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Quantification of the Discontinuity Risk of Pension funds

Jorgo Goossens

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Quantification of the Discontinuity Risk of Pension funds

By:
Jorgo Goossens (715062)
BSc. Tilburg University

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Tilburg School of Economics and Management
Tilburg University

Supervisors:
Dr. S. Sender (Tilburg University)
Dr. M. van Leuvensteijn (APG)
J. de Haan (APG)

Co-reader:
Prof. Dr. B. Melenberg (Tilburg University)

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Abstract

This master thesis explores the sustainability of collective pension funds, if participation in a collective fund is not mandatory but becomes voluntary. It quantifies the discontinuity risk for a typical, collective pension fund in The Netherlands. That is, the risk that new entrants abstain from the collective pension contract and current participants leave the collective fund, since they perceive to be better off in an individual system. Specifically, this thesis researches how participants react during a discontinuity event and, consequently, how these reactions (i.e. dynamic in- and outflow) affect the sustainability of the collective fund. I introduce a new dynamic ALM-approach instead of the current static ALM-approach. The development of a discontinuity event is complex and depends on the reactions of the participants, whereby most likely an information cascade occurs. I quantify the reactions by constructing a reaction-function that takes the funding ratios as input. Using Monte Carlo simulations, it is quantitatively shown how dynamic in- and outflow of participants affects the sustainability of the collective pension fund in the short-term and long-term. Classical ALM analysis shows that the fund becomes more sensitive to shocks, since the fund's population decreases and, therefore, policy instruments work less efficiently. Especially, the fund is less sustainable with respect to the long-run. Generational accounting based on value-based ALM shows that discontinuity risk affects the remaining participants with the same magnitude as benefit reductions and recovery premia do. For this reason, discontinuity risk not only affects the sustainability of pension funds, but also the remaining participants in the fund. If the fund is underfunded, the discontinuity risk with respect to new entrants and near retirees is high (up to 76.2% and 49.3% respectively). The probability of defaulting in 75 years lies between 7.4% and 42.4%, whereas the probability of default is 0% if the in- and outflow is static. Low interest rates increase the probability of default and increase the discontinuity risks even further. Funding ratios between 33% and 71% lead to a Sinking Giant, depending on how the fund transfers the pension rights.

Keywords: classical ALM, collective, discontinuity risk, dynamic inflow and outflow, funding ratio, information, inter-generational risk sharing, participation, pension fund, sinking giant, sustainability, value-based generational accounting.

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Chapter 1

Introduction

RECENTLY, I stumbled upon an everyday example of discontinuity risk. Namely, I participate in a collective car-sharing arrangement, whereby my mother shares her car with my sister and me. All of us pay an equal share of the expenses and are allowed to use the car equally often. However, after a while, my sister complains that every time she needs the car, someone else is using it. In other words, she was getting the bad time-slots, while my mother and I were benefiting from her investment (in my sister's view). Of course, since my sister did not receive for what she paid-in, she was on the edge of withdrawing from this car-sharing arrangement. This would lead to (relatively) higher costs for my mother and me, because we utilise the car in the same way but have to share the expenses with two instead of three. The problem is that the car (i.e. a collective object) belongs to nobody, but has to be shared equally. Such a situation creates winners (my mother and me) as well as losers (my sister), and threatens the sustainability of the sharing-arrangement.

Nowadays, the collective pension funds in The Netherlands face a corresponding division-problem: collective undivided capital (like the car) has to be decomposed over all generations in the pension system. Due to changing demography (fertility and longevity risks), the recent global financial crisis and the low interest rates, funds have become less sustainable and the division-problem is highlighted. Pension funds (and insurance companies) face the problem of promising long-term future cash-flows to their participants. Till the end of the twentieth century, the stock market was doing fine and the interest rate was quite stable (although already decreasing since 1970), resulting in clear expectations of the future cash-flows and, consequently, a stable situation for the participants. Up till this moment in time, pension premia were even decreasing. However, during the start of the new century, the dot-com bubble emerged which lead to drops of 7% on investments in the year 2002. Subsequently, the financial positions of the pension funds worsened (i.e. the funding ratios dropped) and the pension premiums started to rise. To control the costs, the (larger) pension funds in The Netherlands made a transition from final-wage schemes to average-wage schemes at the beginning of the year 2004. Then, the market recovered, but soon it collapsed again due to the credit-crisis in 2008, which lead to drops of 20% on investment returns; the interest and inflation rates dropped till 0% in 2016. Hereafter, many pension funds became underfunded and had no other way to improve their financial position than to cut benefits and charge recovery premia.

Due to those current developments a conflict arose about how shortages should be divided. As a consequence, a fair division of money over the generations is hindered, which erodes the trust between the different generations in the pension system. Namely, current generations might deplete the fund (for example) which is disadvantageous for future new generations. Trust is important, since it lies at the heart of inter-generational risk sharing (also known as solidarity). Inter-generational risk sharing is a common principle in collective funds and it has much support in The Netherlands: it shares negative and positive shocks to the financial position of the fund between different (overlapping) generations — possibly leading to transfers between young and old agents. The benefits of inter-generational risk sharing can be substantial (see the literature review in Section 1.1). The macro (or dependent) risks are, namely, absorbed by agents that can absorb them best.The following major types of dependent risks are commonly shared. Firstly, wage and labour productivity risk. Young generations are exposed to (aggregate) labour market risks, whereas the old are not. If the pension benefits of the old are linked to wages, then the elderly also share in this risk. A second major type of risk is financial market risk. Financial market risks do not have a negative interpretation per se; risk premiums in the financial market create attractive risk-return trade-offs. Thirdly, inflation risk depreciates the value of financial capital, whereby the old are the most vulnerable. Fourthly, (macro) longevity risk affects both the young and old generations. Finally, (macro) fertility risk influences the system directly. The text-box at the end of this introduction describes another from of solidarity, namely subsidising solidarity.

Besides, funds (and insurers) also entail intra-generational risk sharing, which means that risks are shared within one generation itself. In this case, the risks are independent and pooled away with a large number of agents — therefore, also known as risk pooling instead of risk sharing. The risks are idiosyncratic and contain no macro uncertainty. An example is individual (micro) career risk; but, the government insures this risk by social insurance and progressive taxes. A second and third type of micro risk are (micro) fertility and (micro) longevity risk, which can be pooled in pension funds. Intra-generational risk sharing (i.e. pooling of independent micro risks) does not receive any attention in my thesis further, since most citizens agree to share those risk and see it as a beneficial feature. Note, however, that intra-generational risk sharing could still lead to transfers between males and females, higher and lower educated.

Inter-generational risk sharing is only beneficial *if* all generations are committed to the collective pension contract Bovenberg, Nijman, et al. (2007). Compare it with the car-sharing example: if my sister withdraws and chooses to use a car individually, the sharing agreement vanishes, and my mother and I remain with higher costs. Since the conditions were too bad for my sister, she started an own (probably more expensive) individual system; leaving my mother and me with higher costs also. In case the car breaks down, reparations costs are shared less efficient. Such a situation might occur in a collective fund also: if participants perceive the intrinsic benefits of participating (and sharing risks) less than not participating in a collective fund, they abstain from the collective and save rather on their own. A collective Defined-Benefit (DB) system has the advantage of inter-generational risk sharing, while an individual Defined-Contribution (DC) system lacks this property but may seem more attractive from the individual participant's point of view in case e.g. funding positions are low. Hence, the *if* is important and it leads to the classical trade-off: inter-generational risk sharing is welfare enhancing (for the collective, but not by definition per individual), however it leads to discontinuity risks (i.e. the action of my sister). The main drawback of inter-generational risk sharing is discontinuity risk and arises when generations abstain and/or withdraw from the collective contract. For this reason, a collective DB features discontinuity risks.

As stated above, the pension funds have become less sustainable due to the adverse market conditions. Consequently, the risk-sharing principle lead to conflicts and even to discontinuity (i.e. abstaining generations). Thus, a pension fund can only add value if it is sustainable, e.g. the funding ratio is high enough in the perception of the participants. Without the faith that the fund is financially sustainable, risk-sharing with future generations becomes uncertain, because the future generations may become unwilling to accept the risks any longer. If participants believe that the fund may disappear and future cash-flows become uncertain, they become unwilling to further participate in the pension fund (Bovenberg and Mehlkopf, 2014). If this happens, it is said that discontinuity risks prevail which might lead to discontinuation of a fund. Discontinuation means that a fund loses its support from its participants and is no longer able to stay operative (remark that pension funds in The Netherlands cannot default by definition).

A pension fund may discontinue in case the funding ratio of the fund worsens. As a consequence, participants might leave and/or abstain from the fund, which creates a loss of support for the system: the fund's population starts to decline. If the population has decreased too much, it is not worthwhile for the fund to stay operative. In case the fund is underfunded, the fund's population can decline by two factors. First of all, new participants abstain from the collective contract, since they do not want to face a large implicit debt and recovery premia upon entry. Secondly, current participants leave the collective fund, because their benefits are maybe reduced by pension cuts. The new and current participants develop incentives to save for retirement individually if the funding ratio becomes too low, because the intrinsic benefits of participating in the collective fund might be less than the negative effect of underfunding (Ewijk et al., 2007). For this reason, policies of a fund directly influence the sustainability as well as support of the fund. For example, cutting the pension rights increases the financial stability of the fund, but erodes the participants' support. On the other hand, the belief that a pension fund is unsustainable, might also lead to a decline in support. Discontinuity risks rise when more and more participants withdraw from the current pension system. This is partially happening nowadays, see the effect of a rising number of self-employed in The Netherlands (Kocken, 2016).

Note that there are actually two types of discontinuity risk: endogenous and exogenous. The latter arises due to exogenous circumstances, i.e. the pension fund itself has no direct influence on those exogenous factors. For example, political risks such as changing regulation and lack of commitment (promises); or, the fact that civil servants cease to exist. For the (larger) pension funds in The Netherlands political risks could be large, but the fact that some professions cease to exist is assumed to be negligible.[1] Since exogenous discontinuity risk is outside the scope of the pension fund, exogenous discontinuity risk is disregarded in my analysis.

The other type of discontinuity, endogenous discontinuity risk, is born in the pension fund itself and relates to the type of discontinuity described in the car-sharing example (i.e. my sister leaving the arrangement). It emerges, amongst others, due to inter-generational risk sharing. Risk sharing, and thereby discontinuity risk, is present in collective schemes, but absent in individual schemes. However, I take the Netherlands as baseline, where participation in a pension fund is mandatory by law and, thus, participants can not just

[1]In the Netherlands an example of exogenous discontinuity risk is the proposed law during the cabinet of Lubbers in 1982: Wet Brede Herwaardering. The buffers at pension funds were supposed to depleted by taxation. Ultimately, the law did not become functional.

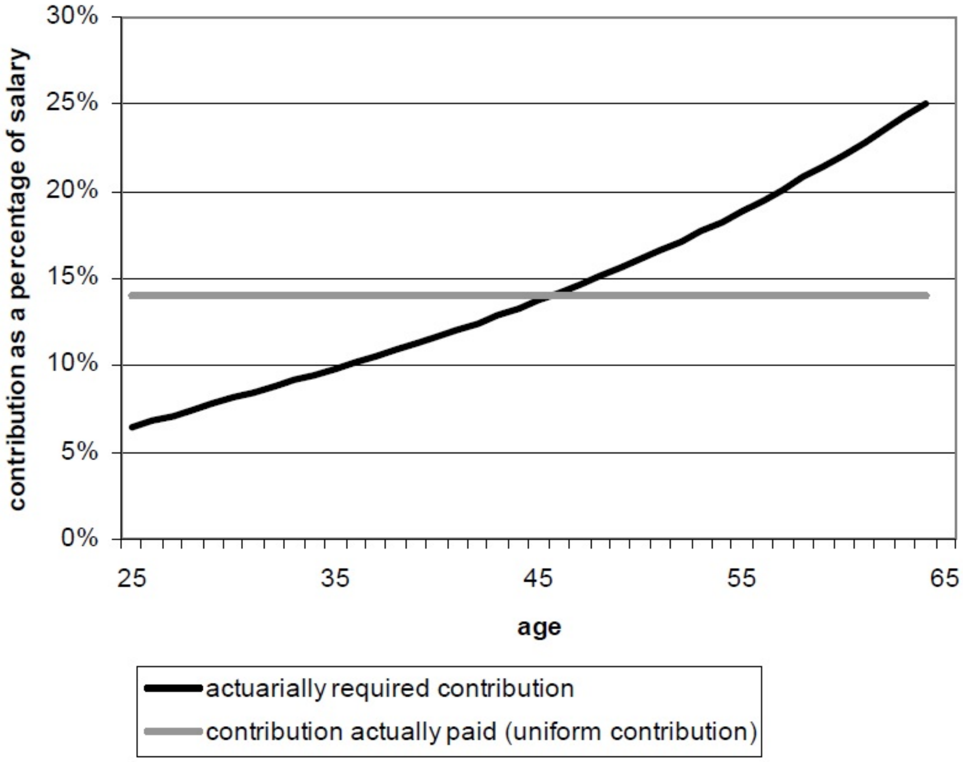
abstain and/or withdraw from the pension contract.[2] However, a theoretical setting is considered whereby individuals are able to abstain/withdraw from the pension contract and pension participation is voluntary — in other words, it is no longer mandatory to participate in a pension fund (i.e. the so-called 'verplichtstelling' is abandoned). It is assumed that agents, then, can choose funds themselves. Participants are initially faced with the option to enter in a collective Defined-Benefit (DB) system, if they exit from the collective contract, they prefer an individual Defined-Contribution (DC) system — a same approach is followed by inter alia Siegmann (2011) and Molenaar et al. (2011). The collective DB fund has the advantage of inter-generational risk sharing, but the individual DC fund does not lead to any discontinuity risk. Like the two mentioned papers, this thesis abstracts from other differences between the two systems such as administration costs (which can significantly be higher in the individual fund or if the fund's population in the collective fund declines).

The next section reviews the existing literature.

Subsidising solidarity

This text-box explains the difference between risk solidarity and subsidising solidarity. All the above is related to risk solidarity, but economists also distinguish subsidising solidarity. In The Netherlands, the young workers subsidise the elderly in the pension system. This is caused by the so-called 'doorsneesystematiek', where uniform accruals and premiums prevail: the young pay the same premium level as the elderly, but both accrue rights at the same level. The young, for this reason, pay actuarially too much (till roughly the age of 46), while the old participants pay too less for their pension — see the Figure below, where the x-axis displays age and the y-axis the premium level of the salary. The constant line represents the uniform premium level, while the curved line presents the actuarially fair premium.[a]

[2] In approximately 80% of the cases, an employee is mandated to accrue a pension; at the other 20% of the employers, it is not mandatory to accrue a pension. An employer must offer (by law) a pension-arrangement in case (i) the firm is registered by an industry-wide pension fund (holds for roughly 75% of the fund in The Netherlands), or (ii) it is stated in the employee's contract.



The young workers, thus, transfer a part of their contributions to the current old generation. The young assume that later, when they themselves are old, there will be new young workers who subsidise them. In this way, the net loss for the young workers equals the net gain for the old workers. So, the uniform contribution and accrual system provides a Pay-As-You-Go (PAYG) element; this system is also known as the doorsneesystematiek and a form of subsidising solidarity. Subsidising solidarity is considered as an exogenous factor for discontinuity, but it lies at the heart of some principles used in my thesis and, therefore worth mentioning.

Trust in the fund and between generations fades away when one group is systematically benefiting at the expense of others (like in the car-sharing example). Even if a pension solution is financially sustainable, systematic redistribution between groups erodes support for the system (Galen et al., Cardano). Nowadays, such transfers occur quite often in the system, with the young systematically paying for the old, or vice versa. The ownership rights are unclear, which makes it difficult how any surplus or deficit should be divided among the different participants. This is stressed in Pay-As-You-Go (PAYG) systems.

The problem in this case is that the young pay partially the pensions for the elderly *and* have to make up possible negative financial shocks. This PAYG system works as long as new generations are willing to enter in the contract and pay actuarially too much. But, if the young generations retreat (thereby causing discontinuity), the last generation that entered the contract, is severely disadvantaged.[b]

[a]The numbers in the graph are for illustrative purposes and do not reflect assumptions or outcomes from my thesis. The source of the Figure is Boeijen et al. (2007).

[b]The Ministry of Social Affairs and Employment has the ambition to abolish the doorsneesystematiek and to replace it with an actuarial fair system, because it acknowledges that the subsidising solidarity is a drawback of the system. *Rijksoverheid, Ministerie van Sociale Zaken en Werkgelegenheid: Kamerbrief hoofdlijnen van een toekomstbestending stelsel (Kamerstuk, 06-07-2015).*

1.1 Literature Review

There is little evidence and literature on how a discontinuity event develops. For example, there are no clear historic discontinuity events. Therefore, it is not easy to find evidence-based material. Moreover, during a discontinuity event a lot of behavioural consequences play an important role, that influence the outcomes of the model. For this reason, the research is complex. It is, though, particularly important because it puts lights on the sustainability of pension funds.

Most of the research so far is performed with regard to the underlying problem: inter-generational risk sharing. But the research is conditional upon the fact that new generations are committed to the pension contract. Then, there are some papers more closely related to discontinuity, which investigate indifferent minimum funding ratios. Besides, some papers mention discontinuity risk, but do not quantify it or only study discontinuity with respect to new entrants on the labour market, whereas I study discontinuity for all age groups. No papers have been found that study the impact of dynamic in- and outflow (i.e. discontinuity) on the financial sustainability of pension funds with the corresponding effects for the remaining participants.

Firstly, it is stated in the literature that inter-generational risk sharing can be welfare-enhancing for participants. Gollier (2008) shows that an inter-generational risk sharing scheme has a positive effect on the welfare of all current and future generations; he mainly scrutinised the effects of stock-market risks. Tyagi (2016) finds similar results with regard to risk and return trade-off, whereby the smoothing of financial-shocks is considered. Cui et al. (2005) find that risk-shifting over time (due to adjustment mechanisms such as indexation and contributions instruments) leads to higher utilities in pension schemes. Even in the case of underfunded collective funds, inter-generational risk sharing can be welfare enhancing, since risk sharing can only be attained in a collective scheme and not in an individual. However, the literature does not take into account the discontinuity risk (recall that this is the important *if*). According to Kocken (2016), when capturing this risk, inter-generational solidarity could still be welfare-enhancing, but not when capitals are transferred between generations. Nevertheless, sharing in macro risks (such as longevity) is still beneficial, since those risks are (still) the same for every generation. Considering the Dutch political climate, it is likely that some inter-generational solidarity will remain in the system because there is still (much) support for it. Concerning the recent developments about a new pension system in The Netherlands, risk sharing seems to be a key ingredient in the Dutch civilisation. Namely, The Social Economic Council advised amongst others on Personal Pensions with Collective Risk sharing as a new possible pension system.[3]

[3]In Dutch: Sociaal Economische Raad (SER).

Secondly, literature that is more closely related to our research, analyses whether low funding ratios create incentives for individuals to leave a pension fund and save on an individual basis. Siegmann (2011) makes a comparison between a collective DB and an individual DC plan. The comparison leads to minimum funding ratios for which new generations are willing to enter a DB-fund, instead of saving on an individual basis in a DC-fund. If the prevailing funding ratio is below the minimum funding ratio, new generations (of 25 years old) abstain from joining the collective DB fund. The analysis is based on utility levels of (future) participants and attained by Monte Carlo simulation. Several utility functions with different risk-aversion levels are computed, along with sensitivities to the investment strategy and equity premium. For an agent with a Constant-Relative-Risk-Aversion (CRRA) utility function — whereby the fund invests $\phi = 50\%$ in stocks — the minimum funding ratios range from 96% till 120% (depending on the level of risk-aversion). A higher equity premium or higher fraction invested in stocks, leads to higher attractiveness of a collective DB scheme. Namely, if the DB fund invests more in risky assets (compared to $\phi = 50\%$), the minimum funding ratios become lower. That is, participants enter the DB fund more quickly (i.e. they are willing to enter at a lower funding ratio). One reason is that a risky investment strategy has a large value for participants in a DB fund; inter-generational risk sharing is the most valuable, when the investment strategy of a fund is more risky. This holds because the downside risk is limited by the risk-solidarity, while the upside potential is unbounded. Individual savings lack this diversification opportunity. A second reason is that a collective fund has an existing pool of assets, while an individual saver starts with an empty fund. For this reason, young generations benefit more from the risky investment strategy of the collective fund. If the equity premium (prevailing in the financial market) is higher, minimum funding ratios are also lower — for the same reasoning as a risky asset mix. Inter-generational risk sharing adds more value in a risky context. Namely, a young generation in a collective fund benefits immediately from the higher equity premium on the already existing (collective) assets upon entering the fund. In contrast, an individual saver does not.

Molenaar et al. (2011) take the analysis one step further and analyse for all ages when individuals are indifferent between joining a DB-plan or DC-plan (as opposed to new generations of 25-years-old only). The paper does this by computing break-even funding ratios for the ages from 25 up to and including 60. Thus, they also consider the possibility of opting-out of the fund, as I will consider also. They find that the minimum funding ratios vary substantially with age but exhibit an U-shaped pattern: young and old cohorts have more incentives to leave a collective fund. For a 25-year-old with CRRA utility and an investment strategy of 50% in risky assets, break-even funding ratios range from 105% till 125%. The range is derived from a sensitivity analysis with regard to the asset mix. The break-even funding ratio for a 40-year-old is 5%, while a 60-year-old leaves by a funding ratio below 80%. No ranges (i.e. sensitivities) have been calculated for other age groups than 25-year-olds. The break-even funding ratio for a 25 years old indicates again — like by Siegmann (2011) — whether an individual enters the contract, or not. The break-even funding ratios for 26-64 years old determine when an individual of age $x \geq 26$ leaves the fund. The minimum funding ratio for a 25-year-old to enter the contract is 120% when the fund invests $\phi = 50\%$ in risky assets, for an agent with CRRA-utility and risk-aversion 5. Note that this differs from the result found by Siegmann (2011). However, Molenaar et al. (2011) advocate that similar results are found if approximately the same parameters and model set-up are used.

The threshold for entering the fund is high, because young agents accrue less pension rights relative to their high contribution payments. This is caused by the subsidising solidarity (also known as "doorsneesystematiek", which is explained in the text-box above). Young generations, for this reason, have an incentive to leave the fund and, thus, the funding ratio must be high. Young generations do not want to make up for a deficit *and* pay already high contributions relative to the actuarial fair premium as well as relatively lower accrual. Young generations on the edge of entering the pension contract, may face substantial deficits, because the fund is able to share risks between generations. If the buffer becomes largely negative (i.e. underfunded), the young members might decide to not enter the contract, which could create discontinuity risk. Such a situation questions the sustainability of collective schemes. On the other hand, due to the uniform premium and accrual, old-cohorts (till the retirement age of 65) pay relatively less premium for their pension rights. Elderly are, hereby, inclined to stay in the fund at higher ages. Both incentives are labelled the "uniform-effect" (Molenaar et al., 2011).

Because of the U-shaped pattern (as shown in Figure 2.2), the funding ratio for young generations must be high, middle-aged cohorts have enough incentives to stay in the fund even if the funding ratio is low, while the elderly are inclined to leave the fund in case the financial position is low. Older participants want to leave the fund, because in case the funding ratio drops, they are the ones that feel the burden. Near retirement most pension rights have been accrued by these generations compared to the young, thus when pension rights are cut, they feel it the most. Besides, older cohorts have a shorter horizon to account for the losses. This is labelled as the "burden-effect" (Molenaar et al., 2011). Notice that the uniform- and burden-effect work in an opposite direction for the older agents. The uniform-effect tends elderly to stay, while the burden-effect forces them to leave. However, the uniform-effect only plays a role till the end of the working period.

The results regarding the investment strategy do not correspond between the two mentioned papers of: Siegmann (2011) and Molenaar et al. (2011). The latter advocates that in case the equity allocation of the fund increases, the break-even funding ratios are higher (collective is less attractive); while the former says the opposite. Molenaar et al. (2011) reason as follows: when the funding ratio is low, young generations do not want to take a lot of risk. More risky investments may lead to (quicker) bankruptcy. The new entrants can only go to an individual scheme, hence to capture the benefits of a DB, the fund must ensure not to bankrupt. Both papers find the same results regarding risk-aversion levers. A lower risk-aversion (i.e. more risk seeking) induces a preference for a risky investment policy so the break-even funding ratios become lower for higher equity exposures. Thus, the individual scheme becomes less attractive at a more risky investment strategy if the individual is less risk-averse.

Both papers, however, do not say much about the discontinuity risk and the steering instruments that influence discontinuity risk. I extend on their research by computing the effects of discontinuity with the aid of their break-even funding ratios. The break-even funding ratios are used to construct the reaction function of the participants; the reaction function determines whether participants enter or leave the collective fund.

Bovenberg, Nijman, et al. (2007) touch upon the topic of discontinuity and indeed advocate discontinuity risk as a main drawback of inter-generational solidarity. The paper advocates that discontinuity risk can be reduced by dynamic investment strategies that reduce the deficits when a new generation enters the labour market and, hence, the pension contract. Bovenberg and Mehlkopf (2014) explore requirements that reduce discontinuity risks and examine discontinuity risk in the case of imperfect commitment, which raises questions about inter-generational fairness and sustainability. Imperfect commitment is comparable with the situation wherein less new generations enter in the pension contract. Frequently, discontinuity risk is defined as opted in Bovenberg, Nijman, et al. (2007): "new generations facing substantial deficits when they start working may decide not to participate in the contract." However, I extend on the research of discontinuity risk and add the probability that older generations might also have incentives to leave the fund (like Molenaar et al. (2011)).

The main source of inspiration for my thesis is described now.

1.1.1 Information Sensitivity

The philosophy for this thesis actually emerged from the paper of Holmstrom (2015). He provides a qualitative framework for calculating the necessary level of collateral against debt in a system, where the payoff of the asset depends on the information sensitivity. The idea is based on how a pawn-shop works; though, for an application to a pension fund, a repo is actually going on. In pawning the initiative comes from the borrower who has a need for liquidity, while in a repo the motive is often the opposite: someone with money wants to park it safely by buying an asset. A worker (someone with money, the seller of the asset) wants to store his financial capital safely by accruing a pension at a pension fund (the buyer of the asset, the 'pawn-shop'). The pension fund keeps the asset (i.e. the financial capital) in custody and returns the items (i.e. the participant's accumulated pension rights, named liability) at the date of retirement, with hopefully some return on the asset (i.e. the financial capital). However, the participant runs the risk that the buyer of the asset (i.e. the pension fund) cannot honour its liability at the termination date (i.e. the retirement date), because the fund may have used the posted collateral of the participant (due to 'bad' market conditions for example). It is irrelevant that the participant's valuation is different from pension fund's valuation about the pension rights (e.g. the individual discount rate exceeds the market rate of return). Like in pawning, it is sufficient that the participant has the confidence and trust that he (she) can redeem his (her) financial capital at retirement.

People often assume that liquidity (i.e. entering a pension fund) requires transparency, but this is a misunderstanding (Holmstrom, 2015). What is required for participation in a pension fund is symmetric information between lender and borrower, thus the fund's participants should have enough confidence that the pension fund is able to repay the posted collateral in the end (i.e. pay the pensions at retirement, its liabilities).

By looking at Figure 1.1, it is clear that debt is information insensitive if it is deep in the money; that is, the distribution of the payoff x is so far out in the right tail that the market value of debt at the time of a future sale will equal its face value D with high probability (the black line hugs the red line).[4] Pension funds actually serve debt claims that are backed by collateral. Hence, if the value of the collateral (i.e. the assets of the fund and, thereby, the funding ratio) is high enough, the debt (i.e. the pension rights) is in the information insensitive region. In other words, participants have the trust that the fund is able to fulfil its liabilities and it does not pay a participant to acquire information about possibly leaving the fund. On the other hand, debt (i.e. the liabilities) is information sensitive if it pays the participant to acquire information about the underlying collateral (i.e. funding ratio) before parking his (her) money at the fund. In this region, it is uncertain whether the participant receives for what he

[4]The graph is for illustrative purposes and does not reflect assumptions or outcomes from my thesis. The source of Figure 1.1 is Holmstrom (2015).

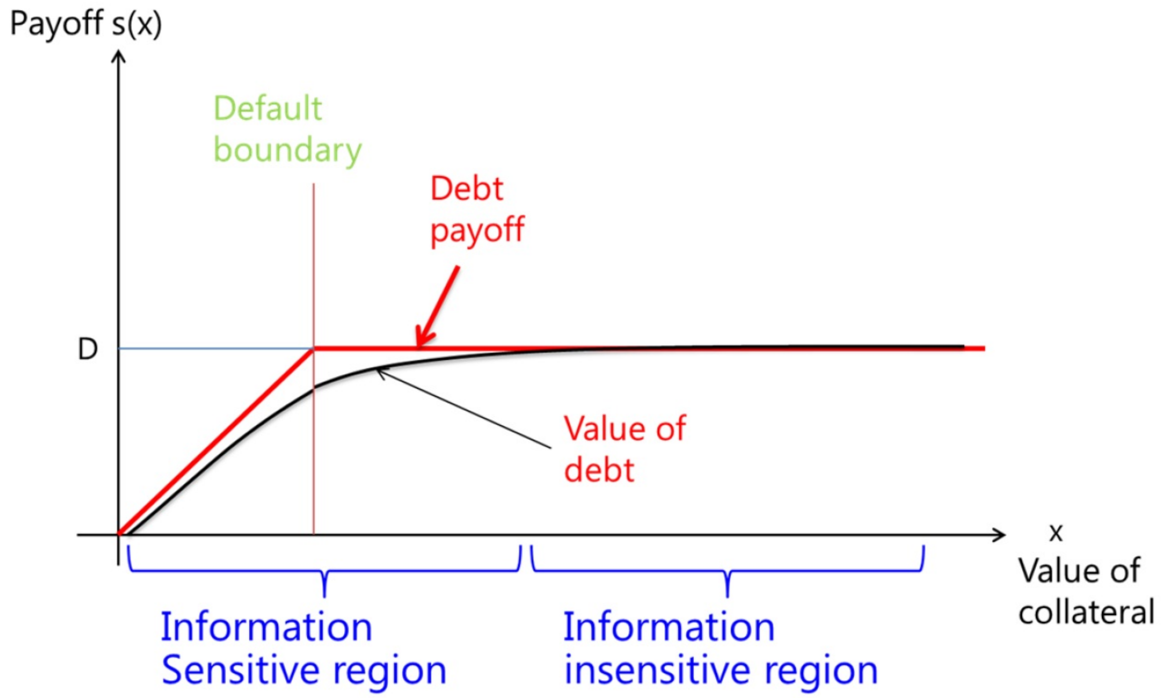


Figure 1.1: Debt and information sensitivity.

(she) will pay in. The desire, of course, for a pension fund is to be in the information insensitive region, because this minimises their discontinuity risk. In the information sensitive region, discontinuity risk might be substantial, since (future) participants are inclined to choose for another fund. Thus, the quite insensitive state is hugely valuable, because in the sensitive region participants might leave. Higher volatility in funding rates can increase the process of reaching the information sensitive region. Note that the moment of becoming information sensitive lies before the moment of actual default. In The Netherlands a pension fund can not default by definition, though.

So, basically, an idea is to identify at which point the information goes from the insensitive region to the sensitive region. If this happens, pension funds run the risk of discontinuity, and this thesis quantifies this risk.

The next section present the definition of discontinuity risk and introduces the research questions.

1.2 Contribution

It is particularly important to consider older cohorts for discontinuity as well, since generations nearing retirement also face incentives to leave the fund, not only new generations that just enter the fund. Older cohorts may want to possibly leave, because in case of low funding ratios they are the ones that feel cuts in the pension rights substantially. On the other hand, young generations do not want to be confronted with high recovery premiums upon entry. Therefore, I come to the following definition for discontinuity risk:

Definition 1. *The probability that new generations (of 25 years old) — when they enter the labour market — abstain from entering the collective pension contract, and/or the probability that older generations (from 26 till 64 years old) — currently in the collective fund — withdraw from the collective pension contract and enter the individual fund.*

Currently, this risk is not considered by risk managers in the computations at pension funds and insurers; also, the literature does not provide such information. Most papers abstain from discontinuity risk by assuming that new generations are committed to the pension contract. However, it is particularly important since discontinuity risk changes the outcomes of the models as my analysis shows. Namely, discontinuity risk turns out to be not negligible. For this reason, the current computations are static. That is, the models do not incorporate uncertainty surrounding dynamic in- and outflow. Here lies the contribution and relevance of my thesis, namely I introduce a new dynamic approach. In other words, my thesis studies the impact of dynamic in- and outflow (i.e. discontinuity risk) on the sustainability of the fund.

Two main elements are deducted for my research. Firstly, in order to assess the financial sustainability of the fund, it is necessary to know how participants react during a discontinuity event. The reaction of a participant is twofold: either enter or exit the collective DB. New generations can decide to not enter, while older generations might decide to exit. Due to the reactions the in- and outflow of the collective fund becomes uncertain, since the abstaining participants go to an individual DC. So, it is of the highest importance to investigate how participants reacts during a discontinuity event. For this reason, I ask myself the following first research question:

1. *How do new entrants and current participants react during a discontinuity event?*

The answer is quantified by the so-called reaction function of the (future) participants. The reaction function, which entails a bank-run effect, determines whether participants want to enter a collective or individual fund which leads to dynamic in- and outflow. Normally the in- and outflow is static since the in- and outflow depend solely on the fertility and survival tables. The fertility rates imply the number of future inflow, while the survival projections determine the outflow (i.e. deceasing retirees).

Secondly, if the answer is known, it is studied how the financial sustainability of the fund is consequently affected by the dynamic in- and outflow. The dynamic in- and outflow includes fertility rates, survival tables and (endogenous) discontinuity. Therefore, the second research question is:

2. *How affects (endogenous) discontinuity risk the sustainability of the (larger) collective pension funds in The Netherlands and how large is the discontinuity risk?*

The answer to second question is particularly helpful in understanding the financial sustainability of pension funds. On the one hand, it answers how dynamic in- and outflow affects the sustainability of pension funds, while on the other hand something can be said about the discontinuity risk funds are facing and the effects for the participants. The answers to both research questions fit properly in the literature, since (to my extent) no such studies exist and, therefore, this study is highly relevant. My research builds on some of the current academic literature and expands this; moreover, it leaves enough possibilities for further research. Especially, research on the behavioural effects during a discontinuity event seems to be important.

The upcoming section presents the research description and methodology to answer the research questions.

1.3 Methodology

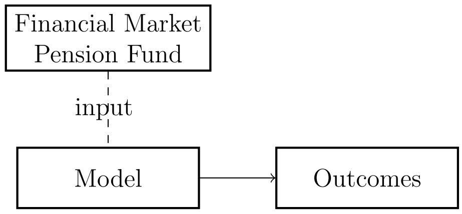
To asses the research questions, the methodology will be described now. Figure 1.3 shows the structure of the research process, being also the structure of my thesis. Recall that it is the objective of the thesis to assess the (endogenous) discontinuity risk of the (larger) pension fund in The Netherlands. The method of analysis in this thesis is the widely spread Monte Carlo simulation technique. To answer the research questions mainly classical Asset-Liability-Management (ALM) is used, together with value-based ALM and generational accounting. The first question is addressed in Chapter 2, while the second question is answered in Chapter 5.

As stated above, discontinuity risk is currently not considered in the calculations; traditionally, the approach is as sketch in Figure 1.2a. There is an ALM-model that takes as input on the one hand the financial market and, on the other hand the pension fund which operates in the generated financial market. Leading to some outcomes, such as funding ratios. This current approach is static, since it does not consider discontinuity and, therefore, does not allow for uncertain in- and outflow.

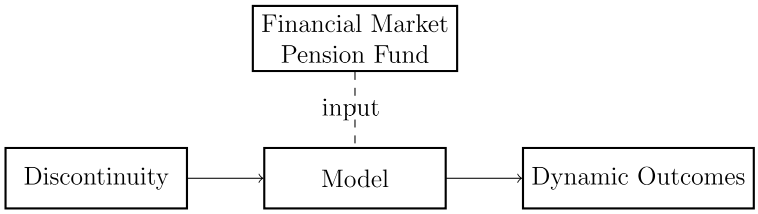
I introduce a new dynamic approach. Still, the ALM-model takes as input the financial market and the pension fund. But, a new third element is introduced: discontinuity. For this reason, the model incorporates three types of input instead of the traditional two. Including discontinuity changes the outcomes of the model, which become dynamic. The main difference with the static approach is that in- and outflow of the fund becomes dynamic; the in- and outflow depends on a reaction function of the participants. The new approach is visualised in Figure 1.2b.

The reaction function quantifies how participants react during a discontinuity event and, thereby, gives an answer to research question one. The rational reactions of the participants are based upon the nominal funding ratios. Namely, the reaction function takes the funding ratios as input. The main advantage is that the nominal funding ratios are published and publicly available in The Netherlands and, therefore, provide an accessible source of information for agents about the sustainability of pension funds. Besides, funding rate are easy to interpret: it quantifies the ratio of assets over liabilities — is the ratio bigger than 100%, there is more capital than liabilities. For certain funding ratios some age-groups might be willing to exit the collective pension contract. For example, young generations do not want to enter in the contract, if the deficit and premium is high, while middle-aged cohorts (of 45 years) are inclined to stay due to the subsidising solidarity. The break-even funding ratios, whereupon certain cohorts want to exit, are based on two papers from the current academic literature: Siegmann (2011) and Molenaar et al. (2011). Moreover, I introduce a snowball/bank-run effect, which intuitively materialises during a discontinuity event. If you see somebody abstaining, you become inclined to abstain also. Participants are assumed to react and behave (rational) as described in the used papers. All of this is elaborated in Chapter 2.

To answer the second research question, an ALM-model is built that takes the three elements as input. The model compromises a financial market analyser, a pension fund analyser and considers discontinuity risk. The financial market, wherein the pension



(a) Current static approach, without discontinuity.



(b) New dynamic approach, including discontinuity.

Figure 1.2: Comparison of the static and dynamic approach.

fund operates, needs to be initialised as first. A generally accepted financial market that suits our needs is the Black-Scholes-Vasicek model, expanded with a random normally distributed inflation rate. The financial market initiates the economic scenarios: the stock prices and short rates follow, based on their respective differential equations. Besides, it produces the pseudo-stochastic inflation rates. With the short rates at hand, the term-structure of interest rates is retrieved, which is used for market consistent discounting of the liabilities. Moreover, the market enables to compute the portfolio returns for the investment strategy of the pension fund.

Given the economic scenarios, the pension analyser computes classical asset-liability output such as funding ratios. The output depends on the investment portfolio, indexation and premium rules as well as composition of the fund's participants — based on the current projected survival probabilities. The pension fund maintains the typical Dutch DB system. Besides, the pension fund analyser enables to calculate generational accounts based on value-based ALM. This generational accounting technique can show differences between pension systems on an age-specific level, whereas classical ALM produces results for the collective. The financial market and pension fund analyser are presented in Chapter 3.

Classical ALM (usually) means that the results are obtained under the real-world measure $\mathbb{P}$ and, hence, have to be used for making policies and reporting about the financial sustainability of the fund. It uses the mean and quantiles as measures of riskiness for a variable, and frequently uses the Monte Carlo simulation technique to make projections of the distributions and to optimise the strategy of the fund. It provides, inter alia, information about probabilities of underfunding and probabilities of (no) indexation. Such information is useful, since it provides an idea about the sustainability of the pension fund (Kortleve et al., 2006). Value-based ALM uses the same economic scenarios as the classical ALM technique, however the future outcomes are discounted back to the present with an appropriate risk adjusted discount factor. This is realised by methods as the pricing kernel technique or the equivalent martingale measure; therefore, the results are obtained under the objective measure $\mathbb{Q}$, which gives a higher weight to 'bad' economic scenarios. Value-based ALM (derived from option pricing theory) enables to price policy instruments and to compute generational accounts in a market-consistent way. The measures to evaluate the pension outcomes of the ALM-model are shown in Chapter 4.

New in my approach is the fact that the model considers also the discontinuity risk (besides the financial market and pension fund characteristics). The outcomes of the model, for this reason, become dynamic on an ad-interim basis. Dynamic in the sense that in- and outflow becomes uncertain, and ad-interim in the sense that at the beginning of each year participants decide to enter or leave. I investigate two main situations: (i) dynamic inflow and (ii) dynamic in- and outflow. The first situation analyses if new entrants are not committed to the contract, while the latter analyses also the possibility that current generations exit. With generational accounting I show the incentives for specific age-groups to opt-out of the fund. Namely, instruments (depending on the funding ratio) such as recovery premia and sustainability cuts are tools — called embedded options — that influence the discontinuity risk of a fund. It helps in maintaining a 'healthy' fund, but those instruments can cause incentives to leave or to not enter the fund. The contribution rate strategy is mainly a steering instrument for the young, while a sustainability cut affects all generations (but especially the elderly, since they accrued most pension rights). This provides another reason for why discontinuity risk should not only be investigated for new generations entering the system, but also for generations already in the system.

The main analysis entails a comparison between the static ex-ante situation and the dynamic ad-interim situation. Besides, sensitivity analyses are performed. The expectation is that funding ratios become lower and more volatile in the case discontinuity is considered, relative to the current static approach. Namely, because of dynamic in- and outflow the fund's population declines throughout time and the policy instruments work less effectively, especially sine the fund maintains too long smoothing periods for shocks. A smaller population is not directly a problem, but it makes the pension fund less sustainable in the long run and more sensitive to shocks. The main results and sensitivity analyses are presented in Chapter 5.

On the next page the outline of the thesis is shown.

1.4 Outline

This section outlines the chapters in the entire thesis in order to make navigation and readability easier for the reader. The structure is built upon the everlasting importance between: research questions, model and results. My thesis contains text-boxes which include relevant practical information with regard to current situations in The Netherlands. After each section in my thesis a small summary with the main conclusions follows. A flow-chart of the thesis is presented in Figure 1.3.

Chapter **2, Discontinuity** describes how a discontinuity event develops and shows the reaction function. Hereby answering the first research question.

Chapter **3, Model** describes firstly the financial market, which mainly rests on the idea of the Black-Scholes-Vasicek model. Secondly, the pension fund participants, characteristics and policies are presented.

Chapter **4, Data & Evaluation** initialises the model parameters and gives an overview of how the model works. Besides, it discusses how to measure pension outcomes and presents the results of the static (benchmark) funding ratios.

Chapter **5, Results** shows the outcomes with dynamic in- and outflow. First of all, the situation of dynamic inflow is presented (as well as a closed fund). Secondly, it investigates how dynamic in- and outflow affects the pension fund and its (remaining) participants.

Chapter **6, Conclusions** summarises the thesis and gives indications for further research, as well as recommendations.

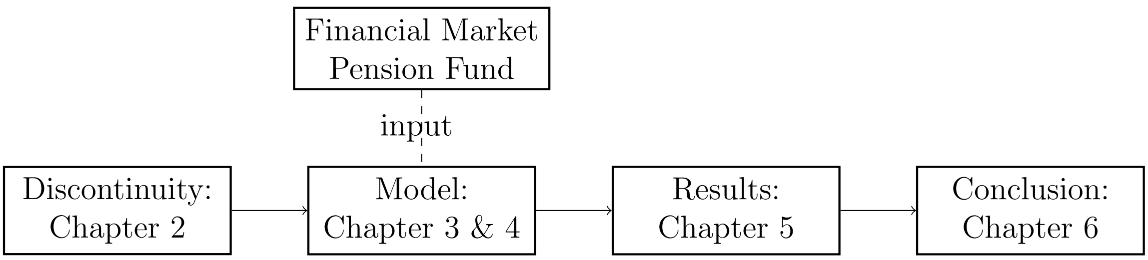


Figure 1.3: Flow-chart of relationships between the various chapters.

Chapter 2

Discontinuity

This chapter describes discontinuity risk and answers the first research question. Firstly, it presents how agents react during a discontinuity event. Secondly, the behaviour of the participants is quantified by a (hypothetical) reaction function, which determines consequently the dynamic in- and outflow.

As stated above, my research is performed with regard to endogenous discontinuity risk, which I just call discontinuity risk for simplicity throughout the rest of the thesis.

2.1 Discontinuity Event

The literature provides arguments that it could be welfare enhancing to share unforeseeable risks with the collective (Boeijen et al., 2007), but the main disadvantage is discontinuity. If inter-generational risk sharing becomes larger, discontinuity risks rise. Moments of discontinuity happen always in the information sensitive region, by definition. If due to less inflow and high outflow, the number of participants in the fund shrinks, the capability of the fund to share risks with other generations decreases. Besides, the loss of support leads to the severe risk of discontinuation.[1]

The development of a discontinuity event for a pension fund is a complicated situation. Analysis is even further complicated, since there are no historical moments of discontinuation known and, hence, there is no evidence-based material. For this reason, I sketch a setting that is to my means made as objective as possible. To describe the reactions of (rational) agents during a discontinuity event, some specific questions and assumptions are made.

Leaving the Fund

First of all, who leave the pension fund and what are the costs of if? In Definition 1 the inflow of new generations of 25 years old is considered as well as the potential outflow of current generations of 26-64 years. So, I abstract from the current rules of mandatory participation. Every year, at the beginning of the career, new employees of 25 years old can decide whether they enter in the collective pension contract or not. If an individual decides to abstain, he (she) is not able to enter the collective fund during the rest of his (her) life again. The new entrant remains in the individual fund for the rest of its life-time. For example, an individual that is 25 years old now (year t), decides to enter the

[1] Remark that discontinuity is not necessarily only born in case of low funding rates, but also at high funding rates. If the funding rates are high, participants may be inclined to liquidate the fund.

collective pension contract. But at year $t + 1$, when the individual is 26 years old, he/she decides to leave the fund because the funding position has worsened. If an individual decides to leave, he (she) leaves the fund at the beginning of year t and he (she) cannot enter the collective fund any more during the rest of his (her) lifetime. In other words, the participants enters the individual fund. So, an individual decides to enter or leave the fund at the beginning of year t and it bases his/her decision on the prevailing funding ratio at the beginning of year t. Thus, the costs of not entering the fund are zero, because no pension rights have been accrued by new entrants and, therefore, there are no transaction costs involved.

Employees of 26 years and older can opt to leave the pension fund. The decision is made on the basis of the financial position of the fund. Participants may generate incentives to save for retirement individually if the funding ratio becomes too low, because the intrinsic benefits of participating in the collective pension fund might be less than the negative effect of underfunding. Therefore, he (she) wants to abstain from further benefits of inter-generation risk sharing. If a current employee (already in the collective fund) exits from the collective, it enters the individual fund for the rest of its life-time also. The costs of leaving the fund are assumed to be zero (e.g. administration costs are neglected), but the fund has to pay-out the accrued rights to the leaving participant, of course. Consequently, the market value of the fund's assets changes because the leaving participant transfers its accrued pension rights from the collective fund to the individual fund.

In case somebody leaves, the fund has two possibilities to transfer the accrued rights. The first option is that the fund pays the individual 100% of the market-consistent discounted accrued pension rights. The accrued pension rights depend on the income of the worker, the uniform accrual rate (expressed as a percentage of the participant's salary) and (indexation) policy of the fund. The rights are discounted with a discount factor which consists of the (projected) survival probabilities and the interest rates following from the term-structure. The rights are transferred as if the funding ratio equals 100%. Thus, no deficit or surplus is shared with respect to the other generations; the leaving participant receives exactly what belongs to him/her (discounted). This transfer option may provide wrong incentives, because participants are inclined to leave at a funding rate of 80% while they still receive all their rights. If this happens, the remaining participants in the collective are disadvantaged. Namely, (i) they have to pay for the current underfunded position and (ii) they pay for the additional 20% the leaving participant receives. The second option is that the fund pays the individual the market-consistent discounted value of pension rights, conditional upon the financial position of the fund. That is, if the funding ratio equals 80%, the individual only receives 80% of their (discounted) pension rights. So, the other 20% belongs to the fund, in order to restore the financial position since the fund is underfunded. In case the funding ratio exceeds 100%, a similar reasoning holds, but then the individuals receive an additional surplus. But, the effect is absent in my thesis since only situations are considered whereby participants want to leave in case of underfunding. In sum, participants from the age of 25-64 can leave and the transfer costs for the fund are market-consistently.

If the transfers are conditional upon the funding ratio, the funding ratio does not worsen in itself and the option is funding ratio neutral. Thus, the decision of the individual to leave, does not affect the funding ratio and, thereby, it also does not affect other participants in the fund. As an example, consider the following simplified setting:

- There are 10 participants in the fund.

- The liabilities equal 1 per participant, so a total liability value of 10.
- The assets of the fund are worth 7.
- The funding ratio (assets over liabilities) equals 70%, consequently.

Now, based on the 70% funding ratio, 80% of the current participants decides to withdraw from the pension contract and leaves. The leaving participants receive 70% of their liability instead of the accrued 1. Then, the following happens (ceteris paribus):

- There are 2 participants left in the fund.
- The total liability value equals 2, since 80% left.
- The assets are worth $7 - 5.6 = 1.4$, since 8 participants leave with 70% of their accrued rights each.
- The funding ratio equals 70%, subsequently.

Hence, the coverage rate remains unaffected for transfers conditional upon the funding ratio, although the population decreased substantially. On the other hand, if the fund would have paid 100% of the pension rights, the fund would have been bankrupt actually. Namely, the assets are worth 7 and the fund would have had to pay out 7 for the transfer value also; resulting in a funding rate of 0% (with still some liabilities). Receiving 100% of the accrued rights depletes the fund's assets rapidly.

Where to Go?

The second question is if participants have the possibility to leave the fund and where they go to? Recall that a voluntary setting is considered whereby participants initially enter a collective DB fund, but are allowed to abstain/withdraw from the collective pension contact. Please note that there is a difference between abstaining and withdrawing from the contract. A new entrant (of 25 years old) abstains from the contract, does not enter the fund and has not accrued any pension rights in the collective fund yet. While a current generation in the fund (of 26 years and older) withdraws from the contract, leaves the fund but has accrued pension rights already. For this reason, new entrants are able to abstain from the contract, upon entering the labour market. In case a new entrant abstains from the contract, there is one option: enter in an individual DC fund and stay there for the rest of your life-time; the abstained participants are not able to enter in the collective fund any more during their lifetime. Current generations may withdraw from the collective contract and, subsequently, leave the collective fund and transfer their accumulated wealth to an individual DC. Also, the withdrawn participants are unable to re-enter in the collective fund and stay in the individual DC. So, yes participants can leave the fund but are consequently (automatically) enrolled in an individual DC.

The theoretical setting assumes that participants are rational and the setting ensures that participants are still mandated to enter a pension contract, however it is not collective anymore. In a collective fund some surpluses can be achieved, which are not attainable in an individual contract. It is assumed that the only difference between a collective DB and individual DC is that a collective DB regulation entails the benefits of inter-generational risk sharing, but has the disadvantage of discontinuity. In a collective fund inter-generational risk sharing is possible, while in an individual DC not. But, an individual system lacks the risk of discontinuity that might pop-up in a collective fund. It is assumed that agents make rational decisions about these consequences during a discontinuity event. In equilibrium, a participant will choose the collective system because it has the advantages of risk sharing. However, ad interim the drawbacks of risk sharing may become clear — due to too large transfers for example — and the rational participant may prefer the individual DC.

The text-box below explains the possibilities for leaving a fund in the current Dutch pension system.

The Dutch case: leaving a fund

This text-box describes the practical options individuals have in The Netherlands to abstain from a collective contract. My thesis considers the theoretical setting whereby an individual is faced with the pure choice between a collective DB or an individual DC. In practice, individuals have actually several options to exit. Firstly, agents can become self-employed. Self-employed in The Netherlands can accrue a pension, but it is on a voluntary basis. Therefore, a self-employed is not obliged to join a collective pension fund (in the second pillar). Hence, a self-employed can determine whether it wants to accrue a pension and how much; it can also join an insurer. Secondly, individuals could switch from job, profession or employer which is affiliated with another pension fund (provider) — of course, the individual should switch to a fund that has a lower deficit than the fund in consideration. Or, they switch from employer that is not obliged to arrange a pension for their employees. Thirdly, an individual could start to work abroad, where another pension system prevails. Fourthly, if a pension fund has a too low 'z-score' (low asset returns) over the last five years, then are employers allowed to leave that pension fund. Besides, individuals are allowed to leave the pension contract if they have at least an equal or better alternative contract. Lastly, an agent could decide to abstain from working at all. Those options are examples of 'voting with your feet'; that is, individuals literally walk away from the pension fund. Another option is 'to vote with their voice', i.e. individuals protest against the current pension contract (by forming groups for example).[a]

Discontinuity risk arises when more and more participants withdraw from the current pension system. In The Netherlands, this is partially happening nowadays for some specific sectors; for example, the fund for construction workers. Due to developments on the labour market, a growing number of construction workers is not longer mandatory to accrue a pension. The number of self-employed has risen from 6% of the total population in 2010 to 10%; boiling down to 1 million self-employed in The Netherlands. Due to this increase, and because employers are leaving as sponsors, the population of participants decreases specifically for this fund. Besides, the fund is ageing and there is a decline in construction work. For this reason, the population of participants decreases and ages.

[a] The effect of possible competition between the funds is disregarded.

Information Cascade

Finally, do generations abstain due to the unsustainable current financial position of the fund, or because other individuals — such as your neighbour — (probably) do not participate? The answer to this question is difficult to quantify and has to be found probably in the behavioural literature. The question grasps back on how policy rules should be taken into account. Cutting pension rights increases the financial sustainability of the pension fund (beneficial for future cohort), but it erodes the current participants' support. On the other hand, an unsustainable financial position leads to a decline in support from new entrant, which may abstain. A pension fund can discontinue for the following main reason — as indicated by the question. Firstly, the financial position of the fund may cause a discontinuation event; such a situation can be triggered by supervising agencies (exogenous factors). Secondly, due to the lower financial position, participants may exit. A decline in participants may lead to discontinuity, because participants leave the fund (or because participants collectively force a transfer to a different pension provider). Longer smoothing periods in case of shocks increase the added value of inter-generational risk sharing, but it increases the discontinuity risk as well.

To include the financial situation and decrease of participants' support, I develop a reaction function. The reaction shows how many participants flow in and out of the fund, whereby the in- and outflow is based upon the break-even funding ratios from the literature. Hence, the participants react on the prevailing funding ratio, which is a main indicator for the financial position. My work, and especially the reaction function, rests on previous work of Siegmann (2011) and Molenaar et al. (2011). The reaction function is explained in the next section, but let me first sketch what is likely to happen during a discontinuity event.

Intuitively, participants act on the actions of other participants (i.e. herd behaviour). Therefore, I include an information cascade effect, leading to a snowball/bank-run effect in the reaction function. Information cascades arise when individuals choose identical actions, despite possibly having different private information (such as expectations about the development of a pension fund). The theory of information cascades assumes that agents are rational and that agents believe others to be rational as well. A cascade happens when (binary) decisions have to be made sequentially, with later people watching the actions of earlier people, and from these actions inferring something about what earlier people knew (Easley et al., 2010). Thus, the theory fits the (binary) decision between joining a collective or individual system neatly. The decisions are also sequential, in the sense that each year a participant chooses to exit the collective contract or not. Moreover, a 25-year-cohort at time t can observe and infer what the previous generation of 25 years old did at time $t-1$ (i.e. last year). During a cascade people imitate each other, but it is not mindless imitation. It is the result of drawing rational inferences from limited information.

Let me illustrate this process by an example, based on an idea of Easley et al. (2010). Suppose that you are choosing a pension fund to enter, and based on your own research about pension funds, you intend to enter the collective DB fund B. However, when you want to enter you notice that almost nobody is currently in fund B, while the individual DC pension fund C has a huge population of participants. If you believe that other agents have preferences similar to yours, and that they too have some information about where to accrue pensions, it may be rational to join the crowd at the individual fund C rather than to follow your own information. The information consists about your expectations of the future development of the pension fund in consideration. To see how this is possible, suppose that each agent has obtained independent but imperfect information about which of the two pension funds is better. Then, if there are already many agents in fund C, the information that you can infer from their choices may be more powerful than your own information, in which case it would in fact make sense for you to join regardless of your own private expectation. In case of leaving a fund, the process is similar but reversed. If many agents are leaving the fund, you can become inclined to leave also, although your private expectations may tell that the fund is not sustainable in the future anymore.

Hence, the difference between some participants entering while others are not, comes from the concept of heterogeneous expectations. The population of few entrants (as well as current participants) may have different expectations about the future development of the pension fund. For this reason, it is possible that 35% of the new entrants abstains from the collective, while the other 65% of the new labour entrants still wants to enter the collective contract.

In sum, this section presented the general development of a discontinuity event. The circumstances during discontinuation are complex and, therefore, some hypothesis and assumptions are introduced. Three main questions are answered sequentially. Generations of 25-64 years can cause discontinuity and may exit the collective contract — whereby 26-year-olds and elderly leave with their market-consistent discounted value of pension rights (either conditional upon the funding ratio, or not). Generations that abstain/withdraw from the collective fund enter in an individual fund, where the benefits of risk-sharing are absent. During a discontinuity event, agents react on each other and it is likely that an information cascade occurs.

The next section uses these results as input for constructing the reaction function.

2.2 Reaction Function

Whether an individuals enters or leaves a fund, is determined by their (age-specific) reaction function that is built upon the corresponding break-even funding ratio. This reaction function, which I also call the discontinuity function, quantifies how individuals react to the financial position of the fund. In case the prevailing funding ratio is high enough compared to the break-even funding, new generations are willing to enter the collective pension fund, otherwise not. Each individual (being of a different age) responds differently on the funding level of the fund. The nominal funding ratio is used as main indicator for the financial performance of the fund and is publicly reported in The Netherlands. Individuals, therefore, are able to acquire and observe them in a relatively easy way; also, funding rates are easy to interpret: assets over liabilities.

For this reason, the behaviour of the participants during a discontinuity event depends on the reaction functions. The function is a key characteristic in my research, since it determines the dynamic in- and outflow f the fund. During my research, I distinguish between two settings. Firstly, a situation where new entrants may abstain from the contract; causing a dynamical inflow. Secondly, a situation where new entrants may abstain and current generations in the fund may leave; leading to a dynamical in- and outflow. Both situations are sequentially discussed now.

Throughout the thesis I consider a homo economicus, unless stated otherwise; all the assumptions are in line with the used literature. However, notice that not all the assumptions (such as in the financial market and pension fund analyser) are completely identical to the used papers from **siegman** and Molenaar et al. (2011). Based on a pension fund that invests $\phi = 50\%$ in risky assets (i.e. a stock) and $1 - \phi = 50\%$ in risk-free assets (i.e. a bond), the agents in the economy behave rational with a Constant-Relative-Risk-Aversion (CRRA) utility function with a risk-aversion level of 5.

2.2.1 Dynamic Inflow

As discussed in the literature review, Siegmann (2011) investigates at which funding ratios new participants have an incentive to join a collective DB system or start saving in an individual DC scheme on the other hand. I use his indifferent funding ratios to determine whether new entrants of 25 years old want to enter the collective pension fund. Siegmann (2011) finds a minimum funding ratio of 96%, which I call the entry level. That is, in case the funding ratio drops below 96% a future participant abstains from the entering the contract, while if the funding ratio exceeds the entry level of 96% the individual enters in the collective fund. Recall that the participants base their reaction on the prevailing nominal published funding rates, possibly combined with some private information heterogeneous expectations. That is, each participant perceives the financial (future) sustainability of the pension fund individually.

Intuitively, a snowball/bank-run effect materialises if individuals decide in avoiding the pension contract, as explained in the paragraph about information cascades. It is not the case that at a drop from a funding level of 97% to 96% everybody abstains immediately, the transition is gradual. In other words, new entrants abstain from year to year in a continuously decreasing way till the entry level of 96% is reached. Hereby, the new labour entrants (at time t) observe the actions of the previous 25-year-cohort (which made their decision at time $t-1$). The new entrants at time t infer information from the actions of the previous cohort and combine this with their heterogeneous expectations. If at time $t-1$ many new cohorts abstained or entered, new labour entrants at time t are likely to use this information. In case the funding position has worsened compared to time $t-1$, even more entrants at time t abstain since the heterogeneous expectations about the sustainability are lower; the discontinuity risk increases. On the other hand, if the funding level has increased compared to time $t-1$, new entrants at time t become willing to enter the pension fund again because the expectation is that the financial sustainability increases even further; the discontinuity risk decreases. Thus, the assumption is that agents are rational in the sense that they can decide individually whether the fund is indeed in an unsustainable position and, besides, that agents react on the behaviour of other individuals.

For example, such a situation happened in Greece during the banking-crisis. A bank-run emerges, since the banks are perceived to be in an unsustainable financial position. Individuals perceive that the banks are unsustainable and could potentially not pay any money to the agents in the economy. Agents react by withdrawing their wealth from the banks and show behaviour as implied by the theory of information cascades. On the first day 10 agents withdraw their wealth, since they expect to not receive their money any more in the future. A week later, 100 agents withdraw their wealth from the saving deposits, since they have seen other participants withdrawing as well, and inferring expectations and information from these actions. Again a week later, 1000 agents retrieve their saved money, etcetera.

In other words, the number of abstaining and entering participants determines the dynamic inflow and, thereby, the discontinuity risk $d_{x,t}$ at time t for participants of age x. The dynamic inflow increases as new entrants enter the collective DB and, therefore, the discontinuity risk decreases. If the dynamic inflow decreases, the discontinuity risk is likely to increase. I assume that the bank-run effect happens over an absolute change in funding ratio of 20%. That is, using the entry level of 96%, all participants of the new generation enter at a funding ratio of 116% (or higher) but all participants abstain if the funding ratio equals the entry level 96% (or lower). In between, an interval of 20% in

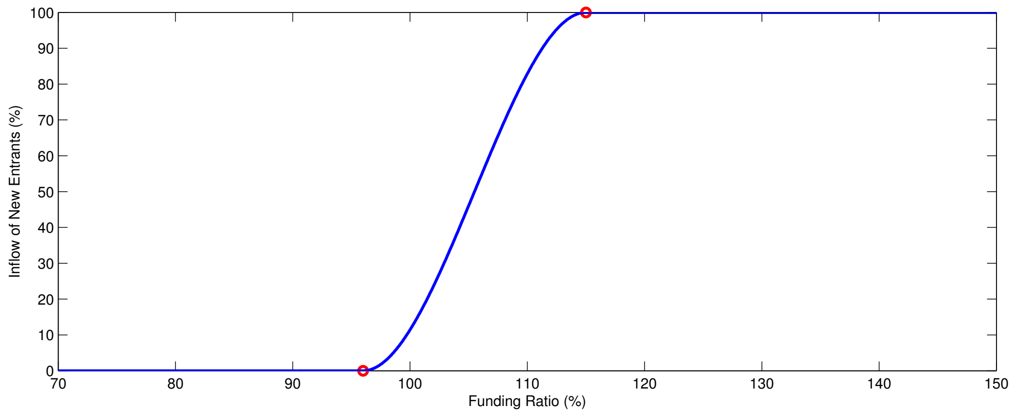


Figure 2.1: Reaction function for a 25-year-old: entry level $FR = 96\%$.
The x-axis shows the funding ratio and the y-axis the discontinuity risk $d_{x,t}(FR_t)$ for $x = 25$, i.e. the percentage of new entrants that is willing to enter in the pension contract. E.g.: if the funding ratio is 100%, 10% of the new labour entrants joins the pension fund. New participants only enter for an entry level of $FR = 96\%$ or higher.

terms of funding ratios, the information cascade happens. So, if the funding ratio is 116% at time t but 113% at time $t + 1$, a fraction of the new entrants abstains. The fraction between the entry level 96% and the entry limit 116% is determined by a piecewise cubic hermite interpolating polynomial $p(FR_t)$, where FR_t represents the funding level at time t. The interval of 20% follows from own analysis and represents a reasonable difference in funding ratios that allows for participants to abstain.

Quantitatively, the reaction function reads as follows for only $x = 25$ (since we consider new entrants):

$$d_{x,t}(FR_t) = \begin{cases} 1 & \text{if } FR_t \leq 96\% \\ p(FR_t) & \text{for } 96\% < FR_t < 116\% \\ 0 & FR_t \geq 116\% \end{cases}, \tag{2.1}$$

where FR_t is the prevailing nominal funding ratio at time t and the function p is the interpolating function that takes the nominal funding ratio as input. The interpretation of $d_{25,t}$ is as follows: if it equals 1, the discontinuity risk is 100% and the full generation of new entrants abstains (inflow is 0%); if it equals 0, the discontinuity risk is 0% and all participants of the new generation enter (inflow is 100%); if it equals $p(FR_t)$, the discontinuity risk is a fraction and that fraction abstains (inflow lies between 0% and 100%). The interpolating function actually represents the information cascade effect.

Thus, the reaction function quantifies the inflow of participants, who make their decision on the nominal funding ratio FR_t. The reaction function for age x determines the level of discontinuity $d_{x,t}$ for a participant of age x at time t; $d_{x,t}$ specifies for new entrants of age $x = 25$ how many do not flow in the fund. For example, if $d_{25,2030} = 1$ nobody of age 25 enters the contract in 2030. It is assumed that males and females behave similarly, so no distinction in discontinuity risk between genders.

Graphically, the reaction function for a 25-year-old looks as shown in Figure 2.1. The hypothesis is that a snowball effect materialises, which is clearly shown. At 96% nobody wants to enter the collective fund anymore. Thus, before that financial position is reached, an information cascade emerges. The inflow of future participants starts to decrease from 116% onwards till the entry level is reached.

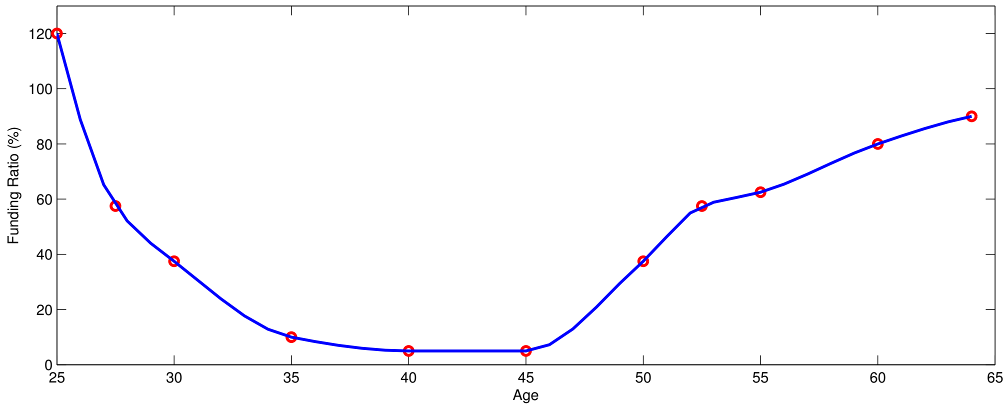


Figure 2.2: Break-even funding ratios for all ages.
On the x-axis the age of the participant is shown, with on the y-axis the corresponding break-even funding ratio. Participants exit if the current funding ratio is below the age-specific break-even funding ratio.

Sinking Giant

Due to a smaller population and, consequently, the lower ability to steer the fund with policy instruments, the pension fund may turn into a Sinking Giant. A Sinking Giant is a term for a pension fund that is unable to recover any more since it has been passing its critical funding ratio for too long (e.g. lower than 75% coverage rate for a substantial period of time) and receives less premium income compared to the pension payments, although it keeps gaining the usual returns on the assets. Upon itself, receiving less premium than flows out, is not really a problem because the future pension payments are covered by capital. However, if the situation persists for too long and the fund does not take any measures, the fund depletes its assets, despite gaining returns on the financial market. Moreover, the policy instrument work less effectively, since there are less participants, wherefore also the operational costs increase per person. In case of a closed fund (i.e. no new inflow at all), the effect is even more extreme.

Next, the situation is explained whereby also older cohorts can exit.

2.2.2 Dynamic In- and Outflow

As previously discussed, it is important to consider discontinuity risk with respect to older cohorts also. Results Molenaar et al. (2011) are used for formulating a reaction function for existing participants (25-64 years). In the paper they find break-even funding ratios for individuals aged from 25 until and including 60. Since my thesis considers participants that retire at the age 65, the break-even funding ratios are extrapolated till the retirement age of 65; I extrapolate by piecewise cubic smoothing spline. In The Netherlands individuals can switch between the funds till and including the age of 64. After retirement, it is not possible to leave the collective fund anymore. Retirees are, thus, assumed to stay at the same pension provider for the rest of their life. For this reason, since The Netherlands is taken as baseline, I make this assumption also. Hence, the last working year is determinative for your retirement provision. For example, if you decide to leave the collective fund at age 64 and transfer your capital to an individual fund, your retirement provision is regulated by the individual DC.

Now, the outflow becomes also dynamic. In other words, new participants decide whether they enter or not, but current participants in the fund may leave also. Each age-group reacts differently on the financial position of the fund, which is due to the burden-effect and uniform-effect (as explained in the literature review, Section 1.1). Consequently, there is not only one entry level as with dynamic inflow. Namely, for each specific age, particular break-even funding ratios hold true. The U-shaped pattern of the break-even funding ratios, including the extrapolation for ages higher than 60, is shown in Figure 2.2. The graph shows that young cohorts have high entry levels, middle-aged cohorts stay even at low funding rates, while near-retirees have high break-even funding rates again. The U-shape stems from the "doorsneesystematiek". To each break-even funding ratio a specific reaction function belongs.

The reaction function for new entrants contains the exact same characteristics and interpretation as for dynamic inflow only, but the reaction function is based upon a higher entry level. Namely, the entry level according to Molenaar et al. (2011), equals 120%. For this reason, the reaction function looks the same as shown in Figure 2.1, but the entry level is 120% (i.e. everybody abstains) while the upper level lies at 140% (everybody enter) and in between the snowball effect prevails.

The reaction function for outflow of current agents works in the reverse direction, as compared to the reaction function for inflow of participants. Consider a 64-year-cohort: the break-even funding ratio for this age-group lies at 90%, as shown by Figure 2.2. Thus, if the funding ratio drops below the 90%, all participants of 64-years-old leave the collective fund and transfer their pension rights to the individual fund. In a continuously decreasing way, the 64-year-olds leave. It not the case (as explained above) that at a coverage rate of 91% every 64-year-old stays and at 90% all leave. Leaving happens conform the information cascade effect, whereby 64-year-olds at time t observe the actions of the previous 64-year-cohort at time $t-1$. The snowball-effect again occurs during an interval of 20% funding ratio. So, at a funding rate of 110% all participants of 64 years old remain and if the funding ratio starts to drop, the 64-year-olds begin to leave. The reaction function is shown in Figure 2.3.

In other words, each cohort of age x has a specific reaction function which is due to the age specific break-even funding ratios BFR_x. The break-even funding ratios for a cohort of age x specify the entry level, i.e. the level whereupon every participant of that cohort exits. The upper level, whereupon everyone from that cohort enters, follows from the break-even funding ratio plus the additional time for the information cascade effect. Between the entry and upper levels the interpolating function determines the fraction of participants that exits. The interpolating function $p(FR_t)$ is not age-specific (so uniform). However, it takes the prevailing funding ratios FR_t as input and interpolates only between the entry level and upper level which are age specific. So, quantitatively the discontinuity function at time t for age $25 \leq x < 64$ is specified by:

$$d_{x,t}(FR_t) = \begin{cases} 1 & \text{if } FR_t \leq BFR_x \\ p(FR_t) & \text{for } BFR_x < FR_t < BFR_x + 20\% \\ 0 & FR_t \geq BFR_x + 20\% \end{cases}, \tag{2.2}$$

where BFR_x is the age-specific break-even funding ratio (constant trough time), FR_t is the prevailing nominal published funding ratio and p is the interpolating function. The outcomes should be interpreted as follows: $d_{x,t}$ specifies for new entrants of age $x = 25$ how many do not flow in the fund, while for current actives of age $x \geq 26$, $d_{x,t}$ determines how many flow out the fund. For example, if $d_{25,2030} = 0.65$ 65% of the participants of

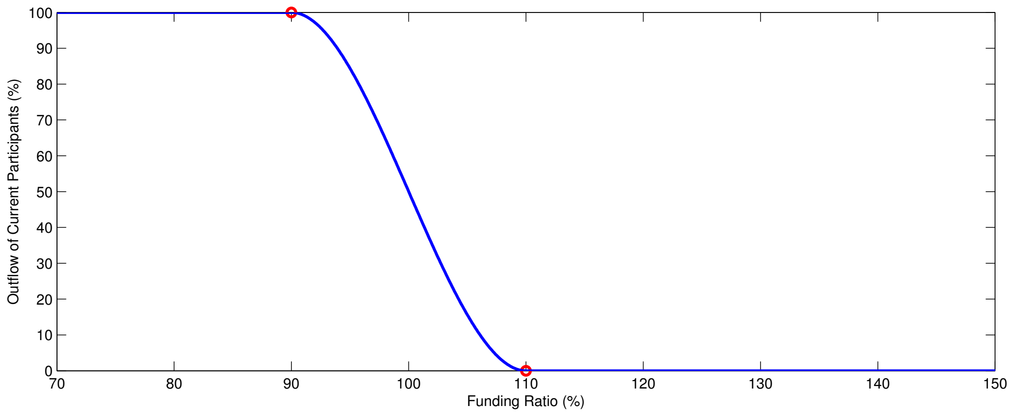


Figure 2.3: Reaction function for a 64-year-old.
The x-axis shows the funding ratio and the y-axis the discontinuity risk $d_{x,t}(FR_t)$ for $x = 64$, i.e. the percentage of participants that leaves the pension fund. E.g.: if the funding ratio is 100%, 50% of the 64-year-olds withdraws from the pension contract.

age 25 does not enter the contract in 2030, while $d_{64,2030} = 0.6$ shows that 60% of the 45-year-olds leaves the fund in 2030.

Then, why do some 64-year-olds remain in the fund at a funding ratio of roughly 100%, while 40% leaves for example? Or, why do some 25-year-olds still enter in the fund, while 30% abstains at a funding ratio of 110%? The effect is due to the information cascade. It is not due the utility function and level of risk-aversion, since those are fixed for the whole population (as in accordance with the used literature). The difference comes from the heterogeneous expectations. To indicate the sensitivity of the results with respect to the underlying discontinuity function and to investigate different papers from the literature, I study discontinuity risk with the aid of both papers, i.e. both entry levels for new participants are tested. Furthermore, cost differences are neglected in both papers; though, a 1% cost difference can bring the break-even funding ratio down to 55% for a 25-year-old (Molenaar et al., 2011).

To sum up, this section described how agents specifically react during a discontinuity event and, thereby, answered the first research question: how do new entrants and current participants react during a discontinuity event? Namely, the behaviour of the agents is quantified by the reaction function (also called the discontinuity function), which takes the funding ratio FR_t as input and is based upon the break-even funding ratios combined with the snowball-effect. Two situations were shown: (i) less inflow of new entrants (leading to possibly a Sinking Giant); (ii) dynamic in- and outflow of new entrants and current generations. In the latter case, it was shown that all age-groups react differently on the prevailing funding ratios.

The next chapter describes the ALM-model with the input elements: (i) financial market and (ii) pension fund analyser.

Chapter 3

Model

This chapter introduces the financial market where the pension fund is operating in and it describes the pension fund as well.

3.1 Financial Market

Before I turn towards the analysis of the pension fund, I present the financial market This section describes the modelling choices and assumptions on the financial market.

A continuous-time stochastic economy with two financial market risk factors is considered. Namely, on the one hand a stock market index and on the other hand a variable interest rate. Those sources of risk are, respectively, identified as stock market risk and interest rate risk, and are often considered to be the dominant risks for a pension fund (upside as well as downside risks). Pension fund boards deliberately take market risk, in order to be rewarded by positive returns. Interest rate risk has to be dealt with by the pension fund, and it is difficult to imagine the interest rate to be modelled as a constant. The stocks and interest rates are assumed to be uncorrelated, while in practice stocks and interest rates show frequently negative correlations (i.e. if interest rates fall, the shares prices rise ceteris paribus). Moreover, it is assumed that the market is free of arbitrage (conditions are given in Section 3.1.1) and, besides, transaction costs are neglected and fees (i.e. the market is frictionless). Next to that, it is assumed that the model generates asset prices that are consistent with deep, liquid and transparent financial markets.

A model that suits those needs, is the frequently used Black-Scholes-Vasicek model which is a generally accepted model. It is a comprehensive (not too complex) model, with a non-deterministic interest rate, that approximates reality fairly well. More specifically, the financial market is driven by two financial market state variables. Firstly, the stochastic behaviour of the stock market index is governed by the Black and Scholes model (Black et al., 1973). S_t denotes the price of the stock (i.e. the value of the stock index) at time t and follows a geometric Brownian motion with constant drift and volatility.[1] The stock

[1]The process $dX_t = \mu X_t dt + \sigma X_t dW_t$ is called a geometric Brownian motion and the use of this model for the evolution of stock prices was initiated by Samuelson. Here, W_t is a standard Brownian motion, also called a standard Wiener process. A continuous-time process W_t ($t \geq 0$) is said to be a (standard) Wiener process if it satisfies the following properties:

- $W_0 = 0$;
- If $t_1 < t_2 \leq t_3 < t_4$, then the increments $W_{t_2} - W_{t_1}$ and $W_{t_4} - W_{t_3}$ are independent;
- For any given t_1 and t_2 with $t_2 > t_1$, the distribution of the increment $W_{t_2} - W_{t_1}$ is the normal distribution with mean 0 and variance $t_2 - t_1$.

does not pay any dividends. Secondly, the evolution of the (nominal) short-term interest rate is described by the one-factor, mean-reverting Vasicek model. This interest rate at time t is denoted by r_t. In the complete objective — real world — probability space $(\Omega, \mathcal{F}, \mathbb{P})$, I denote the processes of these variables with their initial values as

$$dS_t = (r_t + \lambda_S \sigma_S) S_t dt + \sigma_S S_t dW^{\mathbb{P}}_{S,t}, \qquad S_0 = 1, \tag{3.1a}$$

$$dr_t = \alpha(\kappa - r_t)dt + \sigma_r dW^{\mathbb{P}}_{r,t}, \qquad r_0 = 0.02. \tag{3.1b}$$

where $\lambda_S = \frac{\mu - r_f}{\sigma_S}$ is the price of risk, and $W^{\mathbb{P}}_{S,t}$ and $W^{\mathbb{P}}_{r,t}$ are standard Brownian motions (i.e. the risks in the economy) under the objective measure for the stock and (instantaneous) interest rate, respectively. μ denotes the average expected stock return with σ_S the stock's volatility and r_f the risk-free rate; while α denotes the speed of mean reversion, κ is the long-term average of the nominal short rate (i.e. the level of mean reversion) and σ_r is the (instantaneous) volatility of the interest rate.

Furthermore, a pseudo-stochastic price index is assumed. The value of the price index at time t is denoted by I_t and follows a normally-distributed random variable. The price of the short-term risk-free asset (i.e. cash) at time t is denoted by B_t, which depends on the nominal short rate r_t. The processes with their initial values are given by

$$dI_t = \iota I_t dt, \quad \iota \sim N(\mu, \sigma^2) \qquad I_0 = 0.02, \tag{3.2a}$$

$$dB_t = r_t B_t dt, \qquad B_0 = 1, \tag{3.2b}$$

where ι denotes the random variable of the inflation rate which is normally distributed with mean μ and variance σ^2. The literature states that the hypothesis of normally distributed inflation rates is not bad, but it seems that a scaled t-distribution with six degrees of freedom is more appropriate (see e.g. Carlson (1975)). Nevertheless, I do not consider it as a huge violation for now. Both the short rate and inflation rate are initialised with a value of 2%, reflecting the ambition of the European Central Bank (ECB);[2] in the market, the interest and inflation rates are frequently of similar value. The stochastic differential equations (3.1a), (3.1b), (3.2a) and (3.2b) form the so-called Black-Scholes-Vasicek model. Throughout the thesis, a Black-Scholes-Vasicek model is used in order to generate scenarios for (future) values of the nominal short rate (r_t), the stock price (S_t), the price index (I_t) and the money market account (B_t).

Hence, the menu of assets to choose from for the pension fund in the financial market consists of: cash, a default-free nominal bond, a default-free real bond and a stock; which is more than enough to consider, since each asset can be seen as an individual portfolio with one generic price. Those portfolios, then, reflect that the market is deep. The next subsections describe, respectively, the Black-Scholes model and Vasicek model in detail.

3.1.1 Black-Scholes Model

I assumed absence of arbitrage, but this does not come for free; namely, the model actually needs some conditions. In order to assess the value of the assets and liabilities of the pension fund in a market consistent way (i.e. arbitrage-free), it is (for instance) required (i) to use the pricing kernel method or (ii) to change from the objective measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$ by an equivalent martingale measure. Both methods are

[2] See `https://www.ecb.europa.eu/home/html/index.en.html`.

discussed below and a direct result of the fundamental theorem of asset pricing, which provides conditions for arbitrage-free prices. The approach is based upon the assumption of complete markets. In other words, the embedded options in the pension fund can be replicated by the products that exist in the financial market.[3]

Pricing Kernel Method

One way to value the liabilities in a fair way (i.e. an arbitrage-free price), is by using the so-called stochastic discount factor. Let P_t be a price process. Then, the first fundamental theorem of asset pricing states:

Theorem 1. *Absence of arbitrage holds if and only if there is a positive adapted scalar process M_t such that the process $M_t P_t$ is a martingale under $\mathbb{P}$.*

The process M_t is called the pricing kernel, or in other words the stochastic discount factor. To attain the price at time t for the payoff of asset P at time of maturity T, discounting is done in the following way:

$$P_t M_t = E_t^{\mathbb{P}}[P_T M_T] \Leftrightarrow P_t = E_t^{\mathbb{P}}\left[P_T \frac{M_T}{M_t}\right],$$

where the expectation at time t is taken under the real-world probability measure. Hence, the value at time 0 of asset P with payoff at time of maturity T is obtained by

$$P_0 = E_0^{\mathbb{P}}\left[P_T M_T\right],$$

where I use the fact that $M_0 = 1$. This leads to the statement: "price (i.e. value) is expected discounted pay-off."

Equivalent Martingale Method

A second method to determine market-consistent prices, is by using risk-neutral valuation. Assume that it is given that there is a joint process of asset prices P_t, and a numéraire N_t. Then, a second version of the first fundamental theorem of asset pricing states:

Theorem 2. *Absence of arbitrage holds if and only if there is a measure $\mathbb{Q}$, equivalent to $\mathbb{P}$, such that P_t/N_t is a martingale under $\mathbb{Q}$.*

The measure $\mathbb{Q}$ is called the equivalent martingale measure that corresponds to the numéraire N_t.[4] Such a change of measure for a random variable can be established via using the so-called Radon-Nikodym derivative; a change of measure for random processes is called a Radon-Nikodym process.[5] In the case of processes obtained from stochastic

[3] Please note that the liability side may not be completely replicable, especially the inflation linked part, since inflation-linked bonds only exist in a very illiquid form (Kocken, 2008).

[4] Any asset can be a numéraire (i.e. the unit of account), as long as the price of the numéraire is never equal to zero.

[5] The details will not be discussed here any further, but more information can be found in the literature (see e.g. Schumacher, 2015, p. 48-50).

differential equations driven by Brownian motions, the conditions under which two processes are related (both ways, from $\mathbb{P}$ to $\mathbb{Q}$ and vice versa) by a Radon-Nikodym process are provided by the theorem of Girsanov. Girsanov's theorem (1960) states that if we let λ_t a process adapted to the Browmian motion W_t (under mild boundedness conditions), and if we change measure from the original measure $\mathbb{P}$ to a new measure $\mathbb{Q}$ according to the Radon-Nikodym process θ_t defined by

$$d\theta_t = -\lambda_t \theta_t dW_t^{\mathbb{P}}, \quad \theta_0 = 1.$$

Then, under the new measure $\mathbb{Q}$ obtained in this way, the stochastic process $W_t^{\mathbb{Q}}$ defined by

$$dW_t^{\mathbb{Q}} = \lambda_t dt + dW_t^{\mathbb{P}}, \quad W_0^{\mathbb{Q}} = 0 \tag{3.3}$$

is a Brownian motion. Essentially, a change of the probability measure is a change of the drift term in the stochastic process. Note that a constant price of risk λ is assumed throughout the thesis (i.e. independent of time t). The price of asset P at time t for payoff at time of maturity T now satisfies (also known as the numúmeraire-dependent pricing formula)

$$\frac{P_t}{N_t} = E_t^{\mathbb{Q}_N}\left[\frac{P_T}{N_T}\right],$$

where the expectation is taken at time t under the objective measure $\mathbb{Q}_N$ relative to the numéraire N. Now, consider the (risk-free) money market account as numéraire (i.e. $B_t > 0 \quad \forall t$), where the value of the money market account B_t follows the process given in (3.2b). Moreover, if I consider a bond that pays P_T units of currency at time time of maturity T, then the price of this bond at time t is given by the numéraire-dependent pricing formula:

$$\frac{P_t}{B_t} = E_t^{\mathbb{Q}}\left[\frac{P_T}{B_T}\right],$$

under the objective measure with as numéraire the money market account. Taking the risk-free money market account as numéraire, is called risk-neutral valuation, which is a special form of the equivalent martingale measure, obviously. Note that for simplicity the risk-neutral probability measure is denoted by $\mathbb{Q}$ (no subscript for the numéraire anymore). A more explicit expression can be obtained in terms of the short rate, which is expressed as follows: [6]

$$P_t = E_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T r_s ds\right) P_T\right]. \tag{3.4}$$

Here, the price of a bond at any time t with payoff P_T at time of maturity T is established through the use of the interest rate. Thus, the arbitrage-free value of the bond at time 0

[6] Therefore, use equation (3.2b) along with Itô's rule (for the function ($f(x) = \ln(x)$) and obtain

$$B_t = B_0 \exp\left(\int_0^t r_s ds\right),$$

where r_s is the nominal short rate at time s. Subsequently, by substituting B_t as numéraire in the numémeraire-dependent pricing formula, the price of the bond follows.

is determined by

$$P_0 = E_0^{\mathbb{Q}} \left[\exp\left(-\int_0^T r_s ds \right) P_T \right],$$

where r_s still denotes the nominal (short) interest rate. Again, the "market-consistent value equals the expected discounted payoff." For this reason, both the pricing kernel method and equivalent martingale measure lead to the same outcomes. Risk-neutral valuation, however, is used throughout the thesis simply for a preference reason, instead of the stochastic discount factor approach.

Hence, to write the Black-Scholes equation (3.1a) under the risk-neutral measure $\mathbb{Q}$, I change from the measure $\mathbb{P}$ to $\mathbb{Q}$ by using Girsanov's theorem. As stated above, changing from measure leads to a change in drift:

$$\begin{aligned} dS_t &= (r_t + \lambda_S \sigma_S) S_t dt + \sigma_S S_t dW_{S,t}^{\mathbb{P}}, \\ &= (r_t + \lambda_S \sigma_S) S_t dt + \sigma_S S_t (dW_{S,t}^{\mathbb{Q}} - \lambda_S dt), \quad \text{by substituting (3.3)} \\ &= r_t S_t dt + \sigma_S S_t dW_{S,t}^{\mathbb{Q}}, \end{aligned} \tag{3.5}$$

where $W_{S,t}^{\mathbb{Q}}$ is a standard Brownian motion under the risk neutral measure for the stock. It is clear that the risk-neutral model generates scenarios that do not incorporate the risk premium. So, the expected return on the stock equals the risk-free return. In Figure 3.1, for some parameter values, the scenarios and time-evolution of the stock price are shown under both measures $\mathbb{P}$ and $\mathbb{Q}$. Note that under $\mathbb{Q}$ the stock price is indeed lower.

Please note that the processes of the money market account and the inflation rate are the same under the real-world as well as objective measure, since they only depend on time and not on a Wiener process. Now, the Vasicek model is explained.

3.1.2 Vasicek's Model

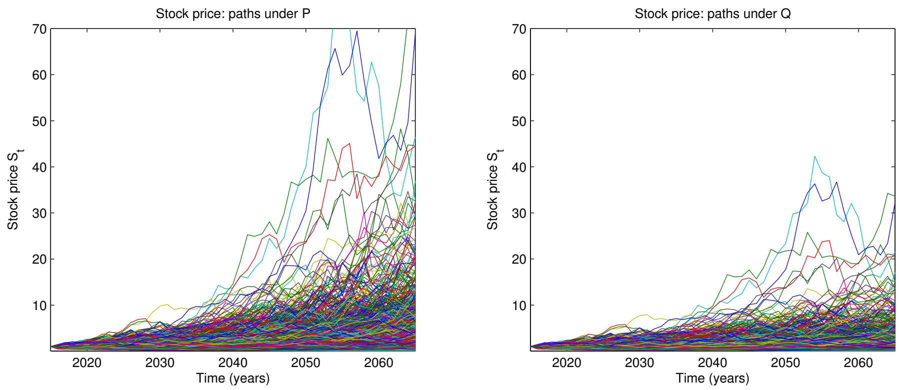
Having described the evolution of the stock, money market account and inflation rate under the risk-neutral measure $\mathbb{Q}$, I now present the evolution and term structure of the interest rate under $\mathbb{Q}$. The term structure is used for computing the (discounted) market-consistent value of the liabilities for the pension fund. The (instantaneous) nominal interest rate in equation (3.1b) follows an Ornstein-Uhlenbeck process.[7] This short rate is used to determine the term structure of the nominal and real interest rate. Before I turn towards the term structures, the evolution of the interest rate under the risk-neutral measure is presented:

$$\begin{aligned} dr_t &= \alpha(\kappa - r_t) dt + \sigma_r dW_{r,t}^{\mathbb{P}}, \\ &= \alpha(\kappa - r_t) dt + \sigma_r (dW_{r,t}^{\mathbb{Q}} - \lambda_r dt), \quad \text{by substituting (3.3)} \\ &= \alpha(\kappa - \frac{\lambda_r \sigma_r}{\alpha} - r_t) dt + \sigma_r dW_{r,t}^{\mathbb{Q}}, \end{aligned} \tag{3.6}$$

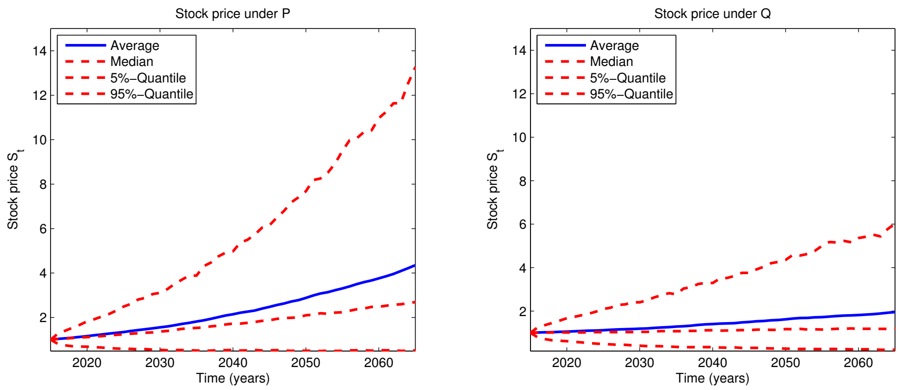
[7]The Ornstein-Uhlenbeck is a frequently used linear stochastic differential equation and the general form (with positive speed of reversion a) is denoted as

$$dX_t = a(c - X_t)dt + bdWt,$$

with W_t a standard Wiener process and the constant c being the mean that the process reverts to.

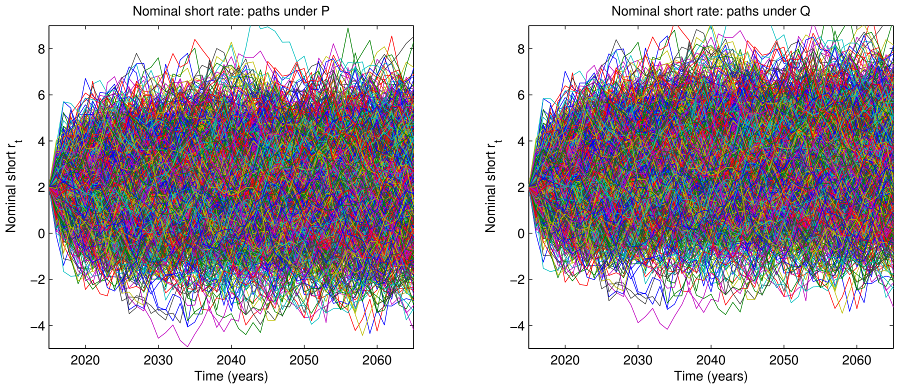


(a) Simulated paths of the stock price.

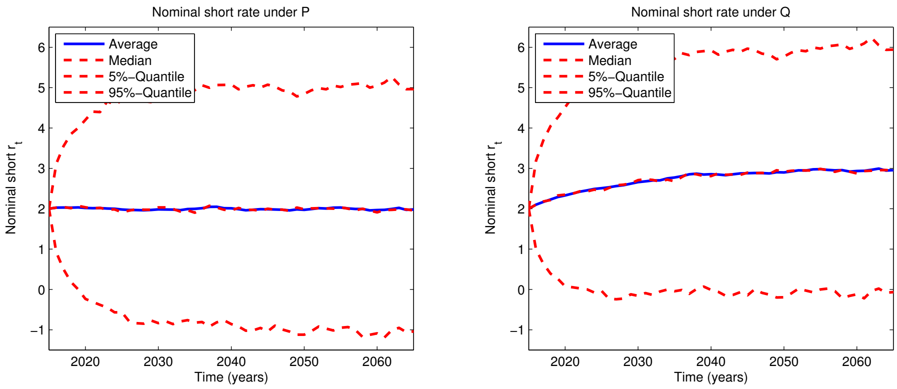


(b) Summary statistics of the stock price.

Figure 3.1: Behaviour of the stock price under $\mathbb{P}$ and $\mathbb{Q}$: $\lambda_S = 4\%$, $\sigma_S = 20\%$. The horizontal axis shows time (in years), while the vertical axis displays the value of the stock index.



(a) Simulated paths of the nominal interest rate.



(b) Summary statistics of the nominal interest rate.

Figure 3.2: Behaviour of the short rate under $\mathbb{P}$ and $\mathbb{Q}$: $\alpha = 0.15$, $\kappa = 2\%$, $\sigma_r = 1\%$. The x-axis displays time and the y-axis the value of the nominal short rate.

where λ_r is the price of risk of the innovations in the nominal short rate and $W^{\mathbb{Q}}_{r,t}$ is a standard Brownian motion under the risk neutral measure for the interest rate. In Figure 3.2 scenarios and the time-evolution of the nominal interest rate are shown, for some parameter values (specified in the figure).

Please recall that the price of a bond at time t with a payoff P_T units of currency at time of maturity T is given by equation (3.4). Consider, from now onwards, a default-free zero-coupon bond that pays one unit of currency at time of maturity T. Then, the price of such a bond at time t is given by

$$P_t = E^{\mathbb{Q}}_t \left[\exp\left(- \int_t^T r_s ds \right) \right].$$

The value of this bond maturing at time T is also called the (riskless) discount factor for maturity T (Schumacher, 2015). Obtaining discount factors for different maturities T leads to the discount curve, which is a (basic) representation of the term structure. A more common representation of the term structure is by means of the yield-curve (other possibilities are the swap and forward curves). The yield curve is obtained by computing the (continuously compounded) yields for different maturities T. The yield at time t for a given maturity $T > 0$ is computed and denoted by

$$R_t = -\frac{1}{T} \ln(P_t). \tag{3.7}$$

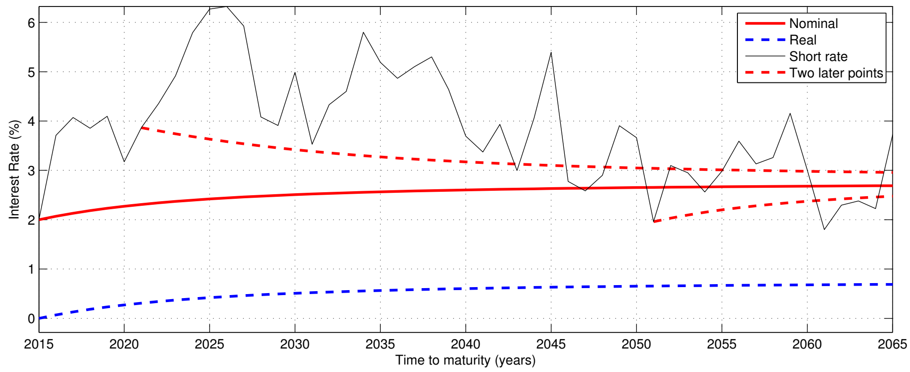


Figure 3.3: Nominal and real term structure in the Vasicek model.
The horizontal axis displays time along with time to maturity (in years) and the vertical axis presents the interest rate in percentage points. The two later points show the nominal term structure that emerges from the nominal short rate.

Thus, to compute the yield, the value of P_t under the risk-neutral measure has to be determined explicitly; there exists a closed-form solution, known as the bond pricing formula.[8] The value of a default-free zero-coupon bond at time t paying one unit of currency at time of maturity T — given the short rate r_t at time t — in the Vasicek model under the measure $\mathbb{Q}$ is

$$P_t = \exp\left(-\left[\left(b_Q - \frac{\sigma_r^2}{2\alpha^2}\right)T + \left(r_t - b_Q + \frac{\sigma_r^2}{\alpha^2}\right)\frac{1-e^{-\alpha T}}{\alpha} - \frac{\sigma_r^2}{2\alpha^2}\frac{1-e^{-2\alpha T}}{2\alpha}\right]\right), \quad (3.8)$$

with $b_Q = \kappa - \frac{\lambda_r \sigma_r}{\alpha}$ and the parameters as described above. So, the yield (i.e. the interest rate) at time t for maturity T is calculated by substituting the closed-form solution of P_t in (3.7).

To get the nominal term structure for each maturity T and each point in time t, the yield is subsequently computed for each maturity T and each point in time t. The real term structure is obtained by subtracting the mean of the random inflation rate ι for each scenario. The term structures at time 2015 ($t = 0$) for each maturity T are shown in Figure 3.3. The mean-reversion effect towards the long-term average κ is nicely demonstrated. Besides, the graph visualises the nominal short rate (for a scenario) in accordance with the (nominal) term structures at two later points in time.

Summarising, the financial market was presented in this section. Firstly, it showed the frequently used Black-Scholes-Vasicek model under the real-world probability measure $\mathbb{P}$. The stochastic processes for the stock and short rate were introduced, as well as the pseudo-stochastic process for the inflation rate and the deterministic process for the money market account. Secondly, the Black-Scholes and Vasicek models were scrutinised. Two methods (resulting from the first fundamental theorem of asset-pricing) for arbitrage-free prices of the stock and market-consistent interest rates were shown, namely: the pricing kernel method and the equivalent martingale measure. The latter approach led to the case of risk-neutral valuation whereby the Black-Scholes-Vasicek model under the risk-neutral

[8]Details about the derivation of this formula can be found in the literature (see e.g. Schumacher, 2015, p.102, p. 139).

measure $\mathbb{Q}$ was obtained:

$$\begin{aligned}
dS_t &= r_t S_t dt + \sigma_S S_t dW^{\mathbb{Q}}_{S,t}, && S_0 = 1,\\
dr_t &= \alpha(\kappa - \frac{\lambda_r \sigma_r}{\alpha} - r_t)dt + \sigma_r dW^{\mathbb{Q}}_{r,t}, && r_0 = 0.02,\\
dI_t &= \iota I_t dt, \quad \iota \sim N(\mu, \sigma^2) && I_0 = 0.02,\\
dB_t &= r_t B_t dt, && B_0 = 1.
\end{aligned}$$

With the aid of this model, the term structure of the interest rate under the risk-neutral measure was found. This term structure enables us to compute the (discounted) market-consistent value of the liabilities. The simulations of the financial market are performed under the objective measure $\mathbb{P}$ as well as the risk-neutral measure $\mathbb{Q}$. Pricing (of e.g. liabilities) is done under the risk-neutral measure $\mathbb{Q}$, while reporting of the funding ratio (to e.g. management) is performed under the objective real world measure $\mathbb{P}$. The parameters of the model are initialised in Chapter 4.

The next section describes the individual participants and the pension fund that operates in this financial market.

3.2 Pension Fund

This section shows the population and characteristics of the participants in the pension fund and, afterwards, it describes the policy of the pension fund. Finally, it presents the differences in computations between the static and dynamic setting. But, first, some general remarks about the pension fund and notation are stated.

Pension funds focus on the second pillar and, thus, by definition disregard the mandatory first pillar (i.e. also no franchise in the model) as well as the voluntary third pillar (i.e. no insurers). Besides, the pension fund is an open fund, as opposed to a closed fund. In other words, each year new participants enter in the fund (in a closed fund there is no inflow of new entrants). The fund maintains the (hybrid) collective Dutch DB system with uniform premiums and uniform accruals. This is the so-called "doorsneesystematiek", which is explained in the text-box in Chapter 1. It is collective in the sense that risks are shared (as opposed to a pure individual DC) and hybrid in the sense that benefits are conditional on the funding rate. Moreover, the pension contract is assumed to be complete.

I use the following convention throughout the thesis (a same convention is used in Tilburg Finance Tool). For variables that are valid over a period (contributions, income, pension payments), year t refers to the variable's value throughout year t. For variables that hold for a specific moment in time (funding ratios), year t refers to the variable's value at the beginning of year t. Initial time $t = 0$ is taken to be the year 2015.

The next subsection introduces the composition and development of the participants in the pension fund.

3.2.1 Demography

To determine the liabilities of the fund (i.e. future pension payments), the fund uses mortality tables and population sizes. To initialise the population of participants in the pension fund, I take the population size of the whole population in The Netherlands provided by the Netherlands Bureau of Statistics (CBS). With participants is meant actives and retirees, so I do not include sleepers in the fund. The population size is specified per age group of 1-year (25-99) and gender specific (male and female), since males and females have different survival rates. Then, the number of participants in the fund equals the population size of the CBS divided by 3.5. This leads to a representative fund for the (larger) pension funds in The Netherlands in 2015. The number of male (female) participants in the fund of age x at time t is denoted by $M_{x,t}$ ($F_{x,t}$) and is, thus, gender- and age-specific. For example, $M_{x,0}$ equals the number of male participants of age x based on the current population in The Netherlands (i.e. 2015). The projections for generations of 25 years old run till 2060 and are assumed to be constant thereafter. Namely, it is expected that at this point in time survival rates do not increase any more (i.e. humans have reached the maximum possible age).

The survival probabilities are also taken from the CBS (for consistency) and projected till 2060 for every age, hereafter the death rates are assumed to become constant.[9] The rates in the data-set of the CBS are cumulative survival rates; each rate denotes the probability of a person surviving to age x in year t. Consider a person aged x years, also called a life aged x and denoted by (x); the total lifetime of this individual in a given population is a random variable. I denote the k-year death probability of (x) by ${}_kq_x$, i.e. the probability that (x) dies within k-years. Then, the probability that (x) survives at least k-years is determined by ${}_ks_x = 1 - {}_kq_x$. To distinguish between males and females, the survival probabilities at time t are denoted respectively by ${}_ks_{x,t}{}^m$ and ${}_ks_{x,t}{}^f$; males and females have different survival probabilities, since females tend to live longer than males.

The population develops from 2015 onwards by the following recursive process:

$$M_{x+1,t+1} = M_{x,t} \cdot s_{x,t}{}^m, \tag{3.10a}$$

$$F_{x+1,t+1} = F_{x,t} \cdot s_{x,t}{}^f, \tag{3.10b}$$

where $s_{x,t}{}^m$ ($s_{x,t}{}^f$) denotes the one-year survival probability. Each year t a new generation of 25 years old enters the market, i.e. at time $t-1$ this generation was younger than 25. The number of new entrants each year t is determined by the projection of the population size of 25-years-olds at time t and, then, the cohort follows the recursive process. The recursive process starts with the initial values $M_{x,0}$ ($F_{x,0}$) which equal the current population sizes of 2015. In case of the dynamic setting, these new entrants can decide to enter the collective fund (further explained in Section 3.2.4).

The (predicted) evolution of the male and female populations is presented in Figure 3.4. First of all, cohorts can be easily distinguished by following the diagonal patterns. Clearly, the generation of Baby Boomers can be observed by the large hump; it are the generations between approximately 50-70 years in 2015. Moreover, the number of females in the population becomes larger relative to the number of males — compare the ages $x \geq 90$ for males and females. Thirdly, from all age groups an equal share of participants arises in the future (i.e. people live longer). The wobbly character of the population structure in 2015 changes to an equally shaped structure in the future. That is, the population pyramid goes from triangular-shaped to a flat-shape, where every cohort is of approximately the same size. The figure shows that till 2040 the number of retirees keeps increasing (due to inter alia the Baby Boomers), while the number of workers decreases relatively. As is additionally clear from Table 3.1, the ratio of active participants versus

[9]The population sizes and survival probabilities are published in 2011. In my data-set, I contain the population sizes and survival rates from the year 2015 onwards.

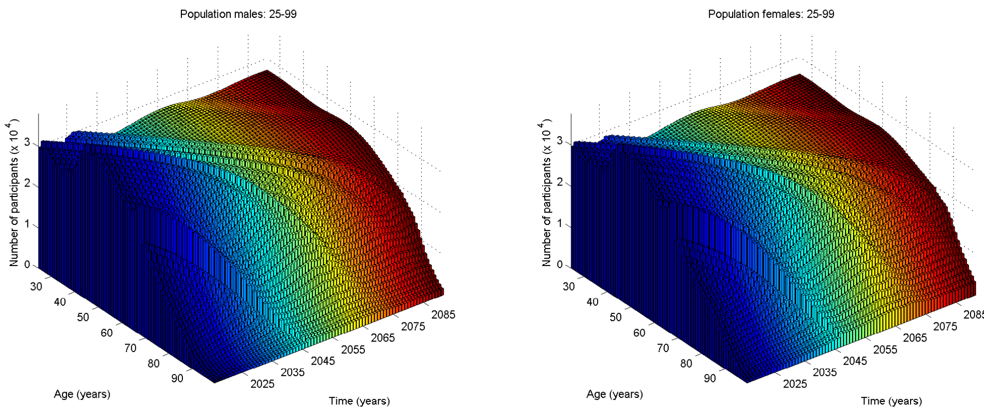


Figure 3.4: Demography of the population through time and age.
The x-axis shows time (in years), the y-axis age (in years) and on the z-axis the number of participants (in ten thousands).

retirees in 2015 equals (approximately) 3:1, while in 2060 this ratio is predicted to be 2:1 — i.e. for one retiree there are two workers, instead of three workers.

So, the pension fund is based on the Dutch population and, therefore, reflects the idea of a national Dutch fund. Next, the characteristics of the participants in the population are described as well as the characteristics of the pension fund.

3.2.2 Characteristics

Now, the characteristics of the active participants (i.e. workers) and retirees are defined. The participants start to work at the age of $S = 25$, retire at $R = 65$ and decease for sure at the maximum age of $D = 100$. The premium payments last until the age of $R - 1 = 64$ and the pension payments start at the beginning of $R = 65$ where the last pension payment is received at the age of $D - 1 = 99$ at most. Participants, thus, work for $R - S = 40$ years and receive pension payments for a period of maximal $D - R = 35$ years. The pension contract is based on an average wage scheme, which is nowadays common in The Netherlands — it replaced the final wage scheme that dominated the market until 15 years ago.[10] That is, the accrual of new pension rights depends on the average salary of the participant during their working career of 40 years. The salary of the participants grows each period with ω_t, which depends on the prevailing (random) price inflation of ι% per year; this is the so-called uniform (economy-wide) wage growth and differs per economic scenario. ω_t follows $iota_t$, however ω_t ensures that wages do not grow at time t if inflation is negative at time t. Besides, each individual experiences a certain career-profile during their working life-time; this is attributable to progression of the individual

[10] Currently the government steers towards CDC's and life-cycle investment strategies. Therefore, it is discussing a transition towards so-called 'premie-regelingen', which will replace the average-wage schemes with life-cycle strategies. An extension is the 'verbeterde premie-regeling' that enables funds to invest also during the retirement phase on behalf of the participant. Nowadays, life-cycle investment strategies must ensure that no investment risks are taken anymore during the retirement phase.

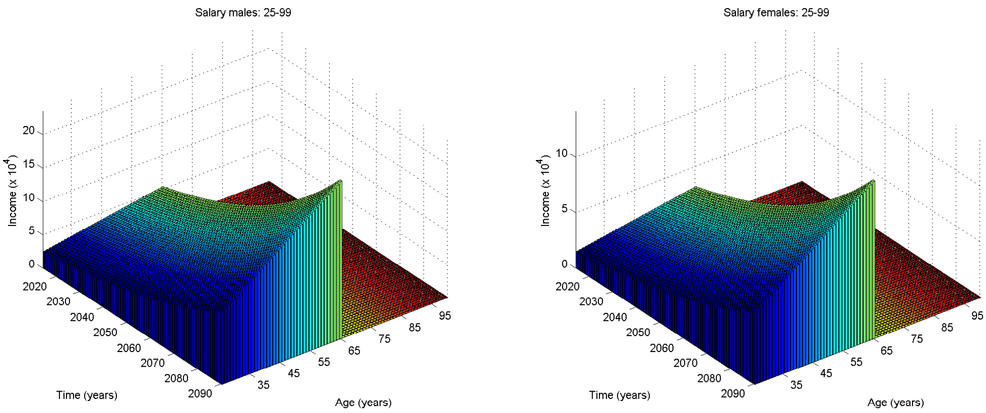


Figure 3.5: Income through time and age.
The x-axis shows age (in years), y-axis time (in years) and z-axis income (in ten thousands). The mean of the salaries is presented (since wage growth is random), whereby the mean is taken over all economic scenarios.

career. The career-profiles differ between males and females, since females tend to work more part-time than males (due to the duty of care for children, amongst others). The incomes are as follows for generations starting in 2015 (to an idea of Wernekinck (2013))

$$w^m_{x,0} = \begin{cases} €25{,}000 + €750 \cdot (x \text{ - } 25) & \text{if } male \text{ and } 25 \leq x < 65 \\ 0 & \text{if } x \geq 65 \end{cases},$$

$$w^f_{x,0} = \begin{cases} €15{,}000 + €450 \cdot (x \text{ - } 25) & \text{if } female \text{ and } 25 \leq x < 65 \\ 0 & \text{if } x \geq 65 \end{cases}.$$

The average income and pension in 2015 ($t = 0$) for males and females is shown in Table 3.1, and these average values are basically similar to the data from the CBS.[11] When a new generation enters the labour market, the new participants start with a salary that equals the salary of the previous age group of 25, increased with the prevailing economy-wide wage growth ω_t. The development of the salaries is shown in Figure 3.5. Clearly, income levels rise with age (due to the career-profile) and time (due to ω_t), and after the retirement age of 65 income levels are zero. To follow a cohort, one proceeds diagonally through the graph.

Under an average wage scheme, the participant's pension forms a reflection of the salary earned during his/her career. Each year the participants accrue new pension rights by $\epsilon = 1.875\%$ of their salary and formerly accrued pension rights are indexed (or cut) — working for 40 years leads consecutively to an ambition of a pension of 75% of the

[11]For 2013 (the most up-to-date year) the CBS reports that the average income in The Netherlands for males and females was €39,600 and €23,400, respectively (http(1), December 12$^{\text{th}}$, 2015). Regarding the average pension income at the age of 65, the CBS reports average values of €17,300 and €8,400 for males and females, respectively, for the year 2012 (http(2), May 28$^{\text{th}}$, 2014).

Variable	*Males*	*Females*	*Total*
Workers (< 65)	1,279,622 (1,087,577)	1,277,190 (1,039,988)	2,556,812 (2,127,565)
Retirees (≥ 65)	400,395 (637,108)	483,236 (704,017)	883,631 (1,341,125)
Accrual rate ϵ (yearly)	1.875%	1.875%	-
Income	€38,848	€23,309	-
Pension at 65	€29,136	€17,482	-
Liabilities (× bln.)	€318	€219	€537
Funding ratio			90%

Table 3.1: Initial (deterministic) pension fund set-up (in parentheses 2060).

average wage. This ambition ensures a continuation of lifestyle after retirement for the pension plan participants. Note that the pension ambition is strongly influenced by the inflation rate, indexation, the career-profile and the accrual rate. Especially at the end of the career (62-65 years) pension rights are vulnerable for indexation, as compared to the earlier stages of life (25-28 years); since more rights have already been accrued (DNB, 2010). Hence, the accrued pension rights are gender-, time- and age-specific. The accumulated pension rights at time t for a male (female) participant of age x are denoted by $r^m_{x,t}$ ($r^f_{x,t}$). Please note that a 25 years old participant has not accrued any rights, but only at the start of age 26. The pension benefits a male (female) retiree receives at time t after the retirement age $x \geq 65$, are denoted by $b^m_{x,t}$ ($b^f_{x,t}$).

The characteristics of the initial pension-fund set-up are summarised in Table 3.1. The total liabilities L_t of the pension fund are the liabilities towards males L^m_t plus the liabilities towards females L^m_t. The liabilities for males and females include workers and retirees, and are attained by summing the individual liabilities over all cohorts $x = [25, 99]$ per gender. Hence, the total liabilities at time t are in general

$$L_t = L^m_t + L^f_t = \sum_{x=25}^{99} L^m_{x,t} + \sum_{x=25}^{99} L^f_{x,t}. \tag{3.11}$$

The individual liabilities of each cohort for males $L^m_{x,t}$ and females $L^f_{x,t}$ express "the discounted present value of future (projected) pension payments to all current male (female) pension fund participants aged x, based on the accrued pension rights up till now, adjusted by the survival probabilities and discounted back to period t", as beautifully phrased by Lekniute et al. (2014). The individual (nominal) liabilities per gender for (x) at time t follow from

$$L^m_{x,t} = r^m_{x,t} \cdot D^m_{x,t} \cdot M_{x,t},$$
$$L^f_{x,t} = r^f_{x,t} \cdot D^f_{x,t} \cdot F_{x,t},$$

where $r^m_{x,t}$ ($r^f_{x,t}$) are the accumulated pension rights at time t for males (females) (x), $M_{x,t}$ ($F_{x,t}$) is the number of males (females) (x) at time t and $D^m_{x,t}$ ($D^f_{x,t}$) is the discount factor at time t for males (females) (x). The liabilities to the near-retirees are the highest, since those generations accrued the most pension rights and start receiving benefits soon for the rest of their lives. The discount factor at time t is gender- and age-specific as well. The (nominal) discount factor is retrieved from the survival probabilities and the (nominal) term structure of interest rates, and specified as

$$D^m_{x,t} = \sum_{k=(65-x,0)^+}^{99-x} \frac{{}_k s_{x,t}{}^m}{\left(1+r_t^{(k)}\right)^k},$$

$$D^f_{x,t} = \sum_{k=(65-x,0)^+}^{99-x} \frac{{}_k s_{x,t}{}^f}{\left(1+r_t^{(k)}\right)^k},$$

where ${}_k s_{x,t}{}^m$ (${}_k s_{x,t}{}^f$) is the k-year conditional survival probability at time t of a male (female) agent (x) and $r_t^{(k)}$ denotes the nominal interest rate with maturity k from the nominal term structure at time t in the market. For this reason, the liabilities are valued in a nominal market-consistent way; note that I am able to value my liabilities under $\mathbb{Q}$, since I disregard longevity risk (which is actually not priced on the market, because I assume absence of longevity-linked bonds). The conditional survival probability denotes the probability that a participant is alive at the time of maturity k of the cash-flow, given that the participant is alive at time t.[12] For $x \geq 65$, the pension payments last at most till the beginning of the year in which the retiree dies, thus for a period of $99 - x$ years. For $x < 65$, the present value of the benefits depends on whether the participant reaches retirement or not. Hence, there will be no pension payments if the worker dies before the retirement age 65, otherwise the pension payments start in $65 - x$ years and continue (at most) for $99 - x$ years. The discount factor is, subsequently, the highest for near retirees, and the lowest for young workers and old retirees — as also the case for the liabilities. Namely, the young have many years ahead before pension payments have to be paid (and also investment returns can still be made), while the old retirees receive pension benefits for a remaining relatively short period of time (and the lower survival rates at those higher ages decrease the value of the liabilities even more). The fund invests in two assets: a stock and a (nominal) 5-year, default-free zero-coupon bond. The net return on the (nominal) 5-year bond is determined by the percentage difference in price between a 5-year bond at time t and a 4-year bond at time $t + 1$. In other words, the fund follows an active trading strategy: it buys a 5-year bond now, holds it for 1 year and then sells it against the prevailing price in the market. Quantitatively, the net return on a 5-year bond at time t is determined by $R^f_t = 1 + (P_t(5)/P_{t-1}(4))$, where $P_t(T)$ is the price of the bond at time t with maturity T and follows from the bond pricing formula in equation (3.8). Thus, at time $t-1$ the bond with maturity $T = 5$ was bought at the price P_{t-1} and sold at time t against the price P_t with a maturity of $T = 4$. For the stock, the net return is computed in a similar way; namely by $R_t = 1 + (S_t/S_{t-1})$, where S_t is the stock price at time (i.e. year) t following from the Black-Scholes model under $\mathbb{P}$, see equation (3.1a). The fund trades again actively and gains from the equity premium (since it operates in the objective real-world measure). The investment strategy is such that each year a constant fraction $\phi = 50\%$ is invested in the stock and a constant fraction in the bond $1-\phi = 50\%$; being a representative investment policy for a pension fund. Moreover, Goossens (2015) shows that a naïve diversification strategy outperforms optimising diversified portfolios, such as the mean-variance strategy. Besides, this investment mix is in line with the used

[12] Note that the survival probabilities in the data-set of the CBS have to be converted to conditional survival probabilities. That is, the cumulative survival probability corresponding to the cash-flow maturity is divided by the cumulative survival probability corresponding to the time period in which the present value is calculated, per cohort.

academic papers for discontinuity and in accordance with the reaction functions. The portfolio return R_t^p in period t on the investment portfolio is thus

$$R_t^p = \phi R_t + (1-\phi) R_t^f,$$

where R_t is the return on equity and R_t^f is the return on a risk-free 5-year zero coupon nominal bond. The portfolio returns determine the total return on the fund's invested capital, i.e. their assets.

The market value of the assets changes with the return on the investment portfolio, as well as with the received contributions and paid pension benefits. The value of the fund's assets at the beginning of next year, A_{t+1}, depends on the asset value of this year multiplied by the portfolio return plus the net money inflow times the portfolio return; the net money inflow equals the difference between the received contributions and paid benefits. So, the market value of the assets follows from

$$A_{t+1} = A_t R_t^p + \left\{ \sum_{x=25}^{64} \left(c_{x,t}^m M_{x,t} + c_{x,t}^f F_{x,t} \right) - \sum_{x=65}^{99} \left(b_{x,t}^m M_{x,t} + b_{x,t}^f F_{x,t} \right) \right\} R_t^p,$$

where $c_{x,t}^m$ ($c_{x,t}^f$) are the contributions paid by working males (females) of age $25 \leq x \leq 64$ and $b_{x,t}^m$ ($b_{x,t}^f$) are the received pension payments by retired males (females) of age $65 \leq x \leq 99$ (at time t). The contributions (as percentage of the salary $w_{x,t}^m$, $w_{x,t}^f$) are the same for every cohort per year, while the benefits differ per age per year (also depending on the policies in the past).

Thus, having the assets and liabilities at hand, the resulting nominal funding ratio — assets A_t over nominal liabilities L_t — follows from:

$$FR_t = \frac{A_t}{L_t}.$$

This is in line with the Dutch regulation, where pension funds have to report nominal funding ratios, and are called under- or overfunded in terms of the nominal funding ratio (Siegmann, 2011). A pension fund's financial position is largely reflected by the coverage ratio. The real funding ratio is obtained by discounting against the real term structure of interest rates, which influences the value of the liabilities because it takes the price inflation into account. The funding ratio in real terms, then, follows by

$$FR_t^R = \frac{A_t}{L_t^R},$$

where L_t^R denotes the value of the real liabilities at time t. The discount factor changes (more specifically $r_t^{(k)}$, since the nominal term structure is replaced by the real term structure of interest rates.

The next subsection specifies the funding policy of the pension fund. A funding policy prescribes how the fund sets premium levels and applies indexation.

3.2.3 Policy

The policy rules specify how the contributions are set and whether the participants receive indexation or cuts, given the financial position of the fund. The pension funds in The Netherlands maintain a hybrid DB-system: pension contributions (i.e. premium) and

⋮	⋮
140%	Full indexation Lower premium Surplus sharing, smoothing period 5 years
140%	Full indexation Cost-covering premium
130%	Linear indexation Cost-covering premium
110%	No indexation Cost-covering premium
105%	No indexation Cost-covering premium Sustainability cut, smoothing period 10 years
95%	No indexation Recovery premium Sustainability cut, smoothing period 10 years
90%	No indexation Recovery premium Sustainability cut, smoothing period 10 years Recovery plan, smoothing period 10 years
⋮	⋮

Table 3.2: Policy of the pension fund depending on the nominal funding ratio

pension benefits (i.e. indexation) are conditional upon the nominal funding rate. The pension system, therefore, is nor purely DB nor purely DC. The financial sustainability of the pension fund is assessed with the aid of the funding ratios and the policies that influence the future funding ratios. The policy instruments of the pension fund are summarised in Table 3.2 and are sequentially explained in detail below. The considered policy is one of the many ways a fund might operate in The Netherlands, nevertheless there is a wide variety in how funds operate and set there instrument-boundaries (DNB — http(3), July, 2015).

Hence, in line with my notational convenience, the indexation and premium levels relevant for year t are based upon the funding ratio at the beginning of year t. Note that indexation applies to all participants in the system (actives and retirees), while the contribution requirements only hold for actives.

Firstly, the premium regulation is presented and, afterwards, four indexation instruments are shown; the four instruments together determine the total level of indexation i_t in period t. Remark that the premium instrument could be a trigger for the young generations to not participate in the fund, because they must pay too high percentages of their salaries. On the other hand, less indexation could be a trigger for the near-retirees to react, since they feel the effect of less indexation in their pension rights because they accrued the most rights. The effects of the policy instruments are investigated later, by means of generational accounting based on value-based ALM.

Premium

The pension fund charges their participants a uniform pension premium across generations which might be different through time; being the result of the "doorsneesystematiek". The premium is expressed as percentage of the participant's wage. Active participants (i.e. workers) pay a pension premium in year t which depends on the nominal funding ratio at the beginning of year t.

The basic premium for year t is based on the cost-covering premium, which is solved in such a way that the contributions in year t equal approximately the accrued pension rights in year t.[13] Therefore, I solve the following for $premium_t^c$ at time t:

$$premium_t^c \cdot \sum_{x=25}^{64} \left(w_{x,t}^m \cdot M_{x,t} + w_{x,t}^f \cdot F_{x,t} \right) = \epsilon \sum_{x=25}^{64} \left(w_{x,t}^m \cdot D_{x,t}^m \cdot M_{x,t} + w_{x,t}^f \cdot D_{x,t}^f \cdot F_{x,t} \right),$$

where the left hand-side equals the contributions during working life and the right hand-side equals the discounted newly accrued pension rights per year during working life. The variables keep the same interpretation as stated in the previous subsection. Hence, the cost-covering premium at time t equals

$$premium_t^c = \frac{\epsilon \sum_{x=25}^{64} \left(w_{x,t}^m \cdot D_{x,t}^m \cdot M_{x,t} + w_{x,t}^f \cdot D_{x,t}^f \cdot F_{x,t} \right)}{\sum_{x=25}^{64} \left(w_{x,t}^m \cdot M_{x,t} + w_{x,t}^f \cdot F_{x,t} \right)}, \tag{3.12}$$

where the superscript c indicates that the premium is cost-covering.

To let the young generations participate in contributing to a healthy fund and not only the elderly, the fund adopts a contribution rate strategy that depends on the (nominal) funding ratio. The premium level is the highest when the financial position is 'bad' (i.e. the funding ratio is low); which are (likely) also the states of the economy where paying a high premium is actually not desirable. Premiums that help recover during a financial unsustainable period are called recovery-premia. When the financial position is 'good' (i.e. the funding rate exceeds the threshold of 140%), the premium level for the (active) participants is lessened. If the fund is performing reasonably well, the fund charges the regular cost-covering premium. However, the fund never charges more than 30% premium of a participant's salary. Therefore, the contribution rate strategy boils down to (based on an idea from Wernekinck (2013))

$$premium_t = \begin{cases} premium_t^c + 0.05 & FR_t \leq 95\% \\ premium_t^c & \text{for } 95\% < FR_t \leq 140\% \\ premium_t^c - 0.05 & FR_t > 140\% \end{cases}, \tag{3.13}$$

where the maximum of the uniform $premium_t$ is 30%. Recovery contributions restore the funding position and, therefore, help in sharing financial market risks and inflation risks. However, if future generations become unwilling to pay recovery contributions (while the current generations did pay them) or get contribution 'holidays' (while the current generations did not), cuts or bonuses become more likely in the future. Such a situation is actuarially unfair and creates incentives to avoid/leave the pension contract, introducing discontinuity risk for participants that rely in the future on recovery contributions of new generations. Recovery premiums yield inter-generational risk, which in itself works welfare enhancing (Bovenberg and Mehlkopf, 2014).

[13] In Dutch: "kostendekkende premie".

But, as advocated in Kocken (2008), the embedded option of a variable contribution is likely to vanish. Namely, the contributions to the fund are very small compared to the total assets of the fund. Charging recovery premiums when the actives are only a small portion of the total participants has little impact. The pension fund is maturing quickly, as shown by the demography development. In a pure funded DB scheme with promised guaranteed benefits, the young bear the market and inflation risks in case contributions are adjusted to restore funding.

The upcoming four paragraphs describe the indexation instruments, which together summed up lead to the total amount of indexation i_t at time t (either positive or negative).

Conditional Indexation

First of all, in order to protect the fund's participants against economic-wide price-inflation, the pension fund has the ambition to indexate the accrued pension rights of their participants (actives as well as retirees). It is an ambition and, thus, not a guarantee. If pension rights get full indexation, they keep up with the price-inflation in the economy and participants do not lose any purchasing power — i.e. the value of the pension rights now is the same as in the future (e.g. the fund ensures that you can buy an equal amount of hamburgers today or over 40 years). Indexation, however, is conditional on the (nominal) funding level of the pension fund. The level of (conditional) indexation for year t is determined by the nominal funding ratio at the beginning of year t:

$$index_t^1 = \begin{cases} 0 & FR_t \leq 110\% \\ \frac{FR_t - 110\%}{130\% - 110\%} \cdot \omega_t & \text{for } 110\% < FR_t \leq 130\% \\ \omega_t & FR_t > 130\% \end{cases} .$$

Hence, if the fund is in a too low funding situation $\leq 130\%$, the pension rights do not keep up with the inflation rate since no indexation is given. The pension rights in real terms decrease with the inflation rate. Providing no indexation means that the pension payments and liabilities are less, which contributes to a better financial position. On the other hand, if the funding ratio exceeds the threshold of 130%, full indexation is granted and the pension rights do not decrease in value with respect to the future. In times of financial prosperity, the fund is able to provide full indexation. In between those boundaries, a linear rule determines the fraction of indexation provided to the participants. Please note that a pension without any indexation is actually half a pension (Janssen, 2012). Pensioners immediately feel the effect of no indexation, while workers do not (but it is for sure not beneficial, if their pension accrual does not keep up with inflation rates).

So, indexation conditional on the performance of the fund lets all generations (young and elderly) share in risk, such as financial-market risks and inflation risks. The old are most vulnerable to the latter type of risk, whereas the young are less vulnerable. In a pure Pay-As-You-Go (PAYG) DB system with indexed benefits, the young provide a guarantee against inflation risk to the old. While in a pure funded DB with conditional indexation, the old participate in financial-market risk.

Surplus Sharing

A second positive instrument of the pension fund is the surplus sharing feature, which is also known as catch-up/recovery indexation. It lets all participants share in the surplus of the fund during financially prosperous times. In case the funding ratio exceeds the threshold of 140%, the fund gives (i) missed indexation to their participants and (ii) additional bonuses to the participants. Missed indexation (i.e. deficiencies) emerges because no indexation could be given in the past or pension benefits have been cut. Surplus sharing works in the follow manner:

$$index_t^2 = \begin{cases} 0 & FR_t < 140\% \\ \frac{1}{n_{sur}} \cdot \frac{FR_t - 140\%}{140\%} & FR_t \geq 140\% \end{cases},$$

with a smoothing period of $n_{sur} = 5$ years. The smoothing period ensures that not only the current generations benefit from the well-being of the fund, but also the upcoming generations. That is, the surplus is smoothed over 5 periods instead of giving all the surplus to the current cohorts. Due to smoothing periods, retirees that die within 5 years may not benefit from all the surplus (which is actuarially unfair). Because of surplus sharing the total indexation levels can be higher than the wage growth ω_t. The instrument gives recovery indexation to all participants, if the indexation rule prescribes this — i.e. whether or not you have a deficiency from past incomplete indexation, you receive recovery indexation. This is done for performance reasons with respect to my model. Note that lower premia, (full) indexation and surplus sharing can be activated simultaneously (the funding ratio must exceed 140%).

Recovery Plan

Thirdly, I discuss a negative adjustment mechanism: the recovery plan. The recent financial crisis decreased the value of the assets and, combined with low interest rates, led to a drop in funding ratios. The low funding ratios indicate that the fund is underfunded and extra regulatory measurements are necessary to bring the fund back to a sustainable situation. Namely, a pension fund must always have enough liquidity to pay the pensions. The Financial Assessment Framework, which is part of the Pensions Act, sets out some requirements for the financial position of the fund. If the coverage ratio drops below the fixed financial position, because of e.g. adverse market conditions, a recovery plan is started.[14] A recovery plan gives a negative percentage of indexation such that it brings the funding ratio immediately back to the fixed value of 90% (i.e. the pension entitlements are cut). I consider an annual (i.e. yearly) recovery plan. Retirees feel the effect of a cut in pension entitlements immediately, while the young do not. The recovery plan is activated if the funding ratio drops below 90%:

$$index_t^3 = \begin{cases} \frac{1}{n_{rec}} \cdot \frac{FR_t - 90\%}{90\%} & FR_t < 90\% \\ 0 & FR_t \geq 90\% \end{cases}.$$

The smoothing period equals $n_{rec} = 10$ years, according to the maximum allowance of the new Financial Assessment Framework regulation (in Dutch: Financieel ToetsingsKader, and abbreviated to FTK). Again, the smoothing period assures that multiple generations share in bearing risk. A longer smoothing period creates more inter-generational risk sharing, however it is expected to cause more discontinuity risk. Namely, longer smoothing periods keep the fund for an extended time in the information sensitive region.

[14] The fixed financial position of the fund actually depends on the amount of investment risk and the average age of a participant in the fund. If there is a shortfall in the funding position (i.e. the coverage ratio is less than 105%), the fund must submit a recovery plan to the Dutch National Bank (DNB) whereby the fund must explain how it attains the 105% level within at most 10 years.

Sustainability Cut

The fourth instrument is a last matter of resort. If the funding ratio is below the threshold of 105% (as advocated in the FTK) for six consecutive years, then the sustainability cut restores the funding position to a value above 105%. The difference between the current funding ratio and the 105% level determines the size of the cut. A sustainability cut depends actually on the so-called policy funding ratio ("beleidsdekkingsgraad"), but since our model is already specified on a yearly basis, the sustainability cut is triggered if the fund is in six subsequent time periods underfunded. So, it follows from

$$index_t^4 = \begin{cases} \frac{1}{n_{cut}} \cdot (FR_t - 105\%) & \text{if } FR_{t-5} < 105\%, FR_{t-4} < 105\%, ..., FR_t < 105\% \\ 0 & \text{otherwise} \end{cases},$$

with again a smoothing period of $n_{cut} = 10$ years, according to the maximum allowance of the FTK-regulation. Remark that the sustainability cut is a more rigorous measure compared to the recovery plan. Namely, the sustainability cut provides negative indexation in a way that the financial position next year is brought back to a level above 105% (though smoothed). E.g. if the current funding ratio is 80%, the benefits are cut (smoothed) by 2.5%; while the recovery plan reduces the benefits by a fraction of $\frac{1}{9}$ and then smoothed. Furthermore, the sustainability cut has the ambition to operate once in the six years, while the recovery plan is yearly. The recovery plan and sustainability cut, along with recovery premia, can act simultaneously in case the coverage ratio is below the level of 90% and the fund is below the 105% level for six consecutive years.

Summing the four indexation mechanisms for each period t amounts to the total level of indexation i_t provided by the fund towards its participants. So, the total amount of indexation at time t boils down to the following sum:

$$i_t = \sum_{i=1}^{4} index_t^i. \tag{3.14}$$

During the following, the situation of dynamic in- and outflow is explained.

3.2.4 Dynamic In- and Outflow

The above computations relate to the static situation wherein no discontinuity arises. In the case discontinuity is taken into account, dynamic in- and outflow affects the financial position of the fund. Discontinuity creates a dynamic instead of static population, therefore all the computations that involve population numbers are affected. Firstly, I describe how discontinuity risk impacts the in- and outflow and, consequently, how dynamic in- and outflow affects the liabilities and assets.

Based on the reactions functions presented in Chapter 2 (see equations (2.1) and (2.2)), the in- and outflow of the participants is regulated, who make their decisions on the nominal funding ratio FR_t. Recall that the reaction function for age x determines the level of discontinuity $d_{x,t}$ for a participant of age x at time t; $d_{x,t}$ specifies for new entrants of age $x = 25$ how many do not flow in the fund, while for current actives of age $x \geq 26$ it determines how many flow out the fund. For example, if $d_{25,2030} = 1$ nobody of age 25 enters the contract in 2030, while $d_{45,2030} = 0.6$ shows that 60% of the 45-year-olds leaves the fund in 2030. It is assumed that males and females behave similarly, so no distinction in discontinuity risk between genders. Once abstained or opted out from the collective pension contract, it is not possible to enter the collective fund anymore; the participant stays in the individual scheme for the remaining years.

The recursive processes in equations (3.10a) and (3.10b) stay the same, however the discontinuity factor $d_{x,t}$ affects the population of participants in the fund. So, the number of participants in the dynamic case $M^d_{x,t}$ ($F^d_{x,t}$) for each age x at time t follows from

$$\begin{aligned} M^d_{x,t} &= M_{x,t} \cdot d_{x,t}, \\ F^d_{x,t} &= F_{x,t} \cdot d_{x,t}, \end{aligned}$$

where $d_{x,t}$ stems from the prevailing funding ratio at the beginning of year t. A new entrant (or current participant) determines at the beginning of year t — based on the published nominal funding ratio FR_t — whether it enters (or leaves) or not. Hence, $M^d_{x,t}$ ($F^d_{x,t}$) denotes the participants that remain in the fund.

Consequently, the total liabilities L^d_t of the fund are affected as compared to the static setting (i.e. without discontinuity). Namely, the individual liabilities in case of discontinuity are calculated as follows

$$\begin{aligned} L^{d,m}_{x,t} &= r^m_{x,t} \cdot D^m_{x,t} \cdot M^d_{x,t}, \\ L^{d,f}_{x,t} &= r^f_{x,t} \cdot D^f_{x,t} \cdot F^d_{x,t}. \end{aligned}$$

The total liabilities L^d_t in case of discontinuity are calculated similarly as stated in equation (3.11), furthermore the pension rights and discount factors are not different than in the static case.

The market value of the assets changes also, since participants that opt out of the fund transfer their discounted accrued pension rights to the individual fund. The amount to be transferred can be transitioned in two ways: (1) as if the coverage rate is 100% (no deficit or surplus inherited), or (2) conditional upon the prevailing coverage rate. For the fund, the transfer is an additional payment. The additional payment a^i_t at time t for transfer method $i = 1, 2$ follows from

$$a^1_t = \sum_{x=26}^{64} \left(r^m_{x,t} \cdot D^m_{x,t} \cdot M^{d^*}_{x,t} \right) + \sum_{x=26}^{64} \left(r^f_{x,t} \cdot D^f_{x,t} \cdot F^{d^*}_{x,t} \right), \tag{3.16a}$$

$$a^2_t = FR_t \cdot \sum_{x=26}^{64} \left(r^m_{x,t} \cdot D^m_{x,t} \cdot M^{d^*}_{x,t} \right) + \sum_{x=26}^{64} \left(r^f_{x,t} \cdot D^f_{x,t} \cdot F^{d^*}_{x,t} \right) \tag{3.16b}$$

where $M^{d^*}_{x,t}$ ($F^{d^*}_{x,t}$) denotes how many participants leave the fund of age x at time t. Note that only current generations of $x \geq 26$ years and older can leave the fund; also, retirees of age $x \geq 65$ cannot switch anymore. For this reason, the sum runs over $x = [26, 64]$. The number of participants that leaves the fund $M^{d^*}_{x,t}$ ($F^{d^*}_{x,t}$) at time t of age x is easily observed by taking the difference between the number of participants of age x last year in the fund at time $t-1$ and at time t.

The additional payment is comparable to extra pension payments and has to be subtracted subsequently from the total capital of the fund. The assets are therefore rebalanced each period according to

$$A^d_{t+1} = \left\{ A_t + \sum_{x=25}^{64} \left(c^m_{x,t} M_{x,t} + c^f_{x,t} F_{x,t} \right) - \sum_{x=65}^{99} \left(b^m_{x,t} M_{x,t} + b^f_{x,t} F_{x,t} \right) - a^i_t \right\} R^p_t,$$

where a_t^i equals one of the transfer options above for $i = 1, 2$ and the other determinants have the same interpretation as before.

Finally, the coverage rate (assets over liabilities) can be computed, both for the nominal-valued liabilities or real-valued liabilities. Discounting against the real term structure of interest rates changes nothing about the computations above, although participants may react differently on a real funding rate than a nominal funding rate.

In summary, this section set out the pension fund characteristics for the static and dynamic setting. Firstly, the development of the population of participants was described by introducing the mortality probabilities. Secondly, the main assumptions underlying the fund were presented. I showed how the fund calculates its liabilities and assets, which cling respectively on the discount factors and investment strategy. Thirdly, the funding policy with a premium steer and four indexations mechanisms was shown. Certain instruments provide incentives for cohorts to stay in or leave the fund. At last, I described the differences between the static and dynamic setting. Specifically, the in- and outflow is calculated differently and the market-value of the assets is directly influenced.

All the above formulae are presented for one scenario $s = 1$ only, but can easily be generalised to more scenarios, however the notation becomes a bit more involved. For simplicity, I have omitted the superscript s for the scenarios in my notation.

The next chapter sketches how the model works and it evaluates the outcomes in a classical ALM and value-based ALM approach.

Chapter 4

Data & Evaluation

This chapter describes the time-line of the simulation and introduces the parameters for the model. Besides, it presents methods to measure pension outcomes and shows the static funding ratios.

4.1 Simulation

The time-line of the simulation is split in two parts, as shown in Figure 4.1: (i) a deterministic period and (ii) a stochastic period. The deterministic period initialises the pension-fund settings before and for the stochastic simulation period. Namely, if the simulation starts in 2015, then a participant of 75 years old has accrued rights in the years before 2015 (i.e. before the stochastic simulation starts). The functioning of the deterministic and stochastic period is explained below.

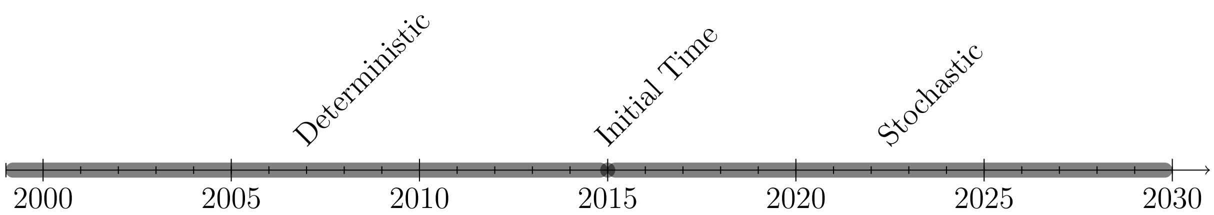


Figure 4.1: Time-line of the simulation.
The deterministic period actually starts in 1940 and the stochastic period ends in 2089.

However, before the deterministic period starts its simulation, the financial market is initialised by the Black-Scholes-Vasicek model. Normally, to program the differential equations, time is discretized and samples are calculated at time-steps of Δt. However, this Euler-approximation method is naïve. As Gillespie (1996) points out, the discrete method is only valid for a sufficiently small Δt. To overcome this problem, I use exact numerical simulation, which holds for any time-interval Δt. The Ornstein-Uhlenbeck processes for the Vasicek model under the real-word measure and risk-neutral measure become, respectively:

$$r_{t+1} = r_t \cdot e^{-\alpha \Delta t} + \kappa \left(1 - e^{-\alpha \Delta t}\right) + \sigma_r \sqrt{\frac{1 - e^{-2\alpha \Delta t}}{2\alpha}} dW^{\mathbb{P}}_{r,t},$$

$$r_{t+1} = r_t \cdot e^{-\alpha \Delta t} + \left(\kappa - \frac{\lambda_r \sigma_r}{\alpha}\right) \left(1 - e^{-\alpha \Delta t}\right) + \sigma_r \sqrt{\frac{1 - e^{-2\alpha \Delta t}}{2\alpha}} dW^{\mathbb{Q}}_{r,t},$$

where the parameters and variables have the same interpretation as before, however this approach is continuous instead of discrete.

4.1.1 Deterministic Period

The deterministic period is a deterministic-simulation, i.e. it contains no randomness. Firstly, it initialises the accrued pension rights for each cohort in the fund. It is assumed that all the accrued pension rights during the deterministic period are fully indexed for wage growth, equalling the price-inflation during the deterministic period.[1] Secondly, based on the pension rights, it computes the value of the liabilities at time zero L_0 (i.e. 2015). Then, given an initial funding ratio FR_0, the asset value A_0 is determined. The initial values of the liabilities and funding ratio are given in Table 3.1 for the static situation. The asset value at time zero A_0 follows from the relationship

$$A_0 = FR_0 \cdot L_0.$$

Hence, the value of the assets and liabilities at time zero are the same for each scenario — therefore, also the funding ratio.

4.1.2 Stochastic Period

After the initial state of the fund has been set, the stochastic simulation begins. Each year t the model proceeds in the following manner.

First of all, given the initial state of the pension fund, the premium and indexation levels are determined at the beginning of period t (a year). That is, the premium and indexation level for period t are based with respect to the funding ratio at the beginning of period t. Secondly, the number of participants that enter/leave the fund is determined based on the prevailing funding ratio and corresponding discontinuity function. Thirdly, the fund receives the premium contributions of the active participants and pays the pension benefits to the retirees. Then, the assets are invested and the asset value for the next period $t + 1$ is computed; based on the investment strategy as well as the financial market, premiums, pension payments, population size and possible transfer payments (for switching from collective DB towards individual DC). Due to discontinuity risk, funding ratios can become very low and combined with continuous smoothing of shocks (at low funding rates) the assets of the fund might be depleted completely; if this happens, the fund goes bankrupt.[2]

Next, the model proceeds to the end of period t and calculates for each age group the new accrued pension benefits, based on their salaries and (uniform) accrual rate. Those new entitlements are added to the existing rights and the rights are indexed (the indexation level is determined at the start of period t). Consequently, the model computes the liabilities of the fund, which leads to a new funding ratio. Note that the total liabilities L_t at time t 'lag' one period with respect to the discontinuity factor. That is, at time t the fund computes it liabilities L_{t+1} for next year based on the population of the fund corrected with the survival probabilities; namely, the fund cannot foresee how many participants possibly leave the fund at the beginning of next year. If a fund goes default (due to complete depletion of assets), it also reduces all the liabilities in such a way that they

[1] A same approach is followed by Lekniute (2011).

[2] Note that it actually is more realistic to change the policy rules if the fund is severely underfunded (i.e. no smoothing anymore).

become zero. Finally, the new funding ratio prescribes the premium and indexation levels for next year.

The stochastic horizon runs for $T = 75$ years into the future, i.e. from 2015 till 2089. For each year, the ALM-model performs 1500 Monte Carlo simulations of possible financial scenarios.[3] The Monte Carlo simulations are done under the real-world measure $\mathbb{P}$, whereby the pricing is done under the risk-neutral measure $\mathbb{Q}$ which is achieved by simulating arbitrage-free scenarios. The latter approach guarantees consistency between the risk model and the valuation model.

In conclusion, this section enumerated how a Monte Carlo simulation of the model works including discontinuity. The classical ALM output (such as coverages rates) is reported under the objective measure, while the pricing and the value-based ALM output (such as generational accounting) is done under the risk-neutral measure.

The next section introduces the model parameters.

4.2 Model Parameters

In order to be able to run the ALM-model, the parameters of the financial market analyser and pension fund analyser need to be specified. At first, the financial market is initialised and, then, the pension fund settings are summarised.

4.2.1 Financial Market

To set the parameters for the financial market, please recall that under the risk neutral measure $\mathbb{Q}$, the Black-Scholes-Vasicek model is

$$\begin{aligned}
dS_t &= r_t S_t dt + \sigma_S S_t dW_{S,t}^{\mathbb{Q}}, && S_0 = 1, \\
dr_t &= \alpha(\kappa - \frac{\lambda_r \sigma_r}{\alpha} - r_t)dt + \sigma_r dW_{r,t}^{\mathbb{Q}}, && r_0 = 0.02, \\
dI_t &= \iota I_t dt, \quad \iota \sim N(\mu, \sigma^2) && I_0 = 0.02, \\
dB_t &= r_t B_t dt, && B_0 = 1.
\end{aligned}$$

where $\lambda_r = -0.15$ is the price of risk of the innovations in the nominal short rate, and $W_{S,t}^{\mathbb{Q}}$ and $W_{r,t}^{\mathbb{Q}}$ are standard Brownian motions under the risk neutral measure for the stock and interest rate, respectively. Note that the choice for the initial values of the differential equations has been advocated in Chapter 3.

The model parameters are set as follows. The (nominal) financial market risk premium $\mu - r_f$ is set to 4%.[4] The volatility of the stock σ_S is taken to be 20%. The speed of mean reversion α is presumed to be 0.15, while the long-term average of the nominal short rate κ is taken to be 2% (i.e. the level of mean reversion). The (instantaneous) volatility of the short rate σ_r equals 1%. The inflation rate during the deterministic period is set to a fixed 2% per year, following from the historical inflation rates in the period 1995-2015 (OECD — http(5), March, 2016) and following from the ambition of the ECB to maintain an inflation rate of 2% per year. The inflation rate during the stochastic period fluctuates in a normally distributed fashion around $\mu = 2\%$, with a variance of $\sigma^2 = 1\%$

[3]Note that at each time point t for every age x the discontinuity function has to be evaluated.

[4]KPMG Netherlands advised to use a market risk premium of 5.75%, however I find this compared with the literature a bit high (http(4), April 12th, 2016).

Parameter		*Value*
$\mu - r_f$	Risk premium	4%
σ_S	Volatility of stock	20%
α	Speed of mean reversion	0.15
κ	Long-term interest rate	2%
λ_r	Price of risk	-0.15
σ_r	Volatility of interest rate	1%
ι	Inflation	$N(\mu = 2\%, \sigma^2 = 1\%)$

Table 4.1: Model parameters for the Black-Scholes-Vasicek model.

per year. The difference between an inflation rate based on a SDE with the Vasicek-model and a pseudo-stochastic inflation rate is that the latter fluctuates more severely than the former. I assume a pseudo-stochastic inflation rate for simplicity, but in such a way that it still reflects reality since it is not constant. The interest rate and inflation rate are close together, as is frequently observed in the market. The values are neatly summarised in Table 4.1.

4.2.2 Pension Fund

Besides the stationary model-parameters for the financial market, the pension fund settings — used in the ALM-model — are shortly summarised in Table 4.2.

Parameter	*Value*
Open, collective DB	
Average wage scheme	
Uniform premium & accrual	
Population	CBS data, scaled
Income	Inflation & career
S, start	25 years
R, retire	65 years
D, death	99 years (following from CBS data)
ϕ, asset mix	50% stock, rest 5-year bond
Horizon	75 years
Initial funding ratio	90%

Table 4.2: Main pension-fund characteristics.

Shortly, this section summarised the main parameters for the financial market and pension fund. The upcoming section presents methods to measure the pension outcomes.

4.3 Pension Outcome

With the results of the ALM-model, the pension contract can be evaluated by different measures. This section describes sequentially the methods which are used to evaluate the main results of my analysis. Firstly, the pension results, replacement rates and generational accounts show the outcomes of the pension contract from the participant's point of view. Secondly, the cumulative weighted indexation evaluates the outcomes more from the fund's perspective. Finally, classical ALM-output such as funding ratios, discontinuity risk and usage of policy instruments show important outcomes for policymakers.

Pension Results and Replacement Rates

In order to measure the effects of the policies of the fund on the participants, I use two widely used indicators to assess the pension outcomes: (i) the pension result and (ii) the replacement rate. The pension result at time t measures the purchasing power of the participants in period t. In other words, it shows how indexation levels in period t keep up with the economy-wide wage growth ω_t in period t and is defined as the cumulative indexation till time t over the cumulative wage growth till time t. Therefore, it assesses the pension outcomes of the active participants and does not make a distinction between genders and age — it is a uniform measure. Mathematically, the pension result at time t follows from

$$PR_t = \frac{\prod_{k=0}^{t}(1+i_k)}{\prod_{k=0}^{t}(1+\omega_k)},$$

where i_k is the fraction of indexation given in period k (as specified in equation (3.14)) and ω_k is the economic wage growth in period k. In case full indexation is granted, the pension result equals 100%. Note that the pension result might exceed 100% if surplus sharing has been active and no cuts have been provided. In my analysis, I report the mean, standard deviation and quantiles (over all scenarios) of the pension result for a cohort which starts to work in 2015 (at age 25) and deceases in 2089 (at age 99).

Next, the replacement rate assesses how the pension rights relate to the previous income of the participant. The replacement rates, since wages differ between males and females, depend on gender as well as generation and time. As pension right I take the first pension payment received at the retirement age 65. Regarding the previous income, I differentiate between two measures: (i) the last earned salary just before retirement, i.e. $w_{64,t}^m$ for males; (ii) the average earned wage during the participants' working life, i.e. $\sum_{x=25}^{64} w_{x,t}^m$ for males that started to work at age $x = 25$. Quantitatively, the replacement rates for males and females follow from

$$RR_t^{m,1} = \frac{b_{65,t}}{w_{64,t}^m} \qquad RR_t^{f,1} = \frac{b_{65,t}}{w_{64,t}^f},$$

$$RR_t^{m,2} = \frac{b_{65,t}}{\frac{1}{40}\sum_{x=25}^{64} w_{x,t}^m} \qquad RR_t^{f,2} = \frac{b_{65,t}}{\frac{1}{40}\sum_{x=25}^{64} w_{x,t}^f},$$

where $b_{65,t}$ is the pension benefit at the moment of retirement at time t and $w_{x,t}$ is the income at age x at time t. In case full indexation has been given throughout time, it holds that $RR_t^{m,1} = RR_t^{f,1} = (65-25)\epsilon\% = 75\%$ for a uniform wage profile, since indexation does not incorporate the wage growth due to a career-profile. When indexation is absent (i.e. 0% indexed benefits), it holds that $RR_t^{m,2} = RR_t^{f,2} = (65-25)\epsilon\% = 75\%$ both for a uniform and career wage-profile. That is, the replacement rate in this example shows that the participant receives 75% as a pension compared to its earned wage. Since I consider a career wage-profile, I use the second replacement rate for assessing the pension rights compared to the average income. In my analysis, I report the mean, standard deviation and quantiles (over all scenarios) of the second replacement rate measure for males $RR_t^{m,2}$.[5] I track a cohort that starts to work in 2049 (at age 25) and retires in 2089 (at age 65).

Ergo, the pension result incorporates the whole simulation horizon and, thereby, a complete life-time of 75 years. While the reported replacement rate incorporates the last part of the simulation horizon only and, thereby, assesses how participants at the end of simulation keep up with their benefits compared to previous earned wages. The pension results and replacement rates are computed under the real-world measure. Note that the pension result does not consider the effect of negative indexation on the accrual of pension rights (which has a different effect on the young than the elderly) and it excludes the effect of full indexation during the deterministic period (for older cohorts); whereas the replacement rate includes both. For this reason, both measures are provided.

Value-Based ALM: Generational Accounts

Another measure indicates a sort of individual 'funding ratio'. Namely, generational accounts assess and value the cash-flows throughout the lifetime of a generation. That is, it records how many premiums the cohort pays and how much benefits they receive, discounted to a specific point in time. The measure is known as generational accounting and it is helpful in understanding how certain pension schemes affect the participants in the fund, which might not be revealed by the other measures. For example: a replacement rate can show excellent results, but maybe the participant has paid a lot of recovery contributions, which is not taken into account by replacement rates. Generational accounting basically values the pension deal and, thus, gives a more complete representation. For this reason, it is a tool that can be used to show how certain policy instruments affect specific cohorts. Since generational accounting is a form of pricing, it is required to work under the risk-neutral measure (Kortleve et al., 2006).

I use the approach as suggested by Lekniute (2011) and report in my results the generational accounts for generations that start in the initial period 2015 (i.e. $t = 0$). The generational accounts give a value of the pension contract for a representative member of each cohort of age x and is therefore gender specific. The generational account for males of age $x < 65$ at time t is as follows

$$V_t^x = \mathrm{Q}_{0.5}\left(GA_t^x\right) = \mathrm{Q}_{0.5}\left[\sum_{k=x}^{64} - \left({}_{t+(k-x)}s_{x,t}{}^{m} c^m_{k,t+(k-x)} \prod_{j=t}^{t+(k-x)} \frac{1}{R_j^f} \right) \right.$$
$$\left. + \sum_{k=65}^{99} \left({}_{(t+65-x)+(k-65)}s_{x,t}{}^{m} b^m_{k,(t+65-x)+(k-65)} \prod_{j=t}^{(t+65-x)+(k-65)} \frac{1}{R_j^f} \right) \right], \tag{4.3}$$

where all the variables and determinants have the same interpretation as previously. Obviously, it values the contributions and benefits by appropriate discounting with respect to the survival probabilities. $\mathrm{Q}_{0.5}$ indicates that the median (i.e. 50%-quantile) is taken over all the simulated scenarios. Hereby, I differ from the approach of Lekniute (2011), since I perceive the median to be a more precise measure for the effects of different pension contracts. The main advantage of the median over the mean is that it is not skewed in data-sets with extreme outliers; for this reason, it provides an idea of a 'typical' value.

[5] The replacement rates for males and females are (near) identical, therefore I report only the results for males.

For retired males of age $x \geq 65$ at time t, the pension contract is valued as follows whereby only the benefits have to be taken into account:

$$V_t^x = \mathrm{Q}_{0.5}\left(GA_t^x\right) = \mathrm{Q}_{0.5}\left[\sum_{k=65}^{99}\left({}_{t+(k-x)}s_{x,t}{}^{m}b_{k,(t+(k-x))}^{m}\right)\prod_{j=t}^{t+(k-x)}\frac{1}{R_j^f}\right]. \tag{4.4}$$

The drawback of generational accounting is that it assumes that a participant stays in the fund for the rest of their life-time, while in case of discontinuity participants might leave. I report the results for males only.

Cumulative Weighted Indexation

To measure the total amount of indexed benefits and cuts, I compute the cumulative weighted indexation. That is, the total amount of cumulative indexation provided to the participants in the fund until time t. It considers per period t the total amount of male and female participants in the fund, multiplied by the cumulative indexation provided till time t and, then, summed through time. Mathematically, it looks as follows:

$$i_t^c = \sum_{i=0}^{t}\left(\sum_{x=25}^{99} M_{x,i} + \sum_{x=25}^{99} F_{i,t}\right)\prod_{k=0}^{i}(1+i_k).$$

The quantity is normalised and expressed in Euros; thus, for €1 of start capital it quantifies the cumulative weighted indexation. The measure is helpful in analysing how much indexation the fund precisely provides, whereby it incorporates the total number of participants and the indexation fraction. Namely, it could be (due to discontinuity for instance) that less participants are in the fund (as compared to the static situation), while the funding rate prescribes that full indexation should be provided (whereas in the static situation maybe cuts are necessary). So, this provides a way to see whether the policy rules work properly; the values are computed under the real-world measure and I report the mean value after 75 years.

Classical ALM

Finally, I report general statistics as is frequently done in classical ALM-analysis. Concerning the funding ratio, the results are reported after 75 years of simulating. It entails the mean, standard deviation, (mean) probability of underfunding and 5%-, 50%- and 95%-quantiles. The (mean) probability of underfunding after 75 years is defined as the underfunded scenarios divided by the total amount of scenarios. Besides, I also include a new measure in the classical ALM-output: characteristics of discontinuity, such as the (mean) probability of default after 75 years (defined similarly as the probability of underfunding). Moreover, it contains the (mean) probability of six consecutive years no inflow, and the mean discontinuity risks with their standard deviations for the 25-, 45- and 64-year-olds (as stated in equations (2.1) and (2.2)) — depicting the three important stages in life. The (mean) probability of six consecutive years no inflow is attained by analysing per scenario how often no inflow occurs and, then, the mean and standard deviation over all scenarios is computed. Moreover, the probabilities of usage of the steering mechanisms during the 75 years is shown: indexation and premium regulation. The mean (with the standard deviation) probability of usage for each instrument is calculated as follows: analyse per scenario when the instrument is used and, consequently, take the mean and standard deviations over all scenarios. I correct for the cases where the fund goes default; that is, if a fund goes bankrupt, all the mechanisms are switched off and the remaining years left in the simulation do not count for the probability of usage. Lastly, the mean number of remaining participants after 75 years is reported along with the standard deviation.

Concluding, this section listed some measures to evaluate the results of the ALM-model. The pension outcomes are assessed through the pension result, replacement rates, generational accounting, cumulative weighted indexation and classical ALM-output. The latter two measures are more from the fund's perspective, while the former three are more from the participant's point of view. The pension results, replacement rates, cumulative weighted indexation and classical ALM-output are commputed under $\mathbb{P}$. The value-based ALM output is produced by generational accounting and, hence, calculated under the risk-neutral measure $\mathbb{Q}$.

The next section shows the results for the current static approach, i.e. without discontinuity.

4.4 Funding Ratios: Static

Since everything is initialised, the basic results can be shown; that is, the funding ratios in case of static in- and outflow. The in- and outflow does not depend on discontinuity risk, but only on the mortality tables (i.e. new generations enter the market and elderly decease). Therefore, this section presents the ALM-results with certain inflow and functions as benchmark. In other words, the static results function as reference to evaluate the consequences for the fund in case of dynamic in- and outflow.

Throughout the thesis, the presentation of the results for each situation (static, dynamic inflow, dynamic in- and outflow) is as follows: (i) development of the funding ratios, (ii) table with ALM-ouput (pension results, replacement rates, cumulative indexation and classical ALM-output) and (iii) generational accounts based on value-based ALM.

Funding Ratio

Figure 4.2 shows the development of the funding ratio throughout time in nominal terms. The 5%- and 95%-quantiles show the confidence bands of the funding rates. In other words, the 5%-quantile shows the scenario whereby 5% of the data shows equal or lower results; the 95%-quantile shows the scenario whereby 95% of the data is worse. During the first decade the fund recovers from underfunding and after 2040 it stabilises and reaches its equilibrium whereby the coverage rate fluctuates just above $FR = 120\%$; this is the steady-state of the fund. The fund recovers from the underfunded position due to the functioning of the two main steering mechanisms, which are shown in Figure 4.3: the premium and indexation instruments. Active participants pay (on average over $T = 75$ years) a cost-covering premium of 18.39%. The plot shows the actual charged premium level to the participants which follows from equation (3.13). The premium levels decrease over time, since initially the premium is high due to the underfunded position in 2015, but throughout time the fund recovers and no recovery premia are necessary anymore. Especially, after 2040 the fund reaches a stable financial equilibrium and, therefore, the premium stabilises. Demography has also some impact on the levied premium level (by means of changes in the cost-covering premium), but the effect is small: till 2040 the

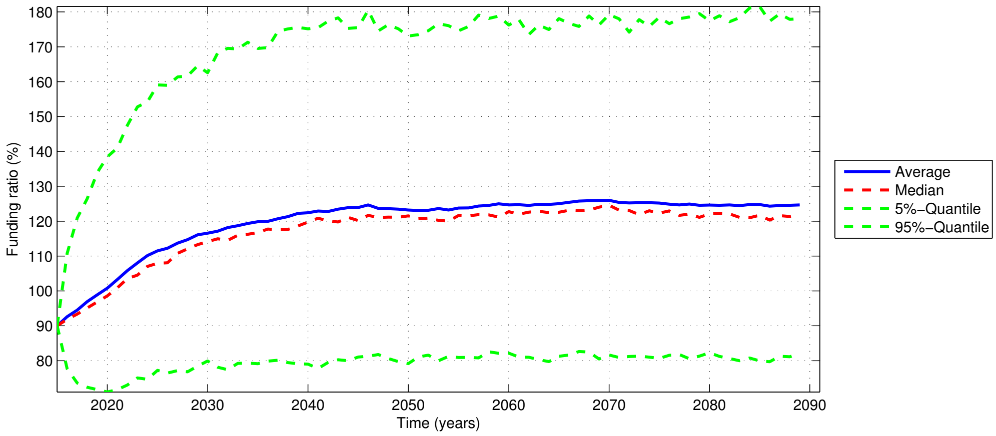


Figure 4.2: Funding ratios: static.
The x-axis shows time (years) and the y-axis the corresponding nominal coverage rate under the real-world measure $\mathbb{P}$.

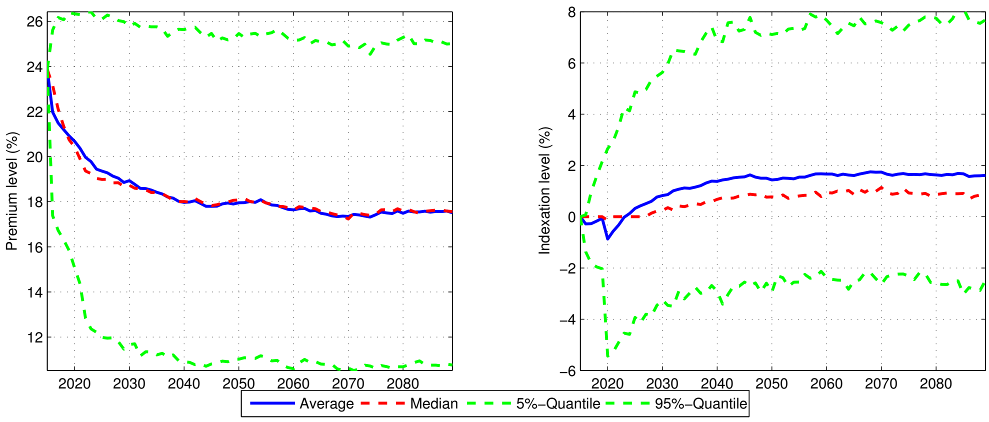


Figure 4.3: Premium and indexation levels.
The x-axis displays time (in years) and the y-axis shows in percentage points the level of premium charged and the amount of indexation provided.

number of workers decreases, so the premium is somewhat higher since there are less workers compared to retirees. The (average) indexation level i_t shows an upward trend, but decreases sharply at 2020. This is the sustainability cut that comes in, because if the fund is six consecutive years underfunded this mechanism trades in. Specifically, at the 5%-quantile it is recognised that severe sustainability cuts take place, after recovery plans have been activate before 2020. After the first decade, indexation levels become positive for the median and mean values. The 95%-quantile exceeds the threshold of the random inflation rate of 2% as a consequence of surplus sharing. The surplus is accrued due to higher returns on the financial market.

ALM-output

The steering mechanisms have various influences on the funding ratio. The recovery plans, sustainability cuts and recovery premia guarantee that the funding rates do not drop too low. For this reason, the 5%-quantile of the funding ratio is (only) 82.06% after

Funding Ratio		Indexation		Population	
$E[FR^N_{75}]$	124.7% (29.74%)	$E[i^c_{75}]$	€482.2 mln.	Participants$_{75}$	3,256,119 (0)
$Q_{0.05}[FR^N_{75}]$	82.06 %				
$Q_{0.5}[FR^N_{75}]$	121.8%	Pr["no indexation"]	0.420 (0.191)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	178.0%	Pr["full indexation"]	0.336 (0.190)	$E[RR^{m,2}_{75}]$	1.041 (0.398)
$\Pr[FR^N_{75} < 100\%]$	0.224	Pr["cond. indexation"]	0.243 (0.089)	$Q_{0.05}(RR^{m,2}_{75})$	0.630
		Pr["surplus sharing"]	0.238 (0.166)	$Q_{0.5}(RR^{m,2}_{75})$	0.941
Discontinuity		Pr["recovery plan"]	0.148 (0.110)	$Q_{0.95}(RR^{m,2}_{75})$	1.799
$\Pr[FR^N_{75} = 0\%]$	0.000	Pr["sustain. cut"]	0.145 (0.121)		
Pr["no inflow"]	0.000 (0.000)			$E[PR_{75}]$	0.797 (0.282)
$E[d_{25}]$	0.000 (0.000)	**Premium**		$Q_{0.05}(PR_{75})$	0.134
$E[d_{45}]$	0.000 (0.000)	Pr["low. premium"]	0.238 (0.166)	$Q_{0.5}(PR_{75})$	0.508
$E[d_{64}]$	0.000 (0.000)	Pr["rec. premium"]	0.222 (0.135)	$Q_{0.95}(PR_{75})$	2.863

Table 4.3: Outcomes static ex-ante funding ratios.
The table reports the ALM-output as explained in Section 4.3.

75 years time, as shown in Table 4.3. The recovery premium is activated in 22.2% of the time, which is higher than the other two negative indexation instruments: recovery plans and sustainability cuts. This is due to fact that recovery plans are only initialised for coverage rates below 90% and sustainability cuts only happen if the fund is below 105% for six consecutive years, while recovery premiums are charged for funding rates below 95%. Intuitively this is correct, since it lets the young share in the risk of an ageing fund. All the negative adjustment instruments (no indexation, recovery premia, recovery plans and sustainability cuts) are mainly used in the early years of the simulation. On the other hand, surplus sharing ensures that coverage rates do not rise extremely high. The positive effect of surplus sharing is noticed in the 95%-quantile of the funding level, which equals 178% (whereas it would rise high in cases without surplus sharing). Note that lower premia and surplus sharing coincide, since they are active for funding ratios above 140%.

The probability of no indexation is higher compared to the other instruments, which is caused by the initial underfunded position and the fact that conditional indexation is only provided above the funding level of 110%. Conditional indexation is an instrument that is used with more certainty than full indexation (the standard deviation is 10% lower), while full indexation occurs with a higher probability. Remark that due to the pseudo-stochastic inflation uncertainty in the indexation levels rise, since inflation levels vary per year. The instruments assure a quite constant funding level with a modest volatility of 29.74% after 75 years and a probability of 22.4% of underfunding. The fund provides €482.2 million of cumulative indexation.

Regarding the population, there are approximately 3.2 million participants left in the fund at the end of the simulation horizon — where the fund started with roughly 3.4 million participants in 2015; the decrease is due to the Baby Boom generation that deceased. Hence, the population remains quite constant throughout time. The indexation level keeps up with wage growth for 50.8% of the time as is clear from the median pension result; the pension result is 100% if full indexation has always been provided, but this is not the case since the steady-state lies at a funding ratio of 121.8% while full indexation is provided above 130% only. The upper quantile shows a level well above the full indexation level (due to surplus sharing), while the lower quantile is clearly less beneficial for the participants due to (severe) benefit reductions. The median replacement rate shows levels above 75% (i.e. the threshold if no indexation has been granted), while the 5%-quantile clearly underwent some benefit cuts and participants in the 95%-quantile enjoy from the surplus sharing. Of course, this situation involves no discontinuity (wherefore the values are zero at discontinuity) and, therefore, the standard deviation of the participants is zero.

The static outcomes of my ALM-model are comparable to the results founds by Janssen (2012) and Lekniute (2011) if similar parameters are set and similar assumptions are made.

Generational Accounts

The negative steering mechanisms contribute to a financially sustainable fund, but also lower the pension results and replacement rates for some of the unlucky participants which may lead to incentives for exiting the collective fund. On the other hand, positive instruments create incentives to stay in the fund, due to contribution 'holidays' (i.e. lower premia), full indexation and surplus sharing. To analyse the effects of the policy instruments on each generation, the interest lies in the changes in generational accounts. Hence, I measure the change in generational account by switching certain policy instruments on and off. To quantify the differences in generational accounts ΔV_t^{x*} at time t for cohort x, I subtract the benchmark value V_t^{x*} of the new pension plan value V_t^x:

$$\Delta V_t^{x*} = V_t^x - V_t^{x*}, \tag{4.5}$$

where V_t^x follows from equations (4.3) and (4.4). Since the results between male and females are quite robust, the change in generational accounts for males only is shown.

Figure 4.4 shows transitions from one policy to another policy. It is for illustrative purposes of the welfare effects that could provide incentives for opting-out of the fund or staying in the fund. I calculate the generational accounts for cohorts starting in the initial time period 2015. I proceed consistently through the examination and explanation of each graph, in order to evaluate the redistribution effects and, thereby, the triggers.

The first graph shows the change in value when the pension fund switches from an unconditional indexation policy with variable premia V_t^{x*} to a conditional indexation policy with variable premia V_t^x. Obviously, conditional indexation instead of full indexation hits all the participants in the fund. But, it contributes to a more sustainable financial position, whereby future cohorts benefit (not shown in the graph). Especially the active participants during the accrual phase are hit.

The second graph presents the change in generational accounts in case the fund charges conditional indexation with fixed premiums V_t^x instead of conditional indexation with variable premiums V_t^{x*}. The fixed premium is set equal to the average cost-covering premium of 18.4%. The fixed premiums are beneficial for the young cohorts, because they do not have to pay any recovery premia during the first decades. For just retired participants of age $x \geq 65$ the effect is somewhat negative since they have to absorb the negative shocks in economic less prosperous scenarios, because contributions as a steering element dropped. The coverage rate, therefore, will be more volatile.

Thirdly, a transition from scheme two V_t^{x*} towards conditional indexation with cuts with fixed premiums V_t^x is made. The premium is kept fixed, so I am able to measure the effects of benefit reductions: i.e. recovery plans and sustainability cuts. As advocated before, the near retirees are hit the severest because they have accrued the most pension rights. Though, it has a negative effect for all generations. As in the case of conditional indexation, the funding position becomes better (as shown in Figure 4.2), but the cohorts in the early decades have to pay the bill. So, the profits are shifted towards the future generations again. The effect for the future-generations is not shown, but it is certainly

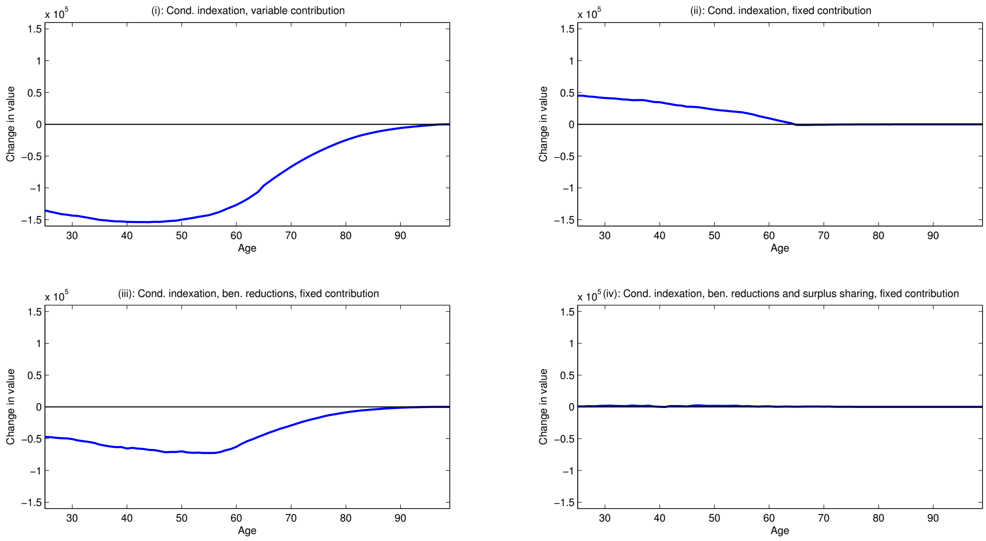


Figure 4.4: Change in generational accounts.

The x-axis presents the age of the participant and the y-axis the change in value of the generational accounts (as stated in equation (4.5)). If the graph lies above zero, the change in pension plan is beneficial, while a value below zero indicates a negative effect for the age-group. Graph 1, in the upper left corner, shows a transition from an unconditional indexation policy with variable premia to a conditional indexation policy with variable premia. Graph 2, in the upper right corner, presents a switch from conditional indexation with variable premiums to conditional indexation with fixed premia. Graph 3, in the left corner below, displays a transition from conditional indexation with fixed premia towards conditional indexation with cuts and fixed premia. Graph 4, visualises the consequences of transitioning from scheme three towards conditional indexation with cuts and surplus sharing, with a fixed premium.

there since a pension fund is a zero-sum game: i.e. one can only gain at the expense of others.

Finally, the last graph visualises the consequences of transitioning from scheme three V_t^{x*} towards conditional indexation with cuts and surplus sharing, with a fixed premium V_t^x. Hereby, the effect of surplus sharing is measured. There is a minimal positive effect for all cohorts, but it is very small because of the initial underfunded position. In case the initial funding rate is higher, the effect of surplus sharing is much stronger. The surplus sharing is beneficial for the cohorts in the fund now, but it depletes the buffer for future generations.

To sum up, the static setting seems to be stable with respect to the financial position and performs as could have been expected. Due to the underfunded position, the fund must charge recovery premiums and reduce benefits to overcome the financially "bad" position. Conditional indexation and, specifically, benefit reductions cause incentives for (elderly) participants to leave the fund in case of less prosperous economic scenarios. Recovery premia trigger new entrants to abstain from the pension contract, especially when the fund is underfunded. Surplus sharing is a positive mechanism for all age groups and provides incentives to stay in the fund, even if the funding ratios become high. Hence, the instruments contribute to a financially stable situation, but it affects generations and policymakers should bear that in mind.

The next chapter gives the main results in case discontinuity risk is incorporated in the computations.

Chapter 5

Results

This chapter answers the second research question, namely how does discontinuity risk affects the financial sustainability of the collective pension fund and how large is this risk.

Firstly, I present the situation of dynamic inflow: i.e. wherein new entrants might abstain from the pension contract. Dynamic inflow is investigated for the two entry levels 96% and 120% shown in Chapter 2: the reactions of the new participants depend on the break-even funding ratios and follow from the equations (2.1) and (2.2) for $x = 25$, respectively. Secondly, the situation of dynamic in- and outflow is discussed: i.e. whereby all generations might react to the prevailing funding rates. Dynamic in- and outflow depends on equation (2.2) for $25 \leq x < 65$. Secondly, this chapter tests the sensitivities of the main results.

5.1 Funding Ratios: Dynamic

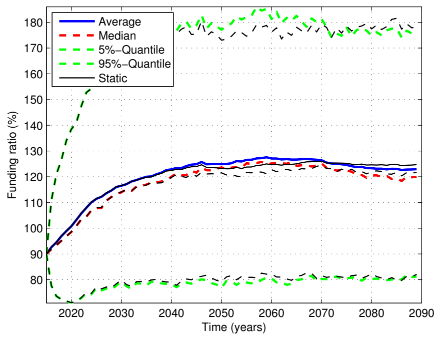
The results in this section are shown in the same systematic way as the static funding ratios: (i) funding ratios, (ii) ALM-output and (iii) value-based generational accounting. All the three manners are compared with the static results. The dynamic funding ratios are plotted together with the static funding ratios. The ALM-output is no longer shown as absolute outcomes — which are presented in Appendix A.1 — but as change with respect to the static outcomes. That is, the ALM-output is presented as

$$\Delta A^{*} = A - A^{*}, \tag{5.1}$$

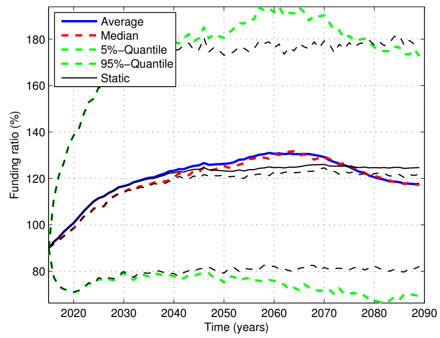
where A^{*} is the benchmark value of the ALM-ouput (i.e. the static funding rates) and A is the value for the cases whereby discontinuity is considered. Thus, it is easy to notice how the dynamic situation affects the current static pension fund outcomes. The change in value of the generational accounts is also measured against the static funding ratio, which functions as a benchmark again. That is, I compute

$$\Delta V_t^{x**} = V_t^{x} - V_t^{x**}, \tag{5.2}$$

where V_t^{x*} is the value for the static funding rates and V_t^{x} is the value for the case discontinuity is considered. Since it is required to work under the risk-neutral measure $\mathbb{Q}$ for value-based generational accounting, it is not correct to run the model under $\mathbb{Q}$ with reaction functions that hold for the real-world funding ratios under $\mathbb{P}$. Namely, under risk-neutral valuation 'bad' states of the world get a higher weight compared to 'good' states of the world; for this reason, the funding ratios under the measure $\mathbb{Q}$ are lower than



(a) Entry level $FR = 96\%$.



(b) Entry level $FR = 120\%$.

Figure 5.1: Funding ratios: dynamic inflow.

The x-axis displays time (in years) whereby the y-axis shows the nominal funding ratio in percentages under $\mathbb{P}$; as clear from the legend, the graph entails the mean and quantiles of the static (grey) and dynamic funding rates. Figure 5.1a presents the nominal funding ratio in case the entry level for new entrants is 96%. That is, if the funding ratio is below $FR = 96\%$, new entrants abstain. Figure 5.1b presents the nominal funding ratio if the entry level is higher, namely 120%.

the funding ratios under the real-world measure $\mathbb{P}$. This would imply that participants react differently if they observe coverage ratios under $\mathbb{Q}$. To avoid such an effect, I compute the dynamic in- and outflow under the real-world measure $\mathbb{P}$ with the reaction functions as presented in Chapter 2 and use these population numbers for calculating the generational values under the risk-neutral measure $\mathbb{Q}$. So, the different pension schemes can be evaluated and compared consistently. On the one hand, the consistency between risk and valuation model is assured (due to appropriate discounting); on the other hand, the participants behave consistently with respect to the objective real-world published funding ratios.

The upcoming two subsections discuss dynamic inflow, and dynamic in- and outflow respectively.

5.1.1 Dynamic Inflow

The inflow is dynamic in this situation. The young generations that enter the labour market may prefer the individual DC fund rather than the collective DB fund with risk sharing. Based on the funding ratio, a new participant decides to enter the collective fund or not. The decision is based on the published nominal funding rate and the reaction of 25-year-olds according to equation (2.1) for an entry level of $FR = 96\%$, while the reaction of 25-year-olds for an entry level of $FR = 120\%$ stems from equation (2.2) for $x = 25$. Recall that a plot of the reaction function for 25-year-olds for $FR = 96\%$ is shown in Figure 2.1. Since the effects of discontinuity are the strongest for an entry level of $FR = 120\%$, I mainly discuss those results in detail; nevertheless, I touch upon the results for an entry level of $FR = 96\%$ as well. By showing the effects for both entry levels (following from two different papers), the sensitivity with respect to the discontinuity function is immediately assessed.

Funding Ratio

Figure 5.1 shows the graphs with the development of the coverage rates for both entry levels. From Figure 5.1b it is readily clear that the lower and upper quantiles show more extreme values than the static funding rates; thus, the sustainability of the pension fund becomes more risky and more volatile. The first fifteen years the dynamic and static funding rates follow a (very) similar pattern, even almost identical. Namely, the fund has to recover from underfunding in both cases. However, in the dynamic case the remaining participants undergo higher premia and more benefit reductions, since the population is smaller than in the static case. That is because till 2040 no new participants enter in the fund, since the prevailing funding ratio is below the break-even funding ratio of 120% (see Figure 2.2 for age $x = 25$). This is mainly due to the underfunded position in 2015. For this reason, a hump emerges around the year 2055 whereby the coverage rate proceeds towards a funding level above 130% — a difference of roughly 10% with the static rate.

Let me elaborate on this hump. Since there is almost no new inflow during the first thirty years, the population of the fund declines and only elderly participants remain (i.e. the fund becomes grey). Young cohorts are not abundantly present anymore, while the older cohorts remain (recall that retirees cannot leave the fund). Therefore, the returns on the assets start to form an additional buffer which cannot be given away (since the fund's population is smaller). The smoothing period of $n_{sur} = 5$ years amplifies this effect; the fund keeps the surplus for too long. Computations show that if the smoothing period equals $n_{sur} = 1$ year in this situation, the hump is almost not present anymore (confirming the intuition). Hence, the capital of assets keeps growing, although less contributions flow in the fund due to less active participants who pay premia. But, the fewer contributions are negligible compared to the total amount of assets. As Kocken (2008) advocates also, the embedded option of a contribution rate strategy is likely to vanish. Changing contributions when actives are only a small portion of total participants has little impact; the pension fund is maturing rapidly namely due to the less inflow.

Besides, the liabilities of the fund decrease due to two effects. Firstly, there is less inflow of new participants and, hence, less pension rights are accrued. Secondly, less pensions have to be paid since older cohorts decease and after 34 years all retirees from 2015 are dead for sure, because people reach at most the age 99. So, in around 2050 (35 years later than 2015), the hump starts to grow and holds on till 2075. Because the funding rate exceeds the entry level of 120% during 2040-2075, new participants enter the fund and the liabilities rise again. Moreover, the assets decline after 2075 since: (i) the additional buffer is depleted and (ii) the fund pays out pensions to the participants that entered the fund 40 years earlier (so in 2040). So, the composition of the fund's population plays an important role.

Especially, the effect of a smaller population is magnified at the 95%-quantile. It shows that in a good economic scenario the additional buffer rises even more due to higher returns on the assets — while simultaneously the surplus is shared with less remaining participants and, therefore, they profit even more. During the first few years, new participants abstain, the fund turns grey and consequently 35 years later, the population is smaller wherefore the bigger additional buffer cannot be given away. As the buffer is depleted (in combination with paying more pensioners), the funding rates start to drop. The lower 5%-quantile proves the opposite: in less prosperous scenarios the fund has more trouble with respect to financial sustainability. Less participants are in the fund, so the premium and indexation mechanisms are less effective and, therefore, it is more difficult for the fund to recover. Since the fund does not recover during the first decades no new participants enter the fund an no contributions flow in. Contributions in the lower quantile are less negligible compared to the assets, since the asset value is lower due to lower returns. Since the return on the assets are lower and the buffer is depleted after 2050 (i.e. no hump), the funding rate continuously decreases. As a consequence, no new participants enter the fund hereafter.

Nevertheless, the fund can overcome such situations. Actually, the problem is twofold: (i) the smoothing periods are too long and (ii) the premium and indexation mechanisms are less effective due to a smaller population. Hence, the consequences are also twofold: either an enormous buffer appears or either a severe burden for the remaining participants rises. As stated, if all smoothing periods are shorter, the quantiles and median values show a stabler pattern because the fund absorbs the (positive and negative) shocks more quickly. Regarding the upper quantile and the median, the fund does not deliver its surplus quickly enough; it holds the buffer too long. A smoothing period of $n_{sur} = 1$ year for surplus sharing brings the upper quantile to less higher funding rates. Concerning the lower quantile, the smoothing periods are also too long. Since the fund does not recover fast enough, new entrants abstain from entering the pension contract and, subsequently, the funding position worsens even further; as a consequence the population becomes smaller and steering becomes even less effective. In case the recovery plans and sustainability cuts are activated within 1 year, the lower quantile shows significant improvements and fluctuates around 90%. So, the risk-sharing rules should depend on the financial position of the fund, otherwise the fund finds itself in a downward sloping spiral.

An entry level of $FR = 96\%$ (Figure 5.1a) shows similar characteristics, but the effects are less strong; only the first 3 years new entrants abstain. Still, at the 50% and 95%-quantiles the hump is visible, but the effect in the lower quantile is less severe since the fund tackles this situation with more recovery premia en benefit reductions.

ALM-output

The classical ALM output provides some helpful insights; for an entry level of 120%, the results are displayed in Table 5.2, while the results for an entry level of 96% are shown in Table 5.1. The tables show for each statistic the changes in values between the dynamic inflow and static inflow.

As Table 5.2 clearly shows, the effects on the funding ratio are negative. The mean funding ratio after 75 years is 7.1% lower (in absolute value) with dynamic inflow as compared to static inflow (i.e. decreases from 124.7% to 117.6%). Note that during 2050-2075 the funding ratios are considerably higher than the static, but decrease to the drop in buffer. Moreover, the volatility of the coverage rate increases by 6.42%, which confirms our previous observations from the graphs of the funding ratio. In each quantile the funding rate is lower also, especially the lower quantile performs worse: a drop of almost 13%. The probability of underfunding increases by 4%, mainly due to the worse 5%-quantile.

The probabilities of the steering mechanisms for an entry level of 96% are similar to the static usage, as shown in Table 5.1. The changes in probability for the indexation and premium instruments are minimal. However, the probabilities differ much more for an entry level of 120%, as shown in Table 5.2. The option of using conditional indexation decreases by 2.2%, whereby it is noticed that the other extreme indexation options are used more often: such as full indexation, surplus sharing, recovery plans and sustainability cuts. This observation is in line with the higher volatility in the fund: either a buffer accrues or a severe burden on the participant emerges. The full indexation option rises

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	-1.800% (-0.740%)	$E[i^c_{75}]$	-€67.30 mln.	Participants$_{75}$	-1,041,725 (641,240)
$Q_{0.05}[FR^N_{75}]$	-0.860 %				
$Q_{0.5}[FR^N_{75}]$	-1.900%	Pr["no indexation"]	-0.001 (0.006)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	-2.800%	Pr["full indexation"]	0.010 (0.002)	$E[RR^{m,2}_{75}]$	-0.005 (-0.008)
$\Pr[FR^N_{75} < 100\%]$	0.007	Pr["cond. indexation"]	-0.008 (0.003)	$Q_{0.05}(RR^{m,2}_{75})$	-0.017
		Pr["surplus sharing"]	0.009 (0.002)	$Q_{0.5}(RR^{m,2}_{75})$	0.002
Discontinuity		Pr["recovery plan"]	0.007 (0.012)	$Q_{0.95}(RR^{m,2}_{75})$	-0.028
$\Pr[FR^N_{75} = 0\%]$	0.000	Pr["sustain. cut"]	0.007 (0.015)		
Pr["no inflow"]	0.073 (0.087)			$E[PR_{75}]$	0.028 (0.019)
$E[d_{25}]$	0.361 (0.420)	**Premium**		$Q_{0.05}(PR_{75})$	-0.021
$E[d_{45}]$	0.000 (0.000)	Pr["low. premium"]	0.009 (0.002)	$Q_{0.5}(PR_{75})$	0.504
$E[d_{64}]$	0.000 (0.000)	Pr["rec. premium"]	0.005 (0.012)	$Q_{0.95}(PR_{75})$	0.115

Table 5.1: Change in output: dynamic inflow v.s. static, entry level $FR = 96\%$.
The table presents the ALM-output as explained in Section 4.3. The reported outcomes represent the absoolute change with regard to the (benchmark) static outcomes; that is, each reported value is computed as stated in equation (5.1) where A is the dynamic value for an entry level of $FR = 96\%$ and A^* the benchmark value. A negative value indicates that the dynamic value decreases compared to the static case, whereas a positive values denotes an increase.

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	-7.100% (6.420%)	$E[i^c_{75}]$	-€122.7 mln.	Participants$_{75}$	-1,932,937 (709,110)
$Q_{0.05}[FR^N_{75}]$	-12.82 %				
$Q_{0.5}[FR^N_{75}]$	-4.600%	Pr["no indexation"]	-0.008 (0.020)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	-5.300%	Pr["full indexation"]	0.030 (0.007)	$E[RR^{m,2}_{75}]$	0.018 (0.085)
$\Pr[FR^N_{75} < 100\%]$	0.041	Pr["cond. indexation"]	-0.022 (0.003)	$Q_{0.05}(RR^{m,2}_{75})$	-0.169
		Pr["surplus sharing"]	0.032 (0.007)	$Q_{0.5}(RR^{m,2}_{75})$	0.046
Discontinuity		Pr["recovery plan"]	0.018 (0.04)	$Q_{0.95}(RR^{m,2}_{75})$	0.160
$\Pr[FR^N_{75} = 0\%]$	0.033	Pr["sustain. cut"]	0.019 (0.039)		
Pr["no inflow"]	0.303 (0.210)			$E[PR_{75}]$	0.082 (0.068)
$E[d_{25}]$	0.633 (0.424)	**Premium**		$Q_{0.05}(PR_{75})$	-0.078
$E[d_{45}]$	0.000 (0.000)	Pr["low. premium"]	0.032 (0.007)	$Q_{0.5}(PR_{75})$	0.132
$E[d_{64}]$	0.000 (0.000)	Pr["rec. premium"]	0.014 (0.038)	$Q_{0.95}(PR_{75})$	0.702

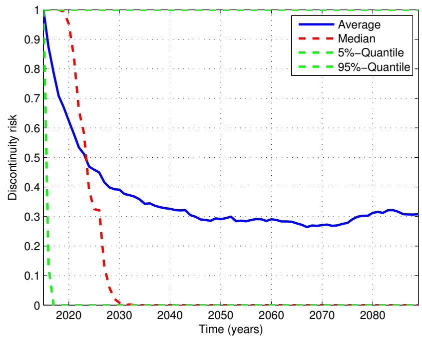
Table 5.2: Change in output: dynamic inflow v.s. static, entry level $FR = 120\%$.
The table presents the ALM-output as explained in Section 4.3. The reported outcomes represent the absolute change with regard to the static (benchmark) outcomes; that is, each reported value is computed as stated in equation (5.1) where A is the dynamic value for an entry level of $FR = 120\%$ and A^* the benchmark value. A negative value indicates that the dynamic value decreases compared to the static case, whereas a positive values denotes an increase.

by 3%, while the surplus sharing probability increases by 3.2%. The increase is because of the higher coverage rates during the years 2050-2075, where the funding rates are higher due to the additional buffer. The fund has more capital and, hence, is able to provide more indexation than the regular conditional indexation in the static case. The negative indexation mechanisms — such as recovery plans and sustainability cuts — have an increase in usage of 1.8% and 1.9% which is cause by the smaller size of the population (as well as the less prosperous scenarios in the lower quantile). Since the population is smaller and the smoothing periods are too long in specifically the lower quantiles, the effect of benefit reductions is smaller than in the static case. For this reason, those negative adjustment mechanisms have to be activated more frequently in order to achieve the same effect as in the static case. Nevertheless, the cumulative indexation decreases by €122.7 million and proves that the fund has to pay out less indexation than in the static case (on average), although the full indexation and surplus sharing option are activated with a higher probability. The lower amount of cumulative indexation is because of the fewer participants.

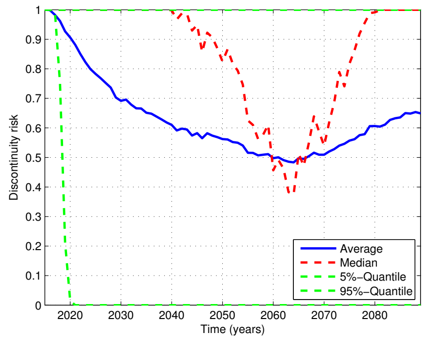
The premium mechanism shows a similar trend, as indicated by the probabilities of usage: lower premiums are charged 3.2% more frequently and the probability of recovery premiums rises by 1.4%. The results again indicate that extreme situations happen more often and just charging the cost-covering premium (as stated in equation (3.12)) occurs less frequently. But, bear in mind that actually less participants are in the fund, which is not beneficial for the fund in terms of the premium strategy. The premium strategy (as stated in equation (3.13)) works less efficiently, since the population is smaller and, therefore, the fund has to charge more frequently higher premium levels to recover. Namely, the premium during the first decades is high due to the underfunded position (as in the static case). However, the premium stays at the level of 22%-23% till 2050 — for comparison: the premium level is already below the 20% at 2022 in the static case (see Figure 4.3). The reason it maintains this high level is because no new cohorts enter the fund and the population mainly contains elderly. So, the remaining participants have to pay the bill: there are active participants than retirees. Then, after 2050, new participants enter and the premium level drops to a level comparable to the premiums in the static case. Remark that the indexation and premium mechanisms all have a higher standard deviation of usage, which shows the higher uncertainty in the dynamic situation.

At the end of 75 years, only 1.35 million participants remain in the fund (a decrease of approximately 2 million participants, as shown in Table 5.2). It is the effect of the (high) break-even funding ratio (for an entry level of $FR = 96\%$ 2.2 million participants remain), which also leads to considerable discontinuity risks. The probability of six consecutive years no inflow (over the full horizon) is 30.3% for an entry level $FR = 120\%$, while it is only 7.3% for an entry level of $FR = 96\%$ — note that in the static case all values for discontinuity were zero, therefore I speak about the probability and not over an increase. Even more worrying is the 3.3% probability of defaulting in case the entry level is $FR = 120\%$. So, in one of the thirty scenarios, the fund defaults in 75 years time.

Furthermore, the probability that new entrants of age $x = 25$ abstain is 63.3% for an entry level of $FR = 120\%$, while it is only 36.1% for an entry level of $FR = 96\%$. The difference comes from the following: during roughly the first three decades and last decade of the simulation horizon, no new generations enter the pension contract for an entry level of $FR = 120\%$; while for an entry level of $FR = 96\%$, only the first two decades no new participants enter. The development of the discontinuity risk together with the inflow of new cohorts is shown, respectively, in Figures 5.2 and 5.3. From Figure 5.2b it is clear



(a) Discontinuity risk: entry level $FR = 96\%$.



(b) Discontinuity risk: entry level $FR = 120\%$.

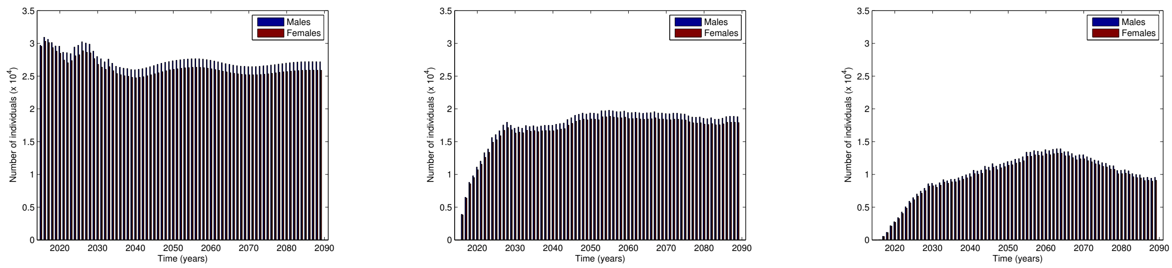
Figure 5.2: Discontinuity risk for 25-year-olds as from equations (2.1) and (2.2). The horizontal axis displays time t (in years) and the y-axis shows the corresponding discontinuity risk $d_{25,t}$ for new entrants of age $x = 25$; the mean and quantiles are displayed. If the discontinuity risk equals $d_{25,t} = 0.9$ at time t, 90% of the new labour entrants abstains from the pension contract at time t. Figure 5.2a shows the discontinuity risk if new entrants abstain at the break-even funding ratio of 96%, while Figure 5.2b shows the discontinuity risk if new entrants abstain at the break-even funding ratio of 120%.

that the discontinuity risk with respect to 25-year-olds is high during the first decades and, then, starts to decrease to 50% since the funding position improves — implying that only 50% of the new participants enter, the other have lower heterogeneous expectations. However, the discontinuity risk rises again after 2065, since the funding position worsens. In the lower quantile, the discontinuity risk equals 100% during all years and no new generations flow in at all ($d_{25,t} = 1 \forall t$). In the upper quantile, only the first few years no new generations enter, but afterwards the funding position is so good that every new cohort is willing to enter the collective fund.

In other words, there is almost no new (mean) inflow of cohorts during the first few years and at the end of the simulation horizon, as shown in Figure 5.3c. By comparing the static case in Figure 5.3a with Figure 5.3c, the severity of discontinuity is visible. Although the funding ratio exceeds the 120% funding level around 2060, not all new entrants of the 25-year-cohort are immediately willing to enter. Still 50% abstains, because they have or perceive possibly other expectations, which is also due to the information-cascade effect. If the breaak-even funding ratio is 96%, the discontinuity risk is only high during the first few years after 2015. Therefore, the number of participants entering the fund catches roughly up with the situation of static inflow as shown in Figure 5.3b.

Having discussed the effects for the fund, I turn towards the effects for the participants, starting with the replacement rate and pension result — as shown in Table 5.2. Both measures show that a remaining participant is worse off in the lower quantile, but gains in the 50%- and 95%-quantiles. Regarding the 5%-quantile, the replacement rate decreases by almost 17% and the pension result by nearly 8%. It is due to the fact that in less prosperous economic scenarios less participants enter the fund and the negative impact on the remaining participants increases, since they have to pay the price for recovery of the fund. The replacement rate decreases more than the pension result, because the replacement rate measures a 25-year-cohort that starts in 2049 and from there onwards the funding rate solely decreases.

The median and upper quantile show improvements. The median replacement rate



(a) No discontinuity. (b) Entry level $FR = 96\%$. (c) Entry level $FR = 120\%$.

Figure 5.3: Inflow of new entrants: static $(M_{25,t}, F_{25,t})$ and dynamic $(M^d_{25,t}, F^d_{25,t})$. The horizontal axis displays time t (in years) and the y-axis shows the mean number of new entrants (of age $x = 25$) that enter the fund (for males and females); the mean is taken over all scenarios, since in each scenario the dynamic inflow differs. Figure 5.3a presents the case of static inflow; Figure 5.3b presents the dynamic inflow if the break-even funding is 96% (discontinuity as in (2.1)); Figure 5.3c presents the dynamic inflow if the break-even funding ratio lies at 120% (discontinuity as in (2.2), for 25-year-olds only).

increases by 4.6%. As said, the replacement rate tracks a cohort that starts working in 2049 and retires in 2089. Thus, the additional surplus during the first working years of such a cohort are clearly beneficial for them. At the upper quantile of the replacement rate, an increase of 16% occurs, which is due to even higher amount of surplus provided by the fund. The median pension result displays an increase of 13.2%, whereby the 95%-quantile shows an increase of 70.2%. Recall that the pension result tracks a cohort that starts working in 2015 and deceases in 2089. For this reason, the 95%-quantile of the pension results is high, because the participant receives full indexation and surplus sharing after 2020 when the funding rate passes the 140%. Hence, in terms of pension results a 25-year-cohort gains a lot under discontinuity; specifically at the upper quantiles. So, the replacement rates and, especially, the pension results show outstanding results.

But, both measures only take the benefits into account and do not weigh this against the premiums paid. Thus, the measures ignore what the workers paid in. The next section reveals if the participants are indeed better off or not; this is done by the method of value-based generational accounting.

Generational Accounts

The changes in generational accounts for the two specific entry levels are shown in Figure 5.4; the graph shows the change in generational accounts with respect to the static situation. Note that I compute the generational accounts for initial time 2015.

It is immediately noticed that the last generations that entered (before others abstained) are severely hit and have a negative change in their generational accounts. That is, the young cohorts are disadvantaged. The generations that entered lastly have to pay the bill (literally), which is not revealed by the classical ALM output such as pension results and replacement rates.

The workers that remain in the fund have to pay frequently higher premia to keep the fund in a sustainable situation; they pay higher premia because there are less active participants left than in the static case to recover from the underfunding. Namely, during the first decades the premium level in the $FR = 120\%$-case is 4% higher than in the static case. This is the main effect that dominates the change in value for the young.

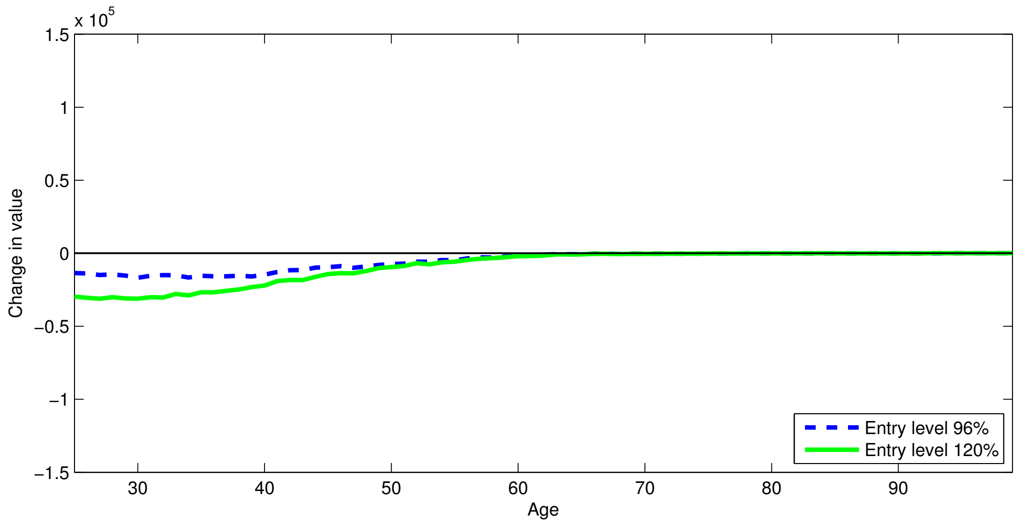


Figure 5.4: Change in generational accounts: dynamic inflow.
The horizontal axis shows the age of the participant (in years), while the y-axis displays the change in generational accounts as stated in 5.2.

The indexation policy during the first years remains relatively the same in the static and dynamic case, only after 2050 it starts to differ substantially since surplus is given away. But, for the generations $x \geq 65$ in 2015 this effect of additional surplus sharing is not captured in this graph, since those cohorts all deceased before 2050. The maximum age is 99, so at most some (lucky) people could have reached 2049. Therefore, it is only for the ages $x = 97, 98, 99$ that we see a small positive change in value. For the other age groups, the small period of additional surplus sharing does not outweigh the extra contributions that have to be paid — also due to the fact that participants are maybe not alive anymore in the future, the benefits are more strongly discounted (as a consequence of mortality rates). In case the entry level is 120% the change in value is almost twice as severe as for an entry level of 96% for the very young generations that have entered.

In sum, dynamic inflow is in the short-run not a problem for the fund. However, for the long-term less inflow causes a less sustainable position for the collective fund. Namely, the fund becomes more unsustainable and more sensitive to positive and negative shock. Intuitively, this is also a clear result. So, the answer to the second research question is: dynamic inflow does not affect the sustainability in the short-run, but it does clearly in the long-run. Especially, the remaining participants that entered the fund lastly are severely hit. The discontinuity risk lies between the 36.1% and 63.3%, with probabilities of default up to 3.3%.

The text-box below presents a situation whereby there is no inflow in the fund anymore. In that case the remaining participants are either severely hit, or benefit tremendously.

No Inflow, but Accrual: Closed Fund

This text-box sketches the situation if no new participants enter in a fund. In The Netherlands some funds are closing, which can lead to even more extreme situations. Some companies started new pension funds and closed their old funds. In those old funds, no new participants enter and no new premia flow in (only from the current active remaining generations). Besides, the fund's population only contains the remaining cohorts, so it turns grey quickly. Furthermore, there are funds in The Netherlands which are linked to a certain profession. The pension fund for construction workers is such a fund and faces also a severe decline of its fund's composition. The fund turns grey due to the effect of ageing, a decline of work in the construction sector and a growing number of self-employed. For this reason, such a fund faces similar consequences.

If a fund closes, enough capital is needed to oblige to the liabilities. Besides, it should be clear how (positive and negative) shocks are redistributed . My analysis shows that either the fund performs very well or very poorly. The 5%-quantile shows that the fund goes bankrupt, while the 95%-quantile shows outstanding performance results.

Due to less prosperous scenarios and smoothing periods that do not adapt to the funding ratio, the fund is default after 50 years in the 5%-quantile — since there is no new inflow in combination with negative financial shocks. Moreover, the steering mechanisms loose their adjustment power due to the decreasing population. In the lower quantiles the remaining participants are severely hit due to high premium levels and benefit reductions. In the 95%-quantile, the assets gain higher returns wherefore the asset value rises, while the liabilities keep decreasing towards zero. As a consequence, the funding ratio rises enormously. On average and in the 95%-quantile, the remaining participants in the fund are better of than in the static case, since the assets gain (on average) positive returns while the fund's population and liabilities keep declining. Subsequently, the funding ratio rises and much indexation is provided.

Since the fund does not adapt their adjustment mechanisms to the changing circumstances, the premium level increases sharply during the first decades because the working population declines compared to the retirees. If the fund closes in 2015, the last premia are received in 2054 (i.e. 40 years later, in 2055 the 25-year-cohort of 2055 retires). The indexation strategy shows a similar trend: either severe cuts occur or indexation levels way above the full indexation option appear (due to surplus sharing). The pension payments peak at 2055 and then start to decline, since all cohorts start to decease slowly.

The recommendation for this situation is, therefore, that a fund should adapt their risk-sharing rules (such as smoothing periods) in case a fund closes. The fund is namely more sensitive to shocks.

Next, the results of dynamic in- and outflow are shown.

5.1.2 Dynamic In- and Outflow

This section presents the results of the situation wherein also the outflow is dynamic. New entrants and current generations might prefer the individual fund over the collective fund with risk sharing. This decision is based on the break-even funding ratios as shown in Figure 2.2, whereby each break-even funding ratio a corresponding reaction function belongs. The reactions are based upon the discontinuity function as presented in equation (2.2); an example of a reaction function for 64-year-olds in shown in Figure 2.3. All the

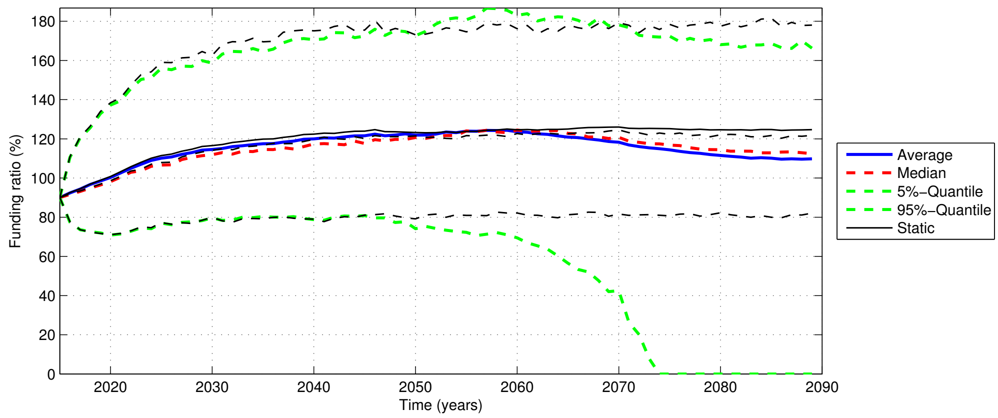


Figure 5.5: Funding ratios: dynamic in- and outflow.
The x-axis displays time (in years) whereby the y-axis shows the nominal funding ratio in percentages under $\mathbb{P}$; as clear from the legend, the graph entails the mean and quantiles of the static (grey) and dynamic funding rates.

results in this section are for the case whereby participants leave the fund with their accrued rights conditional upon the funding ratio (as in equation (3.16b)). Hence, not as if the coverage ratio is 100% (see equation (3.16a)). The latter case is part of the sensitivity analysis. I choose for the former case, since it lets the leaving participants also share in the burdens and surpluses. Namely, if the withdrawing participants get 100% of their accrued rights, they do not share the burden in case of underfunding and do not share in benefits in case the funding rate reaches high levels. For example, if participants get 100% of their rights in case of underfunding, the fund's assets will be quickly depleted and might provide additional incentives to leave the fund, as explained with the example in Chapter 2.

Funding Ratio

I start again with discussing the coverage rate, which is shown in Figure 5.5. Immediately it is clear that the characteristic hump has vanished and that the funding ratios after 2060 lie below the static funding rates. The characteristic hump is gone, since the composition of the fund's population is somewhat more balanced (although smaller) than in the case of dynamic inflow and the fund has to pay the transfer values. Namely, current generations might also leave the fund now. Due to the underfunded position in 2015, new generations abstain from the contract (as with dynamic inflow) but also near retirees withdraw during this period. Since the near retirees (ages around 60 years) withdraw also, the fund's population does not turn extremely grey. Moreover, the leaving participants take their accrued rights with them, which decreases the asset value. All the near retirees withdraw for funding ratios around 90%, as shown in Figure 2.2, but still some of them abstain at funding rates of 110% (as shown by the reaction function in equation (2.2)).

Less inflow does not affect the funding ratio immediately, only after some decades by means of the hump (as seen for dynamic inflow). However, withdrawing cohorts influence the fund's capital directly, since the fund has to pay the leaving participants their accrued rights. Hence, the assets do not build up the same additional buffer, because the fund has to pay the transfer values (from the assets' capital) towards the leaving participants. The liabilities decrease also, wherefore the funding rate rises a bit. However, the decrease in liabilities does not outweigh the immediate decline in the assets. Therefore, the mean and median funding ratios are somewhat lower during the first decades (compared to the static). Since the additional buffer is not accrued, the hump around 2060 is gone. As a consequence, new participants only enter during 2040 till 2065 (while in the dynamic inflow case participants start to enter already at 2035 till 2080). Subsequently, the fund's population grows less quickly compared to the dynamic inflow situation (since the funding rate is lower now), but still new participants enter and current generations remain (because the funding rate exceeds the break-even funding ratios). Due to the new inflow and the remaining participants, the funding rate decreases after 2060. The decline is caused by two effects: (i) the liabilities rise due to the inflow in the years 2040-2060 and (ii) the fund did not had the additional buffer, wherefore the asset value depletes quicker (compared to the dynamic inflow case). The rising liabilities with lower assets directly drive the funding rate down. Subsequently, the coverage rate is substantially lower than in the static case after 75 years. The pension fund is unable to steer the funding ratio upwards, because the population is too small for the instruments to be effective.

The 95%-quantile shows a similar pattern as the mean and median, but the characteristic hump is seen again. The first decades the funding ratio maintains below the static rates, because the fund has to pay transfer values and, consequently, the assets are smaller. Due to no inflow during the first years, but more remaining near-retirees (the break-even funding rates of the older cohorts are lower than the new entrants), the fund turns grey again. Consequently, the hump emerges although somewhat smaller than in with dynamic inflow, since in 2015 near retirees left the fund and, thus, the asset value is lower than with the dynamic inflow. The hump declines for the same reason as in dynamic inflow (i.e. depletion of the additional buffer).Moreover, due to better financial circumstances in the market the funding ratio is higher than the mean and median, and also more participants enter and stay in the fund relatively.

Obviously, the 5%-quantile shows that the funding ratio starts to decline after 2045, and indicates high volatility and severe riskiness for the sustainability of the fund after those years. Due to the continuous underfunding the first decades, nobody enters the fund and also many participants of the ages 55-64 leave till 2045 (as indicated by the break-even funding ratio in Figure 2.2). Then, the fund is almost empty in 2045 (in terms of participants), with a significant lower amount of assets (due to transfers and less contributions) and lower liabilities (due to abstaining/leaving participants); both values declining towards zero in the upcoming years. Due to this snowball effect and combined with a less prosperous economic scenario (driving the asset value even lower), the fund is doomed and unable to recover anymore because the steering mechanisms have no influence any longer since the population is too small. In 2070 the fund is (almost) completely empty (only the ages around 45 years old remain) and it will certainly default if no exogenous measures are taken by e.g. the government (remember that Dutch pension funds cannot default by definition) or by changing the strategy of the steering mechanisms. The premium and indexation instruments keep smoothing the shocks in cases of severe underfunding, while this does not help in recovering since also the fund's populations decreases rapidly. Policymakers should change the smoothing periods in cases of significant worse financial positions, because the steering mechanisms become ineffective. Otherwise, as shown by the lower quantile, a bank-run effect emerges and the fund goes default. In the lower quantile, the fund has many characteristics of a Sinking Giant. The fund has been passing its critical funding ratio of around 75% for too long and the fund is unable to recover, even though it keeps getting their expected returns

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	-14.90% (11.75%)	$E[i^c_{75}]$	-€240.5 mln.	Participants$_{75}$	-2,321,904
$Q_{0.05}[FR^N_{75}]$	-82.06 %				(725,690)
$Q_{0.5}[FR^N_{75}]$	-9.400%	Pr[“no indexation”]	0.032 (0.021)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	-11.70%	Pr[“full indexation”]	-0.029 (0.006)	$E[RR^{m,2}_{75}]$	-0.116 (0.035)
$Pr[FR^N_{75} < 100\%]$	0.097	Pr[“cond. indexation”]	-0.001 (0.000)	$Q_{0.05}(RR^{m,2}_{75})$	-0.630
		Pr[“surplus sharing”]	-0.030 (0.000)	$Q_{0.5}(RR^{m,2}_{75})$	-0.050
Discontinuity		Pr[“recovery plan”]	0.021 (0.032)	$Q_{0.95}(RR^{m,2}_{75})$	-0.154
$Pr[FR^N_{75} = 0\%]$	0.074	Pr[“sustain. cut”]	0.017 (0.025)		
Pr[“no inflow”]	0.335 (0.217)			$E[PR_{75}]$	-0.027 (0.003)
$E[d_{25}]$	0.693 (0.407)	**Premium**		$Q_{0.05}(PR_{75})$	-0.069
$E[d_{45}]$	0.022 (0.101)	Pr[“low. premium”]	-0.025 (0.000)	$Q_{0.5}(PR_{75})$	-0.048
$E[d_{64}]$	0.321 (0.406)	Pr[“rec. premium”]	0.023 (0.030)	$Q_{0.95}(PR_{75})$	-0.266

Table 5.3: Change in output: dynamic in- and outflow v.s. static.

The table presents the ALM-output as explained in Section 4.3. The reported outcomes represent the absolute change with regard to the static (benchmark) outcomes; that is, each reported value is computed as stated in equation (5.1) where A is the value for dynamic in- and outflow, and A^* the benchmark value. A negative value indicates that the dynamic value decreases compared to the static case, whereas a positive values denotes an increase.

recovery is not plausible.

ALM-output

Sequentially, the detailed output is discussed now and presented in Table 5.3. The table shows the changes with respect to the static case.

As stated above, dynamic in- and outflow leads to high volatility and severe riskiness for the pension fund. The values for the funding ratios confirm those findings. The mean and median funding ratios after 75 years are, respectively, 14.9% and 9.4% lower than the static rates; besides, the volatility increases by almost 12%. Those values are almost twice as low as compared to dynamic inflow only; indicating that possible outflow causes even more unsustainability. The upper quantile becomes 11.7% lower and the lower quantile decreases significantly by 82.06% to a funding rate of 0%. The latter is due to the fact that the fund goes bankrupt during the lower quantiles, since the fund has become a Sinking Giant. As the severe probability of default shows (7.4%), bankruptcy occurs (roughly) in one of the thirteen scenarios. Moreover, the probability of underfunding also rises by almost 10% (whereas the dynamic inflow showed an increase of 4.1%, more than twice as low). Both effects are due to the low funding rates at the lower quantiles.

The probabilities of usage of the steering mechanisms show some interesting results also. Namely, all the positive steering mechanisms such as full indexation, conditional indexation, surplus sharing and lower premium show a decrease of usage. In other words, the results show that the fund is frequently in less prosperous financial positions wherefore less bonuses can be provided. Recall that with dynamic inflow, the positive indexation instruments actually shows an increase. Clearly, it can be concluded now that outflow is not beneficial for a fund, since is causes the funding levels to be lower. Logically, the negative steering mechanisms show an evident increase in the probability of usage, since the fund is continuously busy with recovering because it is below the static steady-state of 121.8%. Comparing the results from dynamic in- and outflow (Table 5.3) with the results of dynamic inflow (Table 5.2), the following is noticed: no indexation occurs with 2.5% more, full indexation decreases by 6%, conditional indexation rises by 2%, surplus sharing declines by 6%, the recovery plans and sustainability cuts remain almost the same.

Thus, the no indexation and full indexation and surplus options point out that the fund reaches less often high funding rates levels. This is mainly due to the fact that the characteristic hump is gone. The increase of no indexation comes also from the fact that the fund reaches more often lower funding rates. The increase of conditional indexation (with respect to dynamic inflow) affirms the intuition: the fund reaches less often higher funding rates, wherefore the conditional indexation becomes more attractive. Though, the conditional indexation option relative to the static rates (as shown in Table 5.3) does not change almost, which is due to the following: the funding ratios keep a more modest level (on average), since the fund's population is more balanced. For this reason, conditional indexation is more attractive. The decrease of bonus indexation and increase in conditional indexation confirm our previous observations from the funding ratios graphs: (i) the fund's population is more balanced and (ii) the asset value is lower, so less bonus indexation.

The recovery plans and sustainability cuts are compared with dynamic inflow not used more frequently — the changes are minimal. It is because the number of (near) retirees (having accrued the most pension rights) is lower and, hence, more benefit reductions are not super effective. The number of near retirees is lower, since they can leave the fund and, consequently, the number of retirees in the collective fund is also lower. Recall that the last working year decides where you spent your retirement period. Since less older-cohorts remain in the collective fund before the date of retirement (they switch to the IDC), less retirees are in the collective fund. Nevertheless, less occurrences of benefit reductions does not imply that the benefits are also reduced by a lesser amount. The annual plan and sustainability cut are activated equally in the both dynamic cases, but the reductions in the dynamic in- and outflow setting are more severe. This is also inferred from the lower amount of cumulative weighted indexation.

Thus, the main recovery of the fund has to be established by something else: namely, recovery premia. The young and middle-aged cohorts are inclined to stay in the fund (as shown with the break-even funding ratios in Figure 2.2) and, therefore, are the ones that pay for the recovery. Besides the benefit cuts they experience, they are charged with more recovery premiums and have lower probabilities of paying lower premia. Compared with the dynamic inflow, the recovery premia are charged with 1% more while the option of lower premiums decreases by 2.5%. Since the initial amount of retirees in the fund (i.e. in 2015) remains the same, but also the near retirees leave, the remaining participants have to pay higher premia since the ratio of workers versus retirees decreases even further (besides the demographic effect as shown in Figure 3.4). In case of dynamic inflow, the premium level was already higher than the static since the ratio of workers versus retirees declined. Now, in case of dynamic in- and outflow, this effect is even stronger. In 2015 the current retirees can not leave the fund (by definition), but the older cohorts from 55-64 years leave and the very young cohorts abstain. So, the remaining participants of 30-55 are charged more frequently with recovery premiums which are besides also higher. Namely, the charged premium is higher during the first decades compare to dynamic inflow, then it declines to more reasonable levels since the fund is not so grey any more.

After 75 years only 8k participants is left in the fund (as compared to 3 million in the static case), a decrease of 2.3 million participants. The remaining participants are mostly of the ages between 27 and 55. Namely, the new entrants abstain from the pension contract at all, while the 26-years-old leave the fund (because they have not accrued many rights yet). The generations of 55 years and older leave the fund, because if benefits are

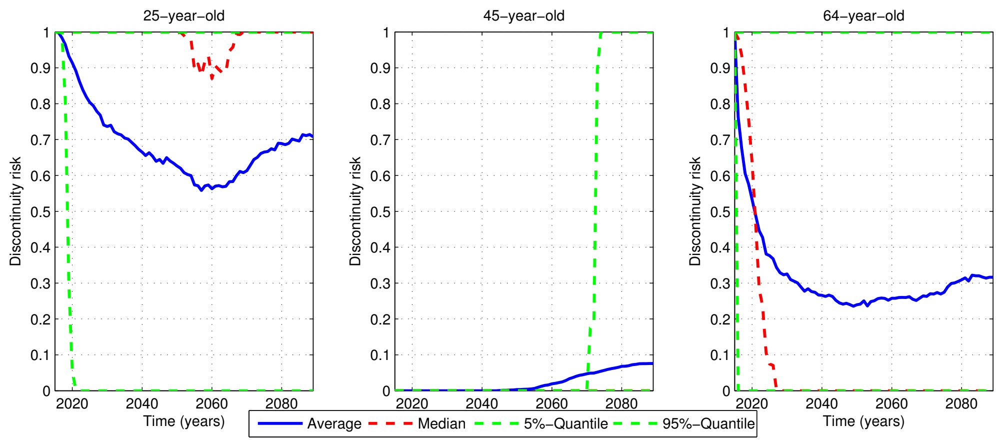


Figure 5.6: Discontinuity risk for different age-groups, following from equation (2.2). The horizontal axis displays time t (in years) and the y-axis shows the corresponding discontinuity risk; the mean and quantiles are displayed. The three figures show the discontinuity risks with respect to three representative cohorts: young, middle-aged and elderly cohorts. The left graph shows the discontinuity risk $d_{25,t}$ for 25-year-olds; the middle graph presents the discontinuity risk $d_{45,t}$ for 45-year-olds; the right graph displays the discontinuity $d_{64,t}$ for near retirees of 64 years old. If $d_{25,t} = 0.9$ at time t, 90% of the new labour entrants abstains from the pension contract at time t. If the discontinuity risk in the middle of right graph equals 0.4, 40% of that age-group leaves the fund.

reduced, they are the ones that feel the cuts. Because those older generations leave, benefit reductions occur less often since it has become a less effective instrument.

The effects are seen in the discontinuity risks in Table 5.3 and in Figure 5.6. The discontinuity risk for the new entrants rises to 69.3% (6% higher than dynamic inflow), indicating that more new entrants abstain from the pension contract. The reason that the average discontinuity risk is higher than with dynamic inflow, comes mainly from the lower funding ratios for the year 2050 and further. The left graph in Figure 5.6 shows that new cohorts abstain from the contract during most of the time, because the funding ratio is not high enough compared to their break-even rates. Only around 2060 more new participants are willing to enter — but, less participants enter than with dynamic inflow. For this reason, the probability of no new inflow rises from 7.3% for dynamic inflow to a significant probability of 33.5% for dynamic in- and outflow. The 5%-quantile (i.e. economically ‘bad’ times) lies for all years at 100% discontinuity risk, while during prosperous economic times (the 95%-quantile) new entrants abstain during the first five years but the discontinuity risk becomes 0% afterwards. The right graph in Figure 5.6 shows that the average discontinuity risk for 64-year-olds is 32.1%, where the risk is the highest during the underfund position in the years after 2015. Those near retirees leave in a decreasing way till approximately 2030, then the cohorts stay somewhat more because the funding ratio is above their minimum funding ratio and the expectations on a recovered fund rise. The middle graph in Figure 5.6 shows that 45-year-olds have enough incentives to stay in the fund (the median is zero) and leave only in some of the worst economic states of the world, i.e. only in the lower quantiles after 2065 where the fund's coverage rates become very low and the fund goes default.

The replacement rate and pension result show unfortunate circumstances for the participants. In all quantiles and the mean the participants are worse off with respect to static inflow as well as dynamic inflow. The reason is that no indexation occurs more fre-

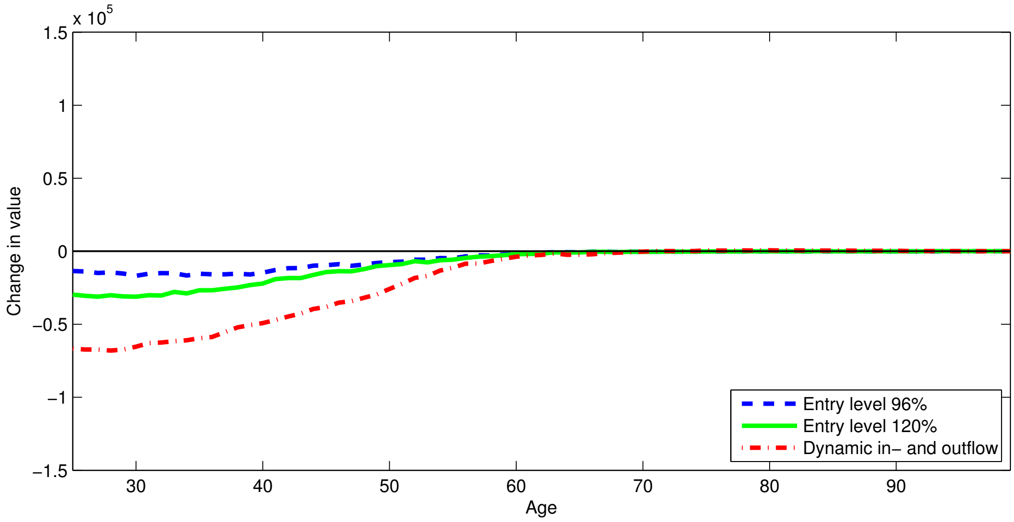


Figure 5.7: Change in generational accounts: dynamic in- and outflow.
The horizontal axis shows the age of the participant (in years), while the y-axis displays the change in generational accounts as stated in 5.2.

quently, and the full indexation and surplus sharing options are also activated less often. As a consequence (and in combination with a substantial lower amount of participants), the fund clearly pays out less in terms of cumulative indexation, which equals only 241.7 million (a decrease of 240 million).

Generational Accounts

The replacement rate and pension result show large decreases, but do not incorporate the premia paid. As the premium mechanism already showed, participants pay more frequently higher premia. Taking that as a given, in combination with lower indexation levels, leads to an undesirable situation for the remaining participants.

Namely, the remaining participants are the ones that suffer, as shown in Figure 5.7. In terms of generational accounts, especially the younger generations are hit, conditional upon the fact that each cohort stays in the fund for the rest of their life-time. The young age-groups have to contribute to the recovery of the fund by paying more frequently high levels of premia during working life as well as absorbing benefit reductions and receiving less indexation during working-life and retirement. The possibility to opt-out of the fund affects the other age groups also more. Specifically, the remaining retirees of 65-70 years have a decrease in their generational value because more severe benefit reductions are necessary. Since retirees are not able to leave the fund, and the group of retirees of 65-70 years is the biggest (older ages occur less due to the higher death rates), the burden of the recovery of the fund partially shifts towards them. The magnitude of the effects of discontinuity in terms of generational accounts are of the same extent as the impact of benefit reductions and recovery premia. This is clearly seen by comparing Figure 5.7 with 4.4.

In conclusion, the dynamic setting seems to cause a highly unsustainable financial situation for the pension fund and increases the riskiness severely. The funding ratios after 75 years show a large decrease. Due to the underfunded position in 2015, participants abstain and leave the fund, while the fund actually needs them to recover and, hereby, the fund loses steering power. There could be up to two million participants less in the fund than in the static case. In response to the second research question: dynamic outflow affects the sustainability in the short-run, since the fund must pay the accrued rights immediately to the participants that want to transfer from the collective fund to the individual fund. In combination with the dynamic inflow, which affects the sustainability in the long-run, the pension fund becomes less sustainable with regard to the short- and long-term. In the short run, leaving participants cause a decrease of the assets, making the fund more sensitive to shocks. In the long-run the fund becomes even more sensitive, especially in financially less prosperous situations (e.g. the lower quantiles) where the fund turns into a Sinking Giant. The fund uses all the instruments at hand, but due to the smaller population the measures are not effective. The discontinuity risks during the voluntary setting with regard to the new entrants could be 69.3% and around 32.1% for near retirees, while the risk for middle-aged cohorts is minimal. Most severe is the probability of default: 7.4%. The effect of discontinuity in terms of generational accounting reaches the same levels as the effects of benefit reductions and recovery premia; which is not shown by the naïve measures as replacement rates and pension results. For this reason, discontinuity is not only disadvantageous for the fund but also for the participants. So, discontinuity risk affects both sides: the fund and participants; while benefit reductions and recovery premia only affect the participants, but improve the financial position. However, the results are quite sensitive to the entry levels.

The next section presents a sensitivity analysis with regard to the above presented results.

5.2 Sensitivity Analysis

In this section the sensitivity of the results from the previous sections is evaluated. The results are shown in the following systematic way: (i) funding ratios and (ii) ALM-output. The funding ratios are plotted along with the static funding rates. The ALM-output is compared with the results of dynamic in- and outflow. That is, the ALM-ouput is computed as follows

$$\Delta A^{**} = A - A^{**}, \tag{5.3}$$

where A^{**} is the absolute value of the ALM-ouput in case of dynamic in- and outflow (as shown in Appendix A.1 in Table A.3), and A is the value for the sensitivity tested. Thus, it is easy to notice how changes in parameters and determinants affect the pension fund outcomes if the in- and outflow is dynamic. The changes in ALM-output ΔA^{**} are calculated by first rounding the absolute outcomes to a total of 4 digits and then subtracting the sensitivity tested.

Firstly, I present how another form of transfers affects the sustainability of the fund. Secondly, the initial funding ratio is changed such that the fund does not start in an underfunded position. Thirdly, I assess how low interest and inflation rates affect the fund. All the three situations are considered by taking discontinuity into account. The absolute outcomes for each of the three cases is shown in Appendix A.2.

5.2.1 Sensitivity: Transfers

Here I show how the transfer values affect the outcomes. Variation in the transfer value makes a difference for the resulting sustainability of the pension fund.

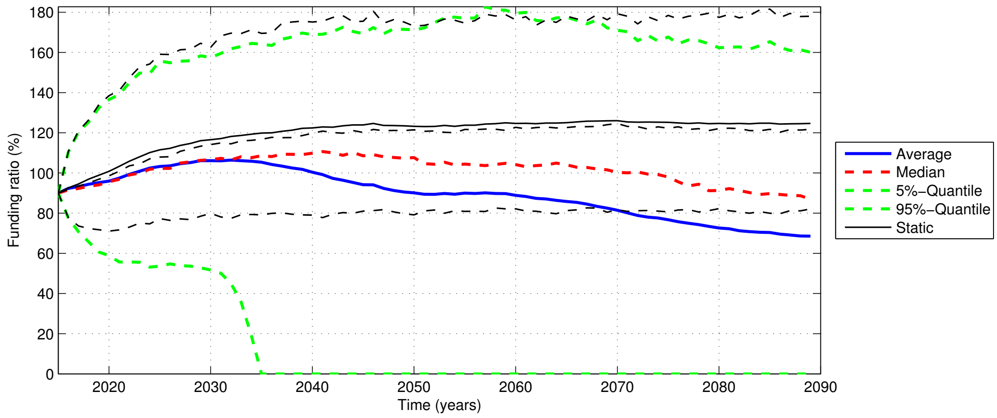


Figure 5.8: Sensitivity funding ratio: transfer value.
The x-axis displays time (in years) whereby the y-axis shows the nominal funding ratio in percentages under $\mathbb{P}$; as clear from the legend, the graph entails the mean and quantiles of the static (grey) and dynamic funding rates. The funding rates are based on the transfer value as specified in equation (3.16a).

The above presented results are based upon a transfer that is conditional upon the prevailing funding ratio. This sort of transfer lets the leaving participants also share in the burden (and surplus) in case anyone withdraws from the fund. Now, suppose that the fund pays the leaving participants their fully accrued benefits as in equation (3.16a). Then, the leaving participants walk happily away with all their rights, but the remaining participants have to pay additionally for the leavers. The expectation is that the fund depletes more rapidly in this case. Namely, the funding ratio automatically decreases if a participant leaves (as shown with the example in Chapter 2), and the remaining participants get into even more troubles. One might also wonder whether transferring 100% of the accrued rights is rightful. Suppose the funding ratio is 80%, participant for sure get the incentive to leave the fund with 100% of their rights instead of staying and receiving benefit reductions for example.

As Figure 5.8 shows, the funding ratios become extremely volatile. The mean, median and quantiles show more extreme situations than seen before and, thereby, the sustainability of the fund becomes even less and more risky. Since the fund gives the participants 100% of their accrued rights (even in cases of underfunding), the fund's assets deplete rapidly although the assets keep gaining their returns on the financial marker. Since the assets decrease so fast, the funding positions become significantly lower. Due to the lower funding rates, new participants become unwilling to enter and current generations leave even faster. Since more new entrants abstain, less contributions are received and because of more leaving participants, the fund has to pay out more transfers. Of course, the liabilities decrease also, but the capital of assets depletes much quicker. The fund is unable to recover because the fund's population is too small and the policy is not effective.

The 95%-quantile shows a development that is comparable to the situation shown in the case of dynamic in- and outflow with conditional transfers (see Figure 5.5). Only in the first few years participants abstain and leave, as in the situation with transfers conditional upon the funding ratio. Now, however, the fund pays out higher transfer values, wherefore the additional buffer of assets is smaller and, consequently, the funding ratios become even lower after 2050. The mean and median have a similar trend, although the average of the funding rates is pulled down by the scenarios in the lower quantiles. During the first decades new participants abstain and near retirees leave. But, since the fund has to pay the leaving participants too much compared to their financial position, the assets deplete fast. Since the assets deplete so fast, the coverage rate is substantially lower and during the rest of the horizon almost no new participants enter, cohorts of 55 years and older leave (since their break-even funding ratio lies around $FR = 80\%$) and the policy of the fund is too ineffective to help recover. Already at 2050 the fund contains merely 1 million participants. In 35 years time 2 million participants left, wherefore the fund reaches such low funding rate levels. Only the middle-aged cohorts stay in the fund. The 5%-quantile shows a very unfortunate scenario: young cohorts abstain and older cohorts leave, too much rights are transferred, due to low returns on the financial market the assets decrease even more and, as a consequence, everybody starts to leave the fund after 2030. Then, 5 years later the fund is bankrupt.

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	-41.15% (21.84%)	$E[i^c_{75}]$	-€36.00 mln.	Participants$_{75}$	-258,505 (60,052)
$Q_{0.05}[FR^N_{75}]$	0.000 %				
$Q_{0.5}[FR^N_{75}]$	-25.37%	Pr["no indexation"]	0.110 (0.072)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	-6.200%	Pr["full indexation"]	-0.307 (0.023)	$E[RR^{m,2}_{75}]$	-0.320 (0.166)
$\Pr[FR^N_{75} < 100\%]$	0.257	Pr["cond. indexation"]	-0.047 (0.026)	$Q_{0.05}(RR^{m,2}_{75})$	0.000
		Pr["surplus sharing"]	-0.042 (0.010)	$Q_{0.5}(RR^{m,2}_{75})$	-0.153
Discontinuity		Pr["recovery plan"]	0.124 (0.099)	$Q_{0.95}(RR^{m,2}_{75})$	-0.118
$\Pr[FR^N_{75} = 0\%]$	0.350	Pr["sustain. cut"]	0.091 (0.070)		
Pr["no inflow"]	0.145 (0.094)			$E[PR_{75}]$	-0.101 (-0.055)
$E[d_{25}]$	0.069 (-0.022)	**Premium**		$Q_{0.05}(PR_{75})$	-0.020
$E[d_{45}]$	0.197 (0.241	Pr["low. premium"]	-0.042 (0.010)	$Q_{0.5}(PR_{75})$	-0.207
$E[d_{64}]$	0.172 (0.056)	Pr["rec. premium"]	0.132 (0.104)	$Q_{0.95}(PR_{75})$	-0.113

Table 5.4: Sensitivity: transfer value.

The table presents the ALM-output as explained in Section 5.2. The reported outcomes represent the absolute change with regard to the outcomes in case of dynamic in- and outflow; that is, each reported value is computed as stated in equation (5.3) where A is the value with other transfers for dynamic in- and outflow (as reported in Table A.4), and A^{**} is the regular dynamic in- and outflow value (as reported in Table A.3). A negative value indicates that the tested sensitivity decreases compared to the dynamic in- and outflow case, whereas a positive values denotes an increase.

Table 5.4 confirms the observations. The standard deviation of the funding rates increases by 21.84% with respect to the dynamic in- and outflow with conditional transfers; the mean, median an 95%-quantile after 75 years are 41.15%, 25.37% and 6.2% lower respectively. It due to the fact that the fund transfers too much rights compared to their underfunded position in the first decades. The 5%-quantile does not change, since the worst case scenario is still that the fund goes default. However, the probability of default increases by 35%, which is due to the fact that the fund's assets decline faster and steering mechanisms are even less efficient. It is noticed that all the positive adjustment mechanisms show a decrease in usage, while the negative adjustment mechanisms show an increase. The reason is that the fund is in financially less prosperous situations and, therefore, more recovery premia are necessary and more benefit reductions. Especially, the premium during the first decades is high and the indexation levels are very low. The fund tries to activate all the steering mechanisms it has, but it is not effective enough due to the continuously decreasing population in combination with constant smoothing of shocks.

The probability of underfunding is 57.8% (thus rises by 25.7% compared to conditional transfers) and the probability of six consecutive years no inflow equals 48% (thus rises by 14.5% compared to conditional transfers). So, in one out of the two scenarios the fund is underfunded after 75 years. Specifically, the discontinuity risks with regard to the middle-aged and older cohorts rise. For the young cohorts it was already high in the dynamic in- and outflow with conditional transfers, but we see that the standard deviation in this case is 2.2% lower, which indicates that it is more certain that new participants abstain.

The population left in the fund after 75 years decreases from 8k to 5k, which is due to higher discontinuity risks. The replacement rate and pension result show both negative values, indicating that participants are worse off in those terms. It is the result of providing less indexation and more benefit reductions with the aid of annual recovery plans and sustainability cuts. This effect is also seen in the lower amount of cumulative indexation.

Summarising, the stability of the pension fund is severely at stake if the collective fund transfers all the accrued rights. The population of the fund is very small (a decline from more than 3 million to only 5k), wherefore the policy of the fund does not work properly any longer. The funding ratios are obviously lower. Intuitively the results are very clear and it shows that the stability of the pension fund depends highly on how the fund transfers the accrued pension rights.

Now, the sensitivity with respect to the initial funding rate is discussed.

5.2.2 Sensitivity: Initial Funding Ratio

Now it is shown how another initial funding affects the aforementioned outcomes. Another start position of the fund influences the results regarding dynamic in- and outflow.

Instead of starting from an underfunded position, the fund starts in its static steady-state. That is, the fund starts with the initial funding rate equal to the mean funding ratio after 75 years in the static case, namely $FR_{2015} = 121.8\%$. Besides, I present at the end of this subsection the initial funding ratio that leads to certain default in the future; if pension funds reach those levels, the sustainability should be highly questioned if policy rules (or other measures) are not changed.

The funding ratio in case the start position equals the steady-state is shown in Figure 5.9. The funding ratios become comparable with the situation if inflow is dynamic please compare Figure 5.1b. After 2060 the funding rates start to decline, which is because of continuously abstaining new generations and less inflow of pension premia. Namely, due to the higher initial funding ratio, older cohorts have enough incentives to stay in the collective the fund (the prevailing funding rate is higher than the break-even funding rate). Only a fraction of the new participants abstains. Recall that the entry level of $FR = 120\%$ states that at that funding ratio for sure no new generations enter (i.e. 100% of the participants of the new labour entrants abstains). However, due to the information cascade effect and heterogeneous expectations, not all new generations enter if the funding rate exceeds $FR = 120\%$. A fraction of the new labour entrants perceives that the funding position is not intrinsically good enough for them to enter. Consequently, during the first 45 years (i.e. till 2060), some of the new participants enter the fund (since the funding rate exceeds the break-even funding ratio of $FR = 120\%$), but still 60%-90% of the new entrants abstains since the equilibrium level is only slightly above the entry level.

Hence, less new participants enter the collective fund than in the static case and less

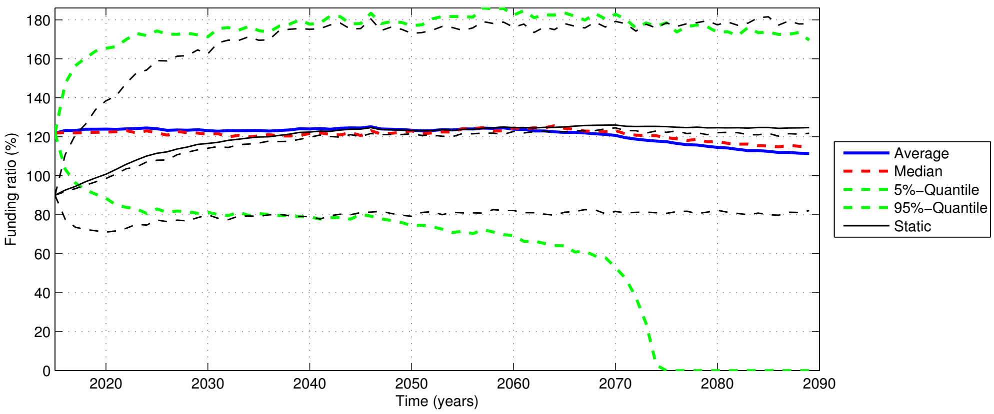


Figure 5.9: Sensitivity funding ratio: initial funding ratio.
The x-axis displays time (in years) whereby the y-axis shows the nominal funding ratio in percentages under $\mathbb{P}$; as clear from the legend, the graph entails the mean and quantiles of the static (grey) and dynamic funding rates. The initial funding ratio equals the equilibrium of 121.8%.

contributions are received. Consequently, the fund's population declines gradually and becomes grey — i.e. only elderly remain (besides the already demographic development of workers versus retirees as shown in Figure 3.4). For this reason, the liabilities of the fund decrease (due to less inflow and retirees that decease). The fund, however, receives less contributions for a period of 45 years and this causes the drop in coverage rates. Namely, since the funding position is high during the first decades, the premium level remains around a level of 19% (compared to the premium level of 23% in the underfunded position). Consequently, the fund receives too few contributions over a period of 45 years, which ultimately leads to a depletion of the assets — although the assets keep gaining their usual returns. Normally, less contribution inflow for several years is not harmful, but less premium inflow over 45 years is just too long. Then, after 2060 the fund is not able to cover its expenses (such as pension payments) any longer and the assets start to decline. Subsequently, more participants abstain and current generations start to leave, wherefore the fund becomes unmanageable again. So, the effect that dominates this graph is due to the grey composition of the fund and, consequently, a depletion of the assets due to too few premium inflow. The fund makes the mistake of charging a too low premium level, while the fund's composition changes gradually from a balanced fund to a grey fund.

In the lower quantile the effect is even stronger, because the assets do not gain the expected returns and the process of depletion is accelerated. In the upper quantile the same characteristic hump is seen as in Figure 5.1b, although smaller, because now the dynamic inflow is more stable than starting from underfunding. For this reason, the fund is less grey in the 95%-quantile than when starting from underfunding.

Although the fund ends up in a worse financial position than compared with the static case, the fund performs much more stable than starting from underfunding with dynamic in- and outflow. The mean funding ratio is 1.7% higher than dynamic in- and outflow with starting from underfunding, as shown in Table 5.5; while the 50%- and 95%-quantile show improvements of 2.5% and 3.3% respectively. The volatility of the funding rate after 75 increases by 1.5%, since it is more unsure how far the assets drop after 2060. Namely, after 2060 the coverage drops and also older cohorts start to leave, which speeds up the process of financial unsustainability. Note that the funding rates show a much stabler

Funding Ratio		**Indexation**		**Population**	
$E[FR_{75}^N]$	1.700% (1.49%)	$E[i_{75}^c]$	€183.8 mln.	Participants$_{75}$	138,973 (76,798)
$Q_{0.05}[FR_{75}^N]$	0.000 %				
$Q_{0.5}[FR_{75}^N]$	2.500%	Pr["no indexation"]	-0.105 (0.002)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR_{75}^N]$	3.300%	Pr["full indexation"]	0.078 (0.017)	$E[RR_{75}^{m,2}]$	0.044 (0.056)
$Pr[FR_{75}^N < 100\%]$	-0.019	Pr["cond. indexation"]	0.026 (0.002)	$Q_{0.05}(RR_{75}^{m,2})$	0.000
		Pr["surplus sharing"]	0.061 (0.020)	$Q_{0.5}(RR_{75}^{m,2})$	0.023
Discontinuity		Pr["recovery plan"]	-0.043 (-0.007)	$Q_{0.95}(RR_{75}^{m,2})$	0.167
$Pr[FR_{75}^N = 0\%]$	0.005	Pr["sustain. cut"]	-0.038 (-0.009)		
Pr["no inflow"]	-0.071 (0.001)			$E[PR_{75}]$	0.290 (0.125)
$E[d_{25}]$	-0.078 (0.041)	**Premium**		$Q_{0.05}(PR_{75})$	0.019
$E[d_{45}]$	0.000 (0.000)	Pr["low. premium"]	0.061 (0.020)	$Q_{0.5}(PR_{75})$	0.209
$E[d_{64}]$	-0.083 (-0.024)	Pr["rec. premium"]	-0.071 (-0.007)	$Q_{0.95}(PR_{75})$	1.807

Table 5.5: Sensitivity: initial funding ratio.

The table presents the ALM-output as explained in Section 5.2. The reported outcomes represent the absolute change with regard to the outcomes in case of dynamic in- and outflow; that is, each reported value is computed as stated in equation (5.3) where A is the value with another initial funding ratio for dynamic in- and outflow (as reported in Table A.5), and A^{**} is the regular dynamic in- and outflow value (as reported in Table A.3). A negative value indicates that the tested sensitivity decreases compared to the dynamic in- and outflow case, whereas a positive values denotes an increase.

pattern however during the whole simulation horizon. The probability of underfunding declines by 1.9%, showing that the lower quantiles are not equally worse as with dynamic in- and outflow from the underfunded position.

The probabilities of the steering mechanisms corroborate the intuition. The positive adjustment mechanisms are triggered more often, since the funding rate reaches higher levels than dynamic in- and outflow with underfunding. Logically the instruments as no indexation, recovery plan, sustainability cut and recovery premiums are used less. The volatility of those negative adjustment mechanisms decreases also, showing that the fund is indeed financially more stable during the first few decades.

The discontinuity risks are lower; this makes sense since the participants react on the funding ratio which is higher in this situation. For the middle-aged cohorts the discontinuity risk remains very small, because the fund does not attain more 'bad' scenarios. The probability of defaulting remains nearly the same, while the probability of six consecutive years no new inflow decreases by 7.1%; this is due to the higher initial funding position, wherefore less new entrants abstain. As previously stated, there are only a few situations whereby all new participants abstain (especially in the lower quantiles). However, on average and in the median, stil 60%-90% of the new entrants abstains. The replacement rate and pension result show higher values compared to dynamic in- and outflow with underfunding, which confirms that the participants are better off and receive more indexation from the positive adjustment mechanisms. Ultimately, the fund ends up with 1.4k participants more than in the dynamic in- and outflow with underfunding. The cumulative weighted indexation measure affirms the observations, since more indexation is provided.

Sinking Giant

As stated in Chapter 2, a fund can turn into a Sinking Giant due to the inability to recover and the situation of less inflow; this paragraph shows the initial funding rate when this happens. After a thorough analysis, I attain the initial funding ratios whereby the fund goes bankrupt in (more than) one of the two scenarios (i.e. I research when the fund goes default for the median funding ratio). Hence, if such a situation occurs, it happens in the lower 50% of the scenarios as well; therefore, I perceive this measure to be consistent for attaining the moment of default.

If the transfers are conditional upon the funding ratio, a certain moment of default occurs if the fund starts with an initial funding ratio of 33%; after 29 years the fund is bankrupt. While if the participants receive 100% of their rights in case they leave, a certain moment of default occurs at an initial funding ratio of 71%; the fund is bankrupt after 73 years. At both initial coverage rates, the fund is not able to recover from the underfunding anymore, since the fund's population is too small and the instruments do not work any more. The fund is a Sinking Giant from 2015 till the default moment in both cases. In other words, if a fund starts with a funding ratio lower than the two aforementioned start positions of 33% and 71%, the probability of default becomes even higher. Clearly, a pension fund runs faster out of capital if it delivers the leaving participants 100% of their accrued rights.

To sum up, the fund underestimates the consequences of less inflow. The outcomes show that policymakers should take the fund's population into account and not only the funding ratio as boundary for activating steering instruments. Dynamic in- and outflow is still harmful for the sustainability of pension funds even if the funding ratio exceeds the level of 120%. Namely, the outflow of pension payments is higher than the inflow of premia, causing the fund to become a Sinking Giant.

Now, the sensitivity with respect to the interest rate is discussed.

5.2.3 Sensitivity: Model Parameters

The sensitivity of the results with respect to the interest and inflation rates is investigated now. Lower interest and inflation rates make the fund less able to adpat to shocks quickly, policy instruments have to be used more often and extreme scenarios happen more.

It is more realistic to assume an interest and inflation rate fluctuating around 0%, as prevails in the market nowadays: on March 10 2016, the ECB set an interest rate of 0%. I assume the situation will hold on in the long-term and, therefore, a Japan scenario is sketched: low interest and inflation rates for a substantial period of time. Concretely, I set the values of the interest rate as $r_0 = 0$ and $\kappa = 0$; while the values of the inflation rate are $I_0 = 0.5$ with $\iota \sim N(\mu = 0.5\%, \sigma = 1\%)$. The rest of the financial market analyser is kept the same. The transfers are conditional upon the prevailing funding ratio.

From Figure 5.10 it is seen that the mean and median funding ratio follow a similar pattern as the static and dynamic situation where a higher interest rate prevails (please compare with the dynamic situation in Figure 5.5). This is due to the policy of the fund. But, since the liabilities of the fund are higher due to the lower interest rate and assets gain lower returns, the steering mechanisms have to be activated more often. Note that the liabilities become higher, since the discount factor increases due to a lower interest rate. Moreover, the premium is also substantially higher than the 2% interest rate economy and equals 27.23% on average over 75 years (compared to 18-19% in the dynamic case with 2% interest). It is because the cost-covering premium depends on the prevailing discount factors (so, directly on the interest rate also).

The 5%- and 95%-quantiles show a different pattern than the 2% interest rate economy,

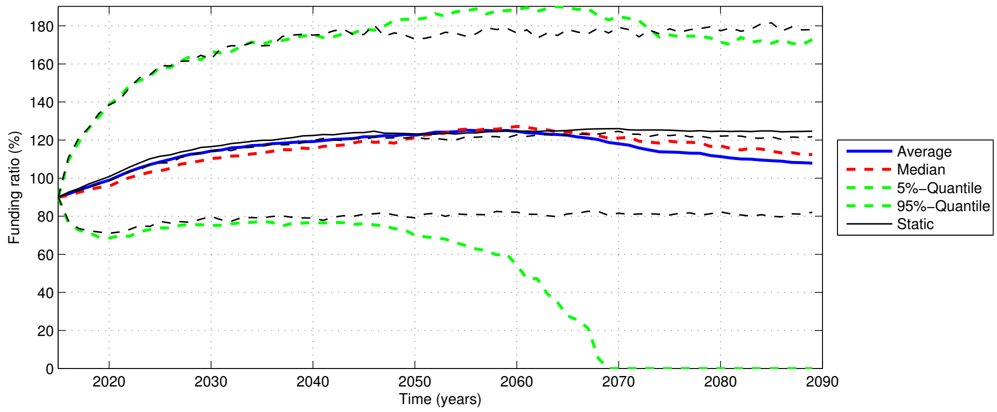


Figure 5.10: Sensitivity funding ratio: interest rate.
The x-axis displays time (in years) whereby the y-axis shows the nominal funding ratio in percentages under $\mathbb{P}$; as clear from the legend, the graph entails the mean and quantiles of the static (grey) and dynamic funding rates. The funding rates stem from the situation whereby the long-term interest rate equals $\kappa = 0\%$ and the inflation rate fluctuates around the level of $\mu = 0.5\%$.

however. Namely, regarding the lower quantile, the funding rates already drop after 2040, while in case of higher interest rates the coverage rates decline after 2050 (as shown in Figure 5.5). This is due to the effect of a lower interest rate, which causes an even less prosperous scenario wherefore the fund's asset gain lower returns and deplete faster due to higher transfer values since the liabilities (i.e. market-consistently discounted pension rights) are higher. The 95%-quantile displays the characteristic hump again. The hump is bigger than the 2%-case since the fund provides less indexation to their participants due to the lower inflation rate. Therefore, the fund keeps more surplus for a longer period of time.

The average funding ratio is almost 2% lower after 75 years with a higher volatility of 4.17% relative to the 2% economy, as Table 5.6 shows. In both economies the funding rate passes the threshold of 120% in 2040. However, in the 0%-case the funding rate reaches at 2068 the level of 120%, while in the 2%-case it reaches the 120% level already around 2064. Thus, the fund keeps the accrued buffer longer, which is because the inflation rate is lower and, consequently, less amount of indexation has to be provided to keep up with wage-growth. Subsequently, the funding rate is higher since the additional buffer is attained for four more years. During those four years new entrants are still willing to enter and the liabilities of the fund increase even more (relatively). In combination with the lower interest rate (causing a higher value of the liabilities), the average funding rate drops quicker after 2070.

Table 5.6 shows that the 5%- and 50%-quantiles remain the same: the lower quantile is still a bankrupt fund (with a funding rate of 0%). The upper quantile shows an improvement of 6.5%: the interest rate is lower, thus the fund can provide less indexation to keep up with economy-wide wage growth and, therefore, keeps a higher asset value (in the upper quantiles) which results in a higher funding rate. However, the mean probability of underfunding increases by 3.3%, which is a consequence of the higher liabilities and lower asset value (on average).

The probability of usage of the steering instruments show that especially the negative adjustment mechanisms are used more. Specifically, recovery plans and sustainability

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	-1.900% (4.17%)	$E[i^c_{75}]$	-€39.30 mln.	Participants$_{75}$	88,200
$Q_{0.05}[FR^N_{75}]$	0.000 %				(110,393)
$Q_{0.5}[FR^N_{75}]$	0.000%	Pr["no indexation"]	0.005 (0.029)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	6.500%	Pr["full indexation"]	0.022 (0.030)	$E[RR^{m,2}_{75}]$	-0.068 (0.001)
$\Pr[FR^N_{75} < 100\%]$	0.033	Pr["cond. indexation"]	-0.028 (0.001)	$Q_{0.05}(RR^{m,2}_{75})$	0.000
		Pr["surplus sharing"]	0.000 (0.031)	$Q_{0.5}(RR^{m,2}_{75})$	-0.063
Discontinuity		Pr["recovery plan"]	0.032 (0.032)	$Q_{0.95}(RR^{m,2}_{75})$	-0.002
$\Pr[FR^N_{75} = 0\%]$	0.017	Pr["sustain. cut"]	0.029 (0.031)		
Pr["no inflow"]	0.011 (0.026)			$E[PR_{75}]$	0.280 (0.130)
$E[d_{25}]$	-0.023 (0.010)	**Premium**		$Q_{0.05}(PR_{75})$	0.051
$E[d_{45}]$	0.009 (0.023)	Pr["low. premium"]	0.029 (0.031)	$Q_{0.5}(PR_{75})$	0.334
$E[d_{64}]$	0.022 (0.013)	Pr["rec. premium"]	0.029 (0.034)	$Q_{0.95}(PR_{75})$	2.415

Table 5.6: Sensitivity: interest rate.

The table presents the ALM-output as explained in Section 5.2. The reported outcomes represent the absolute change with regard to the outcomes in case of dynamic in- and outflow; that is, each reported value is computed as stated in equation (5.3) where A is the value with different interest and inflation rates for dynamic in- and outflow (as reported in Table A.6), and A^{**} is the regular dynamic in- and outflow value (as reported in Table A.3). A negative value indicates that the tested sensitivity decreases compared to the dynamic in- and outflow case, whereas a positive values denotes an increase.

cuts and recovery premia increase by 3%, since the fund has more trouble in recovering due to the lower interest rate. The full indexation option increases by 2.2%, because the inflation rate is also lower in this economy and, therefore, giving full indexation of 0.5% is easier attained than keeping up with a 2% inflation rate. The no indexation and surplus sharing options do not change much in usage, since the development of the funding ratio is similar to the economy with a 2% interest. The coverage rate keeps the same trend due to more benefit reductions. The conditional indexation option decreases since the inflation is lower and full indexation is more quickly attained; besides, it shows that extreme situations occur somewhat more often. The premium strategy confirms that extreme situations happen more often: both lower and recovery premia occur more frequently.

Due to the lower interest rate, the probability of defaulting rises by 1.7% and the probability of six consecutive years no inflow raises by 1.1%. Especially the discontinuity risk with respect to the middle-aged and older cohorts goes up, while the discontinuity risk with regard to new participants decreases. This is due to the fact that at the end of the simulation horizon the mean funding ratio is lower than the economy with 2% interest and, therefore, more 64-year-olds abstain (all the new entrants already abstain for an entry level below 120%). The discontinuity risk of 25-year-olds decreases since the funding rate maintains above the entry level $FR = 120\%$ for four years longer (till 2068) and new entrants enter the fund.

The replacement rate shows that after 2049 indexation does not keep up with wage growth, which is due to the declining funding ratio. This confirms the finding that middle-aged and older cohorts have more incentives to leave during this period, since their pension rights are negatively affected. A cohort that starts working in 2049 and retires in 2089 is worse off in case of lower interest rates. The pension result (tracking a cohort from 2015 till 2089) shows that indexation levels keep better up with the economy-wide wage growth; this confirms the reason about the fact that the probability of full indexation rises and, therefore, the pension result is higher. On average the fund has 88,200 participants more in the fund than in the 2% interest rate economy.

To conclude, lower interest and inflation rates amplify the negative effects on the stability of the fund. The probabilities of default, underfunding and no inflow increase. Besides, the steering mechanisms are activated more frequently to help the fund recover during the period of low interest rates. Note that all the results have an increase in standard deviation, which proves that the conditions become even more volatile and risky.

This section tested the sensitivity of the results in case discontinuity is taken into account. It attained the robustness of the results if the transfer values become 100% of the pension rights, if the fund starts in an equilibrium position, and if the economy shows low interest and inflation rates. Giving participants 100% of their pension rights when they leave, depletes the fund's assets rapidly and causes even lower funding rates, more extreme scenarios and higher discontinuity risks. Although the fund's instruments are activated more, it is not effective due to the smaller population. So, the fund becomes mre sensitive to shocks. Starting in equilibrium is (of course) more beneficial for the fund (higher funding rates, lower default and discontinuity probabilities) and their remaining participants (better replacement rate and pension result). Still, new participants abstain from the collective contract, causing an unexpected drop in the assets after four till five decades. Pension funds should not underestimate the effect of less inflow for a considerable period of time. Besides, funding rates lower than 33% and 71% cause the fund to become a Sinking Giant. Lower interest and inflation rates make the fund even less able to recover quickly and shows that extreme scenarios happen even more.

Chapter 6

Conclusion

This chapter concludes and summarises the main elements of the thesis, whereas it also presents recommendations and options for further research.

6.1 Summary & Conclusions

My thesis quantifies the discontinuity risk of pension funds. Specifically, it researches how participants react during a discontinuity event and, consequently, how these reactions (i.e. dynamic in- and outflow) affect the sustainability of the pension funds in The Netherlands.

The thesis explores the sustainability of collective funds if participation in collective pension funds is not mandatory but becomes voluntary. That is, the probability that new generations (of 25 years old) — when they enter the labour market — abstain from entering the collective pension contract, and/or the probability that older generations (from 26 till 64 years old) — currently in the collective fund — withdraw from the collective pension contract and enter the individual fund. This discontinuity risk arises due to inter-generational risk sharing. The literature provides arguments that inter-generational risk sharing is welfare enhancing, but it also advocates the main drawback: discontinuity risk. Risk sharing, and thereby discontinuity risk, is present in collective schemes, but absent in individual schemes. Participants may generate incentives to save for retirement individually if the funding ratio becomes too low, because the intrinsic benefits of participating in the collective pension fund might be less than the negative effect of underfunding.

I include this discontinuity risk in an ALM-model by researching the complex circumstances surrounding a discontinuity event (e.g. information cascades ar likely to occur). The current approach is namely static, i.e. it does not consider the discontinuity risk in the computations at pension funds. By including discontinuity, I introduce a new approach which is dynamic, i.e. it includes dynamic in- and outflow of participants which enables to research discontinuity risks. The dynamic in- and outflow depends on how participants react, whereby the reaction of the participants depends on the nominal published funding ratios. Namely, the nominal published funding ratio provides information about the financial position of the fund. Moreover, participants can easily interpret the funding ratio: assets over liabilities. For this reason, I construct a reaction function that is based on the prevailing funding ratio. The reaction function determines how participants react during a discontinuity event and, hence, it specifies the dynamic in- and outflow of the fund. The reaction of the participants is twofold: enter in a collective fund or enter in an individual fund. To decide when a participant enters a collective fund or not, current academic literature about break-even funding ratios is used. So, this function answers how participants behave during a discontinuity event.

Using Monte Carlo simulations, it is analysed how dynamic in- and outflow affects the sustainability of the collective fund. The economy, wherein the pension fund operates, is produced by the financial market analyser which is built upon the frequently used Black-Scholes-Vasicek model with a randomly distributed inflation rate. The considered pension fund is a typical, current Dutch hybrid fund that entails two main steering mechanisms: a premium and an indexation strategy conditional on the nominal funding rate. I specifically show how to include discontinuity risk in the computations. Namely, dynamic in- and outflow affects the fund's population. The assets are directly impacted by outflow, since the fund has to transfer the leaving participants their accrued pension rights; also implicitly the liabilities are affected of course.

The analysis shows that the pension fund is financially stable in the static situation. Besides, it is shown with value-based generational accounting for the static setting how certain policy instruments affect the remaining fund's participants. However, the pension fund is substantially less sustainable if the in- and outflow is dynamic. If only the inflow is dynamic, the fund suffers from a buffer-fall because the composition of the fund's population turns grey and becomes smaller. Since the population is smaller, the steering instruments have to be activated more often to keep the fund financially stable. In the short-run, dynamic inflow does not lead to any problems. However, for the long-term the fund becomes more sensitive to shocks and unstabler. If the in- and outflow is dynamic, the older-cohorts leave also besides the young, which is even less beneficial for the fund in terms of sustainability. Namely, the asset values declines due to the transferred pension rights. Consequently, the fund is less able to react to shocks. The number of participants in the fund decreases wherefore the policy instruments become less effective and have to be used more frequently. The discontinuity risks reach severe levels and the funding rates are substantially lower and more volatile; the fund is likely to go bankrupt in one up the thirteen scenarios. Dynamic in- and outflow, for this reason, affects the short- and long-term. Value-based generational accounting shows that discontinuity risk hurts the remaining participants in the same magnitude as benefit reductions and recovery premia. For this reason, discontinuity risk affects the fund's sustainability as well as the participants (while policy instruments affect the participants as well, but contribute to a healthy fund). If the interest and inflation rates in the economy are lower, the fund is less able to recover quickly and more extreme scenarios are likely to happen. Transferring all the pension rights (instead of conditioning upon the funding rate) severely hits the sustainability. If the initial funding ratio shows higher levels, a pension fund should not underestimate the continuous less inflow of new entrants otherwise it turns into a Sinking Giant.

So, I quantified discontinuity by reaction functions, which include an information cascade effect. Including discontinuity in the calculations (i.e. dynamic in- and outflow) shows that pension funds are less sustainable than the current static computations reveal: extreme scenarios are more likely to occur and on average the funding rates are lower. Specifically, the discontinuity risk with respect to new entrants and near retirees is high. Hereby, the research questions are satisfactorily answered.

6.2 Recommendations

First of all, pension fund managers should take both forms of sustainability into account: on the one hand creating a financially stable pension fund, while on the other hand creating a strong support from the participants — as briefly mentioned in Chapter 1. For example, benefit reductions contribute to a financially stable fund and future generations benefit from such a measure. However, near-retirees (who have accrued the most pension rights) become inclined to leave the collective fund since cuts in pension rights affect them the most compared to other generations. Recovery premia have a similar effect but then on young cohorts; nevertheless, steering with premia seems to be very ineffective. Thus, recovery premia do not contribute much to sustainability but creates (large) incentives for new entrants to abstain — as shown in Chapter 4.

Besides, the policy of smoothing shocks seems not to be optimal, if discontinuity is taken into account. Discontinuity risk can be reduced by dynamic smoothing periods that reduce the smoothing periods when a fund has a low coverage rate. If the fund is financially stable — therefore, in the information insensitive region — smoothing of shocks forms no problem. However, smoothing of shocks during periods of low funding rates is highly suboptimal in case of dynamic in- and outflow. Namely, smoothing shocks while attaining a low coverage rate inclines cohorts to abstain or leave and, consequently, the funding ratio becomes volatile. The smoothing of shocks spreads the burden, but the fund recovers less quickly due to the longer smoothing periods. Since the fund recovers less quick and the funding rate stays at a lower level, more participants exit and the fund becomes sensitive to shocks. The fund's population becomes smaller and steering with instruments becomes even less effective. If the smoothing periods are not revised during such a period, the fund faces a high probability of going bankrupt because it reaches a down-ward sloping spiral. So, the recommendation is (i) to make smoothing periods conditional upon the prevailing funding ratio and (ii) if the fund notices that their population turns grey (as with the initial funding ratio of 121.8%), policy makers should revise their mechanisms already. Especially, the smoothing periods may not be too long during periods with low funding rates because the fund's composition may change rapidly: smaller and more elderly. As a consequence, the system becomes even more unstable. If a fund has ten years time to recover from underfunding, while the fund has many elderly, the smoothing period must become shorter. Namely, most of the older participants have only a few remaining years left to live and smoothing over ten years is too long. The problems, then, transfer towards the ages 55-65, but those will leave the fund in case the funding rate drops below the break-even funding ratio.

Thirdly, managers of pension funds should not focus too much on the nominal funding ratio as sustainability indicator. As shown in Chapter 5 with dynamic inflow, the funding ratio displays a (positive) hump, whereby the funding rate starts to rise. This is due to the accrual of an additional buffer, caused by a smaller population and too long smoothing periods. The situation seems all fine for the fund, but it experiences a drop after several years in the coverage rate when new participants enter again: the buffer is depleted and the liabilities increase. The increase in funding rates, leading to the positive hump, shows a financially prosperous situation for the fund, while it actually faces a significantly smaller population due to the lower funding rates the years before.

As a consequence, this leads to the fourth recommendation: do not look at too short horizons, if you want to attain the effects of discontinuity. Namely, by looking at a horizon of say 45 years, everything will seem fine. However, afterwards the effect of less inflow will only be visible. For this reason, I investigate a horizon of 75 years also. In case of real funding ratios the discontinuity risk is likely to become even higher, since participants perceive what they receive for what they pay in — such a conclusion can also be found in the paper of Ewijk et al. (2007).

6.3 Further Research

This section describes options for further research with respect to the topic of discontinuity. But, I start to discuss with the limitations of my assumptions and model. First of all, participants (till the age of 65) can only abstain from the collective pension contract or leave the collective pension fund. For this reason, participants cannot enter the collective fund again once they withdrew and, consequently, they remain in the individual DC fund for the rest of their life-times. Secondly, the Black-Scholes-Vasick model with stochastic stock price and stochastic interest rate gives an approximation of reality, but a model with stochastic inflation rates (such as the Koijen-Nijman-Werker model) would be even more preferred; to achieve randomness in the inflation rate, I model the inflation rate as a normally distributed varaible. Besides, the fund has actually only a few investment options and attains a constant asset mix. It could be more realistic to include uniform life-cycle strategies also, as well as risk-preferences of the participants. Lastly, my research on a discontinuity event rests on current academic literature, but a possibility — which has been seriously considered — was to investigate dynamic in- and outflow via an option-like technique such as Chen et al. (2016). It would have tried to explore the circumstances whereby individual participants exercise the option to leave the collective fund.

Reaction Function

The reaction function is based upon papers from the literature, however the underlying assumptions and conditions in the papers are not identical to the ones in my thesis. I tried to set most of the assumptions in a similar way, but there are still some differences. For example, Siegmann (2011) uses a constant interest rate and Molenaar et al. (2011) consider taxes, while I use the Vasicek model for the short-rate and do not incorporate taxes; furthermore, I consider a non-stationary fund based on survival probabilities. Moreover, the policy rules of the pension funds are not completely identical; the contribution and indexation rules differ. Besides, minimum funding ratios are only computed till the age of 60 and, hence, have to be extrapolated for higher ages. An option would be to reduce my model to the simpler versions in the literature (i.e. such as a constant interest rate), but this seems not an option because you miss some key ingredients that are present in reality — in my opinion. So, further research is necessary to improve the correspondence.

Next, the discontinuity-function is a key input in the new dynamic approach, however a discontinuity event is still abstract wherefore I sketched the hypothetical and intuitive snowball-effect. Namely, it is unclear how and when things precisely happen, so it is of the utmost importance that further research is performed with regard to a discontinuity event. For a complete and concise analysis, an alternative is to consider utility functions for each participants for each setting. Answers might also be found in behavioural economics, since during such an event behavioural incentives play an important role (besides only the information cascade effect). Especially the dynamic outflow may receive more attention. It seems unrealistic that middle-aged cohorts want to stay in the fund even at a funding ratio of 5%. For this reason, a direction for further research lies also in the direction of game theory, because participants have to make intelligent rational decisions under uncertainty between cooperating in a collective fund or not. Game theory deals also with the principle of zero-sum games, which is present in a pension fund as well, of course. Another option for research lies in the field of extreme value theory, since discontinuity events happen only at the outer extremes of the probability distribution underlying the funding ratio.

Thirdly, discontinuity risks may also arise due to information based on premium levels; especially for new entrants this information could be important. Indifferent premium levels are also available in the literature (Molenaar et al., 2011), but since policy rules differ, the break-even premium levels can not be readily applied. Young generations (entering the labour market) could also be triggered by the prevailing (recovery) premium, consequently giving rise to another trigger for discontinuity. Namely, as shown in Chapter 5, the premium mechanism becomes unstable with dynamic in- and outflow.

Fourthly, in case the collective fund is left, the only remaining alternative is an individual fund. This theoretical setting measures purely the effects of inter-generational risk sharing, however in the real world there are more alternatives. For example, individual systems exist which also feature collective risk sharing. Besides, an individual DC is more expensive than a collective system and an individual fund can not maintain an equal advanced investment strategy (e.g. such as illiquid investments).

Finally, policy instruments — such as premium and indexation mechanisms — influence the decisions of the participants. It would, therefore, be interesting to analyse how pension funds can design their set of policy instruments to minimise the likelihood of discontinuity risks (i.e. that any cohort withdraws from the pension scheme). The functioning of the policy instruments is already captured in the reaction functions, but sensitivities are not, though.

Longevity Risk

In this thesis, I abstract from macro longevity risk in the model. There are two types of mortality risk, whereby I follow the definitions of Hári et al. (2008). Micro-longevity risk quantifies the risk related to uncertainty in time of death, if survival probabilities are known with certainty. Macro-longevity risk is due to uncertainty in survival probabilities. Micro-longevity risk, which is by definition diversifiable in collective schemes and not priced in the financial market, is easily taken into account. As I have done in my thesis, since I do not assume the fund's population to be stationary.

However, the projections of the survival probabilities are taken as a given and do not incorporate the uncertainty concerning those estimates, leading to macro-longevity risk. It becomes even more complicated if macro longevity risk is partially hedgeable in the financial market by longevity-linked bonds; this would introduce a new risk factor and, on the other hand, affect the liabilities in a systematic way (Nijman et al., 2013). As Hári et al. (2008) points out, uncertainty in the projections leads to riskiness in the funding ratios, which is not negligible but can be reduced by higher investments fractions in stocks.

Of course, a longevity shock could imply serious disadvantages for the sustainability of the pension fund. In case people live, for example from one day to another, 10 years longer (due to the invention of a pill or DNA transplantation), pension funds will not be able to absorb such a shock. Consequently, discontinuation of the fund becomes plausible. Hence, including the uncertainty in the survival probability projections is a topic for further research, hereby considering macro-longevity risk. Note that the government has already taken some steps in this direction. The statutory retirement age is recently linked to the life expectancy.

Fund's Population

The population of my pension fund represents the national Dutch population with a small effect of turning into a grey population. However, it would be interesting to observe the effects of an even more grey fund, such as occurs in Japan. Namely, in a grey fund there are less workers compared to retirees and, moreover, the elderly have more incentives to stay in the fund (as shown in Chapter 2) then the young. Hence, the effect of a grey fund might lead to an even stronger effect of discontinuity, since the young abstain more easily, while the elderly want to stay. Besides investigating a grey fund, one could also research the effects of a green fund, i.e. many young cohorts are in the fund and new young entrants are frequently entering the fund.

Information Sensitivity

With the aid of the qualitative framework of Holmstrom (2015), pension funds might be classified as being in the information sensitive or insensitive region. The reactions of the participants depend only on one information set in my analysis, namely on the published nominal funding ratio. However, since the Dutch pension system might make a transition towards Personal Pensions with Risk sharing (abbreviated PPR's), more information towards the participants will be provided. In other words, I assume in my thesis that participants have easy access to the nominal published funding rate whereby I assume that most of the civilians understand this measure. Now, if PPR's are introduced, participants will receive a sort mini balance sheet which provides much more information about the participants' pension accrual and is a second information set. This additional information set might provide other incentives for (future) participants to withdraw from the pension contract or not. To research the effect of either reacting on funding ratios or on balance sheets of PPR's, utility functions are necessary. Then, with the aid of the utility outcomes, a diff-in-diff comparison between information sets can be made. This allows to assess whether providing more information towards clients is actually beneficial or not; in other words, providing balance sheets of PPR's may increase the discontinuity risk, since the participant gets more insight and information about the fund's financial position.

Smoothing of Shocks

This final further research topic relates to the second recommendation. The smoothing periods are suboptimal in case of discontinuity, due to the smaller fund's population. I already recommended to make the smoothing periods conditional upon the funding level in combination with the magnitude of the population. Both conditions affect the effectiveness of the steering mechanisms. So, a nice question for further research is: what is the optimal smoothing period, given the probability of dynamic in- and outflow? If you do not smooth shocks, the yearly results of the fund become more volatile. Thus, smoothing of 2 or 3 years is maybe better, however it could be that such a period is also too long and a fund just has to accept a higher volatility in the funding rates?

Bibliography

Black, F. and M. Scholes (1973). “The Pricing of Options and Corporate Liabilities”. In: *Journal of Political Economy* 81.3, pp. 637–654.

Boeijen, D., C. Jansen, N. Kortleve, and J. Tamerus (2007). *Intergenerational solidarity in the uniform contribution and accrual system. In S. G. van der Lecq & O. W. Steenbeek (Eds.), Cost and benefits of collective pension systems*. Berlin: Springer.

Bovenberg, L. and R. Mehlkopf (2014). “Optimal Design of Funded Pension Schemes”. In: *De Economist* 6.1, pp. 445–474.

Bovenberg, L., T. Nijman, R. Koijen, and C. Teulings (2007). “Saving and investing over the life cycle and the role of collective pension funds”. In: *De Economist* 155.4, pp. 347–415.

Carlson, J. (1975). “Are Price Expectations Normally Distributed?” In: *Journal of the American Statistical Association* 70.352, pp. 749–754.

Chen, D.H.J., R.M.W.J. Beetsma, D.W.G.A. Broeders, and A.A.J. Pelsser (2016). “Sustainability of Participation in Collective Pension Schemes: An Option Pricing Approach”. In: *Netspar Academic Series/DNB Working Paper*.

Cui, J., F. de Jong, and E. Ponds (2005). “The value of intergenerational transfers within funded pension schemes”. In: *Netspar Discussion Paper* 022.

DNB (2010). “Indexatie pensioenen blijft achter bij loon- en prijsstijging”. In: *DNB / Statistich Bulletin*, pp. 27–31.

Easley, D. and J. Kleinberg (2010). *Networks, Crowds, and Markets: Reasoning about a Highly Connected World*. Cambridge Univeristy Press, pp. 483–505.

Ewijk, Casper van and Coen Teulings (2007). “Efficiëntie en continuïteit in pensioenen: het FTK nader bezien”. In: *Netspar NEA Papers (Netspar Economische Adviezen)* 3.

Galen, T. van and S. Lundbergh (Cardano). *Principles of Pensiondesign*. Tilburg, Netherlands: Drukkerij Groels.

Gillespie, D.T. (1996). “Exact numerical simulation of the Ornstein-Uhlenbeck process and its integral”. In: *Physical Review E* 54.2.

Gollier, C. (2008). “Intergenerational risk-sharing and risk-taking of a pension fund”. In: *Journal of Public Economics* 92.5-6, pp. 1463–1485.

Goossens, J. (2015). *Design Optimal Annuities (BSc Thesis)*. Tilburg: Tilburg University, Tilburg School of Economics and Management.

Hári, N., A. De Waegenaere, B. Melenberg, and T. Nijman (2008). “Longevity risk in portfolios of pension annuities”. In: *Insurance: Mathematics and Economics* 4.2, pp. 505–519.

Holmstrom, B. (2015). “Understanding the role of debt in the financial system”. In: *BIS Working Papers* 479.

http(1), Central Bureau for Statistics (CBS) (December 12th, 2015). *Gemiddeld inkomen van personen naar kenmerken en naar regio*. Acccessed: May 12th, 2016, URL: `http:`

//statline.cbs.nl/StatWeb/publication/?VW=T&DM=SLNL&PA=80500NED&D1=2-3&D2=l&D3=0-38&D4=0-4&D5=l&HD=100903-1406&HDR=G4,T,G1,G3&STB=G2.

http(2), Central Bureau for Statistics (CBS - Wim Bos) (May 28th, 2014). *Pensioenkloof tussen mannen en vrouwen nog steeds groot*. Acccessed: May 12th, 2016, URL: https://www.cbs.nl/nl-nl/nieuws/2014/22/pensioenkloof-tussen-mannen-en-vrouwen-nog-steeds-groot.

http(3), DNB (July, 2015). *De Nederlandsche Bank*. URL: http://www.dnb.nl/home/index.jsp.

http(4), KPMG Valuations Netherlands (April 12th, 2016). *Equity Market Risk Premium — Research Summary*. Acccessed: March 30th, 2016, URL: http://www.kpmg.com/NL/nl/IssuesAndInsights/ArticlesPublications/Documents/PDF/Transactions-Restructuring/MRP-Summary-12-april-2016.pdf.

http(5), OECD (March, 2016). *Organisation for Economic Co-operation and Development*. URL: http://www.oecd.org/.

Janssen, K. (2012). *The Comparison of Different Implementations of the Holistic Balance Sheet for Pension Funds (MSc Thesis)*. Tilburg: Tilburg University, Tilburg School of Economics and Management.

Kocken, T. (2008). *Curious Contracts: Pension Fund Redesign for the Future*. 's-Hertogenbosch, Netherlands: Tutein Nolthenius.

— (2016). "Een collectieve pot leidt nu eenmaal tot gezeik". In: *Volkskrant, by Yvonne Hoofs*.

Kortleve, N. and E. Ponds (2006). *Pension deals and value-based ALM. In: N. Kortleve, T. Nijman & E.H.M. Ponds (Eds.), Fair value and pension fund management*. Amsterdam, Elsevier, pp. 181–209.

Lekniute, Z. (2011). *A Value-Based Approach to Pension Plan Redesign in the Netherlands: Who Will Gain and Who Will Lose? (MSc Thesis)*. Tilburg: Tilburg University, Faculty of Economics and Business Administration.

Lekniute, Z., R. Beetsma, and E. Ponds (2014). "A Value-Based Approach to Pension Redesign in the US State Plans". In: *Netspar Discussion Paper*.

Molenaar, R., K. Peijnenburg, and E. Ponds (2011). "Should I Stay or Should I Go? Break Even Funding Ratios for DB Pension Plan Participants". In: *Netspar Discussion Papers* 04.027.

Nijman, T., S. Stalborch, J. van Toor, and B. Werker (2013). "Formalizing the new Dutch pension contract". In: *Netspar Occasional Papers*.

Schumacher, J.M. (2015). *Financial Models*. Tilburg University: Tilburg School of Economics and Management.

Siegmann, A. (2011). "Minimum Funding Ratios for Defined-Benefit Pension Funds". In: *Journal of Pension Economics and Finance* 10.03, pp. 417–437.

Tyagi, M. (2016). *Risk in Pension Plans (dissertation)*. Maastricht: Maastricht University School of Business and Economics.

Wernekinck, F. (2013). *The Holistic Balance Sheet: An Analysis of Benefit Reduction and Conditional Indexation (MSc Thesis)*. Tilburg: Tilburg University, Tilburg School of Economics and Management.

Appendix A

Outcomes ALM-model

This appendix shows the precise ALM-outcomes of my ALM-simulations. The first section presents the results for the case of dynamic inflow (entry levels $FR = 96\%$ and $FR = 120\%$), and for the case of dynamic in- and outflow. The second section displays the outcomes for the simulations that are performed for the sensitivity analysis: other transfer value, another initial funding ratio, low inflation and interest rates.

All the results in this thesis — unless stated otherwise — are obtained by an ALM-model which is programmed in `Matlab`; the results are available upon request.

A.1 Dynamic Outcomes

Funding Ratio		**Indexation**		**Population**	
$E[FR_{75}^N]$	122.9% (29.00%)	$E[i_{75}^c]$	€414.9 mln.	Participants$_{75}$	2,214,394
$Q_{0.05}[FR_{75}^N]$	81.20 %				(641,240)
$Q_{0.5}[FR_{75}^N]$	119.9%	Pr["no indexation"]	0.419 (0.197)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR_{75}^N]$	175.2%	Pr["full indexation"]	0.346 (0.192)	$E[RR_{75}^{m,2}]$	1.036 (0.390)
$\Pr[FR_{75}^N < 100\%]$	0.231	Pr["cond. indexation"]	0.235 (0.092)	$Q_{0.05}(RR_{75}^{m,2})$	0.613
		Pr["surplus sharing"]	0.247 (0.168)	$Q_{0.5}(RR_{75}^{m,2})$	0.943
Discontinuity		Pr["recovery plan"]	0.155 (0.122)	$Q_{0.95}(RR_{75}^{m,2})$	1.771
$\Pr[FR_{75}^N = 0\%]$	0.000	Pr["sustain. cut"]	0.152 (0.136)		
Pr["no inflow"]	0.073 (0.087)			$E[PR_{75}]$	0.825 (0.301)
$E[d^{25}]$	0.361 (0.420)	**Premium**		$Q_{0.05}(PR_{75})$	0.113
$E[d^{45}]$	0.000 (0.000)	Pr["low. premium"]	0.247 (0.168)	$Q_{0.5}(PR_{75})$	0.548
$E[d^{64}]$	0.000 (0.000)	Pr["rec. premium"]	0.227 (0.147)	$Q_{0.95}(PR_{75})$	2.978

Table A.1: Outcomes dynamic inflow: entry level $FR = 96\%$.

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	117.6% (36.16%)	$E[i^c_{75}]$	€359.5 mln.	Participants$_{75}$	1,323,182
$Q_{0.05}[FR^N_{75}]$	69.24 %				(709,110)
$Q_{0.5}[FR^N_{75}]$	117.2%	Pr[“no indexation”]	0.412 (0.211)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	172.7%	Pr[“full indexation”]	0.366 (0.197)	$E[RR^{m,2}_{75}]$	1.059 (0.483)
$\Pr[FR^N_{75} < 100\%]$	0.265	Pr[“cond. indexation”]	0.221 (0.092)	$Q_{0.05}(RR^{m,2}_{75})$	0.461
		Pr[“surplus sharing”]	0.270 (0.173)	$Q_{0.5}(RR^{m,2}_{75})$	0.987
Discontinuity		Pr[“recovery plan”]	0.166 (0.150)	$Q_{0.95}(RR^{m,2}_{75})$	1.959
$\Pr[FR^N_{75} = 0\%]$	0.033	Pr[“sustain. cut”]	0.164 (0.160)		
Pr[“no inflow”]	0.303 (0.210)			$E[PR_{75}]$	0.879 (0.350)
$E[d^{25}]$	0.633 (0.424)	**Premium**		$Q_{0.05}(PR_{75})$	0.056
$E[d^{45}]$	0.000 (0.000)	Pr[“low. premium”]	0.270 (0.173)	$Q_{0.5}(PR_{75})$	0.640
$E[d^{64}]$	0.000 (0.000)	Pr[“rec. premium”]	0.236 (0.173)	$Q_{0.95}(PR_{75})$	3.565

Table A.2: Outcomes dynamic inflow: entry level $FR = 120\%$.

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	109.8% (41.49%)	$E[i^c_{75}]$	€241.7 mln.	Participants$_{75}$	934,215
$Q_{0.05}[FR^N_{75}]$	0.000 %				(725,690)
$Q_{0.5}[FR^N_{75}]$	112.4%	Pr[“no indexation”]	0.452 (0.212)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	166.3%	Pr[“full indexation”]	0.307 (0.196)	$E[RR^{m,2}_{75}]$	0.925 (0.433)
$\Pr[FR^N_{75} < 100\%]$	0.321	Pr[“cond. indexation”]	0.242 (0.089)	$Q_{0.05}(RR^{m,2}_{75})$	0.000
		Pr[“surplus sharing”]	0.213 (0.166)	$Q_{0.5}(RR^{m,2}_{75})$	0.891
Discontinuity		Pr[“recovery plan”]	0.169 (0.142)	$Q_{0.95}(RR^{m,2}_{75})$	1.645
$\Pr[FR^N_{75} = 0\%]$	0.074	Pr[“sustain. cut”]	0.162 (0.146)		
Pr[“no inflow”]	0.335 (0.217)			$E[PR_{75}]$	0.770 (0.285)
$E[d^{25}]$	0.693 (0.407)	**Premium**		$Q_{0.05}(PR_{75})$	0.065
$E[d^{45}]$	0.022 (0.101)	Pr[“low. premium”]	0.213 (0.166)	$Q_{0.5}(PR_{75})$	0.460
$E[d^{64}]$	0.321 (0.406)	Pr[“rec. premium”]	0.245 (0.165)	$Q_{0.95}(PR_{75})$	2.597

Table A.3: Outcomes dynamic in- and outflow.

A.2 Sensitivity Analysis

Funding Ratio		**Indexation**		**Population**	
$E[FR_{75}^N]$	0.686% (63.33%)	$E[i_{75}^c]$	€205.7 mln.	Participants$_{75}$	675,710 (785,742)
$Q_{0.05}[FR_{75}^N]$	0.000 %				
$Q_{0.5}[FR_{75}^N]$	87.03%	Pr["no indexation"]	0.562 (0.284)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR_{75}^N]$	160.1%	Pr["full indexation"]	0.243 (0.219)	$E[RR_{75}^{m,2}]$	0.605 (0.599)
$\Pr[FR_{75}^N < 100\%]$	0.578	Pr["cond. indexation"]	0.195 (0.115)	$Q_{0.05}(RR_{75}^{m,2})$	0.000
		Pr["surplus sharing"]	0.171 (0.176)	$Q_{0.5}(RR_{75}^{m,2})$	0.738
Discontinuity		Pr["recovery plan"]	0.293 (0.241)	$Q_{0.95}(RR_{75}^{m,2})$	1.527
$\Pr[FR_{75}^N = 0\%]$	0.424	Pr["sustain. cut"]	0.253 (0.216)		
Pr["no inflow"]	0.480 (0.311)			$E[PR_{75}]$	0.669 (0.230)
$E[d^{25}]$	0.762 (0.385)	**Premium**		$Q_{0.05}(PR_{75})$	0.045
$E[d^{45}]$	0.219 (0.342)	Pr["low. premium"]	0.171 (0.176)	$Q_{0.5}(PR_{75})$	0.253
$E[d^{64}]$	0.493 (0.462)	Pr["rec. premium"]	0.377 (0.269)	$Q_{0.95}(PR_{75})$	2.484

Table A.4: Outcomes for a transfer value as if the funding ratio equals 100%.

Funding Ratio		**Indexation**		**Population**	
$E[FR_{75}^N]$	111.5% (42.98%)	$E[i_{75}^c]$	€425.5 mln.	Participants$_{75}$	1,073,188 (802,488)
$Q_{0.05}[FR_{75}^N]$	0.000 %				
$Q_{0.5}[FR_{75}^N]$	114.9%	Pr["no indexation"]	0.347 (0.214)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR_{75}^N]$	169.6%	Pr["full indexation"]	0.385 (0.213)	$E[RR_{75}^{m,2}]$	0.969 (0.489)
$\Pr[FR_{75}^N < 100\%]$	0.302	Pr["cond. indexation"]	0.268 (0.091)	$Q_{0.05}(RR_{75}^{m,2})$	0.000
		Pr["surplus sharing"]	0.274 (0.186)	$Q_{0.5}(RR_{75}^{m,2})$	0.914
Discontinuity		Pr["recovery plan"]	0.126 (0.135)	$Q_{0.95}(RR_{75}^{m,2})$	1.812
$\Pr[FR_{75}^N = 0\%]$	0.079	Pr["sustain. cut"]	0.124 (0.137)		
Pr["no inflow"]	0.264 (0.216)			$E[PR_{75}]$	1.060 (0.410)
$E[d^{25}]$	0.615 (0.448)	**Premium**		$Q_{0.05}(PR_{75})$	0.084
$E[d^{45}]$	0.022 (0.101)	Pr["low. premium"]	0.274 (0.186)	$Q_{0.5}(PR_{75})$	0.669
$E[d^{64}]$	0.238 (0.382)	Pr["rec. premium"]	0.174 (0.158)	$Q_{0.95}(PR_{75})$	4.404

Table A.5: Outcomes for an initial funding ratio of 121.8%.

Funding Ratio		**Indexation**		**Population**	
$E[FR^N_{75}]$	107.9% (45.66%)	$E[i^c_{75}]$	€202.4 mln.	Participants$_{75}$	1,022,415 (836,083)
$Q_{0.05}[FR^N_{75}]$	0.000 %				
$Q_{0.5}[FR^N_{75}]$	112.4%	Pr["no indexation"]	0.457 (0.241)	**Repl. Rate &**	**Pens. Result**
$Q_{0.95}[FR^N_{75}]$	172.8%	Pr["full indexation"]	0.329 (0.226)	$E[RR^{m,2}_{75}]$	0.857 (0.434)
$\Pr[FR^N_{75} < 100\%]$	0.354	Pr["cond. indexation"]	0.214 (0.090)	$Q_{0.05}(RR^{m,2}_{75})$	0.000
		Pr["surplus sharing"]	0.242 (0.197)	$Q_{0.5}(RR^{m,2}_{75})$	0.828
Discontinuity		Pr["recovery plan"]	0.201 (0.174)	$Q_{0.95}(RR^{m,2}_{75})$	1.643
$\Pr[FR^N_{75} = 0\%]$	0.091	Pr["sustain. cut"]	0.191 (0.177)		
Pr["no inflow"]	0.346 (0.243)			$E[PR_{75}]$	1.050 (0.415)
$E[d^{25}]$	0.670 (0.417)	**Premium**		$Q_{0.05}(PR_{75})$	0.116
$E[d^{45}]$	0.031 (0.124)	Pr["low. premium"]	0.242 (0.197)	$Q_{0.5}(PR_{75})$	0.794
$E[d^{64}]$	0.343 (0.419)	Pr["rec. premium"]	0.274 (0.199)	$Q_{0.95}(PR_{75})$	5.012

Table A.6: Outcomes for an interest rate of 0% and inflation rate of 0.5%.