Variable Annuities in the Dutch Pension System

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Abstract

In this paper we consider the risk-return trade-off for variable annuities in the retirement phase, with a special focus on the Dutch institutional setting. In particular, we study the effect of the so-called Assumed Interest Rate. We also consider in detail the consequences of the possibility to smooth, in a certain sense, financial market shocks over the remaining retirement period. Our analysis is based on an explicit distribution of initial pension wealth over the pension payments at various horizons. We discuss the effects of sharing (micro) longevity risk. Our focus is on variable annuities in an individual Defined Contribution setting.

Keywords Assumed Interest Rate (AIR), Habit formation, Smoothing financial market shocks.

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1 Introduction

Recently[1], the Dutch senate passed a law that enables retirees to invest their pension wealth in risky assets. Before that law it was compulsory to convert pension wealth at retirement in a life-long fixed annuity. Under the new law, life-long variable annuities are allowed as well.

In the present paper we study the risk-return trade-off for variable annuities, focusing in particular on the choice of the assumed interest rate (AIR) and the effect of smoothing financial market shocks. We derive exact analytical expression for the distribution of pension payments at given horizons given a chosen assumed interest rate and possibly using the option to smooth financial market returns.

In particular we study the question of obtaining constant, in expected nominal terms, pension payments. In case of smoothing financial market returns that leads to the so-called BNW discount curve that we re-derive. In the setting of the new Dutch law, we translate this curve into a horizon-dependent "fixed decrease".

The rest of this paper is organized as follows. In Section 2 we describe the Merton financial market that we consider. Section 3 considers the risk-return trade-off for variable annuities without smoothing. Subsequently, Section 4 considers the setting with smoothing of financial market shocks. Section 5 relates our paper to the Dutch institutional setting as it concerns the so-called "fixed decrease" ("vaste daling"). Section 6 concludes the paper. All numerical illustrations are based on an Excel spreadsheet that is available from the authors.

2 The financial market

As financial market we consider the standard Black-Scholes/Merton setting. This means in particular that there is a constant (continuously compounded) interest rate r and a single risky asset whose price at time t we denote by S_t.

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t \tag{2.1}$$

$$= (r + \lambda\sigma) S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t. \tag{2.2}$$

Here, μ denotes the (arithmetically compounded) expected return, σ denotes the (instantaneous) stock volatility, λ denotes the Sharpe ratio $\lambda = (\mu - r)/\sigma$ and Z is a standard Brownian motion.

We abstain from other risk factors. In particular, we also ignore longevity risk, though we discuss it to some extent in Section 3.2. When calculating payments in real terms, we consider a constant (continuously compounded) inflation rate π.

[1] https://www.eerstekamer.nl/stenogramdeel/20160614/initiatief_lodders_wet_verbeterde_2

3 Variable annuities without smoothing

Consider a retiree who enters retirement with total DC wealth W_t at time t and has to finance H annual pension payments at times $t+h$, $h=0,\ldots,H-1$. For now we assume H to be given, i.e., we consider fixed-term, instead of life-long, variable annuities. Think of H as the remaining life expectancy at the retirement age.

The pension payment at each horizon $h=0,\ldots,H-1$ has to be financed from the initial total pension wealth W_t. If we denote by $W_t(h)$ the market-consistent value of the pension payment at horizon h, the budget constraint implies

$$W_t = \sum_{h=0}^{H-1} W_t(h). \tag{3.1}$$

Alternatively stated, at time t we consider an amount of wealth $W_t(h)$ that is available to finance the pension payment at time $t+h$. The actual pension payment will of course depend on the investment strategy that is followed and the financial market returns. We can, conceptually, allow for a different investment strategy for the wealth allocated to each horizon $h=0,\ldots,H-1$. Indeed, this is precisely what happens when financial market returns are smoothed, see Section 4.

The way total pension wealth W_t is allocated over wealth $W_t(h)$, for each horizon $h=0,\ldots,H-1$, determines implicitly the so-called assumed interest rate (AIR) $a_t(h)$. That is, the assumed interest rate is defined through

$$\frac{W_t(h)}{W_t} = \frac{W_t(h)}{\sum_{k=0}^{H-1} W_t(k)} = \frac{\exp\left(-h a_t(h)\right)}{\sum_{k=0}^{H-1} \exp\left(-k a_t(k)\right)}. \tag{3.2}$$

Note that this implies in particular

$$\frac{W_t(h)}{W_t(0)} = \exp\left(-h a_t(h)\right). \tag{3.3}$$

A few comments are in place. First, usually, the assumed interest rate $a_t(h)$ is taken as given and used to determine the allocation of total wealth over the various payments. Equations (3.2) and (3.3) show that both approaches are equivalent. Secondly, observe that in general, $a_t(h)$ need not be constant in h, i.e., there can be an assumed interest rate *term structure*. This will become particularly relevant when discussing the possibility of smoothing in Section 4. Finally, the Dutch institutional setting uses to notions of "projection rate" ("projectierente") and "fixed decrease" ("vaste daling"). We will see, in Section 5, that their sum equals the assumed interest rate. We will ignore the (political) reasons for not using this sum directly but a separate projection rate and fixed decrease.

Suppose that we invest $W_t(h)$ in a continuously re-balanced strategy with a fixed stock exposure w. Then standard calculation show that wealth $W_t(h)$, for the pension payment at time h, evolves as

$$\mathrm{d}W_t(h) = (r + w\lambda\sigma)\, W_t(h)\mathrm{d}t + w\sigma W_t(h)\mathrm{d}Z_t. \tag{3.4}$$

Using Itô's lemma we find

$$\mathrm{d}\log W_t(h) = \left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right)\mathrm{d}t + w\sigma\mathrm{d}Z_t. \tag{3.5}$$

As a result, the pension payment at horizon h, $W_{t+h}(h)$, follows a log-normal distribution with parameters $h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right)$ and $hw^2\sigma^2$. In particular, the expected pension payment at horizon h is given by

$$\begin{aligned} \mathbb{E}_t W_{t+h}(h) &= W_t(h)\exp\left(h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right) + \frac{1}{2}hw^2\sigma^2\right) \\ &= W_t(h)\exp\left(h\left(r + w\lambda\sigma\right)\right). \end{aligned} \tag{3.6}$$

Risk in the pension payment $W_{t+h}(h)$ at horizon h can be determined by calculating the volatility of the payment. However, we even easily get the quantiles for the distribution. The quantile at level α is given by

$$Q_t^{(\alpha)}\left(W_{t+h}(h)\right) = W_t(h)\exp\left(h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right) + z_\alpha\sqrt{h}w\sigma\right), \tag{3.7}$$

where z_α denotes the corresponding quantile of the standard normal distribution.

In case one is interested in the real pension payment, the above expected payoff and quantiles simply have to be multiplied with $\exp(-h\pi)$.

One may be interested in choosing the assumed interest rate $a_t(h)$ in such a way that the expected pension payments are constant with respect to h, i.e., such that $\mathbb{E}_t W_{t+h}(h) = W_t(0)$ (recall that the first pension payment $W_t(0)$ is without investment risk). From (3.6) we find that this implies

$$\frac{W_t(h)}{W_t(0)} = \exp\left(-h\left(r + w\lambda\sigma\right)\right), \tag{3.8}$$

or, using (3.3),

$$a_t(h) = r + w\lambda\sigma. \tag{3.9}$$

This (constant) assumed interest rate leads to, in expectation, nominally constant pension payments. In case our financial market would exhibit interest rate risk (that is, a horizon-dependent risk-free *term structure*) and/or stock market predictability, we would need horizon-dependent assumed interest rates to obtain, in expectation, constant pension payments. We will see that, even in the present financial market, also smoothing financial market returns leads to a horizon-dependent assumed interest rate (Section 4).

3.1 Combining variable and fixed annuities

In the derivations above, we implicitly assumed continuous re-balancing of the risky investment portfolio. Thus if one specifies to invest w of wealth in the risky asset, then movements in this underlying asset cause a change in the relative investment in the risky and the risk-free asset. As a result, either risky assets have to be bought (when their value goes down) or sold (when their value goes up) in order to keep the proportion of wealth invested in the risky asset constant at w.

Thus, if the stock price drops, the total value of wealth goes down and the relative proportion invested in the stock decreases as well. Therefore, one will buy more stocks to keep the investment proportion constant. If the stock price drops further, the total value of wealth invested can theoretically go to zero. This undesired feature can be alleviated by combining a variable annuity with continuous re-balancing with a fixed annuity.

In the current setting, a fixed annuity is simply obtained in case no investment risk is taken for any of the pension payments. That is, each $W_t(h)$ is fully invested in the risk-free asset.

Note that, in a setting with interest rate risk and risk-free term structures, the above can be extended. There will be a guaranteed pension payment at horizon h in case the wealth $W_t(h)$ is fully invested in default-free zero-coupon bonds with maturity h. Essentially such an investment strategy fully hedges interest rate risk. That is, along the way the evolution of $W_t(h)$ is risky, but it is known, ignoring default risk, what the payment at time $t+h$ will be.

Clearly, the above can lead to the suggestion to build a floor in a variable annuity. This means that for each horizon h, a fraction of $W_t(h)$ is invested in default-free zero-coupon bonds with maturity h. The remainder can then be invested in a diversified (risky) return portfolio. This is known as splitting the investment in a hedge demand and a speculative demand. Note that the speculative demand should generally also still be invested partially in bonds as they offer a risk-return trade-off as well and thus lead to diversification benefits.

Let v be the fraction of $W_t(h)$ that is invested in the risk-free asset, hence v is the hedge demand and equals the proportion of the fixed annuity which builds the floor in the mixed annuity. Then $1-v$ is the remainder that is used to buy the variable annuity, thus the fraction $1-v$ of $W_t(h)$ is the speculative demand. The expected pension payment of the mixed annuity at horizon h is given by

$$\mathbb{E}_t W_{t+h}(h) = v \cdot \mathbb{E}_t W^r_{t+h}(h) + (1-v) \cdot \mathbb{E}_t W^{r+w\lambda\sigma}_{t+h}(h), \tag{3.10}$$

where $\mathbb{E}_t W^r_{t+h}(h)$ is the expected pension payment at horizon h of the fixed annuity with $w=0$

and hence $a_t(h) = r$, and $\mathbb{E}_t W_{t+h}^{r+w\lambda\sigma}(h)$ is the expected pension payment at horizon h of the variable annuity with a fixed stock exposure w and an assumed interest rate $a_t(h) = r + w\lambda\sigma$.

This combined annuity induces a minimally guaranteed annuity payment of

$$\begin{aligned} L_v(h) &= v \cdot W_t(h) \cdot \exp(hr) + (1-v) \cdot 0 && (3.11)\\ &= v \cdot \frac{W_t}{\sum_{h=1}^{H} \exp(-hr)}. && (3.12) \end{aligned}$$

The level α quantile at horizon h is

$$\begin{aligned} Q_t^{(\alpha)}\left(W_{t+h}(h)\right) &= v \cdot \frac{W_t}{\sum_{h=1}^{H} \exp(-hr)} + (1-v) \cdot \frac{\exp\left(-ha_t(h)\right)}{\sum_{k=0}^{H-1} \exp\left(-ka_t(k)\right)} \cdot && (3.13)\\ &\quad W_t \cdot \exp\left(h\left(r + w\lambda\sigma - \frac{1}{2} w^2\sigma^2\right) + z_\alpha \sqrt{h} w\sigma\right). \end{aligned}$$

Note that this expression for the quantile will remain valid in the case of smoothing (see Section 4), simply by using the adjusted parameters derived in that section.

3.2 Longevity risk

So far we have ignored longevity risk altogether. We will not discuss macro longevity risk in this paper. Concerning micro longevity risk, the standard mortality credit argument applies. In case the pension payment at horizon h will only be mode with probability $p_t(h)$, independent of the evolution of financial markets, the market-consistent value of the pension payment will be reduced by a factor $p_t(h)$. As a result, a higher assumed interest rate $a_t(h)$ can be used in order to get a higher, in expectation constant, nominal pension.

One may wonder how micro longevity risk can be shared in this setting. In particular, issues may arise in case agents with different investments risks share their longevity risk in a pool. The reason is that, in such case, the wealth that the pool receives upon death of one agent depends on previous financial market returns and, thus, is risky. This is an interesting area for further research.

3.3 Examples

The standard setting for the empirical examples in this paper are $\mu = 6.00\%, \pi = 1.00\%, \sigma = 20.00\%, \lambda = 20.00\%, r = 2.00\%, H = 20$ year and $W_t = €100,000$.

Figure 1 shows the expected pension payment with a fixed stock exposure $w = 35\%$ and the 5%- and 95%-quantiles. The blue line is the fixed annuity in which $W_t(h)$ is fully invested in the risk-free asset.

Figure 1: Variable annuity without smoothing

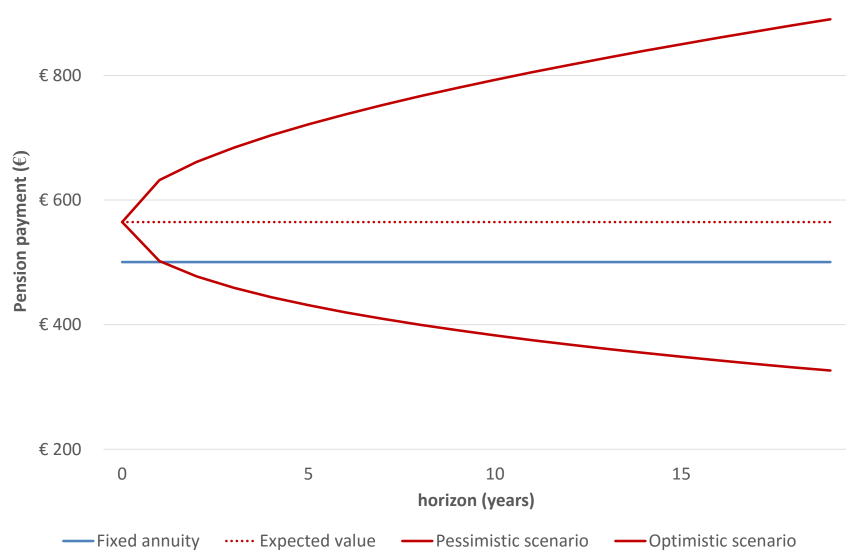
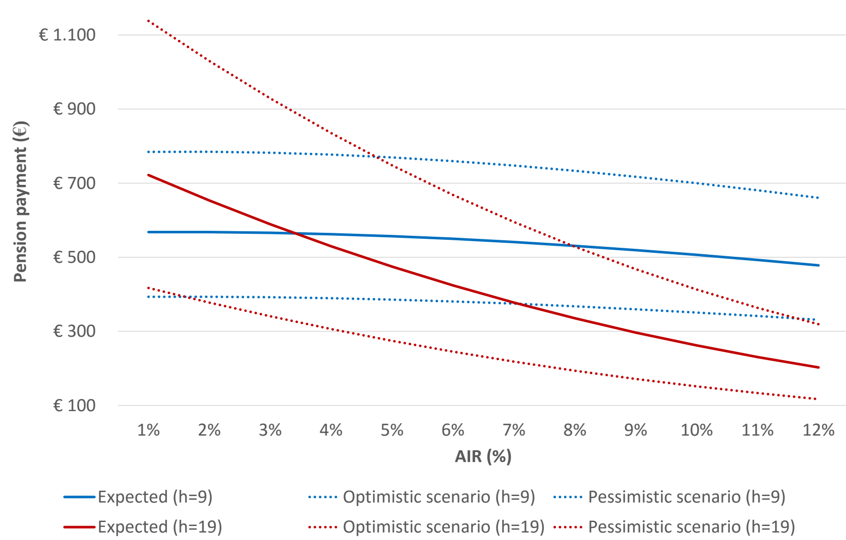


Figure 2 shows the sensitivity of the assumed interest rate on the expected pension payment at $h = 9$ and $h = 19$. The increasing air has hardly an effect on the 10th pension payment

Figure 2: Sensitivity AIR



distribution but all the more on the 20th payment. As a result, for communication purposes, we would advise to show the risk-return of variable annuities over horizons significantly exceeding 10 years.

4 Variable annuities with smoothing

In case agents have habit-formation preferences, they may want to reduce year-to-year volatility in the pension payments. The traditional view to achieve this is to "smooth" financial market returns. That is, in case returns are −20%, instead of reducing the pension payment immediately with 20%, it is only reduced by a fraction. Clearly, this implies that pension payments later in the retirement phase have to be cut by more than 20% to fulfill the budget constraint. Smoothing then leads to smaller year-to-year decreases, but the total decrease is larger.

The view on smoothing above leads, effectively, to an increase in the assumed interest rate following negative financial market returns and, symmetrically, a decrease in the assumed interest rate following positive financial market returns. This leads to a situation where wealth $W_t(h)$ originally reserved for the pension payment at time $t+h$ is redistributed over all future pension payments. The resulting mathematics is complicated and thus we propose an alternative view here, inspired by Bovenberg, Nijman, and Werker (2012)[2].

The reduced year-to-year volatility can also be achieved as follows. Recall that the initial pension payment at time t is given by $W_t(0)$. In order to have a limited risk in the pension payment $W_{t+1}(1)$ we do not invest it according to a stock exposure w, as in Section 3, but with a stock exposure $w_t(1) = w/N$, where N denotes the smoothing period, say, $N = 5$ years. Subsequently, the pension wealth $W_t(2)$ for the pension payment $W_{t+2}(2)$ is invested with exposure $w_t(2) = 2w/N$ the first year and $w_{t+1}(2) = w/N$ the second year. In general, with a smoothing period N and long-term stock exposure w, the pension wealth $W_{t+j}(h)$ for the pension payment at time $t+h$ has stock exposure

$$w_{t+j-1}(h) = w \min\left\{1, \frac{1+h-j}{N}\right\}, \quad j = 1, \ldots, h, \tag{4.1}$$

during the year from $t+j-1$ to $t+j$.

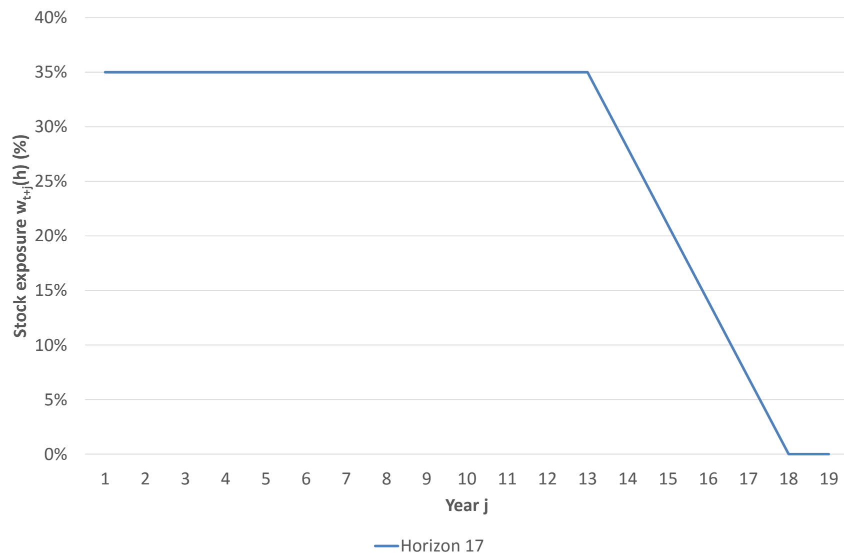
Figure 3 shows the stock exposure $w_{t+j-1}(h)$ against j for $h = 17$.

Note that the horizon-dependent stock exposure $w_{t+j}(h)$ induces a life-cycle investment strategy. That is, with smoothing the investment strategy is no longer constant over time.

As above, we can now calculate the distribution of the pension payment at time $t+h$. Again,

[2] Lans Bovenberg, Theo Nijman, and Bas Werker, "Voorwaardelijke Pensioenaanspraken: Over Waarderen, Beschermen, Communiceren en Beleggen", Netspar Occasional Paper 2012.

Figure 3: Smoothing stock exposure



this distribution is log-normal, but now with parameters

$$\sum_{j=1}^{h}\left(r+w_{t+j-1}(h)\lambda\sigma-\frac{1}{2}w_{t+j-1}^{2}(h)\sigma^{2}\right), \tag{4.2}$$

and

$$\sum_{j=1}^{h} w_{t+j-1}^{2}(h)\sigma^{2}. \tag{4.3}$$

The expected nominal pension payments, and their quantiles, can now be calculated as before.

4.1 The BNW assumed interest rate

The previously mentioned paper Bovenberg, Nijman, and Werker (2012) discussed already the implications of smoothing financial market shocks for market-consistent valuation of pension liabilities, although their focus was more on CDC systems. The present setting allows for an exact derivation of the implied assumed interest rate. The idea is simple: which assumed interest rate $a_t(h)$ leads to a pension payment that is, in expectation, constant in nominal terms. This, essentially, amounts to inverting the expected nominal pension payments derived in the previous section.

With smoothing, the expected nominal pension payment at time $t+h$ is given by

$$W_t(h)\exp\left(\sum_{j=1}^{h}\left(r+w_{t+j-1}(h)\lambda\sigma\right)\right). \tag{4.4}$$

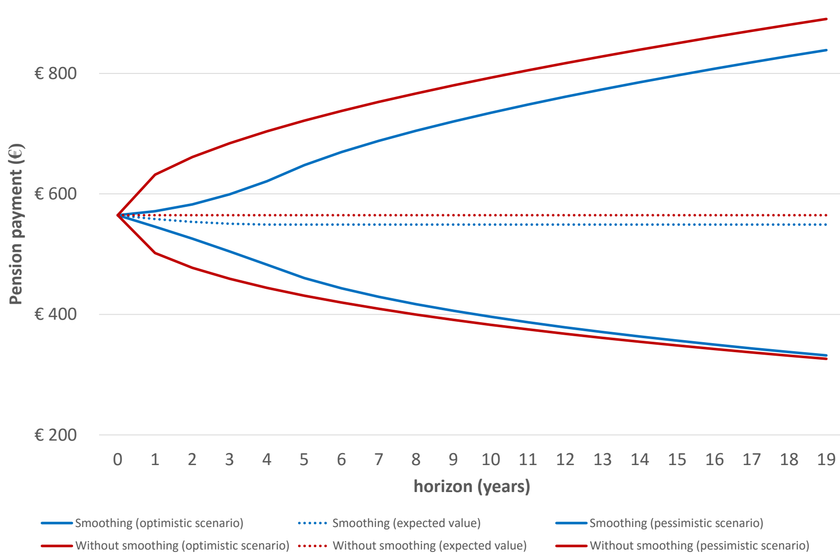
In order to have a constant nominal, in expectation, pension payment, we must choose the assumed interest $a_t(h)$ such that this expectation equals $W_t(0)$ for all h. Thus, we immediately find

$$a_t^{(BNW)}(h) = r + \lambda\sigma\frac{1}{h}\sum_{j=1}^{h} w_{t+j-1}(h). \tag{4.5}$$

4.2 Examples

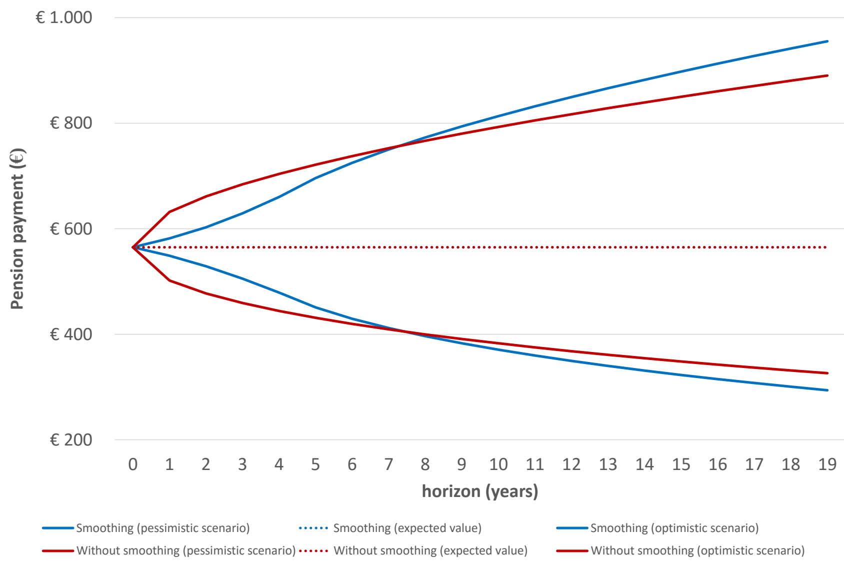
The blue lines in Figure 4 show the expected (dotted) pension payment and the 5%- and 95%-quantiles (solid) with smoothing period $N = 5$ years for a fixed stock exposure $w = 35\%$. The red lines are obtained without smoothing, similarly as in Figure 1.

Figure 4: Smoothing stock exposure



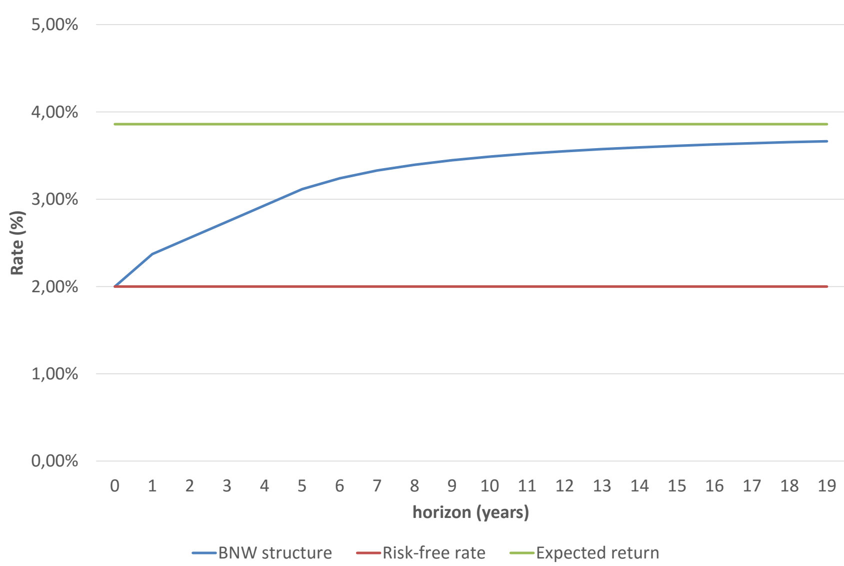
The blue lines in Figure 5 show the expected (dotted) pension payment and the 5%- and 95%-quantiles (solid) with smoothing period $N = 5$ years for a fixed stock exposure $w = 35\%$ and the assumed interest rate equal to the $a_t^{(BNW)}$. The red lines are obtained without smoothing, similarly as in Figure 1.

Figure 5: Smoothing BNW with $N = 5$



The blue line in Figure 6 shows the assumed interest rates as a function of the horizon such that the expected pension payments are constant, hence the BNW assumed interest rate. If the assumed interest rates are set equal to the risk-free rate, as given by the red line, the fixed annuity is obtained. The expected return with a stock exposure of $w = 44.5\%$ is the green line.

Figure 6: BNW structure



5 The concept of fixed decrease ('vaste daling')

For political reasons, the Dutch law "Improved pension payments" ("Verbeterde premieregelingen") does not use assumed interest rates to determine the distribution of pension wealth over the various horizons, but uses the risk-free rate in combination with a so-called "fixed decrease" ("vaste daling"). From a financial point of view, the induced assumed interest rate is simply the sum of the risk-free rate and the fixed decrease. As a result, in order to have a constant nominal pension in expectation, the fixed decrease must be chosen to be horizon dependent. The exact formula simply follows from (4.5). In case the fixed decrease at horizon h is chosen as

$$\lambda\sigma\frac{1}{h}\sum_{j=1}^{h} w_{t+j-1}(h) = w\lambda\sigma\frac{1}{h}\sum_{j=1}^{h}\min\{1,\frac{1+h-j}{N}\}, \tag{5.1}$$

the expected pension is constant in nominal terms.

6 Conclusion

This paper provides analytical expressions for the risk-return trade-off of variable annuities with a special focus on the situation where financial market returns may be smoothed over the remaining retirement period. As far as we know, this has not been documented before in the literature. For the Dutch pension debate, we find two results. First, in order to obtain, in a contract with smoothing, a constant nominal pension in expectation, the so-called fixed decrease has to be horizon dependent. We give an explicit expression. Secondly, we show that an increase in the assumed interest rate (or an increase in the fixed decrease) does affect the initial payoff, but hardly the risk-return trade-off at a horizon of 10 years.