Network for Studies on Pensions, Aging and Retirement

NETSPAR ACADEMIC SERIES

Welfare Analysis of Housing in the Presence of Interest Rate Risk

Emily C. van Erk, Servaas van Bilsen, Theo Nijman

DP 12/2023-073

Welfare Analysis of Housing in the Presence of Interest Rate Risk.*

Emily C. van Erk
Department of Econometrics & OR
Tilburg University
and De Nederlandsche Bank

Servaas van Bilsen
Amsterdam School of Economics
University of Amsterdam
and NETSPAR

Theo E. Nijman
Department of Finance
Tilburg University
CentER and NETSPAR

Abstract

We model the welfare losses of (i) the presence of a mortgage with required repayments, (ii) a minimum pension savings constraint, and (iii) imposing a suboptimal investment strategy. We develop a life-cycle model which considers housing and interest rate risk. For a reasonable set of parameter values, we find welfare losses of up to 2.41% (5.02%) if a homeowner with a 30-year fixed-rate (adjustable-rate) mortgage faces a minimum savings constraint of 10%. These welfare losses increase sharply if a homeowner is obliged to save an even higher percentage to the retirement account. Moreover, we find sizeable welfare losses when imposing the investment strategy of a homeowner with a mortgage on a homeowner without a mortgage.

JEL Classification: C61, D15, G11.

Keywords: Life-cycle investment, Mortgages, Interest rate risk management

*Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank. We are grateful to Bas Werker, Damiaan Chen (discussant), and conference and seminar participants at the Quantitative Finance Seminar Group (Tilburg University), Netspar Pension Day 2022 and UvA PhD Workshop 2022 for their useful comments that greatly improved this paper. These authors acknowledge financial support from Instituut Gak. Email addresses: e.c.van.erk@dnb.nl; S.vanBilsen@uva.nl; nyman@tilburguniversity.edu

1 Introduction

Housing and mortgage wealth are top contributors to total gross household wealth. For instance, OECD (2021) observes that the primary residence for a mean household equals 37% of total gross household wealth. This is also why many retirement studies argue to consider housing in the optimal retirement consumption and investment decision.[1] However, many studies ignore the impact of interest rate changes on monthly mortgage payments as well as total household wealth. Moreover, in many institutional settings, people typically (i) finance their house with a mortgage with required repayments, (ii) contribute a certain percentage of salary to a retirement account, and (iii) invest retirement wealth according to the portfolio strategies set by a pension fund board. Hence, it is of great importance to analyze the effect of these three constraints on people's welfare in an environment with stochastic interest rates.

To obtain welfare losses, we first analytically derive the optimal non-housing consumption and portfolio decisions for a homeowner without a mortgage and without any constraints regarding minimum pension savings and investment strategies. We assume that this homeowner finances her house by her human wealth. The derivation of the optimal (unconstrained) decisions is non-trivial. Subsequently, we derive the analytical closed-form non-housing consumption and portfolio decisions in case a homeowner with a mortgage and required repayments faces a minimum pension savings rate. Next, we compute the welfare costs associated with these two constraints. Finally, we compute the welfare losses associated with a suboptimal investment strategy. In particular, we assume that a pension fund imposes the investment strategy of a homeowner with a mortgage on a homeowner without a mortgage.

We find that the welfare loss can be as large as 2.41% (5.02%) if a homeowner finances its house with a fixed-rate (adjustable-rate) mortgage and faces a minimum pension savings rate of 10%. This welfare loss increases sharply if a homeowner is required to save an even higher percentage of salary to the retirement account. Intuitively, homeowners experience huge costs as they need to pay off their mortgage in a certain time period (usually 30 years), while, at the same time, they are required to save for their retirement. As a consequence, young homeowners have limited budgets available, and due to this, perfect consumption smoothing is usually unattainable.

[1] See, e.g., Campbell and Cocco (2003), Sinai and Souleles (2005a), Cocco (2005), Yao and Zhang (2005a), Corradin, Fillat, and Vergara-Alert (2014), Berger, Guerrieri, Lorenzoni, and Vavra (2018), Oleár, de Jong, and Minderhoud (2017); Kraft, Munk, and Wagner (2018), and Duarte, Fonseca, Goodman, and Parker (2021).

Furthermore, the welfare loss of imposing the investment strategy of a homeowner with a mortgage on a homeowner without a mortgage is up to 1.92%. In case of a 30-year fixed-rate mortgage, the monthly mortgage payments remain the same throughout the mortgage life — which also equals 30 years in our illustrations — regardless of interest rate dynamics. However, if the interest rate increases, the mortgage market value — i.e., the discounted value of future mortgage payments — decreases. This suggests that a fixed-rate mortgage can be seen as a short position in a bond portfolio. Hence, a homeowner with a mortgage should consider its presence when determining the investment strategy of her retirement wealth. As a result, the investment strategy of a homeowner with a mortgage deviates from the investment strategy of a homeowner without a mortgage. In case of an adjustable-rate mortgage, the impact of an interest rate change on the mortgage market value is not directly clear. Indeed, both monthly mortgage payments and the discount factor change following an interest rate change. In general, it is welfare improving for a homeowner with a mortgage to consider the presence of the mortgage in optimizing her investment strategy.

Our work is related to the literature on household heterogeneity in optimal savings and investment behavior (see, e.g., Van Ewijk, Mehlkopf, van den Bleeken, and Hoet (2017), Been, van Ewijk, Knoef, Mehlkopf, and Muns (2021) and Ciurilă, de Kok, Rele, and Zwaneveld (2022)). Van Ewijk et al. (2017) study the main advantages and disadvantages of offering more choice in contributions and asset allocations to participants. They find that the size of the welfare gains is most pronounced for flexibility regarding pension contributions. Our work adds by providing closed-form formulas for optimal non-housing consumption, optimal investment strategies and welfare losses. Been et al. (2021) show to what extent ignoring home ownership results in sizeable welfare costs. They study the welfare gains of liquidating housing wealth to finance consumption — the pension member sells their owner-occupied property for 50% at retirement or borrows against 50% of their housing wealth over the life-cycle — for a representative Dutch household sample. Instead, our work contributes by studying the impact of requiring a homeowner with a mortgage to contribute a certain percentage of salary to a retirement account. Finally, Ciurilă et al. (2022) tailors the pension contributions and pension benefits to three aspects: (i) wage earnings rise over the working life span, (ii) number of children, and (iii) the level of educational attainment. These authors show that the optimal pension scheme has lower pension benefit levels and increasing pension contributions during the working life span. We abstract away from these three aspects but contribute by integrating home ownership in the pension system.

Furthermore, there is a well-developed literature on portfolio choices without mortgages and with time-varying interest rates (see, for example, Campbell and Viceira (2001), Brennan and Xia (2002), Sangvinatsos and Wachter (2005), Koijen, Nijman, and Werker (2010), and Munk and Sørensen (2010)). Portfolio choices with housing (and optimal mortgage design) have been addressed before in Campbell and Cocco (2003), Sinai and Souleles (2005a), Cocco (2005),Yao and Zhang (2005a), Corradin et al. (2014), Berger et al. (2018), Oleár et al. (2017), Kraft et al. (2018), and Duarte et al. (2021). Papers that jointly consider housing and interest rate risk are limited; some notable examples are Van Hemert (2010), Kraft and Munk (2011) and Campbell and Cocco (2015).

Van Hemert (2010) solves a dynamic asset allocation problem in which the individual optimizes her portfolio and housing choices, while also considering bond and mortgage decisions. Kraft and Munk (2011) obtain explicit closed-form solutions for the consumption, housing and investment decisions in a model that features stochastic stock prices, interest rates, labor income and house prices. Kraft and Munk (2011) allow for both renters and home ownership. Campbell and Cocco (2015) study optimal mortgage default and refinancing decisions in a life-cycle model that features labor income, inflation, interest rate and house price risks. We focus on the welfare costs associated with home ownership.

The remainder of the paper is organized as follows. In Section 2, we present the model. Section 3 presents the optimal non-housing consumption and portfolio choice for a homeowner without a mortgage. Section 4 analyzes the impact of requiring a homeowner with a mortgage to contribute a certain percentage of salary to a retirement account. Section 5 provides the welfare analysis. The paper ends with a conclusion.

2 Model

This section explains the main assumptions and introduces the individual's maximization problem.[2]

The individual buys a house at age 20 (i.e., $t = 0$) and lives in the house for her complete remaining lifetime. Hence, she is a homeowner and we are not concerned with the rent-versus-buy decision.[3] This section assumes that the individual buys the house

[2]Our model is closely related to Koijen, Van Nieuwerburgh, and Yogo (2016), Berger et al. (2018), Been et al. (2021), and Duarte et al. (2021).

[3]For example, Yao and Zhang (2005b) and Sinai and Souleles (2005b) focus on the rent-versus-buy

without a mortgage (unconstrained case).[4] In Section 4, we assume that the house is financed either with a fixed- or an adjustable-rate mortgage (constrained case). The home value is constant and we do not consider maintenance costs. Indeed, as the individual lives in the house for her complete remaining lifetime, house price risk is irrelevant in our setting. This motivates the assumption of a constant home value. Furthermore, we abstract away from tax-favorable treatment of home ownership.

We assume that the individual lives for T periods (with $T \geq 0$) in a dynamic life-cycle setting. In our model, t denotes the age of the individual which is equal to the effective age minus age 20. The individual works full-time from $t = 0$ to $t = T_W$. She thus spends $T - T_W$ years in retirement. Moreover, the individual receives income from labor. In this paper, we consider a funded second-pillar pension.

Our main goal is to study the impact of (i) a mortgage with required repayments, (ii) mandatory contributions to a retirement account, and (iii) suboptimal investment strategies on the individual's welfare.

2.1 Financial Market

We consider two state variables: the financial market interest rate $r(t)$ and the stock price $S(t)$.

The financial market interest rate evolves according to:

$$\mathrm{d}r(t) = \kappa(\bar{r} - r(t))\mathrm{d}t + \sigma_r \mathrm{d}Z_r(t), \tag{2.1}$$

where $\kappa \geq 0$ models the speed of mean reversion, $\bar{r}$ represents the expected long-run financial market interest rate, $\sigma_r \geq 0$ captures the interest rate volatility and $Z_r(t)$ is a standard Brownian motion.

The stock price dynamics is given by

$$\mathrm{d}S(t) = S(t)\left(r(t) + \lambda_S \sigma_S\right)\mathrm{d}t + S(t)\sigma_S \mathrm{d}Z_S(t), \tag{2.2}$$

where λ_S represents the expected excess stock return per unit of volatility, $\sigma_S \geq 0$ denotes the stock price volatility, and $Z_S(t)$ is a standard Brownian motion.

The individual invests in three assets: a bank account (with price $B(t)$), a nominal

decision.

[4] She can afford to buy the house at time 0 without a mortgage since her total wealth at time 0 is assumed to be larger than the house value.

zero-coupon bond with time to maturity h (with price $P_h(t)$) and a stock (with price $S(t)$). The price dynamics of the bank account and the nominal zero-coupon bond are given by

$$\begin{aligned} \mathrm{d}B(t) &= r(t)B(t)\mathrm{d}t, \\ \mathrm{d}P_h(t) &= P_h(t)\left(r(t) - \lambda_r\sigma_r D_{\kappa,h}\right)\mathrm{d}t - P_h(t)\sigma_r D_{\kappa,h}\mathrm{d}Z_r(t), \end{aligned} \tag{2.3}$$

where λ_r is the market price of interest rate risk and $D_{\kappa,h} = (1-e^{-\kappa h})/\kappa \in [0,h]$ represents the interest rate duration of the bond.

The stochastic discount factor $M(t)$ satisfies (He and Pearson (1991)):

$$\mathrm{d}M(t) = -r(t)M(t)\mathrm{d}t + \varphi^\intercal M(t)\mathrm{d}Z(t), \tag{2.4}$$

where $\mathrm{d}Z(t) = (\mathrm{d}Z_r(t), \mathrm{d}Z_S(t))^\intercal$ denotes the vector of Brownian motions and $\varphi = (\varphi_S, \varphi_r)^\intercal$ represents the vector of factor loadings. This vector of factor loadings is computed as follows:

$$\varphi = -\rho^{-1}\lambda, \tag{2.5}$$

where $\lambda = (\lambda_S, \lambda_r)^\intercal$ represents the vector of market prices and ρ denotes the correlation matrix of the Brownian increments:

$$\rho = \begin{pmatrix} 1 & \rho_{Sr} \\ \rho_{Sr} & 1 \end{pmatrix}. \tag{2.6}$$

Here, ρ_{Sr} represents the correlation between $\mathrm{d}Z_S(t)$ and $\mathrm{d}Z_r(t)$. We assume that the Brownian motion increments are uncorrelated, i.e., $\rho_{Sr} = 0$.

2.2 Preferences

The individual derives utility from non-housing consumption $c(t)$ and housing consumption $h(t)$ (i.e., square meters of living space). Housing consumption is exogenously chosen and deterministic (i.e., $h(t) = h$ for all t). We do not include a bequest in the utility function.[5]

[5]At the end of her life-cycle T, the house will be bequeathed to her heirs. However, we do not explicitly consider this in the utility function, as we assume the individual cannot control bequeathed wealth.

For $\gamma \in (0, \infty) \setminus \{1\}$, the expected lifetime utility of the individual is described by

$$U = \mathbb{E}\left[\int_0^T e^{-\delta t} \frac{1}{1-\gamma} \left(c(t)^\alpha h^{1-\alpha}\right)^{1-\gamma} \mathrm{d}t\right], \tag{2.7}$$

where $\delta \geq 0$ is the subjective rate of time preference, α is the relative preference for non-housing consumption, and $\theta = 1 + \alpha(\gamma - 1)$ is the relative risk aversion coefficient (RRA).

2.3 Dynamic Budget Constraint

Denote by $\omega(t) = (\omega_P(t), \omega_S(t))^\intercal$ the vector of portfolio weights, with $\omega_P(t)$ the fraction of total non-housing wealth invested in the bond and $\omega_S(t)$ the fraction of total non-housing wealth invested in the stock. Total non-housing wealth is used to finance non-housing consumption. Then, total non-housing wealth evolves according to:

$$\mathrm{d}W(t) = \left(r(t) + \omega(t)^\intercal \left[\mu(t) - r(t)\right]\right) W(t)\mathrm{d}t + \omega(t)^\intercal \Sigma(t) W(t) \mathrm{d}Z(t) - c(t)\mathrm{d}t, \tag{2.8}$$

with

$$\mu(t) = \begin{pmatrix} r(t) - \lambda_r \sigma_r D_{\kappa,h} \\ r(t) + \lambda_S \sigma_S \end{pmatrix} \quad \text{and} \quad \Sigma(t) = \begin{pmatrix} -D_{\kappa,h}\sigma_r & 0 \\ 0 & \sigma_S \end{pmatrix}. \tag{2.9}$$

2.4 Dynamic Optimization Problem

The individual maximizes her utility subject to the dynamic budget constraint (2.8). Hence, the dynamic optimization problem is given by

$$\begin{aligned} \max_{c(t),\omega(t):0\leq t\leq T} & \ \mathbb{E}\left[\int_0^T e^{-\delta t} \frac{1}{1-\gamma} \left(c(t)^\alpha h^{1-\alpha}\right)^{1-\gamma} \mathrm{d}t\right] \\ \text{s.t. } \mathrm{d}W(t) &= \left(r(t) + \omega(t)^\intercal \left[\mu(t) - r(t)\right]\right) W(t)\mathrm{d}t \\ &\quad + \omega(t)^\intercal \Sigma(t) W(t) \mathrm{d}Z(t) - c(t)\mathrm{d}t. \end{aligned} \tag{2.10}$$

To solve the dynamic optimization problem (2.10), we transform the dynamic optimization problem into a static optimization problem. We analyze the optimal solution in Section 3, which we refer to as the unconstrained solution. In the unconstrained case, the individual buys the house at time 0 using her total wealth. Her remaining total wealth is used to finance non-housing consumption.

3 Optimal Unconstrained Policies

We now present the optimal non-housing consumption and portfolio strategy when the individual buys the house at time 0 without a mortgage (unconstrained case). She can afford to buy the house at time 0 since her total wealth at time 0 is assumed to be larger than the house value. Her remaining wealth, i.e., human wealth[6] minus the house value, will be used to finance her non-housing consumption. In other words, in the unconstrained case, the individual, in fact, receives a lower salary and obtains a house in return. Note that the unconstrained case is not realistic, as banks do not allow to exchange future wages for a house. In Section 4, we consider a realistic case.

3.1 Wealth Decomposition

Before we present the optimal unconstrained policies, we first explore the different components of total *non-housing* wealth $W(t)$. Let us introduce total wealth $TW(t)$ which we can decompose into wealth to finance non-housing consumption and wealth to finance housing consumption. Furthermore, it is well-known that total wealth $TW(t)$ is equal to financial wealth $F(t)$ plus human wealth $H(t)$; see Bodie, Merton, and Samuelson (1992). At time 0, total *non-housing* wealth equals $W(0)$ and total *housing* wealth equals $HV(0)$. Hence, total wealth at time 0, i.e., $TW(0)$, is given by

$$TW(0) = W(0) + HV(0) = F(0) + H(0).$$

We observe that total non-housing wealth at time 0, i.e., $W(0)$, is given by

$$W(0) = F(0) + H(0) - HV(0) = F(0) + \left(1 - \frac{HV(0)}{H(0)}\right) H(0) = F(0) + \widetilde{H}(0),$$

with $\widetilde{H}(0) = (1 - HV(0)/H(0))\, H(0)$. Let us define adjusted human wealth as follows:

$$\widetilde{H}(t) = \left(1 - \frac{HV(0)}{H(0)}\right) H(t). \tag{3.1}$$

In other words, in the unconstrained case, the individual, in fact, receives a new lower salary $\widetilde{w}(t) = (1 - HV/H(0))\, w(t)$ and obtains a house in return.

[6]Human wealth is equal to total wealth at time 0.

In what follows, we assume riskless labor income. That is,

$$w(t)=\begin{cases} w & \text{if } t\in[0,T_W],\\ 0 & \text{if } t\in(T_W,T]. \end{cases} \tag{3.2}$$

Human wealth $H(t)$ at time t equals the discounted value of future labor income:

$$H(t)=\int_0^{T-t}\mathbb{E}_t\left[\frac{M(t+h)}{M(t)}w(t+h)\right]\mathrm{d}h=\int_0^{T-t}P_h(t)w(t+h)\mathrm{d}h. \tag{3.3}$$

Human wealth evolves according to

$$\mathrm{d}H(t)=(r(t)-\lambda_r\sigma_r\widehat{D}_H(t))H(t)\mathrm{d}t-\sigma_r\widehat{D}_H(t)H(t)\mathrm{d}Z_r(t)-w(t)\mathrm{d}t, \tag{3.4}$$

where the duration of human wealth is given by

$$\widehat{D}_H(t)=\frac{\int_0^{T-t}D_{\kappa,h}P_h(t)w(t+h)\mathrm{d}h}{\int_0^{T-t}P_h(t)w(t+h)\mathrm{d}h}. \tag{3.5}$$

Appendix A derives (3.4) and (3.5). Finally, we note that financial wealth evolves as follows:

$$\begin{aligned}\mathrm{d}F(t)=&\left(r(t)+\widetilde{\omega}_P(t)\lambda_r\sigma_r D_{\kappa,h}+\widetilde{\omega}_S(t)\lambda_S\sigma_S\right)F(t)\mathrm{d}t\\ &-\widetilde{\omega}_P(t)\sigma_r D_{\kappa,h}F(t)\mathrm{d}Z_r(t)+\widetilde{\omega}_S(t)\sigma_S F(t)\mathrm{d}Z_S(t)+(w(t)-c(t))\mathrm{d}t,\end{aligned} \tag{3.6}$$

where $\widetilde{\omega}_S(t)$ and $\widetilde{\omega}_P(t)$ denote the optimal fractions of the financial wealth invested in the stock and the nominal bond, respectively.

3.2 Optimal Unconstrained Consumption

Theorem 1 states the expression for the optimal unconstrained non-housing consumption choice $c_{NR}(t)$ (here, NR stands for no restrictions).

Theorem 1 *(Optimal unconstrained non-housing consumption) Consider an individual with the CRRA utility function (2.7) who solves the optimization problem (2.10). Then the optimal unconstrained non-housing consumption is given by*

$$c_{NR}(t)=\zeta\left(e^{\delta t}M(t)\right)^{-\frac{1}{\theta}},$$

where $\zeta = y^{-\frac{1}{\theta}}$ and $y \geq 0$ is the Lagrange multiplier associated with the static budget constraint. More specifically, we have that

$$\zeta = \frac{W(0)}{\mathbb{E}\left[\int_0^T \left(e^{\delta t} M(t)\right)^{-\frac{1}{\theta}} M(t)\mathrm{d}t\right]}.$$

Proof. See Appendix B.

As becomes clear from Theorem 1, the individual can achieve a smooth non-housing consumption pattern. Indeed, she does not face any restrictions regarding mortgage payments, minimal pension savings and investment strategies.

3.3 Optimal Unconstrained Asset Allocation

We now present the optimal unconstrained asset allocation.

3.3.1 Portfolio Choice in terms of Total Non-Housing Wealth

Theorem 2 presents the optimal asset allocation as percentage of total non-housing wealth.

Theorem 2 *(Optimal asset allocation as % of total non-housing wealth) Consider an individual who solves the optimization problem (2.10). Then the optimal fractions of total non-housing wealth $W(t)$ invested in the stock and the nominal bond are as follows:*

$$\begin{aligned} \omega_{S,NR}(t) &= \frac{\lambda_S}{\theta\sigma_S}, \\ \omega_{P,NR}(t) &= -\frac{\lambda_r}{\theta\sigma_r D_{\kappa,h}} + \frac{\widehat{D}_A(t)}{D_{\kappa,h}}, \end{aligned} \tag{3.7}$$

where $\widehat{D}_A(t)$ denotes the optimal duration of the conversion factor (C7).

Proof. See Appendix C.2.

If $\alpha = 1$, then our asset allocation (3.7) follows from Brennan and Xia (2002). We observe that the individual invests in the stock to pick up the equity risk premium. Furthermore, we see that the individual holds two bond demands: a speculative demand and a hedging demand. The first term represents the speculative bond demand to pick up the interest rate premium, while the second term denotes the hedging bond demand to hedge against interest rate declines.

3.3.2 Portfolio Choice in terms of Financial Wealth

Theorem 3 states the optimal asset allocation as percentage of financial wealth.

Theorem 3 *(Optimal asset allocation as % of financial wealth) The optimal fractions of financial wealth $F(t)$ invested in the stock and the bond are given by*

$$\begin{aligned} \widetilde{\omega}_{S,NR}(t) &= \omega_{S,NR}(t)\frac{W(t)}{F(t)}, \\ \widetilde{\omega}_{P,NR}(t) &= \omega_{P,NR}(t)\frac{W(t)}{F(t)} - \frac{\widehat{D}_H(t)}{D_{\kappa,h}}\frac{\widetilde{H}(t)}{F(t)}, \end{aligned} \tag{3.8}$$

where $\widehat{D}_H(t)$ denotes the duration of human wealth given by (3.5), $\widetilde{H}(t)$ represents adjusted human wealth given by (3.1), and $\omega_{S,NR}(t)$ and $\omega_{P,NR}(t)$ follow from Theorem 2.

Proof. See Appendix C.3.

As becomes clear from Theorem 3, the individual applies a life-cycle investment strategy, which is well-known from the literature; see, e.g., Bilsen, Boelaars, and Bovenberg (2020). However, due to the homeownership, the impact of human capital on the optimal bond demand is lower. Indeed, since part of human capital is used to buy the house, the value of the bond-like asset labor income becomes, in fact, smaller.

4 Optimal Constrained Policies

This section assumes that the individual finances the house with an adjustable-rate mortgage or a 30-year fixed-rate mortgage. Furthermore, we assume a minimum pension savings rate. Section 3 already derived the first-best solution $c_{NR}(t)$ without a mortgage and a minimum pension savings rate. Although the first-best solution is theoretically optimal, pension participants typically uses a mortgage with required repayments to finance their house. Furthermore, they have to compulsory save for their pension. For example, in the Netherlands, individuals typically finance their house with a mortgage that needs to be paid off within 30 years and save the contribution rate set by their pension fund. Imposing such restrictions can result in sizable welfare losses. This is, in particular, the case if the optimal constrained consumption $c_R(t)$ (here, R stands for restrictions) deviates substantially from the first-best unconstrained consumption $c_{NR}(t)$. We assume that the individual contributes at least $s_{\min}$ of labor income to a retirement account. To obtain welfare losses, we compare the certainty equivalent of the constrained non-housing consumption ce_R with the certainty equivalent of the unconstrained non-housing consumption ce_{NR}.

4.1 Wealth Decomposition

As mentioned before, we can decompose total wealth into wealth to finance non-housing consumption and wealth to finance housing consumption. In this section, we assume that this last wealth component equals the so-called mortgage market value $V_m(t)$. Indeed, money spend on paying off the mortgage can be seen as housing consumption. As a result, wealth to finance non-housing consumption, which we refer to as total non-housing wealth, equals financial wealth plus human wealth minus the mortgage market value. We now define the mortgage market value. We distinguish between two cases: an adjustable-rate mortgage and a fixed-rate mortgage.

4.1.1 Adjustable-Rate Mortgage

Our mortgage valuation framework is in line with Goncharov (2003), Goncharov (2004), and Goncharov (2006).[7] These papers assume that a homeowner continuously pays mortgage payments $v_m(t) > 0$ during the mortgage life-time $[0, T_X]$. These mortgage payments $v_m(t)$ depend on the initial mortgage loan amount, the mortgage interest rate $r_m(t)$ and the mortgage loan term T_X. We assume that the mortgage interest rate is equal to the financial market interest rate at time t, i.e., $r_m(t) = r(t)$. Furthermore, the initial mortgage loan amount equals the house price per square meter times housing consumption h (i.e., square meters of living space). Hence, the loan-to-value ratio at time 0 is 100%.

To derive the mortgage payment $v_m(t)$, we introduce $O(t)$ which represents the outstanding principle at time t. For the remainder of the paper, we assume that the individual cannot pay more than the regular mortgage payments agreed upon in the contract (i.e., no prepayments). The total mortgage payment equals the difference between the mortgage interest payment $r_m(t)O(t)\mathrm{d}t$ and the change in the principle balance as result of principle repayment $\mathrm{d}O(t) < 0$:

$$v_m(t)\mathrm{d}t = r_m(t)O(t)\mathrm{d}t - \mathrm{d}O(t),\ t \in [0, T_X], \tag{4.1}$$

[7] Other papers focusing on the mortgage valuation framework are Pliska (2006) and Gorovoy and Linetsky (2007).

where the initial outstanding principle equals the initial mortgage loan and the mortgage is fully amortized (i.e., $O(T_X) = 0$). This implies that we can write the outstanding principle at time t as the difference between the present value of the initial outstanding principle and the present value of the cumulative mortgage payments:

$$O(t) = O(0)e^{\int_0^t r_m(u)\mathrm{d}u} - \int_0^t v_m(s)e^{\int_s^t r_m(u)\mathrm{d}u}\mathrm{d}s. \tag{4.2}$$

We assume that the arbitrage-free condition holds. Then the mortgage payment at time t is computed as follows (see Appendix D)

$$v_m(t) = \frac{r(t)O(t)}{1 - e^{-r(t)(T_X - t)}}. \tag{4.3}$$

The mortgage market value is defined as

$$V_m(t) = \int_0^{T_X - t} \mathbb{E}_t \left[\frac{M(t+h)}{M(t)} v_m(t+h) \right] \mathrm{d}h. \tag{4.4}$$

We observe that a decline in the interest has two effects on the mortgage market value. On the one hand, a decrease in the interest rate implies a decrease in the mortgage payments, and accordingly, leads to a decrease in the mortgage market value. On the other hand, a decrease in the interest rate implies an increase in the stochastic discount factor which, in turn, implies an increase in the mortgage market value. The individual prefers to consider the interest rate sensitivity of the mortgage market value when determining the investment strategies of financial wealth.

4.1.2 Fixed-Rate Mortgage Contract

Alternatively, the homeowner can finance the house with a fixed-rate mortgage. We assume that the fixed-rate period is set equal to the mortgage term. As a result, the mortgage payments are constant during the entire mortgage term. Moreover, we assume that the arbitrage-free condition holds. We can express the mortgage market value as follows:

$$V_m(t) = \int_0^{T_X - t} \mathbb{E}_t \left[\frac{M(t+h)}{M(t)} v_m \right] \mathrm{d}h = \int_0^{T_X - t} v_m P_h(t)\mathrm{d}h, \tag{4.5}$$

where the constant mortgage payments are given by (see Appendix D)

$$v_m = \frac{r_m O(0)}{1 - e^{-r_m T_X}}. \tag{4.6}$$

Note that we determine the mortgage interest rate $r_m(t) = r_m$ in such a way that the no arbitrage condition holds true. From (4.5), we observe that a decline in the interest rate leads to an increase in the mortgage market value. As in Section 4.1.1, the individual wants to hedge against this interest rate risk.

4.2 Dynamic Optimization in the Constrained Problem

We now discuss the dynamic constrained optimization problem. The unconstrained non-housing consumption $c_{NR}(t)$ is found by solving problem (2.10). The constrained maximization problem for the pension participant is given by

$$\begin{aligned}
&\max_{c(t),\omega(t):0\leq t\leq T} \mathbb{E}\left[\int_0^T e^{-\delta t}\frac{1}{1-\gamma}\left(c(t)^\alpha h^{1-\alpha}\right)^{1-\gamma}\mathrm{d}t\right] \\
&\qquad \text{s.t. } \mathrm{d}W(t) = \left(r(t) + \omega(t)^\intercal\left[\mu(t) - r(t)\right]\right)W(t)\mathrm{d}t \\
&\qquad\qquad\qquad + \omega(t)^\intercal\Sigma(t)W(t)\mathrm{d}Z(t) - c(t)\mathrm{d}t, \\
&\qquad\qquad c(t) \leq w(t) - v_m(t) - s_{\min}w(t),\ t\in[0,T_W],
\end{aligned} \tag{4.7}$$

where $s_{\min}$ denotes the minimum percentage of labor income the individual contributes to a retirement account. We note that labor income $w(t)$, the mortgage payment at time t, $v_m(t)$, and the minimum pension savings constraint $s_{\min}$ are exogenously determined. Also note that $v_m(t) = 0$ after time T_X, i.e., the mortgage need to be fully paid off within $T_X \leq T_W$ years. In the numerical illustrations, the minimum percentage of labor income that is added to the pension account, i.e., $s_{\min}$, is assumed to be equal to 10%. Moreover, if we rewrite the mortgage payment and pension savings constraint, it becomes directly clear that this constraint captures that at least 10% of labor income is added to the pension account: $s(t)w(t) = w(t) - v_m(t) - c(t) \geq 0.10w(t)$ for $t \leq T_W$. After retirement, the individual withdraws money from her pension account. Hence, pension savings after retirement become negative.

4.3 Optimal Constrained Consumption

Theorem 4 states the expression for the optimal constrained non-housing consumption choice $c_R(t)$.

Theorem 4 *(Optimal constrained non-housing consumption) Consider a homeowner with a mortgage with the CRRA utility function (2.7) who solves the optimization problem (4.7). Then the optimal constrained non-housing consumption $c_R(t)$ is given by*

$$c_R(t) = \begin{cases} \min\left\{\zeta\left(e^{\delta t}M(t)\right)^{-\frac{1}{\theta}}, w(t) - v_m(t) - s_{\min}w(t)\right\}, & \text{if } t \in [0, T_W], \\ \zeta\left(e^{\delta t}M(t)\right)^{-\frac{1}{\theta}}, & \text{if } t \in (T_W, T], \end{cases}$$

where $\zeta = y^{-\frac{1}{\theta}}$ and $y \geq 0$ is the Lagrange multiplier associated with the static budget constraint.

Proof. See Appendix E.

We can compute the value of the stochastic discount factor for which $c_R(t) = w(t) - v_m(t) - s_{\min}w(t)$ holds true; intuitively, the mortgage payment and pension savings constraint becomes binding if the optimal unconstrained non-housing consumption $c_{NR}(t)$ is no longer feasible.

We are also interested in the case in which the homeowner with a mortgage is not restricted by a minimum pension savings rate. Then the optimal non-housing consumption is given by

$$c_R(t) = \begin{cases} \min\left\{\zeta\left(e^{\delta t}M(t)\right)^{-\frac{1}{\theta}}, w(t) - v_m(t)\right\}, & \text{if } t \in [0, T_X], \\ \zeta\left(e^{\delta t}M(t)\right)^{-\frac{1}{\theta}}, & \text{if } t \in (T_X, T], \end{cases} \tag{4.8}$$

where $\zeta = y^{-\frac{1}{\theta}}$ and $y \geq 0$ is the Lagrange multiplier associated with the static budget constraint.

4.4 Optimal Constrained Asset Allocation

We note that the optimal constrained investment strategies follow from the principle of hedging. More specifically, the individual chooses the investment strategies such that changes in total non-housing wealth match changes in the market value of optimal constrained non-housing consumption. Let us denote the optimal fractions of total non-housing wealth invested in the stock and the nominal bond by $\omega_{S,R}(t)$ and $\omega_{P,R}(t)$, respectively. Theorem 5 now presents the asset allocation in terms of financial wealth.

Theorem 5 *(Asset allocation as a % of financial wealth) Consider a homeowner with a mortgage who solves the optimization problem (4.7). Then the optimal fractions of financial wealth $F(t)$ invested in the stock and the nominal bond are given by*

$$\begin{aligned} \widetilde{\omega}_{S,R}(t) &= \omega_{S,R}(t)\frac{W(t)}{F(t)}, \\ \widetilde{\omega}_{P,R}(t) &= \omega_{P,R}(t)\frac{W(t)}{F(t)} - \frac{\left(\widehat{D}_H(t)H(t) + \frac{\partial V_m(t)}{\partial r(t)}\right)}{D_{\kappa,h}F(t)}, \end{aligned} \tag{4.9}$$

where $\widehat{D}_H(t)$ is the duration of human wealth given by (3.5).

Proof. See Appendix F.

First, we investigate the impact of a fixed-rate mortgage on the optimal bond demand $\widetilde{\omega}_{P,R}(t)$. In that case, the mortgage payments v_m are not exposed to interest rate changes. However, the mortgage market value $V_m(t)$ increases if the interest rate decreases, i.e., $\partial V_m(t)/\partial r(t) < 0$. Since the mortgage is a loan, we can see the mortgage market value as a short position in a bond. This causes the share of financial wealth invested in the bond to increase.

Second, we investigate the impact of an adjustable-rate mortgage on the optimal bond demand $\widetilde{\omega}_{P,R}(t)$. In that case, the mortgage payments $v_m(t)$ are exposed to interest rate changes: a decrease in the interest rate $r(t)$ implies a decrease in the mortgage payment $v_m(t)$, and accordingly, leads to an increase in the mortgage market value $V_m(t)$. However, this cashflow effect counter interacts with the discount effect. In most cases, the discount effect dominates the cashflow effect, so that $\partial V_m(t)/\partial r(t) < 0$. Indeed, part of the mortgage payment $v_m(t)$ is a repayment which is insensitive to interest rate changes. A lower interest is good news (lower mortgage payments) for an adjustable-rate mortgage while this is not the case for a fixed-rate mortgage. So, an individual with an adjustable-rate mortgage has less need to invest in long-term bonds to hedge against low interest rates.

Finally, we note that an individual invests less in the nominal bond due to the presence of human capital, see Bilsen et al. (2020) for a detailed analysis on this subject matter.

5 Welfare Losses

This section illustrates the welfare losses.

5.1 Additional Assumptions and Baseline Parameters

This section describes the additional assumptions and baseline parameter values needed to illustrate the (un)constrained non-housing consumption.

The individual buys a home at age 20.[8] The individual's remaining working period is 45 years (i.e., $T_W = 45$) and the individual's remaining life expectancy is 65 years (i.e., $T = 65$). We assume that $\gamma = 5$. The homeowner's relative preference of non-housing consumption and the subjective rate of time preference are set at $\alpha = 0.8$ and $\delta = 3.0\%$, respectively.[9] The homeowner finances the house with an adjustable-rate mortgage or a 30-year fixed-rate mortgage. We assume that the mortgage term is 30 years (i.e., $T_X = 30$). For the sake of convenience, the maximum home loan a homeowner is eligible for is capped at five times the annual labor income, which is set equal to unity.[10] In line with Brennan and Xia (2002) and Bilsen et al. (2020), the speed of mean reversion, the expected long-run short-term interest rate, the interest rate volatility, the market price of interest rate risk and the initial financial market interest rate are set at $\kappa = 0.0347$, $\bar{r} = 0.01$, $\sigma_r = 0.01$, $\lambda_r =$ -0.075, and $r(0) = 0.01$, respectively. Moreover, the stock price volatility and the market price of equity risk are set at $\sigma_S = 0.20$ and $\lambda_S = 0.18$, respectively. The illustrations in the following sections are based on a minimum pension savings rate of 10% (i.e., $s_{\min} = 0.10$).

5.2 Welfare Losses due to Constraints

This section documents the welfare losses due to the presence of a mortgage with required repayments and a minimum pension savings constraint. In Appendix G, we discuss the results when home ownership is exogenously chosen at age 30 rather than age 20.

[8] Associaton of Realtors (2021) reports that 82% (48%) of all home buyers in the age category $22-30$ $(31-40)$ are first buyers. Moreover, the median income of a home buyer is \$80,000 for the age group $20-30$ and \$105,600 for the age category $31-40$ in the US.

[9] The weight for non-housing consumption is set at 0.80 in Cocco, Gomes, and Maenhout (2005), Van Hemert (2010), Kraft and Munk (2011) and Chetty, Sándor, and Szeidl (2017), while Campbell and Cocco (2015) and Hambel, Kraft, and Meyer-Wehmann (2020) set the non-housing weight at 0.70 and Cocco (2005) at 0.90. Berger et al. (2018) and Duarte et al. (2021) set the non-housing weight at 0.8875.

[10] E.g., Caloia (2022) shows an average (median) loan-to-income ratio of 3.84 (4.3) in 2018 based on Loan Level Data collected by the Dutch Central Bank (DNB).

Furthermore, this appendix gives more details on how to compute welfare losses.

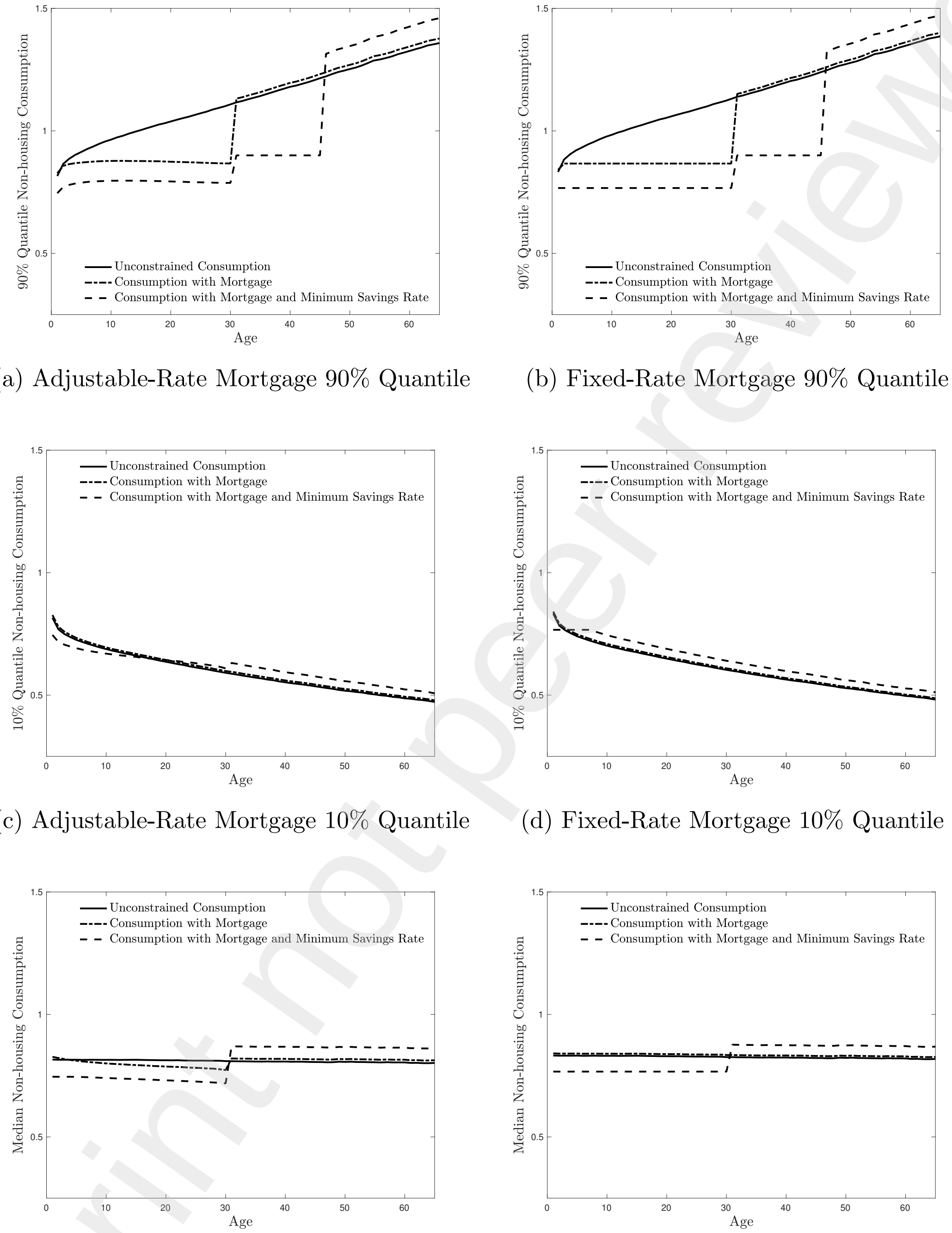
Figure 1 shows the non-housing consumption paths for a homeowner with an adjustable-rate mortgage or a 30-year fixed-rate mortgage. The solid lines indicate the unconstrained non-housing consumption paths (see Theorem 1). In that case, the individual is able to reach a smooth non-housing consumption pattern. Moreover, the dash-dotted lines indicate the non-housing consumption paths for a homeowner with a mortgage with required repayments who does not face any constraints regarding pension savings (see (4.8)). Finally, the dashed lines indicate the non-housing consumption paths for a homeowner with a mortgage with required repayments who faces a pension savings constraint (see Theorem 4).

In a good scenario, non-housing consumption displays a stair-step shape when restricted by both constraints, as shown in Panels (a) and (b). Indeed, the homeowner prefers to have a large unconstrained non-housing consumption, but she cannot reach this large non-housing consumption due to required mortgage payments and required pension savings. The first step is caused by the mortgage combined with the pension savings constraint, while the second step is solely attributable to the 10% pension savings constraint. In a bad scenario, the constraints are, at almost every age, not binding anymore, as shown in Panels (c) and (d). Indeed, the homeowner prefers to have a small unconstrained non-housing consumption, which can be reached even with a mortgage and a pension savings constraint. In a moderate scenario, as seen in Panels (e) and (f), the constraints are less likely to become binding compared to a good scenario.

Table 1 reports the welfare losses (in terms of relative decline of certainty equivalent life-time non-housing consumption) associated with the constraints. Although the presence of a mortgage leads to welfare losses, combining this with the 10% pension savings constraint, results in more substantial welfare losses. These welfare losses become even more pronounced when having a bigger mortgage loan and, in case of an adjustable-rate mortgage, a larger interest rate volatility.

5.3 Welfare Losses due to Suboptimal Investment Strategies

This section documents the welfare losses due to suboptimal investment strategies. In particular, we apply the investment strategy of a homeowner with a 30-year fixed-rate mortgage (see Section 4.4) to a homeowner without a mortgage (see Section 3.3.2). This is relevant because the investment policy of a collective pension fund does typically not



(a) Adjustable-Rate Mortgage 90% Quantile (b) Fixed-Rate Mortgage 90% Quantile

(c) Adjustable-Rate Mortgage 10% Quantile (d) Fixed-Rate Mortgage 10% Quantile

(e) Adjustable-Rate Mortgage Median (f) Fixed-Rate Mortgage Median

Figure 1. The figure shows the optimal non-housing consumption for a homeowner with an adjustable-rate mortgage or a 30-year fixed-rate mortgage. The baseline parameters are described in Section 5.1.

Panel A: Mortgage		
(σ_r, LTI)	Adjustable-Rate Mortgage	Fixed-Rate Mortgage
$(1\%, 4)$	0.63	0.42
$(1\%, 5)$	0.93	0.50
$(1.25\%, 4)$	0.77	0.37
$(1.25\%, 5)$	1.30	0.44
Panel B: Mortgage + 10% Pension Contribution Constraint		
(σ_r, LTI)	Adjustable-Rate Mortgage	Fixed-Rate Mortgage
$(1\%, 4)$	3.09	2.30
$(1\%, 5)$	4.26	2.75
$(1.25\%, 4)$	3.29	2.01
$(1.25\%, 5)$	5.02	2.41

Table 1. This table represents the welfare losses due to the presence of a mortgage with required repayments and due to the presence of both a mortgage with required repayments and a 10% pension savings constraint for two different interest rate volatilities $\sigma_r \in \{1\%, 1.5\%\}$ and two different loan-to-income ratios LTI $\in \{4, 5\}$. The baseline parameters are described in Section 5.1.

depend on individual characteristics such as homeownership and type of mortgage. For example, in the Netherlands, the board of a collective pension fund investigates to what extent pension fund members are willing to take risk and decides on the collective asset allocation. However, pension fund boards typically make no distinction between the investment policy of homeowners with a mortgage and the investment policy of individuals without a mortgage, although buying a house is a major investment for many pension fund members.

Table 2 illustrates the welfare losses (in terms of relative decline of certainty equivalent life-time non-housing consumption) for different mortgage terms, different loan-to-income ratios and different interest rate volatilities. We observe that these welfare losses become more sizeable for longer mortgage terms, higher mortgage loans, and higher interest rate volatilities. For the baseline case (i.e., $\sigma_r = 1\%$, LTI $= 4$), we find a welfare loss of 1.19%. We note that in case of an adjustable-rate mortgage, welfare losses are smaller as the mortgage market value is less sensitive to interest rate changes (see also Section 4.4).

6 Conclusion

This paper investigates the welfare losses of (i) financing a home with a mortgage with required repayments, (ii) saving a certain percentage of salary to a retirement account,

	Fixed-Rate Mortgage Term			
(σ_r, LTI)	10 Year	15 Year	20 Year	30 Year
$(1.0\%, 3.0)$	0.07	0.19	0.36	0.81
$(1.0\%, 4.0)$	0.10	0.27	0.51	1.19
$(1.5\%, 3.0)$	0.09	0.25	0.51	1.28
$(1.5\%, 4.0)$	0.13	0.37	0.74	1.92

Table 2. This table represents the welfare losses associated with imposing the optimal investment strategy for individuals without a mortgage on homeowners with a fixed-rate mortgage. The table provides welfare losses for two different interest rate volatility $\sigma_r \in \{1\%, 1.5\%\}$, two different loan-to-income ratios LTI $\in \{3, 4\}$, and four different fixed-rate mortgage terms $(10, 15, 20, 30$ year$)$. In case of a fixed-rate mortgage, the interest rate is fixed during the entire loan term. The baseline parameters are described in Section 5.1.

and (iii) following the investment strategy set by a collective pension fund board.

We find that welfare losses (in terms of relative decline of certainty equivalent lifetime non-housing consumption) vary between $3.09\% - 5.02\%$ $(2.30\% - 2.75\%)$ for a 30-year fixed-rate (adjustable-rate) mortgage. When the optimal investment strategy for individuals with a 30-year fixed-rate mortgage is imposed on homeowners without a mortgage, this results in welfare losses varying between $0.81\% - 1.92\%$. For lower fixed-rate mortgage terms, the welfare losses become less pronounced.

Our results suggest that offering homeowners with a mortgage the option to contribute less to the pension account or to decide on their investment strategy is welfare enhancing. Although considering homeownership in a collective pension scheme improves welfare, there are still some challenges on how to address the heterogeneity in mortgage conditions.

There are several limitations of our paper. First of all, home ownership is chosen exogenously at age 20.[11] This is relatively young, and the Appendix investigates a later age as well. Second, she remains in the same house over the entire life-cycle. And finally, we do not take house price risk into account. We leave these challenges for future research.

[11] Since the house market is illiquid, we have abstracted away from the buy-versus-rent decision.

A Dynamics of Human Wealth

The human wealth equation is given by (3.3). Applying Itô's Lemma on $H(t) = \psi(t, P_h(t))$, we find that

$$\mathrm{d}H(t) = \frac{\partial \psi}{\partial t}\mathrm{d}t + \frac{\partial \psi}{\partial P_h(t)}\mathrm{d}P_h(t) + \frac{1}{2}\frac{\partial^2 \psi}{\partial (P_h(t))^2}\mathrm{d}[P_h, P_h]_t, \tag{A1}$$

where the partial derivatives are given by

$$\begin{aligned}
\frac{\partial \psi}{\partial t} &= \frac{\partial}{\partial t}\int_0^{T-t} P_h(t)w(t+h)\mathrm{d}h = -w(t), \\
\frac{\partial \psi}{\partial P_h(t)} &= \int_0^{T-t} \frac{\partial}{\partial P_h(t)}\left(P_h(t)w(t+h)\right)\mathrm{d}h = \int_0^{T-t} w(t+h)\mathrm{d}h, \\
\frac{\partial^2 \psi}{\partial (P_h(t))^2} &= 0.
\end{aligned}$$

Hence, the dynamics of the human wealth $H(t)$ evolve according to:

$$\begin{aligned}
\mathrm{d}H(t) =& - w(t)\mathrm{d}t + \int_0^{T-t} w(t+h)\mathrm{d}h\mathrm{d}P_h(t) \\
=& - w(t)\mathrm{d}t + \left(r(t)H(t) - \lambda_r\sigma_r\int_0^{T-t} D_{\kappa,h}w(t+h)P_h(t)\mathrm{d}h\right)\mathrm{d}t \\
& - \sigma_r\mathrm{d}Z_r(t)\int_0^{T-t} D_{\kappa,h}w(t+h)P_h(t)\mathrm{d}h \\
=& - w(t)\mathrm{d}t + \left(r(t) - \sigma_r\widehat{D}_H(t)\lambda_r\right)H(t)\mathrm{d}t - \sigma_r\widehat{D}_H(t)H(t)\mathrm{d}Z_r(t),
\end{aligned} \tag{A2}$$

with the duration of human wealth given by

$$\widehat{D}_H(t) = -\frac{\partial \log H(t)}{\partial r(t)} = \frac{\int_0^{T-t} w(t+h)P_h(t)D_{\kappa,h}\mathrm{d}h}{\int_0^{T-t} w(t+h)P_h(t)\mathrm{d}h}. \tag{A3}$$

B Unconstrained Non-Housing Consumption Choice

This appendix derives the optimal unconstrained non-housing consumption choice for a homeowner without a mortgage. The static optimization problem is given by

$$\begin{aligned}
\max_{c(t):0\le t\le T} \ & \mathbb{E}\left[\int_0^T e^{-\delta s}\frac{1}{1-\gamma}\left(c(s)^\alpha h^{1-\alpha}\right)^{1-\gamma}\mathrm{d}s\right] \\
\text{s.t. } & \mathbb{E}\left[\int_0^T c(t)M(t)\mathrm{d}t\right] \le H(0) + F(0) - HV(0).
\end{aligned} \tag{B1}$$

Here, HV represents the house value. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} =& \mathbb{E}\left[\int_0^T e^{-\delta t}\frac{1}{1-\gamma}\left(c(t)^\alpha h^{1-\alpha}\right)^{1-\gamma}\mathrm{d}t\right] \\ &+ y\left(H(0)+F(0)-HV-\mathbb{E}\left[\int_0^T c(t)M(t)\mathrm{d}t\right]\right) \\ =& \int_0^T \mathbb{E}\left[e^{-\delta t}\frac{1}{1-\gamma}(c(t)^\alpha h^{1-\alpha})^{1-\gamma} - yc(t)M(t)\right]\mathrm{d}t \\ &+ y\left(H(0)+F(0)-HV(0)\right),\end{aligned} \tag{B2}$$

where $y \geq 0$ represents the Lagrange multiplier associated with the static budget constraint. The individual's main purpose is to maximize $e^{-\delta t}\frac{1}{1-\gamma}(c(t)^\alpha h^{1-\alpha})^{1-\gamma} - yc(t)M(t)$. The first-order condition gives

$$e^{-\delta t}\alpha c(t)^{\alpha(1-\gamma)-1}h^{(1-\alpha)(1-\gamma)} = yM(t). \tag{B3}$$

Hence, the unconstrained non-housing consumption is given by

$$c_{NR}(t) = \left(\frac{ye^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}}. \tag{B4}$$

Substituting the optimal unconstrained non-housing consumption in the static budget constraint, we find the Lagrange multiplier

$$y = \left(\frac{W(0)}{\mathbb{E}\left[\int_0^T \left(\frac{e^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}} M(t)\mathrm{d}t\right]}\right)^{-\theta}, \tag{B5}$$

with $W(0) = H(0) + F(0) - HV(0)$ denoting total non-housing wealth at time 0.

C Unconstrained Portfolio Choice

C.1 Market Value of Non-Housing Consumption

To determine the optimal unconstrained portfolio choice, we first need to determine the market value of non-housing consumption at time t. The market value of non-housing consumption at time t is given by

$$
\begin{aligned}
V(t) &= \mathbb{E}_t\left[\int_t^T \frac{M(s)}{M(t)} c_{NR}(s)\mathrm{d}s\right] \\
&= \mathbb{E}_t\left[\int_t^T \frac{M(s)}{M(t)} \left(\frac{ye^{\delta s}M(s)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}} \mathrm{d}s\right] \\
&= \mathbb{E}_t\left[\int_t^T \frac{M(s)^{\frac{-\theta+1}{-\theta}}}{M(t)} \left(\frac{ye^{\delta s}}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}} \mathrm{d}s\right] \\
&= \mathbb{E}_t\left[\int_t^T \left(\frac{M(s)}{M(t)}\right)^{\frac{-\theta+1}{-\theta}} \frac{\left(\frac{ye^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}}}{\left(\frac{ye^{\delta t}}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}}} \left(\frac{ye^{\delta s}}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{-\frac{1}{\theta}} \mathrm{d}s\right] \\
&= c_{NR}(t)\mathbb{E}_t\left[\int_t^T \left(\frac{M(s)}{M(t)}\right)^{\frac{-\theta+1}{-\theta}} e^{\frac{\delta(s-t)}{-\theta}} \mathrm{d}s\right] \\
&= c_{NR}(t)A_{NR}(t).
\end{aligned} \tag{C1}
$$

To solve (C1), we need to derive the market-consistent discount rate:

$$
A_{NR}(t) = \int_t^T \mathbb{E}_t\left[\left(\frac{M(s)}{M(t)}\right)^{\frac{-\theta+1}{-\theta}} e^{\frac{\delta(s-t)}{-\theta}}\right] \mathrm{d}s. \tag{C2}
$$

We have that

$$
\begin{aligned}
\frac{M(s)}{M(t)} = \exp\Bigg\{&-\int_0^{s-t}\left(r(t+u)+\frac{1}{2}\lambda_1^2\right)\mathrm{d}u \\
&-\int_0^{s-t}\left(\lambda_r \mathrm{d}Z_r(t+u)+\lambda_S \mathrm{d}Z_S(t+u)\right)\Bigg\},
\end{aligned} \tag{C3}
$$

with $\lambda_1^2 = \lambda_r^2 + \lambda_S^2$. Substituting (C3) in (C2) to find

$$
\begin{aligned}
A_{NR}(t) = &\int_t^T \mathbb{E}_t \Bigg[\exp\left\{ \frac{\delta(s-t)}{-\theta} - \frac{-\theta+1}{-\theta} \int_0^{s-t} \left(r(t+u) + \frac{1}{2}\lambda_1^2 \right) \mathrm{d}u \right\} \\
&\times \exp\left\{ -\frac{-\theta+1}{-\theta} \int_0^{s-t} \lambda_r \mathrm{d}Z_r(t+u) - \frac{-\theta+1}{-\theta} \int_0^{s-t} \lambda_S \mathrm{d}Z_S(t+u) \right\} \Bigg] \mathrm{d}s \\
= &\int_t^T \exp\left\{ \frac{\delta(s-t)}{-\theta} \right\} \exp\left\{ -\frac{-\theta+1}{-\theta} \int_0^{s-t} \left(\mathbb{E}_t r(t+u) + \frac{1}{2}\lambda_1^2 \right) \mathrm{d}u \right\} \times \\
&\mathbb{E}_t \Bigg[\exp\Bigg\{ -\frac{-\theta+1}{-\theta} \int_0^{s-t} \left(\lambda_r + \sigma_r D_{\kappa,s-t-u} \right) \mathrm{d}Z_r(t+u) \\
&-\frac{-\theta+1}{-\theta} \int_0^{s-t} \lambda_S \mathrm{d}Z_S(t+u) \Bigg\} \Bigg] \mathrm{d}s \\
= &\int_t^T \exp\left\{ \frac{\delta(s-t)}{-\theta} \right\} \exp\left\{ -\frac{-\theta+1}{-\theta} \int_0^{s-t} \left(\mathbb{E}_t r(t+u) + \frac{1}{2}\lambda_1^2 \right) \mathrm{d}u \right\} \times \\
&\exp\left\{ \frac{1}{2} \left(\frac{\theta-1}{\theta} \right)^2 \int_0^{s-t} \left(\lambda_r + \sigma_r D_{\kappa,u} \right)^2 \mathrm{d}u + \frac{1}{2} \left(\frac{\theta-1}{\theta} \right)^2 \int_0^{s-t} \lambda_S^2 \mathrm{d}u \right\} \mathrm{d}s.
\end{aligned}
\tag{C4}
$$

Computations yield

$$
A_{NR}(t) = \int_t^T \exp\left\{ -\int_0^{s-t} d_u(t) \mathrm{d}u \right\} \mathrm{d}s, \tag{C5}
$$

with

$$
\begin{aligned}
d_u(t) = &\frac{-\theta+1}{-\theta} \left(\mathbb{E}_t r(t+u) - \lambda_r \sigma_r D_{\kappa,u} - \frac{1}{2} \frac{-\theta+1}{-\theta} (\sigma_r D_{\kappa,u})^2 \right) \\
&- \frac{\delta}{-\theta} - \frac{-\theta+1}{(-\theta)^2} \lambda_r \sigma_r D_{\kappa,u} - \left[\left(\frac{-\theta+1}{-\theta} \right)^2 - \frac{-\theta+1}{-\theta} \right] \frac{1}{2} \lambda_1^2.
\end{aligned}
\tag{C6}
$$

The optimal duration of the conversion factor is given by

$$
\begin{aligned}
\widehat{D}_A(t) &= -\frac{\partial \log A_{NR}(t)}{\partial r(t)} \\
&= \frac{-\theta+1}{-\theta} \frac{\int_t^T D_{\kappa,s-t} \exp\left\{ -\int_0^{s-t} d_u(t) \mathrm{d}u \right\} \mathrm{d}s}{\int_t^T \exp\left\{ -\int_0^{s-t} d_u(t) \mathrm{d}u \right\}}.
\end{aligned}
\tag{C7}
$$

C.2 Portfolio Choice in terms of Total Non-Housing Wealth

The market value of optimal non-housing consumption $V(t) = f(t, M(t), r(t))$ evolves according to

$$\begin{aligned} \mathrm{d}V(t) &= \frac{\partial f}{\partial t}\mathrm{d}t + \frac{\partial f}{\partial M(t)}\mathrm{d}M(t) + \frac{\partial f}{\partial r(t)}\mathrm{d}r(t) + (\dots)\mathrm{d}t \\ &= (\dots)\mathrm{d}t + \left(-\frac{\partial f}{\partial M(t)}\lambda_r M(t) + \frac{\partial f}{\partial r(t)}\sigma_r\right)\mathrm{d}Z_r(t) - \frac{\partial f}{\partial M(t)}\lambda_S M(t)\mathrm{d}Z_S(t). \end{aligned} \tag{C8}$$

Comparing with the dynamic budget constraint in (2.8), we find that

$$\begin{aligned} \omega_{S,NR}(t) &= -\frac{\partial f}{\partial M(t)}\frac{\lambda_S}{\sigma_S}\frac{M(t)}{W(t)}, \\ \omega_{P,NR}(t) &= \frac{\partial f}{\partial M(t)}\frac{\lambda_r}{\sigma_r D_{\kappa,h}}\frac{M(t)}{W(t)} - \frac{\partial f}{\partial r(t)}\frac{1}{D_{\kappa,h}W(t)}. \end{aligned} \tag{C9}$$

Define the function

$$\begin{aligned} V(t) &= f(t, M(t), r(t)) \\ &= c_{NR}(t)A_{NR}(t) \\ &= \left(\frac{ye^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{\frac{1}{-\theta}} A_{NR}(t). \end{aligned} \tag{C10}$$

First, we derive the partial derivatives of this function with respect to the stochastic discount factor $M(t)$ and the interest rate $r(t)$. By taking the partial derivative of f with respect $M(t)$, we find

$$\begin{aligned} \frac{\partial f}{\partial M(t)} &= \frac{1}{-\theta}\left(\frac{ye^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{\frac{1}{-\theta}-1}\frac{ye^{\delta t}}{\alpha h^{(1-\alpha)(1-\gamma)}}A_{NR}(t) \\ &= \frac{1}{-\theta}\left(\frac{ye^{\delta t}M(t)}{\alpha h^{(1-\alpha)(1-\gamma)}}\right)^{\frac{1}{-\theta}}\frac{A_{NR}(t)}{M(t)} \\ &= \frac{1}{-\theta}\frac{V(t)}{M(t)}. \end{aligned} \tag{C11}$$

Differentiating f with respect to $r(t)$, we arrive at:

$$\begin{aligned} \frac{\partial f}{\partial r(t)} &= -c_{NR}(t)\int_t^T \frac{-\theta+1}{-\theta}\int_0^{s-t} e^{-\kappa u}\mathrm{d}u \exp\left\{-\int_0^{s-t} d_u(t)\mathrm{d}u\right\}\mathrm{d}s \\ &= -c_{NR}(t)\int_t^T \frac{-\theta+1}{-\theta}D_{\kappa,s-t}\exp\left\{-\int_0^{s-t} d_u(t)\mathrm{d}u\right\}\mathrm{d}s \\ &= -\widehat{D}_A(t)V(t). \end{aligned} \tag{C12}$$

Substituting these partial derivatives in (C9), the optimal fractions of total non-housing wealth invested in the stock and the nominal bond are given by

$$\begin{aligned}\omega_{S,NR}(t) &= \frac{\lambda_S}{\sigma_S}\frac{1}{\theta},\\ \omega_{P,NR}(t) &= \frac{1}{-\theta}\frac{\lambda_r}{\sigma_r D_{\kappa,h}} + \frac{\widehat{D}_A(t)}{D_{\kappa,h}}.\end{aligned} \tag{C13}$$

C.3 Portfolio Choice in terms of Financial Wealth

Total non-housing wealth $W(0)$ at time 0 is defined as follows:

$$W(0) = F(0) + H(0) - HV = F(0) + \left(1 - \frac{HV}{H(0)}\right) H(0) = F(0) + \widetilde{H}(0),$$

with $\widetilde{H}(0) = (1 - HV/H(0))\, H(0)$. Let us define adjusted human capital as follows:

$$\widetilde{H}(t) = \left(1 - \frac{HV}{H(0)}\right) H(t).$$

In other words, in the unconstrained case, the individual receives a new lower $\widetilde{w}(t) = (1 - HV/H(0))\, w(t)$ and obtains a house in return.

Financial wealth $F(t)$ evolves as follows

$$\begin{aligned}\mathrm{d}F(t) =& \left(r(t) - \widetilde{\omega}_P(t)\lambda_r\sigma_r D_{\kappa,h} + \widetilde{\omega}_S(t)\lambda_S\sigma_S\right) F(t)\mathrm{d}t\\ &-\widetilde{\omega}_P(t)\sigma_r D_{\kappa,h}F(t)\mathrm{d}Z_r(t) + \widetilde{\omega}_S(t)\sigma_S F(t)\mathrm{d}Z_S(t) + (w(t) - c(t))\,\mathrm{d}t.\end{aligned} \tag{C14}$$

We find that the total non-housing wealth dynamics are given by

$$\begin{aligned}\mathrm{d}W(t) =& \mathrm{d}F(t) + \mathrm{d}\widetilde{H}(t)\\ =& (\dots)\,\mathrm{d}t + \left[-\widetilde{\omega}_P(t)\sigma_r D_{\kappa,h}F(t) - \sigma_r\widehat{D}_H(t)\widetilde{H}(t)\right]\mathrm{d}Z_r(t)\\ &+ \widetilde{\omega}_S(t)\sigma_S F(t)\mathrm{d}Z_S(t).\end{aligned} \tag{C15}$$

Comparing with the dynamic budget constraint

$$\begin{aligned}\mathrm{d}W(t) =& \left(r(t) - \omega_P(t)\lambda_r\sigma_r D_{\kappa,h} + \omega_S(t)\lambda_S\sigma_S\right) W(t)\mathrm{d}t\\ &-\omega_P(t)\sigma_r D_{\kappa,h}W(t)\mathrm{d}Z_r(t) + \omega_S(t)\sigma_S W(t)\mathrm{d}Z_S(t) - c(t)\mathrm{d}t,\end{aligned} \tag{C16}$$

we find that the allocations of financial wealth to the stock and the nominal bond are given by

$$\begin{aligned}
\widetilde{\omega}_{S,NR}(t) &= \omega_{S,NR}(t)\frac{W(t)}{F(t)}, \\
\widetilde{\omega}_{P,NR}(t) &= \omega_{P,NR}(t)\frac{W(t)}{F(t)} - \frac{\widehat{D}_H(t)}{D_{\kappa,h}}\frac{\widetilde{H}(t)}{F(t)}.
\end{aligned} \tag{C17}$$

D Mortgage Payment

This appendix derives the mortgage payments. Recall that an individual buys a house at $t = 0$ and lives in her home for the complete remaining lifetime. The house is financed either with a fixed- or an adjustable-rate mortgage. The fixed-rate mortgage (FRM) refers to a home loan with an interest rate that is fixed over time; the adjustable-rate mortgage (ARM) refers to a home loan with an interest rate that adjusts over time.

Before we start, we recall that the present value of n fixed monthly payments with an amount x and a monthly interest rate i is given by

$$L = \frac{x\left(1 - (1+i)^{-n}\right)}{i}. \tag{D1}$$

From this, it follows that the fixed monthly payments on the loan amount, L, is given by

$$x = \frac{i \cdot L}{1 - (1+i)^{-n}}. \tag{D2}$$

We start with some small adjustments to the formula:

(i) Replace i with $r(t)/N$, where $r(t)$ represents the annual interest rate and N the annual frequency of compounding periods ($N = 12$ in case of monthly payments).

(ii) Replace the total number of periods n with $N \cdot T_X$, where T_X represents the mortgage loan term.

(iii) Replace the loan amount L with the outstanding principle balance at time t, i.e., $O(t)$.

It follows from the adjusted formula that the fixed payment $x(N)$ corresponding to the frequency N is given by

$$x(N) = \frac{r(t)O(t)}{N\left(1 - \left(1 + \frac{r(t)}{N}\right)\right)^{-N \cdot T_X}}. \tag{D3}$$

Note that $N \cdot x(N)$ represents the total amount paid per year. Then the limiting value of the annual mortgage repayment is given by

$$v_m(t) = \lim_{N\to\infty} N \cdot x(N) = \lim_{N\to\infty} \frac{r(t)\cdot O(t)}{1-\left(1+\frac{r(t)}{N}\right)^{-N\cdot T_X}} = \frac{r(t)\cdot O(t)}{1-e^{-r(t)T_X}}. \tag{D4}$$

In case of a FRM, the payment rate v_m is determined at $t = 0$:

$$v_m = \frac{r_m O(0)}{1-e^{-r(0)T_X}}. \tag{D5}$$

E Constrained Non-Housing Consumption Choice

This appendix derives the optimal constrained non-housing consumption. The corresponding Lagrangian $\mathcal{L}$ is given by

$$\begin{aligned}\mathcal{L} =&\mathbb{E}\left[\int_0^T e^{-\delta t}\frac{1}{1-\gamma}\left(c(t)^{\alpha}h^{1-\alpha}\right)^{1-\gamma}\mathrm{d}t\right]\\ &+y\left(H(0)+F(0)-V_m(0)-\mathbb{E}\left[\int_0^T c(t)M(t)\mathrm{d}t\right]\right)\\ =&\int_0^T \mathbb{E}\left[e^{-\delta t}\frac{1}{1-\gamma}\left(c(t)^{\alpha}h^{1-\alpha}\right)^{1-\gamma}-yM(t)c(t)\right]\mathrm{d}t\\ &+y\left(H(0)+F(0)-V_m(0)\right),\end{aligned} \tag{E1}$$

where $y \geq 0$ represents the Lagrange multiplier associated with the static budget constraint. Then the individual's main purpose is to maximize $e^{-\delta t}u(c(t),h) - yM(t)c(t)$ subject to $c(t) \leq w(t) - v_m(t) - s_{\min}w(t)$ for $t \in [0, T_W]$, while it aims to maximize $e^{-\delta t}u(c(t),h) - yM(t)c(t)$ for $t \in (T_W, T]$. For $t \in [0, T_W]$, the optimal non-housing consumption $c_R(t)$ needs to satisfy the following conditions:

$$\begin{aligned}&e^{-\delta t}u'(c_R(t),h) = yM(t) + x(t),\\ &c_R(t) \leq w(t) - v_m(t) - s_{\min}w(t),\\ &x(t)(w(t) - v_m(t) - s_{\min}w(t) - c_R(t)) = 0,\\ &x(t) \geq 0,\end{aligned} \tag{E2}$$

where $x(t)$ represents the Lagrange multiplier associated with the constraint. After solving these conditions, we find the following optimal non-housing consumption

$$c_R(t) = \min\left\{(u')^{-1}\left(ye^{\delta t}M(t)\right), w(t) - v_m(t) - s_{\min}w(t)\right\}. \tag{E3}$$

For $t \in (T_W, T]$, we find the following optimal non-housing consumption:

$$c_R(t) = (u')^{-1}\left(ye^{\delta t}M(t)\right). \tag{E4}$$

In summary, the optimal non-housing consumption can be written as

$$c_R(t) = \begin{cases} \min\left\{(u')^{-1}\left(ye^{\delta t}M(t)\right), w(t) - v_m(t) - s_{\min}w(t)\right\}, & \text{if } t \in [0, T_W], \\ (u')^{-1}\left(ye^{\delta t}M(t)\right), & \text{if } t \in (T_W, T], \end{cases} \tag{E5}$$

with y the associated Lagrange multiplier.

F Constrained Portfolio Choice

We start by deriving the dynamics of the mortgage market value. A homeowner selects to finance the house with an adjustable- or a fixed-rate mortgage:

- The adjustable rate mortgage market value is given by (4.4) and can be rewritten as:

$$V_m(t) = v_m(t)\int_t^{T_X} \mathbb{E}_t\left[\frac{M(s)}{M(t)}\frac{v_m(s)}{v_m(t)}\right]\mathrm{d}s, \tag{F1}$$

  where the adjustable-rate mortgage payments can be described as a function of the interest rate, i.e., $v_m(t) = f(r(t))$. Applying Itô's Lemma to the function $V_m(t) = \psi(t, r(t))$, we obtain

$$\begin{aligned} \mathrm{d}V_m(t) &= \frac{\partial\psi}{\partial t}\mathrm{d}t + \frac{\partial\psi}{\partial r(t)}\mathrm{d}r(t) + \frac{1}{2}\frac{\partial^2\psi}{\partial(r(t))^2}\mathrm{d}[r,r]_t \\ &= \left(\frac{\partial\psi}{\partial t} + \frac{\partial\psi}{\partial r(t)}\kappa(\bar{r} - r(t)) + \frac{1}{2}\frac{\partial^2\psi}{\partial(r(t))^2}\sigma_r^2\right)\mathrm{d}t + \frac{\partial\psi}{\partial r(t)}\sigma_r\mathrm{d}Z_r(t). \end{aligned} \tag{F2}$$

  In other words, we have that

$$\mathrm{d}V_m(t) = (\ldots)\,\mathrm{d}t + \frac{\partial V_m(t)}{\partial r(t)}\sigma_r\mathrm{d}Z_r(t). \tag{F3}$$

- The fixed-rate mortgage market value is given by (4.5). Applying Itô's Lemma to the function $V_m(t) = \psi(t, P_h(t))$, we find that

$$\mathrm{d}V_m(t) = \frac{\partial\psi}{\partial t}\mathrm{d}t + \frac{\partial\psi}{\partial P_h(t)}\mathrm{d}P_h(t) + \frac{1}{2}\frac{\partial^2\psi}{\partial(P_h(t))^2}\mathrm{d}[P_h, P_h]_t, \tag{F4}$$

where the partial derivatives are given by

$$\begin{aligned}\frac{\partial \psi}{\partial t} &= \frac{\partial}{\partial t}\int_0^{T_X-t} P_h(t) v_m \mathrm{d}h = -v_m,\\ \frac{\partial \psi}{\partial P_h(t)} = \int_0^{T_X-t} \frac{\partial}{\partial P_h(t)} \left(P_h(t)v_m\right)\mathrm{d}h &= \int_0^{T_X-t} v_m \mathrm{d}h,\\ \frac{\partial^2 \psi}{\partial (P_h(t))^2} &= 0.\end{aligned}$$

Hence, the fixed-rate mortgage market value $V_m(t)$ dynamics is given by

$$\begin{aligned}\mathrm{d}V_m(t) =& -v_m\mathrm{d}t + \int_0^{T_X-t} v_m \mathrm{d}h\mathrm{d}P_h(t)\\ =& -v_m\mathrm{d}t + \left(r(t)V_m(t) - \lambda_r\sigma_r\int_0^{T_X-t} D_{\kappa,h}v_m P_h(t)\mathrm{d}h\right)\mathrm{d}t\\ & -\sigma_r\mathrm{d}Z_r(t)\int_0^{T_X-t} D_{\kappa,h}v_m P_h(t)\mathrm{d}h\\ =& -v_m\mathrm{d}t + \left(r(t) - \sigma_r\widehat{D}_m(t)\lambda_r\right)V_m(t)\mathrm{d}t - \sigma_r\widehat{D}_m(t)V_m(t)\mathrm{d}Z_r(t),\end{aligned} \tag{F5}$$

with the duration of the mortgage market value denoted by

$$\begin{aligned}\widehat{D}_m(t) &= -\frac{\partial \log V_m(t)}{\partial r(t)}\\ &= \frac{\int_0^{T_X-t} v_m P_h(t) D_{\kappa,h}\mathrm{d}h}{\int_0^{T_X-t} v_m P_h(t)\mathrm{d}h}.\end{aligned} \tag{F6}$$

The total non-housing wealth $W(t)$ is defined as the sum of the financial wealth $F(t)$ (with dynamics (3.6)), human wealth $H(t)$ (with dynamics (3.4)) minus the mortgage market value $V_m(t)$ (with dynamics (F5)). We find that the total non-housing wealth dynamics are given by

$$\begin{aligned}\mathrm{d}W(t) &= \mathrm{d}F(t) + \mathrm{d}H(t) - \mathrm{d}V_m(t)\\ &= \begin{cases}(\ldots)\,\mathrm{d}t + \left[-\widetilde{\omega}_P(t)\sigma_r D_{\kappa,h}F(t) - \sigma_r\widehat{D}_H(t)H(t) - \sigma_r\frac{\partial V_m(t)}{\partial r(t)}\right]\mathrm{d}Z_r(t)\\ +\widetilde{\omega}_S(t)\sigma_S F(t)\mathrm{d}Z_S(t) \qquad \text{(Adjustable-Rate Mortgage)},\\ (\ldots)\,\mathrm{d}t + \left[-\widetilde{\omega}_P(t)\sigma_r D_{\kappa,h}F(t) - \sigma_r\widehat{D}_H(t)H(t) + \sigma_r\widehat{D}_m(t)V_m(t)\right]\mathrm{d}Z_r(t)\\ +\widetilde{\omega}_S(t)\sigma_S F(t)\mathrm{d}Z_S(t) \qquad \text{(Fixed-Rate Mortgage)}.\end{cases}\end{aligned} \tag{F7}$$

Comparing with the dynamic budget constraint

$$\begin{aligned} \mathrm{d}W(t) = & \left(r(t) - \omega_P(t)\lambda_r\sigma_r D_{\kappa,h} + \omega_S(t)\lambda_S\sigma_S\right) W(t)\mathrm{d}t \\ & - \omega_P(t)\sigma_r D_{\kappa,h} W(t)\mathrm{d}Z_r(t) + \omega_S(t)\sigma_S W(t)\mathrm{d}Z_S(t) - c(t)\mathrm{d}t, \end{aligned} \tag{F8}$$

we find that the allocations of financial wealth to stock and nominal bond are given by

$$\begin{aligned} \widetilde{\omega}_{S,R}(t) &= \omega_{S,R}(t)\frac{W(t)}{F(t)}, \\ \widetilde{\omega}_{P,R}(t) &= \begin{cases} \omega_{P,R}(t)\frac{W(t)}{F(t)} - \frac{\left(\widehat{D}_H(t)H(t) + \frac{\partial V_m(t)}{\partial r(t)}\right)}{D_{\kappa,h}F(t)} & \text{(Adjustable-Rate Mortgage)}, \\ \omega_{P,R}(t)\frac{W(t)}{F(t)} - \frac{\left(\widehat{D}_H(t)H(t) - \widehat{D}_m(t)V_m(t)\right)}{D_{\kappa,h}F(t)} & \text{(Fixed-Rate Mortgage)}. \end{cases} \end{aligned} \tag{F9}$$

G Welfare Analysis

G.1 Definition

Proposition 6 *(The welfare analysis). The welfare losses are computed as follows:*

$$WL = \frac{ce_{NR} - ce_R}{ce_{NR}}, \tag{G1}$$

where ce_{NR} and ce_R are the unconstrained certainty equivalent consumption and constrained certainty equivalent consumption.[12]

G.2 Mortgage and Minimum Pension Contributions

This appendix shows the computations due to the presence of a mortgage and minimum pension contributions. For $i \in \{NR, R\}$, the certainty equivalent is found by solving the equation:

$$\mathbb{E}\left[\int_0^T e^{-\delta t} u(c_i(t), h)\mathrm{d}t\right] = \int_0^T e^{-\delta t} u(ce_i, h)\mathrm{d}t. \tag{G2}$$

Using the finding that

$$\int_0^T e^{-\delta t} u(ce_i, h)\mathrm{d}t = \frac{\left[(ce_i)^\alpha h^{1-\alpha}\right]^{1-\gamma}}{1-\gamma} \int_0^T e^{-\delta t}\mathrm{d}t, \tag{G3}$$

[12] The certainty equivalent of a stochastic non-housing consumption stream $c(t)$ ($t \geq 0$) is defined as the amount ce such that the agent is indifferent between $c(t)$ ($t \geq 0$) and receiving ce with certainty.

the certainty equivalent level for $i \in \{NR, R\}$ is given by

$$ce_i = \left[\frac{(1-\gamma)\mathbb{E}\left[\int_0^T e^{-\delta t} u(c_i(t), h)\mathrm{d}t\right]}{h^{(1-\alpha)(1-\gamma)} \int_0^T e^{-\delta t}\mathrm{d}t} \right]^{\frac{1}{\alpha(1-\gamma)}}. \tag{G4}$$

Recall that $u(c_i(t), h) = [(c_i(t))^\alpha h^{1-\alpha}]^{1-\gamma}/(1-\gamma)$, then the certainty equivalent levels for $i \in \{NR, R\}$ simplify to

$$ce_i = \left[\frac{\mathbb{E}\left[\int_0^T e^{-\delta t} (c_i(t))^{\alpha(1-\gamma)}\mathrm{d}t\right]}{\int_0^T e^{-\delta t}\mathrm{d}t} \right]^{\frac{1}{\alpha(1-\gamma)}}. \tag{G5}$$

G.3 Sensitivity Of Welfare Losses To Age Home Buyer

This section performs a sensitivity analysis of our welfare losses with respect to the age of a home buyer. Associaton of Realtors (2021) reports that the typical home buyer is 33-years old in 2021. In previous years, the typical home buyer is somewhat younger, and therefore, we assume that our first-time buyer is 30-years old (i.e., $t = 0$). The home is financed with an adjustable- or a 30-year fixed-rate mortgage. The individual's remaining working period is 35-years (i.e., $T_W = 35$) and the individual's remaining life expectancy is 55-years (i.e., $T = 55$). Initial financial wealth $F(0)$ is set equal to two times the annual labor income. The remainder of the assumption and baseline parameters follow Section 5.1. Table 3 reports the welfare losses.

Panel A: Mortgage		
(σ_r, LTI)	Adjustable-Rate Mortgage	Fixed-Rate Mortgage
$(1\%, 4)$	0.34	0.21
$(1\%, 5)$	0.49	0.24
$(1.25\%, 4)$	0.42	0.17
$(1.25\%, 5)$	0.73	0.20
Panel B: Mortgage + 10% Pension Contribution Mandate		
(σ_r, LTI)	Adjustable-Rate Mortgage	Fixed-Rate Mortgage
$(1\%, 4)$	1.65	1.14
$(1\%, 5)$	2.39	1.34
$(1.25\%, 4)$	1.77	0.89
$(1.25\%, 5)$	2.97	1.05

Table 3. This table represents the welfare losses due to the presence of a mortgage with required repayments and due to the presence of both a mortgage with required repayments and a 10% pension savings constraint for two different interest rate volatilities $\sigma_r \in \{1\%, 1.25\%\}$ and two different loan-to-income ratios LTI $\in \{4, 5\}$. The baseline parameters are described in Section 5.1. The home buyer is aged 30-years rather than 20-years. We assume that initial financial wealth is set equal to two times the annual labor income. The individual's working period is 35-years and the individual's remaining life expectancy is 55-years.

References

Associaton of Realtors, N. 2021. 2021 Profile of Home Buyers and Sellers. https://cdn.nar.realtor/sites/default/files/documents/2021-highlights-from-the-profile-of-home-buyers-and-sellers-11-11-2021.pdf. [Online; accessed 16-November-2022].

Been, J., C. van Ewijk, M. Knoef, R. Mehlkopf, and S. Muns. 2021. Households' heterogeneous welfare gains from liquidating housing wealth over the life-cycle .

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra. 2018. House prices and consumer spending. *The Review of Economic Studies* 85:1502–1542.

Bilsen, S. van., I. A. Boelaars, and A. L. Bovenberg. 2020. The Duration Puzzle in Life-Cycle Investment. *Review of Finance* 24:1271–1311.

Bodie, Z., R. C. Merton, and W. F. Samuelson. 1992. Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model. *Journal of Economic Dynamics and Control* 16:427–449.

Brennan, M. J., and Y. Xia. 2002. Dynamic asset allocation under inflation. *The Journal of Finance* 57:1201–1238.

Caloia, F. 2022. Borrower-Based Measures, House Prices and Household Debt. *De Nederlandsche Bank* .

Campbell, J. Y., and J. F. Cocco. 2003. Household risk management and optimal mortgage choice. *The Quarterly Journal of Economics* 118:1449–1494.

Campbell, J. Y., and J. F. Cocco. 2015. A model of mortgage default. *The Journal of Finance* 70:1495–1554.

Campbell, J. Y., and L. M. Viceira. 2001. Who should buy long-term bonds? *American Economic Review* 91:99–127.

Chetty, R., L. Sándor, and A. Szeidl. 2017. The effect of housing on portfolio choice. *The Journal of Finance* 72:1171–1212.

Ciurilă, N., C. de Kok, H. t. Rele, and P. Zwaneveld. 2022. Optimizing the Life-Cycle Path of Pension Premium Payments and the Pension Ambition in the Netherlands. *De Economist* pp. 1–37.

Cocco, J. F. 2005. Portfolio choice in the presence of housing. *The Review of Financial Studies* 18:535–567.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout. 2005. Consumption and portfolio choice over the life cycle. *The Review of Financial Studies* 18:491–533.

Corradin, S., J. L. Fillat, and C. Vergara-Alert. 2014. Optimal portfolio choice with predictability in house prices and transaction costs. *The Review of Financial Studies* 27:823–880.

Duarte, V., J. Fonseca, A. S. Goodman, and J. A. Parker. 2021. Simple Allocation Rules and Optimal Portfolio Choice Over the Lifecycle. Tech. rep., National Bureau of Economic Research.

Goncharov, Y. 2003. *Mathematical theory of mortgage modeling*. University of Illinois at Chicago.

Goncharov, Y. 2004. On the Mortgage Rates Implied by the Option-Based and Empirical Approaches, proceedings of Hawaii International Conference on Statistics.

Goncharov, Y. 2006. An intensity-based approach to the valuation of mortgage contracts and computation of the endogenous mortgage rate. *International Journal of Theoretical and Applied Finance* 9:889–914.

Gorovoy, V., and V. Linetsky. 2007. Intensity-based valuation of residential mortgages: An analytically tractable model. *Mathematical Finance* 17:541–573.

Hambel, C., H. Kraft, and A. Meyer-Wehmann. 2020. When should retirees tap their home equity? *Available at SSRN 3681834* .

He, H., and N. D. Pearson. 1991. Consumption and portfolio policies with incomplete markets and short-sale constraints: The infinite dimensional case. *Journal of Economic Theory* 54:259–304.

Koijen, R. S., T. E. Nijman, and B. J. Werker. 2010. When can life cycle investors benefit from time-varying bond risk premia? *The Review of Financial Studies* 23:741–780.

Koijen, R. S., S. Van Nieuwerburgh, and M. Yogo. 2016. Health and mortality delta: Assessing the welfare cost of household insurance choice. *The Journal of Finance* 71:957–1010.

Kraft, H., and C. Munk. 2011. Optimal housing, consumption, and investment decisions over the life cycle. *Management Science* 57:1025–1041.

Kraft, H., C. Munk, and S. Wagner. 2018. Housing habits and their implications for life-cycle consumption and investment. *Review of Finance* 22:1737–1762.

Munk, C., and C. Sørensen. 2010. Dynamic asset allocation with stochastic income and interest rates. *Journal of Financial Economics* 96:433–462.

OECD. 2021. Brick by Brick: Building Better Housing Policies. *OECD Publishing, Paris, http://doi.org/10.1787/b453b043-en* .

Oleár, G., F. de Jong, and I. Minderhoud. 2017. Individualized life-cycle investing. *Netspar* pp. 1–36.

Pliska, S. R. 2006. Mortgage Valuation and Optimal Refinancing. In *Stochastic Finance*, pp. 183–196. Springer.

Sangvinatsos, A., and J. A. Wachter. 2005. Does the failure of the expectations hypothesis matter for long-term investors? *The Journal of Finance* 60:179–230.

Sinai, T., and N. S. Souleles. 2005a. Owner-occupied housing as a hedge against rent risk. *The Quarterly Journal of Economics* 120:763–789.

Sinai, T., and N. S. Souleles. 2005b. Owner-Occupied Housing as a Hedge Against Rent Risk. *Quarterly Journal of Economics* 120:763–789.

Van Ewijk, C., R. Mehlkopf, S. van den Bleeken, and C. Hoet. 2017. Welke keuzemogelijkheden zijn wenselijk vanuit het perspectief van de deelnemer. *Netspar Design Paper* 71.

Van Hemert, O. 2010. Household interest rate risk management. *Real Estate Economics* 38:467–505.

Yao, R., and H. H. Zhang. 2005a. Optimal consumption and portfolio choices with risky housing and borrowing constraints. *The Review of Financial Studies* 18:197–239.

Yao, R., and H. H. Zhang. 2005b. Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints. *Review of Financial Studies* 18:197–239.