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Macro Longevity Risk under the New Dutch Pension Deal

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**Hoc autem: qui parce seminat, parce et metet;
et, qui seminat in benedictionibus, in benedictionibus et metet.**

2 Corinthians 9:6

Abstract

After months of negotiation, years of political tension and decades of interdisciplinary research, the Dutch social partners agreed on a pension system reform. The resulting proposed contract allows for more risk taking - in its broadest sense- in employer funded pension plans on an individual level than before. This suggested design, though, comes with challenges on how to allocate the longevity and financial risks among all pension fund participants. I investigate, given this new contract, the implications of systematic longevity risk in the Dutch population on the financial position of pension funds and, consequently, on pension rights of both workers as well as pensioners. After quantifying this risk, I propose a practically feasible way to deal with the faced macro longevity risk using a fund-wide buffer. I find that an adequate buffer policy may help coping with macro longevity risk within a fund.

Keywords: Macro longevity risk, Solidarity, Variable annuities, Pension systems, Buffers

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Contents

1 Introduction

Solidarity in its widest sense has been a key concept in the light of interhuman relations since the beginning of mankind's history. The question to what extent a society should be designed to protect its members against the inability to fulfill their material needs, is a normative rather than a mathematical one. However, insights from both biology and history tell us that humans are in general inclined to feel some kind of responsibility for the well-being of the members belonging to the group they live in. Meltzov (2002) notes presence of specific brain cells which enable us to show empathy towards other human beings. Aristotle characterized man as *a social animal by nature* and used this as a backbone of his motivation for the way the public sphere should be designed in Greek city states ($\pi o \lambda \eta \varsigma$) in the 4th century BC (Miller, 2017).

Though failing to come up with, e.g., an optimal life cycle investment strategy in the context of a defined contribution public pension scheme or, say, a definition of the concept *citizen* which would include women or slaves, even ancient society designers acknowledged to some extent the intrinsic need humans have to be part of a group where members are protected from potential harm. The statement in the latter clause is not too specific yet, but categorizing the concept of poverty within the set of events that cause harm to an individual experiencing it, gives some more room to work with. Let *poverty* be defined by the potential that one might at some point in time not be able to fulfill their material needs. Barientos, Gorman and Heslop (2003) show that elderly people in a society are exposed to this risk to a significant extent. Provision of old-age pension to individuals above a certain age may help alleviating this risk.

Especially after WWII, European governments started establishing pension provision plan legislation and incentivation. The Dutch system is characterized by the so called *three pillars*. The first consisting of a pay-as-you go system to which every citizen is entitled. By the *second pillar* a employer funded pension plan is meant, aimed at those, who have at some point in life been part of the active labor force. Depending on how long and for which wage one has worked, one is entitled to a pension income. The *third pillar* consists of individual savings and investments of any kind one wants to set aside for retirement, in excess of the pension income coming from the first two *pillars*. Within the *second pillar*, the soon to be implemented legislative changes provide potential room for risk-taking on an individual level. Historically, namely, this system has been characterized by its collective nature. Implicitly, pension right holders share both investment and longevity risk with each other across generations. However, in recent years, concerns regarding the extent of desirability of risk-sharing are voiced. Fueled by societal, economical and fiscal developments, a trend of increasing flexibility with regard to labor is observed (Bolhaar, Brouwers and Scheer, 2016). Wren (2013) observes these type of trends on a global level as well, albeit of different cause for different countries and industries. This course incited a need for establishment of a new pension deal, which would be better suited in this new reality, in the hearts of decision makers within in the pension landscape.

Even though the change in pension system design is, mainly within the second pillar, characterized by a shift from collective risk-sharing towards more risk-taking on an individual level, policymakers do not entirely part from the solidarity-based nature of the initial system. In short, solidarity in this context boils down to risk sharing among pension fund participants. I will illustrate the way the in itself rather inexpressive concept of solidarity has been given substance during the (post-)modern era in section 2.

When mentioning solidarity and risk-sharing, it is also important to actually specify which risks are faced by pension funds and its participants. Fund members put in a so called *premium*, which is of a part of labor income, in a pension fund for each period of their working life. These premia are invested in order to generate returns. The basic idea is that the asset value in the fund matches the present value of all promised pension provisions. The returns on investment are in general not guaranteed and there exists a potential that this return does not match its expectation for the period of interest. This potential is referred to as *investment risk*. If this risk materializes, a pension fund's asset value is not at the level it was expected to be, causing possible challenges to keep up with the promises made to individual fund members.

Longevity risk is fundamentally different from investment risk and can be split into two parts: *micro-longevity* and *macro longevity risk*. A pension fund determines the price of a pension right depending on its beliefs with respect to the remaining life expectancies of its fund members. This, namely, determines for what time a pension fund expects to pay out pension payments to its pension right holders. The concept of longevity risk refers to the potential the fund ex-ante expectations regarding mortality of fund participants are not matched in reality. Micro-longevity risk refers to the potential that in a specific year, less (or more) people pass away than expected. This impacts a pension fund's liability value as the amount of members that are entitled to a pension benefit to be paid out, is higher (or lower).

Macro-longevity risk refers to the potential that ex-ante beliefs regarding mortality distributions over time happen to be breached by reality. In other words: if the change in life expectancy over time of fund members is more (or less) severe than expected, this impacts the liability value as well. Namely, if an individual's life expectancy increases more rapidly than expected, so does the present value of his/her pension rights, as the expected payout period has gone up.

Within the light of systemic changes within the Dutch pension contract design, which opens up possibilities for individuals - or the employer who makes the decision on their behalf - to take on some of the risks described above by themselves. This leads to a pension income which is higher in expectation, though more uncertain and also possibly lower than in the case in which individuals are not explicitly exposed to these risks (Balter, Kallestrup-Lamb and Rangvid, 2019).

Instead of fully exposing fund members to investment and longevity risks and the inherent income uncertainty, the new pension deal offers possibility to cover part of materializing risks by means of a fund-wide buffer. Abel (1990) shows that people in general do not like a consumption pattern - which is a *de facto* description of what a pension payout stream matches by design - which decreases along the way. This notion is re-evaluated and endorsed by Van Bilsen, Bovenberg and Laeven (2019) in a life cycle investment context. These findings provide backing for incorporating a mechanism which enables the pension fund to prevent pension cuts to some extent.

The Socio-Economic Council (SER) has proposed a new contract, which is referred to in Dutch by the self-explanatory name 'het nieuwe contract', in which employers get the possibility to let their employees share risks along the whole life cycle. Moreover, individuals build up their own personal pension wealth during the accumulation phase of the life cycle. It becomes possible for an individual to take on more risks himself in order to gain a higher expected pension income, rather than being obliged to insure to some extent as in the old setting. Now the question arises to what extent it is a good idea to actually take on these risks yourself as a participant. Muns and Werker (2019) propose a return allocation mechanism for returns on financial risk exposure within a pension contract. The developed procedure yields an increase in perceived welfare compared to the *status quo* using intergenerational risk sharing (Muns and Werker, 2019).

Regarding non-financial risk, De Waegenaere, Joseph, Janssen and Vellekoop (2019) propose a modelling approach in which specific age groups can take over longevity risk from other age groups within the context of a pension fund. Boeijen, Bonenkamp, Bovenberg, Frehen, de Haan, Joseph et al. (2016) investigate the welfare effects and potential risks the presence of intergenerational risk sharing in a pension contract contains. They argue that intergenerational sharing of macro longevity risk provides a welfare increase (Boeijen et al, 2016). Piggott, Valdez and Detzel (2005) provide modeling framework for benefit dynamics when fund participants are explicitly exposed to investment, micro longevity risk and macro longevity risk.

I will investigate how inclusion of a buffer structure to deal with macro longevity risk impacts pension provisions. To this end, I model both population wide mortality data as well as the impact the risks they embed have on pension benefits. The modelling approach of this thesis will hence consist of a mortality model and benefit model.

I carry out this research as part of an internship at APG, which is the pension service provider for some of the Netherlands' largest pension funds. The motivation behind this research is to gain more insight in the extent to which the new proposed pension deal provides room for coping with macro longevity risk that fund participants face. In order to perform this analysis I will make use of pension fund participant data of a fund of which APG is the executive. Using mortality tables from the Human Mortality Database, I first need to quantify the systematic longevity risk pension funds and their participants actually face. For this I will apply estimation methods to longevity data, with eye for both exploring patterns in the historical data as well as for predicting future developments. Lee and Carter (1992) proposed a forecasting method where mortality distributions over time are characterized by a time-specific as well as an age-specific component. Qiao and Sherris (2013) performed a factor analysis using a Gaussian Makeham model and Booth, Maindonald and Smith (2002) use a autoregressive approach. Li and Lee (2005) extended the factor based approach proposed Lee and Carter (1992) by extracting relevant explanatory factors from multi-population mortality data. Cairns, Blake and Dowd (2008) evaluate performance of different multi-period mortality forecasting methods and use these insights to determine a suitable hedging strategy using mortality-linked products.

After quantifying the extent of macro longevity risk, I will generate scenarios for materializing risk, with respect to systematic longevity risk as the concerning risk factor, in variable annuities under the new Dutch pension contract. Balter and Werker (2016) described how under Dutch law variable annuities can be implemented. They focus on how a certain degree of stock market risk taking influences payment patterns for an individual. To this end they make use of the expected return a specific strategy and show that a higher expected return by a more risky strategy allows an individual to receive a higher initial income stream upon retirement (Balter and Werker, 2016). Balter et al. (2019) propose a way to use the notion of individual exposure to macro longevity risk in the context of unguaranteed pension products. They find that longevity risk can, if it materializes, reduce pension provisions of fund participants significantly (Balter et al., 2019).

A practically feasible way to deal with macro longevity risk can be a buffer approach. Metselaar, Nibbelink and Zwaneveld (2020) propose how this can be carried out for financial risks in the newly proposed Dutch pension contract. Forman and Sabin (2014) propose a way to explicitly account for materializing longevity risk in a defined contribution pension plan.

Within the context of variable annuities, Piggott et al. (2005) propose a model for pooled annuity funds in which longevity shocks are translated to monetary shocks for individual age groups without inter- or intragenerational risk sharing. Richards, Currie and Ritchie (2012) carry out a multi-period approach for examining the impact of macro longevity risks within a pension scheme.

In section 4 I will present how I model and forecast mortality over time. Such a model is needed to be able to pose claims, based on data of past experience, about the nature of future survival probability distributions. In section 5, I will present a model with regard to pension rights. Here I will show how mortality forecasts affect individual pension fund participant's benefit level.

Using these formulated models, I will present my research and simulation methodology in section 6. The results regarding mortality, benefit level and buffer dynamics are presented in section 7. Possible relevant directions for future research are provided in section 9.

In my research I will focus on how changes in an individual's pension benefit level as a result of materializing macro longevity risk can be mitigated using a *solidarity buffer* construction. Assuming a uniform exposure of all participants to the macro longevity risk factor, I find that, in case substantial buffer levels are met, changes in pension income can to some extent be prevented and protect pension fund members to some extent from welfare losses. Buffer requirements become larger in case the assumed expected investment return decreases. Given this setting, also the probability that the buffer is used for softening pension cuts to some degree, increases if one assumes a lower expected investment return.

Provided directions for future research lie in the domain of, i., alternative risk allocation principles among fund participants and, ii., practically feasible buffer dynamics modeling.

2 The concept of solidarity

Within the context of pension systems, *solidarity* basically refers to sharing of risks within a pension fund across its members. Boeijen et al. (2016) argue that implementation of intergenerational risk sharing mechanisms increases welfare. Question remains how the role of the concept of solidarity has been given substance in the public sphere throughout the years and how this relates to social security structures in which a pension system has its place. This section aims to provide interpretation and guidance for grasping the ideas behind pension system designs.

As noted in the *Introduction*, ideas regarding the social nature of the human being were, either implicitly or explicitly, taken into account in societal designs since ancient history already (Miller, 2017). This provides a trace of a motivation with regard to the implicit need for a *social contract* between public institutions and individuals in a society (Rousseau, 1762). However, it does not entirely explain nor prove existence of the need for some sense of solidarity among individuals who are part of a societal group.

When defining solidarity in its widest sense, it is necessary to note a distinction between *normative* and *de facto* solidarity (De Wit, 1999). *Normative solidarity* can be explained as a moral imperative applicable to a citizen to take care of his/her fellow citizens to some extent. Explanation of *de facto* solidarity should be approached from a more semantic point of view. The discussion how display of solidarity should be part of the *social contract*, became explicit for the first time around the era of the French Revolution in the same country that "hosted" these series of historical events. (De Wit, personal communication, March 2020). *De facto* solidarity is based upon a notion of factual interdependence among citizens. There are certain types of risks that can happen to all of us as humans. The most suitable example to illustrate this, is related to longevity risk. The potential that one becomes old, is not able to work anymore and hence fails to provide basic material needs, is a scenario that could - if nothing with regard to insurance of this risk would yet be arranged in a society - happen to everyone. Even though some people are more vulnerable than others to materialization of this risk, noting that this risk is homogeneous in the sense that it could happen to every citizen, is not completely out of line.

In case a consensus exists about the statement that a certain risk is of homogeneous nature, i.e., if there exists consensus that its materialization can happen to harm each individual in the society, the need for incorporation of a way to cope with this specific risk within the social contract becomes apparent. In the Netherlands, this need has been given substance by means of implementation of a wide variety of social security schemes after WWII, among which we find the *AOW*, which makes up the *first pillar* I referred to in the *Introduction*. Recently however, this common stance with regard to the nature of risks has been put under pressure due to societal developments as sketched in Bolhaar et al. (2016). Individuals take on more risks with regard to labor income during their working life and within the public sphere, the discussion with regard to implementing more risk taking on individual level within a pension scheme has intensified as well. [1]

While incorporating risk sharing mechanisms, like pension schemes, within legislation, a government in fact assembles an implicit value framework with regard to solidarity for its citizens. As long as these citizens in general terms implicitly agree with this value framework, a system succeeds in fulfilling the need for incorporating risk sharing schemes within the *social contract* between citizen and its government.

[1] It is worth noting that either of these developments may be fueled by worries regarding increasing costs of the system which may not have anything to do with the developments I sketched so far. This does, however, not make the relation between the described developments non-existent *per se*.

All in all, the concept of solidarity has played an essential role in the discussion with regard to pension system design. When looking beyond the monetary definition of sustainability, a pension system is only viable if its fundamental values with regard to solidarity are underwritten by the participants it is designed to serve. Departing from past values with regard to desirability of risk sharing, which the transition to the new pension contract in fact boils down to, suits changes with regard to the public desires concerning individual risk exposure in recent years. However, it might render the system insufficiently capable of serving the needs or match the values with regard to the nature of solidarity of the participants it serves. Incorporation of a solidarity buffer mechanism within the newly proposed pension contract as proposed within the analysis in this thesis, might help to maintain aspects of the collective nature which characterizes Dutch social security. This may, in combination with the proposed design changes which are more up-to-date with the present labor market and society, add to a more sustainable pension system where pension fund members' needs are fulfilled.

3 Annuity modelling

Given the scope of my research and in the light of the nature of the pension contracts described in section 2, I will elaborate on the way annuity prices will be modelled in this thesis.

In addition to the design structure of Dutch pension contracts, it is also worth noting that there exists empirical evidence that people prefer a consumption stream which does not decrease sharply from one time period to another (Abel, 1990; Bansal and Yaron, 2004). This phenomenon is called *habit formation*, referring to the consumption magnitude that people get used to and do not like to give up.

Summarizing, given the design elements sketched in section 1 and 2, I want the annuity payout model to

- be able to forecast changes in funding rate, because decisions on adjustments of pension payouts in practice are carried out based on the funding rate [2]
- respect the still collective rather than individualistic nature of fundamentals of the Dutch pension system
- give an output resulting in a payout pattern which is approximately stable in expectation
- allow for quantifying shock magnitudes, after which shocks can be redistributed across all fund participants as well as over time
- be able to model pension rights in both the accumulation and decumulation phase, i.e., it should do the job during working life and after retirement [3]

The price of annuity at time s, which starts paying unity as of retirement, for an individual in age group x at time s can be priced at time s as follows:

$$a_{x,s}^{(g)} = \sum_{k=\max\{0, ret.Age-x\}}^{M-x} \frac{{}_k p_{x,s}^{(g)}}{(1+\delta_k)^k} \qquad \forall s \forall x \forall g \in \mathcal{P}, \tag{1}$$

with δ_k being the discount rate at time k corresponding to period k and ${}_k p_{x,s}^{(g)}$ the probability that individual belonging to population age group x and population group g at time s survives another k years. M represents the maximum age in the population, $\mathcal{P}$ is a countable set denoting all combinations of characteristics there exist in a population (e.g., male or female).

Along the course of this thesis, these annuity prices will be estimated, based on available information. Future mortality distributions will, namely, not be known exactly, so in order to come up with an estimation for $a_{x,s}^{(g)}$ for all x, s, g, forecasts for ${}_k p_{x,s}^{(g)}$ will need to be made for all x, s, g. In section 4, this will be given substance along the lines of a modelling approach for mortality. Discount rate δ_k will be referred to as expected investment return within the context of the analysis performed in this thesis.

[2] This is the case in current Dutch legislation. Even though the transition to a more DC-like plan might soften the need for this requirement, it is still the case that provision adaptations are based on asset versus liability value.

[3] In current DB legislation workers acquire pension rights while employed, in a more DC-like setting, workers put money aside in a personal money account.

4 Longevity modelling

4.1 Modelling approach motivation

In order to research the extent of systematic longevity risk a pension fund faces, one first needs to model the nature of mortality events. I.e., one needs to come up with a mathematical way to explain and predict probabilities that an individual survives a particular time period. This in practice boils down to modelling survival probabilities for certain age groups over time. The methodology I use is based on the model proposed by Lee and Carter (1992). I use this model in a multi-period setting such that systematic mortality risk is adequately captured. Systematic risk concerns the risks that come with our nature of modelling. I.e., those risks that materialize when reality happens to violate fundamental ex-ante assumptions. Within the context of longevity modelling, the main systematic risks are *model risk*, *basis risk* and *trend risk*, (Richards, Currie and Ritchie, 2012). By model risk, the potential that the chosen model as a whole is wrong, is meant (Richards et al., 2012). Basis risk refers to the possibility that calibration data is not representative for the research question of interest. I will focus on *trend risk* in this setting, which captures, given the chosen model, there happens to exist a discrepancy between modelled mortality trend and the trend observed in reality (Richards et al., 2012). Note that this observed trend is not inconsistent with the chosen model, though it does not match its expectation.

The reason for this focus is that, given past experience of population survival probabilities, I will assume a trend in future mortality evolution. This trend assumption may seem rather premature at this stage, but within the existing literature there is consensus that period specific one-year survival probabilities tend to increase over time due to mankind's healthcare improvements (Richards et al., 2012; Lee and Carter, 1992). This is also what we have seen in the past from the data (CBS, 2020). Moreover, even if the setting of my research does not exhibit such trend behavior, it simply would become clear after estimating the model that no mortality trend would be observable. However, the mortality model should be flexible enough to capture such a trend in mortality distributions over time in order for statements with regard to systematic mortality risk to make sense. Moreover, if life expectancy decreases, this will also become clear from model estimation.

The idea is that I will derive scenarios for which this trend does not match the ex-ante predictions. I will simulate scenarios for future evolution of mortality and use these to investigate the impact of the shock on pension benefit payments using the benefit model in section 5.

I will start by proposing and explaining the multi-period model as also used in Richards et al. (2012) within a value-at-risk framework. The reason why I use a multi-period mortality model, is because within the context of the new pension deal, it is crucial to evaluate impact of risks over a longer period of time. In practice, namely, parameters and calculation axioms (Dutch: *grondslagen*) are updated every three, for large pension funds, or five years, for smaller pension funds. The prescribed period-specific mortality distributions are updated after two years by the Dutch Actuarial Society. Also, policymakers' major system design change proposals tend to pend for over decades (see section 1). Within the horizon of my analysis, I should hence allow for mortality experience within multiple years in the future in order to draw valuable conclusions on a pension plan's participants' exposure to systematic longevity risk.

Worth noting is that in existing literature, a concept called *stochastic mortality* is covered. Plat (2009) proposes a stochastic way of modelling future mortality probabilities. The idea is to give the predicted trend uncertainty based on observed data. Cairns, Blake, Dowd, Coughlan, Epstein and Khalaf-Allah (2009) evaluate different modelling approaches while using data of several Anglo-Saxon countries. For practical reasons, I choose not to include these proposed extentions in my approach.

For my research it is important to find a way to translate mortality developments to implications on annuity values. To this end it is valuable to note that Cairns et al. (2009) cover this in their applications using stochastic projections of mortality rates. Within the context of systematic mortality risk it is valuable to allow for new information during the simulation process and translate simulated experience into annuity price shock scenario's.

The goal of this model is to quantify the degree of systematic longevity risk within a population. The quantitative output of this longevity/mortality model will serve as *shock scenarios* of which I investigate the impact on pension payments using the benefit model (see 5). This benefit model is used to investigate how shocks, affecting the value of the pension provision liabilities of a fund, are passed on to plan participants. Namely, the presence of, among others, motives of solidarity and buffering in the contract makes the translation of shocks to provisions non-trivial and also different than in a variable annuity setting as in Qiao and Sherris (2013) and Balter et al. (2019).

4.2 Modelling historical human mortality

For the prediction of future mortality, I make use of the model proposed by Lee and Carter (1992). The key notion in this section is that I model the population survival probabilities rather than the observed number of deaths. In order to be able to model this probability the literature suggests modelling of a monotonic transformation of the survival probability in order to prevent violation of usual axioms associated with probability spaces (Lee and Carter, 1992).

For a probability measure $\mathbf{P}$, *force of mortality* is defined as follows (Melenberg, Stevens and De Waegenaere, 2010):

$$\mu_{x,s} = \lim_{\epsilon \to 0} \frac{\mathbf{P}(T_{x,s} < \epsilon)}{\epsilon}, \qquad \forall x, s \tag{2}$$

where $T_{x,s}$ stands for the remaining lifetime of an individual aged x at time point s.

Now $\mu_{x,s}$ from (2) can, for all x and s, be, in approximation, transformed to survival probabilities which are needed to derive the price of a variable annuity as in (1). The survival probabilities are approximated as follows:

$$p_{x,s} \approx 1 - exp(\mu_{x,s}) \qquad \forall x, s. \tag{3}$$

Here $p_{x,s}$ is the probability that an individual aged x in year s survives one more year. Consequently, we derive the h year survival probability $hp_{x,s}$, representing the probability that an individual aged x in time period s survives h more years by the following relation using the one year probabilities as referred to in (3):

$$_hp_{x,s} = \Pi_{j=0}^{h} p_{x+j,s+j} \qquad \forall h \in \mathbf{N}_+ \forall x, s \tag{4}$$

where $p_{x,s}$ is for all x and s computed by (3).

Having defined and explained survival probabilities, question remains how to model a population's mortality distribution over time. Lee and Carter (1992) propose a factor model where human period specific survival probabilities are disentangled in a time period and a age dependent factor. In this approach $\mu_{x,s}$ from (2) is explained by the following relation (Lee and Carter, 1992):

$$ln(\mu_{x,s}) = \alpha_x + \beta_x * \kappa_s + \epsilon_{x,s} \quad \forall x, s \tag{5}$$

subject to:

$$\sum_s \kappa_s = 0, \tag{6}$$

$$\sum_x \beta_x = 1. \tag{7}$$

The vector $\vec{\kappa}$ is centered around zero by (6) and (7) puts a restriction on vector $\vec{\beta}$. This prevents overidentification and guarantees uniqueness of the estimated model solution (Lay, Lay, McDonald, 2016). In (5), κ_s represents the time period specific parameter explaining human mortality for period s; $\vec{\beta}$ is a parameter vector of which the entries indicate the sensitivity of age group x to time trend $\vec{\kappa}$. $\epsilon_{x,s}$ in (5) accounts for all x, s for the aspect of mortality which is not explained by the model. Lee and Carter (1992) assume that $\epsilon_{x,s}$ follows a normal distribution with zero mean.

I useforce of mortality data from the Human Mortality Database which are transformed by (3) in cases where survival probability is needed. (5) is calibrated using past mortality data $s = t_0 = 1978$ till $s = t = 2018$. As $\epsilon_{x,s}$ is assumed to be zero in expectation, the parameters in (5) can be estimated by the maximum likelihood estimator (5) (Pitacco, Denuit and Haberman, 2009):

$$\min_{\hat{\beta}_x, \hat{\kappa}_s} [\sum_s \sum_x ln(\mu_{x,s}) - \hat{\alpha}_x - \hat{\beta}_x * \hat{\kappa}_s]^2. \tag{8}$$

Because $\vec{\kappa}$ and $\vec{\beta}$ are bound to (6) and (7), respectively, α_x is estimated by the method of moment estimator and can be calculated by taking the average over time of $ln(\mu_{x,s})$ for all x (Lee and Carter, 1992; Bain and Engelhardt, 1992). (7) and (6) in combination with a singular value decomposition of $ln(\mu_{x,s}) - \hat{\alpha}_x$ now yields a unique solution for the vectors $\vec{\beta}$ and $\vec{\kappa}$. From (3) and (5) it follows that these parameter estimations can be used to estimate survival probabilities by using the following relation:

$$\hat{p}_{x,s} = 1 - exp(\hat{\alpha}_x + \hat{\beta}_x * \hat{\kappa}_s) \quad \forall x, s. \tag{9}$$

Worth noting is that this model as introduced by Lee and Carter (1992) can be extended by adding variables accounting for the cohort an individual belongs to (Renshaw and Haberman, 2006). Also Li and Lee (2005) propose an extension which takes trends in, e.g., other countries or sex groups than the one of interest into account as well, which leads empirically to performance increase when explaining observed data (Li and Lee, 2005). However, within my scope of research these extensions are not prioritized for practical reasons.

The goal of my research is to quantify the extent of systematic longevity risk within the Dutch pension contract. Part of this contract is that a buffer which can to some extent be used to cover for changes in annuity prices can be present. To this end, I should assess how annuity prices are affected by new information. Let $\mathcal{F}_s$ be a filtration containing all information known at time s for all s. Now given $\mathcal{F}_t$, denoting the information available at time t, I estimate the annuity value[4] for a deferred life annuity starting coupon payments at time $s = t + max(ret.Age - x, 0)$. When investigating systematic longevity risk, it is important to evaluate by

[4]Dutch jargon: *contante waardefactor*

how much this annuity value changes as new information becomes available in the future. As the reader will probably acknowledge, mathematics is quite powerful. However, even mathematical modelling cannot perfectly forecast the future - which is to some extent fortunate as it prevents my research becoming irrelevant due to absence of risk in general. Anyway, to come up with a measure for systematic risk, I need to get a grasp on how the information set $\mathcal{F}_s$ evolves over time for $s = t, t+1, t+2, ...$ Using known data, I will simulate scenarios for key elements within $\mathcal{F}_{t+h}$ where $h \in \mathbf{N}_+$, i.e., I will derive instances for the model parameter(s) of interest within $\tilde{\mathcal{F}}_{t+h}$, where the tilde distinguishes observed information sets from simulated "virtual experience" . These I will use to calculate updated annuity prices, which lead to a measure for macro longevity based *biometric return*.

In section 4.3 I will elaborate on how I use multi-period mortality forecasts to estimate these annuity prices and evaluate the risks materializing if these estimations will not be matched by reality. This will give insight in the possible degrees of severity of the mismatch between forecasts and reality, which can be distilled as so called "shock-scenarios". In section 5 I will propose a model mimicking how these shocks affect liability values for a pension provider responsible for paying out pension rights. This quantified impact on fund-wide liabilities will in turn be redistributed over fund participants. [5]

4.3 Mortality forecasts

First I will make use of past mortality data to derive a process for the time specific parameter, which drives the risk within the process modelling future mortality rates. In section 4.2 I described how to decompose known mortality data into a time-specific and an age specific part. Within the context of mortality distribution modelling in the context of my research regarding systematic risks within the pension system, three main points in time will be of in interest. Let t be the year up till which mortality data is available. Now let h be the number of years ahead I want to investigate: the variable of interest will be $a^{(g)}_{x,t+h}$ and its estimation will be denoted by $\hat{a}_{(g)}{}_{x,t+s}$, which is the time $t+h$ value of an annuity which pays unity to insured after time of retirement for the rest of his life. To measure the macro longevity biometric return, i.e., to measure by how much this annuity price has changed during the period of interest. I introduce, similar to Piggott et al. (2005):

$$CWA^{(g)}_{x,t,t+h} = \frac{\hat{a}^{(g)}_{x+h,t+h}|\tilde{\mathcal{F}}_{t+h}}{\hat{a}^{(g)}_{x+h,t+h}|\mathcal{F}_t} \qquad h = 1, 2, 3, ... \tag{10}$$

Here CWA stands for *contante waardefactor aanpassing*, which translates to "annuity value change"; subscript x denotes the age of an individual.

Time t annuity prices for a life annuity paying unity after year of retirement are, see (1), estimated as follows in this context:

$$\hat{a}^{(g)}_{x,t}|\mathcal{F}_t = \sum_{k=max\{0,RetAge-x\}}^{M-x} \frac{{}_k p^{(g)}_{x,t}}{(1+\delta_k)^k}|\mathcal{F}_t \qquad \forall x, \tag{11}$$

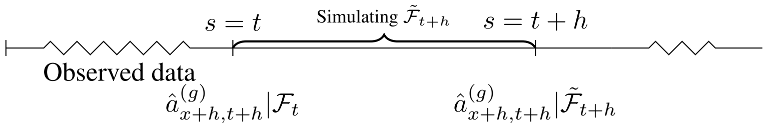
where δ_k represents the discount factor corresponding to year k and *ret.Age* refers to the retirement age and${}_k p^{(g)}_{x,t}$ follows from (3). Note that within the context of this analysis, discount rate δ_k refers to

[5]The extent to which it is legally prohibited to change a participants pension income rights is left out of scope of research for two main reasons: i., Shocks expressed in monetary values on a fund-wide level are - to put it mathematically - robust for changes in legislature regarding redistribution within the fund and ,ii., when modelling impact on an individual level, legal changes can be incorporated within the risk assignment mechanism I will introduce in section 5. The regarding dynamics are also explained in section 5.

Note that in this context, for a time t value of holding an annuity at $t+h$, we get, similarly:

$$\hat{a}^{(g)}_{x+h,t+h}|\mathcal{F}_t = \sum_{k=max\{0,RetAge-(x+h)\}}^{M-(x+h)} \frac{{}_kp^{(g)}_{x+h,t+h}}{(1+\delta_k)^k}|\mathcal{F}_t \quad \forall x; h \in \mathbf{N}_+. \tag{12}$$

Hence to investigate the degree of systematic risk, simulations are performed for the time period between t and $t+h$. In the timeline below this idea is visualized:



Now it is important to be more concrete about which elements of $\mathcal{F}$ we want to simulate. In order to determine a measure for macro longevity biometric return, we need to estimate annuity prices (see (10)). From (11) it follows that for this estimation, survival probabilities are needed. The systematic risk element now lies in the fact that the survival distribution may change between t and $t+h$. At t it is possible to estimate the survival probabilities for all age groups x holding as of time $t+h$, say $\hat{P}_{t+h}|\mathcal{F}_t$. This matrix $\hat{P}_{t+s}$ can be used to calculate annuity prices by (11), which can be plugged into (10).

Now the reason for a change in survival probabilities within our modelling context is that within the period between t and $t+h$, mortality data may be observed which, after re-estimation of $\hat{P}_{t+h}$ at time $t+h$, lead to $\hat{P}_{t+h}|\tilde{\mathcal{F}}_{t+h}$ being different from $\hat{P}_{t+h}|\mathcal{F}_t$,[6] which hence lead to an updated annuity price by (11) which will be reflected in CWA_x for all x by (10).

The entries of the survival probability matrices P, are period and age specific specific survival probabilities. Given our modelling choice, disentangling mortality in age and time specific components (see (5)), (9) shows how parameter estimations of our model can be translated to tangible survival probabilities. As I want to quantify macro longevity risk, I want to find a way to model what actually drives a population's survival probabilities. As we model risk over time, it is most useful to dive more into how the time specific parameter evolves. For this k_s the following process is modelled as follows (Richter and Weber, 2009):

$$k_{s+1} = c + k_s + \eta_s, \quad \forall s \tag{13}$$

where η_s is assumed to follow a prior normal distribution centered around zero with standard deviation σ_{η_s}. The drift term c is estimated by historical mortality data by fitting an ARIMA model (Booth, Maindonald and Smith, 2002):

$$\hat{c}_s = \frac{\hat{\kappa}_s - \hat{\kappa}_{t_0}}{s - t_0}, \quad \forall s \tag{14}$$

where t_0 represents the first point in time for which mortality data is incorporated in the analysis. This so called *random walk with drift* process can be used to draw instances for a series of κ_s for $s = t+1, ..., t+h$. κ_s will in turn determine $\hat{P}_{t+h}|\tilde{\mathcal{F}}_{t+h}$ by (9). This drawing will be dependent on the distribution of η_s. An instance η^i_s will in turn yield a $\hat{\kappa}^i_{s+1}$ by (6).

For all s, we assume for the distribution form of η_s:

$$\eta_s \sim N(0, \sigma_s) \tag{15}$$

The motivation for such an approach is that it does not require nested simulation or variance reduction methods, which might not be feasible given the nature of the relation between κ_s's over time (Tsitsiklis and

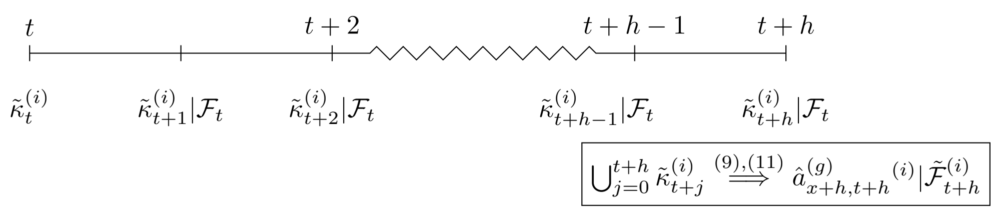
[6] Technically this event happens with probability one as the set of events in which simulated experience exactly matches time s expec-

van Roy, 2001). These techniques might be of use, however, in future research if one wishes to link intermediate observed κ_s to resulting final annuity adjustments, which might speed up simulations (Krah, Nicolic and Korn, 2019). However, Richards et al. (2012) point out that the approach used in this context is by design not affected by the need for nested simulations as the focus is on the macro longevity risk, rather than on using intermediate results to generate scenarios for micro longevity risk.

Then I will use this process to simulate the evolution of this time specific parameter, κ_s over the time. We will simulate trajectories for κ_s for $s = t, t+1, ..., t+h$ where t is the current point in time and h indicates the number of years we simulate ahead. Index s will be chosen such, that the resulting κ_s trajectories for $s = t, t+1, ..., t+h$ serve as input for the annuity value in (11), which in turn yield an adjustment factor as in (10) representing the *biometric return* of a certain time period. This will then be translated to a quantified impact on the pension rights as in section 5.

In the analysis in section 7 I will first show what the impact of systematic longevity risk is for both a one and a two year horizon. The model described in this section is by construction also suitable for use on a larger prediction horizon (Richards et al., 2012).

The timeline below is used to illustrate these series of simulated events, where it is explicitly noted that the intermediate κ_s draws depend on data observed until time $s = t, .., t+h$.



This series of $Tilde\kappa_{t+j}^{(i)}$ for $j = 1, .., h$ is in turn transformed to a survival probability matrix by (9), after which an updated annuity price is calculated by (11). In fact, this instance i for mortality distribution yields an instance for $\hat{a}_{x+h,t+h}^{(g)}{}^{(i)}|\tilde{\mathcal{F}}_{t+h}^{(i)}$ as used in (10). Now this change in mortality distribution over time can be translated to a measure for macro longevity risk by (10), yielding *de facto* instances for $CWA_{x,t,t+h}$, say $CWA_{x,t,t+h}^{(i)}$ for each simulation instance i. These *scenarios* will, in turn, affect benefits the way described in section 5. The simulated instances $\tilde{\kappa}_{t+j}^{(i)}$ for $j = 0, 1, ..., h$ yields a new estimation for the trend parameter $\hat{c}_{t+h}$ as follows from re-estimating (14). Note that $\bigcup_{j=0}^{t+h} \tilde{\kappa}_{t+j}^{(i)} \in \tilde{\mathcal{F}}_{t+h}^{(i)}$ is used in (11) to calculate the annuity price and hence generate an instance for $CWA_{x,t,t+h}^{(i)}$ by (10).

tations has measure zero for continuous distributions. This statement, however, also has zero practical applicability in this context, so that is why I will consider shock scenarios rather than noting that "there is always a discrepancy between estimations and reality".

5 Benefit model

Within the context of the new pension deal we need to come up with a model which gives an output regarding how much pension benefits one can finance with a given pension wealth W_s^x assigned to an individual belonging to the group of people aged x at time period s. Using (11) we can translate these individual wealth levels to corresponding benefit levels upon retirement. Summarizing: both in the context of a model with and in the one without a solidarity buffer, we need to take three variables into account:

- Wealth over time, W_s^x
- Annuity price $a_{x,s}$
- Number of people present of a particular age at time s, say N_s^x

5.1 Wealth dynamics

The modeling approach is based on the model proposed by Piggott et al. (2005) in a variable annuity context.

Wealth levels can be derived recursively from a given W_s^x at all time s by (Piggott et al., 2005):

$$W_{s+1}^{x+1} = (1 + \pi_s^x)(W_s^x - B_s^x * \mathbf{1}_{\{x \geq ret.\ age\}}), \qquad \forall s \tag{16}$$

where W_{s+1}^{x+1} stands for the wealth level of the individual belonging to age group x at s, which has turned one year older. The investment return assigned to the cohort aged x at time s is denoted by π_s^x. Note that this π_s^x is in principle a random variable, though observed at the end of year s. When interested in the wealth level of a cohort further in the future, one can simply iterate forward using (16) since it holds for any s, if one knows corresponding wealth and benefit levels.

Note that the relation in (16) could be extended by including premium payments into the analysis. This is, however, not straightforward as premium payments cannot be expressed as a fixed part of wealth.

Note, moreover, that the benefit level I use in my analysis is the existing level that is determined at year t, which does not coincide with the newly calculated benefit levels determined one year later.

B_s^x in (16) stands for the benefit payment paid out to an individual aged x at time s. It is determined such, that paying out this liability each year after retirement, leads to a stable consumption pattern in expectation. B_s^x is calculated along the lines of section 4.2 as follows:

$$B_s^x = \frac{W_s^x}{a_{x,s}} \forall x, s, \tag{17}$$

where $a_{x,s}$ stands for the price of a pension product, paying out unity each year after retirement in case the individual in age group x at s is still alive and can be found by (11).

An important notion in this setting is that materialized longevity risk affects the fund wide wealth level because there are more (less) people, to whom benefit payments should be paid out, which costs (saves) the pension fund money. In this particular year, the survival probability differs from its ex-ante expectation, which affects the total amount of pension payouts. However, if this unexpected mortality observation causes estimations with regard to future liability values to change, this change of liability valuation does not create or cost extra wealth on a fund level *per se*. In that case, however, benefit levels are adapted by (17) in order to arrive at a suitable payout stream, which is constant after retirement in expectation.

Individual accounts of deceased participants are redistributed across fund participants. On a fund level, the following budget constraint needs to be respected in this regard at each point in time (Piggott et al., 2005):

$$\sum_x N_s^x W_{s+1}^{s+1} = \sum_x B_{s+1}^{x+1} * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1} \qquad \forall s, \tag{18}$$

where N_s^x stands for the amount of people aged x at time s for all x, s. $a_{x+1,s+1}|\mathcal{F}_{s+1}$ is retrieved by (11) and stands for the value of a pension liability paying unity each survived year after retirement for an individual aged $x+1$ at time $s+1$ given the information at hand at times $s+1$.

The intuition behind (18) is that after each year, the total wealth belonging all age cohorts summed together (left-hand side of (18)) is redistributed across the surviving fund members. The resulting allocation of pension rights should in total have the same value as the total wealth present in the fund. In other words: a pension fund's assets at hand should match it's future liabilities, which are the at time s existing benefit levels. This liability value (right-hand side of (18)) is dependent on the $s+1$- value of an annuity for a specific age group, the allocated level of pension rights and the amount of people alive in the regarding age group.

$a_{x+1,s+1}$ is retrieved from the mortality model described in section 4. The risk involved with estimation of this variable is referred to as *macro longevity risk*. The potential that the realized number of deceased individuals differs from its expected value, is called *micro-longevity risk*. In the context of this research, I do not take the latter into account and assume that:

$$N_{s+1}^{x+1} = p_{x,s} * N_s^x. \qquad \forall s, x \tag{19}$$

When looking at (18) at time $s = t$, question remains how to determine B_{t+1}^{x+1} for all x. In contrast to the situation in time period t, it is no longer the case that it follows from (17) as there does not exist an unambiguous answer to the question what W_{t+1}^{x+1} should be for each age group x. There are now in fact x unknown variables, identified by only one equality.

To solve this problem I propose the approaches I will treat in section 5.2 and 5.3. I will first consider a case where shocks in longevity are uniformly allocated. Then I will propose incorporation of a so called *solidarity buffer* in line with the newly proposed Dutch pension deal.

5.2 Uniform allocation of shocks

In this section I will propose a uniform allocation of of longevity shocks at time $s = t$, where t is the last observed data point for calibration data. This means that each age group is allocated the same relative change in their pension income. In mathematical terms: B_{t+1}^{x+1} from (18) is set to:

$$B_{t+1}^{x+1} = \gamma B_t^x \qquad \forall x; \gamma \in \mathbf{R} \tag{20}$$

In case of uniform shock allocation in a fund with different age groups, the benefit payment adaptation $\hat{\gamma}$ needed in order to not breach budget neutrality now follows from filling (20) into (18) and solving for $\hat{\gamma}$:

$$\hat{\gamma} = \frac{\sum_x N_t^x W_{t+1}^{x+1}}{\sum_x B_t^x * a_{x+1,t+1}|\mathcal{F}_{t+1} * N_{t+1}^{x+1}}. \tag{21}$$

Here the numerator stands for the total wealth present in the fund at $t+1$, which is the sum of all personal wealth accounts of the people who were alive at t. The intuition behind the denominator is that it tells us how much it would cost to pay out the benefit level upon retirement, calculated given the wealth accounts and annuity prices at time t by (17), to all people who are still alive at time $t+1$. This is done, using the annuity prices for time $t+1$, $a_{x+1,t+1}|\mathcal{F}_{t+1}$.

In a homogeneous fund with respect to age, sex and wealth level, one can show that in case of simulating an instance i for random variable $a_{x+1,t+1}|\mathcal{F}_{t+1}$, say $a_{x+1,t+1}^{(i)}|\mathcal{F}_{t+1}$, the corresponding instance i for $\hat{\gamma}$ as in (21), the following holds: $\hat{\gamma}^{(i)} = CWA_{x,t,t+1}^{(i)}$ (Piggott et al., 2005). Here $CWA_{x,t,t+1}^{(i)}$ is the annuity value adjustment factor introduced in (10).

The intuition behind this result is that, in case of absence of investment risk, in a fund in which all members have the same age and sex, annuity prices for all fund members are equally affected by updated mortality predictions. As shocks are distributed uniformly and the shock is the same for every fund member, each individual's benefit level is adapted by the same factor as by which annuity prices for the concerning age and sex change.

5.3 Solidarity buffer

When incorporating a solidarity buffer (:=SB) into this approach, one actually introduces an extra asset and an extra liability in the model. The idea is that this buffer is filled and/or paid out each year. A share of each participant's premium and part of the fund-wide excess return is used used to fill the SB. This SB is in turn exposed to a certain investment strategy. Depending on the level of the SB at the end of the year, there might be a payout which can be shared across fund participants. The main characteristics in terms of SB inflow are: i. Each year a maximum of percentage of premia is used to fill SB, and, ii., similarly a certain percentage of excess return is added to the existing SB.

Conversely, the SB can be used to compensate (part of) an occurring negative shock with regard to pension rights. Also, regardless of the extent of materialization of investment and longevity risks, a fixed percentage of the existing SB is paid out each year.

In this research I will focus on how uniform allocation of shocks affects buffer dynamics. The idea is that implementing the adaptation in benefit level derived from (21) which is needed to absorb the shock, might be too harsh too implement upon fund participants. In order to account for this, the introduction of a fund wide solidarity buffer, which can be used to absorb part of the shock, may come in handy. I will look at how this works out at time $s = t$, being the point in time where the last observation with regard to data is made.

The solidarity buffer is filled with both premium payments and excess returns. Let SB_t represent the level of the solidarity buffer on a fund level at the start of year $s = t$, just before has flown out of it to fund participants. Let SB_{t+1} be the level just after payout to the fund members. This SB_{t+1} is hence the initial level of the solidarity buffer in year $t + 1$, before excess returns are added. Note that the investment mix, the solidarity buffer is exposed to, need not necessarily be the same portfolio the wealth accounts are exposed to, which motivates the choice to use a different notation for these variables.

Let Φ_t be the amount of money that flows into the solidarity buffer at the end of year $s = t$ from either positive or negative shocks in longevity. Note that in the latter case Φ_t is negative as the design is intended such, that negative macro longevity shocks are (at least partially) absorbed by extracting money from the SB.

As this research aims to look at the impact of using this buffer in the context of longevity risk, investment returns are assumed to be equal to their expectation. In general, this is not respecting the nature of the investment profile to which the SB is exposed. However, the scope of this research is to investigate the mere impact of incorporating a buffer structure to cover for longevity risk. For follow up research it might be valuable to investigate the extent to which specific allocations of investment returns into the SB can impact the shock smoothing performance of the SB. I will further elaborate on this in (9).

With the idea sketched above in mind, we define for each s for the buffer dynamics:

$$SB_{s+1} = (1 + r_s)SB_s + \Phi_s \geq 0 \ \ \forall s. \tag{22}$$

Now the budget constraint, which should be respected at each time period s becomes:

$$-\Phi_s + \sum_x N_s^x W_{s+1}^{x+1} = \sum_x B_{s+1}^{x+1} * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1}. \ \ \forall s \tag{23}$$

Here each variable has a similar interpretation as in (18). Here all funds which were in the SB are either used to finance benefit liabilities or put back into the SB.

Now question remains how to to distribute the funds flowing out of the SB at the end of the year $s = t$, i.e., how to allocate Φ_t, across fund participants, after it's size has been determined.

The idea is to cap the shocks in benefit level due to materializing macro longevity risk an individual can face. Let $\tilde{\gamma}_t$ be the allocated uniform adaptation in benefit level at time t that all fund participants face, such that, in line with (20), we have that $B_{t+1}^{x+1} = \tilde{\gamma} B_t^x$. Now let the for the value of the adaptation factor be bounded from above and from below, such that:

$$\gamma_{min} \leq \tilde{\gamma} \leq \gamma_{max}, \tag{24}$$

with $\gamma_{min} \in \mathbf{R}, \gamma_{max} \in \mathbf{R}$.

For investigation of suitability of the SB in this context, it is now interesting to check how often (24) is violated and how SB_s evolves over time for $s = t, t+1, ...$

Now it makes sense, as stated above, to determine how Φ_t is defined. To this end, $\hat{\gamma}$ from (21) should be compared to γ_{min} and γ_{max} from (24) to come up with the desired in- and outflow levels. Note that it might happen that, in some case of a negative shock, SB_t is insufficient to satisfy both (23) and (24). For practical and regulatory reasons, the model is not extended to account for sophisticated strategies to smooth shocks over multiple periods or shorting cash to fill up the SB to the desired level. Future research might look into how smoothing shocks over time in a buffer context affects the analysis. Nijman, Van Stalborch, Van Toor and Werker (2013) propose an approach to incorporate such a mechanism within the context of financial risk factors. I will prioritize budget constraint (23) over (24) and hence look at how often the SB_t will not be enough to keep benefit cuts between the boundaries of interest.

Note that this problem does not pop up in case of positive shocks, as the SB is not bounded from above in this model.

For Φ_s the following now holds for all s:

$$\Phi_s = \begin{cases} -\min[(1+r_s)SB_s, \sum_x (\gamma_{min} - \hat{\gamma}) B_s^x * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1}] & \text{if } \hat{\gamma} < \gamma_{min} \\ 0 & \text{if } \gamma_{min} \leq \hat{\gamma} \leq \gamma_{max} \\ \sum_x (\hat{\gamma} - \gamma_{max}) B_s^x * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1} & \text{if } \gamma_{max} < \hat{\gamma} \end{cases} \tag{25}$$

The intuition behind this is that the amount of money flowing out of the SB is equal to the difference between the monetary value of the implied shock allocation from (21) and the desired boundaries in this regard as denoted in (24).

To check whether (25) makes sense, one can split the right hand side of (23) into two parts. Namely, after filling in $B_{s+1}^{x+1} = \hat{\gamma} B_s^x$, one can split $\hat{\gamma} B_s^x$ into an excess part and an allocated part of shock in benefits. For the case of a large positive shock this works as follows. $\hat{\gamma} B_s^x = (\hat{\gamma} - \gamma_{max}) B_s^x + \gamma_{max} B_s^x$. Now the present value on a fund level of the pension rights part in excess of γ_{max}, flows into the SB, i.e., it equals Φ_s. This idea is captured by (25) and works similarly for a negative shock.

For the corresponding realized allocation at time $s+1$, $\tilde{\gamma}_{s+1}$, now the following holds for all s:

$$\tilde{\gamma}_{s+1} = \begin{cases} \frac{(1+r_s)SB_s + \sum_x N_s^x W_{s+1}^{x+1}}{\sum_x B_s^x * a_{x+1,s+1}^{x+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1}} & \text{if } \hat{\gamma} < \gamma_{min} \wedge (1+r_s)SB_s < \sum_x (\gamma_{min} - \hat{\gamma}) B_s^x * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1} \\ \gamma_{min} & \text{if } \hat{\gamma} < \gamma_{min} \wedge (1+r_s)SB_s \geq \sum_x (\gamma_{min} - \hat{\gamma}) B_s^x * a_{x+1,s+1} | \mathcal{F}_{s+1} * N_{s+1}^{x+1} \\ \hat{\gamma} & \text{if } \gamma_{min} \leq \hat{\gamma} \leq \gamma_{max} \\ \gamma_{max} & \text{if } \gamma_{max} < \hat{\gamma} \end{cases} \tag{26}$$

Now note that only in the first case distinguished in (26), we have that $\tilde{\gamma}_{s+1} < \gamma_{min}$. In the other three cases, $(1+r_s)SB_s$ is sufficient to either cover the shock that would have been materialized (second case) or there is no outflow out of the buffer (third and fourth case). Linking these notions to the model described so far, one should note that $\tilde{\gamma}_{s+1}$ from (26) only breaches (24) in the first case of the equation in (26).

Intuitively, by using the notion of uniform shocks in the context of (23), (26) can be generalized as follows:

$$\tilde{\gamma}_{s+1} = \frac{-\Phi_s + \sum_x N_s^x W_{s+1}^{x+1}}{\sum_x B_s^x * a_{x+1,s+1}|\mathcal{F}_{s+1} * N_{s+1}^{x+1}} \quad \forall s \tag{27}$$

where Φ_s follows from (25).

Budget neutrality is verified for each of these cases by filling in $\tilde{\gamma}_s$ into (20) and this result into (23) consecutively. Working out these cases, one gets in cases in which the buffer is used, that right hand side of (23) can be split into a part accounting for the excess shock and a part accounting for the allocated shock. Solving for Φ_s, one should find that (25) is satisfied. In case the buffer is unused, budget neutrality follows from filling (21) into (20) and this result into (23), consecutively.

6 Research and simulation methodology

Using the models described in section 4 and 5, we want to quantify the impact that systematic longevity risk has on benefit levels. In section 6.1 I will describe how to perform this analysis in case the risk is shared equally across all individuals. In section 6.2 I will describe how the methodology works in case one would allow for existence of a solidarity buffer.

6.1 Uniform shock allocation

As said: we want to link mortality results to benefit payments. The simulation approach for mortality results h years ahead, is described by the following steps for an instance i at time period $s = t$ being the last point in time where calibration data is observed:

6.1.1 Mortality

- *Step 1a:* Explain observed mortality data in terms of model components from (5) and estimate $\hat{c}_t$ from (13) by (14)
- *Step 1b:* Derive a distribution for η_t from (15) given observed data
- *Step 1c:* Using this distribution, draw an instance i of a h dimensional vector of $\tilde{\eta}^i_j$ $(j = 1, ..., h)$, say $\tilde{\eta}^i$
- *Step 1d:* Use (13) to translate $\tilde{\eta}^i$ from *step 3* to a corresponding $\tilde{\kappa}^{(i)}$ vector which represents a set of simulated observations for the time specific mortality parameter
- *Step 1e:* Use this instance to estimate corresponding instances for the force of mortality at the simulated time steps. I.e., create $\tilde{\mu}^i_{x,j}$ for $j = t, t+1, ..., t+h$ by using (5)
- *Step 1f:* Perform Step 1a again, though now using time t observed data with column vectors $\tilde{\mu}^i_{x,j}$ for $j = t, t+1, ..., t+h$ appended to it as input data. This results in a corresponding instance $\hat{c}^{(i)}_{t+h}$
- *Step 1g:* Forecast new future mortality rates by forecasting future $\hat{\kappa}_s$ for $s = t+h+1, t+h+2, ...$ by using (28) below

In Step 1g we use (13) and set:

$$\hat{\kappa}_{t+h+z} = \tilde{\kappa}_{t+h} + \hat{c}_{t+h} * z. \; ; z \in \mathbf{N}_+ \tag{28}$$

The resulting estimations for κ_s for $s = t+h+1, t+h+2, ...$ from (28) can be used to make a matrix containing survival probabilities by making use of (9). These are in turn used to make an updated version estimation for the value of the annuity of interest by (11).

Since we are interested in quantifying the impact of macro longevity risk on benefit payments, we have to derive instances for $\hat{a}_{x+h,t+h}|\mathcal{F}_t$, which is the time $t+h$ value of an annuity paying unity after retirement for an individual aged x at time t, estimated at time t and $\hat{a}_{x+h,t+h}|\tilde{\mathcal{F}}_{t+h}$, being the estimated value of the same annuity, though now evaluated at time $t+h$ using forecasts for κ_s for $s = t+h+1, t+h+2, ...$ from Step 1g. These annuity values imply benefit levels by (17). In order to arrive at an instance for the benefit level, corresponding to the experienced mortality shock, we perform the algorithm in section 6.1.2.

6.1.2 Benefit and wealth levels

First note that for the amount of people still alive at time $t+h$, I use the estimation:

$$\hat{N}_{t+h}^{x+h} =_h p_{x,t} * N_t^x \qquad \forall x \tag{29}$$

where $_hp_{x,t}$ are the same as will be used in *Step 2a* and these are computed from single year survival probabilities by (4).

In order to arrive at benefit and wealth levels, corresponding to the mortality results generated from the procedure in section 6.1.1, we now perform the following algorithm:

- *Step 2a*: Use *Step 1a* from the *mortality*-algorithm in section 6.1.1 to arrive at an estimation for future mortality by (28) and (9). The resulting annuity value estimation $\hat{a}_{x,t}$ can be plugged into (17) to arrive at an estimation for the height of the pension payments that can be financed from wealth level W_t^x. Let this result be B_t^x
- *Step 2b*: Fill in B_t^x for all x into (16) to arrive at $\tilde{W}_{t+1}^{x+1}$, denoting the wealth level of an individual alive and aged $x+1$ at time $t+1$ before redistribution of wealth within the fund for all x
- *Step 2c*: Employ (21) with B_t^x, $\tilde{W}_{t+1}^{x+1}$ from Step 2b and $\hat{a}_{x+1,t+1}|\tilde{F}_{t+1}$ resulting from Step 1g as input for all x, which yields $\hat{\gamma}$
- *Step 2d*: Fill in $\hat{\gamma}$ from Step 2c into (20) to arrive at B_{t+1}^{x+1}
- *Step 2e*: Now the wealth level after redistribution of funds W_{t+1}^{x+1} follows from (17) for all x and the result satisfies (18) given B_{t+1}^{x+1} from Step 2d

These benefit and wealth levels enable us, in combination with annuity values based on mortality results from section 6.1.1, to derive quantities of interest with regard to the solidarity buffer, of which the dynamics are proposed in section 5.3.

6.2 Solidarity buffer

In case of inclusion of a solidarity buffer which is aimed to absorb (some) of the longevity shocks, it makes sense to investigate how large this buffer should be given a fund decomposition.

Given the results following from the procedure described in section 6.1 and the model and dynamics proposed in section 5.2 and section 5.3, I will investigate buffer requirements at time $s = t$ within a pension fund composition for a specific cut regime characterized by $\Gamma := (\gamma_{min}, \gamma_{max})$ from (24).

The goal is to arrive at a suitable value to which SB_t can be set to be able absorb cuts to the desired extent. Let $SB_t^* \in \mathbf{R}_+$ be the level SB_t should be set to in order to satisfy (24) given the concerning regime Γ. In other words, we want to find, SB_t^* such, that (24) is satisfied with $1 - \alpha$ probability for some $\alpha \in (0, 1)$. Summarizing, we want:

$$\mathbf{P}(\tilde{\gamma}_{t+1} \geq \gamma_{min}) \geq 1 - \alpha. \tag{30}$$

This equals the probability that $\tilde{\gamma}_{t+1}$ satisfies (24). The goal is to find a value, to which SB_t can be set, in order to satisfy (30). Formalizing this yields:

$$SB_t^* = \inf\{SB_t \geq 0 : \mathbf{P}[(1 + r_t)SB_t \geq \sum_x (\gamma_{min} - \hat{\gamma})B_t^x * \hat{a}_{x+1,t+1}|\tilde{\mathcal{F}}_{t+1} * N_{t+1}^{x+1}] \geq 1 - \alpha\}, \tag{31}$$

which would by (26) satisfy (24). The intuition behind this, is that SB_t is set such, that pension benefit adaptations at the end of year t can be kept such, that these respect the cut regime. Corresponding outflow is in that boundary case intuitively equal to $(1 + r_t)SB_t$, which can be checked by evaluating (25) from the model in section 5.3. From (25) can, consequently, be seen that $\tilde{\gamma} \geq \gamma_{min}$ holds.

A suitable SB_t is, for a set of $\hat{\gamma}$, given the results from the simulation procedure described in section 6.1, determined by:

$$SB_t^* = max\{0, q_{1-\alpha}(\sum_x (\gamma_{min} - \hat{\gamma})B_t^x * \hat{a}_{x+1,t+1}|\tilde{\mathcal{F}}_{t+1} * N_{t+1}^{x+1})\}/(1 + r_t), \tag{32}$$

where r_t is the investment return in time period t. This boils down to setting the buffer level at t, SB_t^*, equal to the $1 - \alpha$ quantile of the realized monetary excess shock at $t + 1$, discounted one period back.

The amount flowing out of, or into, the solidarity buffer, Φ_t at time t follows, in turn, using $\tilde{\gamma}$ from filling in SB_t^* into (25). This outflow is used to finance (part of) the pension liabilities, while respecting (23).

7 Results

In this section I will present the results for the simulation performed as described in section 6. I will first present in-sample estimates for the parameters in the mortality model as presented in section 4. Hereafter I will show the resulting out-of sample forecasts of the mortality distribution, which are used to estimate annuity prices.

I will use these estimations to generate scenario's for observed mortality out-of sample. At first the simulation period will be one year out of sample, after which I will present results for a two year out of sample prediction. This case is intuitively most close to what happens in practice, as the Dutch Actuarial Society presents new life tables biennially.

Each scenario implies a prediction for future survival probabilities and, hence - by (11) - an estimate for the annuity values for each age group. This gives the opportunity to revise initial annuity value predictions,i.e., this yields for a simulation period, as of year $s = t$, of h years ahead an instance i for $a^{(i)}_{x+h,t+h}$, which yields by (10) for every x an instance for $CWA_{x,t,t+h}$, say $CWA^{(i)}_{x,t,t+h}$.

In order to translate these annuity price updates into benefit payment updates, I use (18) to determine the implied update in benefit level. As (18) on its own is overidentified, I will propose multiple examples of restrictions which are in turn congruent with a certain shock transfer regime. I start with the case of uniform allocation of longevity shocks as described in section 5.2. Then I present what it looks like if part of the shock is absorbed in the solidarity buffer as in section 5.3.

For practical reasons, I assume a maximum age of 100 years in my investigation. Unless stated explicitly otherwise, I assume an investment return of 3% wherever applicable, unless explicitly stated otherwise. The retirement age is set at $x = 67$.

7.1 Mortality

In line with the approach performed by the Dutch Actuarial Society, I look at a sample of observations from the period 1978-2018 for individuals as of age 15. I apply the approach as presented section 4.2 on the matrix below to estimate $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\kappa}$. The parameter results are reported in the figures below where the parameter estimation values are plotted with respect to their index. For an overview of these results I refer to the *appendix*.

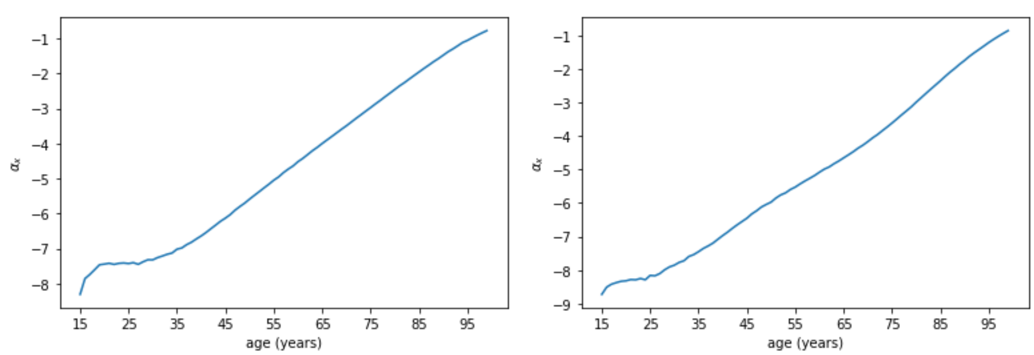


Figure 1: Left: $\vec{\alpha}$ with respect to age for the Dutch male population; Right: idem, for female population

In Figure 1 the age specific component of human mortality is plotted with respect to age. For both men and women, the force of mortality $\mu_{x,s}$ from (5) increases in x, ceteris paribus. In Figure 2 below, results with respect to an age group's sensitivity to time trend $\vec{\kappa}$ are reported. We see that the young male age groups and those just after retirement age are most sensitive to time trends within the evolution of survival probabilities over time.

For the female part of the population, this becomes less clear, though there is a local maximum with regard to time trend sensitivity around the age of 75. In the *appendix* the results for each age group are displayed for both genders in tabular form.

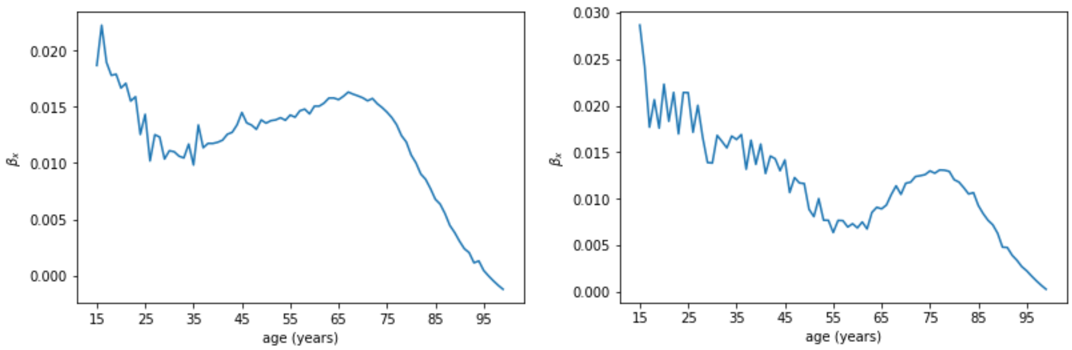


Figure 2: Left: $\vec{\beta}$ with respect to age for the Dutch male population; Right: idem, for female population

For κ_s, capturing the time trend in mortality in sample, I find the following estimations for (13) for both men and women, which are illustrated by Figure 3:

$$\hat{\kappa}_{s+1}^{men} = -1.450 + \hat{\kappa}_s^{men} + \eta_s^{men}, \text{with } \hat{\sigma}_{\eta_s^{men}} = 1.4084 \quad \forall s \tag{33}$$

$$\hat{\kappa}_{s+1}^{women} = -1.143 + \hat{\kappa}_s^{women} + \eta_s^{women}, \text{with } \hat{\sigma}_{\eta_s^{men}} = 2.7201 \quad \forall s \tag{34}$$

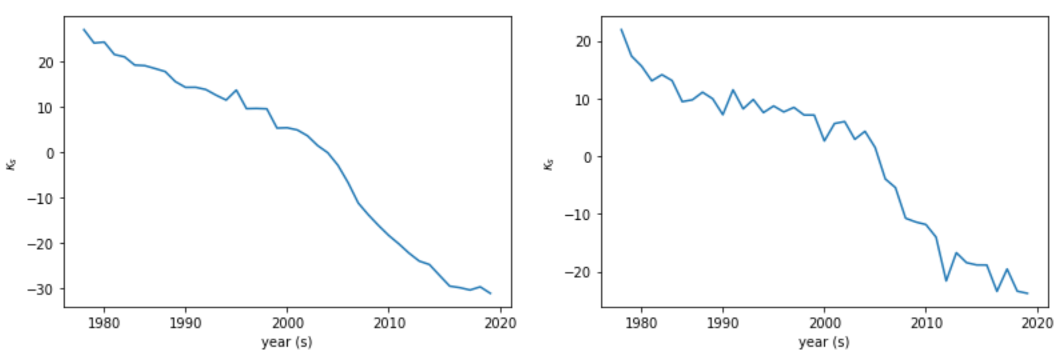


Figure 3: Left: $\vec{\kappa}$ with respect to time period for the Dutch male population; Right: idem, for female population

Estimation values with respect to the entries s of κ_s are reported in the *appendix* for all s.

These parameter estimations allow for forecasting survival probabilities and, in turn, annuity prices ahead in the future by (11) and (12). Based on the sample at hand, I find estimations for the annuity prices for men as presented in Figure 4.

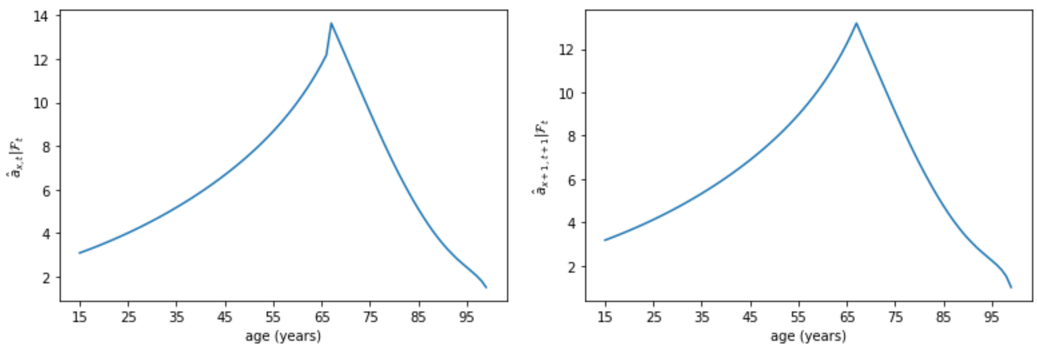


Figure 4: Left: The price a man would have to to pay if one buys a life annuity at time t; Right: The estimated price a man would have to pay if one would want to buy an annuity one year ahead.

For women, these annuity prices for different age groups are visually displayed in Figure 5.

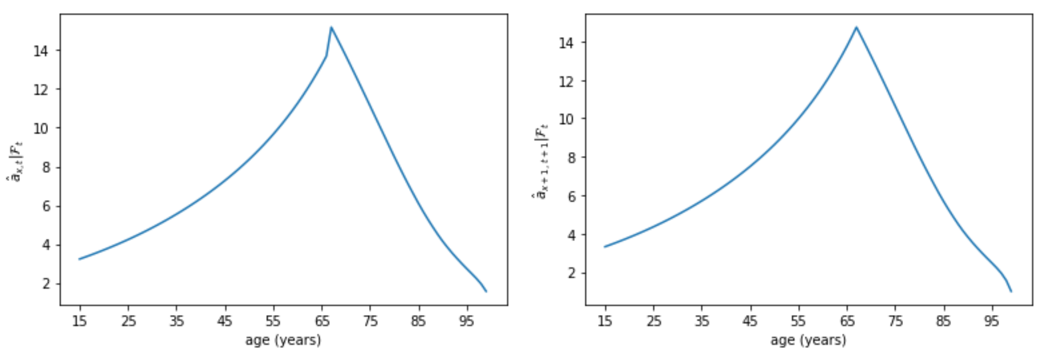


Figure 5: Left: The price a women would have to pay if one buys a life annuity at time t; Right: The estimated price a women one has to pay if one would want to buy an annuity one year ahead.

After simulating instances i for future κ_{t+h} realizations, $\kappa_{t+h}^{(i)}$ for $h = 1, 2$ one can construct corresponding survival probability matrix instances, $P_{t+h}^{(i)}$ by the methodology described in section 4.3. The change in annuity price, $CWA_{x,t,t+h}$ from (10) for $h = 1, 2$, given the new pseudo-data and a net expected investment return of 300 basis points, is characterized by the following quantiles:

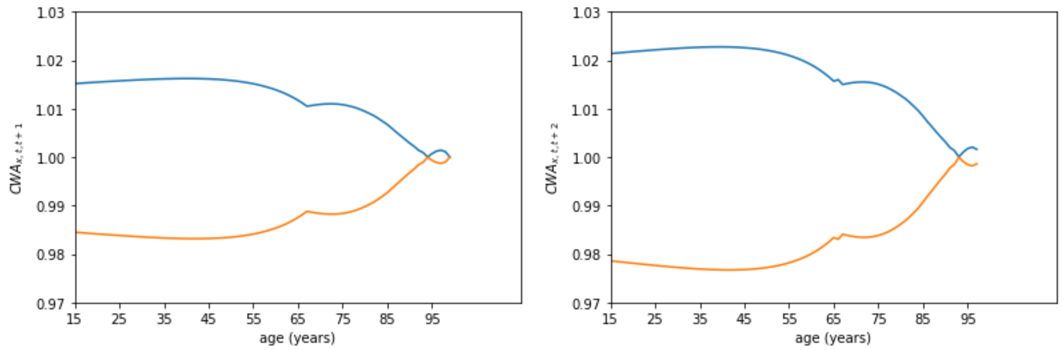


Figure 6: Left: Male annuity value update factor after one year $(h = 1)$. Orange: $q_{2.5\%}$, Blue: $q_{97.5\%}$; Right: idem, after two years $(h = 2)$

For more detailed insight in the value of simulation 2.5% and 5% simulation quantiles for each age group, I refer to the *appendix*.

Similarly, for female fund participants, the analysis yields:

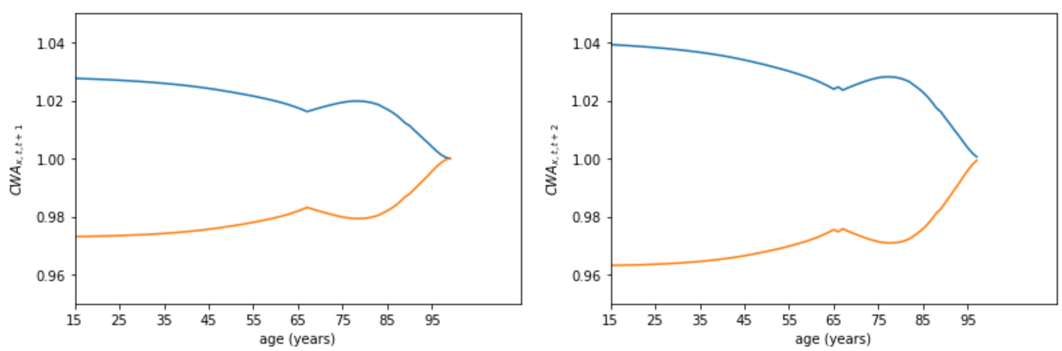


Figure 7: Left: Female annuity value update factor after one year $(h = 1)$. Orange: $q_{2.5\%}$, Blue: $q_{97.5\%}$; Right: idem, after two years $(h = 2)$

Here we see that the tail quantiles for benefit payment adjustment percentages lie given the modelling choices made in section 4, respecitvely, within the 1.5%, for men, and 2.5% ballpark, for women, when considering a one year period. When looking at a two year forecast horizon, these ranges go up to 2% for men and 4% for women, respectively.

7.2 Alternative mortality time series approaches

As visible in Figure 3 the random walk process proposed in (13) might not fit the female data as well as it does for male data. After cross-validation, an arima process with one lagged variable seems to perform better. Forecasts using this type of model fit yield a lower out-of-sample prediction MSE. For practical and explanatory reasons I chose not to depart from the *random walk*-assumption, given that ARIMA(0,1,0) does not perform dramatically worse than the optimal time series process (see Table 1). I have reported the performance of all different time series approaches in the table below.

ARIMA(p,q,d)	MSE($\vec{\kappa}^{men}$)	MSE($\vec{\kappa}^{fem}$)
(0,0,0)	898.5	481.916
(0,0,1)	231.1	172.406
(0,1,0)	1.627	10.662
(0,1,1)	2.414	9.972
(0,1,2)	2.221	10.041
(0,2,0)	1.331	33.051
(0,2,1)	1.908	11.375
(1,0,0)	4.735	13.263
(1,1,0)	2.327	9.053
(1,2,0)	1.371	16.513
(1,2,1)	2.375	-
(2,0,0)	2.200	14.138
(2,1,0)	2.468	10.073
(2,2,0)	1.878	12.142
(2,2,1)	2.281	10.959
(2,2,2)	2.423	-
(3,0,0)	1.269	12.114
(3,1,0)	2.073	10.265
(3,1,1)	2.072	11.100
(3,2,0)	2.164	12.416
(3,2,1)	2.302	14.296

Table 1: Prediction performance of different time series methods for mortality risk driver

If one would use an arima-process with one lagged variable, the results regarding systematic mortality returns yield, when comparing Figure 7 and Figure 8, that the quantiles in the latter approach are a bit tighter.

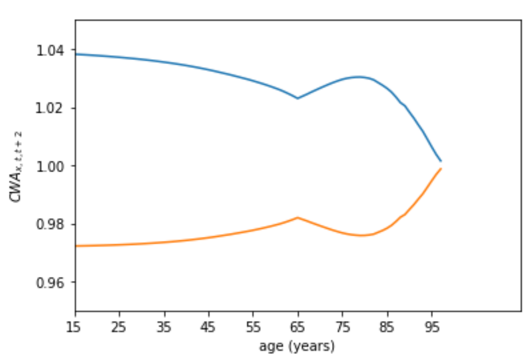


Figure 8: Female annuity value update factor after two years using ARIMA(1,1,0) to forecast κ_s into the future. Orange: $q_{2.5\%}$, Blue: $q_{97.5\%}$

7.3 Uniform allocation of longevity shocks on benefit levels

7.3.1 Setup

In case of a uniform shock allocation, we can translate instances i for $CWA^{(i)}_{x,t,t+h}$ to corresponding instances i for the uniform benefit payment adjustment, say $\hat{\gamma}^{(i)}$. In this section I will present how large shocks are given the ABP fund decomposition. I will look at how large SB should be in several cases for restrictions, characterized by Γ as in section 6.2, on allocated shocks as in (24). We still consider time $s = t$ as the point in time where mortality calibration data is last observed and look at what happens from there with regard to the realisation of macro-longevity risk.

First I will determine $\hat{\gamma}$ in line with (21). Under Dutch law, it is prohibited to distinguish pension cut regime on the basis of an individual's gender. Hence, from a practical and legislative point of view it makes sense to adapt (21) slightly, albeit discarding possible concerns with regard to actuarial fairness, to:

$$\hat{\gamma} = \frac{\sum_x [N^{x,male}_t W^{x+1}_{t+1} + N^{x,fem}_t W^{x+1}_{t+1}]}{\sum_x [B^x_t * \hat{a}^{male}_{x+1,t+1}|\tilde{\mathcal{F}}_{t+1} * N^{x+1,male}_{t+1} + B^x_t * \hat{a}^{fem}_{x+1,t+1}|\tilde{\mathcal{F}}_{t+1} * N^{x+1,fem}_{t+1}]}. \tag{35}$$

$\hat{\gamma}$, where the number of men and women aged x at time t, are, respectively, defined by $N^{x,male}_t$ and $N^{x,fem}_t$. A similar notation is used for gender-specific annuity prices. Other variables and indices have the same interpretation as in (21).

Now given the simulation results for mortality forecasts characterized by (33) and (34) in section 7.1 for both genders, I will present quantiles for the corresponding instances for experienced shock factors, say $\hat{\gamma}^i$ as in (35), where i denotes the instance. Consequently, I will report the corresponding monetary value this shock quantile represents on a portfolio level. This, in turn is used to propose a level for SB_t such that (24) is not violated in $(1-\alpha)\%$ of the cases, i.e., SB^*_t will be suggested as described in section 6.2. I will show to what extent pension cuts can be prevented on an individual level in case the $SB_t = SB^*_t$ is employed as proposed for a specific Γ as in (32).

These required buffer levels are expressed in relative terms with respect to the time t best estimate value of the pension funds liabilities, which are denoted by L. For $L^{BE} := \hat{L}|\mathcal{F}_t$, we denote:

$$L^{BE} = \sum_x [B^x_t * \hat{a}^{male}_{x+1,t+1}|\mathcal{F}_t * N^{x+1,male}_{t+1} + B^x_t * \hat{a}^{fem}_{x+1,t+1}|\mathcal{F}_t * N^{x+1,fem}_{t+1}], \tag{36}$$

where $N^{x+1,male}_{t+1}$and $N^{x+1,male}_{t+1}$ follow from (19).

Note, for the sake of completeness, that within the modelling approach I used in this thesis, the main parameters, which are impacting this liability value, are the assumed expected investment return and the survival probability estimates. The assumed expected investment return influences both the forward annuity value estimates through (12) and, indirectly, the benefit level, determined at $s = t$ by (17), because it affects time t annuity prices through (11). Survival probability estimates also pop up in both (11) and (12). With this in mind, I will also look at the impact on buffer results of using a different value for the assumed expected investment return (see section 7.4).

Furthermore I will illustrate to what extent pension cuts resulting from longevity shocks can be prevented in case the buffer level would correspond to SB_t^* from (32). In cases where pension cuts might be needed, I will present by how much this cut can be prevented in case the solidarity buffer is employed. For this, the following metric will be defined as an indication of gains with respect to purchasing power:

$$\Delta PP_{t+1} = E[\tilde{\gamma}_{t+1} - \hat{\gamma} | \hat{\gamma} < \gamma_{min}]. \tag{37}$$

The motivation for looking into this metric, is that negative changes in benefit levels that an individual is used to, leads to losses in perceived welfare (Abel, 1990). Also, humans are loss averse, which also contributes to loss of perceived welfare in case cuts in benefit level are needed (Mitchell and Utkus, 2006).

Also I will present $q_{0.975}(SB_{t+1})/L^{BE}$, representing the relative level of the solidarity buffer after year t in case of a positive longevity shock, given $SB_t = SB_t^*$. SB_{t+1} follows from (22) and Φ_t follows correspondingly from (25). This gives insight in how the upper tail of the buffer distribution is behaving. Values for Φ_t in each instance i, say $\Phi_t^{(i)}$ provide insight in the extent to which the solidarity buffer is used to mitigate shocks upon realisation. Simulation instances in this regard provide estimates for the probability distribution with regard to buffer usage extent.

7.3.2 Buffer dynamics

The following tables summarize the results I found in this context for the metrics of interest described in section 7.2.1. Using an expected return of 3% is most in line with the investment policy APG uses for the managed funds. After presenting results in this regard, I will illustrate how the results are impacted if one wishes to depart from this assumed return.

Throughout the course of evaluating metrics with regard to the buffer for different restrictions on $\tilde{\gamma}$, I repeatedly use the term *promise*. Even though the design change that has to be implemented, naturally yields a more variable pension income, decision makers still have the opportunity to set restrictions on the impact a materializing risk for a specific risk factor may have on pension benefits. The expected benefit payment is not a guarantee, but more a so called *soft right*. With the notion of *promise* I refer to the mutually cherished desire from both pension fund and participant to arrive at a situation in which the pension benefit payment pattern respects the restrictions with regard to change in consumption level over time. For quantile α as used in (32), it holds that $\alpha = 0.025$, unless explicitly stated otherwise. In Table 2, 3, 5 and 6, results regarding buffer outflow characteristics and purchasing power are based on $SB_t = SB_t^*$ for $\alpha = 0.025$ and the regime Γ denoted in the first row of the table. Buffer size requirements obtained as in section 6.2 are expressed relative to L^{BE} as introduced in (36).

Γ :	$\gamma_{min} = 0.99; \gamma_{max} = 1.01$	$\gamma_{min} = 0.98; \gamma_{max} = 1.02$
$q_{0.975}(\hat{\gamma})$	1.0158	1.0158
$q_{0.5}(\hat{\gamma})$	0.9999	0.9999
$q_{0.025}(\hat{\gamma})$	0.9835	0.9835
SB_t^*/L^{BE}	0.762 %	0 %
SB_t^*/L^{BE}, $\alpha = 0.001$	1.396 %	0.224 %
$q_{0.975}(SB_{t+1})/L^{BE}$	1.507 %	0 %
$q_{0.975}(SB_{t+1})/L^{BE}$, $\alpha = 0.001$	2.161 %	0.230
$P(\Phi_t > 0)$	0.120	0.006
$P(\Phi_t = 0)$	0.765	0.994
$P(0 > \Phi > -(1 + r_t)SB_t)$	0.087	-
$P(\Phi_t = -(1 + r_t)SB_t)$	0.028	-
ΔPP_{t+1}	0.348 %-pt	0.00 %-pt

Table 2: Buffer metric results ($r = 0.03$)

Γ :	$\gamma_{min} = 1; \gamma_{max} = 1.01$
$q_{0.975}(\hat{\gamma})$	1.0158
$q_{0.5}(\hat{\gamma})$	0.9999
$q_{0.025}(\hat{\gamma})$	0.9835
SB_t^*/L^{BE}	1.927%
SB_t^*/L^{BE}, $\alpha = 0.001$	2.569 %
$q_{0.975}(SB_{t+1})/L^{BE}$	2.708 %
$q_{0.975}(SB_{t+1})/L^{BE}$, $\alpha = 0.001$	3.368 %
$P(\Phi_t > 0)$	0.120
$P(\Phi_t = 0)$	0.382
$P(0 > \Phi_t > -(1 + r_t)SB_t)$	0.473
$P(\Phi_t = -(1 + r_t)SB_t)$	0.025
ΔPP_{t+1}	0.644 %-pt

Table 3: Buffer metric results in scenario where all cuts are aimed to be avoided ($r = 0.03$)

In Table 2 we see that in case shocks to pension income are restricted to be at most 1%, a buffer of 0.762% of L^{BE} is needed in order to prevent cuts larger which happen to be larger than promised with a probability of 0.975. In order to prevent these types of cuts with a probability of 0.999, the required time t buffer increases drastically to 1.396 % of L^{BE}, suggesting tail behavior which introduces reasons to concern effectiveness in this case. However, it is worth noting that in cases of a shock event, i.e, in cases in which pension cuts are immanent, use of a buffer leads to a uniform increase in purchasing power of 0.348 %. Also, the probability of not being able to fulfill pension benefit promises as restricted by (24), decreases from 0.115 $(= P[0 > \Phi_t \geq -(1 + r_t)SB_t)])$ to 0.028 in case of employing a SB in this manner compared to a setting in which fund members are not protected by this buffer structure.

In case one imposes looser restrictions on the extent to which pension rights can be altered due to materializing longevity risk, we see that a significantly lower level, namely 0.224 % of L^{BE} will be needed to avoid pension benefit cuts which contradict earlier promises with 0.999 probability (Table 2). In this case, the pension fund allows for a longevity risk induced cut of maximum 2%, which is quite significant, but it becomes also significantly easier to keep up with benefit payment promises in line with (24).

In case a buffer structure is used with the aim to completely offset negative portfolio shocks due to macro longevity risk, a buffer level of 1.927 % of L^{BE} is needed at time t to prevent cuts with 0.975 probability (see Table 3). With a buffer level of 2.569 % of L^{BE}, cuts can be avoided with probability 0.999. Worth noting is that in all cases that cuts would be implemented if there was no restriction $\gamma_{min} = 1$ in place, 94.5% of those can be avoided in case the buffer level is picked at the proposed level of 1.927 % of L^{BE}. Cases of negative macro biometric return consist of both the avoided cuts I referred to earlier and the cases in which the materialized risk is too far in the tail of the distribution to be covered in full by the buffer. In the event there is an increase in pension liability value due to a macro longevity event, having the proposed buffer in place reduces purchasing power decrease by 0.644 %-point.

7.3.3 Benefit adaptation factor

Now suppose one attains SB_t^* for $\Gamma := (0.99, 1.01)$. Table 4 now summarizes the quantiles of $\tilde{\gamma}$ if one uses SB_t^* based on this Γ within the context of a different cutting regime.

Γ	$\gamma_{min} = 0.99, \gamma_{max} = 1.01$	$\gamma_{min} = 0.98, \gamma_{max} = 1.02$	$\gamma_{min} = 1, \gamma_{max} = 1.01$
$q_{0.975}(\tilde{\gamma})$	1.01	1.0158	1.01
$q_{0.5}(\tilde{\gamma})$	0.9999	0.9999	1.0001
$q_{0.025}(\tilde{\gamma})$	0.99	0.9835	0.9900

Table 4: Robustness of buffer performance with respect to different benefit adaptation regimes ($r = 0.03$)

Table 4 gives some insight in how robust the ability to mitigate pension cuts of an initial pick for SB_t^* is with regard to a different cutting regime, say Γ'. We see that for the regime to which its size is calibrated, the performance is as expected and we find the quantiles exactly matching the restriction bounds. For a looser policy with regard to adaptation factor, we see that the availability of this buffer does not affect the tail quantiles of the allocated adaptation factor of interest. For a regime, in which cuts are ought to be avoided, we see that the quantiles of $\tilde{\gamma}$ in Table 4 are tighter than $\hat{\gamma}$ quantiles in Table 3, which is in line with expectations, as the buffer level is touched for every scenario where $\hat{\gamma} < 1$ and the non-negative outflow is used to finance benefits, and increasing allocated adaptation factor $\tilde{\gamma}$ beyond $\hat{\gamma}$ (see (26)).

7.4 Impact of change in expected investment return

7.4.1 Buffer dynamics

In case one wishes to assume a lower expected investment return, which might suit applicable market conditions better, the present value of pension promises increases as a result of (11). This notion may, however, not necessarily directly translate into similar increases in relative buffer requirements. It is hence insightful to perform the simulation analysis for a different level of assumed interest rate as well. In the table below I report the metrics of interest for the case in which an investment return of 1% is assumed in a similar way as in section 7.3.2. SB_t^* is, similar as in section 7.3, calculated from (32), though now calibrated using an assumed expected investment return of $r = 0.01$. [7]

Γ :	$\gamma_{min} = 0.99; \gamma_{max} = 1.01$	$\gamma_{min} = 0.98; \gamma_{max} = 1.02$
$q_{0.975}(\hat{\gamma})$	1.0180	1.0180
$q_{0.5}(\hat{\gamma})$	0.9999	0.9999
$q_{0.025}(\hat{\gamma})$	0.9812	0.9812
SB_t^*/L^{BE}	0.890 %	0 %
SB_t^*/L^{BE}, $\alpha = 0.001$	1.511 %	0.496 %
$q_{0.975}(SB_{t+1})/L^{BE}$	1.733 %	0 %
$q_{0.975}(SB_{t+1})/L^{BE}$, $\alpha = 0.001$	2.365 %	0.501 %
$P(\Phi_t > 0)$	0.153	0.015
$P(\Phi_t = 0)$	0.700	0.985
$P(0 > \Phi > -(1 + r_t)SB_t)$	0.122	-
$P(\Phi_t = -(1 + r_t)SB_t)$	0.025	-
ΔPP_{t+1}	0.428 %-pt	0.00 %-pt

Table 5: Buffer metric results ($r = 0.01$)

We see that the buffer requirements increase by approximately 0.1-0.2 %-point of L^{BE} in almost all cases. The probability that the buffer remains untouched given materializing longevity risk, decreases by around 6.5%-points, given the buffer requirement: 0.7000 (Table 5) versus 0.765 (Table 2). Though, in the cases that cuts can be (partially) offset by outflow of funds out of the solidarity buffer, the buffer policy yields a larger safety net with regard to decrease in purchasing power: 0.428%-point (Table 5) versus 0.348%-point (Table 2). We see that in case of looser restrictions with regards to cuts and indexation, the probability that excess mortality returns flow into the buffer almost triples, but this probability is still not larger than 1.5%. In case the pension benefit promise would be made that cuts/indexations are at max 1%, the probability that this promise cannot be kept drops from 0.147($= P[0 > \Phi_t \geq -(1 + r_t)SB_t)]$) to 0.025 in case one keeps the proposed buffer level in place.

[7] Note, moreover, that freedom of choice with respect to assumed expected investment performance suits the context of the new pension deal with respect to individual risk appetite.

$\Gamma:$	$\gamma_{min} = 1; \gamma_{max} = 1.01$
$q_{0.975}(\hat{\gamma})$	1.0180
$q_{0.5}(\hat{\gamma})$	0.9999
$q_{0.025}(\hat{\gamma})$	0.9812
SB_t^*/L^{BE}	1.895 %
$SB_t^*/L^{BE}, \alpha = 0.001$	2.526 %
$q_{0.975}(SB_{t+1})/L^{BE}$	2.752 %
$q_{0.975}(SB_{t+1})/L^{BE}, \alpha = 0.001$	3.391 %
$P(\Phi_t > 0)$	0.153
$P(\Phi_t = 0)$	0.349
$P(0 > \Phi_t > -(1 + r_t)SB_t)$	0.473
$P(\Phi_t = -(1 + r_t)SB_t)$	0.025
ΔPP_{t+1}	0.7315 %-pt

Table 6: Buffer metric results ($r = 0.01$) in case cuts are aimed to be avoided

In case one wants to completely avoid cuts as a result of materializing macro longevity risk, the increase in relative buffer requirements is less apparent as it is the case for looser regimes (see Table 5), as becomes clear from comparing Table 6 with Table 3. Prevented decrease in welfare is also in this case larger than in the scenario using $r = 0.03$: 0.7315%-point (Table 6) versus 0.644 %-point (Table 3).

7.4.2 Benefit adaptation factor

Now suppose one attains SB_t^* for $\Gamma = (0.99, 1.01)$, based on $r = 0.03$. Table 7 now shows the quantiles of $\tilde{\gamma}$ if one uses this SB_t^* based on $r = 0.03$ within the context several cutting regimes and an assumed expected investment return of $r = 0.01$.

Γ	$\gamma_{min} = 0.99, \gamma_{max} = 1.01$	$\gamma_{min} = 0.98, \gamma_{max} = 1.02$	$\gamma_{min} = 1, \gamma_{max} = 1.01$
$q_{0.975}(\tilde{\gamma})$	1.0085	1.0085	1.0085
$q_{0.5}(\tilde{\gamma})$	0.9915	0.9915	0.9980
$q_{0.025}(\tilde{\gamma})$	0.9804	0.9800	0.9804

Table 7: Robustness of buffer performance with respect to different benefit adaptation regimes ($r = 0.01$)

In Table 7 we see that the quantiles for $\tilde{\gamma}$ are lower than in Table 4. This makes sense, as one can see, when comparing Table 2 and 3 with Table 5 and 6, that within the same pension cut and indexation regime, in case of $r = 0.01$ a higher buffer level is needed than in case $r = 0.03$. The buffer, which level is determined with $\Gamma = (0.99, 1.01)$ and $r = 0.03$ in mind, is insufficiently large to cover cuts, leading to lower bottom quantile values. Hence I find that a lower assumed expected investment return, increases the buffer that you need, given a set of means you wish to accomplish by incorporating it in the pension contract. If one does not have a larger buffer at hand, benefit adaptations $\tilde{\gamma}$ will break the desired restriction, denoted by (24), more easily.

8 Conclusion

The focus of my research has been macro longevity risk within the context of the new Dutch pension deal. It is not yet formally proposed how, within the context of this newly proposed contract, longevity risk may cause changes in individual pension entitlements. I argued how such a contract element would fit in the broader context of developments that drive social security design. Then I illustrated how a buffer can be put in place in case one wishes to reduce fund participants' exposure to macro longevity risk. I showed how this works out in case of a uniform risk allocation rule and determined desired buffer levels needed to cover risks to a certain extent. I performed this analysis for two different expected investment return levels.

Within the fund population we see, one period macro longevity related biometric return tail quantiles $q_{0.025}$ and $q_{0.975}$ in the range of 1.5% for men and 2% for women. Assuming an expected investment return of $r = 0.03$, the required change in benefit level remains smaller than 1% with a probability of 0.765; in case $r = 0.01$, this probability drops to 0.7000. In case one wishes to keep cuts below this 1% figure with 0.975 probability, a buffer of 0.762% of L^{BE} for $r = 0.03$. For a lower expected investment return of $r = 0.01$, this buffer requirement increases to 0.890 % of L^{BE}.

In case the desired bounds with respect to cuts/indexation due to materializing longevity are somewhat looser, i.e., if they are restricted by 2%, the event that the implied adaptation factor $\hat{\gamma}$ from (21) falls outside these bounds happens more rarely. Cuts which would be out of line with the promises can be prevented with a 0.999 probability for a buffer level of 0.224 % of L^{BE} for $r = 0.03$. This requirement increases to 0.496 % of L^{BE} in case of a lower assumed expected return $r = 0.01$.

In the event that proposed cuts, which would have been carried out if no buffer would have been present, exceed the maximum cut promised beforehand, i.e., in case (24) is violated, the buffer can (to some extent) be used to prevent cuts. Welfare losses are mitigated by 0.348%-point if promises included a maximum of 1% change in pension income for $r = 0.03$. This alleviated welfare loss equates to 0.644%-point in case cuts were promised beforehand not to take place at all. For $r = 0.01$, these prevented losses when using the proposed buffer are slightly higher: 0.428%-point for a maximum of 1% change; 0.7315%-point in case of a breached promise to not cut at all.

Hence within the space provided by the new contract to expose individual participants to macro longevity risk to expose individual participants to macro longevity risk, imposing a buffer mechanism may provide added value. Given promises a fund makes to its participants with regard to cuts and indexation, keeping a buffer of the proposed magnitude at hand prevents violation of these terms to some extent. In the cases I looked into, the probability a pension fund could not fulfill its promises to their fund members decreases by a factor 4 to 5. Pension fund board members should determine to what extent they deem setting aside the proposed amount as a solidarity buffer in the fund desirable, given the possible opportunities and challenges inclusion of a buffer offers within the domain of potential welfare increase, protection against shocks fund participants face and explainability of what an individual's personal wealth account, which he is entitled to, actually mean in terms of pension income as of retirement.

Design challenges remain regarding how this buffer should be initialized, filled, invested or how it should be allocated in case it remains untouched during a period of time. In section 9, I will elaborate a little more on this.

The results and recommendations I presented may contribute to a conceptual starting point for implementation of the solidarity buffer in such a way, that it helps alleviate risks with regard to macro longevity that fund members are exposed to.

9 Discussion of used methods and recommendations for future research

While my thesis focuses on the impact macro longevity risk has on pension provisions, it should be noted that investment risk is in general of larger magnitude and hence its outcome and/or sign might not make a difference on a portfolio level (Dees et al., 2019). Such a result might seem a little awkward to, e.g., someone who has spent the majority of his life during the past calendar year researching allocation mechanisms for macro longevity risk. However, within the light of the transition of the pension contract towards a more DC-minded setting, the need for more transparency regarding how the materialization of different risk factors at each point in time affects an individual's pension accounts and the benefits he/she can acquire with this, becomes apparent. Also, insights in these specific effects on pension provision levels enables stakeholders to evaluate to what extent concepts with regard to actuarial fairness, which have gained more and more attention in recent years, are respected. Belloni and Maccheroni (2013) provide such an analysis within the context of Italian pension schemes.

When illustrating societal relevance of the proposed contract design element I used a short genealogical approach. This gives insight in background of abstract terminology and confronts implicit assumptions embedded in structural designs. However, such a method insufficiently covers socioeconomic developments on a broader scale, which future interdisciplinary research might want to look into within the context of systemic sustainability.

The assumptions with regard to expected investment returns within the analysis are far from cast in concrete in case one wishes to move in a true-to-nature direction on the *realism versus practicality* scale. This might, e.g., be of interest in case one wishes to evaluate performance of the proposed mechanisms in a more guarantee-based setting like the old Dutch pension contract.

A similar remark should be made with regard to suitable parameter values when referring to cut and indexation regimes. Depending on future legislative and regulatory developments within the areas in which the newly proposed pension deal is yet to be further worked out, looser or tighter restrictions in this respect can be assumed and analysed using the approach as carried out in section 7.3 and 7.4, which is described in section 6.2

For practical and illustrative purposes, I have imposed uniform shock allocation rules within my research. Relevant for follow-up research might be to look at how a specific age group's exemption from *casu quo* extra exposure to longevity changes buffer dynamics. Such a mechanism would make inter- and intragenerational risk-sharing in the contract more explicit. De Waegenaere et al. (2019) propose a general framework in this regard as a starting point.

When developing alternative allocation rules with regard to macro longevity risk, it can be interesting to take exposure of specific age group to the macro longevity risk factor within the system into account. From results with regard to faced macro longevity risk, which are visualised in Figure 6 and 7, it becomes clear that exposure of younger age groups is larger than that of pensioners, which may motivate investigation of an allocation mechanism where pensioners take over macro longevity risk from younger age groups.

What is also worth noting is that I, in essence, investigated how the newly system can cope with present macro longevity risk, but I did not research nor show in general terms to what extent extra exposure to or protection from macro longevity risk increases a certain participant's perceived welfare as compared to the old system in which macro longevity risk is implicitly shared. Summarizing, I researched the degree of risk and the implications of using a possible coping mechanism. However, a comparative study follow-up study with regard to perceived wealth effect changes when adopting the possibility to have a different exposure to longevity risk might be relevant to know when communicating with fund participants in the context pension plan transitions.

Throughout the whole analysis regarding longevity and buffering results, I simulate and calculate tail quantiles of simulated quantities. In order to arrive at conclusions in this regard, I distributed all available computing capacity across the whole distribution of the simulation variable of interest. In case one is specifically interested in these tail quantities and tail behavior in general (e.g., for risk management purposes), importance sampling techniques increase accuracy (Glynn and Iglehart, 1989). Tsitsiklis and Van Roy (2001) propose an algorithm which, after identifying an explicit link between simulation quantities which are dependent on each other, increases simulation speed also. Richards et al. (2012), however, note that such an approach can be ruled redundant in this practical setting by design of the simulation approach. Researchers looking into this specific domain may hence wish to approach this matter from a broader conceptual point of view rather than an ad-hoc one that suits the context of macro longevity risk in a to some extent variable annuity setting.

For the situation with regard to Dutch social security specifically, it may also be interesting to investigate impact of macro-longevity risk on an individual's consumption pattern, while also taking the impact on *first pillar* provisions of realisations with respect to this risk factor. An individual's legal entitlements to these *AOW* benefits are, namely, directly linked to population life expectancy, which is - basically - what macro longevity risk is all about. It might, hence, be worthwhile to investigate how the interplay between these two *pillars* works out for an individual's total available consumption budget after retirement.

With regard to the chosen mortality modeling approach, performance increase can be gained by: i. taking into account multi-population trends (Li and Lee, 2005) and ii. considering alternative processes fitted for the time specific parameter than (13). In section 7.2 I reported the performance of different time series methods within the Dutch population and I also showed how this affects simulation quantiles for the extent of materializing macro longevity risk. For practical reasons, I did not deviate from the method proposed by Lee and Carter (1992), but future researchers may be interested in how such a deviation impacts results which are dependent on mortality. When coming up with results regarding buffer dynamics and requirements, I determined a suitable level for the buffer and I illustrated how a buffer at hand impacts a fund's ability to meet its promises regarding cuts/indexations. However, the I do not treat how solidarity buffer should be established, funded and how it should be allocated in case it remains untouched. Metselaar et al. (2020) described how this would work within the setting of the new contract for absorbing excess shocks in investment returns. For future researchers it might be valuable to investigate how the spirit of this proposal can be translated to a dynamic buffer model. Especially with regard to macro longevity risk, no explicit formal proposals have been made yet by policymakers or industry stakeholders.

Mayor challenge in this regard is to link the conceptual notion of macro *biometric returns* - which have no effect whatsoever on a funds total asset value - with a real monetary equivalent. Conceptually, it can be of interest to investigate a modeling approach in which during each period, there is a transfer of currency between individual wealth accounts and the fund-wide solidarity buffer. Roughly, this transfer partially coincides with Φ_t from (25), but it is possible to make this buffer model design more general and flexible. The starting point would be a dynamic approach in which a certain part of excess return (or excess-under-performance) with respect to a specific risk factor (e.g., macro longevity risk) equates to the total in- or outflow from the solidarity buffer.

There is still a large gap, though, between this conceptual idea and a corresponding budget neutral dynamic buffer model, which handles these non-trivial wealth transfers transparently and correctly - both mathematically and legally. Also questions with regard to create the buffer in the fund remain open for future research. The rate at which it is filled at and which cash flows money should be coming from could be topic of research with regard to the practical feasibility of establishment of a solidarity buffer in general, and more specifically in the case it is used to mitigate the impact of macro longevity risk faced by pension fund participants.

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Appendices

Parameter estimates Lee-Carter mortality model

In the table below, parameter estimates for the entries x of α and β are reported for each age group x.

Age (x)	α_x^{male}	α_x^{female}	β_x^{male}	β_x^{female}
15	-8.306	-8.72677	0.0186927	0.0286665
16	-7.85687	-8.51224	0.0222743	0.0242324
17	-7.73819	-8.42405	0.0189744	0.0176969
18	-7.60207	-8.37639	0.0178145	0.0206511
19	-7.46157	-8.33596	0.0179118	0.0176023
20	-7.44217	-8.32264	0.0166838	0.0223176
21	-7.41856	-8.28546	0.0171059	0.0183335
22	-7.45063	-8.29266	0.0155244	0.0214363
23	-7.42031	-8.25293	0.0159092	0.0169771
24	-7.41027	-8.29117	0.0125556	0.0214248
25	-7.42645	-8.1613	0.014343	0.0214091
26	-7.40094	-8.1718	0.0101982	0.0171509
27	-7.44859	-8.10565	0.0125309	0.0200453
28	-7.37667	-7.99291	0.0123215	0.016626
29	-7.31851	-7.90698	0.0103658	0.013906
30	-7.32125	-7.8552	0.0111025	0.0138513
31	-7.25627	-7.77243	0.0110157	0.0168177
32	-7.21097	-7.72214	0.0106218	0.0161661
33	-7.16084	-7.59513	0.0104606	0.0154753
34	-7.12559	-7.53968	0.0116965	0.016746
35	-7.01615	-7.45247	0.00983634	0.0163771
36	-6.98137	-7.35532	0.0133959	0.0169112
37	-6.8875	-7.28053	0.0113658	0.013185
38	-6.81725	-7.19483	0.0117558	0.0163008
39	-6.72464	-7.08153	0.0117511	0.0137084
40	-6.63824	-6.97087	0.0118573	0.0158844
41	-6.54057	-6.86852	0.0120436	0.0127282
42	-6.43287	-6.75467	0.0125694	0.0146013
43	-6.32747	-6.64939	0.0127432	0.0143057
44	-6.21894	-6.55044	0.0134065	0.0130237
45	-6.12967	-6.45246	0.0145054	0.0141805
46	-6.02953	-6.32591	0.0135798	0.0106838
47	-5.90272	-6.23112	0.0133866	0.0122914
48	-5.79752	-6.11861	0.0130103	0.0117111
49	-5.70015	-6.0418	0.0138448	0.0116471
50	-5.58615	-5.97156	0.0135575	0.00887404
51	-5.47851	-5.85014	0.0137788	0.00809932
52	-5.37256	-5.75659	0.0138486	0.0100341
53	-5.26468	-5.69515	0.0140406	0.00771279
54	-5.15984	-5.59415	0.0138149	0.00769964
55	-5.04686	-5.52295	0.0142816	0.00638822

Age (x)	α_x^{male}	α_x^{female}	β_x^{male}	β_x^{female}
56	-4.95236	-5.42796	0.0140878	0.00768931
57	-4.83037	-5.34097	0.0146589	0.0076651
58	-4.72846	-5.2598	0.0148065	0.00698077
59	-4.6378	-5.1767	0.0143826	0.00733498
60	-4.5172	-5.07881	0.0150712	0.00688625
61	-4.42328	-4.98714	0.0150712	0.00752131
62	-4.31409	-4.92188	0.0153443	0.00677092
63	-4.20308	-4.82625	0.0157922	0.00855045
64	-4.10606	-4.74339	0.0157966	0.00909799
65	-3.99831	-4.65072	0.0156498	0.00892975
66	-3.89674	-4.5592	0.0159512	0.00934236
67	-3.79581	-4.4649	0.0163217	0.010465
68	-3.69331	-4.35948	0.0161388	0.0114055
69	-3.59062	-4.26987	0.0159849	0.0104875
70	-3.49281	-4.16492	0.0158111	0.011682
71	-3.38684	-4.05222	0.0155472	0.0117933
72	-3.28024	-3.95446	0.0157525	0.0124017
73	-3.17853	-3.83906	0.0152855	0.0124882
74	-3.0741	-3.72763	0.0149466	0.012607
75	-2.9729	-3.61084	0.0145401	0.0130055
76	-2.86776	-3.48962	0.0140579	0.0127575
77	-2.76745	-3.36444	0.0134187	0.0131041
78	-2.66145	-3.24612	0.012451	0.0130803
79	-2.55913	-3.12354	0.0118816	0.0129497
80	-2.45487	-2.98649	0.0107298	0.0120669
81	-2.35071	-2.85809	0.0100223	0.0118016
82	-2.25857	-2.7275	0.00903179	0.0111999
83	-2.15551	-2.5996	0.00855881	0.0105271
84	-2.05533	-2.47393	0.00773532	0.0106711
85	-1.95597	-2.34813	0.00678854	0.00929627
86	-1.85643	-2.21653	0.00635214	0.00842276
87	-1.76177	-2.09063	0.00551204	0.00770885
88	-1.66288	-1.97583	0.00446016	0.00721417
89	-1.57412	-1.85162	0.00382494	0.00630402
90	-1.47689	-1.7402	0.00305696	0.00481859
91	-1.3788	-1.61853	0.002388	0.00477997
92	-1.29735	-1.5121	0.00203319	0.00394634
93	-1.20783	-1.41141	0.00113208	0.00339854
94	-1.11519	-1.30964	0.00129363	0.00272291
95	-1.05392	-1.2056	0.000450679	0.00228123
96	-0.97942	-1.11016	-3.08078e-05	0.00172558
97	-0.908164	-1.019	-0.000471547	0.00120781
98	-0.840242	-0.932342	-0.000870021	0.000731197
99	-0.775715	-0.850324	-0.00122497	0.000298806

Estimations with regard to the time trend parameter κ_s are reported below for any s.

Time period (s)	κ_s^{male}	κ_s^{female}
1978	26.9769	-8.72677
1979	24.028	-8.51224
1980	24.2504	-8.42405
1981	21.5023	-8.37639
1982	20.9907	-8.33596
1983	19.1796	-8.32264
1984	19.0571	-8.28546
1985	18.4211	-8.29266
1986	17.7534	-8.25293
1987	15.5472	-8.29117
1988	14.2821	-8.1613
1989	14.2937	-8.1718
1990	13.8013	-8.10565
1991	12.5702	-7.99291
1992	11.4855	-7.90698
1993	13.6848	-7.8552
1994	9.59389	-7.77243
1995	9.63576	-7.72214
1996	9.55654	-7.59513
1997	5.3192	-7.53968
1998	5.39336	-7.45247
1999	4.91022	-7.35532
2000	3.62951	-7.28053
2001	1.47774	-7.19483
2002	-0.10745	-7.08153
2003	-2.86353	-6.97087
2004	-6.68834	-6.86852
2005	-11.1758	-6.75467
2006	-13.7796	-6.64939
2007	-16.122	-6.55044
2008	-18.3047	-6.45246
2009	-20.1678	-6.32591
2010	-22.2045	-6.23112
2011	-23.9398	-6.11861
2012	-24.6953	-6.0418
2013	-27.0954	-5.97156
2014	-29.4516	-5.85014
2015	-29.788	-5.75659
2016	-30.3072	-5.69515
2017	-29.6087	-5.59415
2018	-31.041	-5.52295

Simulation quantiles macro longevity adaptation factor

Below simulation quantiles with regard to the annuity value update factor are tabulated for a one year horizon for any age group x while using $r = 0.03$. In the table, $q_\alpha^{[gender]}$ stands for the α-simulation quantile with respect to $CWA_{x,t,t+1}$ for the concerning gender.

Age (x)	$q^f_{0.025}$	$q^f_{0.05}$	$q^f_{0.95}$	$q^f_{0.975}$	$q^m_{0.025}$	$q^m_{0.05}$	$q^m_{0.95}$	$q^m_{0.975}$
15	0.973099	0.977433	1.02376	1.02762	0.984507	0.987056	1.0131	1.01518
16	0.973116	0.977449	1.02372	1.02757	0.984432	0.986993	1.01315	1.01524
17	0.973136	0.977468	1.02368	1.02752	0.984359	0.986933	1.01321	1.01531
18	0.97316	0.97749	1.02364	1.02747	0.984288	0.986874	1.01326	1.01537
19	0.973187	0.977515	1.02359	1.02741	0.98422	0.986818	1.01331	1.01543
20	0.973218	0.977544	1.02353	1.02735	0.984153	0.986763	1.01336	1.01548
21	0.973252	0.977574	1.02348	1.02728	0.984088	0.986709	1.01341	1.01554
22	0.97329	0.977608	1.02342	1.02721	0.984023	0.986655	1.01346	1.01559
23	0.97333	0.977644	1.02336	1.02714	0.983959	0.986603	1.0135	1.01565
24	0.973375	0.977683	1.0233	1.02706	0.983896	0.98655	1.01355	1.0157
25	0.973425	0.977728	1.02323	1.02698	0.983834	0.986499	1.0136	1.01575
26	0.973477	0.977773	1.02316	1.0269	0.983772	0.986448	1.01364	1.0158
27	0.973534	0.977824	1.02308	1.0268	0.983712	0.986399	1.01368	1.01585
28	0.973596	0.977877	1.023	1.02671	0.983654	0.986351	1.01372	1.0159
29	0.973661	0.977934	1.02292	1.02661	0.983598	0.986305	1.01376	1.01594
30	0.97373	0.977994	1.02284	1.02651	0.983543	0.98626	1.0138	1.01599
31	0.973807	0.978061	1.02274	1.0264	0.983492	0.986218	1.01384	1.01603
32	0.973889	0.978132	1.02265	1.02629	0.983443	0.986177	1.01387	1.01606
33	0.973977	0.978208	1.02254	1.02617	0.983396	0.986139	1.0139	1.0161
34	0.974072	0.978289	1.02244	1.02604	0.983353	0.986104	1.01393	1.01613
35	0.974173	0.978376	1.02232	1.02591	0.983313	0.986071	1.01396	1.01616
36	0.974282	0.97847	1.0222	1.02577	0.983279	0.986044	1.01398	1.01618
37	0.974393	0.978565	1.02208	1.02563	0.983247	0.986018	1.01399	1.0162
38	0.974515	0.97867	1.02195	1.02547	0.98322	0.985996	1.01401	1.01622
39	0.974643	0.978779	1.02182	1.02531	0.983198	0.985979	1.01402	1.01623
40	0.974783	0.978899	1.02167	1.02514	0.983182	0.985966	1.01402	1.01623
41	0.974927	0.979021	1.02152	1.02497	0.983171	0.985958	1.01402	1.01623
42	0.975085	0.979156	1.02136	1.02478	0.983169	0.985956	1.01402	1.01622
43	0.975254	0.979299	1.02119	1.02458	0.983173	0.985961	1.014	1.01621
44	0.97543	0.979448	1.02101	1.02437	0.983187	0.985974	1.01398	1.01618
45	0.975621	0.979611	1.02082	1.02415	0.983211	0.985994	1.01395	1.01615
46	0.975812	0.979772	1.02064	1.02393	0.983242	0.986021	1.01392	1.01611
47	0.976022	0.979951	1.02043	1.02369	0.983284	0.986057	1.01387	1.01606
48	0.976245	0.980139	1.02022	1.02344	0.983336	0.986101	1.01382	1.01599
49	0.976479	0.980337	1.01999	1.02318	0.983401	0.986157	1.01376	1.01592
50	0.976704	0.980528	1.01978	1.02293	0.98348	0.986223	1.01368	1.01583
51	0.976938	0.980725	1.01956	1.02267	0.983573	0.986302	1.01359	1.01573
52	0.977205	0.98095	1.01931	1.02238	0.983682	0.986394	1.01349	1.01561
53	0.977459	0.981165	1.01907	1.0221	0.983809	0.986501	1.01338	1.01548
54	0.977728	0.981391	1.01882	1.02181	0.983954	0.986622	1.01325	1.01533
55	0.977991	0.981613	1.01858	1.02152	0.984122	0.986763	1.0131	1.01515
56	0.97829	0.981864	1.0183	1.0212	0.984309	0.986921	1.01293	1.01496
57	0.978604	0.982129	1.01801	1.02087	0.984528	0.987104	1.01274	1.01474

Age (x)	$q^f_{0.025}$	$q^f_{0.05}$	$q^f_{0.95}$	$q^f_{0.975}$	$q^m_{0.025}$	$q^m_{0.05}$	$q^m_{0.95}$	$q^m_{0.975}$
58	0.978923	0.982397	1.01773	1.02053	0.984774	0.98731	1.01253	1.01449
59	0.979266	0.982686	1.01742	1.02017	0.985042	0.987535	1.01229	1.01422
60	0.97962	0.982984	1.0171	1.0198	0.985353	0.987795	1.01202	1.01391
61	0.980012	0.983312	1.01675	1.01939	0.985697	0.988083	1.01173	1.01356
62	0.980401	0.983639	1.0164	1.01899	0.986084	0.988407	1.0114	1.01318
63	0.980864	0.984028	1.01599	1.01851	0.986523	0.988774	1.01102	1.01275
64	0.98137	0.984452	1.01555	1.018	0.987007	0.989179	1.01061	1.01227
65	0.981905	0.984901	1.01508	1.01745	0.987545	0.989628	1.01016	1.01174
66	0.982489	0.985391	1.01457	1.01686	0.988144	0.990128	1.00965	1.01116
67	0.983155	0.985948	1.01399	1.01619	0.988813	0.990687	1.00909	1.0105
68	0.982714	0.98558	1.01435	1.01661	0.988643	0.990547	1.00922	1.01066
69	0.982223	0.985171	1.01476	1.01709	0.988496	0.990425	1.00934	1.01079
70	0.981773	0.984795	1.01514	1.01752	0.988371	0.990321	1.00943	1.0109
71	0.981332	0.984427	1.01551	1.01795	0.988278	0.990244	1.0095	1.01098
72	0.980915	0.98408	1.01585	1.01834	0.988237	0.99021	1.00953	1.01101
73	0.98052	0.983751	1.01617	1.01872	0.988232	0.990208	1.00952	1.011
74	0.980152	0.983444	1.01648	1.01907	0.98828	0.990248	1.00947	1.01094
75	0.979843	0.983187	1.01672	1.01935	0.98838	0.990333	1.00938	1.01084
76	0.97957	0.98296	1.01694	1.0196	0.988545	0.990471	1.00924	1.01067
77	0.979398	0.982819	1.01707	1.01975	0.988769	0.990659	1.00904	1.01044
78	0.979309	0.982747	1.01712	1.01981	0.989045	0.990889	1.00881	1.01017
79	0.979322	0.98276	1.01708	1.01976	0.9894	0.991186	1.00851	1.00983
80	0.979401	0.982828	1.01699	1.01965	0.98979	0.991511	1.00818	1.00945
81	0.97962	0.983014	1.01678	1.0194	0.990257	0.991901	1.0078	1.009
82	0.979961	0.983302	1.01646	1.01904	0.990739	0.992303	1.0074	1.00854
83	0.980421	0.983687	1.01605	1.01856	0.991336	0.9928	1.00691	1.00798
84	0.981213	0.984352	1.01535	1.01774	0.991992	0.993346	1.00638	1.00736
85	0.982004	0.985015	1.01466	1.01695	0.992669	0.993909	1.00583	1.00673
86	0.982925	0.985786	1.01387	1.01603	0.993497	0.994598	1.00516	1.00596
87	0.984016	0.986698	1.01295	1.01496	0.994365	0.995321	1.00446	1.00515
88	0.985361	0.987822	1.01181	1.01364	0.995171	0.99599	1.00382	1.0044
89	0.986823	0.989042	1.01059	1.01223	0.996011	0.996689	1.00315	1.00363
90	0.987817	0.98987	1.00978	1.01129	0.996811	0.997353	1.00251	1.0029
91	0.989396	0.991186	1.00848	1.00979	0.997568	0.997981	1.00192	1.00221
92	0.990886	0.992426	1.00727	1.00839	0.998445	0.998709	1.00122	1.00141
93	0.992475	0.993748	1.00598	1.00691	0.998949	0.999127	1.00083	1.00095
94	0.993953	0.994977	1.0048	1.00554	0.999972	0.999976	1.00003	1.00003
95	0.995597	0.996344	1.00348	1.00402	0.999345	0.999435	1.00061	1.00073
96	0.997154	0.997637	1.00224	1.00259	0.998891	0.999042	1.00103	1.00124
97	0.998524	0.998775	1.00116	1.00134	0.998718	0.998893	1.00119	1.00143
98	0.999552	0.999629	1.00035	1.0004	0.999001	0.999137	1.00093	1.00112
99	1	1	1	1	1	1	1	1