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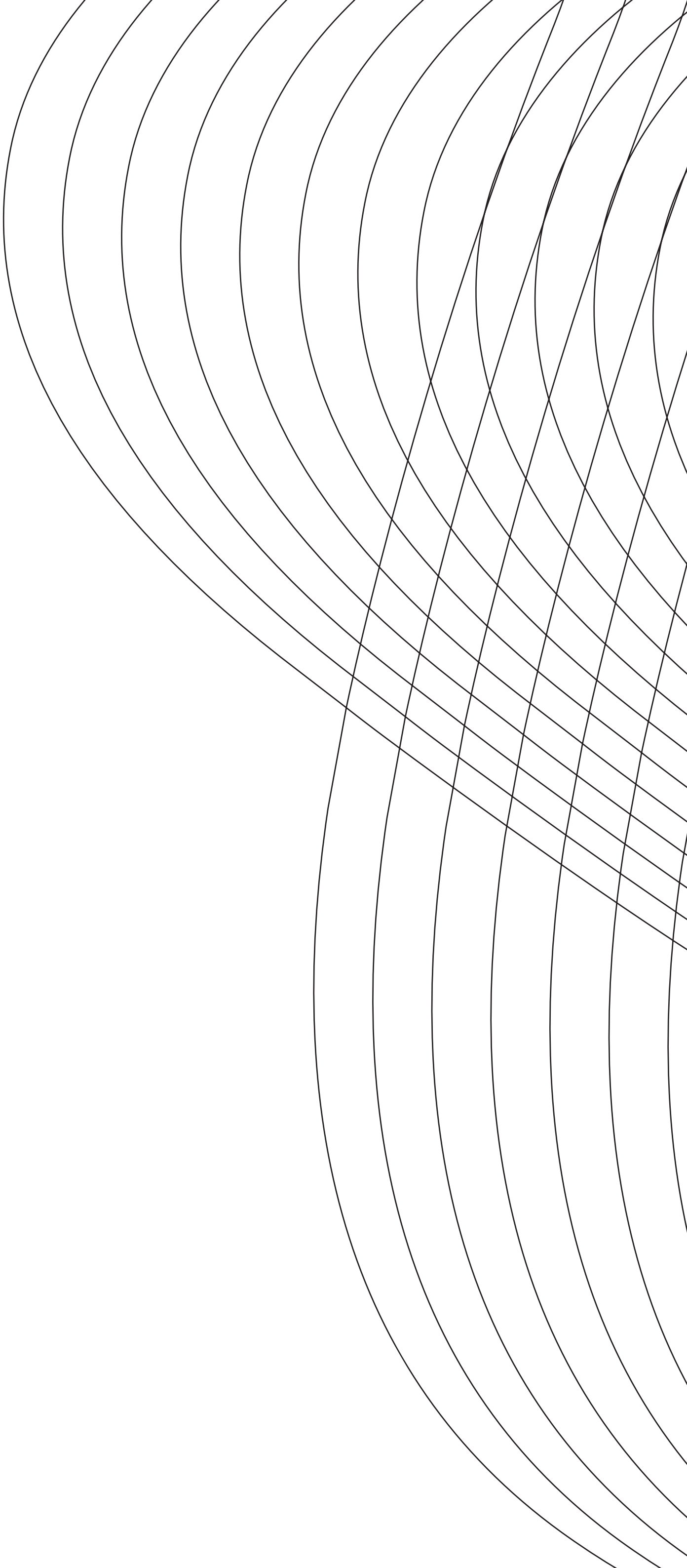
Netspar DESIGN PAPERS

Frank de Jong and Yang Zhou

Guarantees and habit formation in pension schemes

A critical analysis of the floor-leverage rule



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Editorial address
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PO Box 90153, 5000 LE Tilburg
info@netspar.nl

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PREFACE

Netspar seeks to stimulate debate on the effects of aging on the behavior of men and women, (such as what and how they save), on the sustainability of their pensions, and on government policy. The baby boom generation is approaching retirement age, so the number of people aged 65 and over will grow fast in the coming decades. People generally lead healthier lives and grow older, families have fewer children. Aging is often viewed in a bad light since the number of people over 65 years old may well double compared to the population between 20 and 65. Will the working population still be able to earn what is needed to accommodate a growing number of retirees? Must people make more hours during their working career and retire at a later age? Or should pensions be cut or premiums increased in order to keep retirement benefits affordable? Should people be encouraged to take personal initiative to ensure an adequate pension? And what is the role of employers' and workers' organizations in arranging a collective pension? Are people able to and prepared to personally invest for their retirement money, or do they rather leave that to pension funds? Who do pension fund assets actually belong to? And how can a level playing field for pension funds and insurers be defined? How can the solidarity principle and individual wishes be reconciled? But most of all, how can the benefits of longer and healthier lives be used to ensure a happier and affluent society? For many reasons there is need for a debate on the consequences of aging. We do not always know the exact consequences of aging. And the consequences that are nonetheless clear deserve

to be made known to a larger public. More important of course is that many of the choices that must be made have a political dimension, and that calls for a serious debate. After all, in the public spectrum these are very relevant and topical subjects that young and old people are literally confronted with.
For these reasons Netspar has initiated Design Papers. What a Netspar Design Paper does is to analyze an element or aspect of a pension product or pension system. That may include investment policy, the shaping of the payment process, dealing with the uncertainties of life expectancy, use of the personal home for one's retirement provision, communication with pension scheme members, the options menu for members, governance models, supervision models, the balance between capital funding and pay-as-you-go, a flexible job market for older workers, and the pension needs of a heterogeneous population. A Netspar Design Paper analyzes the purpose of a product or an aspect of the pension system, and it investigates possibilities of improving the way they function. Netspar Design Papers focus in particular on specialists in the sector who are responsible for the design of the component.

Roel Beetsma
Chairman of the Netspar Editorial Board

Affiliations

Frank de Jong – Tilburg University
Yang Zhou – Tilburg University

GUARANTEES AND HABIT FORMATION IN PENSION SCHEMES: A CRITICAL ANALYSIS OF THE FLOOR-LEVERAGE RULE

Summary

Scott and Watson (2011) recently introduced a simple "Floor-Leverage" rule for investment if households want to ensure non-decreasing consumption from one year to the next. We show that the leverage in their risky-asset investment policy implies a positive probability of lower consumption than in the previous year. However, for realistically calibrated asset returns, insurance against such bankruptcy risk using put options (at the Black-Scholes prices) is inexpensive and can make the Floor-Leverage rule work. A comparison with standard life-cycle models of consumption and investment shows that the requirement of non-decreasing consumption is very costly in welfare terms, because it results in low early consumption and high consumption growth and works against the desire of households to smooth consumption over time.

1. Introduction

Many pension plans in the Netherlands guarantee that the (nominal) benefits will never decrease. The benefits can increase if the financial position of the fund allows, according to the so-called conditional indexation rule. In exceptional circumstances, benefits can be cut ('afstempelen'), but this is a measure of last resort and considered to be a very painful measure to take. In contrast to this policy, typical optimal consumption and investment models prescribe that consumption should always be adjusted to increases and decreases in wealth. In other words, there is no place for without guarantees that consumption will never decrease.

The recent academic literature suggests that investors regard a large part of their previous consumption as necessary for subsistence, and derive utility only from the excess of consumption above the subsistence level; this is referred to as habit formation.[1] As pension funds invest on behalf of their members (De Jong, Schotman and Werker (2008)), the habit formation of pension participants might have a great impact on the pension design and investment strategy of pension funds. As discussed above, many pension plans contain guarantees and habit formation might explain at least part of the demand for such guarantees. Therefore, it is of interest to examine the impact of habit formation preferences on the optimal portfolio and consumption choice and to explore the implications for pension funds.

A simple but extreme form of habit formation can be seen in

[1] See, for example, Campbell and Cochrane (1999) and Gomes and Michaelides (2003).

"ratchet consumption" preferences, which require non-decreasing consumption over time. Scott and Watson (2011) analyze the portfolio choice problem with a ratchet consumption constraint and propose a rule of thumb---the Floor-Leverage rule for retirement: to ensure non-decreasing spending, a simple strategy for retirees is to invest at least 85% of the wealth in a risk-free asset to set up a *floor portfolio*, and the remaining wealth in a stock to set up a *surplus portfolio* with a leverage factor of three. Money is transferred from the surplus portfolio to the floor portfolio, if the value of the surplus portfolio exceeds 15% of the total portfolio value. However, Scott and Watson overlook the possibility of going bankrupt in the surplus portfolio. To hedge the bankruptcy risk, we propose insuring non-decreasing consumption with put options. Our findings demonstrate that the total costs of buying put options to guarantee nonnegative wealth in the surplus portfolio are fairly low.

We then take into account inflation, and compare nominal guarantees with real guarantees. The type of consumption guarantees plays a negligible role in determining the investment strategy, due to the constraint imposed by the Floor-Leverage rule that the floor portfolio can only be invested in a riskless asset. However, it has a substantial effect on the consumption pattern. The reason is that as inflation erodes future consumption, retirees with nominal guarantees tend to shift their consumption towards the early periods of retirement.

Finally, we compare the outcomes of the ratchet consumption strategies (in terms of welfare) with the classic model of Merton (1969), who considers a continuous-time portfolio-and consumption-choice model with the time-separable CRRA utility preference. In that model, the fraction of wealth invested in risky assets is constant over time, and substantial declines in consumption are possible. We find that the ratchet consumption constraint incurs substantial welfare losses as compared to the optimal strategy in Merton's model. The causes of this efficiency loss are twofold. First, the ratchet consumption model features ineffective smoothing of consumption over time. Second, the ratchet model restricts equity exposure of the retirees in the long run.

The remainder of this paper is organized as follows. Sections 2 and 3 review Merton's life-cycle model and the ratchet consumption model, respectively. Section 4 introduces the Floor-Leverage rule for ratchet retirees and proposes some variants. Section 5 compares the welfare of the various strategies, and section 6 concludes the paper with a few policy recommendations.

2. Benchmark: Life-Cycle Model with CRRA Utility

This section reviews the life-cycle model with the CRRA utility as the benchmark for the analysis that follows. This portfolio-and consumption-choice problem was first analyzed by Samuelson (1969) and Merton (1969). Samuelson (1969) determines the optimal portfolio-and consumption strategies for an investor with discrete-time, time-separable utility. Merton (1969) solves the portfolio-choice problem in a continuous time setting. For simplicity, we focus only on Merton's continuous-time portfolio-choice problem.

The standard life-cycle model uses the expected utility framework to describe the preferences of economic agents. Moreover, the utility function takes the form of CRRA (Constant Relative Risk Aversion). Given the time-separable CRRA utility preferences, the objective function of an investor with a fixed horizon T can be written as

$$\max E\left[\int_0^T e^{-\delta t}\frac{C_t^{1-\gamma}}{1-\gamma}\mathrm{d}t + e^{-\delta T}B(W_T)\right], \tag{1}$$

where γ is the risk-aversion parameter, δ is the subjective time discount factor, C_t is the consumption at time t, $B(W_T)$ is the bequest function, and W_T is the bequeathed wealth. Equation (1) implies that the investor is concerned with maximizing the expected utility from both of the consumption streams over her lifetime and her bequeathed wealth. Note that in Merton's model the economic agent does not have a bequest motive, which means that the model can be viewed as a special case of Equation (1) with $W_T \geq 0$ and $B(W_T) = 0$ for any W_T.

Next, we set up the economy. We assume that there are only two assets available to the investor, a risky asset with constant expected return of μ and volatility of σ, and a riskless asset that carries a fixed interest rate of r. Then, the returns of these two assets follow certain diffusion processes:

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + \sigma \mathrm{d}z_t, \tag{2}$$

$$\frac{\mathrm{d}B_t}{B_t} = r\mathrm{d}t, \tag{3}$$

where S and B denote the price of the risky asset and the price of the riskless asset respectively. Then, the portfolio-and consumption-choice problem for the investor is subject to the budget constraint

$$\mathrm{d}W_t = [(x_t(\mu - r) + r)W_t - C_t]\,\mathrm{d}t + x_t W_t \sigma \mathrm{d}z_t, \tag{4}$$

and the constraints $W_t > 0$ and $C_t > 0$ for $t \in [0, T]$. Here, x_t denotes the fraction of wealth invested in the risky asset at time t. Solving this portfolio-choice problem using the dynamic programming approach yields

$$x_t = \frac{\lambda}{\gamma\sigma}, \tag{5}$$

$$C_t = \left[\frac{v}{1 - e^{v(t-T)}}\right] W_t, \tag{6}$$

with

$$v = \frac{1}{\gamma}\left[\delta + (\gamma - 1)\left(\frac{\lambda^2}{2\gamma} + r\right)\right], \tag{7}$$

where $\lambda = \frac{\mu - r}{\sigma}$ is the Sharpe ratio. These two equations are the optimal portfolio-and consumption strategies, respectively. It is

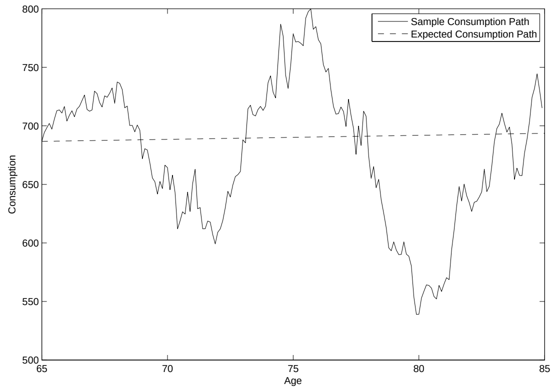

Figure 1: Expected and sample consumption paths.

evident from Equation (5) that the proportion invested in the risky asset is a constant. It is larger, the higher the Sharpe ratio, the lower the volatility, and the lower the risk aversion. Moreover, it is independent of wealth and the investment horizon. This is due to the assumption that investment opportunities are time-invariant. As a consequence, the investor becomes myopic and only has speculative demand in the optimal portfolio.

As for the optimal consumption strategy, Equation (6) implies that the fraction of wealth consumed is only time-dependent, but not state-dependent. However, as the wealth level is volatile due to the stock market risk, the consumption level fluctuates over time. To illustrate this, we turn to a numerical example. We consider a retiree of age 65, risk aversion of $\gamma = 3.5$, a 20-year horizon, a subjective discount factor of $\delta = 0.05$ and initial wealth of €10000. The risk-free rate and equity risk premium are set to 2% and 4%, respectively. The volatility of the stock is 18%. Therefore, by (5) we can determine the constant proportion of wealth invested in the risky asset $x = 35.3\%$. As shown in Figure 1, given this set of parameter values, the expected consumption remains stable over time. In contrast, the sample consumption exhibits considerable fluctuation and declines in some periods. The retiree will suffer substantially from these intermittent drops in consumption, if, as the habit formation literature suggests, she regards a large part of her previous consumption as necessary for subsistence and derives utility only from the excess of consumption above the subsistence level. Therefore, the optimal consumption strategy derived from Merton's portfolio problem does not fit the need of investors with habit persistence. To explore this, the following sections are devoted to the analysis of alternative models with habit formation.

3. Portfolio Choice with Ratchet Consumption

The ratchet consumption preferences are similar to Merton's assumptions, but in addition require nondecreasing consumption over time. Dybvig (1995) first introduced ratchet consumption preferences into the portfolio choice problem with an infinite investment horizon, finding that the optimal investment strategy is to invest part of the wealth in a risk-free asset to guarantee future spending and the remainder in a leveraged portfolio to seek future increases. Watson and Scott (2011) analyze a similar problem with finite horizon and in a discrete time setting.

Watson and Scott (2011) assumed a standard Black-Scholes world: there are only two assets traded on the market and their returns follow (2) and (3). Therefore, the market is complete and there exists a unique pricing kernel. They use the martingale representation approach (Cox and Huang (1989)) to map dynamic portfolio choice problem into the following static problem:

$$\max_{C_t} \sum_{t=0}^{T} E\left[e^{-\delta t}\frac{C_t^{1-\gamma}}{1-\gamma}\right], \tag{8}$$

$$\text{s.t.} \sum_{t=0}^{T} E\left[M_t C_t\right] \leq W_0, \tag{9}$$

$$0 \leq C_0 \leq C_1 \leq ... \leq C_T, \tag{10}$$

where T is the investment horizon, δ is the subjective discount factor and M_t is the pricing kernel. Equation (9) is the static budget constraint and Equation (10) is the constraint that accommodates the investor's need for nondecreasing spending.

Watson and Scott verify that the solution takes the form

$$
\begin{aligned}
Y_t &= \max\left[0, I\left(\frac{\gamma M_t}{e^{-\delta t} y_t}\right)\right] && (11)\\
C_t &= \max(C_{t-1}, Y_t), && (12)
\end{aligned}
$$

where I is the inverse function of the first-order derivative of the utility function and Y_t is the consumption for a Merton-Samuelson investor with the time preference function $e^{-\delta t} y_t$ and multiplier γ. Note that both γ and y_t can be uniquely determined. Clearly, a ratchet investor's optimal consumption at time t (C_t) depends on both her previous period's consumption (C_{t-1}) and her expected future consumption (Y_t); her optimal consumption turns out to be a derivative security on the pricing kernel. The value of the security is a function of three independent variables $V_t(M, C, t)$, where M is pricing kernel, C is current consumption, and t is time. Further, the total portfolio wealth $V(M, C, t)$ is the sum over all V_t,

$$V(M, C, t) = \sum_{n=0}^{T-1-t} V_{t+n}(M, C, t). \quad (13)$$

By delta-hedge formula, the fraction of total wealth invested in the pricing kernel is

$$f(M, C, t) = \frac{\partial \ln V(M, C, t)}{\partial \ln M}. \quad (14)$$

To determine $f(M, C, t)$, one needs to first compute $V(M, C, t)$. This can be done numerically using backward induction. Then, the fraction of total wealth invested in the risky asset follows from the chain rule,

$$s(M, C, t) = \frac{\partial \ln V(M, C, t)}{\partial \ln M} \frac{\partial \ln M}{\partial \ln S} = \left(-\frac{\mu - r}{\sigma^2}\right) f(M, C, t), \quad (15)$$

and the fraction of total wealth invested in the risk-free asset is given by

$$F(M, C, t) = 1 - s(M, C, t). \quad (16)$$

Watson and Scott (2011) claim that a rachet consumer's optimal investment portfolio can be partitioned into a *floor portfolio* and a *surplus portfolio*. The former invests in the risk-free asset to secure spending at the current level, while the latter invests in risky assets to garner future consumption increases. The consumption level is determined annually in the following way. At the beginning of each year, the retiree first calculates the amount of money D_t needed to sustain €1 of spending throughout the remaining retirement years. Note that D_t is the total price of a ladder of riskless zero-coupon bonds that pay €1 at time t to $T-1$. Therefore, D_t is given by

$$D_t = \sum_{i=t}^{T-1} e^{-r(i-t)}, \quad (17)$$

where r is the risk-free interest rate and T is the planning horizon. Second, the retiree needs to determine the minimum floor ratio F_t, which is the minimum fraction of wealth that must be dedicated to sustaining future consumption. The value of F_t is obtained from the optimization solution described above ($F(M, C, t)$). Armed with D_t and F_t, the retiree can calculate C_t, the optimal spending for year t,

$$C_t = \max(C_{t-1}, F_t W_t / D_t), \quad (18)$$

where C_{t-1} is the consumption in the previous year and W_t is the current wealth. In short, each year, the retiree compares the minimum spending implied from the previous year with the spending sustained by investing $F_t W_t$ in riskless bonds and then chooses the larger one. The initial-period consumption is given simply by $C_0 = F_0 W_0 / D_0$.

In the discussion above, we assumed that the inflation rate was zero or, alternatively, that all variables were expressed in real, inflation-adjusted terms. In reality, many pension schemes give only nominal guarantees. Let π denote the inflation rate. With the introduction of inflation, equation (10), which captures the ratchet consumption constraints, can be rewritten in nominal terms as

$$0 \leq C_0 \leq e^{\pi} C_1 \leq ... \leq e^{T\pi} C_T, \tag{19}$$

where the inflation parameter π controls the maximum rate at which real spending C_t is allowed to decrease. If π is zero, inflation is not considered and real consumption never declines. Conversely, if π is greater than zero, nominal spending never declines, but real spending may. The total price of the nominal zero-coupon bonds is

$$\widetilde{D}_t = \sum_{i=t}^{T-1} e^{-(r+\pi)(i-t)}, \tag{20}$$

where r is the real interest rate and the nominal interest rate is the sum of the real interst rate and inflation ($r + \pi$). The optimal consumption policy is

$$C_t = \max(e^{-\pi} C_{t-1}, \widetilde{F}_t W_t / \widetilde{D}_t), \tag{21}$$

where W_t is the current real wealth and $\widetilde{F}_t$ is obtained from the optimization solution.

4. The Floor-Leverage Rule

Scott and Watson (2011) propose a rule-of-thumb to approximate the complex ratchet consumption policy---the *Floor-Leverage* rule. To guarantee non-decreasing spending in the retirement years, a simple strategy for retirees is to initially allocate 85% of their retirement wealth to the floor portfolio ($F_0 = 85\%$) and all remaining wealth to the surplus portfolio with a leverage factor of three. If the stock market rises, money is transferred from the surplus portfolio to the floor portfolio to maintain a higher level of consumption. They assert that even if the stock market falls, spending is sustained and losses are limited to the surplus portfolio. Nonetheless, given the leveraged position, it is natural to doubt whether the Floor-Leverage rule guarantees non-decreasing ratchet consumption under any circumstances.

To simplify analysis, we assume throughout this section that there is no inflation except for subsection 4.4, which discusses the case of sustainable nominal consumption and compares it with that of sustainable real consumption. To begin with, we test the validity of the Floor-Leverage rule and propose some variants. A simulation experiment reveals that there is a positive probability that the value of the surplus portfolio falls below zero, which implies that the transfer of money has to be reversed to keep the surplus portfolio solvent. As a consequence, future consumption is reduced and the Floor-Leverage rule does not guarantee nondecreasing consumption. To remedy this problem, we propose a dynamic trading strategy with put options: to hedge against the downside risk of the stock market, we purchase a series of put options and determine both the strike prices of the put options and put option holdings dynamically.

Our findings demonstrate that in the Black-Scholes world the total costs of buying put options account for only a very small fraction of the initial wealth because the strike prices are set at such low levels that only the zero value of the surplus portfolios is guaranteed.

In addition, we investigate the dynamic portfolio strategies for different leverage factors and equity premium, and find that future spending increases with the leverage factor due to the higher expected return by taking higher equity exposure. Moreover, a comparison of sustainable nominal spending with sustainable real spending reveals that the retirees with the nominal constraint receive a higher consumption stream in the early periods, but have lower consumption growth than their counterparts.

4.1. Bankruptcy Risk and Leverage

Following the Floor-Leverage rule proposed in Scott and Watson (2011), the experiment designed is as follows. First of all, we invest 85% of available assets to purchase a spending floor ($F_0 = 85\%$). Once 85% of the initial wealth is allocated to the floor, the remaining wealth is invested aggressively in equity with a leverage factor of three. After this initial allocation, we check annually whether the surplus portfolio exceeds 15% of the total portfolio. If so, then surplus assets in excess of 15% are reallocated to purchase additional floor spending. Otherwise, no money is transferred from the surplus portfolio to the floor portfolio, and consumption level remains the same as in the previous period. Moreover, we annually rebalance the surplus portfolio to maintain a constant leverage factor of three. The age of the retiree is 65 at the beginning. Moreover, we follow Scott and Watson to assume that the remaining life expectancy for the retiree is 40 years ($T = 40$), but we examine the investment and consumption behavior of the retiree only in the first 20 years. In other words, the investment horizon of the retiree is 20 years, whereas the planning horizon is 40 years. At each annual review, 'scenarios' that have negative surplus portfolio value are eliminated, and are not considered in subsequent periods.

Specifically, we generate 10000 scenarios with equal initial wealth of €10000. We investigate how many scenarios survive in each period and how this survivorship evolves over time. The parameter values are the following. We consider a two-asset economy with a riskless interest rate equal to 2% and a risky asset broadly consistent with developed equity markets: an annual risk premium of 4% with an annual volatility of 18%. 10000 paths of stock prices are simulated over 20 years with initial stock prices of €100. It is assumed that the leverage is taken by borrowing money at the cost of the real rate and that all assets are infinitely divisible. More, there is no inflation.

Figure 2 shows that the survival probability[2] declines almost linearly over time and reaches a level of 84% at the horizon. This outcome contradicts the argument in Scott and Watson (2011) that the Floor-Leverage rule can ensure nondecreasing consumption over time. In fact, as time goes on, in more and more scenarios the retiree runs out of money in her surplus portfolio because of the occurrence of market crashes. In contrast, when the leverage factor is reduced to 2.5, the survival probability remains above 95% throughout, although it still decreases. Thus, reducing the leverage factors remarkably

[2]Survival probability in each period is calculated as the ratio of the number of the scenarios alive in that period to the total number of the scenarios generated at the beginning.

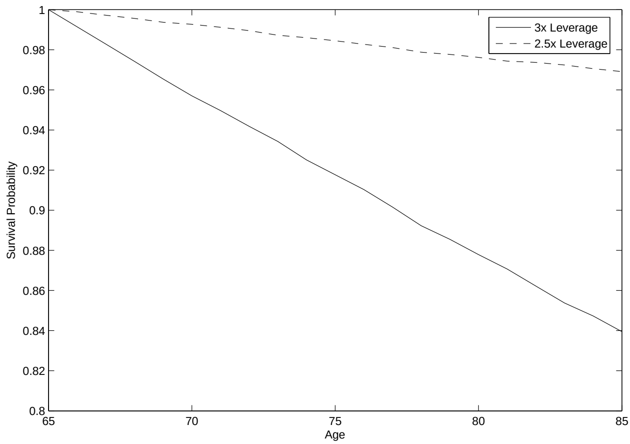

Figure 2: Survival probability for different leverage factors.

increases the chance of keeping consumption non-decreasing over time. Nonetheless, as long as there exists a leveraged position, the surplus portfolio is always likely to go bankrupt in extremely bad states of the world, thereby invalidating the argument that the Floor-Leverage rule is always able to generate non-decreasing spending.

4.2. Insurance with Put Options

One straightforward strategy to overcome the bankruptcy possibility of the Floor-Leverage rule is to buy a series of put options to hedge against the downside risk of the stock market. The trading strategy is dynamic, because at each annual review the put option holdings must be adjusted in order to obtain a full level of insurance against bankruptcy risk. We assume a standard Black-Scholes world with a complete market, so that the prices of the put options can easily be calculated using the Black-Scholes option-pricing formula. Specifically, for each period, we determine the number of shares (N_t^S), the number of options (N_t^P) and the strike prices of put options (K_t) by solving the following system of equations:

$$\begin{cases} N_t^S = N_t^P \\ W_t^{surp} = S_t N_t^S + P_t N_t^P \\ K_t N_t^S = (\frac{L-1}{L} W_t^{surp})\,(1+r)\,, \end{cases} \tag{22}$$

where S_t and P_t denote the prices of the stock and the put option at time t and W_t^{surp}, r and L are the wealth in the surplus portfolio at time t, the borrowing rate and the leverage factor, respectively. Note that $L \geqslant 1$ and in the Floor-Leverage rule $L = 3$. The first equation implies that to fully hedge the stock market risk, the number of the put options must be equal to the number of the stocks. In the second equation, we calculate the value of the surplus portfolio as the sum of the values of each asset class in the surplus portfolio. Finally, we determine the strike price of the put options such that the insured value of the surplus portfolio coincides with the sum of the principal and the interest of the leveraged position, which means that the liquidation value of the surplus portfolio can exactly cover the loan in the event that the stock market plunges. Note that as L, r, W_t^{surp} and S_t are known in advance, and P_t is a function of K_t, we end up with three equations and three unknowns (N_t^S, N_t^P and K_t). Hence, in general, there exists a unique solution for this system of equations. Unfortunately, however, it has to be solved numerically, due to the complexity of the Black-Scholes formula for P_t.

To show how (22) works, we present a numerical example in the following. Suppose we are in the initial period and have

Table 1: Value of put options as fraction of initial wealth for different volatilities

L	$\sigma = 14\%$	$\sigma = 16\%$	$\sigma = 18\%$	$\sigma = 21\%$	$\sigma = 24\%$	$\sigma = 27\%$
3	0.024%	0.082%	0.19%	0.47%	0.88%	1.39%
2.5	0.00013%	0.0079%	0.029%	0.11%	0.26%	0.50%
1	0	0	0	0	0	0
0	0	0	0	0	0	0

This table reports the value of the put options as fraction of initial wealth for different volatilities of the stock. L and σ are the leverage factor and the volatility of the stock, respectively.

€10000 on hand. By the Floor-Leverage rule, we first invest €8500 in the risk-free bond to set up the floor portfolio, which ensures a spending level of €305.65 in every future period. Then, we borrow €3000 to maintain a leverage factor of three, and put all of the money in equity. As a result, the value of the surplus portfolio is €4500 and the leveraged position is two-thirds of it (€3000). The initial stock price is assumed to be €100. Plugging these quantities into (22) yields

$$\begin{cases} N_1^S = N_1^P \\ 4500 = 100 N_1^S + P_1 N_1^P \\ K_1 N_1^S = 3060. \end{cases} \tag{23}$$

Solving for N_1^S, N_1^P and K_1 numerically yields $N_1^S = N_1^P = 44.97$ and $K_1 = 68.04$, which means that in the initial period, the retiree should buy 44.97 stocks and 44.97 put options with the strike price of €68.04, which implies a put option price of €0.06.

Table 1 shows the value of put options as a fraction of the initial wealth, which is calculated using the pricing kernel based on simulations.[3] As the retiree starts with an equal endowment of €10000 in all cases, one can easily translate the cost of put options into euro terms. For example, when $L = 3$ and $\sigma = 18\%$, the retiree needs to pay an average of only €19 (€$10000 \times 0.19\%$) for the option insurance. Not surprisingly, the costs of buying the put options increase with the volatility of the stock. Moreover, the retiree with a leverage factor of 2.5 pays less for the option insurance than does her counterpart with a leverage factor of three, since the former has lower equity exposure and higher survival probability. However, the fraction of wealth allocated to the put options remains small across different strategies[4] and different volatilities. There are two reasons. First, the strike prices are set at such low levels that only zero value of the surplus portfolios is guaranteed, thereby generating rather low prices for the put options in the Black-Scholes model. Second, the average amount invested in the risky asset is shrinking over time because of the one-way cash flow from the surplus portfolio to the floor portfolio. As a result, there is not much money to insure in many of the scenarios. It is important to note that there are some difficulties in implementing the option insurance strategy in practice. First, deep out-of-money (OTM) options are lack of liquidity and potentially have large

[3]The calculation procedure is as follows. We first use the pricing kernel as the discount factor to compute the present values of all of the put options we purchase. Then, we take the mean of the present values of the put options in a certain period as the expected value of the put options in that period. Finally, we sum up the expected value of the put options in each period to obtain the total expected value of the put options.

[4]Retirees with leverage factor of one borrow no money and only invest the wealth in the surplus portfolio in the stock, while retirees with leverage factor of zero invest all of their wealth in the risk-less bond.

counterparty credit risk. Second, deep OTM options are much more expensive in practice than the Black-Scholes prices, which is known as volatility smile.

4.3. Consumption Patterns and Leverage

A vast literature is available on the consumption during retirement. Some studies focus on the behavior of consumption as households transition to retirement and analyze the so-called "retirement consumption puzzle", an abrupt decline in expenditures at retirement.[5] Other papers examine consumption over the life-cycle and provide empirical evidence that the consumption of retirees decreases over the retirement periods, which seems difficult to reconcile with ratchet consumption preferences.[6] In contrast, using an internet survey conducted in the U.S. and the Netherlands, Binswanger and Schunk (2011) find that individuals in both countries aim to achieve a level of retirement spending exceeding 70 percent of working-life spending, and do not want to fall below a certain lower limit of old-age spending, providing evidence in favor of habit persistence. In this subsection we investigate the patterns of consumption after retirement when individuals follow the Floor-Leverage rule for consumption and investments.

Figure 3 illustrates the expected consumption of retirees using different leverage strategies. Again, we look at the leveraged strategies with $L = 3$ and $L = 2.5$, with additional put options to prevent bankruptcy in the investment portfolio. Besides the

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[5]See, for example, Aguiar and Hurst (2005), Hurst (2007), Ameriks, Caplin and Leahy (2007), Hurd and Rohwedder (2003).

[6]See Fernández-Villaverde and Krueger (2011), Bullard and Feigenbaum (2007), Fernández-Villaverde and Krueger (2007), and Gourinchas and Parker (2002).

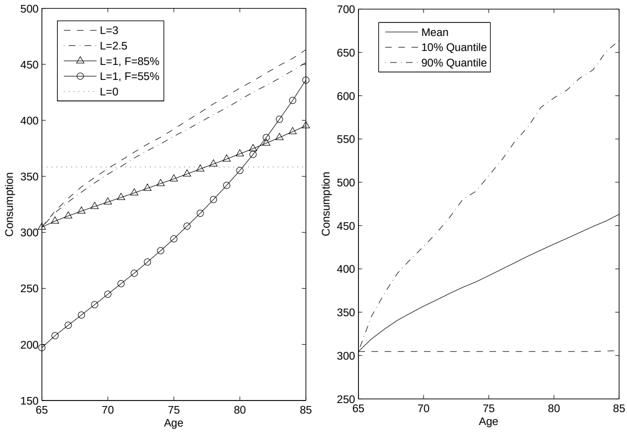

Figure 3: Expected consumption using different strategies and confidence bounds with leverage factor of three. The left panel plots the expected consumption using different strategies, while the right panel illustrates confidence bounds with leverage factor of three. L is the leverage factor. F is the floor ratio.

insurance with put options, another simple strategy to get rid of bankruptcy risk is to take no leverage and use only the cash on hand to invest ($L \leq 1$). For a leverage factor of 1, we analyze two different strategies. One follows the original Floor-Leverage rule and puts 85% of the wealth in the floor portfolio; the other changes the floor ratio to 55% so that the initial stock investment is €4500, which coincides with the initial stockholding of the strategy with leverage factor of three.

Figure 3 (left panel) illustrates that the retirees with stock investment have an increasing consumption pattern over time, whereas the retirees without stock investment have a constant consumption level. This is because the former types of investors benefit from the positive equity premium and pursue a non-decreasing consumption pattern. However, since the retirees without equity exposure have no surplus portfolio and use all their wealth to set up the floor portfolio, they consume more than the other types of retirees in the early periods. As the leverage factor rises, the slope of the consumption curve steepens, which implies that investment strategies with a higher leverage factor generate higher expected future spending for retirees.

Since the retirees with leverage factor of three have the same initial stock investment as those with leverage factor of one and floor ratio of 55%, the distinction in the shape of their expected consumption curves reflects the effect of taking leverage. The retirees without leverage have much lower initial consumption than do their counterparts, because the only way for them to raise funds for larger equity investment is to cut current consumption. On the other hand, the retirees without leverage enjoy higher consumption growth than those with leverage---both because the latter type of retirees gets decreasing benefits from the equity premium and because they have to pay for the put options.

The right panel of Figure 3 shows the dispersion in consumption for the $L = 3$ case. Consumption at the 90% quantile increases rapidly over time, while consumption at the 10% quantile remains almost constant. The distinction follows from the market downturns. When the stock prices go down, the surplus portfolio shrinks and becomes financially incapable of raising the consumption level. Under extremely bad market conditions, the wealth invested in the surplus portfolio may even decline to zero, leaving consumption constant over the remaining periods and financed completely by the floor portfolio.

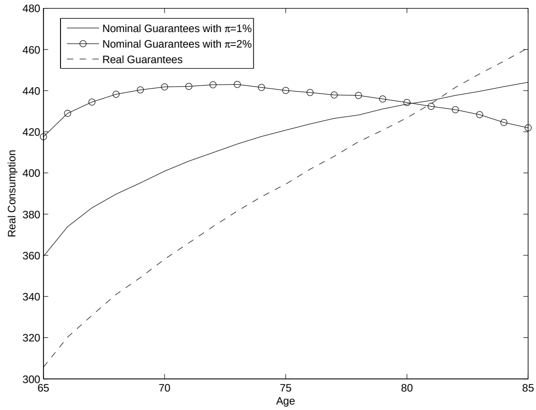

Figure 4: Expected real consumption with different types of consumption guarantees and inflation levels.

4.4. Nominal Consumption Guarantees

As many pension plans provide guarantees only for the nominal benefit, it is of interest to consider the case with the requirement for non-decreasing nominal consumption. In this subsection we therefore relax the assumption of no inflation, set the inflation rate π equal to some constant levels and raise the stock return by the same amount to keep the equity premium the same as the real guarantee case. In the meantime, other parameter values are held unchanged.

Figure 4 illustrates the expected real consumption with different types of consumption guarantees and inflation rates. Obviously, the type of consumption constraint has a great influence on the expected consumption behavior of the retirees: those requiring nominal guarantees enjoy higher real spending in the early periods but have lower spending growth than their counterparts. Equation (20) suggests that the total price of the nominal zero-coupon bonds $\widetilde{D}$ decreases with π. The intuition is that the increase in the inflation rate raises discount rates on the future nominal consumption (nominal interest rate) and lowers the current price of nominal zero-coupon bonds. Therefore, the shift from the real constraint to the nominal one leads to a lower value of D and a higher initial consumption level. However, in the nominal guarantee case, real consumption can be eroded by the inflation in future periods, thus reducing real spending growth. The lower consumption growth follows from the fact that the type of consumption guarantees plays a very limited role in determining the equity exposure of the surplus portfolio, as there is no money transfer from the floor portfolio to the surplus portfolio. Hence, the switch between the two types of guarantees only generates a trade-off between the real spending in the short run and in the long run. On the other hand, for nominal guarantees, the higher the inflation rate, the higher the initial consumption, but the lower the consumption growth. When the inflation rate is set to 2%, the expected real consumption for retirees with nominal guarantees even declines over the late retirement years.

4.5. Historical Simulation

It is tempting to evaluate the Floor-Leverage rule on real-life data. For this purpose, we use annual data on the S&P 500 index from Datastream as the single risky asset and the VIX index from WRDS as the volatility for computing option prices. The sample period is from 1990 to 2010. To simplify the analysis, we keep the risk-free rate constant at the level of 2% and permit only the stock returns to vary over time. Moreover, we assume that there is no inflation and that there are options in place to make sure the leveraged surplus portfolio keeps a nonnegative value.

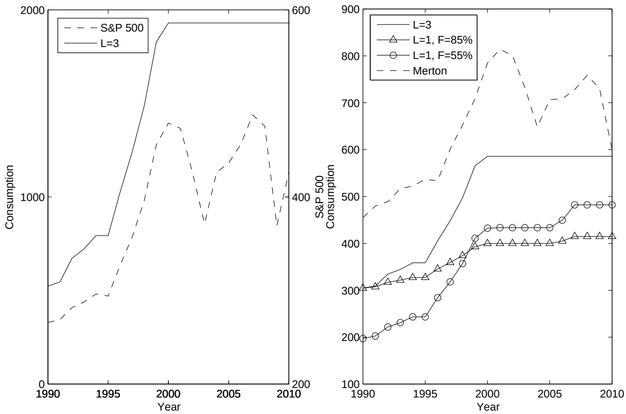

Figure 5: Consumption using different strategies with real data. The left panel plots the expected consumption with leverage factor of three versus the S&P 500 index, while the right panel illustrates the expected consumption using different strategies. The data are obtained from Datastream and WRDS. The sample period is from 1990 to 2010.

The left panel of Figure 5 plots the realized consumption pattern with leverage factor of three versus the S&P 500 index. Consistent with the previous analysis, consumption growth is highly subject to market conditions; note how it paused whenever the stock price went down (for example, the market decline that occurred around 1995) and ceased after the market crash around 2003. The reason for this consumption pattern is that any market decline results in losses of wealth in the surplus portfolio. As a consequence, the surplus portfolio is unable to provide money to the floor portfolio, and the consumption level has to remain the same as in the previous period. In the extreme cases of market crashes, the surplus portfolio fails to pay back the loan and ends at zero value. Consumption is then sustained only by the floor portfolio and therefore stops growing over the remaining periods.

The right panel illustrates the consumption using different strategies. Not surprisingly, it resembles the left panel of Figure 3: the strategy with leverage factor of three produces higher consumption than the strategies with leverage factor of one; for the strategies with the same leverage, the lower the floor ratio, the lower the initial consumption but the higher the consumption growth. However, using real-life data indeed results in some deviations from the average patterns shown in the previous figure. First, for any type of Floor-Leverage strategy, the consumption level remains unchanged for some periods of time. Second, after a market crash, consumption growth for the retiree with leverage factor of three come to an end. For comparison purposes, the consumption associated with Merton's constant investment strategy is also displayed; it is highly affected by the market conditions and fluctuates over time. Moreover, the investors with Merton's strategy enjoy much higher consumption than do their counterparts with the Floor-Leverage rule, as they do not have to reserve substantial funds for future non-decreasing consumption and can therefore invest more aggressively.

5. Welfare Analysis

To examine quantitatively how the leverage factor and equity premium affect the welfare of retirees, we compare the efficiency of different strategies. A welfare criterion is needed for this purpose. Within an expected utility framework, a straightforward method of scoring different strategies is as follows. We use the optimal dynamic investment strategy in Merton's model as the benchmark[7] and first calculate the utility achieved by adopting this strategy with a given initial wealth (€10000 in our example). Following Scott and Watson (2011), we model the utility of a spending sequence as the weighted sum of the single-year utility---a time-separable model with CRRA utility function. Next, we compute how much it would be to attain the same utility as Merton's model. The result is referred to as the efficiency index, which can be used to compare the efficiency of different strategies.

Table 2 reports the efficiency analysis of different consumption guarantees and leverage levels.[8] As shown in Panel (a), in the presence of equity investment, the efficiency index increases with the leverage factor, given the floor ratio of 85%, which is consistent with the consumption behaviors of different agents in Figure 3. However, the efficiency gap declines with the investor's risk aversion. In unreported results, we find that a decrease in the equity risk premium also reduces the efficiency gap. Somewhat surprisingly, the pure bond investment strategy

[7] To ensure the comparability of different strategies, we assume that the retirees in Merton's model have a finite horizon of 40 years, which is identical to the ratchet retirees' planning horizon.

[8] As the optimal strategy in Merton's model is used as the benchmark, its efficiency is 100% in all cases.

Table 2: Efficiency analysis of strategies for different consumption guarantees and leverage levels

(a) Real guarantees with $\pi = 0$ and different γ						
γ	Merton	L=3	L=2.5	L=1, F=0.85	L=1, F=0.55	L=0
2	100%	75.9%	74.8%	71.2%	54.5%	75.6%
3.5	100%	81.8%	81.3%	77.9%	57.3%	83.3%
5	100%	83.7%	83.3%	81.0%	58.1%	87.2%
(b) Nominal guarantees with $\gamma = 3.5$ and different π						
π	Merton	L=3	L=2.5	L=1, F=0.85	L=1, F=0.55	L=0
0	100%	81.8%	81.3%	77.9%	57.3%	83.3%
1%	100%	89.1%	88.6%	85.0%	64.0%	89.6%
2%	100%	94.8%	95.2%	91.3%	70.5%	94.2%

This table reports the efficiency analysis of different consumption guarantees and leverage levels. In panel (a), inflation is not considered and the guarantees are in real terms. In panel (b), inflation rates vary and the guarantees are in nominal terms, while the risk aversion γ is held constant at 3.5. "Merton" refers to the optimal investment strategy in Merton's model. γ, π and L are the risk aversion, the inflation rate and the leverage factor, respectively. The equity premium ($\mu - r$) is 4%.

($L = 0$) dominates the Floor-Leverage strategies with equity investment.

The strategy with the very low floor ($L = 1$ and $F = 55\%$) results in considerable welfare losses. This strategy exhibits higher consumption growth, but generates much lower consumption in the initial period and therefore does a very poor job of consumption smoothing over time. Therefore, the strategy with no leverage and low floor ratio is inferior to any of the other strategies. Futhermore, Merton's strategy dominates all the other strategies in any scenario. The reasons are twofold. First, in contrast to the Floor-Leverage strategies, Merton's strategy is not constrained from taking large equity exposure in the long run.

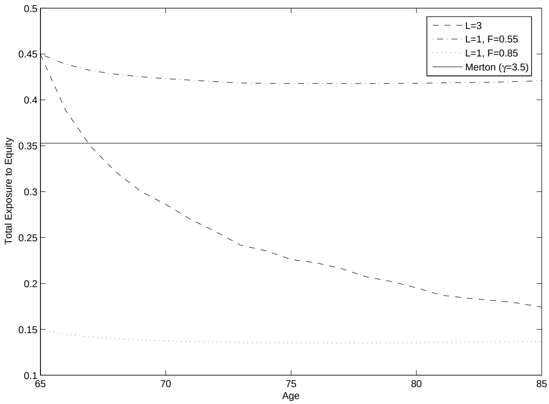

Figure 6: Total equity exposure of different strategies. "Merton" refers to the optimal investment strategy in Merton's model. L and F are the leverage factor and the floor ratio respectively. The consumption guarantees are in real terms. The equity premium ($\mu - r$) is 4%. The inflation rate is zero.

Second, it generates higher consumption streams in the early periods of the retirement than other strategies, because it does not require substantial saving for non-decreasing future spending.

Panel (b) focuses on nominal consumption guarantees. As the inflation rate rises, the utility loss relative to Merton's strategy shrinks for all the other strategies. This welfare improvement follows from the preference of the retirees towards consumption in the early periods of retirement in the presence of inflation.

To better understand the distinctions in efficiency between different strategies, we compare their total equity exposure over the investment horizon. Note that as the type of consumption guarantee has very little effect on the investment strategies, we focus only on the real consumption guarantees. As shown in

Figure 6, the equity exposure of Merton's strategy stays constant over time. By contrast, the strategies with a leveraged position take increasingly less exposure to equity because of both the decreasing survival probability and the intermittent wealth transfer from the surplus portfolio to the floor portfolio. For both strategies with leverage factor of one, the portfolio share allocated to stocks stays nearly constant; the floor ratios (F) of 85% and 55% result in initial equity exposures close to 15% and 45%. This can be explained by the growth of the stock market: in almost all scenarios, money is transferred from the surplus portfolio to the floor portfolio and the surplus ratio remains at the maximum level ($1 - F$).

6. Conclusions and Policy Implications

This paper analyzes two different models for consumption after retirement. The first is Merton's rule, where consumption is always adjusted to changes in wealth; the second formalizes what is known as ratchet consumption, where consumption is guaranteed not to fall over time. Although highly stylized, these rules resemble the benefit rules of the new pension deal in the Netherlands (Merton's rule) and the existing contracts with a nominal floor (ratchet consumption rule).

First, we analyze a simple version of the ratchet consumption model, the Floor-Leverage rule proposed by Scott and Watson (2011). We show that the original Floor-Leverage rule is infeasible and has a high probability of bankruptcy. However, a relatively inexpensive option strategy can insure against such bankruptcy risk. Second, we show that compared to Merton's consumption rule, the requirement for sustaining previous consumption is very costly in welfare terms. The non-decreasing consumption pattern requires that the retiree starts with a very low initial consumption, with an expected increasing consumption pattern. This is very costly in welfare terms because of the desire of households to smooth consumption over time. Third, we investigate a nominal consumption guarantee and compare it with its real counterpart. The less restrictive nominal guarantees lead to a higher initial spending level and lower consumption growth, which gives better consumption smoothing over time and lower welfare losses than the real consumption guarantee.

Based on the previous analysis, we can draw several policy implications for pension funds. First, in terms of investment strategy, if the pension fund members indicate demand for guarantees, there should be a clear separation of the riskless portfolio and the risky portfolio. The reason is that riskless assets are particularly suitable for ensuring future subsistence consumption, while risky assets are used to increase the return of the overall portfolio and generate consumption growth. Second, real guarantees are very costly from a welfare point of view. Therefore, pension boards should take these costs into account when deciding what type of guarantees to offer. In particular, although nominal guarantees allow for declines in real spending, they might be a desirable compromise due to better consumption smoothing and lower welfare losses for the retirees with strong habit persistence.

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Guarantees and habit formation in pension schemes

The recent academic literature suggests that investors regard a large part of their previous consumption as necessary for subsistence, and derive utility only from the excess of consumption above the subsistence level; this is referred to as habit formation. The habit formation of pension participants might have a great impact on the pension design and investment strategy of pension funds. Any pension plan contains guarantees, and habit formation might explain at least part of the demand for such guarantees. In this paper Frank de Jong and Yang Zhou (both TiU) examine the impact of habit formation preferences on the optimal portfolio and consumption choice and explore the implications for pension funds. The authors compare a rule of thumb for ratchet consumption, the Floor-Leverage rule for retirement, with the classic model of Merton.