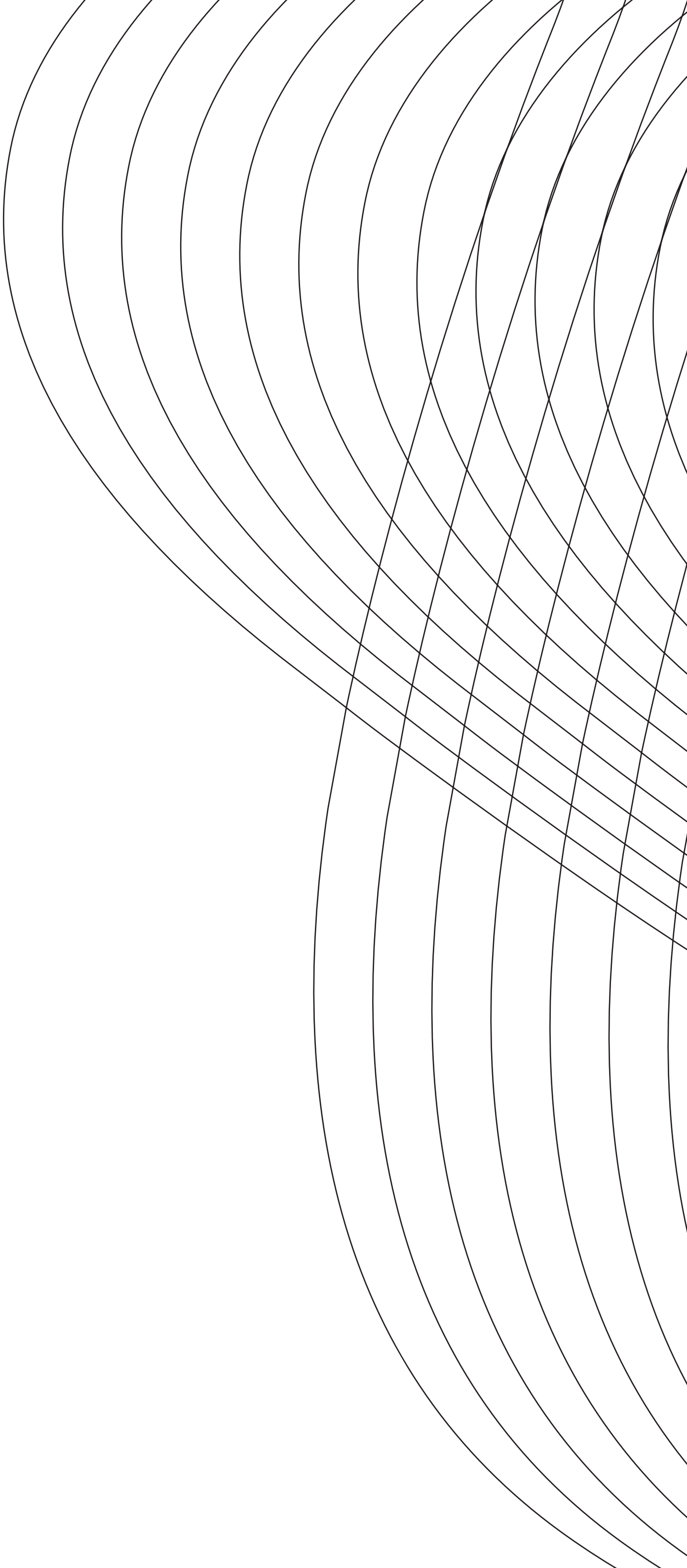
Network for Studies on Pensions, Aging and Retirement

Netspar DESIGN PAPERS

Thijs Markwat, Roderick Molenaar and Juan Carlos Rodriguez

Purchasing an annuity: now or later?

The role of interest rates



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PREFACE

Netspar seeks to stimulate debate on the effects of aging on the behavior of men and women, (such as what and how they save), on the sustainability of their pensions, and on government policy. The baby boom generation is approaching retirement age, so the number of people aged 65 and over will grow fast in the coming decades. People generally lead healthier lives and grow older, families have fewer children. Aging is often viewed in a bad light since the number of people over 65 years old may well double compared to the population between 20 and 65. Will the working population still be able to earn what is needed to accommodate a growing number of retirees? Must people make more hours during their working career and retire at a later age? Or should pensions be cut or premiums increased in order to keep retirement benefits affordable? Should people be encouraged to take personal initiative to ensure an adequate pension? And what is the role of employers' and workers' organizations in arranging a collective pension? Are people able to and prepared to personally invest for their retirement money, or do they rather leave that to pension funds? Who do pension fund assets actually belong to? And how can a level playing field for pension funds and insurers be defined? How can the solidarity principle and individual wishes be reconciled? But most of all, how can the benefits of longer and healthier lives be used to ensure a happier and affluent society? For many reasons there is need for a debate on the consequences of aging. We do not always know the exact consequences of aging. And the consequences that are nonetheless clear deserve to be made known to a larger public. More important of course is that many of the choices that must be made have a political dimension, and that calls for a serious debate. After all, in the public spectrum these are very relevant and topical subjects that young and old people are literally confronted with.

For these reasons Netspar has initiated Design Papers. What a Netspar Design Paper does is to analyze an element or aspect of a pension product or pension system. That may include investment policy, the shaping of the payment process, dealing with the uncertainties of life expectancy, use of the personal home for one's retirement provision, communication with pension scheme members, the options menu for members, governance models, supervision models, the balance between capital funding and pay-as-you-go, a flexible job market for older workers, and the pension needs of a heterogeneous population. A Netspar Design Paper analyzes the purpose of a product or an aspect of the pension system, and it investigates possibilities of improving the way they function. Netspar Design Papers focus in particular on specialists in the sector who are responsible for the design of the component.

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PURCHASING AN ANNUITY: NOW OR LATER?

Abstract
This paper investigates whether the option to delay annuitization in times of low interest rates has value for retirees. The retiree who chooses to wait bets on a fall in the annuity price brought about by a rise in the interest rates; the cost of such a bet is the loss of the mortality credit during the waiting period. We show that an investor who can only invest in bonds during the waiting period will never find waiting ex-ante profitable if annuities are fairly priced, because waiting is costly and buying a fairly priced annuity is a zero-npv project. In contrast, an investor allowed to invest part of her wealth in the stock market will be able to attain more consumption on average, but at the cost of a sharp increase in risk. A retiree may choose to wait, however, if she believes her views are not priced in the term structure. For such a retiree, waiting is optimal if the expected increase in the interest rate is larger than the square of the hazard rate.

1. Introduction

Due to the credit crisis, the Central Banks of major world economies have engaged in unprecedented monetary activism, pushing interest rates down to historically low levels. It seems natural that investors living in such extraordinary times believe that things will eventually go back to normal, where normal means less intervention and higher interest rates. In this context, it makes sense for retirees to ponder over postponing the purchase of an annuity. Although annuities provide a stable income stream and a hedge against longevity risk, retirees for whom interest rates are likely to rise in the near future may find it desirable to wait before buying one. Some practitioners share this view:

"If one firmly believes that interest rates are almost certain to increase significantly in the very near future, there may be good reason to postpone annuitization for at least a few years" (B. Goodman, N. Heller, Annuities: Now, Later, Never?, TIAA-CREF Institute).

Recently, a new law in the Netherlands gave retirees the option to postpone the conversion of accumulated capital into a lifetime annuity. This split-annuity option is known as "pensioenknip". The pensioenknip program, however, was hardly a success: only 100 retirees -out of about 36.600 eligible- enrolled (see Ministerie van Sociale Zaken en Werkgelegenheid (2013)). Since 1 January 2014 the program has been closed to new participants. If waiting to buy an annuity is such a good idea, why so few people sign up?

We investigate whether the option to delay annuitization in times of abnormally low interest rates has ex-ante value for retirees. A retiree who chooses to wait bets on a fall in the annuity price brought about by a rise in the interest rates; the cost of such a bet is the loss of the mortality credit[1] during the waiting period. Waiting is a good decision if the benefits outweigh the costs, both measured ex-ante. We show that waiting is never ex-ante profitable if annuities are fairly priced, because waiting is costly and buying a fairly priced annuity is a zero-npv project. A retiree may find it valuable to wait, however, if she has a view on the evolution of the interest rate not shared by the market (i.e., if she is speculating). We find that such a retiree will find waiting ex-ante valuable as long as the expected increase in the interest rate is larger than the square of the hazard rate.

The first result derives from a standard principle of Finance: buying a fairly priced asset is a zero-npv project, and we hope our paper contributes by helping practitioners and regulators to consider its important consequences. The second result establishes a condition that makes waiting ex-ante profitable to speculators. As it is relatively easy to satisfy, it may explain why, in the current economic climate, some investors consider it a good idea to postpone the purchase of an annuity[2].

In a classic paper on annuity demand, Yaari (1964) argues that retirees not leaving a bequest should invest all of their

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[1]The purchaser of an annuity locks her capital in an insurance pool. When she dies, the remains of her capital are shared by the survivors participating in the pool. The extra yield they get is the “mortality credit”, which represents the capital and interest “lost” by the deceased and “gained” by the survivors (see Milevsky, 2006).

[2]There are other potentially good reasons to postpone the purchase of an annuity, prominent among these the choice of leaving a bequest, but we will not discuss them in this paper. For a discussion in a context similar to ours, see Milevsky and Young (2007).

accumulated assets in mortality-contingent annuities in order to earn the mortality credit, and that they must do it as soon as they retire, because annuities are the optimal instrument for hedging longevity risk. In practice, however, relatively few people voluntarily purchase annuities, a fact that has been dubbed "the Annuity Puzzle"[3].

Many authors have tried to solve the Annuity Puzzle. A line of research of particular interest to us points to the irreversibility of the decision to annuitize, and investigates whether the real option of waiting has value to retirees. One of the first papers to deal systematically with this issue is Milevsky and Young (2007), who allow retirees to invest a fraction of their accumulated wealth in the stock market. They find that waiting is optimal if the risk premium on the stock market is larger than the hazard rate. In their model, relatively young retirees and women are the ones most likely to wait.

Milevsky and Young (2007), and most of the literature that followed, assume constant interest rates and a constant Sharpe ratio. More recently (see Shi (2008) and references therein), some authors have extended Milevsky and Young by incorporating stochastic interest rates in their models. All of these authors, however, maintain that what motivates retirees to postpone annuitization is the hope that a large risk premium compensates them for the mortality credit lost by waiting. Our paper, in contrast, is the first in which retirees are motivated to wait because they perceive interest rates as too low.

We discuss several models of increasing generality, in which we try to make precise the statement of Goodman and Heller

[3]See Cannon and Tonks (2008) for a comprehensive survey on the Annuity Puzzle.

cited at the beginning of this introduction. In our first model, a simple one with risk-neutral retirees, we find that for a retiree who "firmly believes" that the interest rate is going to rise, waiting is ex-ante valuable as long as her expected increase in the interest rate is larger than the square of the hazard rate. This is a relatively mild condition that many realistic situations would satisfy. Moreover, the option to wait is still valuable when we incorporate risk-averse retirees in our model.

"Firmly believes" means here that the retiree has views about the evolution of the interest rate that may not be shared by the market. We find, indeed, that the key issue in the decision to wait is whether the retiree believes that the future change in the interest rate is priced in the current annuity value. When this is not the case, waiting to buy the annuity makes her ex-ante better off if the expected rise in the interest rate is large enough to compensate for the loss of the mortality credit during the waiting period. In contrast, a retiree who believes that the future change in the interest rate is fully priced in[4] will never wait.

We further explore these results in a more realistic model in which the short rate follows a CIR process and the Gompertz law of mortality describes the hazard rate. We show that even if the short rate is very low today, but the term structure is upward sloping (reflecting market expectations that rates will

[4]Investigating whether annuities are fairly priced goes beyond the scope of this paper, but we can point to some references suggesting that in countries like the US, the UK and the Netherlands they indeed are. See Cannon and Tonks (2008, Chapter 6) for an extensive survey on this issue. Murthi, Orszag, and Orszag (2004), Finkelstein and Poterba (2002, 2004, 2006), and Cannon and Tonks (2004) study the American and British markets. Cannon, Stevens and Tonks (2013) focus on the Dutch case.

go up in the future) there is no value in waiting: a low short rate does not necessarily mean that annuities are expensive.

We also investigate whether investing in the stock market during the waiting period makes waiting more attractive. We find that relatively younger retirees can attain a higher average consumption by waiting, but, not surprisingly, at the cost of a sharp increase in risk. Our results are consistent with Milevsky and Young (2007), who studied the issue in a model with constant interest rates, and we contribute to the debate by measuring the volatility of consumption and quantifying the probability of regretting the decision to wait in a context of stochastic interest rates.

Finally, we discuss the Dutch experience regarding the issue of waiting to annuitize, and provide a description of instruments used to that end.

The paper is structured as follows. Section 2 describes a simple model of waiting to annuitize. Section 3 studies a more realistic case in which the change in the interest rate is fully priced in. Section 4 discusses the Dutch experience. Section 5 concludes.

2. A simple model of waiting to annuitize

In times of very low interest rates, some retirees may decide to wait before annuitizing. Buying an annuity is an irreversible decision. Retirees who consider waiting are betting on an increase in the interest rate that will make the value of the annuity cheaper at the purchasing time. The cost of the bet is the mortality credit lost, because the retiree has to consume her own resources during the waiting period. This is the trade-off they face: waiting gives a chance of buying a cheaper annuity, but it consumes resources, so the annuity has to be purchased with less wealth. The model discussed below tries to make this trade-off precise in a very simple setting.

We start discussing the case of a risk-neutral retiree[5], which we use as a benchmark, and then generalize to a risk-averse retiree.

Consider an economy in which the interest rate is the same for all maturities and is equal to $r_0 = 0$. The hazard rate, denoted by λ, is constant. The retiree is risk-neutral and retires with initial wealth[6] $W_0 = 1$, which she must use to buy an annuity. Throughout this section we assume, as it is common in the literature (see Milevsky (2006)), that the consumer's subjective rate of time preference is equal to the risk free rate.

For simplicity, we assume that there is no state pension. We also assume no bequest motives, because if the retiree gets

[5] A risk-neutral investor may not be interested in buying an annuity in the sense of Yaari (1964). We start with this case for simplicity, and to establish a benchmark. We generalize in the next section.

[6] We take the wealth of the investor as given, and do not discuss how the investor accumulated it. For a discussion of this issue, see Koijen, Nijman and Werker (2008).

utility out of leaving a bequest she will have a strong preference for waiting no matter the current state of the interest rates.

Under these assumptions, the value of an annuity at time zero is[7]:

$$\begin{aligned} a_0 &= \int_0^\infty e^{-\lambda t} dt \\ &= \frac{1}{\lambda} \end{aligned}$$

If the retiree annuitizes immediately, she consumes:

$$C^{nw} = \frac{1}{a_0} = \lambda$$

until she dies. Because the interest rate is zero, this is pure mortality credit. If the retiree buys the annuity immediately, the expected present value of her consumption is:

$$\begin{aligned} PVC^{nw} &= \lambda \int_0^\infty e^{-\lambda t} dt \\ &= 1. \end{aligned}$$

This means that, under our current assumptions, buying the annuity immediately is for the retiree a zero-npv project.

Suppose, instead, that the retiree believes that the interest rate may go up at time 1, and that this change is not fully incorporated in the current annuity price. Her problem now is: shall I buy the annuity at time 0 anyway, or wait one period? By waiting she may be able to buy the annuity more cheaply. But she has to consume at time 1 out of her own wealth, so the cost of the gamble is the loss of the mortality credit during one

[7]We assume that the load is zero.

period. In the first example (we generalize later), the retiree believes that the interest rate will go up in next period from 0 to ε with probability p, or stay the same with probability $1 - p$. This is to capture the "firm belief" in rising interest rates mentioned by Goodman and Heller. For this investor, the next period interest rate is a random variable with mean $\mu = p\varepsilon$, and variance $\sigma^2 = p\,(1 - p)\,\varepsilon^2$, where p is a subjective probability expressing the retiree's views. There are other retirees in the economy thinking that the interest rate will stay the same, or even go down and become negative[8]. The aggregate behavior of the retirees results in the current price of the annuity: $a_0 = \frac{1}{\lambda}$.The retiree who considers waiting thinks that the probable increase in the interest rate is not (at least not fully) priced in. Otherwise, waiting makes no sense[9]. In this example we concentrate on the retiree who may find it valuable to wait.

It may be thought that, under the assumptions of this section, the retiree would never wait, but rather would implement the following strategy: she would buy the annuity at time 0 (cost = $1/\lambda$), consume λ between time 0 and time 1, and sell the annuity at time 1 for $1/\lambda$; wait until the interest rate moves: if it goes up, she would repurchase the annuity at a cost $\frac{1}{\lambda+\varepsilon}$; if it stays the same, she would repurchase the annuity at a cost $1/\lambda$. This strategy, according to the retiree's views,

[8]We need to assume that for some investors a negative interest rate may happen with positive probability because of our initial assumption $r_0 = 0$. Starting with $r_0 > 0$ reinforces the results of this section, because an investor can deposit her money at the initial interest rate to recover part of the mortality credit lost in waiting.

[9]Later in this section we show formally that waiting is not profitable if the expected change in the interest rate is fully priced in.

makes money in one state of nature with zero risk, so she will never wait. But this strategy does not take into account that buying an annuity at time 0 is an irreversible decision. Once the investor has purchased the annuity, there is no way back. This is central to our discussion, because only when the decision to buy the annuity at time 0 is irreversible, it does make sense to discuss the value of the option to wait.

As in Milevsky and Young (2007), we assume that a regulator forces the waiting retiree to consume λ at time 1, the same amount she would have consumed had she purchased the annuity at time 0[10]. This means that at time 1 the retiree has only $1-\lambda$ to buy the annuity.

If the rate goes up, the value of the annuity at time 1 will be:

$$a_1 = \frac{1}{\lambda + \varepsilon}$$

and after annuitizing the retiree consumes:

$$\begin{aligned} C^w &= \frac{1-\lambda}{a_1} \\ &= (1-\lambda)(\lambda+\varepsilon) \end{aligned}$$

per period.

The expected present value of waiting for this retiree is:

$$\begin{aligned} E\left(PVC^w\right) &= \lambda \int_0^1 e^{-\lambda t} dt + \left[(1-\lambda)\lambda + (1-\lambda)p\varepsilon\right] \int_1^\infty e^{-\lambda t} dt \\ &= 1 - e^{-\lambda} + \left[(1-\lambda) + \frac{p(1-\lambda)\varepsilon}{\lambda}\right] e^{-\lambda} \end{aligned}$$

[10]Suppose the investor who waits has to deposit λ at time 0 and consume from there λdt instant by instant untill time 1: $\int_0^1 \lambda dt = \lambda$.

Waiting is optimal for our risk-neutral retiree iff $E(PVC^w) > PVC^{nw}$, or:

$$1 - e^{-\lambda} + \left[(1-\lambda) + \frac{p(1-\lambda)\varepsilon}{\lambda}\right] e^{-\lambda} > 1$$

Which happens iff:

$$p\varepsilon > \frac{\lambda^2}{(1-\lambda)}. \tag{1}$$

Even though this is too stylized a model, the last inequality provides interesting insights. First, and most importantly, what the retiree compares to $p\varepsilon$ (the expected increase in the interest rate) is not λ (the cost of waiting), but $\frac{\lambda^2}{(1-\lambda)}$. This is because if the retiree waits and the interest rate indeed goes up, she will get $(1-\lambda)\varepsilon$ every period until she dies, with present value $\frac{(1-\lambda)\varepsilon}{\lambda}$, and she will have to pay the cost, λ, only once. This tilts the retiree's decision towards waiting: $p\varepsilon$ does not have to be too large to make waiting seem profitable. But it has to be large enough -even for the retiree who thinks the interest rate can never go down- to compensate for the mortality credit lost in waiting.

Suppose, for example, that a retiree with hazard rate of 6% thinks that there is 70% probability of a rate increase of 100 basis points over the next period. This retiree will not annuitize immediately:

$$p\varepsilon = 0.007 > 0.0038 = \frac{\lambda^2}{(1-\lambda)}.$$

Retirees who are confident that the interest rate will go up (large p), younger retirees (low λ), and women (low λ, relative to men the same age), are more likely to wait.

2.1. The model with a risk-averse retiree

This subsection investigates the case of a risk-averse retiree. Note that, even though the retiree believes that the interest rate cannot go down, waiting -as opposed to not waiting- is a gamble for her. If the retiree annuitizes at time 0, she consumes λ in all states of nature until she dies. If the retiree waits one period, she consumes $(1-\lambda)(\lambda+\varepsilon)$ until she dies with probability p, or $(1-\lambda)\lambda$ with probability $1-p$. In the bad state of nature she is clearly worse off, because $(1-\lambda)\lambda<\lambda$. In the good state of nature she may be better off, provided that ε is sufficiently large. In the previous subsection we found a condition on ε that makes waiting profitable for a risk-neutral retiree, given her views on the future interest rate. In this subsection we extend that result to the risk-averse retiree.

For the retiree, the future value of the interest rate is (as before) a random variable X with mean

$$\mu = p\varepsilon,$$

and variance

$$\sigma^2 = p(1-p)\varepsilon^2.$$

Now consider a retiree with an exponential utility function over ex-post consumption:

$$u = -e^{-ac},$$

where a is the coefficient of absolute risk aversion and c is future consumption.

The expected utility of not waiting is

$$E[U(C^{nw})] = -\left[e^{-a\lambda}\int_0^1 e^{-\lambda t}dt + e^{-a\lambda}\int_1^\infty e^{-\lambda t}dt\right].$$

Note that the integral has been split to facilitate comparison with the waiting case.

The expected utility of waiting is

$$\begin{aligned} E\left[U\left(C^{w}\right)\right] &= -e^{-a\lambda}\int_{0}^{1} e^{-\lambda t}dt + ... \\ &\quad - \left[pe^{-a[(1-\lambda)\lambda+(1-\lambda)\varepsilon]} + (1-p)\,e^{-a(1-\lambda)\lambda}\right] \\ &\quad \int_{1}^{\infty} e^{-\lambda t}dt. \end{aligned}$$

The risk-averse retiree will see waiting as profitable if:

$$\left[pe^{-a[(1-\lambda)\lambda+(1-\lambda)\varepsilon]} + (1-p)\,e^{-a(1-\lambda)\lambda}\right] < e^{-a\lambda}.$$

Cancelling $e^{-a\lambda}$ on both sides of the inequality, the last expression is equivalent to

$$e^{a\lambda^{2}}\left[pe^{-a(1-\lambda)\varepsilon} + (1-p)\right] < 1.$$

After operating inside the brackets, the last inequality becomes

$$e^{a\lambda^{2}}\left[1 + p\left(e^{-a(1-\lambda)\varepsilon} - 1\right)\right] < 1.$$

Taking logs on both sides we get

$$a\lambda^{2} + \log\left[1 + p\left(e^{-a(1-\lambda)\varepsilon} - 1\right)\right] < 0. \qquad (2)$$

Now, $e^{-a(1-\lambda)\varepsilon}$ is close to 1 if ε is small and a is not too high[11]:

$$\log\left[1 + p\left(e^{-a(1-\lambda)\varepsilon} - 1\right)\right] \approx p\left(e^{-a(1-\lambda)\varepsilon} - 1\right) \qquad (3)$$

[11] $\log(1+x) \approx x$ if $x \approx 0$.

and[12]

$$e^{-a(1-\lambda)\varepsilon} - 1 \approx -a\,(1-\lambda)\,\varepsilon + \frac{1}{2}a^2\,(1-\lambda)^2\,\varepsilon^2. \qquad (4)$$

These approximations are accurate for reasonable values of the parameters. Consider, for example, $a = 5$; $\varepsilon = 0.01$; $p = 0.7$; $\lambda = 0.06$. Then

$$\begin{aligned} \log\left[1 + p\left(e^{-a(1-\lambda)\varepsilon} - 1\right)\right] &= -0.0327 \\ p\left[-a\,(1-\lambda)\,\varepsilon + \frac{1}{2}a^2\,(1-\lambda)^2\,\varepsilon^2\right] &= -0.0321. \end{aligned}$$

Replacing (3) and (4) in (2), we see that the risk-averse retiree waits iff:

$$\begin{aligned} p\varepsilon &> \frac{\lambda^2}{(1-\lambda)} + \frac{pa\,(1-\lambda)\,\varepsilon^2}{2} \qquad (5) \\ &= \frac{\lambda^2}{(1-\lambda)} + \frac{a\,(1-\lambda)\,\sigma^2}{2\,(1-p)}. \end{aligned}$$

The condition described by inequality (5) is more demanding than the condition described by inequality (1). The right hand side increases by a positive term $\left(\frac{pa(1-\lambda)\varepsilon^2}{2}\right)$ depending on the investor's risk aversion coefficient a. The larger a, the larger must be the expected rate increase to satisfy inequality (5). Note, however, that the new term is a second order effect depending on ε^2, a very small quantity. Consider, for example, the same parameter values as in the previous section, and a moderately risk-averse retiree with $a = 5$. This retiree will still find it optimal to wait if $\varepsilon = 0.01$:

$$p\varepsilon = .007 > .004 = \frac{\lambda^2}{(1-\lambda)} + \frac{pa\,(1-\lambda)\,\varepsilon^2}{2}.$$

[12] $e^x - 1 \approx x + \frac{1}{2}x^2$ if $x \approx 0$.

That is, a risk-averse retiree "fairly convinced" that the interest rate will go up over the next period will behave similarly to a risk-neutral retiree, and find it attractive to wait.

2.2. The model when the change in the interest rate is fully priced-in

The model in the previous subsection investigates the polar case of a retiree who has a view on the evolution of the interest rate and thinks that this view is not priced in the current value of the annuity. In this case it makes sense to talk about cheap or expensive annuities: for the retiree, the current annuity is expensive, and the future annuity may be cheaper.

In this section we investigate another polar case, in which the evolution of the interest rate is fully priced in the current value of the annuity. In this case, talking about cheap or expensive annuities no longer makes sense: annuities are fairly priced, and incorporate all available information about the evolution of the interest rate. We show that, under the assumptions of this section, waiting is never optimal. Suppose that, as in the last subsection, the interest rate is a random variable X that can go up to ε with probability p, or stay the same with probability $1 - p$. Once the interest rate moves, it stays at the new level forever. For brevity, we discuss only the risk-neutral case.

To obtain straightforward solutions, we ignore second order effects related to convexity. In the next section we discuss a richer model that takes into account effects of all orders, and we obtain essentially the same results. The simplest model, however, allows us to show why waiting is not profitable ex-ante when the interest rate change is fully priced-in.

Our convexity assumption is equivalent to the expectations hypothesis of the yield curve, and allows us to approximate

$E\left(e^X\right)$ with e^μ, where $\mu = p\varepsilon$ is the expected interest rate. When ε is small relative to λ, this approximation is accurate[13].

Suppose that the current interest rate is zero for all maturities, and that the market knows that after a period of length Δ the interest rate may move up from 0 to $\varepsilon > 0$, although it does not know whether it will move or not. If the rate moves, it will stay at its new value forever. The yield $r(0, T)$ to discount to $t = 0$ risk-free payments to be received at $t = T$, satisfies the following equation:

$$e^{-Tr(0,T)} = e^{-\Delta r(0,\Delta)} E\left(e^{-(T-\Delta)r(\Delta,T)}\right).$$

According to our convexity assumption,

$$E\left(e^{-(T-\Delta)r(\Delta,T)}\right) = e^{-(T-\Delta)\mu}.$$

Also note that, because the current rate is zero and it is known that it will not change until $t = \Delta$:

$$e^{-\Delta r(0,\Delta)} = 1.$$

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[13]Consider, for example, $\varepsilon = 0.01$; $p = 0.7$, $\lambda = 0.06$. Under these assumptions, the annuity price is

$$\begin{aligned} a_0^R &= p\int_0^\infty e^{-(\lambda+\varepsilon)t}dt + (1-p)\int_0^\infty e^{-\lambda t}dt \\ &= 15. \end{aligned}$$

The approximation, instead, gives

$$\begin{aligned} a_0 &= \int_0^\infty e^{-(\lambda+\mu)t}dt \\ &= 14.93, \end{aligned}$$

a difference of just 0.5%.

Therefore, we obtain the following term structure at time 0:

$$r(0,T) = \begin{cases} 0 & \text{for } T \leq \Delta \\ \frac{T-\Delta}{T}\mu & \text{for } T > \Delta, \end{cases}$$

where T is maturity time. The slope of the yield curve is determined by Δ: the smaller Δ, the steeper the yield curve. In the limit, as $\Delta \to 0$, the yield curve jumps from 0 for all maturities, to μ for all maturities.

The value of the annuity is

$$\begin{aligned} a_0 &= \int_0^{\Delta} e^{-\lambda t} dt + \int_{\Delta}^{\infty} e^{-(\lambda+\mu)(t-\Delta)} dt \\ &= \frac{1}{\lambda}\left(1 - e^{-\lambda\Delta}\right) + \frac{1}{\lambda+\mu}. \end{aligned}$$

This price can be written as follows:

$$a_0 = \frac{1}{\lambda}\left(1 - e^{-\lambda\Delta} + \frac{1}{1+\frac{\mu}{\lambda}}\right)$$

Assuming that λ, Δ and $\frac{\mu}{\lambda}$ are small, the annuity price can be approximated as:

$$a_0 \approx \frac{1}{\lambda}\left(1 + \lambda\Delta - \frac{\mu}{\lambda}\right)$$

Note that, relative to the example discussed in section 2.1, the annuity price is increased by $\lambda\Delta$ and reduced by $\frac{\mu}{\lambda}$. If the market believes that the rate will change very soon $(\Delta \to 0)$, the annuity price will be reduced by $\frac{\mu}{\lambda}$, which is exactly what the retiree of section 2.1 -the one who thought her views were not priced-in- tried to capture by waiting.

The retiree who buys the annuity at time zero consumes a certain amount per period until she dies:

$$C^{nw} = \frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}.$$

The expected present value of consumption (split to facilitate comparisons with the waiting case) is

$$\begin{aligned} E\left(PVC^{nw}\right) &= \frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\int_0^{\Delta} e^{-\lambda t}dt + ... \qquad (6)\\ &+ \frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\int_{\Delta}^{\infty} e^{-(\lambda+\mu)(t-\Delta)}dt \qquad (7)\\ &= 1. \end{aligned}$$

As in Section 2, buying the annuity immediately is a zero-npv project.

Again, we assume that a regulator forces the waiting retiree to consume over a period of length Δ the same amount she would have consumed had she purchased the annuity at time 0: $\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta$. The retiree deposits the rest of her wealth at the rate $r(0,t)=0$, $0\le t\le\Delta$, which means that at time Δ she has $\left[1-\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta\right]$ to buy the annuity.

If the rate goes up, the value of the annuity at time 1 will be

$$a_1 = \frac{1}{\lambda+\varepsilon},$$

and after annuitization the retiree consumes

$$\begin{aligned} C^{wu} &= \frac{1-\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta}{a_1} \\ &= \left[1-\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta\right](\lambda+\varepsilon) \end{aligned}$$

per period. By the same reasoning, if the rate stays the same, after annuitization the retiree consumes

$$C^{wd} = \left[1-\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta\right]\lambda.$$

The expected present value of waiting for this retiree is

$$\begin{aligned} E\left(PVC^{w}\right) &= \frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\int_0^{\Delta} e^{-(\lambda+\mu)t}dt+\ldots \\ &+\left[1-\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta\right](\lambda+\mu) \\ &\int_{\Delta}^{\infty} e^{-(\lambda+\mu)(t-\Delta)}dt \end{aligned} \tag{8}$$

For waiting not to be profitable ex-ante, we must have

$$1-\frac{(\lambda+\mu)}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta < \frac{1}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}. \tag{9}$$

We show that this is always the case for $\Delta > 0$. First, we multiply both sides of inequality (9) by $1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)$ to get

$$1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)-(\lambda+\mu)\Delta < 1. \tag{10}$$

Cancelling 1 on both sides and taking $(\lambda+\mu)$ as common factor yields:

$$(\lambda+\mu)\left[\frac{1-e^{-\lambda\Delta}}{\lambda}-\Delta\right]<0. \tag{11}$$

So for inequality (9) to be valid we need

$$\frac{1-e^{-\lambda\Delta}}{\lambda}-\Delta<0.$$

But this is true for $\Delta>0$. To see this, we define a new function $f(\Delta)$ as follows:

$$f(\Delta)=\frac{1-e^{-\lambda\Delta}}{\lambda}-\Delta.$$

This function is obviously differentiable. Observe, in particular, that

$$\begin{aligned} f(0) &= 0 \\ f'(\Delta) &= e^{-\lambda\Delta}-1<0, \text{ for } \Delta>0. \end{aligned}$$

So for $\Delta>0$, inequality (9) is valid, and therefore

$$E\left(PVC^{nw}\right)>E\left(PVC^{w}\right).$$

By waiting, the retiree consumes $\frac{\lambda+\mu}{1+\frac{\lambda+\mu}{\lambda}\left(1-e^{-\lambda\Delta}\right)}\Delta$ of her wealth, and because the change in the interest rate is already priced-in, she has nothing to win beyond what the non-waiting retiree already gets. The more she waits, the worse this trade-off becomes; as a result, she never waits.

To conclude: a waiting retiree must think that the current annuity is expensive and that it will become cheaper in the future. She waits to capture the increase in consumption derived from the fall in the annuity price. The annuity price moves because the interest rate changes, and to the extent that the interest rate change is not fully priced-in, the waiting retiree is better-off on average, if the rise in the interest rate is large enough. Otherwise, she is worse-off.

3. A general model

We also investigate a more general model in which the short interest rate follows a Cox-Ingersoll-Ross (CIR) process and the hazard rate depends on the retiree's age. This model is consistent with investor's risk aversion, takes into account effects of all orders, and implies an upward sloping term structure.

The short rate satisfies the following stochastic differential equation:

$$dr_t = -\kappa\left(r_t - \overline{r}\right)dt - \sigma\sqrt{r_t}dB_t,$$

where r_t is the current short rate, κ is the mean reversion speed, $\overline{r}$ is the long-term short rate, σ is the volatility parameter, and B_t is Brownian motion. We choose the CIR process because we are interested in initial states in which the short rate starts at a very low level, and we want to avoid scenarios in which the rate becomes negative. Another advantage of the CIR process is that, as r_0 is close to zero, the dynamics of r_t are dominated by the drift, capturing the notion that there is a strong expectation that the short rate rises in the near future.

Following Deelstra Grasselli, and Koehl (2000), we define the market price of interest rate risk as

$$\phi_t = \frac{\varphi_1\sqrt{r_t}}{\sigma},$$

where φ_1 is a constant.

To give the retiree the possibility of investing part of her wealth during the waiting period, we also assume the existence of a risky asset, with the following dynamics:

$$\frac{dS_t}{S_t} = \left(r_t + \theta\left(r_t\right)\sigma_S\right)dt + \sigma_S\left(\rho dB_t + \sqrt{1-\rho^2}dZ_t\right),$$

where S_t is the asset price, σ_S its instantaneous volatility, Z_t is Brownian motion, uncorrelated to B_t, and ρ is the coefficient of correlation between stock returns and changes in the short rate. $\theta\left(r_t\right)$ is the stock's Sharpe ratio, defined as:

$$\theta\left(r_t\right) = \rho\frac{\varphi_1\sqrt{r_t}}{\sigma} + \sqrt{1-\rho^2}\varphi_2,$$

where φ_2 is also a constant.

This specification of expected returns captures the fact that times of low interest rates may also be times of high stock prices (or low expected returns). This specification is relevant for us, because if retirees are given the option to wait before annuitazation (because annuity prices are perceived too expensive), they may end up during the waiting period investing in a stock market whose prices are also too expensive.

The survival probability follows the Gomperz-Makeham (GoMa) law of mortality, with the following hazard rate:

$$\lambda\left(x\right) = \lambda + \frac{1}{b}e^{\left(\frac{x-\eta}{b}\right)}.$$

According to the GoMa law, the hazard rate is a constant λ plus a time-dependent exponential function. The constant λ captures the component of the death rate attributable to accidents, while the exponentially increasing portion reflects natural causes of death. The hazard rate increases with age and goes to infinity as $t \to \infty$. $x = \eta$ is the modal value of the curve: when the individual is exactly $x = \eta$ years old, the hazard rate is $\lambda(\eta) = \lambda + 1/b$; when the individual is younger $(x < \eta)$, it is $\lambda(x) < \lambda + 1/b$, and when the individual is older $(x > \eta)$, it is $\lambda(x) > \lambda + 1/b$.

We assume the existence of a stochastic discount factor that makes all the asset prices in the economy consistent, and value the annuity according to the following formula:

$$\begin{aligned} a_x &= E_0 \int_0^{T_x} e^{-\int_0^t r_s ds} dt \\ &= E_0 \int_0^{\infty} \mathbf{1}_{\{T_x \geqq t\}} e^{-\int_0^t r_s ds} dt, \end{aligned}$$

where E_0 denotes risk-neutral expectation and T_x is the random time of death. Assuming that the remaining random lifetime and the short rate are independent, a_x becomes:

$$a_x = \int_0^{\infty} F(t)\, G(t)\, dt,$$

where

$$\begin{aligned} F(t) &= \exp\left\{ -\lambda (s - t) - \int_t^s \frac{1}{b} e^{\left(\frac{(u-t)-\eta}{b}\right)} du \right\} \\ G(t) &= \exp\left\{ \upsilon \log\left(A(t-s)\right) + B(t-s)\, r_t \right\}, \end{aligned}$$

and

$$
\begin{aligned}
\upsilon &= \frac{2\widehat{\kappa}\overline{r}}{\sigma^2} \\
A(t-s) &= \frac{2he^{(\widehat{\kappa}+h)(s-t)/2}}{2h+(\widehat{\kappa}+h)\left(e^{h(s-t)}-1\right)} \\
B(t-s) &= \frac{2\left(e^{h(s-t)}-1\right)}{2h+(\widehat{\kappa}+h)\left(e^{h(s-t)}-1\right)} \\
h &= \sqrt{\widehat{\kappa}^2+2\sigma^2} \\
\widehat{\kappa} &= \kappa-\varphi_1.
\end{aligned}
$$

We calibrate the models using the following parameters:

$$
\begin{aligned}
r_0 &= 0.005, \widehat{\kappa}=0.2, \overline{r}=0.025, \sigma=0.02, \\
\varphi_1 &= 0.0535. \sigma_S=0.2. \\
\lambda &= 0, \eta=88.15, b=10.5.
\end{aligned}
$$

The CIR parameters are chosen to replicate the current shape of the European (AAA) term structure[14]; they imply a risk premium for a ten-year zero-coupon bond of 70 bps, consistent with Hellerstein (2011). The parameters of the GoMa law are taken from Milevsky (2006). The parameters of the stock price are meant to reproduce the behavior of a typical index; given the values of the other parameters, φ_2 is chosen to give an unconditional stock Sharpe ratio of 0.3, which implies an average stock risk premium of 6%.

This example is an extension of section 2.2, because the dynamics of the short rate are already priced-in in the term

[14]http://www.ecb.europa.eu/stats/money/yc/html/index.en.html

structure. Under the listed parameters, the term structure is upward sloping, as Figure 1 shows (see Appendix).

The value of an annuity for a 65 year-old retiree with the CIR term structure is 15.85. In contrast, the value of the same annuity assuming $r_0 = 0.005$, and a flat term structure, is 19.14. The difference is due to the upward-sloping shape of the term structure.

Tables 1 and 2 show the average consumption achieved by a retiree who buys an annuity immediately after retirement, or waits one to five years to annuitize. Table 1 considers a retiree who invests her residual wealth at the one-year risk-free rate during the waiting period. Table 2 considers a retiree who invests 70% of her residual wealth at the one-year risk-free rate and the remaining 30% in the stock market. All tables show average consumption for individuals who retire at the ages of 60 (panel A), 65 (panel B), and 70 (panel C). Each panel has three rows. The first row shows average consumption, the second row shows the volatility of consumption, and the third row shows "regret": the probability that the consumption achieved after waiting is lower than the consumption the individual would have obtained by annuitizing immediately after retirement. The averages are computed over 10000 simulations.

Table 1 shows that waiting and investing the residual wealth at the risk-free rate is never profitable, even though interest rates are going up on average. The increase in the interest rate is not enough to compensate the retiree for the loss of the mortality credit. The problem worsens the older the retiree is, and the longer she waits.

Investing in the stock market during the waiting period is an attractive alternative for waiting investors, so in table 2 we assume that the retiree is allowed to invest 30% of her residual wealth in stocks. This is similar to Milevsky and Young (2007), who studied the issue in a model with constant interest rates; we extend their results by assuming a stochastic interest rate. We compute consumption volatility and quantify the probability of regretting the decision to wait. Table 2 shows that waiting up to five years is profitable on average for relatively young retirees, but consumption volatility is high, and the regret probability is also high (on the order of 50%).

4. Dutch practice

This section discusses how the Dutch DC market has dealt with the problems previously discussed in the paper. We restrict ourselves to the mandatory DC pension savings in the second pillar. We do not take into account joint annuities.

Participants in Dutch DC systems have two options. In the default option they invest the pension contributions according to a carefully designed life cycle path; in the alternative option they can choose an opting-out variant in which they take responsibility for the investments. The large majority chooses the default option, in which more of the participants' wealth is allocated to equities when they are young, and moved, as they grow older, towards a matching portfolio that replicates the nominal or real annuities they have to convert their final wealth into at retirement. Figure 2 shows an example of such a life cycle path (see Appendix).

This pattern of decreasing weights for risky assets over the life cycle is explained by the theory of human capital[15]; see e.g. Bodie, Merton & Samuelson (1992) and Campbell & Viceira (2002). These authors state that in the allocation to the risky (or return) and riskless (or matching) portfolios, investors must take into account not only financial but also human capital. When a participant is young, her financial wealth is low, while her human capital is high. As the participant ages, the relative weight of the two sources changes; for an older participant the financial capital will form a larger or even the main part of her total wealth (see Figure 3). It can be assumed that future labor

[15]Human capital is defined as the discounted value of all future labor income.

income is relatively riskless, so human capital can be regarded as having the same characteristics as a long (real) bond. The result of this assumption is that the allocation to the matching portfolio is high for a young participant and that she should invest all of her financial wealth in the risky portfolio.

In 2007, the Dutch supervisor AFM introduced the prudent person rule, advising investors in the default option to reduce the investment risk on nominal or real annuities towards the retirement date. Investors can implement this advice by investing in long bonds and/or swaps in order to reduce the inflation and interest rate risk. According to the rule, they must also reduce their investments in the most risky assets (as e.g. stocks). Pension funds should verify at least once a year that there is a match between the portfolios participants in the opting-out variant have chosen, and the ones belonging to their specific risk profile. The fund should also inform the clients about the results of these tests.

At retirement, investors in the Netherlands have to convert their accumulated wealth into a lifetime annuity. The annuity must be fixed in principle[16]. It is currently not possible to have a variable annuity, where the height depends (partly) on the return of financial assets (such as e.g. stocks). Nor is it possible to receive a lump-sum payment at retirement, a feature much more common in the rest of Europe. For an overview of the various regimes within Europe, please refer to table 5.3 in Dietvorst et al. (2010). There are, however, exemptions to the

[16]Currently, discussions are under way to make it possible that the pensions will be partly `in natura', i.e. to include living and care requirements in the way the pensions will be consumed.

rule that the lifetime annuity should be fixed[17]:

- The annuity can be (conditionally) increased yearly with, for example, the price inflation (e.g. a real annuity).
- It is possible to have some variability in the size of the annuities, although there are clear rules: the variability should be determined at or before the retirement age and the minimum annuity should be at least 75% of the maximum annuity.
- Currently the retirement age can be adjusted to an age between ten years before and five years after the official retirement age (which is currently 65 years); the height of the lifetime annuity will be adjusted on actuarially neutral manner. It is also sometimes possible that a participant chooses to reduce the number of days he or she will work and instead receives an annuity for the days he or she will not work.
- If the participant wants to retire before the retirement age of 67 it is possible to increase the lifetime annuity by an amount that is equal to twice the state pension (`AOW') for couples.
- A final option is the `pensioenknip', a split-annuity option that is related to the research in this paper.

4.1. Pensioenknip

To help investos cope with losses incurred during the credit crisis, lawmakers in the Netherlands allowed investors to partly postpone the moment of converting accumulated capital into a lifetime annuity. This split-annuity option is known as `pensioenknip'. Participation in this structure is allowed only if

[17] There are some other options possible, which we will not discuss here. We refer to Dietvorst et al. (2010) for details.

the retirement date is after 31 December 2008 and the capital is not yet used to buy a lifetime annuity. Since 1 January 2014, this temporary arrangement is no longer feasible for new participants. The pensioenknip works as follows: At retirement, the capital is split into a part that will be used to buy an annuity for a maximum of five years (`knipperiode') while the remaining capital is invested to be converted into a lifetime annuity in the future. The height of the intermediate annuity is equal to that of an immediately starting lifetime annuity which could be bought at the retirement date. The participant can decide during the knipperiode to buy with the remaining capital a deferred lifetime annuity starting at the end of the knipperiode. If she does not make a choice, the remaining capital will be mandatorily converted into the lifetime annuity as per the end of that period. During the knipperiode the remaining capital has to be invested according to a risk profile that is similar or even lower than the profile before the retirement date.

As in advanced default life cycles the portfolio near the retirement date is already relatively riskless versus the nominal or real lifetime annuities, the return on the remaining capital during the knipperiode mimics the development of the prices of the annuities. So even if the interest rate increases in line with the participants' expectations, the postponement will not result in a much higher lifetime annuity at the end of the knipperiode. Moreover, the participant cannot profit optimally from price increases in the risky portfolio as the allocation to this category is already minimal at retirement age. An additional risk is that an increase in life expectancy can lower the final lifetime annuity as per the end of the `knipperiode' even if the prices of financial assets (bonds and equities) have not changed during the period.

If the participant has chosen for the opting-out variant during the accumulation phase, then the approach in the previous sections could be worthwhile pursuing although the pension fund should warn the participant against the additional risk of her portfolio vs. that of the default life cycle path during the knipperiode.

The pensioenknip was hardly a success: only a small minority of the eligible people -100 out of a possible 36,600- applied for it (see also Ministerie van Sociale Zaken en Werkgelegenheid (2013)).

There are two other types of risk during the decumulation phase, which are not discussed in this paper: inflation risk and longevity risk[18]. If the annuity takes the form of a real annuity the impact of inflation on the purchasing power will be reduced. There can still be a small impact, as the Dutch government does not issue inflation linked bonds at this moment. This will introduce a possible mismatch as the reference inflation by which the nominal value of the annuity will be increased yearly, could differ from the Dutch inflation. The reference inflation for inflation linked bonds or inflation swaps is in general related to the Euro HICP, the French or the German inflation.

Longevity risk is harder to hedge. It can be hedged by an insurer (or any other risk carrier such as a reinsurer, capitive or pension fund) who insures the risk of an increasing longevity. However, the associated costs (i.e. premium to be paid to risk carrier) will decrease the annuity payout for the retiree. Due to

[18]Another risk that we will not discuss is the (small probability) risk that the insurance company that has guaranteed the annuity will (partly) default.

foreseen new regulation (Solvency II and IORP2), insurers and other risk carriers in most EU countries will need higher buffers if they provide the hedge for the longevity risks underlying the lifetime annuities. Next to Solvency requirements, the premium costs are expected to rise in future over time (i.e. annuity pay out for retirees is expected to decrease) in view of the aging population and subsequent increasing longevity risks for the risk carrier.

Some initiatives to issue longevity bonds (see e.g. Thomsen & Andersen (2007) and Antolin & Blommestein (2007)) have not been successful, partly because governments are already exposed to longevity risk in their public pension funds and social security systems (see Antolin & Blommestein (2007)). In 2005 an initiative by the European Investment Bank to issue a longevity bond whose coupons were tied to realized mortality rates of English and Welsh males aged 65 in 2002, was withdrawn due to insufficient demand, because pension funds indicated that the price for the coverage of longevity risk was too high (see Blake et al. (2006)).

Another solution to take care of longevity risk, the `levensverwachtingsaanpassingsmechanisme' (LAM) was introduced in 2010 in the proposal for adjustments of the Dutch pension contract (see Stichting van de Arbeid (2011)). With the LAM it should be possible to adjust the retirement age and the size of both the new and existing claims in a defined-benefit pension fund to an increase of the longevity. The adjustment for the existing claims would be implemented by a decrease of the future indexation over a ten year period (see De Waegenaere et al. (2012)) for an overview of the other effects of the LAM). This same methodology can, however, also be used to adjust existing lifetime annuities for an increase in the longevity. This adjustment would, however, result in a decrease of the nominal value of the annuity and is thus at this point in time not yet allowed by the authorities.

4.2. Reduction of the conversion risk at retirement date

In the Dutch DC system the accumulated wealth of a worker has to be converted upon retirement age into a fixed nominal or real lifetime annuity. The conversion at a specific date results in a possibly large risk, as the annuity value depends almost entirely on the level of the interest rates at the retirement date: lower (higher) rates results in lower (higher) annuities. This so-called conversion risk is in general hedged in the more recent carefully designed DC propositions by means of a specific matching portfolio. The aim of this portfolio is to replicate the liabilities as best as possible in the period leading up to the retirement date. The portfolio composition is optimized using well known techniques developed in Asset Liability Management (ALM), an area in which the Dutch have extensive expertise. There are several possible approaches. The most common is to match the overall interest rate exposure in combination with additional restrictions on the allocations to the various maturity buckets in order to reduce the impact of non-linear interest rate movements. This approach is in fact in line with the well-known methods applied within the Liability Driven Investments (LDI) portfolios that are part of the portfolios of the DB pension funds; the optimal hedge is derived using a combination of long-term sovereign (euro) bonds, interest rate as well as inflation swaps, and in some cases also high-quality credits. The hedge can also be implemented by means of a cash-flow matching portfolio, in which each of the future cash flows at retirement date is matched as closely as possible. A third approach to reduce the conversion risk is the use of deferred annuities: in this approach, the participant buys during the build-up phase (and thus before the retirement date) deferred annuities that will start paying as of the retirement date. In this way she will average the interest rate and also the price of the annuities during the build-up phase.

These methods to reduce the conversion risk have become more common since 2007. In that year, the Dutch supervisor AFM introduced the "prudent person" rule, which advises that in the default option of DC systems the investment risk versus annuities should be reduced towards the retirement date (see AFM (2007)). During the last few years before the retirement date, the DC provider should focus on maximum security of the lifelong annuity instead of on a maximum pension. This could be implemented by means of investing, among others, in long-term bonds and interest rate swaps.

Up until a few years ago it was also possible to buy at the retirement date annuities against a predetermined fixed interest rate; the participant could convert her accumulated wealth into annuities at a rate of e.g. 4%. In these environments the matching portfolio should have a low duration as there was no interest sensitivity of the annuities. However these types of offers are rapidly disappearing due to (among other reasons) the current low market rates.

5. Conclusion

This paper has investigated under what conditions a retiree will find it valuable to delay annuitization in times of abnormally low interest rates. In deciding to wait, a retiree faces a trade-off: if interest rates go up, the annuity price will fall, and she will be able to buy it more cheaply; but she will, however, lose the mortality credit during the waiting period.

Waiting is a good decision if the benefits outweigh the costs, both measured ex-ante. We showed that for an investor who can only invest in bonds during the waiting period, waiting is never ex-ante profitable if annuities are fairly priced, because waiting is costly and buying a fairly-priced annuity is a zero-npv project.

We also investigated whether investing in the stock market during the waiting period makes waiting more attractive, and found that relatively younger retirees can attain a higher average consumption by waiting, but, not surprisingly, at the cost of a sharp increase in risk. Our results are consistent with Milevsky and Young (2007), and we contribute to the debate by measuring the volatility of consumption and quantifying the probability of regretting the decision to wait in a context of stochastic interest rates.

Furthermore, we showed that a retiree who speculates (i.e., who has views on the evolution of the interest rate not shared by the market), will find waiting ex-ante valuable as long as the expected increase in the interest rate is larger than the square of the hazard rate.

The first result derives from a standard principle of Finance: buying a fairly priced asset is a zero-npv project. We hope that our paper contributes by drawing the attention of practitioners and regulators to its important consequences. The last result establishes a condition that makes waiting ex-ante profitable to speculators. As it is relatively easy to satisfy, it may explain why, in the current economic climate, some investors consider it a good idea to postpone the purchase of an annuity.

Finally, we discussed the Dutch experience regarding the option to wait before buying an annuity, with special emphasis on the Pensioenknip, and provided a description of financial instruments suitable to that end.

Appendix: figures and tables

Figure 1: Term Structure of Interest Rates (CIR model, parameters as described on page 35)

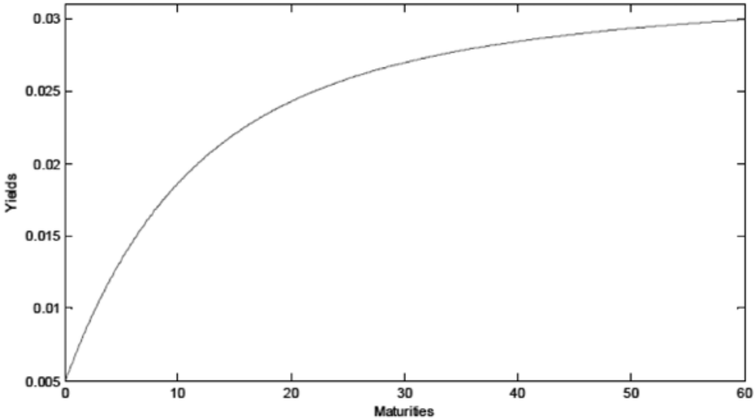
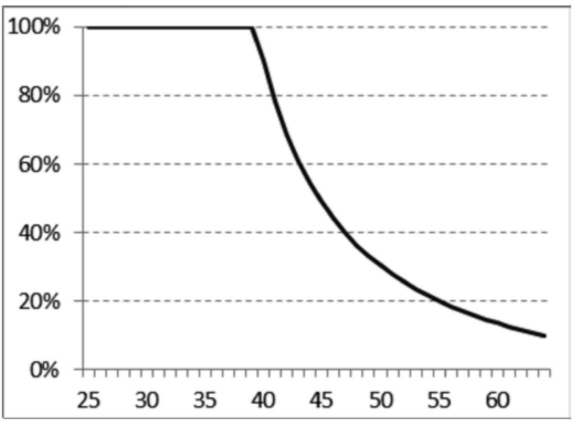
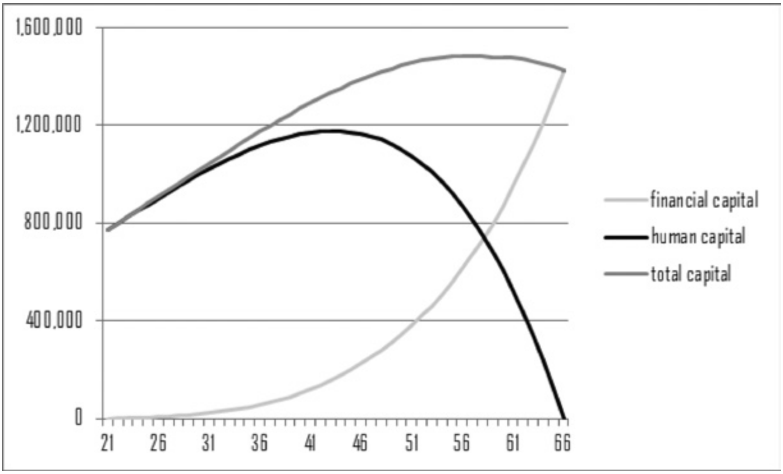


Figure 2: Allocation to equities in a life cycle.



Source: Robeco

Figure 3: Financial, human and total capital.



Source: Robeco

Table 1. Average consumption of the waiting retiree. Residual wealth invested at the risk-free interest rate

	Ret. Age	60	w=0			
Waiting	0	1	2	3	4	5
Av Consumption	5.528	5.480	5.423	5.354	5.274	5.183
Vol Con	0.000	0.076	0.118	0.156	0.189	0.220
Regret	0.000	0.760	0.824	0.867	0.901	0.929

	Ret. Age	65	w=0			
Waiting	0	1	2	3	4	5
Av Consumption	6.311	6.230	6.133	6.018	5.884	5.731
Vol Con	0.000	0.083	0.129	0.167	0.202	0.235
Regret	0.000	0.842	0.905	0.943	0.967	0.981

	Ret. Age	70	w=0			
Waiting	0	1	2	3	4	5
Av Consumption	7.391	7.247	7.074	6.870	6.633	6.360
Vol Con	0.000	0.092	0.142	0.182	0.218	0.248
Regret	0.000	0.927	0.973	0.989	0.997	0.999

Top, midddle and bottom panels show results for individuals retiring at 60, 65 and 70, respectively. In each panel the first row (Waiting) is the time after retirement at which the annuity is purchased, the second row (Av Consumption) is the average consumption achieved after annuitizing, the third row (Vol Con) is the volatility of that consumption, and the fourth row (Regret) is the probability that the consumption achieved after annuitizing is below the one the retiree would have achieved had she purchased the annuity immediately after retirement.

Table 2. Average consumption of the waiting retiree. 30% of residual wealth invested in the stock market.

	Ret. Age	60	w=0.3			
Waiting	0	1	2	3	4	5
Av Consumption	5.528	5.571	5.609	5.647	5.689	5.721
Vol Con	0.000	0.347	0.512	0.649	0.791	0.917
Regret	0.000	0.485	0.470	0.462	0.461	0.456

	Ret. Age	65	w=0.3			
Waiting	0	1	2	3	4	5
Av Consumption	6.311	6.335	6.350	6.355	6.350	6.329
Vol Con	0.000	0.390	0.580	0.745	0.890	1.040
Regret	0.000	0.516	0.508	0.513	0.521	0.537

	Ret. Age	70	w=0.3			
Waiting	0	1	2	3	4	5
Av Consumption	7.391	7.366	7.322	7.260	7.174	7.051
Vol Con	0.000	0.456	0.674	0.855	1.025	1.198
Regret	0.000	0.567	0.576	0.595	0.615	0.645

Top, middle and bottom panels show results for individuals retiring at 60, 65 and 70, respectively. In each panel the first row (Waiting) is the time after retirement at which the annuity is purchased, the second row (Av Consumption) is the average consumption achieved after annuitizing, the third row (Vol Con) is the volatility of that consumption, and the fourth row (Regret) is the probability that the consumption achieved after annuitizing is below the one the retiree would have achieved had she purchased the annuity immediately after retirement.

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Purchasing an annuity: now or later?

Due to the credit crisis, Central Banks of major world economies have engaged in unprecedented monetary activism, pushing interest rates down to historically low levels. It seems natural that investors believe that things will at a certain point go back to normal. In this context, it makes sense for retirees to ponder over postponing the purchase of an annuity. Thijs Markwat, Roderick Molenaar (both Robeco), and Juan Carlos Rodriguez (TiU) investigate whether this option has value for retirees. Two situations are evaluated: retiree can only invest in bonds, and investment in the stock market is (partly) possible.