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The effect of the assumed interest rate and smoothing on variable annuities

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Abstract

In this paper we consider the risk-return tradeoff for variable annuities, focusing on defined contribution retirement facilities in the Dutch institutional setting. In particular, we study the effect of the assumed interest rate. We also consider in detail the consequences of the possibility to smooth financial market shocks over the remaining retirement period. Our analysis is based on an explicit distribution of initial pension wealth over the pension payments at various horizons. We briefly discuss the effects of sharing micro longevity risk.

Keywords

assumed interest rate (AIR), habit formation, smoothing financial market shocks

Samenvatting

In dit paper onderzoeken we de trade-off tussen risico en rendement in de verbeterde premieregelingen voor variabele uitkeringen in de uitkeringsfase, met de nadruk op de Nederlandse institutionele setting. Onze focus ligt op variabele uitkeringen in een individuele DC regeling. Daarbij bestuderen we het specifieke effect van de zogenaamde projectierente en de wettelijk mogelijke vaste daling. Ook analyseren we de consequenties van de mogelijkheid tot het uitsmeren van financiële markt schokken over de resterende pensioenperiode. Onze analyse is gebaseerd op een expliciete toewijzing van het initiële pensioenvermogen aan pensioenuitkeringen op verschillende horizonnen. Beknopt bediscussiëren we de effecten van het delen van (micro) langlevenrisico.

1 Introduction

Recently[1], the Dutch parliament passed a law that enables retirees to invest their pension wealth in assets that involve risk. Prior to the enactment of this law it was compulsory to convert defined contribution pension wealth at retirement into a life-long *fixed* or indexed[2] annuity. Under the new law, life-long *variable* annuities are allowed as well. This new law pertains to Dutch retirees who, during the accumulation phase, participate in a defined contribution retirement plan ("premieovereenkomst").

In this paper we study the risk-return tradeoff for variable annuities, focusing in particular on the choice of the assumed interest rate (AIR) and the effect of smoothing financial market shocks. Both aspects have received much attention in the Dutch policy debate. We derive exact analytical expressions for the distribution of pension payments at various horizons given an assumed interest rate, with the option of smoothing financial market returns.

As to the assumed interest rate, this in essence determines the allocation of initial retirement wealth over pension payments at various horizons. Thus, a higher AIR leads to higher immediate pension payments at the cost of lower later pension payments. In particular we study the question of obtaining pension payments that are constant in expected nominal terms. In the Dutch institutional setting the assumed interest rate is modeled on an exact basis using a "projected interest rate" (which by law equals the risk-free rate) and a "fixed decrease". For the purpose of this paper, only the sum of the two is relevant. We call this the assumed interest rate in line with the extant academic literature. Section 3 discusses the details.

As to the Dutch policy discussion, we also explicitly study the possibility of combining fixed and variable annuities. In that case, the combined product provides a guaranteed minimum pension level, unlike products that re-balance continuously. We discuss this possibility in Section 3.1. We also point out several issues with sharing longevity risk in Section 3.2. Finally, and relevant for the current debate, we show that pension payments with a horizon of "only" ten years are fairly insensitive to the choice of the AIR (or fixed decrease). As a result, communication about the effect of choosing a fixed decrease is preferably based on results for horizons closer to 20 years; see Section 3.3.

The possibility of "smoothing" financial market returns has been incorporated in Dutch law in order to allow for preferences that exhibit habit formation. Smoothing financial market returns essentially means that a decrease of, say, 10% in pension wealth needs not be translated immediately into a decrease of 10% in pension payments. This implies that pension payments will decrease, due to this adverse financial market development, at a slower pace, but ultimately by more than 10%. We explicitly model this

[1] https://www.eerstekamer.nl/stenogramdeel/20160614/initiatief_lodders_wet_verbeterde_2

[2] The "indexed annuity" allowed under Dutch law is one where the yearly increase is predetermined and not dependent on investment returns.

possibility and show how it influences the risk-return tradeoff for variable pension payments; see Section 4.

In case constant nominal expected pension payments are preferred, smoothing leads to a horizon-dependent assumed interest rate, the so-called BNW[3] discount curve. We re-obtain this result in the present setting and, using the terminology of the new Dutch law, this translates into a horizon-dependent "fixed decrease".

An important issue in the Dutch pension debate is the conversion risk. This is the risk that, when accumulated DC wealth needs to be converted into an annuity, interest rates are very low. In such a situation, the DC capital leads to low pension payments. We document that, unlike popular belief, variable annuities do not provide a (real) solution to this problem. In the setting of this paper, low interest rates have the same negative effect on pension payments for variable as for fixed annuities. In other words, variable annuities initially lead to higher pension than fixed annuities, but their interest rate sensitivity is the same. The only viable way of dealing with conversion risk is to use appropriate life-cycle investment strategies (in terms of bond duration) during the accumulation phase. We discuss this in detail in Section 6.

The rest of this paper is organized as follows. In Section 2 we describe the Merton financial market that we consider. Section 3 considers the risk-return tradeoff for variable annuities *without* smoothing and deals in particular with guarantees, longevity risk, and communication. Section 4 then considers the setting *with* smoothing of financial market shocks. Section 5 relates our paper to the Dutch institutional setting in terms of the abovementioned "fixed decrease", and in Section 6 we discuss the possible presence of interest rate risk. Section 7 concludes the paper. All numerical illustrations are based on an Excel spreadsheet that can be obtained from the authors.

[3] Lans Bovenberg, Theo Nijman, and Bas Werker, "Voorwaardelijke Pensioenaanspraken: Over Waarderen, Beschermen, Communiceren en Beleggen", Netspar Occasional Paper 2012.

2 The financial market

For the financial market we consider the standard Black-Scholes/Merton setting. This means in particular that there is a constant (geometrically compounded) interest rate r and a single high-risk asset whose price at time t we denote by S_t.

$$\begin{aligned} \mathrm{d}S_t &= \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t & (2.1) \\ &= (r + \lambda\sigma) S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t. & (2.2) \end{aligned}$$

In this equation μ stands for the (arithmetically compounded[4]) expected return, σ is the (instantaneous) stock volatility, λ is the Sharpe ratio $\lambda = (\mu - r)/\sigma$, and Z is a standard Brownian motion with standard normal distribution, i.e. $Z \sim N(0,1)$. This implies that stock returns are log-normally distributed and, in particular, do not exhibit jumps or time-varying volatility. This is, deliberately, the simplest financial market that can be studied.

We leave out other risk factors. In particular, we ignore longevity risk, although we discuss this to some extent in Section 3.2. When calculating payments in real terms, we consider a constant, geometrically compounded, inflation rate π.

[4]Note that for this specification the expected arithmetically compounded return is $\mathbb{E}\left[\frac{S_{t+1}}{S_t} - 1\right] = e^{\mu - \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2} - 1 = e^{\mu} - 1 \approx \mu$, while the expected geometrically compounded return equals $\mathbb{E}\left[\log\left(\frac{S_{t+1}}{S_t}\right)\right] = \mu - \frac{1}{2}\sigma^2$.

3 Variable annuities without smoothing

Consider a retiree who enters retirement with total DC wealth W_t at time t and, who needs to finance H annual pension payments at times $t+h$, $h=0,\ldots,H-1$. For now we assume H to be given, i.e., we consider fixed-term instead of life-long variable annuities. Think of H as the remaining life expectancy at retirement age. Longevity risk is discussed in Section 3.2.

The pension payment at each horizon $h=0,\ldots,H-1$ has to be financed from the initial total pension wealth W_t. If we denote by $W_t(h)$ the market-consistent value of the pension payment at year t available for horizon h, the budget constraint implies

$$W_t = \sum_{h=0}^{H-1} W_t(h). \tag{3.1}$$

Stated otherwise, at time t we consider an amount of wealth $W_t(h)$ that is available to finance the pension payment at time $t+h$. The actual pension payment will of course depend on the investment strategy that is followed and the financial market returns. We can, conceptually, allow for a different investment strategy for the wealth allocated to each horizon $h=0,\ldots,H-1$. This is indeed precisely what happens when financial market returns are smoothed, see Section 4.

The way total pension wealth W_t are allocated over wealth $W_t(h)$, for each horizon $h=0,\ldots,H-1$, determines implicitly the assumed interest rate (AIR) $a_t(h)$. That is, the assumed interest rate is defined through

$$\frac{W_t(h)}{W_t} = \frac{W_t(h)}{\sum_{k=0}^{H-1} W_t(k)} = \frac{\exp\left(-h a_t(h)\right)}{\sum_{k=0}^{H-1} \exp\left(-k a_t(k)\right)}. \tag{3.2}$$

Note that this implies in particular

$$\frac{W_t(h)}{W_t(0)} = \exp\left(-h a_t(h)\right). \tag{3.3}$$

Intuitively, relation (3.3) determines the relative size of the wealth allocated to the various horizons h. Jointly with the total wealth W_t available in (3.1), this determines each $W_t(h)$. Below we introduce a running example to illustrate this allocation of initial total pension wealth W_t to each wealth $W_t(h)$ reserved for the pension payment at horizon h, see Figure 1.

A few comments are in place. Firstly, the assumed interest rate $a_t(h)$ is usually taken as given and is used to determine the allocation of total wealth over the various payments rather than the other way around as suggested here. Equations (3.2) and (3.3) show that the two approaches are equivalent. Secondly, note that $a_t(h)$ generally does not need to be constant in h, i.e., there can be an assumed interest rate *term structure* or, equivalently, a horizon-dependent assumed interest rate. This will become particularly relevant when discussing the possibility of smoothing in Section 4. Finally, the Dutch institutional setting uses the notions of projected rate ("projectierente") and fixed decrease ("vaste daling"). We will discuss, in Section 5, that their sum equals the assumed interest rate.

We introduce a running example that is used as illustration throughout this paper. We use the following standard parameters in this example: the expected (arithmetically compounded) return is $\mu = 6.00\%$, the stock volatility is $\sigma = 20.00\%$, the risk-free (geometrically compounded) interest rate is $r = 2.00\%$, which leads to a Sharpe ratio of $\lambda = 0.20$.

We fix the horizon at $H = 20$ year; in Section 3.2 we show how to incorporate mortality risk. The initial wealth is assumed to be equal to $W_t = €100,000$. Figures 1 and 2 show, for two different AIRs, the percentage of total wealth W_t that is allocated to each horizon, i.e., we depict $W_t(h)/W_t$. These figures are calculated using (3.2).

Let us, for the sake of illustration, assume that the wealth $W_t(h)$ for each horizon h will be completely invested in a risk-free bank account. Using an assumed interest rate of 2.00% implies that each subsequent bucket that is allocated to a specific year contains 2.00% less of the initial wealth than the previous bucket; see Figure 1. Since each bucket will grow each year at the risk-free rate, we ultimately get a constant stream of pension payments, i.e., a fixed annuity. The initial allocation to the first bucket will be largest since it will not earn any interest income, while the second bucket can contain 2.00% less since the additional income from interest for one year is also 2.00%.

Now consider Figure 2, which is based on an AIR of $a = 3.00\%$. In this case each subsequent bucket will contain 3.00% fewer initial wealth than the previous one. As a result, more wealth is allocated to earlier payments and less to later payments. Stated differently, since the initial total wealth W_t is given, the higher earlier payments will lead to lower later payments. This is visible in Figure 2, where, compared to Figure 1, the first payment is 9.1% higher while the last payment is 9.8% lower. In terms of pension payments, an entirely risk-free investment would lead to a $r = 2.00\%$ return every year. As a result, pension payments will decrease by $3.00\% - 2.00\% = 1.00\%$ every year.

Now, let us consider what happens in case each of the buckets is not invested risk-free but partly in the risky stock S_t. That is, we invest each $W_t(h)$ in a continuously rebalanced strategy with a stock exposure w. Standard calculations then show that wealth $W_t(h)$, for the pension payment at time h, evolves as

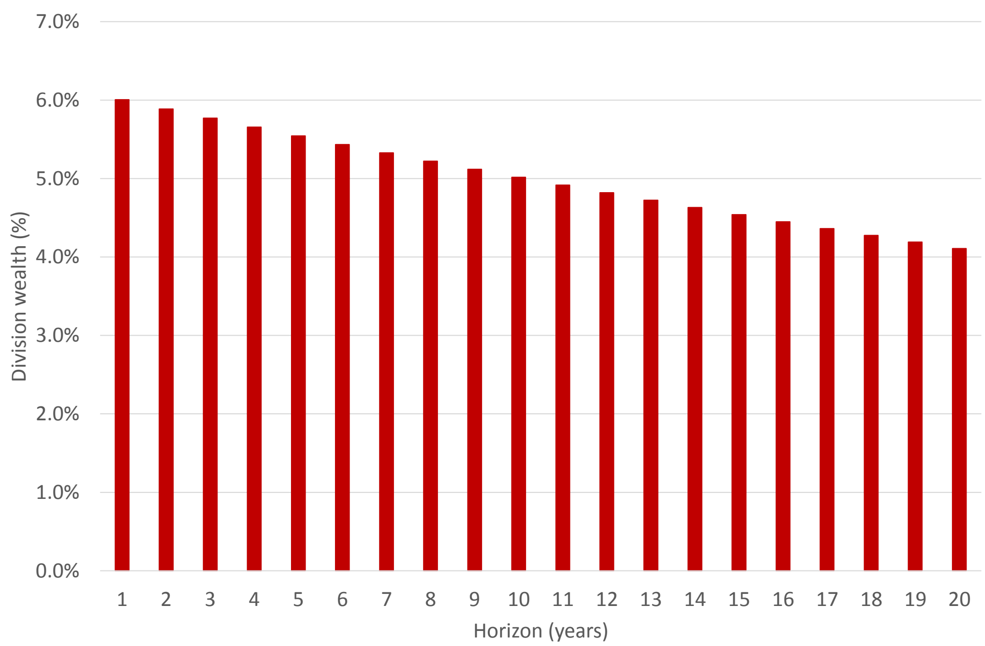
$$\mathrm{d}W_t(h) = (r + w\lambda\sigma)\, W_t(h)\mathrm{d}t + w\sigma W_t(h)\mathrm{d}Z_t. \tag{3.4}$$

Using Itô's lemma we find

$$\mathrm{d}\log W_t(h) = \left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right)\mathrm{d}t + w\sigma\mathrm{d}Z_t. \tag{3.5}$$

As a result, the pension payment at horizon h, $W_{t+h}(h)$, follows a log-normal distribution with parameters

Figure 1: *Wealth division for* $a_t(h) = 2.00\%$ *for all* $h \in [0, \ldots, 20]$



$h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right)$ and $hw^2\sigma^2$. In particular, the expected pension payment at horizon h is given by

$$\begin{aligned} \mathbb{E}_t[W_{t+h}(h)] &= W_t(h)\exp\left(h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right) + \frac{1}{2}hw^2\sigma^2\right) \\ &= W_t(h)\exp\left(h\left(r + w\lambda\sigma\right)\right). \end{aligned} \tag{3.6}$$

Risk in the pension payment $W_{t+h}(h)$ at horizon h can be determined by calculating the volatility of the payment. However, we even easily get the quantiles for the distribution. The quantile at level α is given by

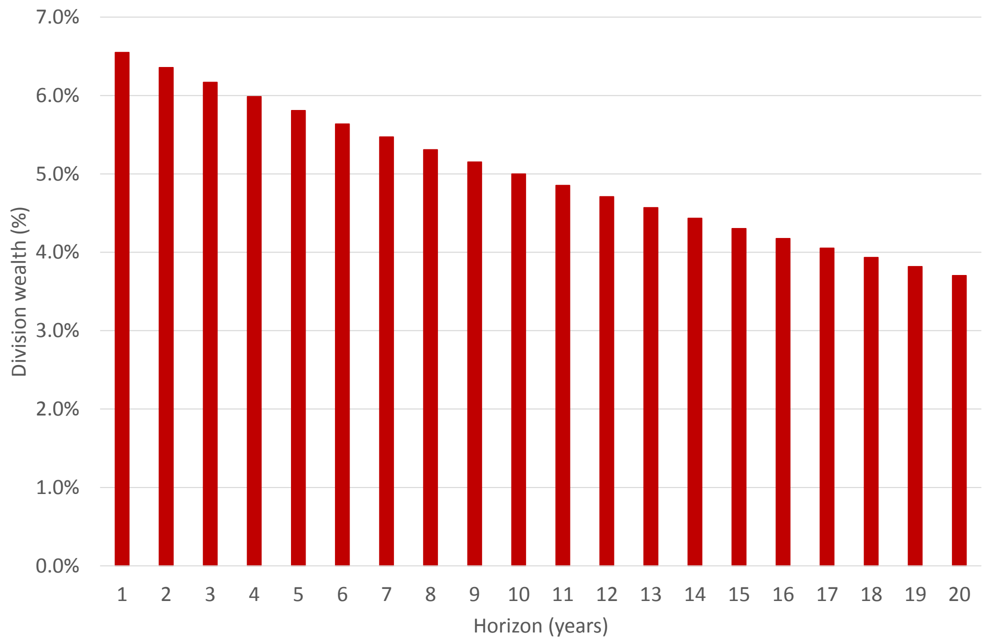
$$Q_t^{(\alpha)}\left(W_{t+h}(h)\right) = W_t(h)\exp\left(h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right) + z_\alpha\sqrt{h}w\sigma\right), \tag{3.7}$$

where z_α denotes the corresponding quantile of the standard normal distribution.

In case one is interested in the real pension payment, the above expected payoff and quantiles simply need to be multiplied by $\exp(-h\pi)$, assuming that the rate of inflation is constant and equal to π.

One may be interested to choose the assumed interest rate $a_t(h)$ in such a way that the expected pension payments are constant with respect to h, i.e., such that $\mathbb{E}_t[W_{t+h}(h)] = W_t(0)$ (recall that the first pension payment $W_t(0)$ is without investment risk). From (3.6)

Figure 2: *Wealth division for* $a_t(h) = 3.00\%$ *for all* $h \in [0, \ldots, 20]$



we find that this implies

$$\frac{W_t(h)}{W_t(0)} = \exp\left(-h\left(r + w\lambda\sigma\right)\right), \tag{3.8}$$

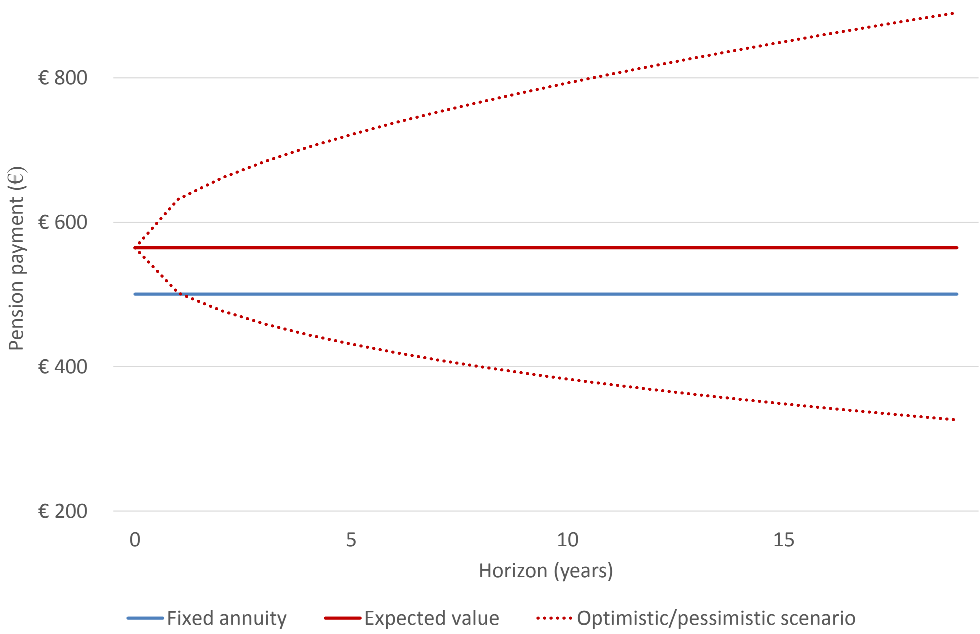
or, using (3.3),

$$a_t(h) = r + w\lambda\sigma. \tag{3.9}$$

This constant assumed interest rate is expected to lead to nominally constant pension payments. In case our financial market would exhibit interest rate risk (that is, a horizon-dependent risk-free *term structure*) and/or stock market predictability, we would need horizon-dependent assumed interest rates to obtain expected constant pension payments. We will see that, even in the present financial market, also smoothing financial market returns leads to a horizon-dependent assumed interest rate (Section 4).

Figure 3 shows, in red, the expected pension payment with a stock exposure $w = 35\%$ and the 5% and 95% quantiles. The running example with the assumed interest rate equal to the expected return as in (3.9) leads to a constant stream of expected pension payments equal to €565, whereas the constant stream of certain pension payments equals €501. The difference between the variable and the fixed annuity is caused by the risk included in the variable annuity. As a result, the variable annuity leads to higher *expected* payments, but there is the risk that the realized payments are actually lower than the fixed annuity. The blue line is the fixed annuity in which $W_t(h)$ is fully invested in the risk-free asset.

Figure 3: *Variable annuity without smoothing*



3.1 Combining variable and fixed annuities

In the derivations above, we implicitly assumed continuous re-balancing of the risky investment portfolio. Thus if one decides to invest w of wealth in a risky asset, then movements in this underlying asset will cause a change in the relative investment in the risky and the risk-free asset. As a result, either risky assets need to be bought (when their value goes down) or sold (when their value goes up) in order to keep the proportion of wealth invested in the risky asset constant at w.

Thus, if the stock price drops, the total value of wealth goes down and the relative proportion invested in the stock decreases as well. Therefore, one will need to buy more stocks to keep the investment proportion constant. If the stock price drops further, the total value of wealth invested can theoretically go to zero. This undesired feature can be alleviated by combining a variable annuity with continuous re-balancing with a fixed annuity.

In the current setting without interest rate risk, a fixed annuity is simply obtained if no investment risk is taken for any of the pension payments. That is, each $W_t(h)$ is fully invested in the risk-free asset. In a setting with interest rate risk and risk-free term structures, the above can be extended. There will be a guaranteed pension payment at horizon h in case the wealth $W_t(h)$ is fully invested in default-free zero-coupon bonds with maturity h. Essentially such an investment strategy fully hedges interest rate risk. That is, along the way the evolution of $W_t(h)$ is risky, but ignoring default risk it is known what the payment at time $t+h$ will be.

Clearly, the above can lead to the suggestion to establish a floor in a variable annuity. This means that, for each horizon h, a fraction of $W_t(h)$ is invested in default-free zero-coupon bonds with maturity h. The remainder can then be invested in a diversified risky return portfolio. This is known as splitting the investment into a hedge demand and a speculative demand. Note that the speculative demand should generally still be partially invested in bonds as they offer a risk-return tradeoff and thus lead to diversification benefits.

If v is the fraction of $W_t(h)$ that is invested in the risk-free asset, then v is the hedge demand and equals the proportion of the fixed annuity which represents the floor in the mixed annuity. Then $1-v$ is the remainder that is used to buy the variable annuity, thus the fraction $1-v$ of $W_t(h)$ is the speculative demand. The expected pension payment of the mixed annuity at horizon h is given by

$$\mathbb{E}_t[W_{t+h}(h)] = v \cdot \mathbb{E}_t[W^r_{t+h}(h)] + (1-v) \cdot \mathbb{E}_t[W^{r+w\lambda\sigma}_{t+h}(h)], \tag{3.10}$$

where $\mathbb{E}_t[W^r_{t+h}(h)]$ is the expected pension payment at horizon h of the fixed annuity with $w=0$ and hence $a_t(h)=r$, and $\mathbb{E}_t[W^{r+w\lambda\sigma}_{t+h}(h)]$ is the expected pension payment at horizon h of the variable annuity with a stock exposure w and an assumed interest rate $a_t(h)=r+w\lambda\sigma$. This combined annuity induces a guaranteed annuity payment of at least

$$\begin{aligned} L_v(h) &= v \cdot W_t(h) \cdot \exp(hr) + (1-v) \cdot 0 &(3.11)\\ &= v \cdot \frac{W_t}{\sum_{h=1}^{H} \exp(-hr)}. &(3.12)\end{aligned}$$

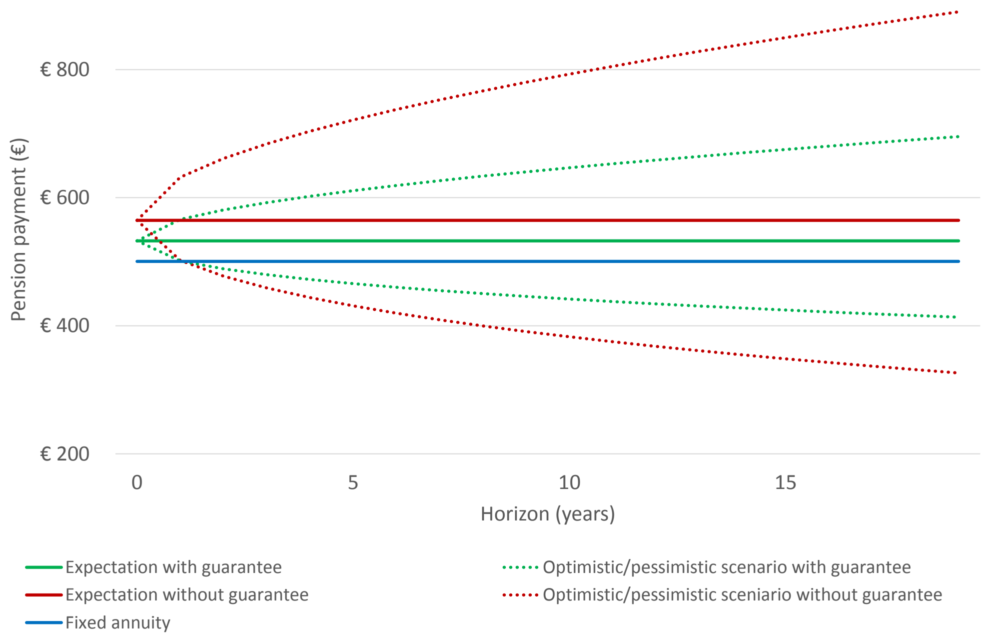
The level α quantile at horizon h is

$$\begin{aligned} Q_t^{(\alpha)}(W_{t+h}(h)) = {} & v \cdot \frac{W_t}{\sum_{h=1}^{H} \exp(-hr)} + \\ & (1-v) \cdot \frac{\exp(-h a_t(h))}{\sum_{k=0}^{H-1} \exp(-k a_t(k))} \cdot \\ & W_t \cdot \exp\left(h\left(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2\right) + z_\alpha \sqrt{h} w\sigma\right). \end{aligned} \tag{3.13}$$

Note that this expression for the quantile will remain valid in the case of smoothing (see Section 4), simply by using the adjusted parameters derived in that section.

Figure 4 shows the expected pension payment with a stock exposure $w=35\%$ and the 5% and 95% quantiles and the fixed annuity in which $W_t(h)$ is fully invested in the risk-free asset, similar to Figure 3. Additionally the combination of a variable and a fixed annuity is shown for a guarantee of $v = 50.00\%$. The mixed annuity has a minimum guarantee of €250.

Figure 4: *Variable annuity with guarantee*



Indexed annuities are similar as fixed annuities since they both guarantee a predetermined cash flow pattern. Hence, the term *fixed* indicates that the cash flow is known ex ante and does not contain uncertainty due to the risk-return tradeoff. An indexed annuity guarantees an indexation on the cash flows. Combinations of indexed annuities and variable annuities can be made analogously to the mixed annuity discussed.

3.2 Longevity risk

So far we have completely ignored longevity risk. We will not discuss macro longevity risk in this paper. As to micro longevity risk, the standard mortality credit argument applies. If all pensioners in a pool have the same risk exposure and AIR, then micro longevity risk can be shared if the pool is large enough. The value allocated to each horizon can be decreased in direct proportion to the mortality rate. So far the buckets could be interpreted both on an individual and on a collective basis since the horizon is fixed to H. However, when introducing mortality, $W_t(h)$ represents the value that the collective reserves for the individuals. Imagine the survival rate of a 65-year old to reach the age of 82 to be

$p_0(17) = 30\%$. The bucket $W_t(17)$ can be decreased by 70% since only 30% of the payments actually need to be made. By analogy all calculations can be updated for mortality.

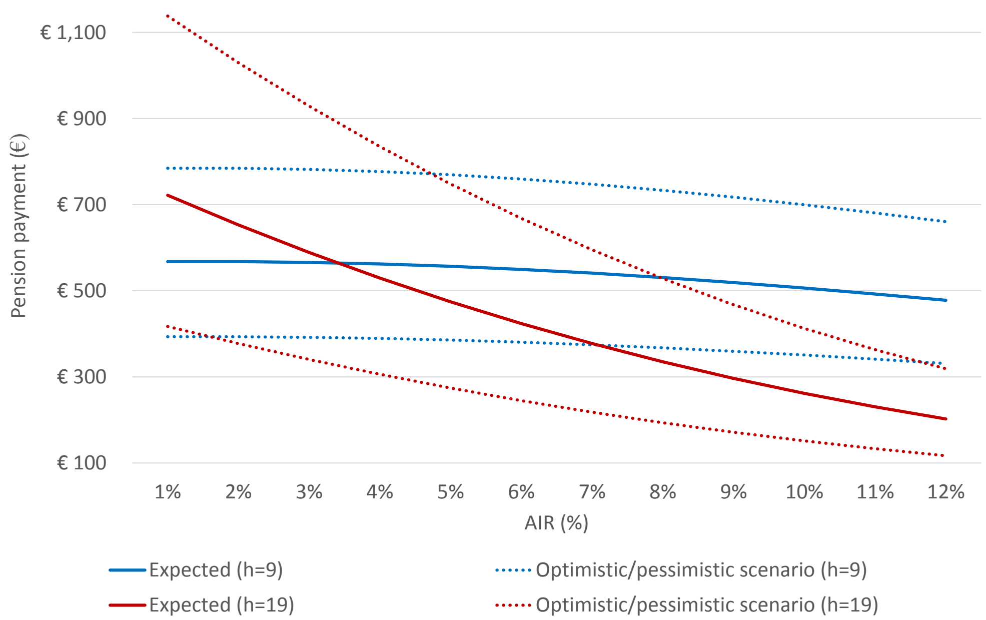
Issues may arise in case agents with different investment risks share their longevity risk in a pool. The reason is that, in such case, the wealth that the pool receives upon death of one agent depends on previous financial market returns and is thus risky. This is an interesting area for further research.

In case the pension payment at horizon h will only need to be made with probability $p_t(h)$, independent of the evolution of financial markets, the market-consistent value of the pension payment will be reduced by the factor $p_t(h)$. If the probability of dying next year is $1 - p_t(h)$, then the collective can allocate a fraction $1 - p_t(h)$ less to each subsequent h. As a result, the probability of death $1 - p_t(h)$ can be added to the AIR $a_t(h)$. Hence, mortality risk implies that a higher AIR can be used, while payments to those who survive are not affected.

3.3 Communication of future pension benefits

Figure 5 shows the sensitivity of the expected pension payment at $h = 9$ and $h = 19$ to the assumed interest rate. We observe that an increase in the AIR has hardly any effect

Figure 5: *Sensitivity AIR*



on the distribution of the 10th pension payment, but all the more on the 20th payment.

As a result, for communication purposes, we advise showing the risk-return tradeoff of variable annuities over horizons significantly exceeding 10 years.

For an AIR of $a_t(h) = 2.00\%$ the first pension payment is €501, the 10th expected payment is €568 and the last payment is €653, while for $a_t(h) = 8.00\%$ the first pension payment is €803, the 10th expected payment is €531 and the last payment is €335. Hence, the higher the AIR the larger the expected payments in the near future and the lower the expected payments in the distant future, with the expected payments in the middle of the horizon being hardly influenced by the choice of AIR.

4 Variable annuities with smoothing

If agents have habit-formation preferences, they may want to reduce year-to-year volatility in the pension payments. The traditional view to achieve this is to “smooth” financial market returns. That is, in case portfolio returns are -20%, instead of reducing the pension payment immediately by 20%, it is only reduced by a fraction. This clearly implies that pension payments later in the retirement phase need to be cut by more than 20% to fulfill the budget constraint. Smoothing then leads to smaller year-to-year decreases, but the total decrease is larger.

The traditional view on smoothing leads, effectively, to an increase in the assumed interest rate following negative financial market returns and, symmetrically, to a decrease in the assumed interest rate following positive financial market returns. This leads to a situation where wealth $W_t(h)$ originally reserved for the pension payment at time $t+h$ is redistributed over all future pension payments. The resulting mathematics is complicated, so we propose an alternative view here, inspired by Bovenberg, Nijman, and Werker (2012).

The reduced year-to-year volatility can also be achieved as follows. Recall that the initial pension payment at time t is given by $W_t(0)$. In order to have limited risk in the pension payment $W_{t+1}(1)$ we do not invest it according to a stock exposure w, as in Section 3, but with a stock exposure $w_t(1) = w/N$, where N denotes the smoothing period, say, $N = 5$ years. Subsequently, the pension wealth $W_t(2)$ for the pension payment $W_{t+2}(2)$ is invested with exposure $w_t(2) = 2w/N$ the first year and $w_{t+1}(2) = w/N$ the second year. In general, with a smoothing period N and long-term stock exposure w, the pension wealth $W_{t+j}(h)$ for the pension payment at time $t+h$ has stock exposure

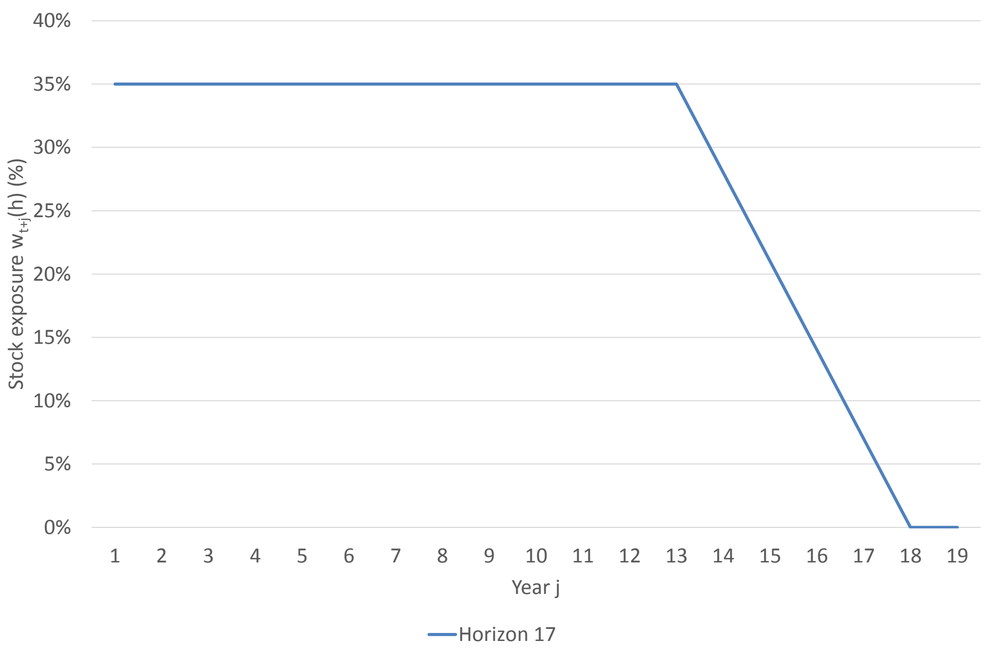
$$w_{t+j-1}(h) = w \min\left\{1, \frac{1+h-j}{N}\right\}, \quad j = 1, \ldots, h, \tag{4.1}$$

during the year from $t+j-1$ to $t+j$.

Figure 6 shows the stock exposure $w_{t+j-1}(h)$ against j for $h = 17$.

The horizon-dependent stock exposure $w_{t+j}(h)$ induces a life-cycle investment strategy. That is, with smoothing the investment strategy is no longer constant over time. Note that classical reasoning behind a life-cycle argument does not hold for the retirement phase. The idea is that human capital, future wealth from income that is still to be earned, decreases with age. As such, the fraction of savings that is allocated to investments is higher for younger workers because the absolute value of savings is relatively low. In the retirement phase the future income does not decrease since the income at old age continues until death. However, smoothing causes a horizon-dependent investment strategy, as illustrated above. We can now calculate the distribution of the pension

Figure 6: *Smoothing stock exposure*



payment at time $t+h$. Again, this distribution is log-normal, but now with mean

$$\sum_{j=1}^{h}\left(r + w_{t+j-1}(h)\lambda\sigma - \frac{1}{2}w_{t+j-1}^2(h)\sigma^2\right), \tag{4.2}$$

and variance

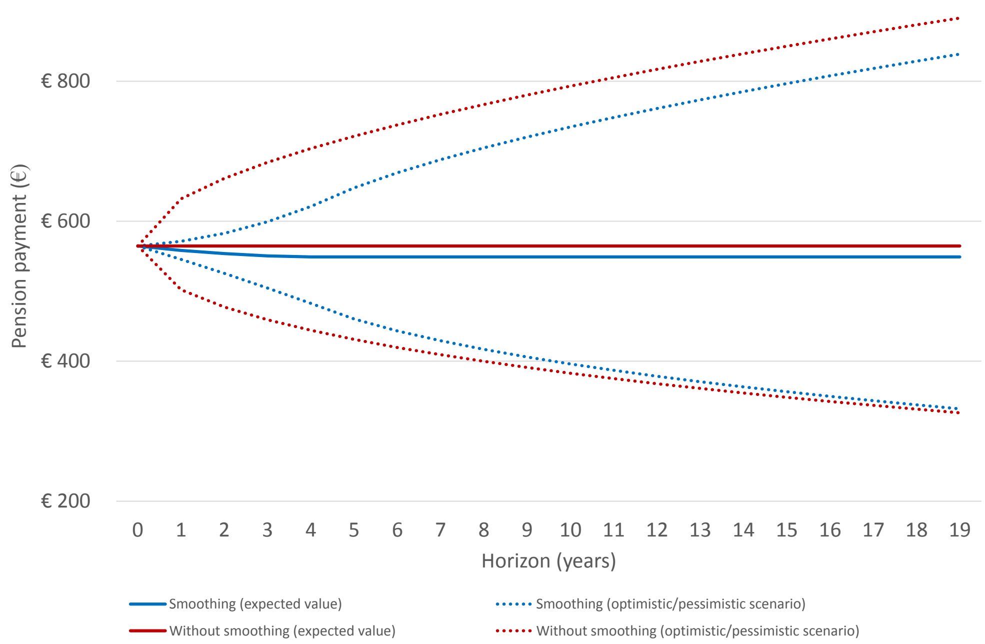
$$\sum_{j=1}^{h} w_{t+j-1}^2(h)\sigma^2. \tag{4.3}$$

The expected nominal pension payments, and their quantiles, can now be calculated as before; a detailed implementation is available in Excel from the authors.

The solid blue lines in Figure 7 show the expected pension payment, and the dotted blue lines show the 5% and 95% quantiles with smoothing period $N=5$ years for a stock exposure $w=35\%$. The first payment from the variable annuity is equal to the fixed annuity payment. However, since the AIR is not adjusted for the smoothing, the remaining expected payments decrease. Smoothing leads effectively to a lower risk exposure, thus a lower risk premium and thus lower expected pension payments. For comparison we also show, in red, the pension payments obtained without smoothing as in Figure 3.

Returning to the issue of mortality/longevity risk, note that our formalization of smoothing (by using a horizon-dependent exposure to the risky asset) also shows that longevity

Figure 7: *Smoothing stock exposure*



risk can be dealt with as before. As long as all agents in the pool share the same investment strategy, the proceeds from those who die can be reallocated to the survivors. Sharing longevity risk between agents with heterogeneous investment strategies is left for future research.

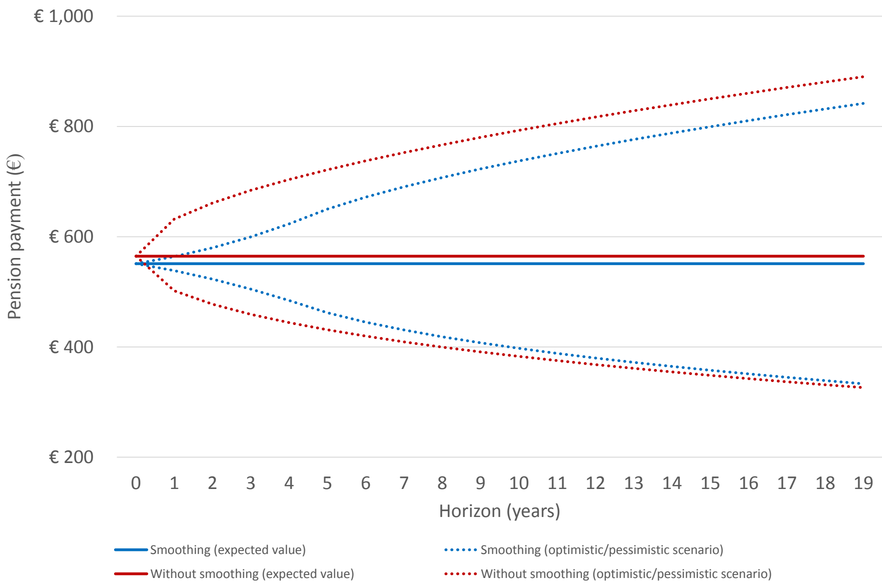
4.1 The BNW assumed interest rate

The previously mentioned paper by Bovenberg, Nijman, and Werker (2012) already discussed the implications of smoothing financial market shocks for market-consistent valuation of pension liabilities, although their focus was more on collective defined contribution (CDC) systems. The present setting allows for an exact derivation of the assumed interest rate that leads to pension payments that have a constant expectation. The idea is simple: which assumed interest rate $a_t(h)$ leads to a pension payment that is expected to be constant in nominal terms? This essentially amounts to inversion of the expected nominal pension payments obtained in the previous section.

With smoothing, the expected nominal pension payment at time $t+h$ is given by

$$W_t(h) \exp\left(\sum_{j=1}^{h} \left(r + w_{t+j-1}(h)\lambda\sigma\right)\right). \tag{4.4}$$

Figure 8: *Smoothing BNW with* $w = 35\%, N = 5$



In order to have a constant expected nominal pension payment, we must choose the assumed interest rate $a_t(h)$ such that this expectation equals $W_t(0)$ for all h. Thus, we immediately find

$$a_t^{(\mathrm{BNW})}(h) = r + \lambda\sigma \frac{1}{h} \sum_{j=1}^{h} w_{t+j-1}(h). \tag{4.5}$$

The solid blue lines in Figure 8 show the expected pension payment and the dotted blue lines show the 5% and 95% quantiles with smoothing period $N = 5$ years for a stock exposure $w = 35\%$ and the assumed interest rate equal to the $a_t^{(\mathrm{BNW})}$. The red lines are obtained without smoothing, similar to Figure 3. Because the total risk exposure is lower due to smoothing, the expected payments are lower than without smoothing. Therefore, a comparison in terms of the scenario spreads based on an equal expected stream of payments is accomplished by a stock exposure of $w = 46.5\%$ with smoothing and $w = 35\%$ without smoothing, as shown in Figure 9.

The blue line in Figure 10 shows the assumed interest rate as a function of the horizon such that the expected pension payments are constant, hence the BNW assumed interest rate. If the assumed interest rates are set equal to the risk-free rate, as given by the red line, the fixed annuity is obtained. The expected return with a stock exposure of $w = 46.5\%$ is the green line.

Figure 9: *Smoothing BNW with* $w = 46.5\%, N = 5$

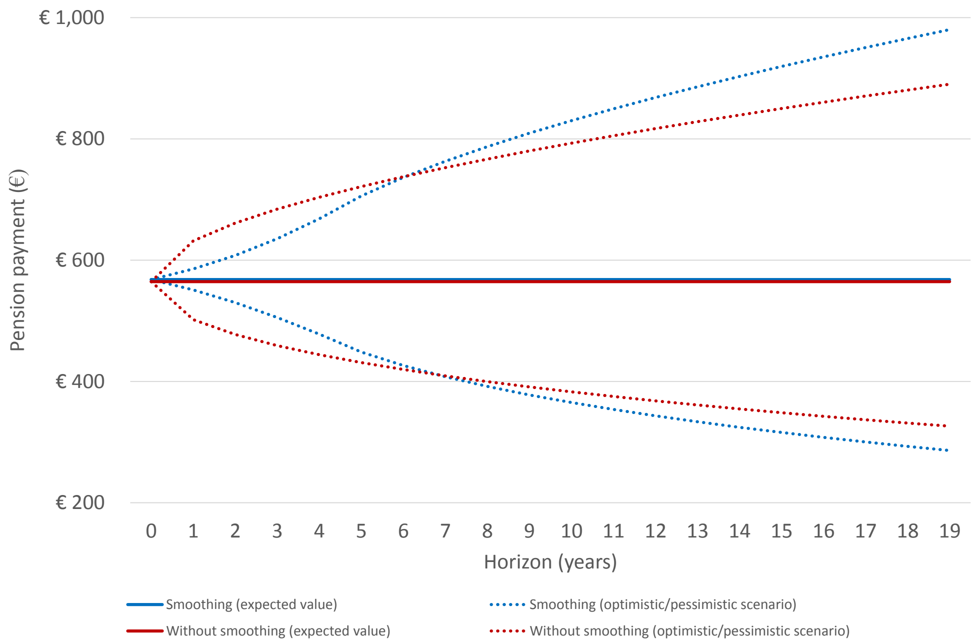
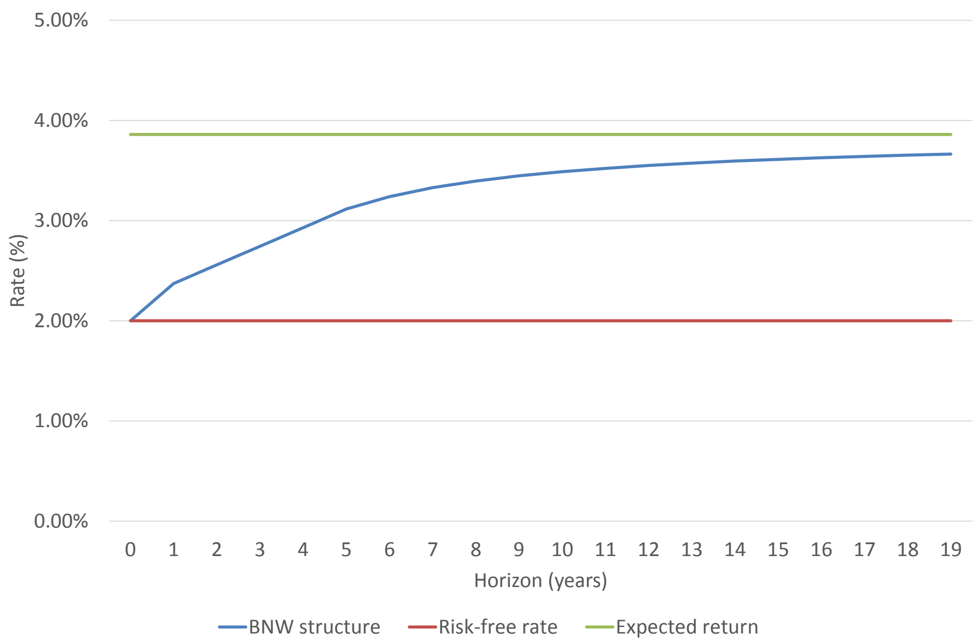


Figure 10: *BNW structure*



5 The concept of fixed decrease ("vaste daling")

For political reasons, the Dutch Improved Pension Payments Act ("Wet verbeterde premieregelingen") does not use assumed interest rates to determine the distribution of pension wealth over the various horizons. Instead it uses the risk-free rate in combination with a "fixed decrease" ("vaste daling"). From a financial point of view, the induced assumed interest rate is simply the sum of the risk-free rate and the fixed decrease. As a result, in order to have a constant expected nominal pension, the fixed decrease must be chosen to be horizon dependent. The exact formula simply follows from (4.5). If the fixed decrease at horizon h is chosen as

$$\lambda\sigma\frac{1}{h}\sum_{j=1}^{h} w_{t+j-1}(h) = w\lambda\sigma\frac{1}{h}\sum_{j=1}^{h}\min\left\{1, \frac{1+h-j}{N}\right\}, \tag{5.1}$$

then the expected pension is constant in nominal terms.

6 Conversion risk

Contrary to popular belief[5], a variable annuity does not provide protection against interest rate risk upon conversion of initial pension wealth W_t into an annuity. Similarly, for a fixed annuity the level of the interest rates at the moment of conversion plays an important role. We illustrate this with a numerical example. Imagine that the risk-free rate decreases from 2.00% to 1.00% and the AIR is $a_t(h) = r$. Then the fixed annuity decreases from €501 to €457, a decrease of 8.61%. The first payment of the variable annuity also decreases from €501 to €457, the last expected pension payment from €653 to €597, and the 5% quantile from €377 to €345 and the 95% quantile from €1030 to €941. All these numbers exhibit the same decrease of 8.61%. Hence, if the risk-free rate decreases, then the expected return is 1.00% lower for the remaining lifetime of the retiree. Therefore all payments decrease proportionally.

If we assume that the AIR is chosen such that it generates constant expected payments as given by equation (3.9), then $a_t(h) = r + w\lambda\sigma$. A decrease of 1.00% of the risk-free rate causes a 1.00% decrease of the AIR as well. This implies that we assume a constant risk premium λ. Also, all expected pension payments and quantiles then decrease by 8.61%.

[5]There is only a small effect that *may* lead to some protection. Under the empirically hard to defend assumption that lower interest rates r imply increased prices-of-risk λ, some protection is provided. But even if this assumption is true, such protection only pertains to the risky investment in the variable annuity. The only generally effective protection against conversion risk is to buy long-maturity bonds during the accumulation phase.

7 Conclusion

This paper provides analytical expressions for the risk-return tradeoff of variable annuities, with special focus on the situation where financial market returns may be smoothed over the remaining retirement period. As far as we can determine, this has not been documented before in the literature. For the Dutch pension debate, we find several results. Firstly, in order to obtain, in a contract with smoothing, a constant expected nominal pension, the fixed decrease has to be horizon dependent. We give an explicit expression. Secondly, we show that an increase in the assumed interest rate (or an increase in the fixed decrease) does affect the initial payoff, but hardly the risk-return trade-off at a horizon of 10 years. However, beyond a horizon of 10 years the effect on the initial payment is seen to go in the opposite direction. Thirdly, we show that variable annuities do not provide a hedge against interest rate conversion risk.

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