Network for Studies on Pensions, Aging and Retirement

NETSPAR ACADEMIC SERIES

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A Comparative Analysis

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DP 06/2023-033

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June 26, 2023

Abstract

Using an overlapping generations model, two new indicators of public pension system sustainability are proposed: the pension space, which measures the capacity to pay for pension expenditures out of labour taxation, and the pension space exhaustion probability reflecting demographic uncertainties. These measures reveal that the pension spaces of advanced economies are strikingly different. Most nations have little scope to further finance pensions out of labour income taxation over the next thirty years. There is no one-size-fits-all solution. Risk-equivalent pension reforms enhance welfare in the long run, particularly for rapidly ageing nations, but also entail non-negligible transitional costs.

Keywords: Ageing, Fiscal Space, Public Pension Sustainability, Overlapping Generations Model.
JEL Codes: E62, H55, H20.

[*]The authors express their gratitude for the financial support provided by the NETSPAR Comparative Research Grant No. CRG2022.02 towards this research. Views and opinions presented in the paper are solely those of the authors and do not necessarily represent the perspectives or stances of NETSPAR.

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1 Introduction

A long-standing question confronting advanced economies is whether they have enough room to sustain their unfunded old-age public pension systems. Public pensions occupy a substantial, often double-digit, share of both national income and the social expenditures of major advanced economies, entailing costs and benefits that affect people for most of their lives. The main threats facing public pensions arise from ageing populations and changing working patterns. Increasingly, these are resulting in a larger share of the population demanding state pensions but a smaller share of active workers responsible for funding them. Weak growth, rising interest rates and soaring government indebtedness compound the problem, adding to the uncertainty surrounding the size and timing of future public pension expenditures and the ability of policymakers to meet these obligations.

The long-term sustainable policy interventions available to governments to address impending public pension imbalances are, in practice, circumscribed, IMF (2022a). One option is to increase direct taxation. This may, however, become infeasible because it discourages work, reduces economic growth and can even result in lower government tax revenues. Additional steps may therefore be required. These include partial financing through indirect taxation, lower pension payments and raising the retirement age.[1] Each of these interventions has very different economic implications for the general government budget, for labour force participation and for private saving. Their distributional effects will also differ depending on the economic conditions of a country when the reforms are implemented and on the pace of implementation.

Against this background, this paper presents two new measures that offer a means to

[1]These four categories of policy interventions encompass the type of reforms normally enacted in advanced economies in recent years, see IMF (2022b) or OECD (2021). In the absence of long-term policy reforms like these, there may be a temptation to look for temporary fixes by, for example, increased government borrowing. At the present time, with many advanced economies reaching levels of public debt not seen since WWII due to the Covid-19 pandemic, further borrowing to finance pension expenditures is unlikely to be a viable option. Nor is it likely to be sustainable in the longer term. Privatizing public pensions has also been proposed. The challenges in doing this, which include the significant transition expenses involved have been studied by, for example, Nishiyama and Smetters (2007).

examine and compare the viability of public pension systems across nations, track their long-run feasibility and address the effectiveness of policy interventions aimed at enhancing pension performance. The first, named the *pension space* (PS), measures how much scope a government has to finance public pensions out of direct (labour income) taxation in any given period of time. The concept of PS is an adaptation to public pensions of the widely-used notion of the fiscal space applied to the general government budget, see among others Bi (2012), Ghosh et al. (2013) and Romer and Romer (2019). The second metric, named the *pension space exhaustion probability* ($PSEP$), measures the probability that the pension space reaches zero at some point in the future as a consequence of demographic uncertainties. The main advantage of the proposed PS for policy analysis is that it can condense diverse macroeconomic and demographic information related to public pensions into simple measures useful for cross-country comparisons. $PSEP$ can be employed primarily as a pension-risk warning indicator, but also as a target for risk-equivalent calibration of different types of public pension reforms.

PS and $PSEP$ are derived from a general equilibrium, life-cycle model featuring overlapping generations of households, a competitive production sector and a government that finances consumption and transfers using distortionary labour, capital and consumption taxation, issues non-contingent debt and runs a pay-as-you-go pension system. The model, which is developed along the lines of the large-scale OLG framework pioneered by Auerbach et al. (1983) and Auerbach and Kotlikoff (1987), makes three novel methodological contributions. First, it explicitly recognizes that an increase of the social security tax rate can result in either more or less revenue to finance pensions, depending on the initial tax rate and the responsiveness of taxpayers to its change. This so-called Laffer effect has been long-studied in the context of infinitely-lived agent models, see for example, Davig et al. (2010), Trabandt and Uhlig (2011) and Polito and Wickens (2015). We show that, applied to public pensions in a life-cycle environment, the Laffer effect determines an upper limit on a government's ability to pay for public pensions out of labour income taxation. Second, the taxation of labour income contributes to two streams of government revenue, one financing the general government budget, the other financing the social security budget. The two budgets are therefore connected via the aggregate supply of labour. It follows that, an increase in the social security tax rate, while potentially increasing revenue available for aggregate pension expenditure, creates a deficit in the general government budget. The maximization of the social security budget that we undertake entails the solution of a non-trivial Laffer problem, because the general government budget needs to be accounted for as an additional constraint. This is different from the standard Laffer solution used in the literature, where one of the variables of the general government budget is solved as a residual. In effect, we search for a solution that maximises one of the two government budgets, namely the social security budget, while maintaining the other, the general budget, unaltered. We refer to this solution as the pension limit (PL) and show how it can be quantified as a general equilibrium outcome. PS then follows naturally as the gap between PL and aggregate pension expenditures, measured as a proportion of PL in any given period of time. The size of PS depends, among others, on the demographic structure of the population. The third methodological contribution consists in estimating the kernel distribution of PS at a future time horizon using available statistics on demographic uncertainty. $PSEP$ is extrapolated from this distribution as the probability of PS being equal to zero.

The remaining contribution of the paper is quantitative. The model is calibrated to a sample of twelve advanced economies (Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, Spain, the United Kingdom and the United States). These countries differ significantly in their macroeconomic characteristics, public pension arrangements and demographic trends. These differences are reflected in the range of values obtained for PS and $PSEP$ across countries. The quantitative analysis is therefore more revealing of the usefulness of the two metrics than would a focus on just a single country, often the United States, as customary in the fiscal policy literature based on large-scale OLG models. The calibration attempts to mimic the stationary equilibrium for each country based on macroeconomic aggregates up to 2020. PS is calculated for each economy. Its determinants are examined and its robustness to alternative model parameterizations are evaluated. Using the latest demographic projections from United Nations (2022), we then extend the calculation of PS beyond 2020, estimating its empirical distribution and $PSEP$ up to 2050. The analysis of policy reforms is in two parts. First we measure how PS and $PSEP$ respond to three stylized interventions: an increase in consumption taxation by 5%, a reduction of the pension replacement rate by 10% and an increase of the retirement age by two years. Second, we calculate the required change in each country's consumption taxation and payment to retirees that yields the same $PSEP$ in 2050 as a two-year increase in the retirement age. This enables us to undertake a risk-equivalent evaluation of the macroeconomic and welfare effects of pension reforms that have the same probability of exhausting the PS at some point in the future. We also evaluate the short-run distributional effects of the three reforms and how these depend on the pace at which they are implemented.

Four headline results emerge from the quantitative analysis:

1. The current size of PS varies significantly across countries, ranging in 2020 from roughly 5 to 57%. Nonetheless, three country groups can be identified depending on their PS being either large (above 40%, United States, United Kingdom and Ireland), medium (between 10 and 25%, Finland, Germany, Netherlands, Portugal and Spain) or small (below 10%, Belgium, Austria, France and Italy). Two factors largely determine these rankings: the extent of labour income taxation and the generosity of the pension system.

2. For most countries there is little scope to finance public pensions out of further labour income taxation over the next 30 years. The speed of the PS decline is especially concerning for Spain, Finland and for the four countries with a small PS in 2020; all of these economies are projected to entirely erode their PS before 2050. In nine of the twelve economies $PSEP$ is estimated to become positive and increasing by 2050. Austria, France and Italy are the most likely to face early exhaustion, as their $PSEP$ is estimated to reach close to 100% well before 2050.

3. Pension policy reform cannot be a one-size-fits-all solution as different policy interventions affect countries' PS and $PSEP$ differently, depending on the extent of the

policy change, its impact on the labour supply, and the country-specific economic and demographic circumstances. We find that a 5% increase in indirect taxation and a two-year increase in the retirement age yields the largest increase in PS for six countries and reduction in $PSEP$ for four countries. For the remaining two countries, a 5% increase in indirect taxation and a 10% reduction in pension benefits are almost equally effective.

4. Risk-equivalent pension reforms bring large long-term gains, particularly for those countries that are projected to age more rapidly, but also short and medium term costs. In the long run, partial financing and lower pension payments generate larger welfare gains compared to postponing retirement. These gains are higher for countries that tend to age faster. However in the short and medium run reforms entail consumption losses that only in part can be alleviated through a more gradual implementation.

Two important considerations must be taken into account when evaluating the quantitative outcomes. First, the PS metric calculates the economy's capacity to meet pension commitments by raising income taxes. It does so without taking into account the government's willingness or the feasibility of implementing such a policy. Thus the PS value should be regarded as an estimated upper limit on the true pension space available to the economy. The second caveat is that the calculated $PSEP$ solely accounts for demographic uncertainty and does not incorporate economic and political risks, which are difficult to quantify and assess due, notably, to their size and interdependence. Even so, focusing solely on demographic uncertainty lends greater transparency and clarity to the proposed $PSEP$ metric.

1.1 Related Literature

The paper complements in three ways a long standing macroeconomic literature on public pension sustainability and demographic change, typified by the works of Auerbach et al. (1983), De Nardi et al. (1999), Diamond and Orszag (2005), Imrohoroğlu and Kitao (2012), Kitao (2014) and Heer (2018). Heer (2019) provides an up-to-date overview. First, these studies share the premise that it is costly to maintain public pensions because the required increase of income taxation over time reduces labour supply, private saving and national output. This paper takes a stronger stance, arguing that economies will soon be unable to generate the revenue from labour income taxation required to meet pension obligations. Our quantitative analysis shows this is actually the case for many countries. Second, many of these studies compare different policies that would make the social security system self-financing in the long run. But demographic uncertainty is large when projecting so far into the future. The $PSEP$ metric reflects this uncertainty, allowing policy comparison to be made from a novel risk-equivalent perspective. Third, despite the significance and the many lessons that can be learned from international comparisons of public pension systems, see for example Disney (2000) and Diamond (2006), most of the OLG literature focuses on a single-country, often the United States. The present paper fills this gap providing instead a multi-country analysis, based on synthetic indicators of the pension space of a nation and of the likelihood of this to be exhausted in the future merely as a result of ongoing demographic trends.

There is a growing fiscal policy literature using infinitely-lived agent models to quantify Laffer effects and the fiscal space for the general government budget, see for example Davig et al. (2010), Trabandt and Uhlig (2011), Bi (2012), Ghosh et al. (2013), Polito and Wickens (2015). The paper complements this literature by employing a life-cycle model and focusing on a specific area of fiscal policy, public pensions, which gives rise to the novel concept of a pension space. Recent works have considered Laffer effects in large-scale OLG models. For example, Guner et al. (2016) and Holter et al. (2019) study how fiscal constraints restrict scope for tax progressivity and redistribution policy. Heer et al. (2020) exploit fiscal limits to identify a demographic threshold beyond which income taxation can no longer fund general government spending. In these studies the Laffer solution is computed deriving one of the variables included in the government budget constraint as a residual. The present paper is different in terms of both methods and objectives. First, the Laffer problem solved in the paper is different, as the revenue-side of the social security budget is maximized taking the size of the general government budget as a binding constraint. Of course, it is the expenditure-side of the social security balanced budget that is implicitly adjusted in the aggregate. Second, the Laffer effect is exploited to derive two synthetic metrics useful to evaluate the pension space of a nation and its likelihood of depletion in the future.

Our quantitative analysis abstracts from (i) potential for productivity change driven by either the decline in the labour force, see Heer and Irmen (2014), or variation in education attainments, see Conesa et al. (2020a) and (ii) altruistic motives among households members, see Fuster et al. (2003). The former could result in higher figures for the PS and lower estimates of the $PSEP$, as long as ageing leads to a greater incentive to invest in labour-saving technology or education attainments increase. The latter is likely to offset some of the transient distributional costs of public pension reforms at the aggregate level. Further, our focus on public pension reforms consider policies enacted recently in advanced economies, aiming at increasing contributions, reducing benefits and postpone eligibility, see IMF (2022a). We do not consider privatization or debt financing. The former is known to entail large short run costs, see for example Nishiyama and Smetters (2007) or Barr and Diamond (2008). The latter appears to be an impracticable route given the large debt accumulated by advanced economies in the aftermath of the Covid-19 pandemic. We consider however consumption taxation. This has long been known to be less distortive than income taxation, see Milesi-Ferretti and Roubini (1998) or Conesa et al. (2020b), and it is increasingly considered as a realistic option to finance the burden of public pensions, see for example İmrohoroğlu et al. (2016) and Ruppert et al. (2021). For computational tractability, given our multi-country focus, the quantitative analysis does not include idiosyncratic (income) uncertainty, as for example in İmrohoroğlu et al. (1995), Storesletten et al. (1999) or Fehr et al. (2008). This would result in higher precautionary savings of the households and, as a consequence, larger PS and lower $PSEP$. The proposed analysis is positive, being purely focused on the sustainability of existing public pension systems in the longer terms and related reforms. We therefore abstract form normative issues concerning the existence and role of public pensions, see Diamond and Orszag (2005), as well as their optimal size, see Heer (2018), and financing, see Fehr et al. (2013).

The paper proceeds as follows. Section 2 describes the model environment, derives the PS and the $PSEP$ metrics, and illustrates their qualitative properties and derivation. Section 3 describes the calibration. Section 4 presents the quantitative results. Section 5 concludes.

2 The Model

2.1 Environment

The economy is described by a perfect foresight life-cycle model comprising three sectors: households, a representative firm and the government.

In every period $t \geq 0$ a new generation of households is born. The age of newborns is $s = 1$. All generations retire at the end of age $s = T^W > 1$ and live up to a maximum age $s = T > T^W$. Population growth varies over time and it is denoted as n_t. The total population in a given period t is N_t and the number of households of age s is $N_t(s)$. Consequently the share of the s-year-old cohort in the total population in period t is defined as $\mu_t^s \equiv N_t(s)/N_t$. All agents of age s survive until age $s+1$ with probability ϕ_t^s. Thus, $\phi_t^0 = 1$ and $\phi_t^T = 0$.

Each household comprises one individual. In every period t, newborn households maximize the expected intertemporal lifetime utility:

$$U_t = \sum_{s=1}^{T} \beta^{s-1} \left(\prod_{j=0}^{s-1} \phi_{t+j-1}^{j} \right) u(c_{t+s-1}^{s}, l_{t+s-1}^{s}), \tag{1}$$

where β, c and l denote the household's discount factor, consumption and labour supply, respectively. Following Trabandt and Uhlig (2011), the instantaneous utility $u(.)$ is specified as:

$$u(c,l) = \frac{1}{1-\eta} \left(c^{1-\eta} \left[1 - \kappa(1-\eta) l^{1+1/\varphi} \right]^{\eta} - 1 \right), \tag{2}$$

where η is the inverse elasticity of intertemporal substitution, φ is the Frisch labour elasticity, κ is a scaling parameter. In every period t, the labour income accrued by a working household is given by the product of the wage rate w_t, the aggregate productivity A_t, the age-productivity of the s-year-old household $\bar{y}^s$ and the working hours l_t^s. Labour income is subject to both an income tax and a social security contribution levied at the rates τ_t^w and τ_t^p, respectively. Pension income received by the household upon retirement is denoted as pen_t. Thus the household's income y_t^s is given by:

$$y_t^s = \begin{cases} (1 - \tau_t^w - \tau_t^p) w_t A_t \bar{y}^s l_t^s & s = 1, \ldots, T^W, \\ pen_t & s = T^W + 1, \ldots, T. \end{cases} \tag{3}$$

In any period t, the budget constraint of an household of age $s = 1, \ldots, T$ is given by:

$$(1 + \tau_t^c) c_t^s = y_t^s + [1 + (1 - \tau_t^k) r_t] a_t^s + tr_t^s - a_{t+1}^{s+1}, \tag{4}$$

where τ_t^c is the tax rate on consumption, τ_t^k is the tax rate on capital income, a_t^s denotes the stock of assets held by the s-year-old household at the beginning of period t, r_t is the rate of return on assets, tr_t^s denotes non-pension-related transfers from the government received by the household of age s in period t. As the household does not work during retirement, $l_t^s = 0$ for $s = T^{W+1}, \ldots, T$. Further the household starts and terminates life without assets, thus $a_t^1 = a_t^{T+1} = 0$. In equilibrium the household is indifferent between holding assets in the form of either physical capital or government bonds, since both yield the same (certain) after-tax return. With a single household living for two periods the proportion of asset holdings would be the same at the household and the aggregate level, but with many periods, the portfolio allocation is indeterminate. Consequently, without loss of generality, we assume that each household holds the two assets in the same fixed proportions. This is determined in the aggregate as the share of capital in total assets $K_t/(K_t + B_t)$, where K_t and B_t denote aggregate capital and government bonds, respectively. In every period t, maximization of lifetime utility in (1) subject to (3)-(4) yields equilibrium conditions for the optimal allocation of consumption, labour and assets given by:

$$u_{c,t}^{s} = \lambda_t^s(1+\tau_t^c), \qquad s = 1, \ldots, T, \tag{5}$$

$$-u_{l,t}^{s} = \lambda_t^s(1-\tau_t^w-\tau_t^p)w_t A_t \bar{y}^s, \qquad s = 1, \ldots, T^W, \tag{6}$$

$$\lambda_t^s = \beta\phi_t^s\lambda_{t+1}^{s+1}[1+(1-\tau_{t+1}^k)r_{t+1}], \qquad s = 1, \ldots, T-1, \tag{7}$$

where $u_{c,t}^{s}$ and $u_{l,t}^{s}$ denote the derivatives in period t of the household's utility at age s with respect to consumption and labour, respectively, whereas λ_t^s is the Lagrange multiplier of the budget constraint in equation (4).

The representative firm uses aggregate capital and labour to maximize profits, while operating a Cobb-Douglas production function with labour-augmenting technological progress given by $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, where L_t denotes aggregate labour and α the share of capital in output. Technology grows over time at the exogenous rate $g_A \geq 0$, which is also equal to the balanced-growth rate of the economy. Since production is perfectly competitive, in equilibrium labour and capital are remunerated at their marginal products, that is:

$$w_t = (1-\alpha)K_t^{\alpha}(A_t L_t)^{-\alpha}, \tag{8}$$

$$r_t = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} - \delta, \tag{9}$$

where δ denotes the rate of physical capital depreciation.

The government allocates expenditure to public consumption G_t, transfers Tr_t and interest payments on public debt $r_t^b B_t$, with r_t^b denoting the after-tax return on government bonds. Government expenditure is financed through taxation Tax_t and new debt $B_{t+1} - B_t$. The government also collects assets from those who do not survive at the end of any given period t. The revenue accruing from these accidental bequests is denoted as Beq_t. In the aggregate, fiscal policy is subject to the sequence of general government budget constraints. In any period t, this is formulated as:

$$G_t + Tr_t + (1 + r_t^b)B_t = B_{t+1} + Tax_t + Beq_t. \tag{10}$$

The canonical solvency condition requires that government debt must grow at a rate lower than r^b in the long run, i.e. $\lim_{T\to\infty} B_{T+1}/\prod_{j=0}^{T}(1+r_j^b) \leq 0$. Following Kindermann and Krueger (2022), accidental bequests collected by the government include the after tax gross return on assets and are formulated as:

$$Beq_{t+1} = \sum_{s=1}^{T-1} N_t(s)(1-\phi_t^s)[1+(1-\tau_{t+1}^k)r_{t+1}]a_{t+1}^{s+1}. \tag{11}$$

Revenue from taxation of aggregate consumption C_t, capital and labour is given by:

$$Tax_t = \tau_t^c C_t + \tau_t^w w_t A_t L_t + \tau_t^k r_t K_t. \tag{12}$$

The government sector includes also a pay-as-you-go pension system. Pension payments made by the government to retired households are calculated as a constant fraction of the after tax average labour income:

$$pen_t = \theta_t(1-\tau_t^l)w_t A_t \bar{l}_t, \tag{13}$$

where θ_t denotes the replacement rate of pensions, $\tau_t^l = \tau_t^w + \tau_t^p$ and $\bar{l}_t$ is the average working hours in period t (of all workers), defined as $\bar{l}_t = \sum_{s=1}^{T^w}\mu_t^s l_t^s / \sum_{s=1}^{T^w}\mu_t^s$. In the aggregate, pension expenditure is the sum of pension payments made to retired households:

$$Pen_t = \sum_{s=T^W+1}^{T} N_t(s)pen_t \tag{14}$$

and the social security budget is formulated as:

$$Pen_t = \tau_t^p w_t A_t L_t, \tag{15}$$

where $\tau_t^p w_t A_t L_t$ is the aggregate revenue raised by the government to finance pension expenditure from the taxation of labour income.[2]

At the aggregate level, consumption, labour in terms of efficiency units, assets and transfers are determined as the sum of the corresponding individual variables, thus:

$$C_t = \sum_{s=1}^{T} N_t(s) c_t^s, \quad L_t = \sum_{s=1}^{T^W} N_t(s) \bar{y}^s l_t^s, \quad \mathcal{A}_t = \sum_{s=1}^{T} N_t(s) a_t^s, \quad Tr_t = \sum_{s=1}^{T} N_t(s) tr_t^s. \tag{16}$$

Equilibrium in the goods market requires that aggregate output is equal to aggregate demand:

$$Y_t = C_t + G_t + K_{t+1} - (1-\delta)K_t, \tag{17}$$

whereas equilibrium in the capital market requires that aggregate assets purchased by the households are equal to the sum of aggregate capital and government bonds demanded by firms and the government, respectively:

$$\mathcal{A}_t = K_t + B_t. \tag{18}$$

The no-arbitrage condition implies that in equilibrium all assets pay the same after tax rate of return:

$$r_t^b = (1 - \tau_t^k) r_t. \tag{19}$$

The dynamics of all endogenous variables in the model are non-stationary because both population and aggregate productivity grow over time. Appendix A describes how we render

[2]The social security budget is specified to measure old-age pension expenditure. All other welfare-related expenditures incurred by the government, such as for unemployment, health and disability insurance, are implicitly accounted for by Tr_t defined below in (16).

the model dynamics stationary and presents the stationary counterparts of all equilibrium conditions from (3) to (19). We conclude this section observing that the specification of the economic environment could be further enhanced by accounting for individual and aggregate income uncertainty, or by developing the nominal side of the economy to consider monetary policy. Although incorporating some of these extensions could help sharpening the quantitative outcomes, it would also increase the computational load of the quantitative analysis, making it impractical for cross-country comparisons. Therefore, the specified model environment reflect our attempt to strike a balance between the precision of the quantitative analysis and its computational feasibility for multi-country evaluations.

2.2 Pension Limit, Pension Space and Exhaustion Probability

The economic environment is not sufficiently specified to uniquely identify fiscal policy. This is because in any period t the government has access to a number of policy instruments (τ_t^c, τ_t^w, τ_t^p, τ_t^k, θ_t, G_t, Tr_t, B_{t+1}) that is larger than the number of constraints it faces, namely the general and social security budget constraints in (10) and (15), respectively. It is this indeterminacy that we exploit to measure the pension limit (PL), which is defined as the maximum amount of aggregate pension expenditure the government can finance by increasing the revenue raised from the labour income taxation through the social security tax rate τ_t^p. To see this, consider first the following definition of a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *Given an initial aggregate endowment of assets* K_0+B_0*, the demographic structure of the economy* $\left(n_t, (\mu_t^s)_{s=1}^T, (\phi_t^s)_{s=1}^T\right)_{t=0}^{\infty}$ *and a government fiscal policy* $\left(\tau_t^c, \tau_t^w, \tau_t^k, \theta_t, G_t, B_{t+1}\right)_{t=0}^{\infty}$*, a competitive equilibrium is a sequence of relative prices* $(w_t, r_t)_{t=0}^{\infty}$ *and individual allocations* $\left(\left(c_t^s, l_t^s, a_{t+1}^s\right)_{s=1}^T\right)_{t=0}^{\infty}$*, such that for* $t \geq 0$*:*

1. *The sequence of individual allocations satisfies* (3) - (7)*,*
2. *The sequence of relative prices satisfies* (8) *and* (9)*,*
3. *The sequence of fiscal policy instruments satisfies* (10) - (15)*,*

4. Aggregate equilibrium conditions (16) - (19) *hold.*

Definition 1 highlights a standard approach to compute the competitive equilibrium of a multi-period OLG model. Given the indeterminacy noted above, the aggregate transfers Tr_t and the tax rate τ_t^p are excluded from the set of fiscal policy instruments taken as given, as these can be computed endogenously from (10) and (15) in the third step of Definition 1.[3] PL can be computed as a particular type of competitive equilibrium. This arises because there is a positive correspondence between the tax rate τ_t^p and the aggregate level of pension expenditure Pen_t in equation (15). To see this, consider a government implementing a new tax policy that delivers higher revenue through an increase of the tax rate τ_t^p. For this new policy to be supported as a competitive equilibrium, Pen_t needs to increase in order that the social security budget in equation (15) still holds. Suppose that there is an upper bound on the revenue that the government can generate through this policy and denote the tax rate that delivers this maximum revenue as $\overline{\tau}_t^p$. The social security budget in equation (15) then implies the existence of an upper bound on aggregate pension expenditure which is denoted as $\overline{Pen}_t$. We call this upper bound the pension limit. Thus $PL_t \equiv \overline{Pen}_t$. PL emerges naturally in an OLG model with distortionary income taxation, because the aggregate supply of labour shrinks as the taxation of income from labour increases, due to the so-called dynamic Laffer effect. In particular, when $\tau_t^p < \overline{\tau}_t^p$, the increase in tax revenue generated from the higher τ_t^p more than compensates the revenue loss caused by the reduction in the supply of labour. As a result, tax revenue increases, leading to an expansion of the social security budget through (15). As τ_t^p reaches $\overline{\tau}_t^p$, PL is attained. At that point, further increase of the social security tax rate would reduce the social security budget.

[3]The equilibrium could also include transfers among the set of instruments taken as given, but leave out either τ_t^c, τ_t^w, τ_t^k, G_t or B_{t+1}. Similarly, the tax rate τ_t^p could replace θ_t. Quantitative analyses of fiscal policy based on dynamic general equilibrium models typically treat transfers as residuals from the government budget constraint to ensure more accurate calibration of the remaining instruments, namely the tax rates on income and consumption, government consumption and debt. The choice of leaving out τ_t^p is often made because replacement rates of pensions can be more accurately calibrated from the data. The initial level of debt B_0 is calibrated to match government indebtedness observed from the data, typically some historical average presumed to measure the steady state value of debt.

Definition 2 (Pension Limit, PL) *Given an initial aggregate endowment of assets $K_0 + B_0$, the demographic structure of the economy $\left(n_t, (\mu_t^s)_{s=1}^T, (\phi_t^s)_{s=1}^T\right)_{t=0}^{\infty}$ and a government fiscal policy $\left(\tau_t^c, \tau_t^k, G_t, Tr_t, B_{t+1}\right)_{t=0}^{\infty}$, the pension limit PL is a competitive equilibrium given by a sequence of relative prices $(w_t, r_t)_{t=0}^{\infty}$ and individual allocations $\left(\left(c_t^s, l_t^s, a_{t+1}^s\right)_{s=1}^T\right)_{t=0}^{\infty}$, such that for $t \geq 0$:*

1. *The sequence of individual allocations satisfies (3) - (7),*
2. *The sequence of relative prices satisfies (8) and (9),*
3. *The sequence of tax rates $(\tau_t^p, \tau_t^w)_{t=0}^{\infty}$ maximizes the revenue side of (15) subject to (10), and (13) - (15) hold,*
4. *The sequence of fiscal policy instruments satisfies (10) - (12),*
5. *Aggregate equilibrium conditions (16) - (19) hold.*

Two features of Definition 2 are worth highlighting. First, PL is computed as a competitive equilibrium in which the social security budget is entirely endogenous. This is because the revenue side of the social security budget is determined from the solution of the maximization problem in step 3, which in turn pins down the expenditure side through equation (15). For this reason, neither τ_t^p nor θ_t are included in the set of fiscal policy instruments that are taken as given in Definition 2. Second, the income tax rate τ_t^w is also excluded from the set of fiscal policy instruments available to the government. One of the remaining policy instruments also needs to adjust in order to satisfy the general government budget constraint in equation (10) because the aggregate labour supply changes as τ_t^p varies to reach $\overline{\tau}_t^p$. Of course, any one instrument from τ_t^c, τ_t^k, τ_t^w, G_t, Tr_t and B_{t+1} can be adjusted for this purpose. The advantage of using τ_t^w, and computing PL according to Definition 2, is that it allows a direct comparison between aggregate pension expenditure and its upper bound when all other sources of government expenditure and revenue, other than the labour income taxation, are taken as given. To ensure this solution, the general government budget is taken as a binding constraint in the maximization problem of step 3. Thus PL is the maximum size of the social security budget the government can achieve by increasing τ_t^p, without altering the size of the general government budget. This is different from a canonical Laffer effect where the expenditure side of the general government budget expands as a result of increase in tax revenue. Given PL, we determine the pension space as follows.

Definition 3 (Pension Space, PS) *Given the aggregate pension expenditure Pen_t and the pension limit $PL_t \equiv \overline{Pen}_t$ determined according to Definitions 1 and 2, respectively. The pension space PS_t is measured in any period t as $PS_t = 100(\overline{Pen}_t - Pen_t)/\overline{Pen}_t$.*

PS is the difference between PL and the equilibrium level of aggregate pension expenditure, expressed as a percentage of PL. Thus, PS indicates how close an economy is to reach PL or, equivalently, how much scope there is to finance public pensions by varying the taxation of income from labour, while keeping all other sources of government expenditure and revenue unchanged. PS is conditional upon the demographic structure of the economy postulated in Definitions 1 and 2. For this reason it is not invariant to demographic change. As a result, PS can be computed over a range of different demographic projections reflecting the uncertainty about the future composition of the population. The resulting range of PS can then be used to fit the kernel distribution of PS at a future time horizon. The probability of PS being equal to zero, which we term pension space exhaustion probability ($PSEP$) can then be extrapolated from the cumulative density function implied by this estimated distribution.

Definition 4 (Pension Space Exhaustion Probability, $PSEP$) *Given a distribution of the demographic triplet $H\left(n_{t+h}, \left(\mu_{t+h}^s\right)_{s=1}^T, \left(\phi_{t+h}^s\right)_{s=1}^T\right)$, $t \geq 0$, $h \geq 1$. Let $G(PS_{t+h})$ be the corresponding distribution of the pension space. The pension space exhaustion probability is defined as $PSEP \equiv Pr(PS_{t+h} \leq 0) = Pr(\overline{Pen}_{t+h} - Pen_{t+h} \leq 0)$.*

$PSEP$ defined in 4 can be easily computed as follows. Consider a finite range of projected values of the population growth rate n_{t+h}, population shares $\left(\mu_{t+h}^s\right)_{s=1}^T$ and survival probabilities $\left(\phi_{t+h}^s\right)_{s=1}^T$ at a future date $h \geq 1$. Using Definition 3, PS_{t+h} can be evaluated for each $t+h$. The set of computed PS_{t+h} can then be used to infer the empirical distribution of the pension space at date $t+h$. The exhaustion probability can then be estimated from the implied cumulative density function. The algorithms used to compute numerically the competitive equilibrium, PL, PS and $PSEP$ are described in more detail in Appendix B.

2.3 A Two-Period Model

Further insight into how the PL equilibrium is calculated and on the determinants of PS and $PSEP$ can be obtained from a two-period version of the full model that can be solved analytically and illustrated diagrammatically.[4] Households work in the first period, retire in the second. Lifetime utility from consumption and leisure is $U = \sqrt{c^y} + \beta\sqrt{c^o} + l$. The household budget constraints when young and old are given by $c^y + b = (1-\tau^l)wl$ and $c^o = (1+r)b + \theta wl$, respectively, with b denoting government bonds. For tractability, pension income accrued during the old age is proportional to labour income during the young age, worker productivity is normalized to one; there is no physical capital and technology is linear in labour, i.e. $y = Al$. The general government budget includes consumption expenditure financed through bonds and labour income taxation. Public pensions are proportional to labour income and financed through a social security contribution tax levied on labour income. Thus tax revenue raised for the general government and the social security budgets in per-worker terms is given by $t^G = \tau^w wl$ and $t^P = \tau^p wl$, respectively, whereas the two budgets are given by $e = t^G$ and $\theta wld = t^P$, where $e = g + [(1+r)d-1]b$, is total government expenditure per worker and $d_t = N_{t-t}/N_t$ is the dependency ratio. In equilibrium, the labour supply l is determined as $l = \bar{l}(1-\tau^w-\tau^p\Phi)$, with $l = \bar{l}(1-\tau^w-\tau^p\Phi)$ denoting the labour supply in the absence of taxation ($\tau^w = \tau^p = 0$) and $\Phi = 1-[d(1+r)]^{-1}$. The Lagrangian for the problem of choosing the rate τ^p that maximizes the social security budget subject to the general government budget is formulated as:

$$\max_{\tau^p,\tau^w,\lambda} L = \tau^p w\bar{l}(1-\tau^w-\tau^p\Phi) + \lambda[\tau^w w\bar{l}(1-\tau^w-\tau^p\Phi) - e].$$

[4] We describe here the main items of the two-period model, full details are in Appendix C.

The solution requires:

$$\tau^p = \frac{1-\tau^w-\lambda\tau^w\Phi}{2\Phi},\ \tau^w = \frac{\lambda-\tau^p(1+\lambda\Phi)}{2\lambda},\ \lambda \geq 0, e \leq t^G,\ \lambda(e-t^G)=0,$$

where the first two conditions are obtained from the differentiation of the Lagrangian with respect to τ^p and τ^w, respectively, whereas the last three are conventional Kuhn-Tucker conditions. When the general government budget is not a binding constraint, $\lambda = 0$, the income tax rate is taken as given and the tax rate that maximizes the social security budget is $\widehat{\tau^p} = (1-\tau^w)/(2\Phi)$. In contrast, PL is attained by the tax rate $\overline{\tau^p}$ that maximizes the size of the social security budget while maintaining the current size of the general government budget, thus $\lambda > 0$ and $e = t^G$. The income tax rate is no longer invariant but decreases as τ^p increases to expand the social security budget. In addition, $\overline{\tau^p} < \widehat{\tau^p}$ when $\lambda > 0$ because in the constrained maximization the supply of labour falls as τ^p increases to expand the social security budget, and the government needs to adjust the income tax rate to maintain the size of the general government budget. This curtails the labour supply even further, thus reducing the social security tax rate that maximizes the pension budget compared to the unconstrained case. Given the equilibrium solutions determined above, PS is computed as:

$$PS = 100\Big(1 - \frac{\tau^p l}{\overline{\tau^p} l_{PL}}\Big),$$

where l_{PL} is the labour supply when $\tau^p = \overline{\tau^p}$. Figure 1 illustrates the determination of PS and $PSEP$. The top left panel shows that the equilibrium supply of labour declines (left axis, in blue) and the income tax rate τ^w (right axis, red) increase as τ^p is augmented while maintaining unchanged the size of the general government budget. The green dots indicate the equilibrium solution obtained from a calibration of τ^p that yields a supply of labour of 0.25. This equilibrium pair is denoted as (τ^{p*}, l^*). The red dots show the corresponding social security tax rate and minimum labour supply, the pair $(\overline{\tau^p}, l_{PL})$ when the economy reaches PL, with $(\overline{\tau^w}$ being the income tax rate at the PL equilibrium. The top right panel shows how the social security budget expands as τ^p increases from zero to $\overline{\tau^p}$. The aggregate pension

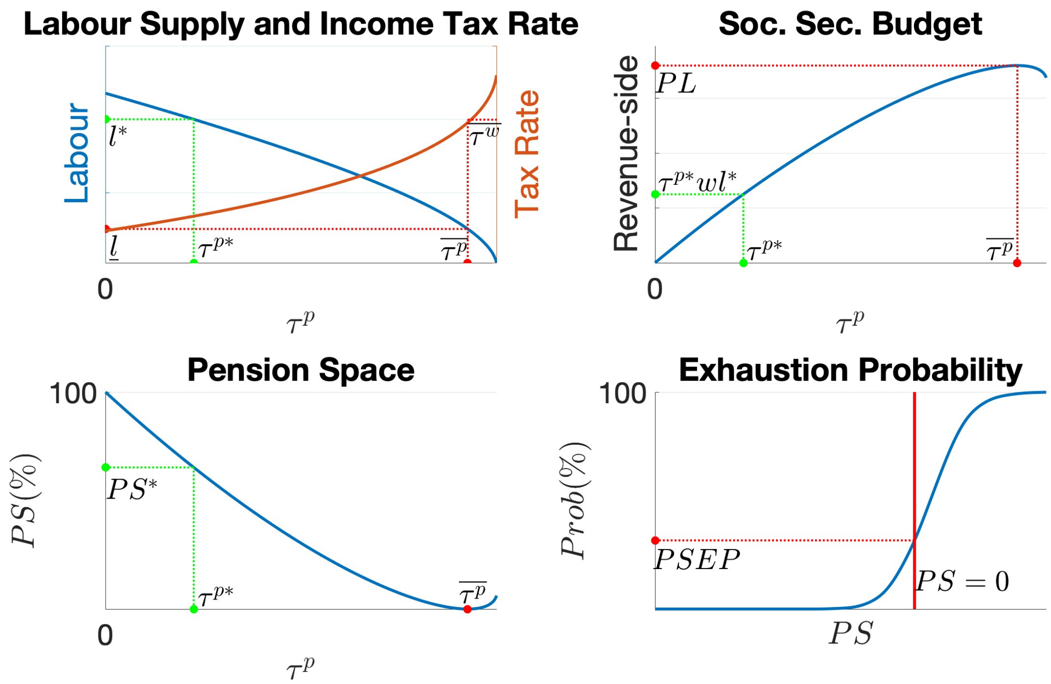

Figure 1: PS and $PSEP$ determination in two-period OLG model.

expenditure (APE) implied by the equilibrium τ^{p*} and PL are shown on the vertical axis. These denote the social security budget sizes implied by (τ^{p*}, l^*) and $(\overline{\tau^p}, l_{PL})$, respectively. The bottom left panel shows how PS changes in response of variation of the equilibrium tax rate τ^{p*}. Given the parameters describing the demographic structure of the population, preferences, technology and the general government budget, PL is uniquely identified. Thus PS shrinks as the equilibrium τ^{p*} increases. The bottom right panel of Figure 1 shows the estimated cumulative density function obtained from 1000 draws of the population growth rate taken from an i.i.d distribution with mean and standard deviation equal to 0.2. The vertical red line marks the point where $PS = 0$. The projection of this point on the vertical axis gives $PSEP$, i.e. the risk due to the demographic uncertainty that PS is exhausted given the economic environment and fiscal and pension policy arrangements.

Table 1: Parameters calibrated using external data sources

	n	T^w	α	δ	g_a	τ^l	τ^k	τ^c	g/y	b/y	θ
AUT	0.41	65	0.39	7.10	1.37	42.09	21.90	19.21	22.70	85.86	76.23
BEL	0.40	63	0.39	8.40	1.36	39.20	31.57	17.53	25.03	125.17	45.60
FIN	0.36	65	0.34	7.00	1.69	42.78	22.05	22.63	25.69	46.42	56.30
FRA	0.45	63	0.41	6.90	1.05	39.97	31.98	17.82	26.92	75.36	58.57
DEU	0.17	65	0.37	6.70	1.03	35.69	19.16	15.61	21.74	67.25	39.23
IRE	0.94	66	0.36	8.60	4.31	27.19	15.06	20.11	19.06	67.44	30.47
ITA	0.14	64	0.39	7.00	0.83	39.65	29.69	15.33	22.01	117.90	74.53
NDL	0.52	66	0.38	7.70	1.42	32.99	22.96	18.08	27.09	72.82	77.03
PRT	0.14	66	0.39	9.80	1.56	26.61	17.34	18.04	21.14	103.78	74.37
ESP	0.58	65	0.42	8.50	1.46	30.98	21.28	14.12	21.40	67.85	76.10
GBR	0.43	65	0.36	6.40	1.40	22.67	26.79	13.13	21.04	64.22	30.77
USA	1.02	66	0.35	8.30	1.53	21.75	22.78	6.15	19.44	72.75	37.93
Common parameters											
$\eta = 2.0,\ \varphi = 0.6$											

Source: See main text for more details. All data in percentage except T^W, α, η and φ.

3 Calibration

3.1 Benchmark

The benchmark calibration is designed to approximate a steady state for each country based, whenever possible, on data averages from 1980 to 2020.

Table 1 presents parameter values calibrated for each country using external data sources. A period t in the model corresponds to one year. Newborns are assumed to have a real-life age of 20, corresponding to $s = 1$, and live up to a real-life age of 94, thus T=75. Retirement ages T^W are calculated as the average of the effective age of retirement of men for the years 2014, 2018 and 2020 obtained from OECD (2021). Population growth rates n are calculated as the averages of the annual population growth rates, from 1980 to 2020, obtained from United Nations (2022). Survival probabilities ϕ_t^s are computed from annual data on life expectancy for both sexes combined (from age 20 to age 95) which are also obtained

Table 2: Parameters calibrated from model equilibrium solution.

	AUT	BEL	FIN	FRA	DEU	IRE	ITA	NDL	PRT	ESP	GBR	USA
κ	9.19	9.55	8.39	10.09	8.50	10.61	9.70	11.74	11.35	11.68	9.86	10.86
β	0.99	0.98	0.97	0.99	0.98	0.91	1.00	0.99	0.98	0.99	0.96	0.96
$\frac{tr}{y}$	1.51	1.35	2.40	0.22	2.40	2.97	-0.31	-2.55	-2.42	-2.26	0.72	-0.69
τ^p	22.39	15.53	16.65	20.84	12.31	6.50	27.32	20.52	21.85	22.62	8.79	8.19

Source: Author's calculations. See main text for more details. All data in percentage except κ and β.

from United Nations (2022). The calibration employs for each country the average survival probabilities from 1980 to 2020. Given the large amount of data involved, these are reported in Appendix D together with additional details on their computation.

The parameters α and δ vary across countries, being taken from Trabandt and Uhlig (2011). The growth rate of productivity g_A is set to match the average growth rate of real GDP per capita in each country during 1980-2020. This is calculated using data obtained from OECD (2022), upon scaling the annual nominal value of the gross domestic product at market prices by the corresponding deflator, and then dividing by the total population.[5]

We calculate the tax rates on income from labour, τ^l, capital, τ^k, and consumption, τ^c, using the revision of the method of Mendoza et al. (1994)'s proposed by Carey and Rabesona (2003). The results in Table 1 refer to averages from 1995 to 2020, because most of the data required to calculate the tax rates are unavailable prior to 1995. Details on these computations and the annual tax rates calculated in each country are reported in Appendix D. The government expenditures to GDP ratios g/y are calculated as the average from 1980 and 2020 of the sum of nominal general government final consumption expenditure and fixed capital formation as a proportion of the nominal value of gross domestic product at market prices. The debt-to-GDP ratios b/y are measured using the average of general government gross financial liabilities as a percentage of GDP from 1980 to 2020.[6] Pension replacement

[5]Due to limited availability, GDP per capita averages start from 1992, 1990, 1981 and 1982 for Germany, Ireland, Portugal and Spain, respectively.

[6]Data for g/y and b/y is taken from OECD (2022). Due to limited data availability, the b/y time series for Austria, Germany, Ireland and Portugal start from 1995, 1991, 1997 and 1995, respectively. The g/y time series for Germany and Ireland start from 1991 and 1990, respectively.

rates θ are computed using the average for the years 2014, 2018 and 2020 of the the gross pension replacement rates of men with mean income obtained from OECD (2021).

Empirical estimates often find the intertemporal elasticity of substitution to be close to 0.5, see for example Trabandt and Uhlig (2011). Thus we calibrate $\eta = 2$ in each country. Calibration of the Frisch labour elasticity and of individual life-cycle productivity is less clear cut. For example, Prescott (2006) calibrates the Frisch labour supply elasticity to be 3, Trabandt and Uhlig (2011) use a value of 1. The survey of Keane (2011) suggests a lower value of about 0.2. More recently, Kindermann and Krueger (2022) argue in favour of a value of 0.6, this being in the medium range of available empirical estimates on the Frisch elasticity for both men and women. We follow this latest measure and set $\varphi = 0.6$ in each country. The empirical evidence on life-cycle individual productivity for the United States indicates that this is hump-shaped, Hansen (1993). Given the lack of consistent data suitable for a multi-country comparison, we use a flat age-productivity profile, $\bar{y}^s \equiv 1.0$, $s = 1, \ldots, T^W$, for all countries. The impact of alternative calibrations of both the Frish elasticity and the age-productivity profile is evaluated in Section 4.

Table 2 reports the values of the four remaining parameters that are calibrated endogenously to ensure a stationary equilibrium solution of the model in each country. These are the utility parameter κ, which is set to yield average working hours $\bar{l} = 0.25$, the discount factor β, set to yield a real interest rate $r = 4.0\%$, government transfers as a proportion to GDP, TR/y, set to ensure that the general government budget constraint in equation (10) is satisfied, and the tax rate τ_p, set to ensure that the social security budget in equation (15) is met. The calibrated values of κ range from about 8.4 for Finland to about 11.7 for the Netherlands and Spain. These are almost twice as large as the corresponding values used by Trabandt and Uhlig (2011). This is because the calibrated parameter κ is derived from the intra-temporal equilibrium condition relating consumption and labour in equations (5) and (6). When $\bar{l} = 0.25$, a reduction in the Frisch labour elasticity requires a higher value of κ for the intra-temporal condition to hold. As noted above, Trabandt and Uhlig (2011)'s calibration uses a Frisch labour elasticity of one, while we use 0.6. Apart from Italy, the calibrated values of the discount factor β are below one for all countries, ranging from 0.96 for the United States to 0.99 for Austria. These are within the normal range of discount factors at the annual frequency. The discount factor calibrated for Italy is 1.00. Contrary to an infinitely-lived agents model, the stability of an OLG model does not require β to be less than one. This is because discounting is adjusted by the mortality risk, measured from the survival probability, which is less than one. The calibrated values of government transfers as a proportion to GDP reported in Table 2 reflect cross-country differences in the average levels of taxation and expenditure. In general, for any given level of taxation, the resulting tr/y is lower the larger is the level of government expenditure on consumption and interest payment on public debt. The calibrated values of the tax rates τ^p reflect cross-country differences in the levels of pension expenditure.

3.2 Counterfactual

To compute PS up to 2050 and estimate the evolution of $PSEP$ over this period, we employ demographic projections on population growth and survival probabilities from United Nations (2022).[7] Figure 2 displays the evolution of the population growth rates n in each country together with that of the world rate from 2020 to 2050. To facilitate visual comparison, the scale of the vertical axis is the same for all countries. Data after 2021 refer to median projections with a 95% prediction interval. Three patterns are clearly visible. First, most of the selected countries are among those with the oldest populations in the world and they are projected to age very rapidly over the next 30 years. Second, the projected speed of ageing is very heterogeneous among countries. Germany, Italy, Portugal and Spain are expected to have a negative population growth rate by 2050. All other countries are expected to experience a decline in population growth of between roughly 20 and 87% over the same period of time. Third, there is significant uncertainty regarding the population dynamics

[7] Given the large volume of data involved, we do not include either in the main paper or in the Appendix the country estimates for the survival probabilities. These data are available on request from the authors.

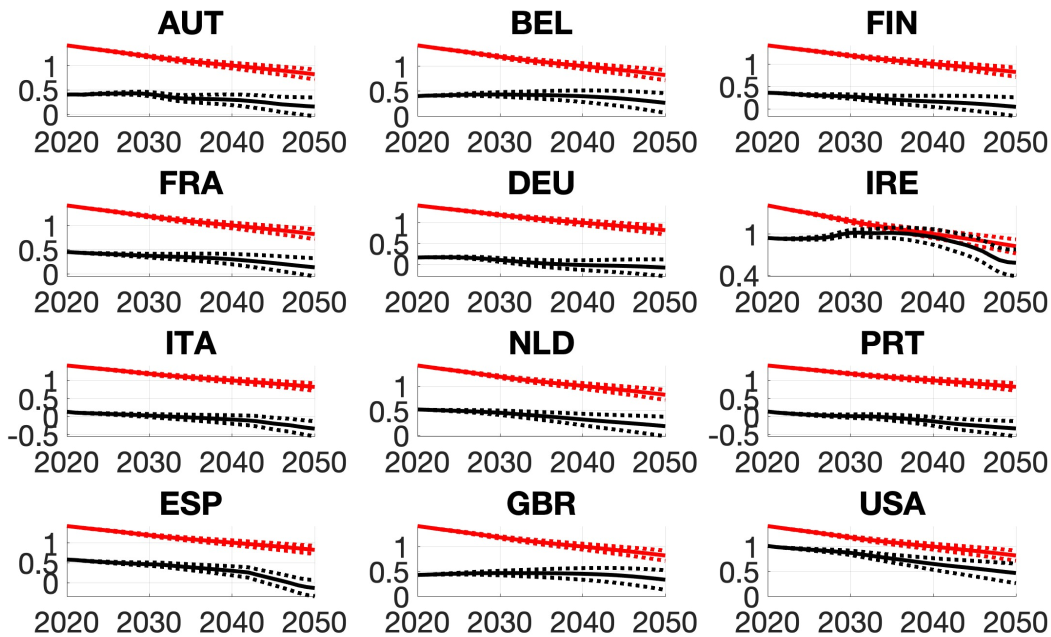

Figure 2: Percentage population growth rate of a country (black) against the world population growth rate (red), 40 years rolling averages from 2020 to 2050. Data after 2021 include median projections with 95% prediction intervals. Source: United Nations (2022).

over the next 30 years. For example, the number of countries expected to experience negative population growth by 2050 varies from 2 to 8 when considering either the upper or the lower bound of the 95% prediction interval, respectively.

The demographic projections shown in Figure 2 are used to quantify how PS evolves in each country under four different policy scenarios during 2020-2050. The first is a scenario of no policy change. This assumes that (i) the growth rate of the population and the survival probabilities change each year according to the median projections from United Nations (2022) and (ii) each country maintains over time pension payments to households, pen_t, and the retirement age, T^W, at their 2020 levels, while financing the higher cost of pensions by raising the tax rate τ^p. The corresponding evolution of $PSEP$ is calculated in each country as follows. First we compute the PS over the period 2020 to 2050 under the benchmark calibration, but using the population growth rates and survival probabilities in each year from the 5th, 20th, 80th and 95th percentiles of the projections from United Nations (2022). These are combined with the time series of PS based on the median demographic projections to estimate a kernel cumulative density function of the PS in each year.[8] From this distributions we then extrapolate the probability that PS will be equal to zero, i.e. the $PSEP$.

The other three scenarios consider counterfactual policy interventions that capture the essence of the types of reforms that are often advocated to improve, if not restore, the sustainability of public pensions. The first, $P1$, considers the effect of partially financing pension expenditure by increasing indirect taxes. We study a 5% increase in τ^c. The other two policies are designed to decrease the burden of aggregate pension expenditure, by either reducing the replacement rate of pension θ by 10%, $P2$, or by increasing the age of retirement T^W by two years, $P3$. At this stage, the extent of change in the three policy parameters τ^c, θ and T^W is clearly ad-hoc, since the sole objective is to gauge evidence about the PS and the $PSEP$ responses under each counterfactual. Policies $P1$ and $P3$ are implemented by replacing for each country the value of τ^c and T^W reported in Table 1 with their corresponding counterfactual, and then leaving these new values unchanged while PS is computed over the 2020 to 2050 horizon. $P2$ is implemented by reducing for each country the value of θ reported in Table 1 by 10%, computing the new value pension entitlement according to equation (13), and then keeping this constant during 2020-2050.

This section is concluded with an observation that we deem important. The data used for the calibration demonstrate substantial differences in macroeconomic characteristics, public pension arrangements, demographic structures, ageing projections and uncertainty across

[8] These five percentiles are the only statistics made available by the United Nations (2022)'s database about the empirical distribution of the population growth rate projections. Arguably, more accurate estimates of $PSEP$ could be obtained using additional data points on these distributions. This would however add to the already large computational time of the algorithm for $PSEP$, which ranges from four to six hours in each instance. We estimate a 3.6 percent probability that the approximation error (due to using the five above mentioned percentiles instead of the true distribution) is larger than 0.5 percent. For more details see Appendix B.

Table 3: Pension space PS_t and its main determinants, benchmark calibration, $t = 2020$

	PS_t	Benchmark $\frac{Pen_t}{Y_t}$	τ_t^p	τ_t^l	Pension Limit, PL $\overline{Pen_t}/Y_t$	$\overline{\tau}_t^p$	$\overline{\tau}_t^l$	Difference $\frac{\overline{Pen_t}-Pen_t}{Y_t}$	$\overline{\tau}_t^p - \tau_t^p$	$\overline{\tau}_t^l - \tau_t^l$
USA	56.77	5.32	8.19	21.75	12.31	28.99	66.18	6.99	20.80	44.44
IRE	49.31	4.16	6.50	27.19	8.21	18.39	62.30	4.05	11.89	35.10
GBR	46.22	5.62	8.79	22.67	10.46	23.37	58.64	4.83	14.58	35.96
PRT	24.62	13.33	21.85	26.61	17.68	39.15	60.23	4.35	17.30	33.62
DEU	22.42	7.76	12.31	35.69	10.00	20.28	59.62	2.24	7.96	23.93
NDL	20.40	12.72	20.52	32.99	15.98	34.68	63.04	3.26	14.16	30.05
ESP	19.43	13.12	22.62	30.98	16.28	37.31	57.74	3.16	14.70	26.76
FIN	13.48	10.99	16.65	42.78	12.70	23.07	60.74	1.71	6.42	17.96
BEL	8.08	9.47	15.53	39.20	10.30	19.45	52.48	0.83	3.93	13.27
AUT	5.95	13.66	22.39	42.09	14.52	26.99	53.78	0.86	4.61	11.69
FRA	5.24	12.29	20.84	39.97	12.97	25.33	52.45	0.68	4.49	12.48
ITA	4.74	16.67	27.32	39.65	17.50	32.96	52.53	0.83	5.64	12.87

Source: Author's calculations. All data in percentage. PL and PS are based on Definition 2 and 3, respectively. Aggregate pension expenditure Pen_t is calculated from (14), Y_t is nominal GDP. The values for τ_t^l and τ_l^p are from Table 1 and 2, respectively.

countries. This heterogeneity is of great significance since the macroeconomic and welfare effects of pension policy reforms are contingent upon the particular economic conditions in which they are implemented. The presence of a diverse range of countries is crucial for capturing the potential wide-ranging effects of policy reforms. At the same time, the observed heterogeneity also renders the evaluation of public pension sustainability and reforms a challenging task given the various competing factors that may determine the final outcomes. The advantage of PS and $PSEP$ is that they condense this diverse range of information into two numbers that have simple interpretations, as we can see in the next section.

4 Quantitative Results

4.1 Differences in PS Magnitude among Nations

Table 3 presents PS and its determinants computed under the benchmark calibration for 2020. The first column ranks countries according to the size of their PS, given in column

2. The next six columns show how aggregate pension expenditure as a proportion to GDP, Pen_t/Y_t, the overall labour tax rate, τ^l, and the social security tax rate, τ^p, change when switching from the benchmark to the PL equilibrium, i.e to $\overline{Pen}_t/Y_t$, $\overline{\tau}^l_t$ and $\overline{\tau}^p_t$, respectively. The final three columns report the differences between them. The values of PS_t, Pen_t/Y_t and $\overline{Pen}_t/Y_t$ have a clear interpretation. For example, in the United States, aggregate pension expenditure is 5.32% of GDP, while the corresponding PL is 12.31%. PS is the proportion of the PL unspent, i.e. 56.77%.

Table 3 shows the first headline result of the quantitative analysis, namely that there is considerable variation in the magnitude of PS among different nations, with values ranging from approximately 5 to 57% in 2020. Three groups can be clearly identified depending on the size of PS in 2020 being either large (above 40%, United States, United Kingdom and Ireland), medium (between 10 and 25%, Finland, Germany, Netherlands, Portugal and Spain) or small (below 10%, Belgium, Austria, France and Italy). The final three columns show that PS differentials among countries are explained by two main factors: labour income taxation and the generosity of the pension system. The three countries with the highest PS values have the least generous replacement rates and the lowest social security tax rates; the four countries with the lowest PS values have among the most generous replacements rates and the highest social security tax rates. In general, PS is larger for countries with lower taxation of labour income (τ^l) as they have more room to finance higher pension payments, and with less generous pension system (Pen/Y) which have less need to finance higher pension payments.[9] The differences in the social security tax rates (τ^p) reflect a combination of these two factors. Further, the country ranking in Table 3 indicates that the income tax rate $\tau^w_t = \tau^l_t - \tau^p_t$, measured by the difference in the last two columns, rises as τ^p_t approaches PL. This has a clear explanation. If pension payments increase, aggregate savings and hence capital fall. This reduces the supply of aggregate labour, in turn leading to lower aggregate consumption, income and tax revenues. Thus the tax rate τ^w needs to increase to ensure

[9] Pen_t is the sum of individual pension payments made by the government according to equation (13), These, in turn, depend on the generosity of the pension system, as measured by the replacement rate θ, and the demographic structure of the population, as measured by $N(s)$.

that the size of the general government budget in equation (10) is unchanged while the size of the social security budget is maximized at the PL.

Importantly, we find the country ranking in Table 3 to be robust to (i) the employment of alternative calibrations of the Frisch labour elasticity φ and the age-productivity profile, $\bar{y}^s$, $s = 1, \ldots, T^W$ and (ii) change in the parameter values for economic growth and government indebtedness.[10] It was noted above that the calibration of the Frisch labour elasticity and of the age-productivity profile is not clear cut. In the benchmark calibration, we set $\varphi = 0.6$ and $\bar{y}^s = 1$, $s = 1, \ldots, T^W$. To assess the significance of alternative parameterisations, we re-calibrated the model for all countries using either a higher Frisch elasticity of labour, $\varphi = 1$ as in Trabandt and Uhlig (2011), or the hump-shaped age productivity profiles estimated by Hansen (1993). Both a higher Frisch elasticity and hump-shaped productivity profile lead to a reduction of PS. The effects however are numerically small and the ranking reported in Table 3 is largely unchanged. In both cases the model is re-calibrated, thus the equilibrium values of Pen_t, τ_t^l and τ_t^p are unaffected.[11] PL declines because the higher is the Frisch elasticity, the more the labour supply decreases in response to a given tax increase. On average across countries PS falls by about 4.7%, despite the Frisch labour elasticity increasing by about two thirds, i.e. from 0.6 to 1. The impact of hump-shaped productivity is even more negligible, as PS falls by only about 1.8% on average across countries. This is because, with a hump-shaped productivity profile, some cohorts of households, namely those aged between 20 and 39, contribute less to tax revenues compared to others, those aged between 40 and 64, that have productivity higher than average. These two effects partially offset each other, leaving PS almost unaltered. To assess the sensitivity of the results to change of economic growth and government indebtedness we recalculated the model equilibrium for each country upon increasing either g_A by 1 percentage point or b/y by 20 percentage points. Neither

[10] These robustness results are fully reported in Appendix E.

[11] This is because the parameter κ adjusts to variation of the Frisch labour elasticity so that average hours amount to 0.25. Further, the discount factor β is calibrated so that in equilibrium the rate of interest is equal to 4%. Thus aggregate labour, capital and the real wage in the benchmark equilibrium are invariant to changes in the Frisch labour elasticity or age-productivity.

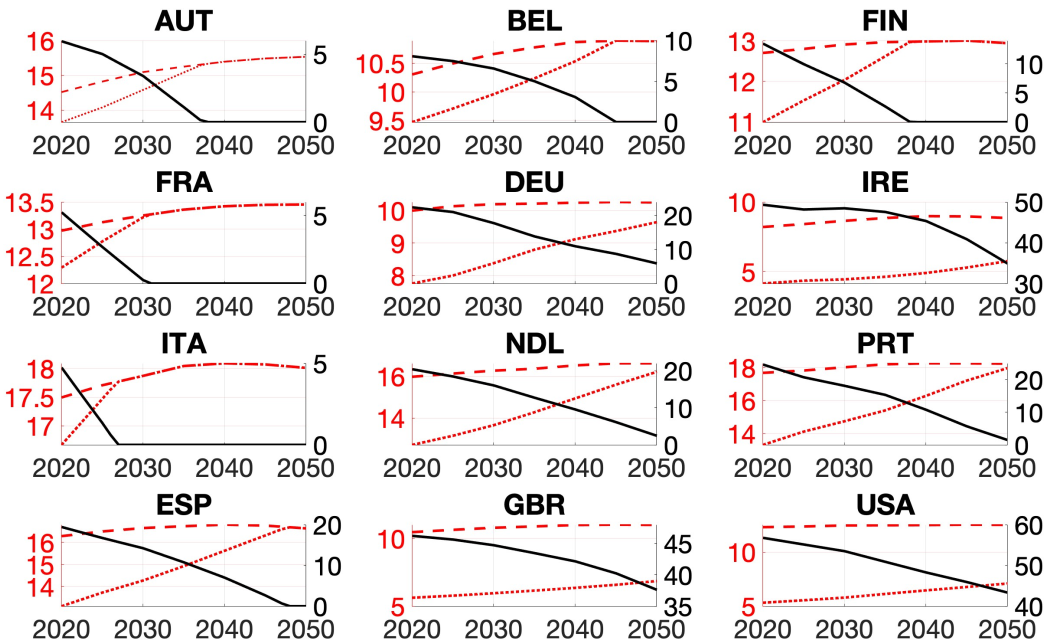

Figure 3: Evolution of PS (black lines), aggregate pension expenditure, Pen/Y (dotted-red lines) and PL (dashed-red lines) during 2020-2050.

of these two changes alters significantly the country ranking in Table 3. Higher economic growth increases PS in all countries and compresses country differentials particularly at the bottom. Higher debt reduces the PS more or less evenly in all countries.

4.2 Limitations of Income Taxation for Public Pensions

Over time the cost of public pensions is expected to rise due to population ageing. Consequently, PS is expected to shrink and the likelihood of exhaustion may increase depending on projected demographic trends. Figure 3 shows how PS (black lines) varies in each country under a scenario of no policy change during 2020-2050. The plots also include the two main determinants of PS, aggregate pension expenditure and PL. Figure 4 plots $PSEP$ estimated for each country during 2020-2050. For reference, each plot includes PS computed over the same period of time. Figures 3 and 4 show the second headline result of the quantitative analysis, namely that for most nations there is little scope to finance public pensions out of further labour income taxation over the next 30 years.

According to the results in Figure 3, in the absence of policy change, PS is projected to rapidly decline due to population ageing in all countries over the next 30 years. The speed of reduction is particularly fast for the countries with smallest PS in 2020 - Austria, Belgium, France and Italy - as well as for Spain and Finland, as these countries are projected to entirely erode their PS before 2050. All other countries, while able to maintain a positive PS by 2050, would also rapidly deplete their PS over the next 30 years, with reductions ranging between roughly one-third (Ireland) to three-quarters (Netherlands and Portugal). The reduction in PS over time is mostly driven by changes in aggregate pension expenditure (dotted-red lines) rather than in PL (dashed-red lines). Over time ageing increases the proportion of retirees in the population and, therefore, aggregate pension expenditure. The slope of the Pen/Y lines for each country reflects the pace at which the population is projected to age. In contrast, the PL lines for most countries are flatter. This is due mainly to two factors. First, at the PL and over the period 2020-2050, the social security tax rate is almost constant at the maximum level $\overline{\tau}_p$. Second, over time the aggregate supply of labour L declines and its marginal value, i.e. the wage rate w, increases at the PL. Depending on which of these two dominates, the tax base for public pensions, i.e. aggregate labour income wL, might marginally increase or decrease. It is the combination of these two factors that determines the relatively flat PL dynamics observable from Figure 3.

According to Figure 4, in nine of the twelve countries $PSEP$ is estimated to be positive and increasing by 2050. Austria, France and Italy are most likely to face early exhaustion, as their $PSEP$ is estimated to be close to 100% well before 2050. Figure 4 also shows the usefulness of $PSEP$ as a pension-risk warning indicator as, for all countries, $PSEP$ becomes positive and increasing well before PS reaches zero. For example, in the six countries projected to fully exhaust their PS before 2050 $PSEP$ becomes positive and increasing about 5 and a

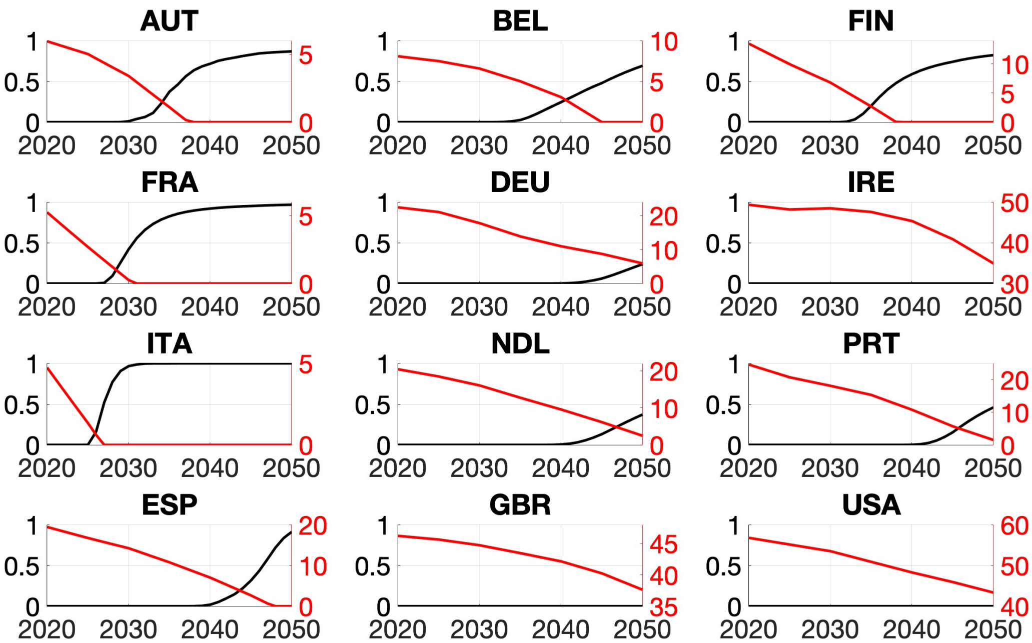

Figure 4: Evolution of $PSEP$ (black lines) and of PS (red lines) during 2020-2050.

half years before PS reaches zero.

4.3 Pension Reforms Cannot be a One-Size-Fits-All Solution

Next we consider how pension reforms can alter the size of PS, its evolution over time and the risk of it being exhausted at some point in the future. Figure 5 shows how PS changes under the three stylised policy reforms $P1$, $P2$ and $P3$ from 2020 to 2050. For comparison, each plot includes the dynamics of PS under the scenario of no policy change, previously shown in Figures 3. Figure 6 presents the evolution of $PSEP$ under the three policy reforms, including for comparison $PSEP$ under no policy change, shown already in Figure 4.

Figures 5 and 6 illustrate the third headline result of the quantitative analysis, that pension policy reform cannot be a one-size-fits-all solution, as the same type of reform has a different

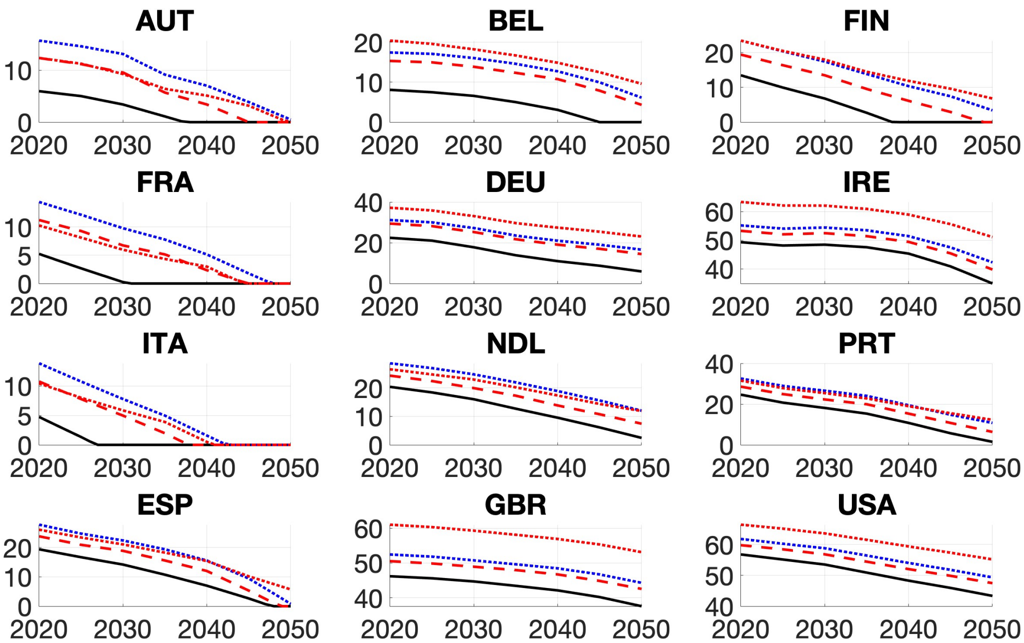

Figure 5: PS under partial financing with 5% increase of consumption tax ($P1$, dashed-red), 10% reduction of the replacement rate ($P2$, dotted-red) and two-year increase of retirement age ($P3$, dotted-blue), based on the median population growth projections for 2020 to 2050 from United Nations (2022). Black lines denote the PS under no policy change.

effect on PS and $PSEP$ for each country. In four countries (Austria, France, Italy and the Netherlands) the largest PS is attained when increasing the age of retirement by two years as under $P3$. In two countries, (Portugal and Spain), the largest PS is also attained under $P3$, but this is quite close to that achieved by reducing the replacement rate of pension by 10%, as under $P2$. For the remaining six countries, the largest PS is attained by raising consumption taxation by 5%, as under $P1$.

Two features of Figure 5 stand out. First, each of the three stylized reforms increases PS relative to the no policy change scenario because it reduces distortionary income taxation and boosts the labour supply, thereby reducing aggregate pension expenditure. $P1$ and $P2$ increase the supply of labour in every period at the intensive margin. $P3$ affects the labour supply mostly at the extensive margin, because households need to work for two additional years. In all of these cases total output in the economy increases in each year. Thus, aggregate pension expenditure as a proportion to GDP falls, which is why PS increases under each reform relative to the benchmark. The second noticeable feature in Figure 5, further indicating that pension reforms cannot be one-size-fit-all, is that there are several instances when the upward shift of PS under $P2$ accelerates at some point during 2020-2050. This is evident for Austria, Finland, France, Italy and Spain, and, in part, also for the Netherlands and Portugal. The discontinuities in the PS dynamics under $P2$ occur in years when there is a large drop in the projected median population growth. Policy reforms that reduce the replacement rate therefore appear to make the pension system more resilient to ageing, compared to those that extend the age of retirement or require additional financing. This is due to ageing having two main effects in general equilibrium. First, it impacts on aggregate saving, as workers save more than retirees. However, private saving falls more when pension entitlements are unchanged, (i.e. higher consumption taxation or age of retirement under $P1$ and $P3$), than when pension replacement rate is lower (under $P2$). Second, the taxation of labour income necessary to finance pension expenditure through τ^p has to increase more under $P1$ and $P3$ than under $P2$, because the first two policies demand higher pension payments to retirees than the latter. The combination of these two general equilibrium effects leads to a larger reduction of aggregate saving and capital under $P1$ and $P3$ compared to $P2$. In addition, the wage rate w falls more under $P1$ and $P3$ than under $P2$. This provides a secondary effect on savings, as it amplifies the impact of a higher number of retirees in reducing the capital stock under all of the three reforms. As a result, ageing reduces the aggregate labour supply L more in policy regimes with higher pension entitlements such as $P1$ and $P3$, compared to $P2$. The combined effects of these different responses of τ^p, w and L to ageing is that aggregate pension expenditure, which depends on these three variables through equation (14), increases less over time due to ageing under a regime of lower pension payments to retirees. These differences would become more marked the larger is the reduction of the replacement rate due to the reforms.

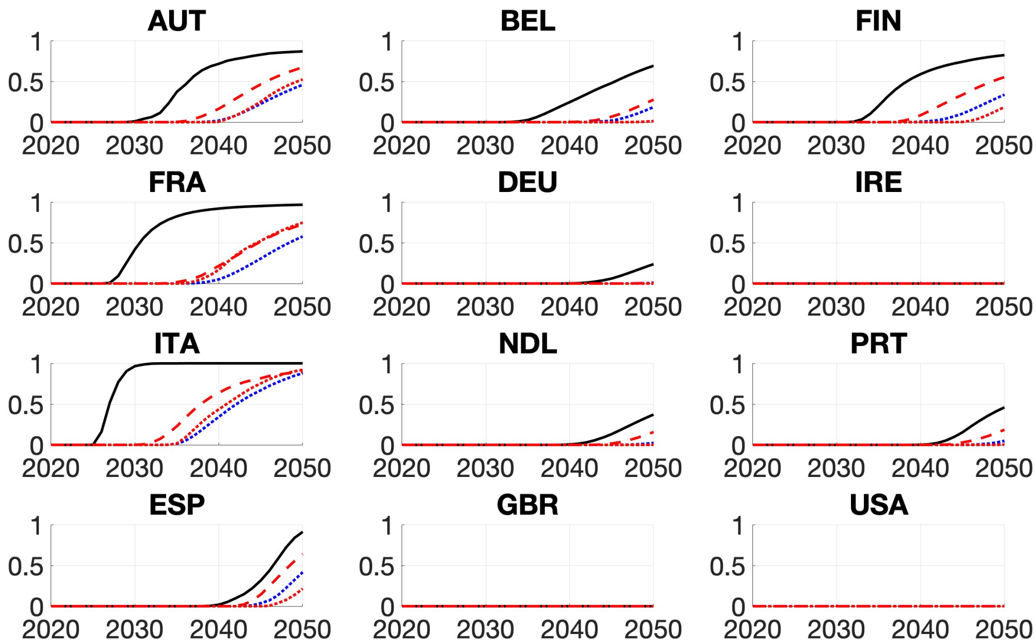

Figure 6: $PSEP$ under benchmark calibration (solid-black), partial financing with 5% increase of consumption tax ($P1$, dashed-red), 10% reduction of the replacement rate ($P2$, dotted-red), two-year increase of retirement age ($P3$, dotted-blue), based on the empirical distribution of the population growth projections for 2020 to 2050 from United Nations (2022).

As shown in Figure 6, the effect of the reforms on $PSEP$ is to cause it to decrease in each country. $PSEP$ diverges less across $P1$, $P2$ and $P3$ compared to what is observed for PS in Figure 5. This is mainly because the evolution of PS during 2020-2050 depends only on the median projections of the population growth rate over that period. In contrast, the estimates of $PSEP$ depend on the entire empirical distribution of the projected population growth rate from 2020 to 2050. Consequently, the different effects of population ageing across policies tend to be ironed out for $PSEP$ compared with PS. Arguably, this makes $PSEP$ a more reliable metric of the effects of policy intervention compared to PS, because it accounts for the impact of demographic uncertainty and depends on the entire empirical distribution of the population, rather than only one realization (the median in the case of PS).

Table 4: Welfare effects of three risk-equivalent pension reforms that achieve the same $PSEP$ in 2050

	$P3$				$P1^*$		$P2^*$		TWD		
Country	τ^c	θ	$PSEP$	$\widetilde{U}$	τ^c	CEC	θ	CEC	(1^*)	(2^*)	$(2')$
AUT	19.21	76.23	45.86	-549.0	27.04	1.49	62.36	0.51	-0.10	-5.87	-8.88
BEL	17.53	45.60	18.49	-379.7	23.86	0.01	37.40	1.57	-0.90	0.81	-1.80
FIN	22.63	56.30	33.68	-378.6	31.11	0.37	46.15	1.95	-0.75	0.02	-2.64
FRA	17.82	58.57	57.67	-518.5	24.93	2.76	48.15	-0.27	0.18	-9.24	-12.60
DEU	15.61	39.23	0.57	-283.2	21.86	0.50	33.21	-0.24	0.03	1.23	-1.50
IRE	20.11	30.47	0.00	-149.3	27.67	0.52	26.06	0.42	-0.05	2.35	-1.12
ITA	15.33	74.53	87.90	-718.7	22.50	-0.51	60.31	2.76	-0.70	-5.69	-9.00
NET	18.08	77.03	2.48	-587.9	28.35	1.30	64.82	1.27	-0.33	3.17	-1.00
PRT	18.04	74.37	4.88	-511.2	27.41	1.52	63.32	1.77	-0.47	3.46	-0.41
ESP	14.12	76.10	41.78	-651.4	22.82	1.45	64.76	0.43	0.18	0.75	-2.40
GBR	13.13	30.77	0.00	-194.9	20.08	1.38	26.67	0.12	0.06	1.68	-2.29
USA	6.15	37.93	0.00	-227.1	13.70	1.31	33.02	0.52	-0.18	2.52	-2.29

Source: Author's calculations. See main text for more details. TWD is Tax Wedge Differential. (1^*) is the difference between the overal tax wedge under $P1^*$ and $P3$, (2^*) is the difference between the overal tax wedge under $P2^*$ and $P3$, $(2')$ is the difference between labour tax wedge under $P2^*$ and $P3$. All data in percentage except $\widetilde{U}$.

4.4 Risk-Equivalent Reforms: Long Run

An implication of the results in Figure 6, is that different types of public pension reforms could be designed to achieve the same $PSEP$ at some point in the future. $PSEP$ can therefore be used as a policy target against which to compare the macroeconomic and welfare effects of pension reforms. This can be implemented by requiring the change in consumption taxation or payments to retirees to have the same $PSEP$ as a given increase in the retirement age in a given year. This enables a risk-equivalent evaluation of different types of public pension reforms, in the sense that each policy can be calibrated to have the same probability of exhausting PS in a given year.

Table 4 shows how we construct the risk-equivalent reforms and their welfare effects. As a target, we consider for each country the estimate of $PSEP$ achieved by the two-year increase of the retirement age, i.e. $P3$, for the year 2050. The first four columns in the table report the values of τ^c, θ, $PSEP$ and $\widetilde{U}$ - the average lifetime utility of a newborn calculated using equation (1) - corresponding to this target.[12] The next four columns report the required change of either τ^c or θ that would give a $PSEP$ as close as possible to that under $P3$ in 2050.[13] We term these scenarios $P1^*$ and $P2^*$, respectively. As a welfare comparison we calculate the consumption equivalent compensation (CEC) that in 2050 would, under each of these two policies, yield the same $\widetilde{U}$ as under $P3$. A positive (negative) CEC means that the policy considered yields a welfare gain (loss) compared to that of increasing the age of retirement by two years.

According to the results in Table 4, $CECs$ are generally positive for each country. This provides (the first part of) the fourth headline result of the quantitative analysis, that risk-equivalent pension reforms enhance welfare in the long run. Two main patterns are evident from the results in Table 4. The first concerns the unambiguous direction required of the policy reforms. For all countries there would need to be a decrease in the replacement rate to achieve a $PSEP$ similar to that obtained from increasing the age of retirement by two years in 2050. This varies across countries from roughly 3 to 14 percentage points. Alternatively, there would need to be an increase in consumption tax rates in all countries, ranging from roughly 6 to 10 percentage points. The second result is that in none of the twelve countries would an increase in the age of retirement be the preferred policy from a welfare viewpoint. Increasing indirect taxation, scenario $P1^*$, yields welfare gains for eleven countries, ranging from 0.01 to 2.7% of consumption per-capita. Italy is the only country recording a welfare loss. Similarly, reductions of pension payments to households, scenario $P2^*$, would produce welfare gains for ten countries, ranging from around 0.12 to 2.8% of consumption per-capita. Welfare losses, albeit marginal (0.27 and 0.24%), are found only for France and Germany.

These findings prompt the questions of what drives country differences in $CECs$, and why for the majority of countries the two policies of increasing the rate of consumption tax and reducing the replacement rate are generally preferred to increasing the age of retirement?

[12] The tilde above the symbol U indicates that lifetime utility is computed from equation (1) using the normalised counterpart of household consumption $\widetilde{c}^s$ defined in Appendix A.

[13] The algorithm designed to match $PSEP$ under the three different policies is described in Appendix B.

To address the first question, we calculate the correlation coefficients of the positive $CECs$ for each policy ($P1^*$ and $P2^*$) with the following indicators for 2050: θ under $P1^*$ or τ^c under $P2^*$, the 2050 population growth rate, the 2050 PS, $PSEP$ and tax wedge differential, measured as the difference between the intra-temporal tax wedge under $P1^*$ and $P3$ and under $P2^*$ and $P3$.[14] We find that the tax wedge differential has the largest correlation with the measured $CECs$, being of -0.74 for $P1^*$ and -0.33 for $P2^*$.[15] These regression results have a very clear economic interpretation: $CECs$ from a given pension policy reform are higher, the greater is the associated reduction in the tax burden. Given equations (5)-(7) this is not surprising, as a reduction in the tax burden results in higher levels of the labour supply and equilibrium output, as well as easing tax distortions on households' optimal decisions. The answer to the second question can be seen from the results in the last three columns of Table 4. The columns corresponding to (1^*) and (2^*) report the intra-temporal tax wedge differentials implied by $P1^*$ and $P2^*$ relative to $P3$, respectively. The column corresponding to $(2')$, reports the labour-only tax wedge differential between $P2^*$ and $P3$. For the majority of countries increasing consumption taxation or reducing pension payments are the preferred policies because these entail lower levels of distortionary labour income taxation compared to a risk-equivalent increase in the retirement age.

4.5 Risk-Equivalent Reforms: Transitional Effects

A possible criticism of the welfare analysis in Table 4 is that it overlooks the intergenerational distribution effects that public pension reforms may have in the short run, which in turn depend on the pace at which reforms are implemented. To address this we switch focus

[14]These results are fully reported in Tables E3 and E4 in Appendix E. In all policy simulations τ^k is unchanged. Thus, the only pertinent tax wedge is that for the intra-temporal margin, which depends on τ^l and τ^c. Using equations (5) and (6), this is formulated as: $TW = 1 - (1 - \tau^l)/(1 + \tau^c)$. Under $P1^*$, the tax rate τ^c is unchanged, as only $\tau^l = \tau^w + \tau^p$ is adjusted to balance the government budgets in equations (10) and (15) respectively. Under $P2^*$, τ^c varies to match $PSEP$, while again τ^l is adjusted to balance the two government budgets.

[15]Upon re-running the regressions using all (positive and negative) $CECs$, the tax wedge differential is still the most significant explanatory variable for $P1^*$, having a correlation coefficient of -0.78, but the correlation reduces significantly under $P2^*$ to 0.04 (second largest coefficient), indicating that the CEC measured for Italy is possibly an outlier.

Table 5: Welfare effects of risk-equivalent reforms that achieve the same $PSEP$ by 2100

No Policy Change ($PS = 25.89\%$, $PSEP = 7.93\%$)							
		$P3$		$P1^*$		$P2^*$	
τ^c	θ	$PSEP$	$\tilde{U}$	τ^c	CEC	θ	CEC
6.15	37.93	1.48	-266.86	13.60	1.43	32.62	1.36

to a single country, the United States, and to a longer evaluation horizon, 2100. We also consider reforms that are implemented either immediately or gradually, and quantify welfare gains and costs as the economy settles to the new longer term.[16] We focus the transition analysis on a single country and choose the United States for two main reasons. First, the results in Table 4 unambiguously indicate that in the long run all countries, despite their demographic, economic and policy differences, would be better off either by curtailing pension entitlements to households or by resorting to partial financing using indirect taxation, instead of increasing the retirement age. We therefore conjecture that the outcomes of a single-country transition experiment can still be pertinent for a wider range of economies. Second, transition simulations in OLG models often require at least as many years as the life span of a new born generation. But, as Figure 4 shows, for several countries, equilibrium under the no policy change scenario cannot be computed beyond 2050, as their PS is exhausted by then. Thus focusing on the United States allows the inclusion in the transition experiment of the no policy scenario as the reference benchmark against which to compare all other policies.

Table 5 reports the policy parameters targeted for the 2100 equilibrium.[17] These can be compared with the 2050 figures for the United States in Tables 3 and 4. Under the no policy change scenario PS is reduced from 43.34% to 25.89% and $PSEP$ is increased from 0 to 7.93% between 2050 and 2100. Increasing the retirement age by two years under $P3$ reduces $PSEP$ to 1.48% (and increases PS to 33.42%) by 2100. The same $PSEP$ by 2100 can be

[16] Additional details are provided in Appendix F.

[17] The demographic data employed are the 5th, 20th, 50th, 80th and 95th percentiles of the population growth rate projections available from the United Nations (2022)' database, as well as the median projection of the survival probabilities. Given the large volume of data involved, we do not include them neither in the main paper nor in Appendix, but these are available upon request from the authors.

achieved by either increasing the consumption tax rate from 6.15% to 13.6% under $P1^*$ or reducing the replacement rate of pensions under $P2^*$ from 37.93% to 32.62%. Both $P1^*$ and $P2^*$ yield larger welfare gains (positive CEC) compared to $P3$. Comparison of these results with those in Table 4 for the United States in 2050, shows that the policy ranking is unaffected: the welfare gains from $P1^*$ are higher than $P2^*$ and, in turn, than $P3$, though these are larger as the $PSEP$ target is implemented over a longer period.

Given these results, the transition simulation is undertaken according to the following protocol. The benchmark calibration is altered setting $\tau^c = 13.60\%$ under $P1^*$, $\theta = 32.62\%$ for $P2^*$ and $T^W = 68$ for $P3$. All policy changes are unanticipated, but implemented either immediately in the year 2021, or gradually during 2021-2040. In the latter case, τ^c is increased linearly from 6.15 to 13.70% between 2021 and 2040, θ is reduced linearly from 37.93 to 32.62% between 2021 and 2040, and T^W is increased by 1 year in 2025 and 2035, respectively.[18] Population growth and survival probabilities evolve according to the median projections of United Nations (2022) during 2021-2100, remaining constant afterwards. Figure 7 shows how the main macroeconomic aggregates (capital, labour, output and the rate of interest) and the tax rates levied on labour income adjust over time in response to the simulated pension policy reforms. The red, blue and green indicate the dynamics under $P1^*$, $P2^*$ and $P3$, respectively, implemented either gradually (dotted lines) or immediately (solid lines). Black lines refer to the evolution of the variables under the no policy change scenario.[19] We highlight three main results: (i) under the no policy scenario, ageing leads to a reduction of capital, labour and output in the long run, and an increase in labour income taxation. The rate of interest also declines in the long run, which is consistent with the secular stagnation hypothesis of Eggertsson et al. (2019). As often found in OLG models, the dynamic evolution of capital is hump-shaped, and it is the main driver of the observed output and interest rate dynamics. (ii) Public pension reforms $P1^*$ and $P2^*$ lead to higher capital accumulation, lower labour supply and interest rates compared to $P3$. Policy differences are less marked on the

[18] In all scenarios, we keep government consumption and transfers constant at the benchmark equilibrium level prevailing in the year 2020.

[19] In each plot, the dynamics of a given variable are shown up to 2225 to ensure stationarity.

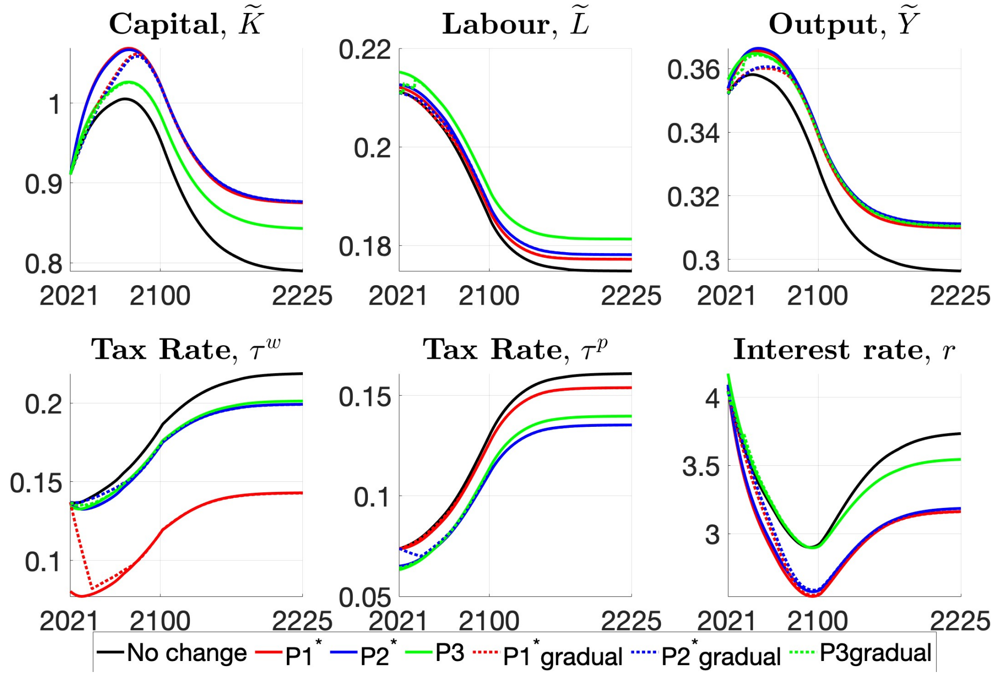

Figure 7: Transition Dynamics 2021- 2225

output dynamics since these reflects a combination of those of capital and labour. Higher consumption taxation under $P1^*$ results in a larger reduction in the income tax rate, while lower pension payments and later retirement under $P2^*$ and under $P3$ imply a lower social security contributions. The increase of τ^c under $P1^*$ determines a larger direct reduction on the tax rate τ^w compared to the indirect reduction, due to partial financing, observable for τ^p. In contrast, the effects of $P2^*$ and $P3^*$ are more or less similar on τ^w and τ^p. (iii) Because they adjust sluggishly, the effects of a gradual policy implementation on the macroeconomic aggregates are not noticeable, aside from lower short-term capital accumulation and output under $P1^*$ and $P2^*$, and the gradual labour supply increase in 2025 and 2035 instead of a sudden jump in 2021. Gradual policy implementation naturally leads to a more gradual reduction of τ^w under $P1^*$ and τ^p under $P2^*$, which is consistent with the lower capital accumulation noted just above.

Figure 8 illustrates the welfare implications, measured in terms of $CECs$ of the three policy

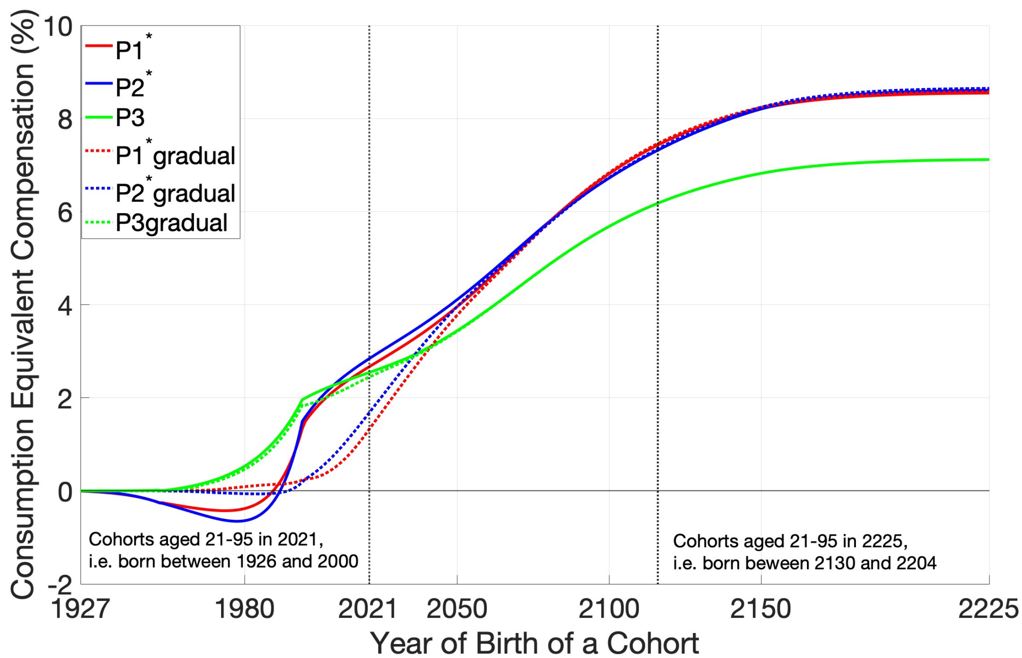

Figure 8: Generational welfare

reforms ($P1^*$ in red, $P2^*$ in blue and $P3$ in green) relative to the no policy change scenario, for all cohorts alive in the years 2021-2225.[20] The vertical line corresponding to the year 2021 marks to the left all cohorts alive when the first policy change is implemented, whereas that after the year 2100 marks to the right cohorts that will be alive once all reforms are phased in. The main result from Figure 8 - which illustrates an important caveat to the fourth headline result of the quantitative analysis - is that while risk-equivalent pension reforms enhance welfare in the long run, transitional effects vary across policies and depending on the pace of implementation. In particular, if pension reforms are implemented instantaneously (solid lines), all generations alive benefit from policy P3, while all generations born prior to 1989 under policy $P1^*$, or born prior to 1991 under $P2^*$, incur a welfare loss between 0-0.65%.[21]

[20] The horizontal axis starts in 1927, because households in the model live up to age 94 and for this reason the oldest cohort alive in 2021 is that of those born in the year 1927.

[21] The result that an increase in the retirement age benefits all generations currently alive, as under $P3$, diverges from Kitao (2014). This is because our transition simulation assumes that pension benefits remain constant at the 2020 level, while they decline in the transition analysis of Kitao (2014). Our result on pension policy $P2^*$ are in accordance with those of Kitao (2014).

To see why welfare increases under $P3$, consider the retirees aged 69 and above in the year 2021 who are born between 1926-1952. They receive the same pension payment under $P3$ as in the case of no policy change, but they do not have to work extra years. Therefore the only change in their lifetime utility is from a general equilibrium effect on the interest rate r. Figure 7 shows that the interest rate is (marginally) higher under $P3$ (sudden change) than it is under the no policy change case during the initial phase of the transition, i.e years 2021-2092. Only in the long run the interest rate is higher in the benchmark than under $P3$.[22] As a consequence, the capital income of retirees and, hence, their welfare increase. In the case of policy $P1^*$ there is a consumption tax increase, while under policy $P2^*$ pensions decline. The welfare of the retirees therefore decreases under $P1^*$ and $P2^*$. When implementation is gradual (dotted lines), all policies cause consumption losses for the oldest generations, but these are quite small and as can be seen in Figure 8. The reason why even $P3$ generates welfare losses for the oldest generations is that a gradual increase of the retirement age has a reduced effect on aggregate labour until 2035 and, hence, a smaller impact on savings and the capital stock. Consequently, the interest rate also increases less relative to the no policy change scenario. Gradual implementation of both policies $P1^*$ and $P2^*$ reduces the welfare losses of the generations born prior to 1992. The younger and still unborn households in 2021, however, benefit from an instantaneous implementation of $P1^*$ and $P2^*$ as it reduces labour market distortions to a greater extent during their employment years. Overall, of the six simulated policy reforms, only one entails no loss at any time horizon, namely, the sudden increase of the age of retirement under $P3$. It is often argued that pension policy reforms should be implemented gradually, see Barr and Diamond (2008). According to these results this is not always the case.

How sensitive are the welfare effects of pension reforms with respect to the uncertainty surrounding demographic projections? To answer this question we re-run the transition simulations for the case of instantaneous policy implementation under two alternative demo-

[22]The marginal product of capital increases under $P3$, because the aggregate labour supply increases due to the larger labour force, but aggregate capital is predetermined in 2021.

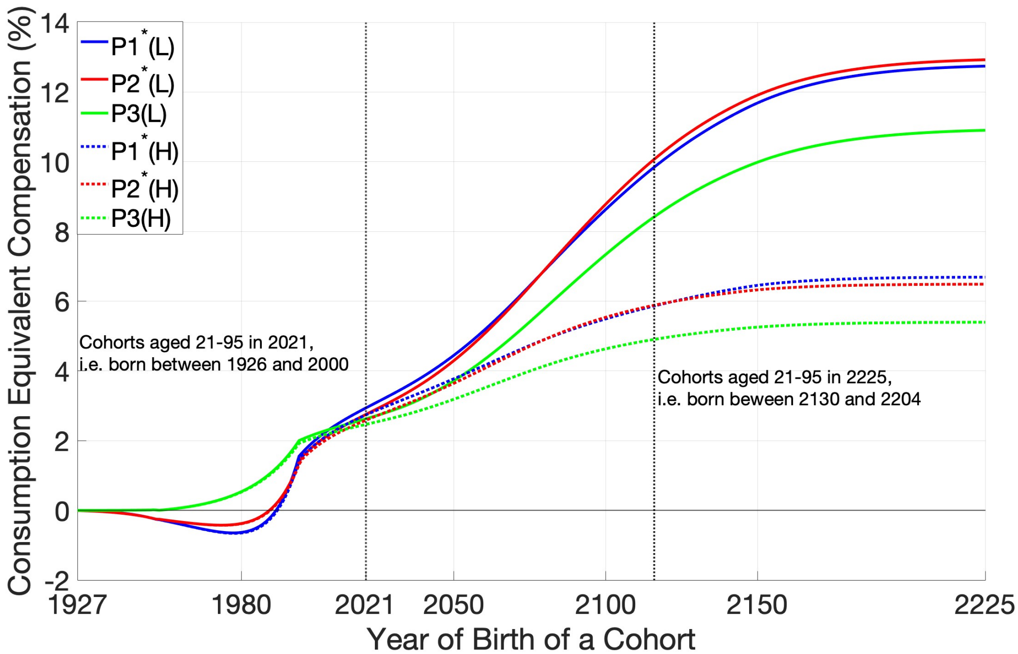

Figure 9: Generational welfare effects under a scenario of low (20th percentile of the population growth distribution) and high (80th percentile) fertility. Consumption equivalent compensations are measured relative to the benchmark. All policy changes are implemented once for all in 2021.

graphic scenarios, using the 20th and the 80th percentiles from the empirical distribution of the projected population growth rate.[23] Figure 9 shows the generational welfare effects of policies $P1^*$ (blue line), $P2^*$ (red line) and $P3$ (green line) relative to the benchmark case under low (solid) and high (dotted) population growth. The figure reveals that the welfare gains from public pension reforms increase as the demographic outlook becomes bleaker. According to these results, for the high fertility scenario, see $P1^*(H)$, $P2^*(H)$ and $P3(H)$, the welfare gains range from roughly 10 to 13% of household consumption. In contrast, for the low fertility scenario, see $P1^*(L)$, $P2^*(L)$ and $P3(L)$, they range from roughly 4 to 6%. This is because a rapidly aging population with low fertility rates implies larger distortionary taxation costs if pension policy is not reformed. Thus a further caveat to our fourth headline

[23]The macroeconomic dynamics during these transitions are presented in Appendix F, together with additional comments on these results.

result of the quantitative analysis is that the benefits of implementing pension policy reforms are greater when the population is expected to age more quickly.

5 Conclusion

The issue of whether or not developed economies can maintain their publicly-funded old-age pension systems without running out of money has been a long-standing concern. This paper proposes two new indicators, the pension space and its exhaustion probability, to study the risks associated with public pension systems and the impact on them of reforms. These measures provide a means of examining and comparing the viability of pension systems in different countries, monitoring their long-term feasibility, and evaluating the effectiveness of policy interventions aimed at improving their performance. The main advantage of these metrics is that they offer synthetic indicators that take into account a wide range of factors that influence the state of a country's public pension system. Our methodology is based on a multi-period OLG model that recognizes that the pension system is financed through labour income taxation that is subject to a dynamic Laffer effects when the social security contribution rate is increased. The pension space depends on the solution of a Laffer problem in which the social security budget is maximized subject to the constraint of the general government budget. By recalculating the pension space under different demographic scenarios, its distribution can be estimated, from which its exhaustion probability can be obtained. This probability is an appropriate pension-risk warning indicator and can be used to compare pension reforms on a risk-equivalent basis. We use these measures to evaluate the public pension systems of twelve countries that differ in terms of economic outlook, public pension provisions and demographic trends. According to our results, in most of these countries, despite significant differences in the current size of their pension space, there is limited room for financing public pensions through further labour income taxation over the next thirty years. Moreover, reforms cannot be a one-size-fits-all solution, as their impact on pension sustainability varies across countries. From a welfare perspective, reforming public pensions provides large long-term gains, particularly for those countries that are projected to age more rapidly, but it also entails costs in the short and medium term. It is not always the case that a gradual implementation of public pension reforms is preferable to a more rapid one.

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Appendix to Pension Systems(Un)sustainability and Fiscal Constraints: A Comparative Analysis

Burkhard Heer, Vito Polito and Mike Wickens

A Stationary Equilibrium

The OLG model described in Section 2 is nonstationary because the aggregate technology A_t and the population N_t grow over time at the rates g_A and n, respectively. Consequently, to derive an equilibrium solution, it is necessary to rescale individual variables by the aggregate technology and aggregate variables by both the aggregate technology and the size of the population. It is then necessary to rewrite all equilibrium conditions in terms of the rescaled variables. Individual variables are rescaled using $\widetilde{x}_t^s \equiv x_t^s/A_t$, for $x_t^s = c_t^s, y_t^s, k_t^s, b_t^s, tr_t^s$, $\widetilde{pen}_t \equiv pen_t/A_t$ and $\widetilde{\lambda}_t \equiv \lambda_t/A_t^{-\eta}$. Similarly, aggregate variables are rescaled using $\widetilde{X}_t \equiv X_t/(A_t N_t)$, where $X_t = Y_t, C_t, K_t, B_t, G_t, Tr_t, Beq_t, Tax_t$, and $\widetilde{L}_t \equiv L_t/N_t$. Table A1 presents the normalised counterparts of all equilibrium conditions in equations (3)-(19). The households first-order conditions are solved using the instantaneous utility function in equation (2). Despite being unaffected by the rescaling, the last two conditions are included for completeness.

B Numerical Algorithms

The four definitions for the Competitive Equilibrium, the Pension Limit, the Pension Space and the Pension Space Exhaustion Probability provided in Section 2.2 can be reformulated in terms of the normalised variables using the same Definitions 1-4 but referring instead to the equations in Table A1. In any period t, the stationary equilibrium counterparts for the

Table A1: Stationary counterparts (left column) of the OLG model equilibrium conditions (right column).

Stationary counterpart		Eq. No.
$\widetilde{y}_t^s = (1-\tau_t^w-\tau_t^p)w_t\bar{y}^s l_t^s$	$s=1,\dots,T^W$	(3)
$\widetilde{y}_t^s = \widetilde{pen}_t$	$s=T^W+1,\dots,T$	(3)
$(1+\tau_t^c)\widetilde{c}_t^s = \widetilde{y}_t^s + [1+(1-\tau_t^k)r_t]\widetilde{a}_t^s + \widetilde{tr}_t^s - (1+g_A)\widetilde{a}_{t+1}^{s+1}$	$s=1,\dots,T$	(4)
$(\widetilde{c}_t^s)^{-\eta}[1-\kappa(1-\eta)(l_t^s)^{1+1/\varphi}]^{\eta} = \widetilde{\lambda}_t^s(1+\tau_t^c)$	$s=1,\dots T$	(5)
$\widetilde{\lambda}_t^s\bar{y}^s(1-\tau_t^w-\tau_t^p)w_t = \kappa\eta(1+\frac{1}{\varphi})(\widetilde{c}_t^s)^{1-\eta}[1-\kappa(1-\eta)(l_t^s)^{1+1/\varphi}]^{\eta-1}(l_t^s)^{1/\varphi}$	$s=1,\dots,T^w$	(6)
$(1+g_A)^{\eta}\widetilde{\lambda}_t^s = \beta\phi_t^s\widetilde{\lambda}_{t+1}^{s+1}[1+(1-\tau_{t+1}^k)r_{t+1}]$	$s=1,\dots,T-1$	(7)
$w_t = (1-\alpha)\widetilde{K}_t^{\alpha}\widetilde{L}_t^{-\alpha}$		(8)
$r_t = \alpha\widetilde{K}_t^{\alpha-1}\widetilde{L}_t^{1-\alpha} - \delta$		(9)
$\widetilde{G}_t + \widetilde{Tr}_t + (1+r_t^b)\widetilde{B}_t = (1+g_A)(1+n)\widetilde{B}_{t+1} + \widetilde{Tax}_t + \widetilde{Beq}_t$		(10)
$(1+n)\widetilde{Beq}_{t+1} = \sum_{s=1}^{T-1}\mu_t^s(1-\phi_t^s)[1+(1-\tau_{t+1}^k)r_{t+1}]\widetilde{a}_{t+1}^{s+1}$		(11)
$\widetilde{Tax}_t = \tau_t^c\widetilde{C}_t + \tau_t^w w_t\widetilde{L}_t + \tau_t^k(r_t-\delta)\widetilde{K}_t$		(12)
$\widetilde{pen}_t = \theta_t(1-\tau_t^l)w_t\bar{l}_t$		(13)
$\widetilde{Pen}_t = \sum_{s=T^W+1}^{T} N_t(s)\widetilde{pen}_t$		(14)
$\widetilde{Pen}_t = \tau_t^p w_t\widetilde{L}_t$		(15)
$\widetilde{C}_t = \sum_{s=1}^{T}\mu_t^s\widetilde{c}_t^s,\ \ \widetilde{L}_t = \sum_{s=1}^{T^w}\mu_t^s\bar{y}^s l_t^s,\ \ \widetilde{\mathcal{A}}_t = \sum_{s=1}^{T}\mu_t^s\widetilde{a}_t^s,\ \ \widetilde{Tr}_t = \sum_{s=1}^{T}\mu_t^s\widetilde{tr}_t$		(16)
$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{G}_t + (1+g_A)(1+n)\widetilde{K}_{t+1} - (1-\delta)\ \widetilde{K}_t$		(17)
$\widetilde{\mathcal{A}}_t = \widetilde{K}_t + \widetilde{B}_t$		(18)
$\bar{l}_t = \frac{\sum_{s=1}^{T^w}\mu_t^s l_t^s}{\sum_{s=1}^{T^w}\mu_t^s}$		
$r_t^b = (1-\tau_t^k)r_t$		(19)

Competitive Equilibrium of Definition 1, the Pension Limit of Definition 2, the Pension Space of Definition 3 and the Pension Space Exhaustion Probability of Definition 4 are derived by equating all subscripts to t in Table A1. These stationary equilibrium conditions are used to derive the results reported in Table 2 and in Sections 4.1-4.4.

The stationary equilibrium includes a sequence of $T-2$ variables for households asset holdings, $\widetilde{a}_t^s$, $s \in \{1,\dots,T-2)$, a sequence of T^W variables for households labour supply, l_t^s, $s \in \{1,\dots,T^W)$, and the aggregate variables $\widetilde{K}_t$, $\widetilde{L}_t$, $\widetilde{\mathcal{A}}_t$, τ_t^p and $\widetilde{Tr}_t$. After replacing the wage and interest rate from equations (8) and (9) into equations (3)-(7), the non-linear system of equations defining the stationary equilibrium solution of the model consists of $T-2$ intertemporal equations obtained by combining (5) with (7), T^W intratemporal equations (6), plus five further equations consisting of the government budget constraint in (10), the social security budget in (15), the aggregate consistency equation (16) for aggregate labour supply

$\widetilde{L}_t$, the feasibility condition in (17) and the aggregate stock of assets in (18). In addition, we insert (11)-(15) together with (19) and the definition of $\bar{l}_t$ in (10) to eliminate the variables $\bar{l}_t$, r_t^b, $\widetilde{Pen}_t$, $\widetilde{Tax}_t$ and $\widetilde{Beq}_t$ from equation (10) and use the stationary equilibrium condition $\tilde{K}_{t+1} = \tilde{K}_t$.

To determine the results in Tables 2-5 and in Figures 3-6, we solve, under different calibrations across countries and over time, the system of non-linear equations defining the model equilibrium with a modified Newton-Rhapson algorithm applied to a large-scale OLG model, as described in Heer and Maußner (2009). The main challenge for the solution is to determine a good initial value for the individual and aggregate variables. To this end, we start with a simplified version of the model with exogenous labour and no retirement, including only T^W periods. The labour supply is set equal to 0.25 and the initial value of the aggregate capital stock is set equal to the value that is implied by the Euler equation and the first-order condition for the firm with respect to capital in the steady state. Thereafter, we add one additional cohort of retirees in each step and use the solution of the model in the previous step as an input for the initial value of the following one. Next, we introduce endogenous labour. During these initial computations, we calculate the solution for the household optimization problem in an inner loop and update the aggregate capital variables in an outer loop with a dampening iterative scheme, as described in Judd (1998), to ensure convergence. Given the initial values, for the final calibration of the equilibrium, the whole system of nonlinear equations is computed with the modified Newton-Rhapson algorithm. The computation time of each PS and $PSEP$ number reported in in Tables 2-5 and in Figures 3-6 is about 4-5 minutes using an Intel(R) Xeon(R), 2.90 GHz machine.

To compute $PSEP$ in Figure 4, Figure 6, Table 4 and Table 5, under different calibrations across countries and over time, we calculate the stationary equilibrium solution from 2020 to 2050 using the median projection of the growth rate of the population as well as the upper and lower 80th and 95th percentiles. These are then used to compute PL and aggregate pension expenditure under each population growth projection during 2020 to 2050. Note that in any of these cases, it is always feasible to extend the OLG model simulation under PL until 2050, whereas the simulation under the benchmark calibration to compute aggregate pension given the current policy may terminate earlier if PL is reached before 2050. For any given country and policy counterfactual, the algorithm for calculating $PSEP$ includes the following four steps:

1. Consider simulated trajectories of PL and aggregate pension expenditure for each available date, i.e. 2020, 2025, 2030 and so on until 2050.

2. Interpolate the five years trajectories into annuals over the entire 2020-2050 horizon. Note that the annual interpolation extends until 2050 even in instances when the model simulation under the benchmark calibration terminates before 2050. Thus for the purpose of computing $PSEP$ we allow aggregate pension expenditure to exceed PL, if this occurs.

3. Using the annual interpolated data compute PS. At this stage, we do not truncate PS to be non-negative.

4. Repeat steps 1 to 3 for all five population growth percentiles. This yields five points of the PS distribution, corresponding to the five available population percentiles, in each year during 2020-2050.

5. Take for each year the five values of PS and use them to estimate an empirical kernel distribution. Convert the estimated probability density function into a conditional density and compute from that the probability of zero PS, i.e. $PSEP$.

The kernel density estimator described in step 5 is the estimated pdf of PS, once this is treated as a random variable. For any real values of PS, the kernel density estimator's formula is given by:

$$\hat{f}_h(PS) = \frac{1}{n_s h} \sum_{1=1}^{n_s} K\left(\frac{PS - PS_i}{h}\right),$$

where $PS_1, PS_2, \ldots, PS_{n_s}$ are random samples from an unknown distribution, n_s is the sample size, $K(.)$ is the kernel smoothing function and h is the bandwidth. The fitted kernel estimator uses $n_s = 5$, a normal-shape smoothing function and a bandwidth determined according to the theoretically optimal for estimating densities for the normal distribution, as in Bowman and Azzalini (1997). As described above and in the main paper the kernel estimation of the $PSEP$ distribution relies on five points from the empirical distribution of the population, namely percentiles 5, 20, 50, 80 and 95. This is because these are the only statistics made publicly available by the United Nations (2022)'s database. A further limiting factor is that the computation time of the algorithm requires roughly 4-6 hours for each $PSEP$ depending on country-specific parameters. Nevertheless, to gauge some indication regarding the possible approximation error we simulated the two-period model described in Section 2.3 500 times and computed each time two distributions of the PS. The first is derived from 1000 draws of population growth rate from an assumed true i.i.d. distribution with mean and standard deviations equal to 0.2. The second is obtained using instead only the 5th, 10th, 50th, 80th and 95th percentiles of the assumed true distribution (approximated distribution). The approximation error is calculated as the difference between the $PSEP$ implied by these two distributions. Figure B.1 plots the approximation errors calculated from the 500 simulations. Only 18 instances, that is 3.6% of cases, return an error greater than 0.5 percent, as marked by the horizontal red line.

The algorithm used to compute the required change in the consumption tax rate and replacement rate under $P1^*$ and $P2^*$, respectively, to achieve the same $PSEP$ as under $P3$, and then generate the results in Tables 4 and 5 includes the following steps:

1. In a given year, start from $PSEP$ given by $P3$.

2. Take two initial values for τ^c (and likewise for θ), and compute $PSEP$.

3. Used the secant method (Section 11.5.1 in Heer and Maußner (2009)) and update original guess until convergence.

In cases where the $PSEP$ is close to zero, the above algorithm is first iterated to match

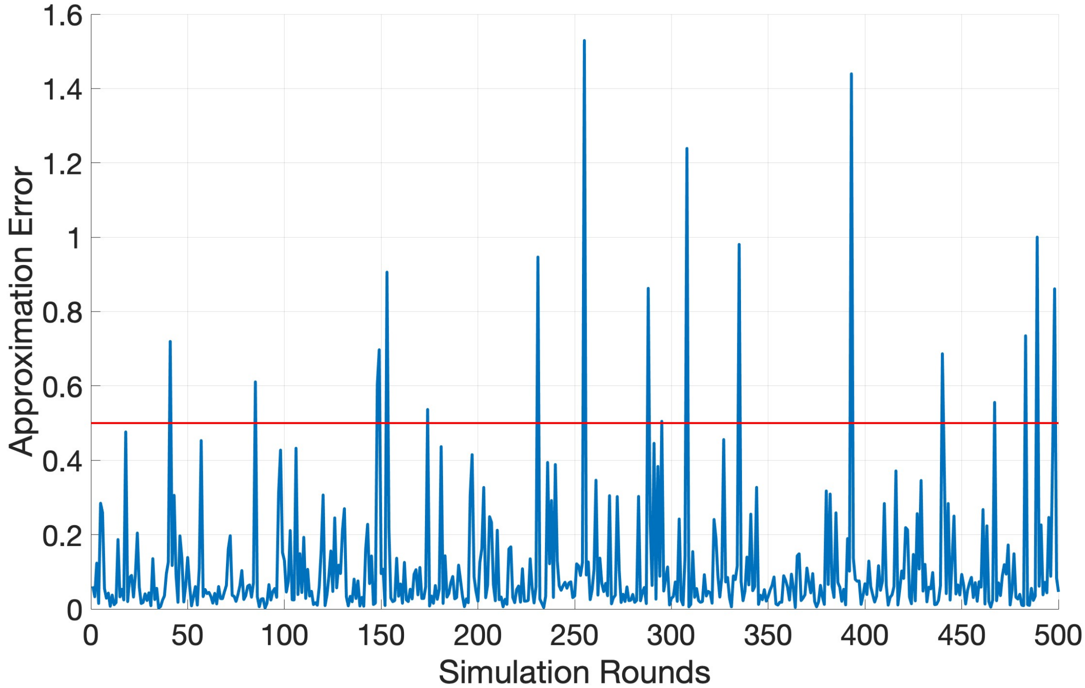

Figure B.1: Differential between $PSEP$ estimated from true PS distribution and distribution estimated from 5 percentiles. Instances above the red line indicate approximation erros greater that 0.5%. See thext for more details.

PS instead of $PSEP$. As initial values, we use those of τ^c and θ computed for the 2050 equilibrium under $P1$ and $P2$, respectively, as well as $\tau^c + 5\%$ and $\theta - 10\%$. The tolerance level for convergence with respect to the targeted value of $PSEP$ is +/- 1%. The computation time ranges from 16 to 22 hours for each $PSEP$ depending on country-specific parameters.

C Two-Period Model: Equilibrium Conditions

We consider a simpler version of the model that can be solved analytically. The equations describing lifetime utility of a newborn household (i), the household budget constraints when young (ii) and old (iii), the general government budget (iv), the social security budget (v), aggregate tax revenue raised from general income taxation (vi) the aggregate revenue side of the social security budget and (vii) the social security budget are given by: $U = \sqrt{c^y} + \beta\sqrt{c^o} - l$, $c^y + b = \left(1 - \tau^l\right) wl$, $c^o = (1 + r)\, b + \theta wl$, $G + (1 + r)\, bN_{t-1} = T^G + bN_t$, $T^P = \theta wlN_{t-1}$, $T^G = \tau^w wlN_t$ and $T^P = \tau^p wlN_t$, respectively. The tax rates are linked by $\tau^l = \tau^w + \tau^p$. The Lagrangian and first-order conditions for a new born household are:

$$\begin{aligned}
\max_{c^y,c^o,l,\lambda} L_t &= \sqrt{c^y} + \beta\sqrt{c^o} - l - \lambda\left\{c^y + \frac{c^o}{1+r} - \left(1-\tau^l\right) wl - \frac{\theta wl}{1+r}\right\} \\
c^y &: \frac{1}{2\sqrt{c^y}} = \lambda \\
c^o &: \frac{\beta}{2\sqrt{c^o}} = \frac{\lambda}{1+r} \\
l &: -1 + \lambda\left[\left(1-\tau^l\right) w + \frac{\theta w}{1+r}\right] = 0.
\end{aligned}$$

Solving these yields:

$$\begin{aligned}
c^y &= \frac{1}{4}\left[w\left(1-\tau^l\right) + w\frac{\theta}{1+r}\right]^2 \\
c^o &= \frac{\beta^2 (1+r)^2}{4}\left[w\left(1-\tau^l\right) + w\frac{\theta}{1+r}\right]^2 \\
l &= \left(1-\tau^l\right)\frac{1}{4}w\left[1+\beta^2(1+r)\right] + \frac{\theta}{1+r}\frac{1}{4}w\left[1+\beta^2(1+r)\right].
\end{aligned}$$

The social security budget implies: $\theta wlN_{t-1} = \tau^p wlN_t$, from which it follows that $\theta = \frac{\tau^p}{d}$. The government cannot choose both θ and τ^p, but given τ^p there is a unique θ. Thus the equilibrium labour supply can also be written as $l = \underline{l}\,(1 - \tau^w - \tau^p\Phi)$, where $\underline{l} = \frac{1}{4}w\left[1+\beta^2(1+r)\right]$ and $\Phi = 1 - \frac{1}{d(1+r)}$. Given the equilibrium solution for labour, the revenue side of the social security budget, in per worker terms, can be formulated as $t^P = \tau^p w\underline{l}\,(1 - \tau^w - \tau^p\Phi)$ and the government budget constraint as $e = \tau^w w\underline{l}\,(1 - \tau^w - \tau^p\Phi)$, where $e = g + [(1+r)\, d - 1]\, b$. The Lagrangian of the social security budget maximization problem is reported in Section 2. The optimality conditions from the maximization are:

$$\tau^p \;:\; w\underline{l}\left[(1-\tau^w) - \tau^p\Phi\right] - \tau^p w\underline{l}\Phi - \lambda\tau^w w\underline{l}\Phi = 0,$$

$$\begin{aligned} \tau^w &: \quad -\tau^p w\underline{l} + \lambda w\underline{l}\left[(1-\tau^w) - \tau^p\Phi\right] - \lambda\tau^w w\underline{l} = 0, \\ \lambda &: \quad \tau^w w\underline{l}\left[(1-\tau^w) - \tau^p\Phi\right] - e = 0, \end{aligned}$$

together with the Kuhn-Tucker requirements $\lambda \geq 0$ and $\{\tau^w w\underline{l}\left[(1-\tau^w) - \tau^p\Phi\right] - e\}\,\lambda = 0$. The solution depends on whether the government budget constraint is binding or not. Case 1: $\lambda = 0$, τ^w given. Thus $\tau^p = \frac{1-\tau^w}{2\Phi}$ and $e' = \tau^w w\underline{l}\left[(1-\tau^w) - \tau^p\Phi\right]$, where e' is a residual from the government budget constraint. Case 2: $\lambda > 0$. The values for τ^p, τ^w and λ can only be determined jointly from the solution of the system comprising the first three optimality conditions. When $\lambda > 0$ the social security tax rate is lower than that achieved when the government budget constraint is not binding.

The results in Figure 1 are based on the following calibration of the two-period model: $n = (1+0.02)^{30}$, $r = (1+0.04)^{30} - 1$, $\beta = [1/(1+0.04)]^{30}$, $w = 1$, $b = 0.05$, $\tau^p = 0.1$, $\theta = 0.2811$, $\tau^w = 0.2537$ and $g = 0.038$. The solution for the PL equilibrium returns: $\overline{\tau^p} = 0.4103$, $\overline{\tau^w} = 0.3615$, $PL = 0.072$ and $PS = 65.26$. The recursion for the bottom left panel of Figure 1 is constructed using a grid for τ^p ranging from 0 to 0.4429, with step of 0.00001. No equilibrium exists as τ^p exceeds 0.4429. $PSEP$ in the bottom right panel of Figure 1 is constructed by taking 1000 draws of the population growth rate from a random normal distribution with mean and standard deviation 0.2. This yields 1000 values of PS, which are regarded as a random sample from an unknown distribution that can be fitted using a kernel distribution, i.e. a nonparametric representation of the probability density function (pdf) of a random variable.

D Additional Data

D.1 Survival Probabilities

Tables D1 and D2 report the average annual survival probabilities from 1980 to 2020 for people aged 20 to 94. The survival probabilities ϕ_t^s are calculated using annual data on life expectancy, LE_t^s, for both sexes combined (from age 20 to age 95) obtained from the United Nations (2022) database. Available data there include estimates of the the annual life expectancy for age s ranging from 0, i.e. a newly born, to age 100, for all countries from 1950 to 2021. To convert life expectancy into survival probability we started from the definition of life expectancy in a given year t for a person of age s: $LF_t^s = \left(1 + \phi_t^s + \phi_t^s \phi_{t+1}^{s+1} + \dots\right)$. Upon updating one period, we obtain $\left(LF_{t+1}^{s+1} = 1 + \phi_{t+1}^{s+1} + \phi_{t+1}^{s+1} \phi_{t+2}^{s+2} + \dots\right)$, which implies: $LF_t^s - \phi_t^s LF_{t+1}^{s+1} = 1$, or equivalently:

$$\phi_t^s = \frac{LF_t^s}{1 + LF_{t+1}^{s+1}}.$$

Thus the data on life expectancy from age from 20 to 95 are used in the formula above to compute the survival probabilities from age 20 to 94 reported in Tables D1 and D2 below.

Table D1: Survival Probabilities of those aged 20-58, average 1980-2020.

AGE	AUT	BEL	FIN	FRA	DEU	IRE	ITA	NDL	PRT	ESP	GBR	USA
20	99.93	99.93	99.93	99.93	99.94	99.94	99.95	99.96	99.92	99.95	99.95	99.91
21	99.93	99.93	99.93	99.93	99.94	99.93	99.95	99.96	99.92	99.95	99.95	99.90
22	99.93	99.93	99.93	99.93	99.94	99.94	99.95	99.96	99.91	99.94	99.95	99.90
23	99.93	99.93	99.93	99.93	99.94	99.94	99.95	99.96	99.91	99.94	99.95	99.90
24	99.93	99.93	99.93	99.93	99.94	99.93	99.95	99.96	99.91	99.94	99.95	99.90
25	99.94	99.93	99.92	99.92	99.94	99.94	99.94	99.96	99.91	99.94	99.95	99.90
26	99.93	99.93	99.92	99.92	99.94	99.94	99.94	99.96	99.91	99.94	99.94	99.89
27	99.94	99.93	99.92	99.92	99.94	99.93	99.94	99.96	99.91	99.93	99.94	99.89
28	99.93	99.93	99.92	99.92	99.94	99.93	99.94	99.95	99.90	99.93	99.94	99.89
29	99.93	99.93	99.92	99.92	99.94	99.93	99.94	99.95	99.90	99.93	99.94	99.89
30	99.93	99.92	99.91	99.91	99.93	99.93	99.94	99.95	99.89	99.92	99.93	99.88
31	99.93	99.92	99.90	99.91	99.93	99.94	99.93	99.94	99.89	99.92	99.93	99.88
32	99.92	99.92	99.90	99.91	99.92	99.93	99.93	99.94	99.89	99.92	99.93	99.87
33	99.92	99.91	99.90	99.90	99.92	99.92	99.93	99.94	99.88	99.91	99.92	99.86
34	99.91	99.91	99.89	99.89	99.91	99.92	99.93	99.93	99.87	99.91	99.92	99.86
35	99.91	99.90	99.89	99.89	99.91	99.91	99.92	99.93	99.87	99.91	99.91	99.85
36	99.90	99.90	99.87	99.88	99.90	99.91	99.92	99.92	99.86	99.90	99.90	99.84
37	99.89	99.88	99.86	99.87	99.89	99.91	99.91	99.92	99.85	99.90	99.90	99.83
38	99.88	99.88	99.85	99.86	99.88	99.90	99.90	99.91	99.84	99.89	99.89	99.82
39	99.87	99.86	99.84	99.85	99.87	99.89	99.90	99.90	99.83	99.88	99.88	99.81
40	99.86	99.85	99.83	99.83	99.85	99.87	99.89	99.89	99.81	99.87	99.87	99.80
41	99.84	99.84	99.81	99.82	99.84	99.86	99.88	99.88	99.80	99.86	99.85	99.78
42	99.82	99.82	99.80	99.80	99.82	99.85	99.87	99.86	99.78	99.85	99.84	99.76
43	99.81	99.80	99.78	99.78	99.81	99.84	99.85	99.85	99.77	99.83	99.82	99.75
44	99.79	99.79	99.76	99.76	99.79	99.82	99.84	99.83	99.75	99.82	99.81	99.73
45	99.76	99.77	99.74	99.74	99.76	99.80	99.82	99.82	99.73	99.80	99.79	99.70
46	99.74	99.74	99.71	99.71	99.74	99.77	99.80	99.80	99.70	99.78	99.76	99.68
47	99.72	99.72	99.69	99.68	99.71	99.75	99.78	99.77	99.68	99.76	99.74	99.65
48	99.69	99.69	99.67	99.65	99.68	99.73	99.76	99.75	99.65	99.74	99.72	99.61
49	99.66	99.66	99.63	99.62	99.65	99.70	99.73	99.72	99.62	99.71	99.69	99.59
50	99.63	99.63	99.59	99.59	99.61	99.65	99.71	99.69	99.59	99.68	99.65	99.55
51	99.59	99.59	99.56	99.55	99.58	99.64	99.68	99.66	99.56	99.66	99.62	99.50
52	99.55	99.56	99.52	99.52	99.54	99.57	99.64	99.62	99.53	99.62	99.58	99.46
53	99.51	99.51	99.49	99.48	99.49	99.54	99.61	99.59	99.48	99.59	99.53	99.41
54	99.45	99.48	99.44	99.44	99.44	99.47	99.57	99.54	99.45	99.56	99.49	99.37
55	99.42	99.42	99.40	99.40	99.40	99.44	99.52	99.50	99.41	99.52	99.43	99.31
56	99.36	99.37	99.33	99.35	99.34	99.38	99.48	99.45	99.37	99.47	99.38	99.25
57	99.30	99.31	99.29	99.31	99.28	99.31	99.42	99.38	99.31	99.43	99.31	99.19
58	99.25	99.24	99.22	99.26	99.22	99.22	99.37	99.32	99.25	99.38	99.24	99.10

Source: Author's calculations. See main text for more details.

Table D2: Survival Probabilities of those aged 59-94, average 1980-2020.

AGE	AUT	BEL	FIN	FRA	DEU	IRE	ITA	NDL	PRT	ESP	GBR	USA
59	99.19	99.18	99.15	99.20	99.15	99.15	99.31	99.26	99.19	99.32	99.16	99.04
60	99.10	99.10	99.07	99.14	99.06	99.05	99.24	99.18	99.12	99.26	99.07	98.95
61	99.02	99.02	98.99	99.08	98.98	99.00	99.16	99.10	99.05	99.20	98.98	98.86
62	98.95	98.92	98.91	99.01	98.89	98.82	99.08	99.01	98.98	99.13	98.88	98.75
63	98.85	98.84	98.81	98.94	98.80	98.73	99.00	98.91	98.88	99.05	98.77	98.66
64	98.75	98.72	98.70	98.86	98.70	98.60	98.90	98.80	98.79	98.97	98.64	98.57
65	98.64	98.60	98.58	98.78	98.57	98.46	98.79	98.68	98.66	98.87	98.51	98.44
66	98.51	98.47	98.47	98.69	98.45	98.29	98.67	98.55	98.55	98.78	98.37	98.32
67	98.38	98.35	98.34	98.59	98.31	98.18	98.55	98.41	98.40	98.66	98.21	98.19
68	98.23	98.18	98.19	98.47	98.15	97.96	98.41	98.24	98.24	98.54	98.04	98.03
69	98.08	98.00	98.01	98.35	97.98	97.74	98.26	98.08	98.07	98.38	97.85	97.88
70	97.90	97.81	97.81	98.20	97.78	97.46	98.08	97.89	97.83	98.21	97.63	97.68
71	97.67	97.60	97.60	98.04	97.56	97.32	97.88	97.67	97.63	98.04	97.41	97.49
72	97.45	97.34	97.37	97.85	97.32	96.89	97.66	97.43	97.37	97.82	97.13	97.25
73	97.20	97.09	97.08	97.65	97.06	96.64	97.41	97.16	97.08	97.59	96.86	97.02
74	96.90	96.81	96.83	97.42	96.75	96.20	97.13	96.87	96.71	97.31	96.54	96.80
75	96.57	96.46	96.46	97.16	96.40	95.82	96.83	96.54	96.35	97.02	96.22	96.51
76	96.19	96.08	96.12	96.86	96.02	95.39	96.47	96.17	95.91	96.69	95.84	96.20
77	95.75	95.67	95.69	96.52	95.60	95.07	96.10	95.77	95.44	96.29	95.45	95.87
78	95.30	95.21	95.25	96.13	95.11	94.46	95.65	95.33	94.90	95.87	95.01	95.53
79	94.81	94.69	94.72	95.70	94.57	93.90	95.16	94.83	94.30	95.37	94.51	95.11
80	94.08	94.08	94.13	95.18	93.95	93.18	94.61	94.26	93.86	94.84	93.91	94.52
81	93.43	93.47	93.51	94.62	93.25	92.85	94.01	93.63	93.21	94.26	93.34	94.03
82	92.65	92.74	92.79	94.00	92.52	91.78	93.34	92.97	92.44	93.57	92.67	93.44
83	91.82	91.93	92.00	93.29	91.70	91.13	92.58	92.20	91.59	92.86	91.95	92.81
84	90.91	91.09	91.19	92.54	90.80	90.09	91.77	91.36	90.73	92.00	91.18	92.12
85	89.84	90.14	90.22	91.68	89.87	89.43	90.87	90.45	89.72	91.12	90.34	91.36
86	88.72	89.10	89.20	90.73	88.83	88.04	89.89	89.41	88.64	90.14	89.40	90.52
87	87.58	87.98	88.16	89.73	87.71	87.04	88.84	88.32	87.57	89.08	88.45	89.62
88	86.34	86.88	86.89	88.63	86.51	86.20	87.69	87.18	86.41	87.94	87.43	88.69
89	85.06	85.56	85.67	87.43	85.28	84.96	86.48	85.91	85.12	86.76	86.26	87.64
90	83.53	84.21	84.27	86.19	83.91	83.70	85.15	84.58	83.80	85.33	85.01	86.51
91	82.15	82.85	82.88	84.84	82.47	82.74	83.82	83.07	82.57	84.14	83.79	85.35
92	80.58	81.44	81.36	83.48	81.01	80.68	82.40	81.65	81.13	82.72	82.34	84.00
93	79.08	80.00	79.94	82.02	79.52	79.47	80.91	80.16	79.86	81.34	80.92	82.62
94	77.48	78.39	78.31	80.52	78.04	78.05	79.52	78.56	78.55	79.91	79.53	81.21

Source: Author's calculations. See main text for more details.

D.2 Effective Tax Rates

The effective tax rates on consumption, income from labour and income from capital are calculated using the revision of Mendoza et al. (1994)'s methodology proposed by Carey and Rabesona (2003). Unless otherwise specified, all calculations are based on annual data from the OECD National Accounts and the OECD Global Revenue Statistics Database, from 1995 to 2020.

Following the method of Mendoza et al. (1994) the effective tax rates are calculated as:

$$\begin{aligned}
\tau^h &= \frac{1100}{(OSPUE + PEI + W)}, \\
\tau^l &= \frac{(\tau_h W + 2000 + 3000)}{(W + 2200)}, \\
\tau^c &= \frac{(5110 + 5121)}{(CP + CG - CGW - 5110 - 5121)}, \\
\tau^k &= \frac{[\tau_h(OSPUE + PEI) + 1200 + 4100 + 4400]}{OS},
\end{aligned}$$

where τ^h is the effective tax ratio on total household income; τ^l the effective tax ratio on labour income; τ^c the effective tax ratio on consumption; τ^k the effective tax ratio on capital income; 1100 denotes taxes on income, profits and capital gains of individuals or households; OSPUE is unincorporated business net income (including imputed rentals on owner-occupied housing); PEI denotes interest, dividends and investment receipts; W is wages and salaries of dependent employment; 2000 is total social security contributions; 3000 is taxes on payroll and workforce; 2200 is total social security contributions paid by employees; 5110 is general taxes on goods and services; 5121 is excise taxes; CP is private final consumption expenditure; CG is government final consumption expenditure; CGW is government final wage consumption expenditure; 1200 is taxes on income, profits and capital gains of corporations; 4100 is recurrent taxes in immovable property; 4400 is taxes on financial and capital transactions; OS measures net operating surplus of the overall economy.

The revised method of Carey and Rabesona (2003) accounts for the deductibility of social security contributions in the calculation of the effective tax rates. When social security contribution is non-deductible, τ^h is the same as according to Mendoza et al. (1994)'s method, while the remaining tax ratios are given by:

$$\begin{aligned}
\tau^l &= \frac{(\tau_h W + 2100 + 2200 + \alpha 2400 + 3000)}{(WSSS + 3000)}, \\
\alpha &= \frac{W}{(OSPUE + PEI + W)}, \\
\beta &= 1 - \alpha, \\
\tau^c &= \frac{(5110 + 5121 + 5122 + 5123 + 5126 + 5128 + 5200 - 5212)}{(CP + CG - CGW)}, \\
\tau^k &= \frac{[\tau_h(OSPUE + PEI) + 2300 + \beta 2400 + 1200 + 4000 + 5125 + 5212 + 6100]}{(OS - 3000)},
\end{aligned}$$

where 2100 is total social security contributions paid by employees; 2400 is total social security contributions unallocated; WSSS is compensation of employees including private employers' contributions to social security and to pension funds; α is the share of labour income in household income; β is the share of capital income in household income; 5122 is taxes on profits of fiscal monopolies; 5123 is customs and import duties; 5126 is taxes on specific services; 5128 is other taxes on specific goods and services; 5212 is taxes on motor vehicles paid by others (i.e. other than households); 2300 is total social security contributions by the self-employed and persons outside of the labour force; 5125 is taxes on investment goods; 4000 is taxes on property; 6100 is other taxes paid solely by businesses.

In contrast, when the social security contribution is deductible, τ^c is the same as in the non-deductible case while the remaining tax ratios are given by:

$$\begin{aligned}
\tau^h &= \frac{1100}{(OSPUE + PEI + W - 2100 - 2300 - 2400)}, \\
\tau^l &= \frac{[(\tau_h W - 2100 - \alpha 2400) + 2100 + 2200 + \alpha 2400 + 3000]}{(WSSS + 3000)}, \\
\alpha &= \frac{(W - 2100)}{(OSPUE + PEI + W - 2100 - 2300)},
\end{aligned}$$

$$\tau^k = \frac{[\tau_h(OSPUE + PEI - 2300 - \beta 2400) + 2300 + \beta 2400 + 1200 + 4000 + 5125 + 5212 + 6100]}{(OS - 3000)}.$$

The calculation of the effective tax rates for the twelve countries covered in the quantitative analysis is based on the formulae described above, considering that social security contributions are non-deductible in Portugal, the United Kingdom and the United States. The resulting tax rates on labour income, consumption and capital income for each country from 1995 to 2020 are reported below in Tables D3, D4 and D5, respectively.

Table D3: Tax Ratios on Labour Income, τ^l.

	AUT	BEL	FIN	FRA	DEU	IRL	ITA	NLD	PRT	ESP	GBR	USA
1995	40.03	39.61	46.05	39.68	36.17	25.04	36.49	33.05	22.94	29.82	21.53	22.79
1996	40.98	39.58	46.98	39.82	35.34	24.74	40.34	31.74	22.63	29.50	20.92	23.41
1997	41.91	39.95	44.94	39.79	35.75	24.97	41.80	30.74	22.76	28.83	20.67	23.90
1998	41.65	40.07	44.72	39.55	36.08	24.54	38.51	31.17	22.41	29.06	21.99	24.18
1999	41.70	39.64	43.54	39.84	35.89	24.81	38.77	31.53	24.16	29.17	21.94	23.28
2000	41.15	39.42	43.83	39.47	35.59	25.67	38.33	31.44	25.08	28.88	21.98	23.48
2001	42.12	39.23	43.69	38.90	35.89	24.99	38.31	29.85	24.75	29.69	22.21	23.06
2002	41.88	39.68	43.37	38.53	35.14	24.96	38.25	29.78	24.73	30.06	21.66	21.20
2003	41.82	39.61	42.41	38.87	34.85	25.13	38.46	30.15	24.53	29.84	22.02	20.37
2004	41.60	39.89	41.66	38.62	34.06	26.18	38.35	29.71	24.44	29.94	22.52	20.28
2005	41.19	38.81	42.22	39.55	33.87	26.12	38.21	30.11	24.56	30.55	22.90	20.97
2006	41.02	38.14	42.38	39.56	34.37	27.11	38.29	33.33	25.33	31.28	23.09	21.24
2007	41.16	38.20	42.19	39.12	34.59	27.10	39.59	33.49	26.49	32.03	23.03	21.50
2008	41.73	38.24	41.80	39.03	34.87	25.25	40.56	34.31	26.84	30.67	23.49	21.50
2009	41.26	38.31	40.85	38.74	35.09	26.76	40.47	33.55	25.99	29.72	22.98	19.48
2010	41.79	38.62	40.09	38.63	34.73	27.04	40.63	34.60	25.91	30.10	22.90	19.88
2011	42.16	39.01	40.74	39.34	35.23	28.83	40.18	34.94	27.95	30.67	23.50	19.74
2012	42.27	39.23	41.39	40.21	35.64	28.62	40.87	35.67	27.26	31.49	23.19	19.60
2013	43.11	39.91	41.90	41.41	35.84	29.31	41.19	35.19	30.16	31.51	22.79	21.81
2014	43.70	40.16	42.99	41.78	36.08	30.05	40.67	35.58	30.91	32.30	22.56	21.91
2015	44.41	40.31	43.23	41.85	36.46	30.25	40.60	36.22	30.81	31.72	22.70	22.35
2016	42.94	39.26	43.48	41.85	36.87	29.82	39.93	33.88	30.27	32.02	23.23	22.17
2017	42.88	39.01	42.61	41.95	37.26	29.23	39.89	34.70	30.06	32.69	23.94	22.09
2018	43.29	38.72	41.92	41.92	37.40	29.80	40.09	34.43	30.24	33.45	23.79	21.64
2019	43.51	38.14	41.71	40.71	37.57	30.51	40.71	34.51	30.14	34.13	23.98	21.80
2020	42.98	38.54	41.63	40.45	37.35	30.25	41.51	34.11	30.58	36.34	23.98	21.78
Mean	42.09	39.20	42.78	39.97	35.69	27.19	39.65	32.99	26.61	30.98	22.67	21.75

Source: Author's calculations. See main text for more details.

Table D4: Tax Ratios on consumption, τ^c.

	AUT	BEL	FIN	FRA	DEU	IRL	ITA	NLD	PRT	ESP	GBR	USA
1995	18.96	16.96	23.20	18.46	15.13	21.33	15.33	17.08	18.07	13.10	13.39	6.86
1996	19.36	17.17	23.00	18.93	14.76	21.11	14.73	17.43	18.16	13.31	13.53	6.72
1997	19.91	17.66	24.52	18.98	14.62	21.44	14.85	17.47	18.03	13.71	13.37	6.72
1998	19.93	17.39	24.30	18.80	14.75	21.75	15.39	17.34	18.75	14.84	13.40	6.66
1999	20.25	18.18	24.37	18.92	15.29	21.88	15.76	17.67	18.86	15.24	13.55	6.56
2000	19.77	17.93	24.05	17.98	15.35	22.08	15.90	17.52	17.97	14.87	13.29	6.42
2001	19.63	17.17	23.09	17.45	15.10	20.25	15.27	17.98	18.11	14.53	13.03	6.20
2002	20.02	17.45	23.22	17.39	15.14	20.64	15.24	17.25	18.36	14.39	13.03	6.08
2003	19.60	17.43	23.34	17.04	15.17	20.58	14.55	17.36	18.35	14.63	13.13	6.04
2004	19.52	18.04	23.14	17.15	15.34	21.50	14.75	17.83	18.10	14.89	13.07	6.14
2005	19.08	18.23	22.93	17.09	15.25	22.50	14.56	18.04	18.66	15.21	12.60	6.22
2006	18.57	18.23	22.72	16.90	15.38	22.38	15.09	18.43	18.85	15.12	12.47	6.20
2007	18.84	18.29	22.11	16.65	16.49	21.34	15.01	18.42	18.07	14.38	12.47	6.02
2008	18.84	17.66	21.43	16.31	16.41	19.13	14.54	18.33	17.24	12.21	12.12	5.87
2009	18.58	17.14	20.78	15.92	16.37	17.71	14.28	17.42	15.59	10.15	11.39	5.57
2010	18.50	17.57	20.71	16.78	16.12	17.95	14.97	17.92	16.44	12.65	12.69	5.69
2011	18.83	17.36	21.97	17.32	16.46	17.63	15.20	17.47	17.30	12.27	13.83	5.83
2012	19.00	17.71	21.96	17.59	16.17	18.05	15.59	17.31	17.34	12.83	13.58	5.89
2013	18.83	17.27	22.34	17.62	15.98	18.43	15.38	17.58	17.18	14.09	13.49	6.06
2014	18.82	17.19	22.01	17.88	16.02	19.43	15.83	18.01	17.61	14.50	13.52	6.09
2015	19.11	17.13	21.60	18.13	16.17	19.75	15.90	18.27	18.12	15.15	13.59	6.04
2016	19.24	17.45	22.13	18.29	15.95	19.62	16.26	19.11	18.43	15.06	13.49	5.96
2017	19.20	17.48	22.15	18.70	15.62	19.95	16.26	19.16	18.94	15.05	13.51	5.93
2018	19.05	17.45	22.36	19.02	15.91	19.28	16.11	19.33	19.16	15.19	13.45	6.09
2019	19.14	17.33	22.37	19.15	15.88	19.31	16.21	20.04	19.19	15.09	13.50	6.07
2020	18.81	17.01	22.50	18.84	15.11	17.82	15.62	20.27	18.16	14.72	12.87	5.98
Mean	19.21	17.53	22.63	17.82	15.61	20.11	15.33	18.08	18.04	14.12	13.13	6.15

Source: Author's calculations. See main text for more details.

Table D5: Tax Ratios on Capital Income, τ^k.

	AUT	BEL	FIN	FRA	DEU	IRL	ITA	NLD	PRT	ESP	GBR	USA
1995	19.12	30.09	21.30	28.09	18.20	17.72	26.49	24.99	13.82	18.57	22.47	25.05
1996	21.43	31.00	23.20	30.34	18.14	18.53	27.13	26.45	14.77	18.46	22.34	25.33
1997	22.30	31.96	23.68	31.50	17.62	18.08	28.18	26.48	15.19	20.56	24.54	25.48
1998	22.98	33.71	24.51	31.87	17.84	18.03	28.21	24.68	15.61	20.84	26.29	25.64
1999	22.24	32.87	24.43	33.21	18.86	18.39	29.69	25.70	18.28	21.10	27.10	24.83
2000	21.68	32.54	28.65	32.54	19.11	17.74	28.70	25.48	19.41	21.78	29.69	24.96
2001	25.10	32.92	23.51	32.88	16.14	16.24	29.69	23.87	17.13	20.93	28.88	22.91
2002	22.21	32.40	23.64	31.21	15.96	15.00	29.45	22.95	17.45	21.92	27.24	20.92
2003	22.08	30.99	21.71	30.70	16.66	16.27	30.82	21.72	15.18	21.94	25.81	21.38
2004	22.03	30.91	21.44	31.81	16.88	17.05	29.40	22.18	15.98	23.74	27.61	22.21
2005	21.29	31.59	21.35	32.01	17.59	17.33	28.24	22.42	16.37	25.69	28.76	24.00
2006	20.96	31.96	21.41	33.45	18.84	19.65	30.44	21.37	17.35	27.00	29.73	24.46
2007	21.25	31.09	21.85	32.52	19.18	18.48	32.05	20.84	19.33	27.69	30.08	24.94
2008	21.99	32.55	21.10	32.11	19.54	16.20	31.26	20.87	19.63	20.97	27.38	23.17
2009	20.34	29.85	19.14	29.61	19.19	13.95	31.35	18.80	16.52	18.73	26.64	19.93
2010	20.53	29.44	20.05	29.73	18.11	13.55	29.86	18.92	16.02	18.76	28.29	20.02
2011	20.74	30.56	20.73	31.71	18.97	13.36	29.76	18.57	17.59	18.48	27.70	20.56
2012	21.76	31.81	20.42	32.91	20.11	14.24	32.91	18.13	16.11	19.88	26.37	20.70
2013	22.97	33.14	21.85	34.07	20.40	14.83	32.78	19.66	19.05	20.35	26.51	21.77
2014	22.84	32.53	21.21	33.04	20.08	14.80	31.60	21.13	18.46	20.73	25.96	22.21
2015	23.37	31.35	21.74	32.04	20.54	10.66	30.83	20.82	18.85	20.79	25.96	22.60
2016	21.66	30.77	21.71	32.04	21.52	10.92	28.85	26.00	18.11	20.34	26.67	22.54
2017	22.16	32.36	21.95	33.63	21.73	10.36	28.53	25.97	18.23	20.35	26.97	24.87
2018	22.96	32.69	21.24	32.72	22.62	10.72	28.18	26.47	19.22	21.17	26.94	20.59
2019	23.55	30.37	21.32	32.05	22.53	10.14	28.51	26.97	18.35	20.58	26.21	20.70
2020	19.90	29.31	20.14	33.73	21.67	9.42	29.01	25.47	18.89	21.90	24.31	20.52
Mean	21.90	31.57	22.05	31.98	19.16	15.06	29.69	22.96	17.34	21.28	26.79	22.78

Source: Author's calculations. See main text for more details.

E Robustness Results

Table E1 presents the results from our evaluation of how the country ranking in Table 3 changes once we re-calibrate the model for all countries using either a higher Frisch elasticity of labour, $\varphi = 1$ as in Trabandt and Uhlig (2011), or the hump-shaped age productivity profiles estimated by Hansen (1993). Table E2 presents the results from our evaluation of how the country ranking in Table 3 is affected once increasing either g_A by 1 percentage point or b/y by 20 percentage points. In both tables, column headings are the same as in Table 3, except for the last one, labelled as ΔPS, which reports the change in the value of PS relative to that for the benchmark calibration presented in Table 3.

Tables E3 and E4 present the results from the regression analysis of the $CECs$ under $P1^*$ and $P2^*$ reported in Table 4, respectively. The first five columns of Table E3 report the regressions of the $CECs$ under $P1^*$ on a constant and one of the following regressors: the difference between the value of θ under $P1^*$ and $P3$ ($x1$), the population growth rate in 2050 ($x2$), PS in 2050 ($x3$), $PSEP$ in 2050 ($x4$), the difference between the intra-temporal tax wedge under $P1^*$ and $P3$ ($x5$). The last column presents the results from the multivariate regression of the $CECs$ on all five x variables as well as an intercept. For each regression we report the estimated OLS coefficients, their t-statistics in squared bracket and R-squared diagnostics. The upper panel of Table E3 presents the regression results obtained once excluding the negative $CECs$ for France and Germany under $P1^*$. The lower panel of Table E3 presents the regression results obtained once considering all $CECs$. The results in Table E4 are presented in a similar format except that the dependent variable refers now to the $CECs$ under $P2^*$, the regressor $x1$ is the difference between the value of τ_c under $P2^*$ and $P3$, the regressor $x5$ is the difference between the intra-temporal tax wedge under $P2^*$ and $P3$.

Table E1: Pension space PS_t and its main determinants, benchmark calibration, $t = 2020$: Recalibration of the Frish labour elasticity and of labour productivity

		Benchmark			Pension Limit, PL			Difference			ΔPS
	PS	$\frac{Pen_t}{Y_t}$	τ_t^p	τ_t^l	$\overline{Pen}_t/Y_t$	$\overline{\tau}_t^p$	$\overline{\tau}_t^l$	$\frac{\overline{Pen}_t - Pen_t}{Y_t}$	$\overline{\tau}_t^p - \tau_t^p$	$\overline{\tau}_t^l - \tau_t^l$	
	Frisch elasticity of labour $\varphi = 1$										
USA	51.74	5.32	8.19	21.75	11.03	26.17	62.73	5.71	17.98	40.98	-5.03
GBR	39.51	5.62	8.79	22.67	9.30	20.54	54.41	3.67	11.75	31.73	-6.70
IRE	39.48	4.16	6.50	27.19	6.87	15.12	56.86	2.71	8.62	29.66	-9.84
PRT	21.34	13.33	21.85	26.61	16.94	37.83	58.81	3.62	15.98	32.20	-3.28
NDL	17.43	12.72	20.52	32.99	15.41	32.78	59.67	2.69	12.26	26.67	-2.97
ESP	16.74	13.12	22.62	30.98	15.75	37.03	58.57	2.64	14.42	27.59	-2.70
DEU	14.78	7.76	12.31	35.69	9.10	17.47	52.61	1.34	5.16	16.91	-7.64
FIN	7.01	10.99	16.65	42.78	11.82	20.45	54.78	0.83	3.80	12.00	-6.47
BEL	4.05	9.47	15.53	39.20	9.87	17.81	47.52	0.40	2.29	8.31	-4.03
AUT	2.74	13.66	22.39	42.09	14.04	25.07	49.40	0.38	2.69	7.31	-3.22
FRA	2.69	12.29	20.84	39.97	12.63	23.61	48.06	0.34	2.77	8.10	-2.55
ITA	2.64	16.67	27.32	39.65	17.12	31.12	48.55	0.45	3.80	8.90	-2.10
	Hump-shaped labour productivity, Hansen (1993)										
USA	55.61	5.32	8.19	21.75	11.99	27.88	64.43	6.67	19.69	42.69	-1.16
IRE	47.40	4.16	6.50	27.19	7.91	17.32	59.65	3.75	10.82	32.45	-1.92
GBR	44.88	5.62	8.79	22.67	10.20	22.69	57.95	4.58	13.90	35.28	-1.34
PRT	23.17	13.33	21.85	26.61	17.34	38.26	59.24	4.02	16.41	32.63	-1.46
DEU	20.87	7.76	12.31	35.69	9.80	19.62	58.38	2.05	7.31	22.69	-1.54
NDL	18.44	12.72	20.52	32.99	15.60	32.79	59.31	2.88	12.27	26.31	-1.96
ESP	17.77	13.12	22.62	30.98	15.95	36.12	56.10	2.83	13.50	25.12	-1.66
BEL	6.88	9.47	15.53	39.20	10.17	18.84	50.64	0.70	3.32	11.43	-1.20
FIN	6.57	12.02	18.21	48.08	12.86	21.69	58.97	0.85	3.48	10.90	-6.91
AUT	5.01	13.66	22.39	42.09	14.38	26.52	52.99	0.72	4.13	10.91	-0.94
FRA	4.10	12.29	20.84	39.97	12.82	24.18	49.34	0.52	3.35	9.37	-1.14
ITA	3.77	16.67	27.32	39.65	17.32	31.95	50.44	0.65	4.62	10.79	-0.97

Source: Author's calculations. All data in percentage. ΔPS is the change in the value of PS relative to that for the benchmark calibration reported in Table 3.

Table E2: Pension space PS_t and its main determinants, benchmark calibration, $t = 2020$: Higher economic growth and government debt

		Benchmark			Pension Limit, PL			Difference			ΔPS
	PS	$\frac{Pen_t}{Y_t}$	τ_t^p	τ_t^l	$\overline{Pen}_t/Y_t$	$\overline{\tau}_t^p$	$\overline{\tau}_t^l$	$\frac{\overline{Pen}_t - Pen_t}{Y_t}$	$\overline{\tau}_t^p - \tau_t^p$	$\overline{\tau}_t^l - \tau_t^l$	
	Increase of GDP growth rate g_A by 1 percentage point										
USA	67.62	4.18	6.42	13.62	12.89	35.90	67.20	8.72	29.48	53.58	10.85
IRE	64.29	3.27	5.10	17.76	9.14	23.32	59.29	5.88	18.22	41.53	14.97
GBR	61.55	4.22	6.60	12.38	10.98	30.25	60.15	6.76	23.66	47.77	15.34
DEU	47.16	5.60	8.89	19.75	10.60	26.78	59.18	5.00	17.89	39.43	24.74
POR	42.23	10.24	16.79	14.39	17.74	47.27	62.41	7.49	30.47	48.02	17.61
BEL	40.68	6.29	10.31	18.56	10.60	27.46	53.38	4.31	17.16	34.82	32.60
NET	40.02	9.37	15.11	19.15	15.62	53.20	91.27	6.25	38.09	72.12	19.62
ESP	38.84	9.42	16.24	17.54	15.41	45.19	60.33	5.98	28.95	42.79	19.41
FIN	36.83	8.10	12.27	26.52	12.82	28.90	61.13	4.72	16.63	34.61	23.35
FRA	34.67	7.73	13.10	17.99	11.83	33.02	53.47	4.10	19.93	35.48	29.43
AUT	32.52	9.15	14.99	20.88	13.55	34.65	56.36	4.41	19.66	35.48	27.28
ITA	30.85	10.72	17.57	17.50	15.50	40.67	53.31	4.78	23.10	35.81	26.12
	Increase of debt-GDP ratio b/y by 20 percentage points										
USA	52.97	5.39	8.30	22.45	11.47	25.98	63.53	6.08	17.68	41.07	-3.80
IRE	47.40	4.18	6.53	26.68	7.94	17.42	60.41	3.76	10.90	33.73	-1.92
GBR	41.98	5.72	8.94	23.87	9.87	21.42	57.38	4.14	12.47	33.50	-4.23
POR	20.72	13.52	22.16	27.80	17.05	35.87	56.07	3.53	13.71	28.26	-3.90
DEU	17.43	7.95	12.61	37.77	9.62	18.50	56.71	1.68	5.89	18.94	-4.99
NET	16.39	12.95	20.88	34.32	15.49	31.73	58.90	2.54	10.85	24.58	-4.01
ESP	15.68	13.38	23.07	32.34	15.87	34.96	55.74	2.49	11.90	23.40	-3.76
FIN	10.36	11.19	16.95	44.26	12.48	22.03	59.56	1.29	5.08	15.31	-3.12
BEL	4.37	9.79	16.05	42.00	10.24	18.55	51.50	0.45	2.49	9.50	-3.71
AUT	3.40	14.07	23.07	44.59	14.57	26.18	53.22	0.49	3.11	8.63	-2.56
ITA	2.07	17.33	28.41	43.06	17.70	31.31	50.18	0.37	2.90	7.12	-2.67
FRA	1.73	12.90	21.87	43.90	13.13	23.59	49.12	0.23	1.73	5.22	-3.51

Source: Author's calculations. All data in percentage. ΔPS is the change in the value of PS relative to that for the benchmark calibration reported in Table 3.

Table E3: Determinants of $CECs$ under $P1^*$

Regressors	Negative $CECs$ excluded					
Intercept	-0.0671	0.8173	1.3456	0.5136	0.4854	-0.1796
	[-0.1009]	[2.9559]	[6.2395]	[1.2815]	[2.4238]	[-0.1180]
$x1$	-0.1270					-0.0512
	[-1.9345]					[-0.5622]
$x2$		-1.6272				-1.4171
		[-2.1076]				[-2.3015]
$x3$			-0.0277			0.0092
			[-2.8929]			[0.2969]
$x4$				1.2075		-0.3429
				[1.9267]		[-0.3319]
$x5$					-1.7071	-1.7398
					[-4.7825]	[-3.6814]
R-squared:	0.3187	0.3570	0.5113	0.3170	0.7409	0.9347
Adj. R-Squared:	0.2335	0.2767	0.4502	0.2316	0.7085	0.8531
Sample size	10	10	10	10	10	10
	All $CECs$ included					
Intercept	-0.3503	0.6003	1.0240	0.4325	0.3407	-1.2905
	[-0.5005]	[1.9182]	[3.8296]	[0.9473]	[2.0884]	[-2.1305]
$x1$	-0.1354					-0.1072
	[-1.9101]					[-1.9838]
$x2$		-1.5146				-1.1502
		[-1.6569]				[-2.3103]
$x3$			-0.0207			0.0298
			[-1.6165]			[2.0600]
$x4$				0.8881		0.4241
				[1.2656]		[0.6345]
$x5$					-1.9018	-1.9411
					[-6.0155]	[-8.0768]
R-squared:	0.2673	0.2154	0.2072	0.1381	0.7835	0.9421
Adj. R-Squared:	0.1941	0.1369	0.1279	0.0519	0.7618	0.8938
Sample size	12	12	12	12	12	12

Source: Author's calculations. In each regression, the dependent variable is the CEC under $P1^*$ reported in Table 4, $x1$ is the differential in θ under $P1^*$ and $P3$, $x2$ is the population growth rate in 2050, $x3$ is PS in 2050, $x4$ is $PSEP$ in 2050, $x5$ is the tax wedge differential on the intra-temporal margin under $P1^*$ and $P3$. t-statistics are reported in squared brackets. The upper panel refers to regressions that exclude the two negative $CECs$ for France and Germany. The lower panel refer to regressions that include all $CECs$.

Table E4: Determinants of $CECs$ under $P2^*$

Regressors	Negative $CECs$ excluded					
Intercept	0.0373	1.0669	1.1834	0.8919	1.1561	-1.9193
	[0.0239]	[3.8130]	[4.5400]	[2.3227]	[5.8305]	[-1.7333]
$x1$	0.1413					0.3484
	[0.7191]					[2.7051]
$x2$		-0.5185				-1.8440
		[-0.5429]				[-2.3022]
$x3$			-0.0044			0.0331
			[-0.3543]			[1.8267]
$x4$				0.5269		-0.4284
				[0.8356]		[-0.4155]
$x5$					-0.1087	-0.2522
					[-2.0864]	[-4.3211]
R-squared:	0.0543	0.0317	0.0138	0.0720	0.3260	0.8409
Adj. R-Squared:	-0.0507	-0.0759	-0.0958	-0.0311	0.2511	0.6818
Sample size	11	11	11	11	11	11
	All $CECs$ included					
Intercept	-0.5667	1.1037	0.9797	1.0073	0.9911	-2.7681
	[-0.3306]	[3.4658]	[3.5988]	[2.2435]	[3.8419]	[-1.2287]
$x1$	0.2021					0.4536
	[0.9294]					[1.7346]
$x2$		0.4766				0.1091
		[0.5124]				[0.0783]
$x3$			0.0049			0.0274
			[0.3794]			[0.7351]
$x4$				0.0032		0.0612
				[0.0047]		[0.0289]
$x5$					-0.0450	-0.1655
					[-0.6969]	[-1.4539]
R-squared:	0.0795	0.0256	0.0142	0.0000	0.0463	0.4335
Adj. R-Squared:	-0.0125	-0.0719	-0.0844	-0.1000	-0.0491	-0.0386
Sample size	12	12	12	12	12	12

Source: Author's calculations. In each regression, the dependent variable is the CEC under $P2^*$ reported in Table 4, $x1$ is the differential in τ^c under $P1^*$ and $P3$, $x2$ is the population growth rate in 2050, $x3$ is PS in 2050, $x4$ is $PSEP$ in 2050, $x5$ is the tax wedge differential on the intra-temporal margin under $P2^*$ and $P3$. t-statistics are reported in squared brackets. The upper panel refers to regressions that exclude the negative CEC for Italy. The lower panel refer to regressions that include all $CECs$.

F Transitional Dynamics and Demographic Uncertainty

In this Appendix, we study the sensitivity of the results in Figure 8 with respect to the uncertainty in the population growth rate projections. We find that our results are robust, in the sense that the path of generational welfare is found to be qualitatively similar to those presented in Figure 8. When assuming instantaneous implementation, policy $P1^*$ ($P2^*$) dominates $P3$ in terms of generational welfare after the birth year 2009 (2014) under the scenario of low fertility (20th percentile of the population growth rate), and after the year 2010 (2015) under the opposite scenario of high fertility (80th percentile of the population growth rate). The quantitative welfare effects are stronger by a factor of approximately 2 in the case of low rather than high fertility. In addition, we find that $P1^*$ dominates $P2^*$ in the long run for the case of low fertility, though only to a small extent.

Figure F.1 presents the transitional dynamics under policy $P3$ (instant change) for the three fertility scenarios low (blue line), medium (red line) and high (green line) corresponding to the 20th, 50th and 80th percentiles of the population growth distribution. The effects of demographic uncertainty on the dynamic behaviour of the aggregate variables is qualitatively the same for the policies $P1$ and $P2$ and for this reason it is not presented.[24] Evidently, aggregate labour supply $\tilde{L}$ mirrors the fertility projection and fans out with increasing time horizon. Both labour income and social security taxes have to rise with declining fertility. The aggregate capital stock is hump-shaped and peaks around the year 2070. The initial increase in the capital stock during 2020-2050 is caused by a composition effect in the population. In the initial phase of the demographic transition, the share of the 50-60-years old cohorts increases. These cohorts are characterized by the highest savings rate of the population so that aggregate savings even increase. As the share of retirees rises, however, the number of households with negative savings rates increases and aggregate capital stock starts to decline

[24] Braun and Joines (2015) consider the effects of different fertility scenarios on the transition dynamics for the case of Japan (for constant retirement age, though). In their analysis, the consumption tax adjusts to balance the fiscal budget. In addition, they emphasize the increase in health expenditures in a graying economy. Somehow surprisingly, they find that a higher fertility may exacerbate Japan's short- to medium-run fiscal imbalances due to the forward-looking behaviour of the individuals.

again. Unsurprisingly, capital accumulation declines for lower fertility. Production mimics the behaviour of the aggregate capital stock to a smaller extent and already peaks in the year 2045 due to the decline in labour. As a consequence of the factor price dynamics, the real interest rate declines and even undershoots its long-run value.

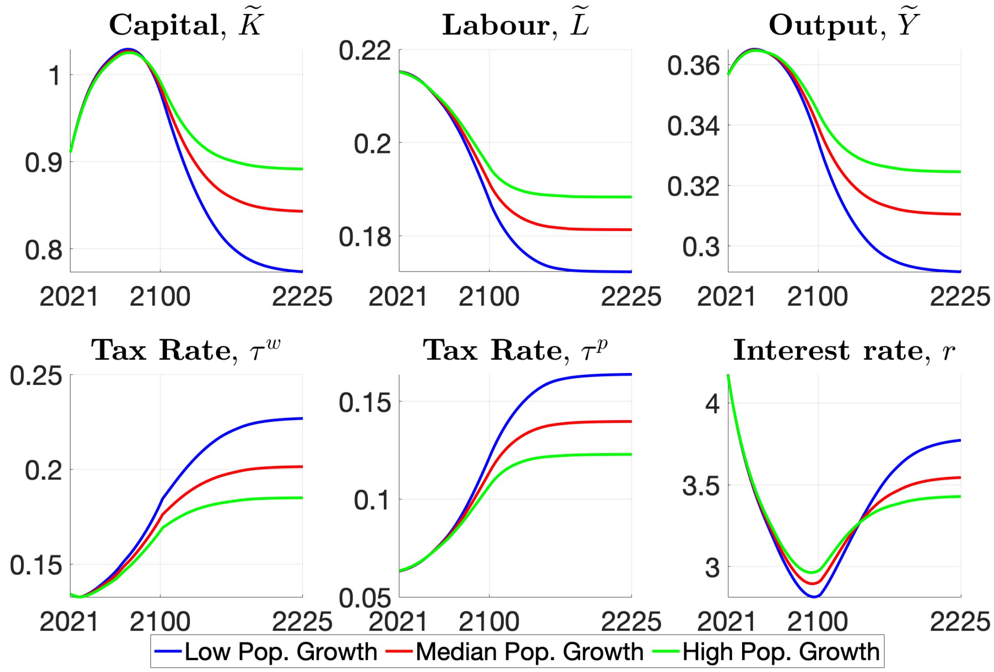

Figure F.1: Transition dynamics under $P3$ for fertility scenarios low (20th percentile of the population growth distribution), medium (50th percentile) and high (80th percentile). Policy change implemented once for all in 2021.

Figure 9, which is reported in the main paper, displays how the generational welfare effects of pension reforms change as a result of the variation in the demographic outlook. Welfare effects over time almost have the same shape to those prevailing in the medium fertility scenario presented in Figure 9. In particular, all generations born after 2009-2010 (2014-2015) prefer policy $P1^*$ ($P2^*$) to policy P3. In addition, the biggest loser from a change in policy among the generations currently alive amounts for those born around the year 1980 under policy $P1^*$. Notice, however, that the welfare effects of the policies are more pronounced in the low fertility case where the long-run consumption equivalent gains of policies $P1^*$, $P2^*$

and P3 relative to the benchmark amount to 12.7%, 12.9% and 10.9%, respectively (compared to 6.7% 6.5% and 5.4% in the high fertility case). Further, in the case of low fertility, $P2^*$ dominates $P1^*$ in the long run, while the opposite holds in the case of higher fertility.