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# The value and risk of intergenerational risk sharing

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**Preliminary note**

A preliminary version of this paper was published in December 2016 as supplement to the publication in Dutch: Boeijen et al.: "De meerwaarde van risicodeling met toekomstige generaties nader bezien: Rapportage van bevindingen van een Netspar-werkgroep."
This paper is based on exact calculations and an open source Excel file. Sander Muns detected an error in formula (19) in the original version which leads to some minor changes in the numerical examples. These have been corrected in the present version.

**Noot vooraf**

Een voorlopige versie van dit paper is in december 2016 gepubliceerd op de Netspar-website als bijlage bij Boeijen et al.: "De meerwaarde van risicodeling met toekomstige generaties nader bezien: Rapportage van bevindingen van een Netspar-werkgroep."
Dit paper is gebaseerd op exacte berekeningen en een open source Excel bestand. Sander Muns heeft een fout in formule (19) in de oorspronkelijke versie ontdekt die tot enige aanpassing in de numerieke resultaten leidt. Deze zijn in de huidge versie aangepast.


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## Summary

This brief note discusses the value and risk of Intergenerational Risk Sharing in a very simple setting. It mainly serves to explain where value and risk of IGR in current Dutch pension contracts comes from. More complicated settings will lead to different numerical results, but conceptually these are the same.

Part of the Dutch discussion also refers to so-called “borrowing constraints”. At a young age, pension participants may actually want to invest more in stocks than the wealth they have at that time. This is very similar to the notion of IGR we discuss here: a specific pension contract may lead to exposure to risky returns that lead to an increase in utility. Within the Dutch setting, choosing optimal exposures and discussing in what institutional settings these are most easily obtained probably deserves more attention.

This paper is accompanied by an Excel sheet that implements the formulas. The reader is invited to consider several concrete parameter settings and to check the sensitivity of the risk and value of IGR for various parameter configurations.

## Samenvatting

De toegevoegde waarde van Intergenerationele Risicodeling (IGR) is een veelbesproken onderwerp in pensioenland. Het concept is onder verschillende termen bekend: ”Solidariteit met toekomstige opbouw”, ”Dempen van pech- en gelukgeneraties”, ”Genoeg is genoeg”, ”Doorschuiven van schokken”, ”Verlengen van de life cycle”, ...

In dit paper tonen we aan dat de politieke afruil betreffende IGR in feite een klassieke risico-rendementsafweging is. Ten gevolge van IGR staan deelneners al bloot aan (aandelen)risico voordat ze in het pensioensysteem instromen. Dit leidt, bij instroom, tot zowel een risico als een risicopremie. In het debat wordt vaak alleen de welvaartswinst genoemd, maar blijft het expliciete risico buiten beeld.

In dit paper worden beiden besproken en daarmee wordt de voorliggende politieke afweging verhelderd. Veel IGR betekent veel welvaartswinst, maar ook veel (discontinuiteits)risico. In het bijzonder worden de resultaten vergeleken met de berekeningen van het Centraal Planbureau betreffende dit onderwerp.

# 1 Introduction

In this note we quantify the *value* and *risk* associated to Intergenerational Risk Sharing (IGR) in pension contracts. We re-derive some results from the academic literature and apply these to the Dutch pension debate. In the Dutch debate, various terms are used to describe the notion of IGR, in particular ”Solidariteit met Toekomstige Opbouw”, “Dempen van pech- en geluksgeneraties”, “Genoeg is genoeg”, “Doorschuiven van schokken”, “Verlengen van de life cycle”, and “Vorming van (onverdeelde) buffers”, to name just a few. We demonstrate that the choice whether to adopt IGR in a pension scheme is ultimately a political one, based on a risk/return trade-off. Policy implications of the present note are discussed in more detail in Section 1.3.

The analysis leads to analytical formulas for both the value and the risk associated to IGR. Quantitatively, the results are in line with other studies that are summarized in [1]. The advantage of a fully transparent simple model is that ambiguities about the precise methodology employed can be avoided. An accompanying Excel sheet contains an implementation of the formulas derived in this note. Moreover, in Section 1.2 we compare our results to those in [3].

We first derive, in Section 3.1, the value of IGR in a first-best setting, i.e., one where future generations are optimally and fully exposed (given the financial market and preferences that we study) to IGR. This leads to a value of IGR equal to

$$B\frac{\lambda^2}{2\gamma}, \tag{1}$$

where $B$ denotes the number of years that new participants are exposed to financial market shocks *before* entering the pension contract, $\lambda$ equals the Sharpe ratio of risky investment opportunity (whose volatility we denote by $\sigma$), and $\gamma$ equals the constant relative risk aversion (CRRA) of the agent. Result (1) is also used in [4]. Report [1] uses $\lambda = 20\%$, $\sigma = 20\%$, and $\gamma = 5$. This leads, for $B = 10$, to a certainty equivalent value of first-best IGR of $10 \times (20\%)^2/(2 \times 5) = 4\%$. A naive interpretation would be that IGR leads to a welfare gain equal to $4\%$ of total (2nd pillar) pension wealth.

It is important to realize that (1) overestimates the value of IGR in the Dutch institutional setting. The reasons for this are twofold:

- Formula (1) assumes that the exposure of participants to financial market shocks that occur before they enter the pension contract is optimally chosen. In reality, due to the smoothing mechanism used in the Dutch pension contract, the exposure is suboptimal.
- Formula (1) assumes that the *full* pension wealth to be accumulated over the *entire* life cycle of the individual is exposed to IGR. In reality, participants gradually accu-

mulate pension entitlements over their life cycle and only the contributions early in the life cycle are exposed to IGR[1].

For these two reasons, we study, in Section 3.3, the value of IGR in a recently proposed pension contract, known as the “SER I-B variant”. This pension contract works as follows. Each year a funding ratio is calculated by dividing the total value of pension fund assets by the market-consistent value of pension fund liabilities (i.e., using the prevailing default-free term structure). If we denote this funding ratio by $F$, then entitlements are adjusted by a factor $1 + (F - 100\%)/B$, where $B$ denotes the duration of the smoothing period. In the “SER I-B variant” $B = 10$ has been chosen.

The contribution of the present note is that, besides a more precisely calculated value of the welfare gains due to IGR in the Dutch setting, we also calculate the *risk* associated with IGR. In the above setting, new participants may actually enter the pension fund when it is underfunded. Indeed, it is precisely the fact that funding ratios may deviate from $100\%$ that generates IGR. For instance, if a new participant enters a fund with a funding ratio of $80\%$, it means that, of the first contributions, $20\%$ is used to reduce the deficit and only $80\%$ (in terms of economic value) leads to new entitlements. We formalize the risk of IGR as the loss that participants can incur, with a given probability (of say $2.5\%$).

It is important to correctly understand the numerical values that we present below. We quantify the value and risk of IGR as a percentage of (the net present value of) lifetime pension contributions. Alternatively, some papers express it as a percentage of human capital or life-time consumption. This affects the results by a significant factor of about $10$.[2] Also, this document does not address any other advantages or disadvantages of IGR other than the numerical value and risk in a stylized setting. For a more comprehensive overview, we refer to [1] (in Dutch).

## 1.1 Numerical examples

The risk-return trade-off in IGR in the SER I-B pension contract is, for a baseline case, summarized in Table 1. This table clearly shows the political trade-off that is available when considering a pension system with IGR. Extending the smoothing period increases the value of IGR, but also its risk. More precisely, for the parameter settings used, a five-year smoothing period ($B = 5$) in the SER I-B contract leads to a certainty equivalent

[1] The uniform contribution and accrual system further dampens the value of IGR since entitlements received early in the life-cycle are less than the pension contribution. This effect is ignored in this and other studies.

[2] This factor is based on the idea that about $10\%$ of labor income is paid into 2nd pillar pensions in the Netherlands. Note, however, that this figure may vary quite a bit over individuals.

| Smoothing period ($B$) | Value of IGR | Risk of IGR |
|---|---|---|
| 5 | 0.9% | -2.3% |
| 10 | 3.8% | -5.6% |
| 20 | 10.6% | -11.5% |

Table 1: *The value and risk associated to IGR for the SER I-B contract as a function of the smoothing period ($B$) for the parameter setting used in [1], i.e., $\lambda = \sigma = 20\%$ and $\gamma = 5$. Moreover, the fund invests $w = 50\%$ in the risky asset. The calculations are based on the model in this note using the companion Excel sheet.*

gain of $3.8\%$. Formulated casually, this means that participants get, on average, a $3.8\%$ higher pension due to IGR. The other side of the coin, however, is that participants also run a risk in the sense that, with a probability of $2.5\%$, they actually incur a loss of $5.6\%$ due to IGR. In other words, they get a $5.6\%$ lower pension than without IGR. Increasing the amount of IGR, i.e., increasing $B$, increases both the average gain and the losses that may occur.

| Smoothing period ($B$) | Value of IGR | Risk of IGR |
|---|---|---|
| 5 | 0.9% | -2.3% |
| 10 | 3.1% | -5.6% |
| 20 | 6.0% | -11.5% |

Table 2: *The value and risk associated with IGR for the SER I-B contract as a function of the smoothing period ($B$). Parameters are equal to those in Table 1, but now for more risk-averse agents with $\gamma = 10$.*

Table 2 considers the baseline parameter settings, but now for more risk-averse agents with $\gamma = 10$. As the investment strategy of the fund does not change, the risk of IGR is identical. However, as agents are more risk-averse, they appreciate risk less. As a result, the welfare gains are lower compared to Table 1.

Table 3 considers the case where the Sharpe ratio of the risky investment opportunity is increased to $\lambda = 25\%$. As this leads to more reward for risk, without additional risk, the welfare gains go up. Also the risk of IGR is decreased due to the higher expected return on the risky investment.

Finally, in Table 4, we consider the situation where volatility is increased to $\sigma = 25\%$. The risk of IGR obviously increases compared to the baseline case, but the value of IGR also increases. This is due to the fact that we have fixed the portfolio allocation to $w = 50\%$ of the fund. With this increased volatility, this fixed exposure is closer to the optimal

| Smoothing period ($B$) | Value of IGR | Risk of IGR |
|---|---|---|
| 5 | 1.2% | -2.0% |
| 10 | 4.9% | -4.5% |
| 20 | 14.7% | -8.2% |

Table 3: *The value and risk associated with IGR for the SER I-B contract as a function of the smoothing period ($B$). Parameters are equal to those in Table 1, but now for a higher Sharpe ratio $\lambda = 25\%$.*

| Smoothing period ($B$) | Value of IGR | Risk of IGR |
|---|---|---|
| 5 | 1.1% | -2.8% |
| 10 | 4.6% | -6.9% |
| 20 | 11.9% | -14.4% |

Table 4: *The value and risk associated with IGR for the SER I-B contract as a function of the smoothing period ($B$). Parameters are equal to those in Table 1, but now for a higher volatility $\sigma = 25\%$.*

one.

## 1.2 Comparison to CPB2016 results

In [3], the CPB Netherlands Bureau for Economic Policy Analysis provides an analysis of the welfare gains of IGR within various pension systems, including the SER I-B contract discussed above. It concludes that, due to IGR, gains in the order of magnitude of $7\%$ are possible[3]. The scenarios used to calculate these numbers are provided by APG from a more complicated model than the one used in this note.

The risk associated with IGR receives less attention in [3]. However, based on the funding ratios provided by CPB for this analysis, the probabilities shown in Table 5 have been calculated. Table 5 should be read as follows. There is a $100\% - 25\% = 75\%$ probability that the funding ratio will fall below $90\%$ at some point over the horizon studied. One may argue that if the funding ratio falls too low, the pension contract will be renegotiated and that the system will not continue. If this renegotiation level were to lie at a funding ratio of $90\%$, this means that there is only a $25\%$ probability that the system will actually survive for the entire horizon (of 100 years) studied.

This leads to another interpretation of the numbers in Table 5. If we want to make

[3]This number includes gains due to additional stock exposure at the beginning of the life cycle, in addition to pure IGR gains due to exposure before entry into the system. Both effects are similar in magnitude.

| Minimum funding ratio | Probability |
|---|---|
| 100% | 0% |
| 90% | 25% |
| 80% | 57% |
| 50% | 97.6% |

Table 5: *The probability that over the horizon studied in [3] the funding ratio remains above the given level.*

sure (say, with $97.6\%$ probability) that the pension contract will survive the horizon studied, then we must be convinced that even funding ratios as low as $50\%$ will not lead to renegotiation of the contract.

All in all, the CPB analysis in [3] leads to the same fundamental trade-off for IGR as in this note. There are no welfare gains without risks. The optimal choice between the two is a political one.

## 1.3 Policy implications

The debate about the use and abuse of Intergenerational Risk Sharing in the Dutch pension system has been heated at times. This note hopes to shed some light on the fundamental question that needs to be answered. IGR allows pension fund participants to be exposed to the stock market risk-return trade-off even *before* they enter the pension system. This leads effectively to a leveraged stock market risk position. This leads to both welfare gains and risk. The gains come from the risk premium associated with stock investments, the risk comes from its risk.

Policymakers could ask themselves: “What funding ratio level do I find acceptable in the sense that the system will not be threatened when this level occurs?”. When that question is answered, a bound on the risk associated with IGR is defined and we may find the maximum possible welfare gains possible. This note gives some idea of the numbers that are feasible and links these to the analysis in [3].

## 2 The financial market

The financial market model that we use is standard. This means that there is a single risk factor which we refer to as stock market risk. Exposure to this risk factor induces risk and an expected return that need to be balanced.

Our model thus excludes interest rate risk. Analytical expressions with interest rate risk are likely to be available for standard Vasicek or more general affine interest rate risk models, see e.g., [2]. However, these formulas are not necessarily very insightful when it comes to the fundamental IGR trade-off. As discussed in the introduction, IGR exposes pension fund participants to risk and return *before* they actually enter the fund. For this trade-off, it does not make much difference whether it comes from stock market risk or (speculative) interest rate risk. In practical situations a fund can simply look at the overall risk/return trade-off of its full portfolio.

We also exclude longevity risk and non-traded inflation risk. Generally speaking, these risks are often considered to be much smaller than investment risk, so their effect on IGR is limited as well.

Let's define the parameters of interest. All returns and interest rates in this paper are geometrically compounded.

- There is a constant interest rate $r$;
- There is a systematic risk factor $Z$ with price $\lambda$;
- There is stock, with exposure to $Z$, that has volatility $\sigma$;
- Agents have CRRA utility with risk aversion $\gamma$;
- The (continuously rebalanced) stock exposure is denoted by $w$.

If we denote the stock price at time $t$ by $S_t$, it evolves according to

$$\mathrm{d}S_t = (r + \lambda\sigma)\, S_t \mathrm{d}t + S_t \sigma \mathrm{d}Z_t. \tag{2}$$

From this expression follows an expected (arithmetic) return on stocks of approximately (see appendix for details) $\mu = r + \lambda\sigma$ so that $\lambda$ can be identified with the Sharpe ratio. The risk premium on the stock is $\lambda\sigma$.

# 3 Modeling IGR

As explained in the introduction, we assume that agents actually have access to stock risk *before* they enter the labor market (at time $t = 0$). Thus, instead of starting to invest one unit of wealth at time $t = 0$, we assume that an agent can invest an amount $\exp(-Br)$ at time $t = -B$.

This paper and the accompanying Excel sheet consider the following three possibilities of *how much* of the pension contributions is exposed to IGR.

**First-best exposure** The *full* pension contribution over the life-cycle of the agent is *optimally* exposed to risks before entering the labor market.

**Full exposure with smoothing** The *full* pension contribution over the life-cycle of the agent is exposed to risks before entering the labor market according to a smoothing mechanism.

**Gradual exposure with smoothing** Only pension contributions at the *start of the life-cycle* of the agent are exposed to risks before entering the labor market and they are exposed according to a smoothing mechanism.

The case with first-best exposures is presented mainly for reasons of illustration purposes and it leads to the often-used $B\lambda^2/\gamma$ formula, e.g., in [4]. This case means that (all future) pension premiums are paid instantaneously upon entering the labor market and are exposed optimally to previous shocks. In particular, no smoothing of shocks is applied.

In the second case ("full exposure with smoothing") we still assume that all life-time pension premiums are paid at once upon entering the labor market, but that they are exposed to IGR using the SER I-B mechanism. This situation occurs when pension wealth is transfered from one pension contract to another ("invaren"). As a result, exposures decrease with the horizon. Note, however, that such a form of smoothing is suboptimal in the current setting.

In the third and last case ("gradual exposure with smoothing") we assume that agents pay pension contributions only gradually over their life-time so that contributions early in life have more IGR exposure than contributions later in life. This is the most realistic setting for agents who enter the labor market at a young age.

In all cases, we do not only derive the *value* of IGR, but also the associated *risk*.

## 3.1 IGR: First-best exposures

We first derive the first-best exposures to shocks before entering the labor market. This will also lead to some formulas that are needed later. As this is still a Merton problem, we know that it is optimal to have a constant stock exposure $w$ over the investment period. Wealth at time $t$ will then equal[4]

$$\begin{aligned} W_t &= \exp(-Br)\exp\left((r+w\lambda\sigma)(B+t)-\frac{1}{2}w^2\sigma^2(B+t)\right. \\ &\quad\left. +w\sigma(Z_t-Z_{-B})\right) \\ &= \exp\left(rt+\left[w\lambda\sigma-\frac{1}{2}w^2\sigma^2\right](B+t)+w\sigma(Z_t-Z_{-B})\right). \end{aligned} \tag{3}$$

Note that, for $B=0$, we indeed get the standard expression for the evolution of wealth in a Merton model. The above expression is essentially the same, only starting at $t=-B$ with an initial wealth of $\exp(-Br)$.

Now, assume that the agent wishes to maximize utility of wealth at time $t=T$. With CRRA utility, the agent thus maximizes

$$\begin{aligned} EW_T^{1-\gamma} &= \exp\left((1-\gamma)\left(rT+\left[w\lambda\sigma-\frac{1}{2}w^2\sigma^2\right](B+T)\right)\right. \\ &\quad\left.+\frac{1}{2}(1-\gamma)^2w^2\sigma^2(B+T)\right) \\ &= \exp\left((1-\gamma)rT+w(1-\gamma)\lambda\sigma(B+T)\right. \\ &\quad\left.-\frac{1}{2}(1-\gamma)\gamma w^2\sigma^2(B+T)\right). \end{aligned} \tag{4}$$

One easily verifies (by solving the first-order condition $\lambda\sigma=\gamma w\sigma^2$), that the optimal stock investment is given by the classical expression

$$w^*=\frac{\lambda}{\gamma\sigma}=\frac{\mu-r}{\gamma\sigma^2}. \tag{5}$$

A well-known consequence of this standard Merton setting is that optimal investments do not depend on the investment horizon nor on accumulated wealth. This is immediate from the above results.

### 3.1.1 First-best exposures: The value of IGR

Note that the utility of wealth at horizon $T$ factorizes in the following way. We have

$$\begin{aligned} EW_T^{1-\gamma} &= \exp\left((1-\gamma)B\left[w\lambda\sigma-\frac{1}{2}\gamma w^2\sigma^2\right]\right) \\ &\quad\exp\left((1-\gamma)\left[rT+w\lambda\sigma T-\frac{1}{2}\gamma w^2\sigma^2 T\right]\right). \end{aligned} \tag{6}$$

The utility gain of IGR is thus given by the proportionality factor

$$\exp\left((1-\gamma)B\left[w\lambda\sigma-\frac{1}{2}\gamma w^2\sigma^2\right]\right), \tag{7}$$

[4] This easily follows from Itô's lemma. Also note that $Z$ is in this case a two-sided Brownian motion with $Z_0=0$, no drift, and unit variance.

which, in terms of certainty equivalents, translates into

$$\exp\left(B\left[w\lambda\sigma - \frac{1}{2}\gamma w^2\sigma^2\right]\right). \tag{8}$$

Under the optimal investment $w^* = \lambda/(\gamma\sigma)$, this expression simplifies to the well-known certainty equivalent wealth formula

$$\exp\left(\frac{B}{2}\frac{\lambda^2}{\gamma}\right). \tag{9}$$

For future reference, note that the *additional* value of IGR of having an (additional) exposure $\alpha$ to shocks at time $t = -B$ equals

$$\exp\left(\left[\alpha\lambda\sigma - \frac{1}{2}\gamma\alpha^2\sigma^2\right]\right). \tag{10}$$

### 3.1.2 First-best exposures: The risk of IGR

We measure the risk of IGR by the distribution of wealth at time $t = 0$. From (3), we have

$$W_0 = \exp\left(\left[w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right]B - w\sigma Z_{-B}\right). \tag{11}$$

Clearly, for $B = 0$ we have $W_0 = 1$, but, for $B > 0$, $W_0$ can be either larger or smaller than $1$. $W_0$ follows a log-normal distribution. More precisely,

$$W_0 \sim LN\left(\left[w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right]B; w^2\sigma^2 B\right). \tag{12}$$

Again plugging in the optimal stock exposure $w^* = \lambda/(\gamma\sigma)$ leads to

$$W_0 \sim LN\left(\left[\frac{1}{\gamma} - \frac{1}{2\gamma^2}\right]\lambda^2 B; \frac{\lambda^2}{\gamma^2}B\right). \tag{13}$$

For future reference, note that the *additional* risk of IGR of having an (additional) exposure $\alpha$ to shocks at time $t = -B$ equals

$$LN\left(\left[\alpha\lambda\sigma - \frac{1}{2}\alpha^2\sigma^2\right]; \alpha^2\sigma^2\right). \tag{14}$$

## 3.2 IGR: Full exposure with smoothing

In the Dutch pension system, shocks are smoothed over a period of ten years. We formalize this such that every year a fraction $\rho$ of shocks is transferred to the funding ratio, and a fraction $1 - \rho$ is transferred to entitlements. This effectively means that the exposure of pension entitlements to a financial market shock at time $t = -B$ is given by $\alpha = w\rho^B$, where $w$ denotes the fund exposure to the financial market. The exposure to shocks before entry thus decreases with time. As mentioned above, this is suboptimal in the present Merton setting with CRRA utility.

As a result, each exposure to shocks before labor market entry, i.e., to shocks at $t = -1, -2, -3, \ldots$ has to be addressed separately. Each shock leads to some additional value and some additional risk of IGR. As the exposures are suboptimal, it may actually be that this value is negative.

Substituting the effective exposure $\alpha = w\rho^B$ in (10) we find for the *additional* value of IGR due to exposure to shocks at $t = -B$

$$\exp\left(\left[w\rho^B\lambda\sigma - \frac{1}{2}\gamma(w\rho^B)^2\sigma^2\right]\right), \tag{15}$$

and for the *additional* risk

$$LN\left(\left[w\rho^B\lambda\sigma - \frac{1}{2}(w\rho^B)^2\sigma^2\right]; (w\rho^B)^2\sigma^2\right). \tag{16}$$

It is important to note that these are the *additional* value and risk due to exposure to shocks at time $t = -B$. Note that, for $\rho = 1$, this leads to the formulas in Section 3.1 that are linear in $B$. In order to obtain the total value and risk, the above effects need to be accumulated. More precisely, the total value of IGR is given by

$$\exp\left(\sum_{B=1}^{\infty}\left[w\rho^B\lambda\sigma - \frac{1}{2}\gamma(w\rho^B)^2\sigma^2\right]\right), \tag{17}$$

with a risk of

$$LN\left(\sum_{B=1}^{\infty}\left[w\rho^B\lambda\sigma - \frac{1}{2}(w\rho^B)^2\sigma^2\right]; \sum_{B=1}^{\infty}(w\rho^B)^2\sigma^2\right). \tag{18}$$

These formulas might be simplified, but that does not seem to lead to additional insights. We refer to the accompanying Excel sheet for an implementation. Also note that, for small values of $w\rho^B$, the second-order terms in (15) and (16) are negligible compared to the first-order terms. This allows even simpler analytical expressions. Finally note that in the present Dutch system, IGR is organized via increases or decreases of entitlements, not via accrued value. If the value of entitlements does not coincide with the value of pension premiums paid[5], this would lead to an additional correction factor.

## 3.3 IGR: Gradual exposure with smoothing

The results in Section 3.2 still assume that the total life-time pension contribution would be fully exposed to IGR shocks. This is generally not the case, with the exception of value transfer into a new pension system.

Suppose pension premiums of size $1/H$ are paid over the course of $H$ years that people participate in the pension system, say $H = 40$. The effective exposure to shocks at time $t = -B$ is now given by

[5] For instance due to the Dutch uniform contribution and accrual system.

1. the premium payment of $1/H$ at $t = 0$ has an exposure of $w\rho^{B}$;
2. the premium payment of $1/H$ at $t = 1$ has an exposure of $w\rho^{(B+1)}$;
3. the premium payment of $1/H$ at $t = 2$ has an exposure of $w\rho^{(B+2)}$;
4. ...

As no more premium payments will be made after and including $t = H$, the total exposure (to shocks at $t = -B$) becomes

$$\alpha = \frac{1}{H} w \sum_{t=0}^{H-1} \rho^{B+t} = \frac{1}{H} w \frac{\rho^{B} - \rho^{H+B}}{1-\rho}. \tag{19}$$

Again, substituting these exposures into (10) and (14) and calculating the total value and risk can be done analytically. Details can be found in the accompanying Excel sheet.

## A Geometric versus arithmetic returns

Actual calculations of the gains of IGR require parameter estimates, in particular for $r$, $\lambda$, and $\sigma$. Under the assumptions imposed, we have

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + S_t \sigma \mathrm{d}Z_t, \tag{20}$$

with $\mu = r + \lambda\sigma$. This SDE implies that (gross) arithmetic asset returns $S_{t+1}/S_t$ are log-normally distributed with parameters $\mu - \sigma^2/2$ and $\sigma^2$. More precisely, we have

$$\log \frac{S_{t+1}}{S_t} \sim N\left(\mu - \frac{\sigma^2}{2}; \sigma^2\right). \tag{21}$$

As a result, the geometric returns satisfy

$$E \log \frac{S_{t+1}}{S_t} = \mu - \frac{\sigma^2}{2}, \tag{22}$$

$$V \log \frac{S_{t+1}}{S_t} = \sigma^2. \tag{23}$$

Similarly, (net) arithmetic returns satisfy

$$E\left\{\frac{S_{t+1}}{S_t} - 1\right\} = \exp\left(\mu - \frac{\sigma^2}{2} + \frac{\sigma^2}{2}\right) - 1 \approx \mu, \tag{24}$$

$$V\left\{\frac{S_{t+1}}{S_t} - 1\right\} = \exp\left(2\left[\mu - \frac{\sigma^2}{2}\right] + \sigma^2\right)\left(\exp\left(\sigma^2\right) - 1\right).$$

The Dutch Committee Parameters (2014), estimates are

$$\hat{\mu} = 8.5\%, \tag{25}$$

$$\hat{\mu} - \frac{1}{2}\hat{\sigma}^2 = 7.0\%. \tag{26}$$

This means that they implicitly estimate $\hat{\sigma}^2 = 2 \times 1.5\% = 3\%$, i.e., $\hat{\sigma} = 17.5\%$. All these parameters are nominal. Taking $r = 2\%$ real interest and $\pi = 2\%$ inflation, we would thus, to be consistent with the Dutch Committee Parameters (2014) estimates, have

$$\hat{\sigma} = 17.5\%, \tag{27}$$

$$\hat{\mu} = 8.5\%, \tag{28}$$

$$\hat{\lambda}\hat{\sigma} = 8.5\% - 2.0\% - 2.0\% = 4.5\%, \tag{29}$$

$$\hat{\lambda} = 4.5\%/17.5\% = 25.7\%. \tag{30}$$

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