), who analyze optimal consumption and portfolio structure in a partial equilibrium life-cycle model with fixed labor supply, lifespan uncertainty and alternative assumptions about bequest motives, borrowing constraints and educational attainment. Instead of, households now optimize

$$V(z_j) = \max_{c_j, \pi_j} \left\{ u(c_j)^{1-\frac{1}{\gamma}} + \delta \left[ \psi_{j+1} E[V(z_{j+1})^{1-\eta}] + (1-\psi_{j+1}) k B_{j+1}^{1-\eta} \right]^{\frac{1-\frac{1}{\gamma}}{1-\eta}} \right\}^{\frac{1}{1-\frac{1}{\gamma}}}$$

subject to the budget constraint

$$a_{j+1} = a_j(1+r_j^p) + b_j + q_j - T_j - c_j,$$

where  $\pi_j$  and  $r_j^p$  denote the portfolio share of risky assets and the return of the portfolio consisting of risky stocks and riskless assets such as treasury bonds. If  $k > 0$  in, then individuals derive utility from leaving bequest  $B_{j+1} = (1+r_{j+1}^p)a_{j+1}$  after death. The portfolio return in is defined by

$$r_j^p = \pi_{j-1} r_i^s + (1-\pi_{j-1}) r^b,$$

with  $r_i^s$  denoting the  $i$ -th realization of the stochastic gross real return of stocks and  $r^b$  the constant return of the treasury bond. The investor can choose between riskless bonds and risky stocks facing both a borrowing and a short-sale constraint that ensure non-negative aggregate wealth and equity holdings (i.e.  $\pi_j \in [0,1]$ ). The excess return of the risky asset is given by

$$r_i^s - r^b = \mu + \varepsilon_i \quad \text{with } \varepsilon_i \sim N(0, \sigma_\varepsilon^2),$$

where the innovation  $\varepsilon_i$  is assumed to be independently and identically distributed. For their benchmark calibration Cocco et al. (2005) assume a real return  $r^b$  of the riskless asset of 2 percent, a mean equity premium  $\mu$

of 4 percent and a standard deviation of the stock returns  $\sigma_\varepsilon$  of 15.7 percent. The discount rate is 4 percent annually (i.e.  $\delta = 0.96$ ) and risk aversion is set at  $\eta = \frac{1}{\gamma} = 10$ . As in the previous section, the labor income process reflects the hump-shaped age-earnings profile with permanent and transitory income shocks. Finally, they assume a zero correlation between permanent labor income shocks and stock returns, since their estimates for different educational groups are surprisingly low and insignificant. The solid line in Figure 4 shows the mean profile of optimal portfolio shares for risky assets over the life cycle from 10,000 simulation runs with the benchmark calibration.

Early in life, most agents invest fully in stocks and hit the borrowing constraint. Only in the very first years of the life cycle do some investors choose to hold the riskless asset. In midlife, with still-rising financial wealth and already declining human capital, the investor tilts the portfolio towards the risk-free asset so that the share invested in stocks reaches 50 percent when the investor retires. When financial wealth is run down quickly after retirement, the optimal stock share increases again slightly, since pension benefits are assumed to be constant and certain. Cocco et al. (2005) discuss in detail the sensitivity of the profile with respect to alternative preference parameters as well as labor and capital income processes. In addition, they compare the welfare cost of non-optimal investment behavior. Figure 4 shows four such arbitrary rules of thumb. The so-called "100-age" heuristic suggests an equity fraction equal to 100 minus the investor's age (i.e.  $\pi_j = (100 - j)/100$ ).

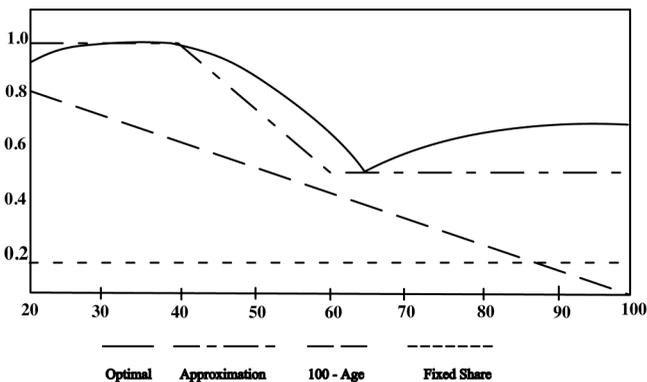


Figure 4: Four rules of thumb. (Source: Cocco, Gomes and Maenhout (2005), p. 523.)

The second strategy, derived by Samuelson (1969) and Merton (1969) in a complete–markets setting without labor income, is independent of age and wealth and relates the stock share to risk aversion and the moments of the excess return so that  $\pi = \frac{\mu}{\eta\sigma_{\tilde{e}}}$ , implying a constant equity share of roughly 16 percent in the benchmark. The third strategy simply invests all wealth in bonds, so that  $\pi = 0$ . The final strategy tries to approximate the optimal portfolio rule by starting with a risky investment share of  $\pi_j = 1.0$  until age 40, and then reducing the share linearly each year until it reaches a fraction of 50 percent at age 60 (where it is kept constant afterwards). Table 7 reports the welfare losses in consumption–equivalent variations.

Table 7: Welfare losses from suboptimal portfolio structures (in %)

	100-Age	Fixed share	Only bond	Approximation
Benchmark	0.64	1.53	2.11	0.08
No high school	0.71	1.76	2.34	0.12
College	0.28	0.67	0.89	0.03
$\eta = 2$	1.30	0.55	2.39	0.67
$\mu = 5.75\%$	1.07	2.14	3.31	0.19
$\delta = 0.98$	0.80	1.81	2.55	0.05

Source: Cocco, Gomes and Maenhout (2005), p. 524.

For the benchmark parameters, the welfare loss in the case of the “100–age” investment rule amounts to 0.64 percent of lifetime consumption. As one would expect from Figure 4, this investment rule dominates the “fixed–share” rule, which is in turn preferred to nonparticipation in equity markets. The associated welfare losses from a fixed equity share of 16 percent and zero percent are substantial and amount to 1.53 percent and 2.11 percent of lifetime consumption, respectively. Finally, the approximation of the optimal strategy in the last column of Table tab7 does very well in terms of welfare costs. Of course, optimal savings rates and equity shares differ for educational groups. Since the income profile is steeper and less risky for college graduates, they save less initially but invest more in equity than other groups. Therefore, suboptimal equity positions are “cheaper” in terms of welfare losses for college graduates than for agents without a high school degree. The optimal equity position increases when either agents are less risk averse or they have a higher risk premium. Consequently, welfare costs of the “100–age” and the “only bond” strategies rise compared to the benchmark case. Since the optimal

equity position of the "fixed share" strategy changes with  $\eta$  and  $\mu$ , the reported welfare losses in Table 7 are difficult to interpret. Finally, a lower discount rate increases savings so that suboptimal equity positions have a higher welfare cost.

Subsequent studies have extended this analysis in various directions. Both Alan (2006) and Gomes and Michaelides (2005) introduce fixed entry costs for risky investment in order to simultaneously match the empirically observed stock market participation rates and asset allocation conditional on participation. Davis, Kubler and Willen (2006) model a wedge between the cost of borrowing and the risk-free investment return in order to explain the simultaneous equity holding and borrowing observed in the data. Gomes, Kotlikoff and Viceira (2008) compare the welfare losses of suboptimal portfolio structures in a model with variable labor supply. Since variable labor supply may act as a buffer against income uncertainty, optimal equity holdings are increased compared to the fixed labor supply case.<sup>20</sup> Love (forthcoming) constructs a two-gender model with marital risk similar to that discussed in section 3.5 above in order to analyze how marital-status transitions and children affect the portfolio choice of men and women. Predictably, if labor income becomes more risky due to divorce or widowhood, the equity share decreases significantly.

Although all of these studies differ in specific aspects of the model structure, they all confirm the optimality of a falling equity share during the life cycle for a wide range of parameter values. The question, therefore, is whether this behavior is also found in the data. Unfortunately, existing empirical studies on this issue are very inconsistent. As Ameriks and Zeldes (2004) point out, this might be due to the fact that the observed portfolio structures combine age, cohort and time effects, which are difficult to isolate. Consequently, the study can only confirm a "humped-shaped" profile for the equity share during employment years for specific time periods.

#### *4.2.2 Asset Allocation in Taxable and Tax-Deferred Accounts*

All of the portfolio-choice studies discussed so far have abstracted from the taxation of asset returns. There is no doubt that the differential taxation of risky and riskless asset returns will affect optimal portfolio choice

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<sup>20</sup> The same argument could be applied to pension funds that could bear more equity return risk in the case of a flexible retirement age.

significantly. Differences in taxation may even explain (at least partly) the inconsistencies of existing empirical studies on life-cycle portfolio choice mentioned above. Dammon, Spatt and Zhang (2001) consider the taxation of realized capital gains and losses in combination with dividend and interest taxes in a portfolio-choice model. Investors' asset holdings at the time of death are liquidated without payment of the capital gains tax, which is consistent with the so-called "reset provision" of the U.S. tax code. Not surprisingly, depending on the strength of the bequest motive, the risky asset position increases after retirement up to 100 percent, since investors either benefit from the realization of capital losses if they occur, or avoid paying the capital gains tax by deferring sales of equity until death.

Including taxes also permits us to analyze government provisions that aim to boost private retirement savings. Many governments around the world allow the tax deferral of interest from specific retirement accounts (which typically contain some withdrawal restrictions reducing their liquidity during employment years). Campbell, Cocco, Gomes and Maenhout (2001) assume a mandatory contribution to such an account during a household's employment years and a conversion of retirement wealth to a riskless annuity stream after retirement. The government varies the portfolio structure of the social security trust fund and the contribution rate. The study finds significant welfare gains from a policy that increases the equity share of the trust fund and reduces the social security tax rate simultaneously. Since households adjust the equity position of their liquid wealth holdings and at least partially offset the government policy, these welfare gains are mainly due to improved consumption smoothing and reductions in liquidity constraints.

However, since Campbell et al. (2001) abstract from the taxation of financial returns and only consider mandatory contributions, their study does not capture several important features of retirement accounts in the U.S. or Western Europe. In practice, investors have to optimize the funds held in liquid taxable accounts (TAs) and mostly illiquid tax-deferred accounts (TDAs). This problem includes a decision on optimal contributions to tax-deferred saving accounts  $s_j$  and the optimal asset location (i.e. the optimal equity shares  $\pi_j^T$  and  $\pi_j$  in TAs and TDAs). More formally, the individual state now distinguishes between wealth in TAs and TDAs  $a_j^R$ , so that  $z_j = (a_j, a_j^R, \tilde{a}_j, e_j)$  and the budget constraint changes to

$$a_{j+1} = a_j(1 + \tilde{r}_j^p) + b_j + q_j - s_j - T_j - c_j$$

$$a_{j+1}^R = a_j^R(1 + r_j^p) + \min[s_j + s_e; \hat{s}],$$

where

$$\tilde{r}_j^p = \pi_{j-1}^T \tilde{r}_i^s + (1 - \pi_{j-1}^T) \tilde{r}^b$$

denotes the after-tax real return of the portfolio, with  $\tilde{r}_i^s$  and  $\tilde{r}^b$  as after-tax real returns on equities and bonds, respectively. During employment years, households choose a contribution  $s_j$  to the retirement account that may be increased by employer contribution  $s_e$  up to a specific contribution limit  $\hat{s}$ . Contributions to accounts reduce tax payments  $T_j$ , while withdrawals during retirement increase the tax burden.<sup>21</sup> While liquid wealth  $a_j$  accumulates with after-tax rates of return  $\tilde{r}_j^p$ , illiquid retirement wealth  $a_j^R$  increases with gross rates of return  $r_j^p$  and contributions from employees and employers. After retirement, the additional restriction  $s_j \leq 0$  is applied, securing the reduction of account balances. The optimal investment decision  $\pi_j$  and  $\pi_j^T$  has to balance the tax benefit against the liquidity benefit. Since bonds are usually taxed more than equity, taxable bonds should be allocated in the TDA, while equity should be accumulated in the TA. If the equity value declines sharply, however, the investor may be forced to liquidate a proportion of the tax-deferred account before retirement, which may result in a severe penalty payment. The liquidity benefit thus induces the investor to hold taxable bonds in the TA. Dammon, Spatt and Zhang (2004) introduce tax-favored retirement accounts in the previous model of Dammon et al. (2001) and find that in the U.S. the tax benefit always dominates the liquidity benefit. Consequently, it is optimal to allocate taxable bonds only in TDAs and equity in TAs (i.e.  $\pi_j^T = 1$ ). For high wealth levels, equity may also be held in tax-deferred accounts. However, this investment behavior contrasts sharply with the observed asset location in the U.S., where TDAs are the preferred location for stocks.

Two recent studies extend the approach of Dammon et al. (2004) in order to solve this so-called "asset location puzzle". Zhou (2009) introduces a progressive tax code so that the marginal tax rate declines after retirement, which in turn reduces the tax burden of bonds. In combination with a high capital gains realization rate, the model

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<sup>21</sup> In order to simplify the discussion, I abstract from early withdrawal penalties, the computation of the tax burdens and (possible) annuitization after retirement.

generates realistic stock market participation rates and equity shares for participants in TDAs with modest risk-aversion coefficients. Alternatively, Gomes, Michaelides and Polkovnichenko (forthcoming) keep the proportional capital income tax structure, but distinguish between direct and indirect stockholders. The former accumulate more wealth (especially in the taxable account) and hold equity in their TA (and may also own stocks in their TDAs), while the latter accumulate less wealth and only hold stocks in the TDAs. The study estimates the preference parameters for the two groups in order to match their respective median TA and TDA wealth accumulation over the employment years. The estimated structural model is then used to study the impact of TDAs on savings for the two groups. The findings suggest that TDAs have only a modest effect on net savings. While households with high savings rates mainly shift funds from TAs to TDAs, those households that were not accumulating significant retirement wealth will continue to do so.<sup>22</sup>

In my opinion, all of the above studies on asset location share the problem that they assume a very sophisticated investor who is able to optimize both contributions and the asset structures of TAs and TDAs. In reality, however, many people simply follow the given defaults in their savings behavior (see Choi et al. 2006). In the U.S., the most common default contribution rate is 3 or 4 percent of earnings, and the most common investment default is a zero equity share. For this reason, a recent study by Cui (2008) quantifies – similar to Cocco et al. (2005) or Gomes et al. (2008) – the welfare losses of alternative contribution limits and suboptimal portfolio structures. As one would expect, the study finds significant welfare losses from a suboptimal investment behavior. But it also shows that restrictions on the contribution choice may have a stronger impact than portfolio-choice decisions. However, since Cui (2008) also applies a standard life-cycle model with portfolio choice of a sophisticated agent, the study does not explain why people in reality choose the default choices so often. We therefore still need quantitative models with non-standard preferences where the default option is derived from individual

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22 The impact of tax-deferred retirement accounts on net savings is also analyzed in stochastic general equilibrium models that abstract from portfolio choice and financial risk; see Imrohoroğlu, Imrohoroğlu and Joines (1998) as well as Fehr, Habermann and Kindermann (2008b). These studies find that roughly 10 percent of TDA contributions constitute additional savings.

optimization. Such a setup would allow us to evaluate the consequences of alternative default designs.

#### 4.2.3 Annuitized vs. Non-annuitized Assets

During the last decade the general awareness of longevity risk has increased worldwide – since governments are moving from public paygo-financed schemes to privately funded pension systems, and employers are shifting from defined-benefit plans to defined-contribution plans. In order to balance the reduction of automatic longevity insurance, some countries in Europe (including the U.K. and Germany) require mandatory annuitization of tax-favored savings plans at some terminal age. Consequently, individuals have to search for optimal annuitization strategies to insure longevity risk. At first glance this seems to be a simple task. The seminal study of Yaari (1965) demonstrated, for example, that the optimal behavior of a non-altruistic individual is to hold all assets in the form of annuities when the probability of death is positive. However, despite this strong theoretical result, only a few households buy annuities at retirement – and even those who participate hold only small fractions of their wealth in the form of actuarial notes. Various explanations have been brought forward in the literature to explain the observed thin capitalization on annuity markets. Nevertheless, Davidoff, Brown and Diamond (2005), who relax the restrictive set of assumptions of Yaari (1965), still find that the observed limited-annuity purchases can hardly be explained with rational behavior. They presume that the so-called “annuity market participation puzzle” might be attributed to psychological or behavioral biases.<sup>23</sup>

In the following I will discuss studies that endogenize the annuitization decision as well as the asset allocation of variable payout annuities in the dynamic portfolio choice framework developed in the previous subsections. In the most general case, the investor has to decide at each age  $t$  about the amount and structure of his liquid savings (i.e.  $a_{j+1}$  and  $\pi_j$ ) and illiquid annuity premiums (i.e.  $a_{j+1}^A$  and  $\pi_j^A$ ). At the same time, he/she receives from previously purchased annuities the sum of annuity payments  $L_j$ , so that the budget constraint changes to

$$a_{j+1}^A + a_{j+1} = a_j(1 + r_j^p) + b_j + q_j + L_j - T_j - c_j.$$

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<sup>23</sup> This is not undisputed. For example, Inkmann, Lopes and Michaelides (2007) demonstrate that the annuity data can be replicated with a fairly standard life-cycle model, and conclude that the puzzle might not be as deep as previously thought.

Annuity payments are computed from the recursive definition

$$L_j = \left[ \frac{L_{j-1}}{1+\hat{r}} + \frac{a_j^A}{\xi_j} \right] (1 + \pi_{j-1}^A r_i^s + (1 - \pi_{j-1}^A) r^b) \quad \text{with} \quad \xi_j = (1 + \kappa) \sum_{i=j}^j \frac{\prod_{u=j}^i \psi_u^a}{(1 + \hat{r})^{i-j}},$$

where  $\hat{r}$  defines the deterministic "assumed interest rate" that controls the shrinkage of the annuity fund and  $\xi_j$  denotes the annuity factor of annuity asset  $a_j^A$  bought at age  $j-1$ . Finally, the parameters  $\kappa$  and  $\psi_j^a$  define the so-called "expense factor" (measuring specific overhead costs) and the survival probabilities applied by the life annuity provider, which may deviate from the average population survival rate  $\psi_j$ . Overhead costs and the longer-than-average lifespan of people who purchase annuities are responsible for the so-called "load factor", which describes the fact that private annuities are roughly 15 to 20 percent more expensive than average mortality would suggest. Note that if we assume a fixed annuity payment, abstract from the load factor and consider only a bond portfolio (i.e.  $\pi_j^A = 0$  so that  $\hat{r} = r^b$ ), we obtain the classical result that the present value of expected annuity payments is equal to the annuity premium (including interest payments).

Horneff, Maurer and Stamos (2008, 2008b) abstract from equity-linked annuities (i.e.  $\pi_j^A = 0$ ) and consider only so-called constant payout or fixed annuities. They analyze the optimal consumption and investment decision over the life cycle and quantify the welfare losses if individuals either cannot annuitize or are forced into suboptimal annuitization strategies. In this framework, complete annuitization is not optimal – even without a bequest motive – since the investor would lose the financial flexibility needed to absorb other shocks (income, health etc.) and would have to accept at younger ages a much lower return. Consequently, in the unrestricted case where the individual can decide each year how much to invest in stocks, bonds and constant life annuities, he starts with a low fraction of wealth invested in annuities and a high fraction invested in stocks. With rising age the equity exposure decreases (as usual) and the fraction invested in annuities increases. This benchmark case, which is referred to as gradual annuitization (GA), is compared with restricted annuitization strategies, where the investor is only allowed to switch once from a phased withdrawal plan into life annuities. In the complete switching (CS) case, the entire savings must be shifted into annuities at the optimal date. The partial switching (PS) case is less restricted, since the

investor can choose an optimal fraction of savings used for purchases of annuities. The benchmark calibration is broadly consistent with Cocco et al. (2005), where they assume standard CRRA preferences with  $\eta = \frac{1}{\gamma} = 5$ , abstract from a bequest motive as well as administrative costs and apply the average population mortality table (i.e.  $\psi_j^a = \psi_j$ ).

Horneff, Maurer and Stamos (2008) consider only retirees at age 65 with liquid financial wealth normalized to the income from public pensions. They first show that the optimal annuity fraction increases with higher financial wealth and with age. The first effect is due to the fact that pension wealth is an implicit annuity holding. Since people like to keep a certain fraction of total wealth in annuities, the annuity share increases with financial wealth. With rising age, the mortality credit increases so that annuities crowd out stocks and bonds completely. If retirees only have the option to annuitize once in the CS and PS cases, it would be optimal to postpone the annuity purchases into future periods until the mortality credit is high enough to justify a complete or partial switch. Table 8 compares the welfare losses (measured in percent of initial wealth) associated with the two restricted annuitization strategies and the cases of complete annuitization at the initial age 65 and not having access to annuities at all.

Table 8: Welfare losses from suboptimal annuitization strategies (in %)

	Parameter values				Wealth	Partial	Complete	Complete	No
	$\eta$	$\gamma$	$k$	$\kappa$	level	switching	switching	annuitization	annuities
(1)	5	0.2	0	0	6	2.5	2.8	6.0	21.1
					12	3.2	3.7	3.8	32.2
(2)		0.5			6	42.2	53.9	77.8	113.7
					12	32.7	41.7	50.0	94.1
(3)			2		6	1.4	6.9	n/a	6.9
					12	2.1	17.3	n/a	17.3
(4)				0.1	6	0.9	1.1	8.9	21.1
					12	2.4	2.8	5.7	32.2

Source: Horneff, Maurer and Stamos (2008), p. 1034.

Two initial wealth levels (with financial wealth either six times or twelve times pension wealth) are distinguished. In the benchmark simulation (1), the retiree experiences a utility loss (compared to utility in the unrestricted GA case) in the range of 20 to 30 percent of initial wealth if he has no access to annuity markets. Of course, the welfare loss rises with wealth, since the optimal annuity fraction in the unrestricted GA case also

increases. This also explains why welfare losses decrease with the wealth level in the case of complete annuitization at age 65. Welfare losses must be smaller in the CS and PS cases, with the PS strategy always dominating the CS strategy (since it has fewer restrictions). In simulation (2), utility losses become huge when the elasticity of intertemporal substitution (EIS) is increased to 0.5. The extreme figures are very surprising and counterintuitive. The considered restrictions distort intertemporal consumption especially at medium ages. For young retirees the annuity premium is rather low, and for old retirees it is optimal to convert to annuities anyway. I would, therefore, expect only small effects from varying the EIS.<sup>24</sup> Of course, the introduction of a bequest motive dampens the demand for annuities so that complete annuitization never becomes an optimal strategy. Consequently, welfare losses in the PS and the "No annuities" cases decrease significantly, but they increase in the CS and "Complete annuitization" case. Finally, in order to reduce the attraction of annuity purchases, simulation (4) introduces a fee of 10 percent and applies the mortality tables of the U.S. annuity industry. As a consequence, the utility losses are slightly reduced in the PS and CS strategies and are slightly higher in the "Complete annuitization" case.

Horneff, Maurer and Stamos (2008b) extend this approach in various directions. They include the employment phase of the life cycle and a stochastic labor process so that annuities could be purchased already before retirement (which is indeed optimal for reasonable wealth levels). In contrast to the previous study, they compute only the welfare gain (measured in percent of labor income) of introducing an unrestricted annuity market. While applying parameters quite similar to those in their previous study, they also check the sensitivity of the welfare calculations with respect to alternative survival probabilities (due to gender or health), income processes (due to educational attainment), looser borrowing restrictions and equity-linked annuities where  $\pi_j^A = 1$ . The welfare gains from annuitization rise dramatically when annuity wealth is invested on the stock market. Therefore, Horneff, Maurer, Mitchell and Stamos (forthcoming) extend the model from Horneff, Maurer and Stamos (2008) by considering so-called variable payout annuities in which part of annuity

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24. Horneff et al. (2008) don't provide a really satisfactory explanation for the results. They argue that retirees benefit from the GA strategy to a greater extent because they can continuously repurchase new annuities from the income provided by annuities purchased earlier.

wealth is invested in stocks (so that  $\pi_j^A$  becomes a choice variable). With variable annuities, the “assumed interest rate”  $\hat{r}$  controls the payout stream during retirement. If it is set low relative to the investor's time and risk preference, then partial annuitization at the beginning of retirement is still optimal in order to smooth the consumption path. For the benchmark calibration (where  $\hat{r}$  is set at 4 percent), full annuitization is delayed to around age 75. As before, the investor will initially hold the majority of his annuity and financial wealth in equities, with the stock fraction falling during retirement. In order to quantify the welfare gains from the equity premium of the stock market and the survival credit from the annuity market, the study compares the optimal variable payout annuity strategy with a phased withdrawal strategy without annuities, where assets are invested either in bonds (i.e. where  $\pi_j = 0$ ) or in an optimal stock/bond portfolio (i.e. where  $\pi_j$  is chosen optimally). The calculations for a median wealth level indicate a welfare gain from the access to an annuity market that amounts to 13 percent of initial wealth. If only bonds are allowed initially, then the welfare gain in the unrestricted case increases to roughly 34 percent. These results are fairly robust for alternative assumed rates of interest. As one would expect, welfare gains increase with risk aversion and decrease with a bequest motive. As before, Horneff, Maurer, Mitchell and Stamos (2008) extend this study by including the entire life cycle in order to assess the impact of labor income, health and housing expenditure shocks on investment levels and structures. Their welfare analysis compares the gains from access to fixed annuities versus variable annuities at different ages when households are able to optimize the asset allocation without annuities. The study finds a substantial positive welfare effect from variable annuities. For example, in the benchmark case without loads and no bequest motive, the welfare gain increases from 21 percent to 52 percent of initial wealth when variable instead of fixed annuities become available at age 40.

Summing up this subsection, I would like to stress two points that seem to be important. First, even without loading factors and bequest motives, these studies indicate that a deferred annuitization of retirement wealth is optimal for a wide range of parameter choices. This finding may at least partly explain the annuity participation puzzle mentioned above. In addition it also confirms the policies of governments that do not force households to annuitize tax-favored retirement wealth too early. Second, general equilibrium studies such as Fehr and Habermann (2008b) (which

quantify the impact of forced annuitization policies but abstract from financial risk and endogenous investment choice) miss significant aspects of the individual decision problem. For that reason, the computed welfare gains from annuitization (or losses due to reduced longevity insurance) might be overstated.

#### 4.3 Aggregate Risk in Computable General Equilibrium Models

The approaches presented in the two preceding subsections suffered from two distinct inconsistencies. On the one side, general equilibrium models with demographic uncertainty include a substantial and systematic bias due to forecasting errors made by individuals who do not take into account uncertainty when making decisions. On the other side, households take into account aggregate risk in partial equilibrium models with portfolio choice, but the repercussions of their behavior on aggregate variables are neglected. For example, the median risk premium  $\mu$  was always specified exogenously in the previous section. Of course, these deficiencies are mainly due to technical reasons. A consistent stochastic model requires that individual decisions reflect the underlying uncertainty structure and that factor and financial prices are derived endogenously. Consequently, wages and equity returns are derived from the production technology

$$Y_t = Z_t K_t^\alpha N_t^{1-\alpha} + K_t(1 - \delta(Z_t)),$$

where  $Y_t, K_t, N_t$  denote aggregate output, capital and labor in period  $t$ ,  $\alpha$  is the capital's share of output and  $Z_t$  defines the aggregate productivity shock. At the firm's optimum, the return on stocks and wages equals the respective net marginal products:

$$r_t^s = Z_t \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta(Z_t) \quad \text{and} \quad w_t = Z_t(1-\alpha) K_t^\alpha N_t^{-\alpha}.$$

The stochastic depreciation rate for capital makes it possible to vary the return from capital independently from the wage rate. Finally, the riskless interest rate from bonds  $r^b$  is derived from individual bond demand and supply and the deficit behavior of the government. This subsection discusses some initial stochastic general equilibrium models with overlapping generations, which in my opinion give important directions for future research.

I start with two studies that aimed to solve the so-called "equity premium puzzle". As demonstrated by Mehra and Prescott (1985), the standard representative-consumer exchange economy is only able to

furnish a mean equity premium ( $\mu$ ) that is far below the historical level in the U.S. of 4–6 percent. In order to analyze the impact of idiosyncratic risk on the equity premium, Storesletten, Telmer and Yaron (2007) developed a realistically-calibrated stochastic life-cycle economy with idiosyncratic and aggregate risk. While they find a significant impact of life-cycle effects and idiosyncratic risk on the equity premium, they fail to generate a realistic premium with standard preferences. Gomes and Michaelides (2008) extended this approach by introducing Epstein–Zin (1989) preferences, participation cost in the stock market, preference heterogeneity, public debt and alternative borrowing constraints. In the benchmark calibration they assume two household types with low ( $\eta=1.1, \gamma=0.1$ ) and high ( $\eta=5, \gamma=0.4$ ) risk and intertemporal substitution characteristics. This generates a risk premium of 3.83 percent and a quite realistic consumption volatility for both stockholders and non-stockholders.

Of course, these two studies are only very indirectly related to pension issues. But especially the study of Gomes and Michaelides (2008) is a direct extension of the partial equilibrium portfolio choice models from the previous section. One could easily imagine that it will be applied in the future to analyze intergenerational risk-sharing issues of various pension systems. This is exactly what the two following studies have in mind with respect to aggregate demographic and productivity risk. Demographic risk refers to a situation with a stochastic  $N_t$  in, so that individuals face the uncertainty of being born into a large or a small cohort. Since  $N_t$  affects factor prices via someone, who is born into a baby-boom cohort can expect lower wages during the working periods and lower interest rates during retirement. The opposite happens to a cohort born during a baby-bust period. Consequently, a pure private economy favors small cohorts and burdens the large ones. From an ex-ante point of view, individuals would like to insure themselves against the risk of being born into a large cohort. However, since trade with the unborn is not possible, such an insurance is not available in a private market economy. At least to some extent, the paygo-financed social security system may provide such an insurance implicitly. In a defined-benefit (DB) system, contribution rates decline when a large cohort enters the labor market, and they increase in the opposite case of a small cohort. Consequently, the adjustment of the contribution rate dampens the factor-price effect so that the DB system offers an implicit insurance against demographic risk. The question remains, however, whether these theoretical factor-price and insurance

effects are really significant in terms of welfare changes. If welfare gains from improved intergenerational risk-sharing of the DB paygo system are significant, then the question arises whether they compensate the welfare losses from the crowding-out of capital. This first issue is taken up by Bohn (2001) within a stylized two-period overlapping-generations economy. The optimal dynamic response to a demographic shock is derived by solving a log-linear approximation of the economy's equilibrium conditions. The analysis confirms that especially DB pension schemes may improve ex-ante utility considerably compared to a purely private-market scheme.

Bovenberg and Uhlig (2008) incorporate endogenous growth in such a stylized model to explore the optimal intergenerational risk-sharing in a world with various productivity and demographic shocks. Sánchez-Marcos and Sánchez-Martín (2006) extend the approach of Bohn (2001) by analyzing a model with four overlapping generations and simulating the short- and long-run implications of an unfunded DB system. It turns out that now the improved intergenerational risk-sharing cannot compensate the long-run crowding-out of the capital stock due to the introduction of social security. However, the simulations also show clear welfare improvements for those generations living in the short run. Consequently, similar to the discussion in the third section above, it would be interesting to neutralize the intergenerational income redistribution in order to isolate the welfare effects from intergenerational risk sharing (although this is not done in this study).

Alternatively, Krueger and Kubler (2006) assume constant population growth but model a stochastic process for the aggregate productivity term  $Z_t$  in the production function and the depreciation rate  $\delta_t$ . Without social security, wage- and interest-rate shocks affect young and old generations separately. Consequently, if returns to capital and wages are imperfectly correlated, the consumption variance of all generations can be reduced if they can pool their labor and capital incomes (see Shiller 1999). A social security system in which retired households implicitly receive a claim to labor income provides such an insurance device. The important point is that private markets are incomplete, in the sense that they cannot provide intergenerational risk-sharing. Krueger and Kubler (2006) carefully calibrate a model with nine overlapping generations where aggregate uncertainty is driven by a four-state Markov chain that reflects the different combinations of factor productivity and depreciation shocks. The

initial market equilibrium without social security is dynamically efficient.<sup>25</sup> In the small open economy without capital crowding-out, the introduction of social security improves the welfare of newborn cohorts between 0.5 and 2.8 percent in terms of consumption-equivalent variation. Welfare gains are increasing with the risk aversion of the household; the aggregate shock of the household's initial year plays only a minor role. Since all existing cohorts benefit even more strongly, the introduction of social security is Pareto-improving – despite the fact that the economy without social security is dynamically efficient. Next, social security is introduced in a closed-economy model where the capital stock is endogenous. In this case, welfare losses due to the long-run crowding-out of the capital stock dominate the positive insurance effects of social security for plausible parameter combinations. Future generations lose roughly 2 percent of resources with the benchmark calibration. Only if the capital share in production is fairly low are crowding-out effects substantially smaller, so that future generations also benefit from the reform.

Summing up, the quantitative results from these studies indicate that the risk-sharing benefits from social security are smaller than the costs due to lower wages implied by the crowding-out of capital. However, these results are only derived by considering future cohorts. In both studies, younger cohorts experience welfare gains from improved insurance and due to paygo benefits received. Therefore, as discussed above, long-run welfare losses are mainly due to intergenerational redistribution. If income effects are neutralized by appropriate transfers, then the positive insurance effects from social security could be isolated. For the future it is important to develop transition strategies that minimize intergenerational redistribution but capture the benefits from intergenerational risk-sharing.

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25 In a model with incomplete markets and an equity premium, it must be sufficiently likely that the riskless interest rate in the economy exceeds the growth rate of the population plus the growth rate of technological progress; see Demange (2002).

## 5. What Have we Learned and Where Should we Go Next?

This survey attempts to cover a broad selection of recent policy-relevant applications based on stochastic life-cycle and overlapping-generations models. Given the formidable size of the literature, the paper is selective even in terms of "computational models", so that not all studies that are important for the social security reform debate could have been covered. In closing this survey I would like to draw some policy conclusions and highlight various directions for future research.

What is the central advantage of stochastic CGE models compared to analytical or small-scale numerical models? In my opinion, it is not that these models generate valuable quantitative numbers that could be used by government officials. Accounting for behavioral reactions and general equilibrium usually comes at the cost of institutional details. Consequently, the government is often better-off with first-round studies (i.e. without behavioral reactions) that disaggregate the economy much further. I also do not think that the main benefit of stochastic CGE studies is to highlight the potential positive effects from intra- and intergenerational risk-sharing or the commitment device offered by social security and other government programs. This is typically already done in both analytical and small-scale numerical models. The latter approaches even offer the advantage that they clearly uncover the underlying economic structure that is responsible for the derived effects. However, stylized models typically isolate a specific aspect of a given government policy such as risk-sharing, labor supply distortions or liquidity consequences. Stochastic CGE models, on the other hand, allow the analysis of policy within a rich institutional setting (i.e. with variable labor supply, borrowing constraints etc.) that accounts for interactions of various (and perhaps even offsetting) effects that can be isolated and quantified. And here, in my opinion, are the main advantages of and central contributions made by this approach. Results from recent stochastic CGE studies suggest that for many (or even most) preference- and technology parameter combinations, the (positive) insurance and commitment effects of social security dominate the (negative) incentive and liquidity effects! If future studies verify the robustness of this conclusion, it will have enormous consequences for public policy consulting. It seems to me that the social security reform debate that has occurred in many countries focussed heavily on labor market and savings distortions while it downplayed the insurance

properties and the role of social security as a commitment device. As a consequence, the introduction of funded pension elements and the move towards contribution-related benefits is the widely believed optimal policy response to population ageing. Taking seriously the results from stochastic CGE studies changes this reform perspective dramatically. Due to the benefits from risk-sharing and the commitment technology it may be efficient to keep the pay-as-you-go financed pension system even with a strong flat benefit part – despite the implied labor market distortions. In my opinion, only stochastic CGE studies can come up with such a strong conclusion.

However, at the moment we're not yet in a position to offer such clear-cut policy advice. The quantitative results from the existing simulation exercises present only some initial evidence, and they should not be taken too literally. They may be derived in an institutional setting that lacks important details or may be based on assumptions about market incompleteness that are not realistic. For example, general equilibrium models with uncertain lifespan usually assume that private annuity markets are missing. Consequently, social security funding reduces welfare, since the formerly provided implicit longevity insurance is no longer available. However, partial equilibrium models with financial risk and portfolio choice tell quite another story. There, it is not optimal to insure against longevity (even at retirement ages) – even without bequest motives and loading factors. This may indicate that existing general equilibrium studies exaggerate the insurance effects from social security. Similarly, it may be useful to model explicitly why people are constrained in borrowing or why there is a wedge between the cost of borrowing and the investment return. Public policy may directly alter these institutional constraints, which in turn could have a significant impact on the quantitative results.

Since the publication of the first stochastic CGE study described in this survey (Imrohoroglu et al. 1995), we have made great progress in analyzing uncertainty in simulation studies. Future research has to integrate the different strands of this literature in order to improve our understanding as to why private markets are not available and how public policy affects the market incompleteness. Particularly, I expect innovative studies in the following directions.

First, the institutional structure of models with idiosyncratic risk will be developed further. This means, for example, that currently exogenous

features such as the human capital profile or the fertility decision of the household will be endogenized. This may also include such features as means testing of retirement benefits or endogenous family formation, which are currently, technically speaking, very demanding. I also expect, however, that future numerical models will consider more complicated risk structures, such as marital or entrepreneurial risk, which allow us to disaggregate the cohorts in various directions.

Second, with respect to partial equilibrium portfolio models, I expect more institutional features to be implemented, such as a detailed modeling of the tax and transfer system and the integration of non-standard preferences. The former will allow us to quantify the impact of policy reforms on the individual household level in greater detail. The latter will improve the representation of important empirical patterns such as default options in financial decisions.

Third, I'm optimistic that future advances in computer technology and solution algorithms will further advance the analysis of aggregate uncertainty in fully specified numerical models. While at the moment theoretical models mainly focus on aggregate risk and intergenerational risk sharing, future numerical work has to connect more closely to the theoretical discussion. Stochastic CGE studies with aggregate risk that isolate efficiency effects will further highlight (similar to models with idiosyncratic risk) the insurance properties of social security systems.

Each approach is designed for a specific purpose, of course, and therefore has its own advantages. For that reason, even the most advanced future stochastic CGE models can't substitute for small-scale numerical models or even theoretical studies. However, I'm convinced that such studies used as a supplement to other approaches will improve our understanding and judgment of government policy. With respect to pensions, it will be interesting to see whether unfunded pension schemes will be valued higher or lower in the public and the scientific debate in fifteen years compared to the present.

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Hans Fehr

# COMPUTABLE STOCHASTIC EQUILIBRIUM MODELS AND THEIR USE IN PENSION- AND AGEING RESEARCH

This paper by Hans Fehr (University of Wuerzburg) surveys recent advances in the field of computable general and partial equilibrium models. These advances take into account various aspects of uncertainty. He introduces the basic structure of an overlapping-generations model with idiosyncratic labor income and mortality risk. Within this framework Fehr discusses the quantitative results of studies dealing with reform of the social security system. Next to that, models are brought up that focus on aggregate factor-productivity risk and/or demographic uncertainty.

