Network for Studies on Pensions, Aging and Retirement

Netspar THESES

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Hyperbolic Discounting and Pension Design

The Case of Germany

MSc Thesis 2010

Hyperbolic Discounting and Pension Design: The Case of Germany

Master Thesis

to achieve the degree

Master of Science (M.Sc.) in Economics

at the Faculty of Economics and Management

at Tilburg University

submitted by

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Berlin, August 17th, 2010

Abstract

Both data and people's self-reports reveal that there is a undersaving problem. Behavioral economics seeks to explain this phenomenon with the concept of hyperbolic discounting. In essence, short-term actions are inconsistent with long-term goals. This is applied to the German pension system in this text. The results lean on a theoretical life-cycle model that is simulated in Matlab, whereby the parameters are calibrated to match the German economy. It is shown that myopic preferences lead to deviations from outcomes that would be desirable from a normative point of view. The savings rate is considerably lower for hyperbolic discounters, compared to standard discounters. Moreover, a fully funded pension scheme seems preferable to the current Pay-As-You-Go system.

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1 Introduction

A recent observation is that the savings rates in the industrialized countries have been very low and continually declining over the last decades. It is interesting to see that people report they would like to save more, but are for some reason unable to do so. Moreover, those who do not save enough must be bailed out by the tax payers. So there are two motives for policy intervention, namely paternalism and the correction of negative externalities. Modern welfare states do not adopt the ideology of "Social Darwinism". That is, they do not want citizens to starve or suffer, even if it is their own fault, and thus provide money in order to make them meet a minimum standard of life. Given the demographic development in industrialized countries, the aforementioned undersaving becomes especially relevant in the domain of old-age retirement. Due to declining birth rates and a fast increase in life expectancy the burden for the young generation that has to provide the means for the pension system, becomes higher and higher. This development is especially true for Germany, where the population is expected to shrink in the subsequent decades, unless a considerable inflow of immigrants is realized. Furthermore, it is the biggest country in the European Union, and until now, also the main provider of financial aid to members of the euro-zone, exemplified by the recent crisis in Greece. It is hence very important that it can shoulder its own problems and consequently a sustainable pension system should be a priority goal. To illustrate the issue, it should be noted that, following Deutsche Rentenversicherung Bund (2010), in 2009 17.54 out of 82.0 million Germans were retirees, which accounts for 21.4 percent of the population. In 2008 the old-age dependency ratio[1] was 30.9 percent according to Statistisches Bundesamt (2010*a*) and Eurostat (2009) estimates it will increase to 55.8 percent by 2050.[2] Roughly speaking, this means that currently three workers support one retiree, whereas in 2050 two workers have to assure one retiree's living standard. It is apparent that this burden cannot be shouldered endlessly.

Of course, this development poses a lot of questions. How to achieve a sus-

[1]The old-age dependency ratio is defined as the number of elderly persons at an age when they are generally economically inactive divided by the number of persons of working age. Here I apply the definition of Eurostat (2009) (variant 1), in which elderly people are defined as aged 65 and more and persons of working age between 15 to 64 years.

[2]The German age of retirement entry is currently 65, gradually increasing to 67 in 2029. Note that in 2008 there were 17.39 million retirees but only 16.82 million people older than 65 years. This implies that the old-age dependency ratio defined in footnote 1 does not necessarily capture the true share of those who are receiving money. Moreover, there is a huge gap (about 4.5 million) between the number of retirees and the number of pensions, because several retirees receive different kinds of pensions.

tainable pension system? Are the guaranteed pension benefits of the governmental old-age provision inefficiently high? Should the entering age for the retirement phase be elevated and if yes to which age? And finally, are the incentives to invest into the second and third pillar sufficient? Obviously, these issues have already been covered by the literature, not only in general, but also especially for the German case. It is uncontroversial that there is a basic necessity for a systematic pension reform that ensures sustainability in Germany.

In this thesis, I therefore look for an angle to address the issue that has not been done so far. One branch of economics, known as behavioral economics uses insights from psychology and neuroscience. By using ideas from these areas the issue of pension reform is approached in a novel way. The general idea of the paper is to construct a three-period life-cycle model that is different from the standard model, because behavioral aspects are included as well. By doing so, it is possible to show that such slight modifications (hyperbolic discounting) of the standard model can have a significant and sizable impact on the predicted state of the economy. Additionally, I compare the Pay-As-You-Go (PAYG) system that is currently in place to the fully funded (FF) pension system that has been advocated by several economists and politicians. It is then interesting to see what conclusions can be drawn for the German pension system.

The remainder of the text is organized as follows. In the next section I give a short review of the relevant behavioral literature and present some facts concerning the German pension system. Section 3 explains the three period life-cycle model without a pension system as well as with the two pension system candidates, PAYG and FF. The calibration is done in section 4 and is followed by the simulation of the model in section 5. The resulting findings are then discussed in section 6. Section 7 concludes.

2 Survey

At first this section presents some facts concerning the non-sustainability of the German pension system. Secondly, the major aspects of the behavioral literature are summarized to put the latter model into a broader perspective.

2.1 Facts about the German Pension System

The basic concept of the German pension system is the three-pillars system, where the first pillar is the public pension provision, the second the complemental pension provision (operational provisions and additional provisions for functionaries) and thirdly the private pension provision. The pillars cover 89 percent, 5 percent and 6 percent of the whole pension provision, respectively. The Riester-Rent and the Rürup-Rent, which can be assigned to private pensions, are unique to the German system.[3] On the one hand, those rents already reach a substantially amount of people and are attractive to different income groups. On the other hand, low transparency and high administrative costs are problematic and demand improvements. The following numbers illustrate the non-sustainability of the system. According to Deutsche Rentenversicherung Bund (2010), the expenditures in 2009 were 245.83 billion euro, but only 181.33 euro could be collected through regular contributions. The remaining share of 64.5 billion euro, accounting for 26 percent, was paid by the German government. As a response to this deficit and to a further increase in life expectancy, a reform to the German pension system was proposed in 2007. It elevated the entering age for retirement from 65 to 67 years.

2.2 Behavioral Aspects

Behavioral economics is an emerging discipline, so still only a small fraction of behavioral papers makes it into the publication lists of the top economic journals. One key element of the rational choice literature is the "homo oeconomicus", even if this paradigm does not seem to fit the reality very well. That is why behavioral economists seek to apply the assumptions of bounded rationality, bounded will-power and bounded self-interest, sacrificing parsimony for objectivity and explanatory power. Della Vigna (2007) summarizes that non-standard beliefs and non-standard preferences lead to non-standard decision-making. For example, consumption smoothing, implied by the permanent income hypothesis by Friedman (1957), which is a rational choice idea, is challenged by observations of human ac-

[3] The Riester-Rent is a privately financed rent, promoted by the state through bonuses. It is directed at low-income individuals. The contributions into the Riester account amount to 4 percent of the pre-tax income, of which the government shoulders 154 euro for the insured person and 300 euro for each child. A guaranteed interest rate is provided. The Rürup-Rent is directed at self-employed and high-income individuals. Participation is incentivized through the contributions being tax-deductible. Yearly contribution is voluntary and subject to personal choice up to a maximum of 20,000 euro per year. Withdrawings are impossible before reaching retirement. It is paid as a monthly rent as long as the agent is alive.

tions.

The utility function of the standard model with exponential discounting was already introduced in the middle of the 20th century by Samuelson (1937). Later on, the idea was extended by Strotz (1956), Phelps and Pollak (1968), Akerlof (1991) and recently by Laibson (1997) capturing self-control problems and overconfidence.[4] Ainslie (1992) and Loewenstein and Prelec (1992) also find that both animals and human beings' discount functions take the form of a hyperbole, formally expressed by $(1+\alpha\tau)^{-\gamma/\alpha}$. I make use of those findings by utilizing quasi-hyperbolic discounting in this thesis.[5]

Normally, economic models assume time-consistent behavior, meaning that if from the perspective of today I want to start saving more money in one month, I still want to do so when the month is over. Or more formally: If a person in period t=0 finds a certain action optimal for t=30, then the person still sticks to this preference at t=29, that is right before implementing the action. However, reality disconfirms this assumption, rather implying time-inconsistency. This kind of preference reversal can be illustrated by the following example. Imagine you are not quite satisfied with your body shape and you plan to start regular workouts a month ahead. From the perspective of today, it seems very likely that you will start then and motivation is high. However, after the month passed and it is actually time to start you might have the feeling that you could start rather a few weeks later. In that sense you might procrastinate and eventually end up never starting the workout or starting with a strong delay at a time when you get really obese. For life-cycle decisions there is always an intertemporal trade-off between immediate consumption and delayed consumption. Patient, long-term decisions are controlled by the prefrontal cortex (new brain), the planning authority of the human cerebric. In contrast, the present-bias [6] stems from the ventral striatum (old brain) a rudiment of our ancestors. In McClure et al. (2008) it is explained how these interacting forces influence human behavior. Accounting for that, Kirby (1997) collects evidence on how the hyperbolic model is superior to the standard one.

[4]Note that self-control problems are classified as bounded will-power, whereas overconfidence is rather belonging to bounded rationality.

[5]The term quasi-hyperbolic was coined by Laibson, because preferences are not really hyperbolic, but capture the key characteristics of hyperbolic discounting. Krusell and Smith (2000) also refer to this kind of preferences as quasi-geometric.

[6]Note that the terms self-control problem, lack of will-power, procrastination, preference for immediate gratification, present-biased preferences, myopic preferences and myopia are equally appropriate to describe the same context, namely inconsistent time preferences.

The "β-δ-discounting" or "quasi-hyperbolic discounting" function for T periods looks as follows:

$$U_t(c_0, c_1, ..., c_T) = E_t[u(c_t) + \beta \sum_{i=1}^{T-t} \delta^i u(c_{t+i})]$$

In this way it is possible to address the empirically observed time-inconsistent behavior, especially in the area of life-cycle decision-making. The problem can be modeled as a intra-personal game among multiple selves, who display different behavior in different periods.[7] O'Donoghue and Rabin (2001) and O'Donoghue and Rabin (2006) find heterogeneity among the agents. There exist three kinds of people, notably sophisticates, fully naives and partly naives. The sophisticates know about their procrastination and self-control problem, whereas the naives do not. Partly naives are aware of their self-control problem, however underestimating its magnitude. This group lies between the polar cases. One could also say that naives are overconfident when it comes to evaluating their own future will-power.[8] Most commonly the agents are modeled as sophisticates when it comes to hyperbolic discounting in life-cycle decisions. However, I presume that decision-makers are partly naives (or partly sophisticates), because even assuming virtually sophistication seems escapist. On the other hand, there are certainly a lot of situations where people foresee their lack of control. It remains to mention, that even among the partly naives there is heterogeneity, because their degree of overconfidence varies and might also be context-dependent. Nevertheless, for reasons of tractability only the fully naives' decision problem is solved here. Moreover, one should keep clearly in mind that hyperbolic discounting is still sidestepping important aspects of human decision making. Probably most important, visceral factors and emotions as described in Loewenstein (2008), bounded rationality reviewed by Kahneman (2003), bounded ethicality as in Kern and Chugh Dolly (2009) and Banaji, Bazerman and

[7] Alternatively, Thaler and Shefrin (1981) model self-control problems with the conflicting sets of preferences of a farsighted planner and a myopic doer. The difference is that they model a conflict of those two "energy systems" at any single point in time, in contrast to the hyperbolic model where tastes change over time. Already Smith (1759) and more recently Schelling (1960), Buchanan (1975), Freud (1958) and Berlin (1969) devoted effort to understand internal conflicts in the human psyche when considering self-control problems.

[8] In analogy with the later model, where β is the (actual) short-term discount rate, the formal way to differentiate between the three types of agents is the following: For sophisticates $\hat{\beta} = \beta$, for partly naives $\hat{\beta} > \beta$ and $\hat{\beta} = 1 > \beta$ applies for fully naives. Thereby $\hat{\beta}$ represents the expected short-term discount rate.

Chugh (2003), bounded self-interest discussed by Frey and Benz (2002), mental accounting described by Thaler (1999) and the utilization of heuristics and rule of thumbs as in Tversky and Kahneman (1978).

The policy implications in the hyperbolic discounting model can be dramatically different from the ones of the standard model. Taking the sub-optimally low savings in Germany and the general gap between reported savings goals and the actual savings there should be room for welfare improving policy interventions. Apart from hard paternalism there is also the possibility of libertarian paternalism. The key element of libertarian paternalism is that policy-makers try to help people without limiting their freedom of choice.[9] This can be done by using defaults, framing, labeling and frequency effects. In addition, active choice should be stimulated, incentives to invest in the other pillars must be boosted and commitment systems like the SMarT plan developed by Thaler and Benartzi (2004) should be used. Furthermore, countries rely too much on the PAYG system and I will therefore look at the question whether a movement to the FF plan is more promising.[10]

Finally, it is important to recognize that sophisticates, although anticipating their self-control, are not better off than naives if no commitment mechanism is in place. With a well-designed policy it is however possible to improve the prospects of both groups. Although the naives are not aware of their bounded self-control and do not "ask" for commitment devices, they can still benefit from them.

3 The Life-Cycle Model - LCM

In this section the stylized two-period LCM model is extended by incorporating insights from behavioral economics. Therefore, a three-period LCM model with quasi-hyperbolic discounting is presented. Three periods is the minimal requirement for modeling hyperbolic preferences and time-inconsistent behavior, in a two-period setup this would not be possible. In each period three generations are alive. Normally, Germans retire with 65 years and this is also how the model is handled. Thus, the age structure is as follows: The young working generation (y) is aged 25

[9] For detailed information, see Sunstein and Thaler (2003) who explain why libertarian paternalism is not an oxymoron. Camerer et al. (2003) describe that this kind of paternalism is asymmetric, because fully rationals undergo no (or only small) harm, whereas non-rationals receive large benefits.

[10] In an idealistic case the return of the PAYG system is determined by the growth rate of population and wage. Clearly, this cannot be very high if the population is shrinking, unless wages are growing incredibly fast. Contrary, the return in the FF scheme is determined by the real rate of interest. Thus, for a shrinking society a FF system might be more promising.

to 45, followed by the middle-aged working generation (m) aged 45 to 65. After that, the old people (o), also dubbed retirees, receive old-age pensions until they die at the age of 85. Children (0-25 years) are not treated explicitly, since they neither earn a considerably amount of wages, nor receive pension benefits. It is essential to recognize that one period is set equal to 20 years. That allows me to concentrate on the 60 decision-relevant years of a persons life, that is from 25 to 85 years. The model is kept in discrete time and focuses on the demand side, since individuals are more prone to mistakes and thus deviate more strongly from the standard economics prediction than firms.[11] That is why all agents are thought of as **fully naives**. Moreover, I abstract from uncertainties in the future, therefore suppressing the expectation operator.

3.1 The Life-Cycle Model without a Pension Scheme

I start by laying out the basic model without a pension scheme, which is supposed to provide the baseline for further modifications in the next subsection. People live for a finite amount of time, are born without any savings (assets) and logically die without leaving any savings to successors. The notation is kept as intuitive as possible and close to other economic papers. Despite that, in table 3 all variables and parameters are summarized for a quick overview.

The population (N) is growing with rate n and the wage (w) increases with rate ω over time, as shown in the following equations.

$$N_t^y = N_{t-1}^y(1+n) = N_{t-2}^y(1+n)^2 = N_0(1+n)^t \tag{1}$$

$$N_t^o = N_{t-1}^m = N_{t-2}^y \tag{2}$$

$$w_t = w_{t-1}(1+\omega) = w_0(1+\omega)^t \tag{3}$$

3.1.1 Behavior of Private Agents

There are three aspects that determine the final consumers' behavior, namely **preferences**, **constraints** and ultimately the arising **consumption-saving decisions**.

[11] The supply side - not treated here - could be thought of as closer to the "standard" model. Since firms ideally are experts in their field it is reasonable to assume the firms to act (nearly) rational. Anyhow, firms are managed by human beings and even highly skilled people (experts) are not reluctant to irrationality. This can be shown by the famous story of Samuelson's colleague in Benartzi and Thaler (1999). On the other hand, it is legitimate to argue that firms can exploit their knowledge of bounded consumer rationality and acquire higher profits by intelligent marketing, due to their expertise.

The quasi-hyperbolic preference function assumes additive separability and takes the following form,

$$U_t = u(c_t^y) + \beta\delta u(c_{t+1}^m) + \beta\delta^2 u(c_{t+2}^o) \tag{4}$$

where β ($0<\beta\leq 1$) is the short-term discount factor and δ ($0<\delta\leq 1$) is the long-term factor of time preference. If β is equal to one, the quasi-hyperbolic discount function coincides with the normal exponential discount function.

I assume a CRRA utility function of the following form:[12]

$$u(c) = \frac{c^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \tag{5}$$

It should be noted that η represents the elasticity of intertemporal substitution, where $\eta = 1$ represents a logarithmic function of the form $u(c) = \ln c$. The budget constraints take the following form:

$$w_t = c_t^y + s_t^y \tag{6}$$

$$w_{t+1} + s_t^y R_{t+1} = c_{t+1}^m + s_{t+1}^m \tag{7}$$

$$w_{t+2} + s_{t+1}^m R_{t+2} = c_{t+2}^o \tag{8}$$

Note that I use the gross interest rate in order to achieve more clarity. The gross interest rate is defined as $R_t = 1 + r_t$, which means it equals one plus the net interest rate . Moreover, the gross interest rate is kept constant in this thesis, that means $R = R_t = R_{t+1} = R_{t+2}$. Despite this assumption, the interest rate is still indexed to allow an easier extension, for example when attempting to put the model in an equilibrium analysis (DGE or DSGE), where the interest rate would be determined endogenously.

Throughout this thesis, I assume that the labor force stems from the young and middle-aged generation, whereas the old generation does not work anymore and thus does not earn any wages. That is why I set the term w_{t+2} equal to zero, reducing equation (8) to

$$s_{t+1}^m R_{t+2} = c_{t+2}^o \tag{9}$$

[12] The function has the typical characteristic of a positive, but falling marginal utility ($U'(c) > 0$ and $U''(c) < 0$).

The household's lifetime budget constraint can be written as:

$$\Omega_t = w_t + \frac{w_{t+1}}{R_{t+1}} \geq c_t^y + \frac{c_{t+1}^m}{R_{t+1}} + \frac{c_{t+2}^o}{R_{t+1}R_{t+2}} \tag{10}$$

Implicitly, this expression demands perfect capital markets, otherwise I would have to impose saving constraints ($s_t^y \geq 0$ and $s_{t+1}^m \geq 0$) for each period. Without those, the periodic savings rate can become negative, because the households borrow more than they earn, in early periods, in order to enlarge the immediate consumption level. In period t, the young households' optimization problem is to maximize the lifetime utility (4) subject to the budget restriction (10), with respect to the three consumption variables c_t^y, c_{t+1}^m, c_{t+2}^o.

$$\underbrace{max\ U_t}_{c_t^y,\ c_{t+1}^m,\ c_{t+2}^o} = u(c_t^y) + \beta\delta u(c_{t+1}^m) + \beta\delta^2 u(c_{t+2}^o)$$

$$s.t.\ \Omega_t \geq c_t^y + \frac{c_{t+1}^m}{R_{t+1}} + \frac{c_{t+2}^o}{R_{t+1}R_{t+2}}$$

As a first step the corresponding Lagrange function is written down.

$$\begin{aligned} L = u(c_t^y) + \beta\delta u(c_{t+1}^m) + \beta\delta^2 u(c_{t+2}^o) \\ +\lambda(w_t + \frac{w_{t+1}}{R_{t+1}} - c_t^y - \frac{c_{t+1}^m}{R_{t+1}} - \frac{c_{t+2}^o}{R_{t+1}R_{t+2}}) \end{aligned} \tag{11}$$

In the second step the partial derivatives are taken with respect to the consumption of the young in t, middle-aged in t+1 and the retirees' consumption in t+2.

$$\frac{\partial L}{\partial c_t^y} = (c_t^y)^{-1/\eta} - \lambda = 0 \tag{12}$$

$$\frac{\partial L}{\partial c_{t+1}^m} = \beta\delta(c_{t+1}^m)^{-1/\eta} - \lambda\frac{1}{R_{t+1}} = 0 \tag{13}$$

$$\frac{\partial L}{\partial c_{t+2}^o} = \beta\delta^2(c_{t+2}^o)^{-1/\eta} - \lambda\frac{1}{R_{t+1}R_{t+2}} = 0 \tag{14}$$

The last step combines the equations (12) and (13) which results in the following relationship between consumption of period t and t+1

$$c_{t+1}^m = [\beta\delta R_{t+1}]^\eta c_t^y \tag{15}$$

and the equations (12) and (14) to observe the relationship between consumption in period t and t+2.

$$c^o_{t+2} = [\beta\delta^2 R_{t+1}R_{t+2})]^\eta c^y_t \tag{16}$$

Combining and rearranging (15) and (16) shows the **expected** relation in period t between c^m_{t+1} and c^o_{t+2}.

$$c^m_{t+1} = \frac{c^o_{t+2}}{[\delta R_{t+2}]^\eta} \tag{17}$$

Apparently this is different to the **actual** relation in period t+1. The difference is that in the latter (24) the myopic discount factor is included.

Taking the lifetime budget constraint from (10) together with (15) and (16) the consumption of the young generation in period t can be obtained.

$$c^t_y = \frac{\Omega_t}{1 + [\beta\delta R_{t+1}]^\eta \frac{1}{R_{t+1}} + [\beta\delta^2 R_{t+1}R_{t+2}]^\eta \frac{1}{R_{t+1}R_{t+2}}} \tag{18}$$

In period t+1 the middle-aged individual has to reoptimize his decision taking s^y_t as given. Therefore, it is denoted as $\widehat{s}^y_t$, whereby the hat indicates the irreversibility of the action. The relevant utility function is

$$U_{t+1} = u(c^m_{t+1}) + \beta\delta u(c^o_{t+2}) \tag{19}$$

whereas from the perspective of period t, the fully naive agent expected the utility function to take the form, $U_{t+1} = u(c^m_{t+1}) + \delta u(c^o_{t+2})$, without the short-run discount factor. Here, one can clearly observe the naivety of the agent, because he will put less weight on the future consumption when t+1 is finally reached (as long as $\beta < 1$). The arising new "remaining lifetime budget constraint" for the middle-aged is derived from the corresponding budget constraints (7) and (9).

$$\Omega_{t+1} = \widehat{s}^y_t R_{t+1} + w_{t+1} \geq c^m_{t+1} + \frac{c^o_{t+2}}{R_{t+2}} \tag{20}$$

In period t+1, the middle-aged household's optimization problem is to maximize the utility (19) subject to the budget restriction (20), that is,

$$\underbrace{max\ U_{t+1}}_{c^m_{t+1},\ c^o_{t+2}} = u(c^m_{t+1}) + \beta\delta u(c^o_{t+2})$$
$$s.t.\ \Omega_{t+1} \geq c^m_{t+1} + \frac{c^o_{t+2}}{R_{t+2}}$$

Again, I write down the corresponding Lagrange function which takes the following form:

$$L = u(c^m_{t+1}) + \beta\delta u(c^o_{t+2}) + \lambda(\widehat{s}^y_t R_{t+1} + w_{t+1} - c^m_{t+1} - \frac{c^o_{t+2}}{R_{t+2}}) \tag{21}$$

The next step is to take the partial derivatives with respect to consumption of the middle-aged in t+1 and the retirees' consumption in t+2.

$$\frac{\partial L}{\partial c^m_{t+1}} = {c^m_{t+1}}^{-1/\eta} - \lambda = 0 \tag{22}$$
$$\frac{\partial L}{\partial c^o_{t+2}} = \beta\delta(c^o_{t+2})^{-1/\eta} - \lambda\frac{1}{R_{t+2}} = 0 \tag{23}$$

By equating the lambdas from both equations, I obtain the following relationship between consumption in t+1 and t+2:

$$c^m_{t+1} = \frac{c^o_{t+2}}{[\beta\delta R_{t+2}]^\eta} \tag{24}$$

Once again, note that the **actual** relationship is different from what the agent **expected** the relationship to be (17), one period earlier. It is crucial to stress this point, since it perfectly demonstrates the time-inconsistency.

Plugging this (rearranged) consumption rule (24) into the middle-aged lifetime budget constraint (20), the optimal consumption for the middle-aged can be found.

$$c^m_{t+1} = \frac{\Omega_{t+1}}{1 + [\beta\delta R_{t+2}]^\eta \frac{1}{R_{t+2}}} \tag{25}$$

Finally, by inserting (25) into (24) the old-age consumption is obtained.

$$c^o_{t+2} = [\beta\delta R_{t+2}]^\eta \frac{\Omega_{t+1}}{1 + [\beta\delta R_{t+2}]^\eta \frac{1}{R_{t+2}}} \tag{26}$$

Here, no optimization is necessary, since life ends at the end of the period and everything left can be consumed. The retirees have to accept what they accumulated over their life span.

3.2 The Life-Cycle Model with a Pension Scheme

This thesis aims to examine the German pension scheme. Therefore, it is necessary to extend the model developed earlier to incorporate the systematics of contributions and benefits. I differentiate between a FF and a PAYG scheme. Pension contributions, denoted by p_t, are paid by the young and middle-aged generation, whereas the old generation receives the pension benefits b_t. The pension contribution is defined as $p_t = w_t\tau$, where τ is the contribution rate. Now, the budget constraints look different, compared to equations (6), (7) and (9) of the non-pension case.

$$w_t(1-\tau) = c_t^y + s_t^y \tag{27}$$

$$w_{t+1}(1-\tau) + s_t^y R_{t+1} = c_{t+1}^m + s_{t+1}^m \tag{28}$$

$$c_{t+2}^o = s_{t+1}^m R_{t+2} + b_{t+2} \tag{29}$$

Those constraints lead to a new equation for the young individual's lifetime income.

$$\Omega_t = w_t(1-\tau) + \frac{w_{t+1}(1-\tau)}{R_{t+1}} + \frac{b_{t+2}}{R_{t+1}R_{t+2}} \geq c_t^y + \frac{c_{t+1}^m}{R_{t+1}} + \frac{c_{t+2}^o}{R_{t+1}R_{t+2}} \tag{30}$$

The middle-aged individuals' remaining lifetime income in t+1 takes the following form,

$$\Omega_{t+1} = w_{t+1}(1-\tau) + \hat{s}_t^y R_{t+1} + \frac{b_{t+2}}{R_{t+2}} \geq c_{t+1}^m + \frac{c_{t+2}^o}{R_{t+2}} \tag{31}$$

where b_{t+2} is defined differently for the two pension plans.

3.2.1 Pay-As-You-Go Pension - PAYG

The pension system is modeled to be sustainable, that is, in each period the earnings are equal to the expenditures. Consequently, the government is only redistributing money from the two working generations to the retirees and thus not making any debts. This means that the relationship is described by the following equation,

$$N_t^o b_t^{PAYG} = L_t p_t = L_t w_t \tau = N_t^y w_t \tau [\frac{2+n}{1+n}] \tag{32}$$

which helps to obtain the equations for the benefits in t and t+2.

$$b_t^{PAYG} = p_t(2+n)(1+n) = w_t\tau(2+n)(1+n) \tag{33}$$
$$b_{t+2}^{PAYG} = w_{t+2}\tau(2+n)(1+n) \tag{34}$$

By plugging b_{t+2}^{PAYG} into (30) Ω_t^{PAYG} is obtained.

$$\Omega_t^{PAYG} = w_t(1-\tau) + \frac{w_{t+1}(1-\tau)}{R_{t+1}} + \frac{w_{t+2}\tau(2+n)(1+n)}{R_{t+1}R_{t+2}} \tag{35}$$

Inserting this into (18) yields the optimal consumption for the young generation in period t which looks as follows:

$$c_t^y = \frac{w_t(1-\tau) + \frac{w_{t+1}(1-\tau)}{R_{t+1}} + \frac{w_{t+2}\tau(2+n)(1+n)}{R_{t+1}R_{t+2}}}{1 + [\beta\delta R_{t+1}]^\eta \frac{1}{R_{t+1}} + [\beta\delta^2 R_{t+1}R_{t+2}]^\eta \frac{1}{R_{t+1}R_{t+2}}} \tag{36}$$

The middle-aged budget constraint is derived by plugging b_{t+2} in (31).

$$\Omega_{t+1}^{PAYG} = w_{t+1}(1-\tau) + \widehat{s}_t^y R_{t+1} + \frac{w_{t+2}\tau(2+n)(1+n)}{R_{t+2}} \geq c_{t+1}^m + \frac{c_{t+2}^o}{R_{t+2}} \tag{37}$$

The optimal consumption for the middle-aged is found by plugging (37) into (25). Lastly, this result must be plugged into (24) in order to get an equation for the retirees' consumption.

$$c_{t+1}^m = \frac{w_{t+1}(1-\tau) + \widehat{s}_t^y R_{t+1} + \frac{w_{t+2}\tau(2+n)(1+n)}{R_{t+2}}}{1 + [\beta\delta R_{t+2}]^\eta \frac{1}{R_{t+2}}} \tag{38}$$
$$c_{t+2}^o = [\beta\delta R_{t+2}]^\eta \frac{w_{t+1}(1-\tau) + \widehat{s}_t^y R_{t+1} + \frac{w_{t+2}\tau(2+n)(1+n)}{R_{t+2}}}{1 + [\beta\delta R_{t+2}]^\eta \frac{1}{R_{t+2}}} \tag{39}$$

3.2.2 Fully Funded Pension - FF

Like the PAYG system, the FF pension scheme is also modeled to be sustainable. However, in this case the working generations are not paying for the retirees, but each individual is contributing to the pension scheme as long as it works and earns the pension benefits after it goes into retirement. This means, that expenditures in t are independent of the earnings in t. Rather, the expenditures in t are linked to the earnings of the two preceding periods.

$$N_{t+2}^{o} b_{t+2}^{FF} = N_t^y w_t \tau R_{t+1} R_{t+2} + N_{t+1}^m w_{t+1} \tau R_{t+2} \tag{40}$$

Since $N_{t+2}^o = N_{t+1}^m = N_t^y$ those terms cancel out and an expression for the benefits in t+2 is found.[13]

$$b_{t+2}^{FF} = w_t \tau R_{t+1} R_{t+2} + w_{t+1} \tau R_{t+2} \tag{41}$$

By inserting b_{t+2}^{FF} into the lifetime budget constraint (30), it strikes that this looks similar to the non-pension case (10), but different to the PAYG pension scheme (35).

$$\Omega_t^{FF} = w_t + \frac{w_{t+1}}{R_{t+1}} \geq c_t^y + \frac{c_{t+1}^m}{R_{t+1}} + \frac{c_{t+2}^o}{R_{t+1} R_{t+2}} \tag{42}$$

The budget constraint from the perspective of the middle-aged is found by plugging b_{t+2}^{FF} into (31). Again, this looks different from the PAYG pension scheme (37).

$$\Omega_{t+1}^{FF} = \widehat{s}_t^y R_{t+1} + w_{t+1} + w_t \tau R \geq c_{t+1}^m + \frac{c_{t+2}^o}{R_{t+2}} \tag{43}$$

In fact the non-pension case and the FF one are once again equivalent, because the contribution rate τ is zero if no pension system is in place and consequently the term $w_t \tau R$ is not present in equation (20). The consumption levels of the different generations are obtained in the same way as in the PAYG case and reported hereafter.

$$c_t^y = \frac{w_t + \frac{w_{t+1}}{R_{t+1}}}{1 + [\beta \delta R_{t+1}]^{\eta} \frac{1}{R_{t+1}} + [\beta \delta^2 R_{t+1} R_{t+2}]^{\eta} \frac{1}{R_{t+1} R_{t+2}}} \tag{44}$$

$$c_{t+1}^m = \frac{\widehat{s}_t^y R_{t+1} + w_{t+1} + w_t \tau R}{1 + [\beta \delta R_{t+2}]^{\eta} \frac{1}{R_{t+2}}} \tag{45}$$

$$c_{t+2}^o = [\beta \delta R_{t+2}]^{\eta} \frac{\widehat{s}_t^y R_{t+1} + w_{t+1} + w_t \tau R}{1 + [\beta \delta R_{t+2}]^{\eta} \frac{1}{R_{t+2}}} \tag{46}$$

Apparently, the consumption levels of the non-pension case and the FF pension are identical. This is due to the perfect capital markets assumed here. Under a fully-

[13]The above equation states that the number of persons that are young in t, middle-aged in t+1 and old in t+2 is the same. Obviously, this assumes that nobody dies within this time frame, otherwise the number of people of the same cohort would decline over time. People of one cohort are assumed to remain alive until the end of the old-age period is reached.

funded system the agent does not care about the contribution rate, he can always ensure his optimal consumption by borrowing the appropriate amount of money. Contrary, in practice (outside this model) there are liquidity constraints and thus people cannot simply borrow as much as they want.[14]

The standard model's explanation is that there is a simple one-to-one trade-off between private savings and illiquid savings, held in the pension account. The German legislation forbids access to the pension account until the requirements for old-age pension are met. However, there is not necessarily a one-to-one trade-off as implied by the standard model, because of different mental accounts. The successive lab and field experiments Kooreman (2000), Card and Ransom (2007), Abeler and Marklein (2008), Epley, Mak and Idson (2006) and Heath and Soll (1996) illustrate how frames and labels affect different mental accounts. Moreover, they demonstrate that the marginal propensity to consume varies from savings account to savings account.

4 Calibration

Calibration is the attempt to find the parameter values that capture the key features of the economy. Sometimes, the estimation of parameters with normal econometric methods is not feasible, since there is only a limited amount of data points and/or the model identification is problematic. For the simulation in the next section it is therefore necessary to parameterize the model, so that the parameters correspond especially to the German characteristics. In the following I describe how the model is calibrated and thereafter summarize the results in table 1 for facility of inspection. It should be mentioned that some parameters are calculated from the German data, whereas others are taken from the existing literature. Of course a myriad of other constellations is possible and parameters are likely to change with time. In order to test whether different parameter values lead to strong deviations from the baseline, a sensitivity analysis is provided in a latter section.

4.1 Short-Run Discount Factor - β

When the concept of quasi-hyperbolic discounting is applied, both the short-run and the long-run discount factor have to be estimated. For the exponential case the

[14]The term "as much as they want" does not mean that they can borrow without any limits in the model. Rather, it implies that there are no restrictions due to imperfect capital markets.

short-run discount factor is simply set equal to 1. In a recent study by Wang, Rieger and Hens (2010) short-term discount factors for different countries and cultures are estimated. For Germany they find a short-term discount factor of .6, which is adopted here.

4.2 Long-Run Discount Factor - δ

Wang, Rieger and Hens (2010) also estimate the German long-term discount factor of .84 for a 10-year period. This is consistent with other findings of a .98 yearly long-term discount factor. Taking those findings for granted, I end up with about .67 ($\delta^{20} = .98^{20} \approx .67$) for one period. In the standard case without hyperbolic discounters the annual long-term discount factor is often chosen to be .96, such as in Campbell et al. (2001). This would break down to .44 for one model period ($\delta^{20} = .96^{20} \approx .44$). Note that for similar fittings to the economy δ must be larger in the hyperbolic case. Intuitively this makes sense, since the hyperbolic agent has two discount rates.

4.3 Elasticity of Intertemporal Substitution - η

The estimation of the EIS is very controversial. Guvenen (2006) tried to reconcile two contradictory views about the subject. While observations on growth and aggregate fluctuations suggest a value of EIS close to 1 the empirical consumption literature focuses on the co-movement between aggregate consumption and interest rates and thus finds a EIS close to zero. Moreover there is heterogeneity between stock-holders (high EIS) and non-stock-holders (low EIS). Considering this ambiguity, a value of .5 which lies in the middle of different estimates is considered as a baseline here.[15]

4.4 Effective Contribution Rate - τ

The contribution rate indicates the percentage a person has to deposit into the pension system out of his wage. Not everyone in the real world labor force is obliged to pay contributions, thus I have to adjust the data so that it fits the model. Therefore the effective contribution rate is calculated. This is done by multiplying the participation rate with the contribution rate, where the participation rate is defined

[15]This implies a coefficient of relative risk aversion of 2, which is simply the reciprocal of the elasticity of intertemporal substitution.

as the quotient of those who contribute to the system and the total labor force. Logically, the participation rate is smaller than one, unless everyone in the labor force participates. However, special occupational groups, for instance self-employed do not have to. The gross contribution rate is set by the German government and is adjusted from time to time.[16] In table 4 the results can be found for the years 1991 to 2008. Statistic offices always lag behind when it comes to providing up-to-date information. That is why data has been collected only until 2008. All in all, data from 18 years is sufficient for the current analysis. Finally, the average of the effective contribution rates is taken and an approximate value of .17 is obtained.

4.5 Growth Rate of Population - n

In table 5, I present the German population from 1991 to 2008.[17] The average population growth rate from one year to the following year is roughly .00125. Assuming the future development follows this trend, for one period the population growth rate is calculated to be roughly .03. [18]

4.6 Growth Rate of Wage - ω

In table 5 the yearly nominal gross wage for Germany between 1991 and 2008 is presented, as well as a deflator, for the purpose of transforming the nominal wages to real wages. After the real wages are obtained, assuming a constant growth rate, the development for future wages can be computed. Then, the yearly real wages are summed up over the 20 years. To keep the data transparent, the obtained real wage per period is then normalized, so that the starting value is 100 for the period 1991-2010. Finally, after the real wages for the periods 2011-2030 and 2031-2050 are calculated, the average growth rate of those normalized real wages can be found. This results in an approximate value of .24 for ω.

[16]Sometimes the contribution rate was changed within one year. In this case I calculated a weighted average in order to get one numerical value per year.

[17]Due to the German Reunification of the BRD and GDR it is not possible to provide earlier data for Germany. Moreover, it would not make any sense to sum up the values for both countries, since at least the former East-German economic system is not at all comparable to the recent one. Not to mention that obtaining data on a country that does not even exist is problematic.

[18]As an alternative specification forecasts of the Statistisches Bundesamt (2006) can be used. This study makes certain assumptions on life expectancy, net migration and birth rate. For example in variant 1 (p.64), a population of 68.74 million is forecasted for 2050. Taking the current population of 82.0 million in 2008, indicates a strong decline (-.16 population growth rate) of German inhabitants for this time frame. On average this implies a population growth rate of roughly -.08 for one model period.

4.7 Net Rate of Interest - r

According to Deutsche Bundesbank (2001), the annual real rate of net interest is on average .03 from 1961 to 2000. There was a sharp increase after the German Reunification, however this jump ebbed away fastly. So the long-lasting average can be adopted without distorting the analysis. The calculation $(1 + 0.03)^{20} - 1$ leads to .81 for one period. In table 1 the selected parameter scenarios are summarized.

5 Simulation

In this section the simulations are presented, taking the parameters in table 1 as baseline. The analysis contrasts the two pension systems (PAYG, FF) and the two types of discounters (exponential, hyperbolic). The computations are done in Matlab 2008. The results and corresponding graphs are presented in the appendix, covering a wide spectrum of values. This is done in order to test the sensitivity of the outcomes with respect to the parameter values. Of course, parameter changes lead to quantitatively different results, but qualitatively different results are only obtained for a few parameters, or very extreme values that are not relevant for practical implications.

One goal is to find the optimal contribution rate, denoted τ^*, that maximizes the utility of the agents. This can be compared to the calibrated one. I look for the optimal value for each type and pension system only within the range of politically feasible contribution rates, which I assume to lie between 0 and 20 percent. The results of the hyperbolic and exponential discounters are only comparable after finding a normative utility (NU) function. That is, I set the short-run discount factor in the lifetime utility function equal to one for the hyperbolic discounters, in order to see what would happen, if the self-control problem was absent. For the exponential discounters this is obviously not necessary, since there is no short-run discount factor. The NU function looks as follows:

$$NU = \frac{c_0^{y\ 1-1/\eta}}{1-1/\eta} + \delta\ \frac{c_1^{m\ 1-1/\eta}}{1-1/\eta} + \delta^2\ \frac{c_2^{o\ 1-1/\eta}}{1-1/\eta} \tag{47}$$

I differentiate between four scenarios: hyperbolic discounters in the PAYG scheme, exponential discounters in the PAYG scheme, hyperbolic discounters in the FF scheme and exponential discounters in the FF scheme. After the optimal consumption levels for a given scenario are obtained, those can be plugged into the NU and one can see for which contribution rate those utilities take the highest value. A word of caution is the order of the day here. Since the pension benefit calculation of the German old-age pension is much more complicated than modeled, one should be careful not to draw the wrong conclusions.

5.1 Modus Operandi in Matlab

This section shortly describes how the calculations are carried out in Matlab 2008. As a first step an m-file is created, that can be run in the Matlab command window later on. Secondly, the baseline parameters are defined and assigned their proper values from table 1. The next step is to write down the initial value for the wage in period t, so that an evolvement over time is possible. A value of 100 is used, as I described in the calibration section. For the PAYG system the relevant equations in the right order are (34), (30), (36), (27), (31), (38), (39), (28), (5) and (4). Similarly, for the FF pension scheme the equations (41), (30), (44), (27), (31), (45), (46), (28), (5) and (4) are relevant. Moreover, within the code I assign subscripts to differentiate between exponential and hyperbolic discounters. By eliminating the short-run discount factor in the lifetime utility function, which is equivalent to setting it to 1, I get the equation for the normative utility as in (47) .[19] Additionally, some automatic check routines are integrated, giving direct feedback if errors occur. That is, for example (29) must be equal to (39) for hyperbolic discounters in the PAYG system, otherwise the program reports a miscalculation. Running the described code plus some further lines for automatic plot functions yields the results and graphs for the baseline scenario.

Moreover, I look for the optimal contribution rate. For this purpose, I substitute the baseline value of τ by a vector, containing several values. Then, Matlab checks for which value within the vector the NU is maximized and displays the corresponding value of τ. Finally, I am not only interested in what τ^* looks like for this particular case, but also when some parameters change. That is why the described code is put into a number of *for loops*, which enables me to carry out the sensitivity check. Several *if conditions* allow for automatic plotting and saving for defined parameter values. For convenience, a couple of values, vectors and small matrixes are joined into bigger ones, allowing easier data evaluation and presentation in the end.

[19] Of course, there are more sophisticated methods, such as a weighted average of the utilities in period t and t+1, for calculating normative utility functions.

5.2 Results and Sensitivity Analysis

First of all, note that this framework is not grounded in a general equilibrium and thus wages and interest rates are not determined in the model. The assumption of perfect capital markets could be lifted, probably resulting in lower borrowing and higher initial savings. Nevertheless, in the following I present the findings that arise from the framework covered in this thesis. While the pension contribution rate affects the outcome of the PAYG system, the FF scheme is totally unaffected. Intuitively this makes sense, since both the contributions and the savings are subject to the same return in the FF system. Therefore an increase in τ implies a higher percentage taken away from disposable income, but this effect can be offset, by a rational person, through more borrowing on the capital markets. Due to this fact, talking about the contribution rate always refers to the PAYG in the remainder of the text. The results will be supported by figures and tables, whereby the figures are intended to convey the general message and the tables are included for the sake of completeness. If not mentioned otherwise, all conclusions are drawn for the baseline values.

For the baseline parameterization the optimal contribution rate is zero and the NU is strictly higher in the FF scheme, both for exponential and hyperbolic discounters. Clearly, a NU of -.0259 (PAYG) is smaller than -.0241 (FF), comparing the hyperbolic discounters, and -.0250 (PAYG) is smaller than -.0233 (FF), comparing the exponential discounters. The NU of the exponential discounters is always higher than the hyperbolic one's, when comparing the same pension scheme. Note that for $\tau = 0$ the results are similiar for both systems. This is because no contribution rate is the same as no pension system at all. The ordinal utility function reveals which system is better, not by how much it is better. Therefore, the small numerical difference is no problem at all.

Figure 2 plots the agent's consumption for the baseline. The consumption values next to the data points are rounded to full numbers. Consumption is measured on the vertical axis, while the horizontal one displays the model period. Each graph tracks the development of one representative agent over time. On the horizontal axis 1 represents the agent when young, 2 when middle-aged and 3 upon reaching retirement age. Also, the lines between the data points have no natural interpretation. However, in this way it is easier to follow the data trend over time, especially when lines are crossing in later plots. Apparently, there is a downward trend for the hyperbolic agent in terms of consumption. Contrary, the consumption for an exponential agent is increasing over time, so that a retiree can consume more than in his youth. Another point is that agents enjoy a higher consumption in the FF scheme for all three periods. Clearly, this implies a higher lifetime utility, which reestablishes the earlier numerical finding.

In figure 3 two lines are plotted for the exponential cases. The black line is the same as before, but for the red line δ was set to .44. Clearly, the shape of the red, exponential lines looks more similar to their hyperbolic counterparts. This can also be seen by comparing the numbers near the data points, now belonging to the red lines in the exponential graphs. I mentioned earlier that the long-term discount rate must be lower in the exponenial case than in the hyperbolic case when looking for a similiar appeareance. However, this thesis wants to point out what the lack of self-control means for the individuals. To make the systems comparable I therefore have to choose the same long-term discount rate. Otherwise, my NU function would always find higher utilities for the PAYG scheme, since the discount rate would be much lower in the exponential case, all else being equal.

It is then interesting to look at the arising savings and consumption rates, while three variants are covered. Firstly, the periodic savings are set in relation to the wage of the corresponding period. Expressed formally this is:

$$
\begin{aligned}
SR_t &= \frac{s_t^y}{w_t} \\
SR_{t+1} &= \frac{s_{t+1}^m}{w_{t+1}} \\
SR_{t+2} &= \frac{0}{w_{t+2}}
\end{aligned}
$$

Secondly, the net savings rate (NSR) is calculated. For the young and old individual there is no difference, since the old generation's saving is zero and the young generation is only endowed with wage income when working life starts. For the middle-aged agent this is different, because for him there are savings augmented by interest from the young period, in addition to the wage income. The NSR excludes this addtional income term, which looks as follows:

$$NSR_{t+1} = \frac{s_{t+1}^m - s_t^y R}{w_{t+1}}$$

The third variant applies a broader definition of savings rate (SR) and is related to the disposable income. Formally expressed this takes the following form,

$$\begin{aligned} SR_t &= \frac{s^y_t + w_t\tau}{w_t} \\ SR_{t+1} &= \frac{s^m_{t+1} + w_{t+1}\tau}{w_{t+1} + s^y_t R} \\ SR_{t+2} &= \frac{0}{b_{t+2} + s^m_{t+1} R} \end{aligned}$$

where the numerator aggregates the voluntary and coercive savings and the denominator reflects the disposable income. The consumption rate (CR) is also calculated by deviding through the disposable income in the third variant.

For the first and second variant consider figure 4 and tables 6 and 7. The CR is quite high for the young individuals and falling throughout lifetime in all scenarios. However, the difference between CR while young and old is smaller for the exponential graphs, this indicates a higher tendency for consumption smoothing when the self-control problem is excluded. The data points over the red dotted line correspond to the NSR, those under the blue line to the SR. In the first period both are close to zero or negative. Except for the exponential PAYG subplot, the NSR is higher than the SR. This is due to the fact the that negative term from the first period is not included. It seems that the agent, while middle-aged, does the best job when it comes to saving decisions.

The third variant, shown in figure 5 and tables 8, 9, 10, 11, also points to the middle-aged saving the most. Note, that for this definition the SR and CR perfectly add up to one and that the SR is strictly positive in the working years and zero for retirees (no bequest). Or to put it differently, the SR and CR are perfect mirror images, relative to a horizontal line that crosses the vertical axis at .5.

After all, the FF scheme looks superior for the baseline values, but it is necessary to find out how robust this finding is. Figure 1 displays the the normative utilities, each main beam representing one scenario. From left to right, each main beam consists of 21 sub beams. These sub beams emphazise the dependence of the utility function on the contribution rate, ranging from .0 to .2 in substeps of .01. The graphs support the idea that the FF scheme is superior for the baseline values, independent of the contribution rate. The value of the NU is simply higher (less negative) for the FF compared to the PAYG counterpart. Table 12 summarizes the numerical values.[20] I look at values of annual interest rates between zero and five percent, to check whether the FF dominates for all values.[21] I find that for low interest rates (r $<$ .12) the PAYG is superior, but once the threshold level is reached the FF dominates in terms of NU. As long as r is below .12 the NU in the PAYG is maximized for the highest possible contribution rate, namely .20. This is straightforward, because the return of the PAYG is higher than the market return in this case.[22] Here an arbitrage gain is possible, because individuals can contribute to the pension system and borrow back at the same time at the capital market, but paying a lower interest there. It is important to see that the FF is the candidate of choice for r being greater than .12. The growth of real wages and population is assumed to be constant here. For falling growth rates the interest for FF being superior would be even lower.[23]

Figures 6, 7 and 8 are supposed to illustrate the influence of the annual interest rate on the development of the consumption over time. The results are plotted for the whole range of τ, which can be seen by the overlapping of lines and data points. Obviously, the annual interest rate determines whether consumption is increasing or decreasing over the life path. For r = .01 consumption is declining for all scenarios. For r = .03, which is the baseline, the case is ambiguous, decreasing consumption for the hyperbolic discounters, but increasing consumption for the exponential discounters. Finally, for r = .05 consumption is increasing each period for all scenarios. Also, the spread of consumption increases for the different values of τ when a higher value of r is chosen. For the baseline it strikes that households could enjoy higher consumption after retirement if they could avoid their lack of willpower.

I also gave a look to the question how the choice of the elasticity of intertemporal substitution affects the individuals' consumption. Figures 9, 10 and 11 do this for the same annual interest rates as above, now only for the baseline value of τ. This is done to isolate the effects. The numbers near the data points correspond to the baseline value of the EIS (blue data points). Two aspects are worth mentioning. Firstly, for low values of r the spread of consumption due to a change in the EIS is

[20] In table 13 the pension benefits are presented. Note that here the PAYG system is independent of r, in contrast to the FF scheme.

[21] Normally, one is used to work with yearly interest rates, not interest rates based on a 20-year period. Therefore, I describe the annual rates for the reader's convenience, but did not forget to scale up the interest rates in the calculations.

[22] For r $>$ .12 the optimal contribution rate is always zero, since savings on the capital market have a higher return.

[23] Falling growth rates does not necessarily refer to negative growth rates, but growth rates smaller than the baseline.

higher for hyperbolic discounters. The opposite is true for high values of r. Secondly, for r being low, a higher EIS translates into higher consumption in the young years, but lower consumption when getting older. Again, the opposite holds for high values of r. For the baseline r, altering the EIS results only in minor changes.

All in all, small deviations from the baseline values do not seem to distort the general findings. Not to mention that an increase/decrease of the annual interest rate by 2 percent on average over a 20-year period is actually a quite strong deviation. Also, I covered a wider spectrum of deviations with Matlab, but certainly some selections must be made. I did not find values within a reasonable area that would dismiss what was said so far. Of course, deviations lead to other numeric values, but the predominance of the fully funded pension scheme over the Pay-As-You-Go pension scheme is unlikely to change.

5.3 Model Extensions

While the 3-period model can give first implications, a 6-period model can provide more insight. Allowedly, one could also argue that 55 periods, such as in Petersen (2004) provide a lot more insights and it is an eligible question to ask where to stop. To me the 6-period model is appealing, because it allows to cover a decade as a period which is very intuitive. Moreover, databases are more likely to provide material for 10-year samples. Basically the derivation of the model is the same as before. Merely, the baseline parameterization, the indexation of generations and the number of pension periods have to be changed. Table 2 provides the baseline parameters. The indexation is pragmatic: 1 = 25-35 years, 2 = 35-45 years, 3 = 45-55, 4 = 55-65, 5 = 65-75 and 6 = 75-85 years. Both 5 and 6 receive pensions in this setup.

Figure 12 shows the different shapes of the exponential and (quasi)-hyperbolic discount functions. Here, the power of the short-term discount factor is more apparent than it would have been in the 3-period case. Lastly, the figures 13, 14, 15 and 16 are intended to make the baseline graphs of the 3-period model comparable to the 6-period model. The general properties are the same, but the extended model has two advantages. Firstly, there are more data points and consequently a more detailed analysis can be done. Second and more important, a slight concave curvature for hyperbolic discounters is observed in the extended model. This section's only intention was to show that small extensions can improve the gained insights and that there is room for future work.

6 Implications and Further Issues

So far, the analysis shed light on the question of what pension system is preferable for the current situation in Germany, but it does not give any recommendations on how the transition path should be organized. Feldstein (1997) addresses the question on how to move from a PAYG scheme to the FF one. Once again, it needs to be said that the presented "black-box" simulation has a certain value, but should be handled carefully when it comes to practical solutions. But even if the results presented here were unchallengeable, politically decision-making is a complex and complicated task. Additionally, contracts are rigid to a high degree and thus every transition takes its time.

Both the theoretical part, as well as the computational one indicated that the FF system is not different to no pension scheme at all. The results for the FF are the same for every contribution rate, even zero. According to this, people are better off with no pension scheme, compared to the PAYG pension scheme. This thesis has also shown how consumption and savings differ between hyperbolic and exponential discounters when no commitment device is in place. For the period 1992 to 2009 the German savings rate was on average 10.6 percent, according to Statistisches Bundesamt (2010*b*). The definition used there is closest to the variant three of the savings rate discussed in this thesis. Consequently, a comparison is possible. While the model traced the development of one individual over time, it is now necessary to find the overall savings rate of young, middle-aged and old in the same period. Note that this implies focusing on different agents. It is assumed that the savings rate of one middle-aged individual in period t+1 is the same as the savings rate of a middle-aged agent in period t. This assumption can be justified, because even when savings and disposable income are different from cohort to cohort, the ratio can still remain the same. For the hyperbolic PAYG scheme (see figure 5), averaging of .08, .26 and .0 yields a periodic savings rate of 11.3 percent.[24] Apparently, the measured savings rate and the computed one are quite close. This lends some credit to the explanatory power of the model, because PAYG is the system that is actually in in place in Germany and it was claimed that real world agents behave like hyperbolic discounters. Indeed a periodic savings rate of .17, as predicted for exponential discounters in the PAYG plan, deviates by a considerable 6 percent

[24]Normally, it is necessary to correct for the fact that each generation consists of a different number of people. However, the population growth rate is so low, that ignoring this does not affect the results by much.

from the measurements. Consequently, hyperbolic discounting seems a far better approach to explain reality than standard discounting.

Another fact is that sophisticates could achieve the same result as exponential discounters, if a perfect commitment mechanism could be found. While this model did not allow for a commitment device, it can and it is applied in the real world. For example, sophisticates seek for illiquid assets, so that they cannot spend too much of their property. An additional advantage of commitment devices is that those that are ignorant of their myopic preferences benefit as well. I do not mention this to discredit my own work, but to put it into a broader perspective. Here, it is shown that those without present-bias preferences enjoy a considerably higher utility. Ultimately, it explains why it is so important, to nudge people in a way that circumvents suboptimal short-term driven actions.

Imagine an economy with a pension plan that does not guarantee the minimal requirements for a human life. I suppose the vast majority of people would describe such a system as immoral, unsocial and unacceptable. While this is not the place for moral questions, I still think a pension system should provide enough to satisfy the basic needs of everyone. I claim that the basic needs are relatively equal among humans, apart from fluctuations due to disability and disease. But these are not part of old-age pension considerations. For me a combination of PAYG and FF is most promising. A slimmed-down version of the PAYG system could provide the minimal requirement to everyone. This would eliminate the necessity of coupling the individual pension to individual wages during working life. Such a system could dramatically lower the systemic and bureaucratic costs as well as redistribution. Every lifestyle desire above the minimum is covered by the FF scheme. Here, people should be free to do whatever is in their own best interest and conform with their aspirations. Obviously, this is where behavioral economics can be applied to help humans get what they want within the possible set of options.

Finally, it remains to discuss further issues and ideas for adjacent work. Admittedly, only solving for the naives is a strong simplification that should be extended. Real people do not just save, but use different saving types e.g. cash, savings accounts, stocks, bonds and derivatives. This is important and must be dealt with, because the fungibility of money does not hold. Rather, there is an asset-specific marginal propensity to consume. There is also evidence that people get positive utility from leaving bequest to ancestors which is a good example of altruistic preferences (bounded self-interest). Moreover, one could consider a framework in continuous time and stochastic shocks to move to more realism. Especially, with regard to unexpected liquidity constraints, uncertainty about future income and labor market developments as in Angeletos et al. (2001) stochastic shocks are justified. Laibson, Repetto and Tobacman (2000) develop a model where three different levels of education result in different wages in the labor market. This is only one idea on how to move away from a plain average value that is true for everyone in the economy. Another way are age-dependent wages. Empirically observed patterns show that in general older workers earn more than their younger counterparts in the same period. It is also helpful to consider probabilities of dying. A government equation could account for debts in the pension system and growth rates can be modeled endogenously. Interactions with other policies and areas must be watched carefully. Feasibility goals covered by Binswanger (2007) can help to give decision-making more realistic features. All other shortcomings discussed throughout the thesis are worthwhile, too.

7 Conclusion

While the numerical results are calculated for Germany, a lot of central issues are equivalently applicable to other countries. At first, it was shown that hyperbolic discounting indeed deviates from what the standard economic model would predict. Lack of willpower leads to lower utility and lower savings for retirement when compared to exponential discounters. This can be summarized by overconsumption in youth and undersaving for the old days. Secondly, a fully funded pension scheme is expected to improve the standard of living in Germany. These findings seem to be robust to variations from the baseline calibration. A third message is that individuals are not consistent in their decisions. This means that there is a disparity between their expected actions and the actual implementation. Finally, it is important for economists to realize that there is no "homo oeconomicus" out there. Therefore, adopting (more) knowlegde from neuroscience and psychology is overdue and inevitable if future work is supposed to have practical relevance.

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Appendices

A Tables

Table 1: Baseline Parameterization 3-Period LCM (1 Period = 20 Years)

Parameter Name	Symbol	Value
short-run discount factor	β	.60
long-run discount factor	δ	.67
elasticity of intertemporal substitution	η	.50
effective contribution rate	τ	.17
growth rate of wage	ω	.24
growth rate of population	n	.03
net rate of interest	r	.81

Table 2: Baseline Parameterization 6-Period LCM (1 Period = 10 Years)

Parameter Name	Symbol	Value
short-run discount factor	β	.60
long-run discount factor	δ	.82
elasticity of intertemporal substitution	η	.50
effective contribution rate	τ	.17
growth rate of wage	ω	.12
growth rate of population	n	.01
net rate of interest	r	.34

Table 3: List of Indexes, Variables and Parameters

Shortcut	**Explanation**
Indexes	
y	young
m	middle-aged
o	old
t	time
Variables	
b	pension benefits
c	consumption
p	pension contribution
s	savings
w	wage (real)
N	population
Ω	lifetime income
Parameters	
β	short-run discount factor
δ	long-run discount factor
η	elasticity of intertemporal substitution
τ	pension contribution rate
ω	growth rate of wage (real)
n	growth rate of population
r	net interest rate (real)
R	gross interest rate (real)

Table 4: Calculation of the Effective Contribution Rate

Year	Participation Rate	Contribution Rate	Effective Contribution Rate
1991	.907	.180	.163
1992	.904	.177	.160
1993	.898	.175	.157
1994	.893	.192	.171
1995	.891	.186	.166
1996	.893	.192	.171
1997	.889	.203	.180
1998	.875	.203	.178
1999	.896	.197	.176
2000	.895	.193	.173
2001	.895	.191	.171
2002	.896	.191	.171
2003	.894	.195	.174
2004	.876	.195	.171
2005	.879	.195	.171
2006	.881	.195	.172
2007	.882	.199	.176
2008	.886	.199	.176

*Source: Statistisches Bundesamt (2010*a*), DIA (2008)*

Table 5: Population, Wage and Deflator

Year	Population	Annual Gross Wage	Deflator
1991	80274564	21984	75.9
1992	80974632	24036	79.8
1993	81338093	25236	83.3
1994	81538603	26220	85.6
1995	81817499	27372	87.1
1996	82012162	28128	88.3
1997	82057379	28668	90.0
1998	82037011	29364	90.9
1999	82163475	30216	91.4
2000	82259540	30612	92.7
2001	82440309	31404	94.5
2002	82536680	32412	95.9
2003	82531671	33396	96.9
2004	82500849	34152	98.5
2005	82437995	34812	100.0
2006	82314906	35400	101.6
2007	82217837	36276	103.9
2008	82002356	37236	106.6

*Source: Statistisches Bundesamt (2010*a*)*

Table 6: Savings Rates

	Hyperbolic PAYG		Exponential PAYG		Hyperbolic FF		Exponential FF	
τ	$\frac{s_t^y}{w_t}$	$\frac{s_{t+1}^m}{w_{t+1}}$	$\frac{s_t^y}{w_t}$	$\frac{s_{t+1}^m}{w_{t+1}}$	$\frac{s_t^y}{w_t}$	$\frac{s_{t+1}^m}{w_{t+1}}$	$\frac{s_t^y}{w_t}$	$\frac{s_{t+1}^m}{w_{t+1}}$
0.00	0.01	0.33	0.12	0.44	0.01	0.33	0.12	0.44
0.01	0.01	0.31	0.11	0.43	0.00	0.30	0.11	0.42
0.02	0.00	0.29	0.11	0.41	-0.01	0.28	0.10	0.39
0.03	-0.01	0.28	0.10	0.40	-0.02	0.25	0.09	0.37
0.04	-0.01	0.26	0.09	0.38	-0.03	0.23	0.08	0.35
0.05	-0.02	0.25	0.09	0.36	-0.04	0.20	0.07	0.32
0.06	-0.02	0.23	0.08	0.35	-0.05	0.18	0.06	0.30
0.07	-0.03	0.22	0.07	0.33	-0.06	0.15	0.05	0.27
0.08	-0.04	0.20	0.07	0.31	-0.07	0.13	0.04	0.25
0.09	-0.04	0.18	0.06	0.30	-0.08	0.11	0.03	0.22
0.10	-0.05	0.17	0.05	0.28	-0.09	0.08	0.02	0.20
0.11	-0.05	0.15	0.05	0.27	-0.10	0.06	0.01	0.17
0.12	-0.06	0.14	0.04	0.25	-0.11	0.03	0.00	0.15
0.13	-0.06	0.12	0.04	0.23	-0.12	0.01	-0.01	0.12
0.14	-0.07	0.11	0.03	0.22	-0.13	-0.02	-0.02	0.10
0.15	-0.08	0.09	0.02	0.20	-0.14	-0.04	-0.03	0.08
0.16	-0.08	0.07	0.02	0.18	-0.15	-0.07	-0.04	0.05
0.17	-0.09	0.06	0.01	0.17	-0.16	-0.09	-0.05	0.03
0.18	-0.09	0.04	0.00	0.15	-0.17	-0.12	-0.06	0.00
0.19	-0.10	0.03	0.00	0.14	-0.18	-0.14	-0.07	-0.02
0.20	-0.11	0.01	-0.01	0.12	-0.19	-0.17	-0.08	-0.05

Source: Author's Computation

Table 7: Consumption Rates

τ	Hyperbolic PAYG $\frac{c_t^y}{w_t}$	$\frac{c_{t+1}^m}{w_{t+1}}$	$\frac{c_{t+2}^o}{w_{t+2}}$	Exponential PAYG $\frac{c_t^y}{w_t}$	$\frac{c_{t+1}^m}{w_{t+1}}$	$\frac{c_{t+2}^o}{w_{t+2}}$	Hyperbolic FF $\frac{c_t^y}{w_t}$	$\frac{c_{t+1}^m}{w_{t+1}}$	$\frac{c_{t+2}^o}{w_{t+2}}$	Exponential FF $\frac{c_t^y}{w_t}$	$\frac{c_{t+1}^m}{w_{t+1}}$	$\frac{c_{t+2}^o}{w_{t+2}}$
0.00	0.99	0.69	0.48	0.88	0.73	0.65	0.99	0.69	0.48	0.88	0.73	0.65
0.01	0.98	0.69	0.47	0.88	0.73	0.64	0.99	0.69	0.48	0.88	0.73	0.65
0.02	0.98	0.69	0.47	0.87	0.72	0.64	0.99	0.69	0.48	0.88	0.73	0.65
0.03	0.98	0.68	0.47	0.87	0.72	0.64	0.99	0.69	0.48	0.88	0.73	0.65
0.04	0.97	0.68	0.47	0.87	0.72	0.64	0.99	0.69	0.48	0.88	0.73	0.65
0.05	0.97	0.68	0.47	0.86	0.71	0.63	0.99	0.69	0.48	0.88	0.73	0.65
0.06	0.96	0.67	0.46	0.86	0.71	0.63	0.99	0.69	0.48	0.88	0.73	0.65
0.07	0.96	0.67	0.46	0.86	0.71	0.63	0.99	0.69	0.48	0.88	0.73	0.65
0.08	0.96	0.67	0.46	0.85	0.70	0.62	0.99	0.69	0.48	0.88	0.73	0.65
0.09	0.95	0.67	0.46	0.85	0.70	0.62	0.99	0.69	0.48	0.88	0.73	0.65
0.10	0.95	0.66	0.46	0.85	0.70	0.62	0.99	0.69	0.48	0.88	0.73	0.65
0.11	0.94	0.66	0.45	0.84	0.70	0.62	0.99	0.69	0.48	0.88	0.73	0.65
0.12	0.94	0.66	0.45	0.84	0.69	0.61	0.99	0.69	0.48	0.88	0.73	0.65
0.13	0.93	0.65	0.45	0.83	0.69	0.61	0.99	0.69	0.48	0.88	0.73	0.65
0.14	0.93	0.65	0.45	0.83	0.69	0.61	0.99	0.69	0.48	0.88	0.73	0.65
0.15	0.93	0.65	0.45	0.83	0.68	0.61	0.99	0.69	0.48	0.88	0.73	0.65
0.16	0.92	0.65	0.44	0.82	0.68	0.60	0.99	0.69	0.48	0.88	0.73	0.65
0.17	0.92	0.64	0.44	0.82	0.68	0.60	0.99	0.69	0.48	0.88	0.73	0.65
0.18	0.91	0.64	0.44	0.82	0.67	0.60	0.99	0.69	0.48	0.88	0.73	0.65
0.19	0.91	0.64	0.44	0.81	0.67	0.60	0.99	0.69	0.48	0.88	0.73	0.65
0.20	0.91	0.63	0.44	0.81	0.67	0.59	0.99	0.69	0.48	0.88	0.73	0.65

Source: Author's Computation

Table 8: Savings and Consumption Rates, Hyperbolic, PAYG

	Young		Middle-Aged		Old		Average	
	SR	CR	SR	CR	SR	CR	SR	CR
τ								
0.00	0.0120	0.9880	0.3206	0.6794	0.0000	1.0000	0.1291	0.8709
0.01	0.0161	0.9839	0.3176	0.6824	0.0000	1.0000	0.1309	0.8691
0.02	0.0202	0.9798	0.3146	0.6854	0.0000	1.0000	0.1328	0.8672
0.03	0.0243	0.9757	0.3116	0.6884	0.0000	1.0000	0.1347	0.8653
0.04	0.0284	0.9716	0.3085	0.6915	0.0000	1.0000	0.1365	0.8635
0.05	0.0325	0.9675	0.3054	0.6946	0.0000	1.0000	0.1385	0.8615
0.06	0.0366	0.9634	0.3022	0.6978	0.0000	1.0000	0.1404	0.8596
0.07	0.0407	0.9593	0.2989	0.7011	0.0000	1.0000	0.1423	0.8577
0.08	0.0448	0.9552	0.2956	0.7044	0.0000	1.0000	0.1443	0.8557
0.09	0.0489	0.9511	0.2922	0.7078	0.0000	1.0000	0.1463	0.8537
0.10	0.0530	0.9470	0.2888	0.7112	0.0000	1.0000	0.1483	0.8517
0.11	0.0571	0.9429	0.2853	0.7147	0.0000	1.0000	0.1504	0.8496
0.12	0.0612	0.9388	0.2817	0.7183	0.0000	1.0000	0.1524	0.8476
0.13	0.0654	0.9346	0.2781	0.7219	0.0000	1.0000	0.1545	0.8455
0.14	0.0695	0.9305	0.2744	0.7256	0.0000	1.0000	0.1566	0.8434
0.15	0.0736	0.9264	0.2706	0.7294	0.0000	1.0000	0.1587	0.8413
0.16	0.0777	0.9223	0.2668	0.7332	0.0000	1.0000	0.1608	0.8392
0.17	0.0818	0.9182	0.2628	0.7372	0.0000	1.0000	0.1630	0.8370
0.18	0.0859	0.9141	0.2588	0.7412	0.0000	1.0000	0.1652	0.8348
0.19	0.0900	0.9100	0.2547	0.7453	0.0000	1.0000	0.1674	0.8326
0.20	0.0941	0.9059	0.2506	0.7494	0.0000	1.0000	0.1696	0.8304

SR = Savings Rate, CR = Consumption Rate

Source: Author's Computation

Table 9: Savings and Consumption Rates, Exponential, PAYG

	Young		Middle-Aged		Old		Average	
	SR	CR	SR	CR	SR	CR	SR	CR
τ								
0.00	0.1182	0.8818	0.3785	0.6215	0.0000	1.0000	0.1684	0.8316
0.01	0.1219	0.8781	0.3762	0.6238	0.0000	1.0000	0.1699	0.8301
0.02	0.1256	0.8744	0.3738	0.6262	0.0000	1.0000	0.1714	0.8286
0.03	0.1292	0.8708	0.3714	0.6286	0.0000	1.0000	0.1729	0.8271
0.04	0.1329	0.8671	0.3690	0.6310	0.0000	1.0000	0.1744	0.8256
0.05	0.1366	0.8634	0.3665	0.6335	0.0000	1.0000	0.1759	0.8241
0.06	0.1402	0.8598	0.3640	0.6360	0.0000	1.0000	0.1775	0.8225
0.07	0.1439	0.8561	0.3614	0.6386	0.0000	1.0000	0.1791	0.8209
0.08	0.1476	0.8524	0.3588	0.6412	0.0000	1.0000	0.1807	0.8193
0.09	0.1512	0.8488	0.3562	0.6438	0.0000	1.0000	0.1823	0.8177
0.10	0.1549	0.8451	0.3535	0.6465	0.0000	1.0000	0.1839	0.8161
0.11	0.1586	0.8414	0.3507	0.6493	0.0000	1.0000	0.1856	0.8144
0.12	0.1622	0.8378	0.3479	0.6521	0.0000	1.0000	0.1872	0.8128
0.13	0.1659	0.8341	0.3451	0.6549	0.0000	1.0000	0.1889	0.8111
0.14	0.1695	0.8305	0.3422	0.6578	0.0000	1.0000	0.1906	0.8094
0.15	0.1732	0.8268	0.3392	0.6608	0.0000	1.0000	0.1923	0.8077
0.16	0.1769	0.8231	0.3363	0.6637	0.0000	1.0000	0.1941	0.8059
0.17	0.1805	0.8195	0.3332	0.6668	0.0000	1.0000	0.1958	0.8042
0.18	0.1842	0.8158	0.3301	0.6699	0.0000	1.0000	0.1976	0.8024
0.19	0.1879	0.8121	0.3269	0.6731	0.0000	1.0000	0.1994	0.8006
0.20	0.1915	0.8085	0.3237	0.6763	0.0000	1.0000	0.2012	0.7988

SR = Savings Rate, CR = Consumption Rate

Source: Author's Computation

Table 10: Savings and Consumption Rates, Hyperbolic, FF

	Young		Middle-Aged		Old		Average	
	SR	CR	SR	CR	SR	CR	SR	CR
τ								
0.00	0.0120	0.9880	0.3206	0.6794	0.0000	1.0000	0.1392	0.8608
0.01	0.0120	0.9880	0.3107	0.6893	0.0000	1.0000	0.1339	0.8661
0.02	0.0120	0.9880	0.3005	0.6995	0.0000	1.0000	0.1286	0.8714
0.03	0.0120	0.9880	0.2901	0.7099	0.0000	1.0000	0.1233	0.8767
0.04	0.0120	0.9880	0.2793	0.7207	0.0000	1.0000	0.1179	0.8821
0.05	0.0120	0.9880	0.2682	0.7318	0.0000	1.0000	0.1124	0.8876
0.06	0.0120	0.9880	0.2567	0.7433	0.0000	1.0000	0.1068	0.8932
0.07	0.0120	0.9880	0.2449	0.7551	0.0000	1.0000	0.1012	0.8988
0.08	0.0120	0.9880	0.2327	0.7673	0.0000	1.0000	0.0955	0.9045
0.09	0.0120	0.9880	0.2201	0.7799	0.0000	1.0000	0.0897	0.9103
0.10	0.0120	0.9880	0.2070	0.7930	0.0000	1.0000	0.0839	0.9161
0.11	0.0120	0.9880	0.1936	0.8064	0.0000	1.0000	0.0779	0.9221
0.12	0.0120	0.9880	0.1796	0.8204	0.0000	1.0000	0.0719	0.9281
0.13	0.0120	0.9880	0.1652	0.8348	0.0000	1.0000	0.0659	0.9341
0.14	0.0120	0.9880	0.1502	0.8498	0.0000	1.0000	0.0597	0.9403
0.15	0.0120	0.9880	0.1347	0.8653	0.0000	1.0000	0.0535	0.9465
0.16	0.0120	0.9880	0.1187	0.8813	0.0000	1.0000	0.0471	0.9529
0.17	0.0120	0.9880	0.1020	0.8980	0.0000	1.0000	0.0407	0.9593
0.18	0.0120	0.9880	0.0847	0.9153	0.0000	1.0000	0.0342	0.9658
0.19	0.0120	0.9880	0.0667	0.9333	0.0000	1.0000	0.0276	0.9724
0.20	0.0120	0.9880	0.0480	0.9520	0.0000	1.0000	0.0210	0.9790

SR = Savings Rate, CR = Consumption Rate

Source: Author's Computation

Table 11: Savings and Consumption Rates, Exponential, FF

	Young		Middle-Aged		Old		Average	
	SR	CR	SR	CR	SR	CR	SR	CR
τ								
0.00	0.1182	0.8818	0.3785	0.6215	0.0000	1.0000	0.1939	0.8061
0.01	0.1182	0.8818	0.3707	0.6293	0.0000	1.0000	0.1897	0.8103
0.02	0.1182	0.8818	0.3627	0.6373	0.0000	1.0000	0.1854	0.8146
0.03	0.1182	0.8818	0.3545	0.6455	0.0000	1.0000	0.1810	0.8190
0.04	0.1182	0.8818	0.3460	0.6540	0.0000	1.0000	0.1766	0.8234
0.05	0.1182	0.8818	0.3374	0.6626	0.0000	1.0000	0.1722	0.8278
0.06	0.1182	0.8818	0.3285	0.6715	0.0000	1.0000	0.1677	0.8323
0.07	0.1182	0.8818	0.3193	0.6807	0.0000	1.0000	0.1632	0.8368
0.08	0.1182	0.8818	0.3099	0.6901	0.0000	1.0000	0.1586	0.8414
0.09	0.1182	0.8818	0.3003	0.6997	0.0000	1.0000	0.1540	0.8460
0.10	0.1182	0.8818	0.2903	0.7097	0.0000	1.0000	0.1493	0.8507
0.11	0.1182	0.8818	0.2801	0.7199	0.0000	1.0000	0.1446	0.8554
0.12	0.1182	0.8818	0.2696	0.7304	0.0000	1.0000	0.1398	0.8602
0.13	0.1182	0.8818	0.2588	0.7412	0.0000	1.0000	0.1350	0.8650
0.14	0.1182	0.8818	0.2476	0.7524	0.0000	1.0000	0.1301	0.8699
0.15	0.1182	0.8818	0.2362	0.7638	0.0000	1.0000	0.1252	0.8748
0.16	0.1182	0.8818	0.2243	0.7757	0.0000	1.0000	0.1201	0.8799
0.17	0.1182	0.8818	0.2121	0.7879	0.0000	1.0000	0.1151	0.8849
0.18	0.1182	0.8818	0.1995	0.8005	0.0000	1.0000	0.1100	0.8900
0.19	0.1182	0.8818	0.1865	0.8135	0.0000	1.0000	0.1048	0.8952
0.20	0.1182	0.8818	0.1730	0.8270	0.0000	1.0000	0.0995	0.9005

SR = Savings Rate, CR = Consumption Rate

Source: Author's Computation

Table 12: Normative Utility Values

τ	Hyperbolic PAYG	Exponential PAYG	Hyperbolic FF	Exponential FF
0.00	-0.0241	-0.0233	-0.0241	-0.0233
0.01	-0.0242	-0.0234	-0.0241	-0.0233
0.02	-0.0243	-0.0235	-0.0241	-0.0233
0.03	-0.0244	-0.0236	-0.0241	-0.0233
0.04	-0.0245	-0.0237	-0.0241	-0.0233
0.05	-0.0246	-0.0238	-0.0241	-0.0233
0.06	-0.0247	-0.0239	-0.0241	-0.0233
0.07	-0.0248	-0.0240	-0.0241	-0.0233
0.08	-0.0249	-0.0241	-0.0241	-0.0233
0.09	-0.0250	-0.0242	-0.0241	-0.0233
0.10	-0.0251	-0.0243	-0.0241	-0.0233
0.11	-0.0252	-0.0244	-0.0241	-0.0233
0.12	-0.0253	-0.0245	-0.0241	-0.0233
0.13	-0.0255	-0.0246	-0.0241	-0.0233
0.14	-0.0256	-0.0247	-0.0241	-0.0233
0.15	-0.0257	-0.0248	-0.0241	-0.0233
0.16	-0.0258	-0.0249	-0.0241	-0.0233
0.17	-0.0259	-0.0250	-0.0241	-0.0233
0.18	-0.0260	-0.0252	-0.0241	-0.0233
0.19	-0.0261	-0.0253	-0.0241	-0.0233
0.20	-0.0263	-0.0254	-0.0241	-0.0233

Source: Author's Computation

Table 13: Pensions for PAYG and FF with Varying Annual Interest Rates

τ	Pay-As-You-Go for all r	Fully Funded r=.00	r=.01	r=.02	r=.03	r=.04	r=.05
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.01	3.215	2.240	3.002	4.051	5.502	7.518	10.330
0.02	6.430	4.480	6.004	8.101	11.003	15.036	20.660
0.03	9.645	6.720	9.006	12.152	16.505	22.554	30.990
0.04	12.860	8.960	12.008	16.202	22.006	30.072	41.320
0.05	16.075	11.200	15.009	20.253	27.508	37.590	51.650
0.06	19.290	13.440	18.011	24.304	33.010	45.108	61.980
0.07	22.505	15.680	21.013	28.354	38.511	52.626	72.311
0.08	25.720	17.920	24.015	32.405	44.013	60.144	82.641
0.09	28.935	20.160	27.017	36.456	49.515	67.662	92.971
0.10	32.150	22.400	30.019	40.506	55.016	75.180	103.301
0.11	35.365	24.640	33.021	44.557	60.518	82.698	113.631
0.12	38.580	26.880	36.023	48.607	66.019	90.216	123.961
0.13	41.795	29.120	39.025	52.658	71.521	97.734	134.291
0.14	45.010	31.360	42.027	56.709	77.023	105.252	144.621
0.15	48.225	33.600	45.028	60.759	82.524	112.770	154.951
0.16	51.439	35.840	48.030	64.810	88.026	120.288	165.281
0.17	54.654	38.080	51.032	68.860	93.527	127.806	175.611
0.18	57.869	40.320	54.034	72.911	99.029	135.324	185.941
0.19	61.084	42.560	57.036	76.962	104.531	142.842	196.271
0.20	64.299	44.800	60.038	81.012	110.032	150.360	206.602

Source: Author's Computation

B Figures

Figure 1: Illustration of Normative Utility Functions

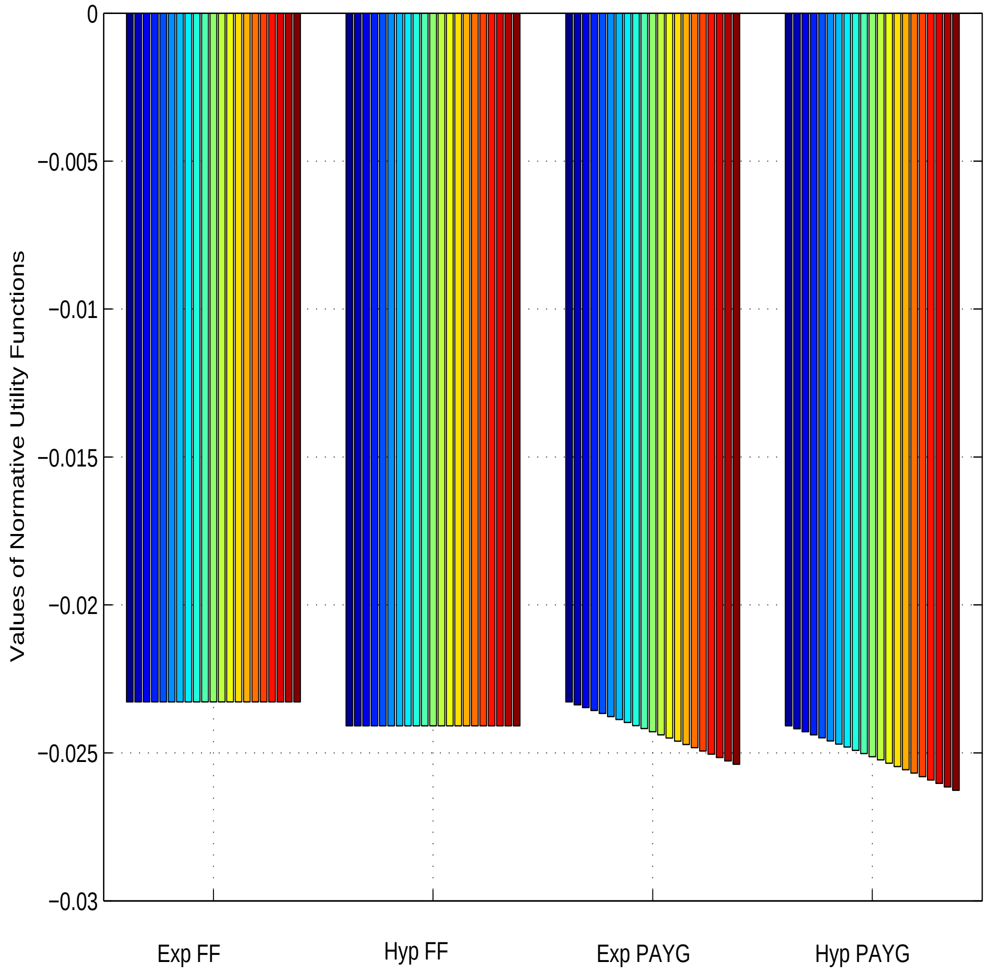


Figure 2: Baseline Value Simulation

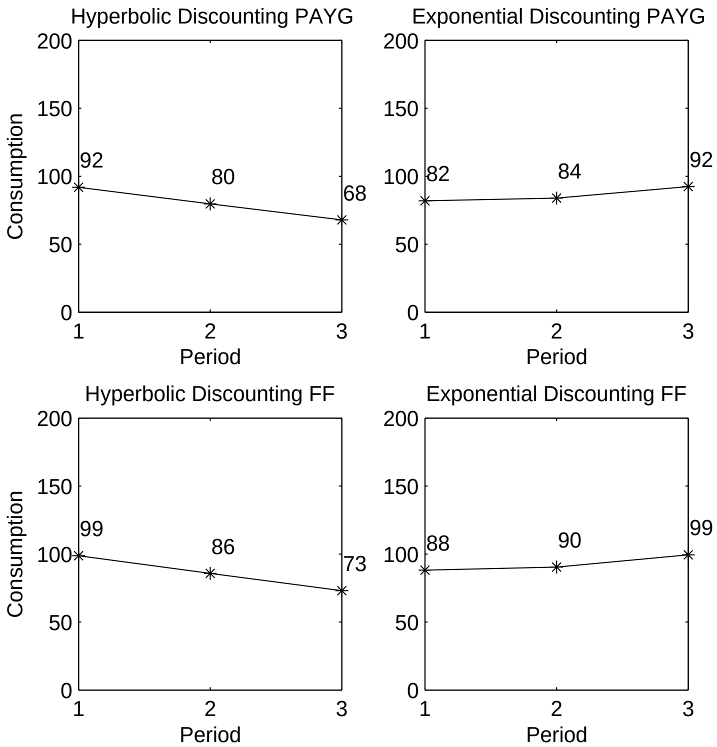


Figure 3: Two Different Exponential Lines

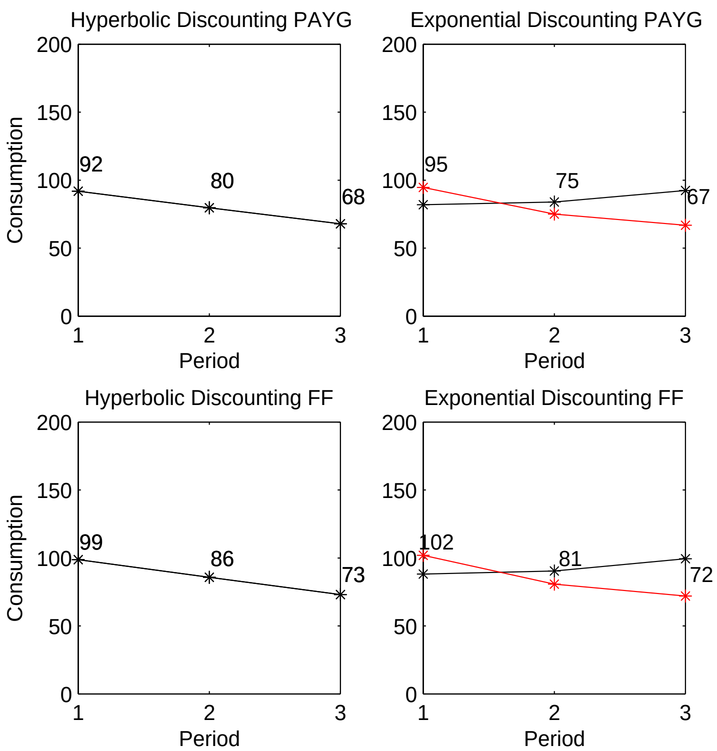


Figure 4: Savings Rate, Net Savings Rate and Consumption Rate

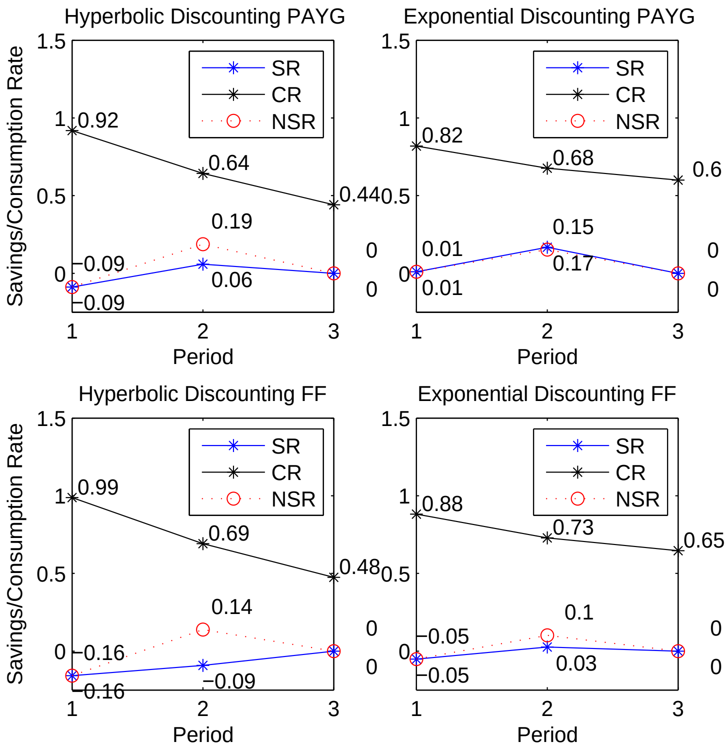


Figure 5: Broader Definition of Savings and Consumption Rate

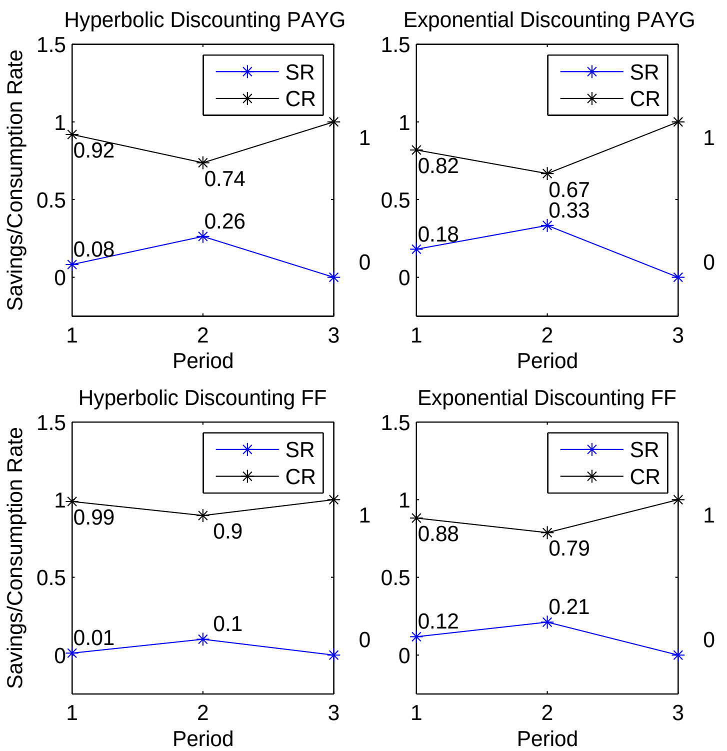


Figure 6: Real Gross Rate of Interest $= 1.01^{20} = 1.22$

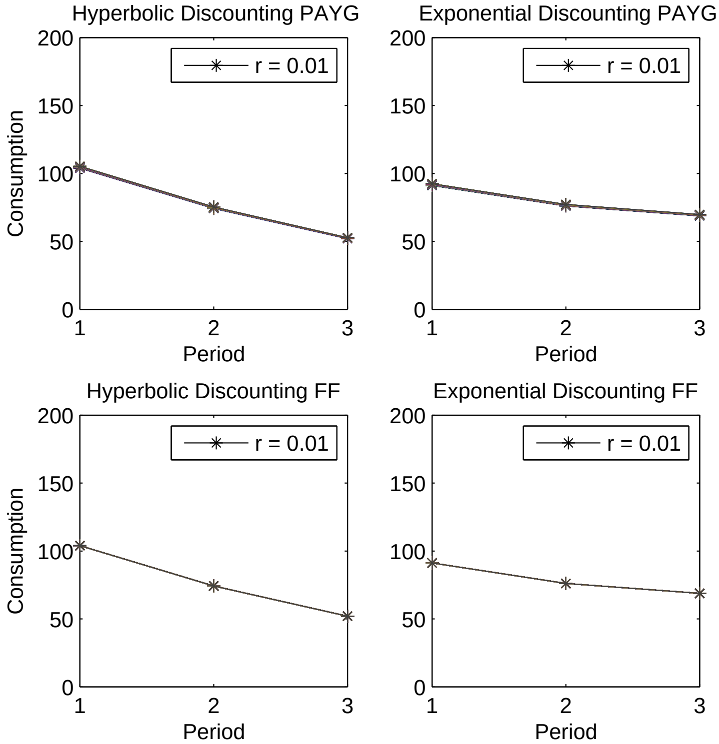


Figure 7: Real Gross Rate of Interest $= 1.03^{20} = 1.81$

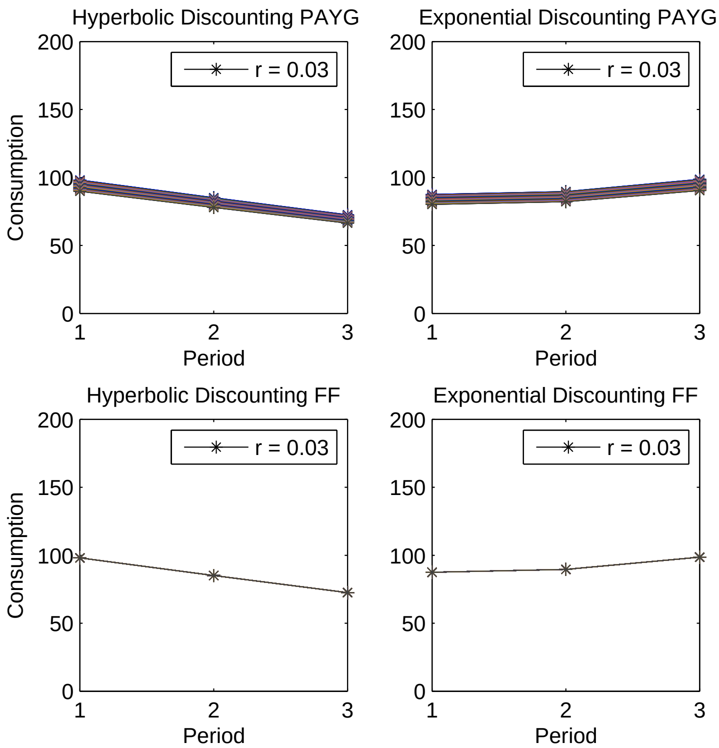


Figure 8: Real Gross Rate of Interest $= 1.05^{20} = 2.65$

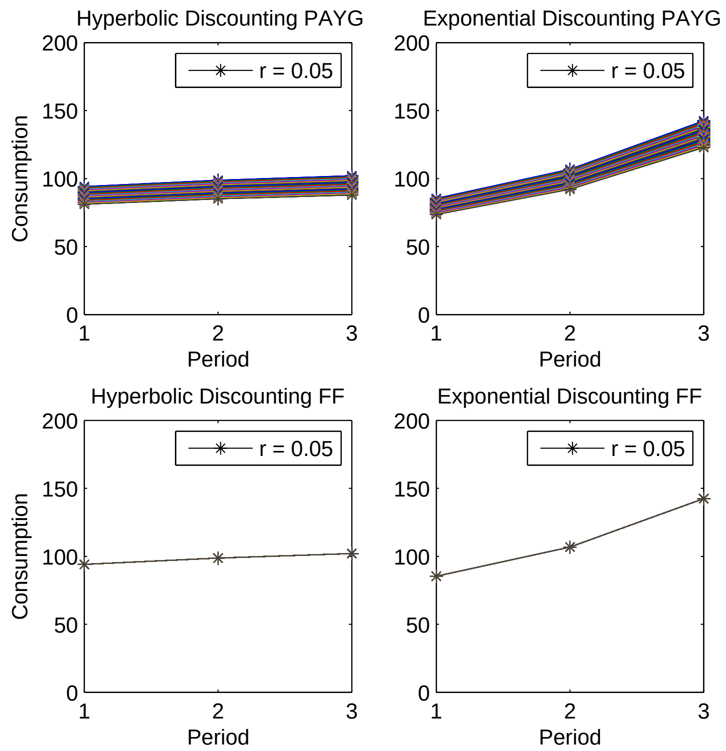


Figure 9: Annual R $= 1.01$, Three Different Values for η

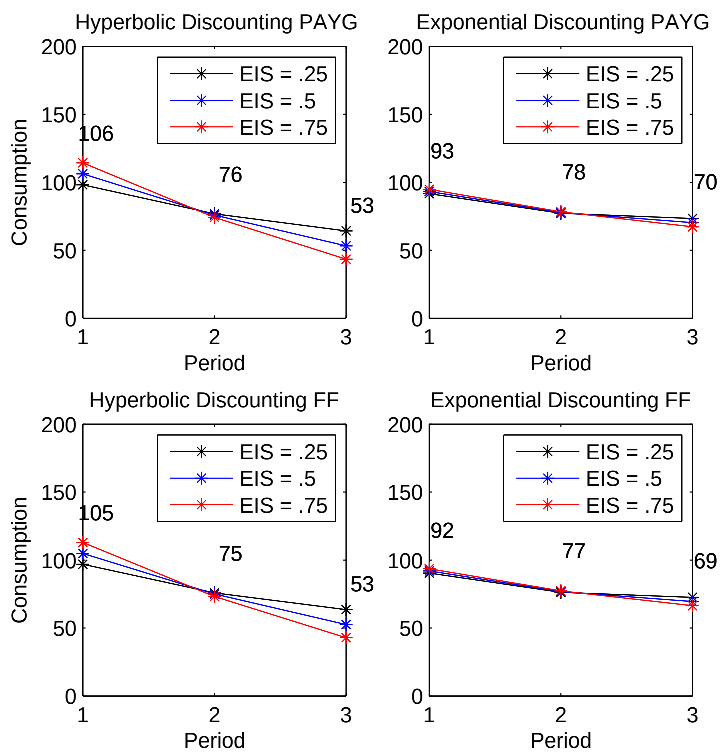


Figure 10: Annual R = 1.03, Three Different Values for η

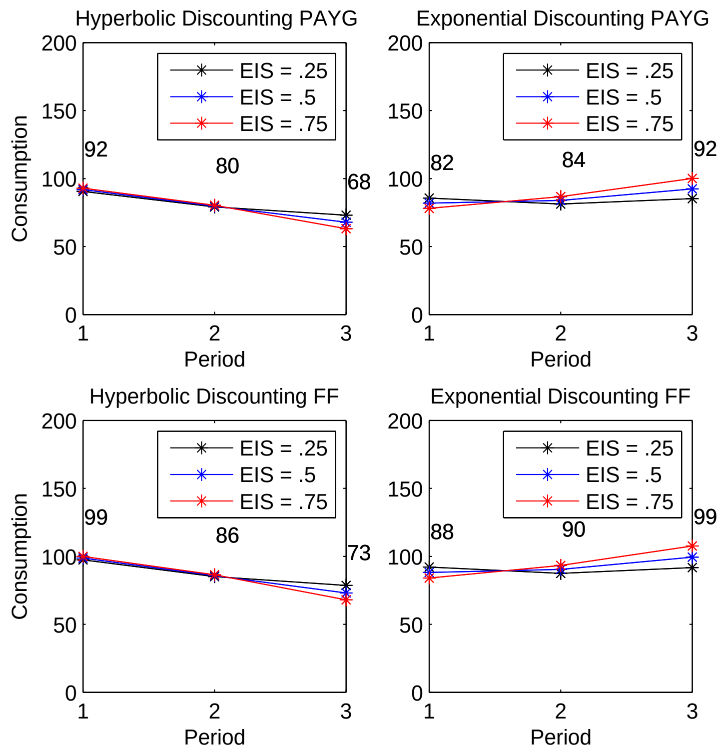


Figure 11: Annual R = 1.05, Three Different Values for η

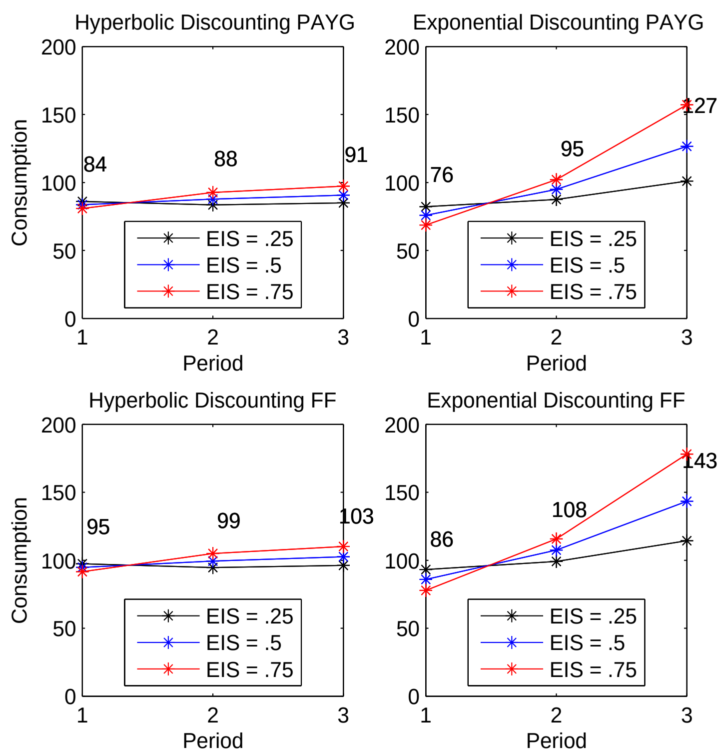


Figure 12: The Shape of Different Discount Functions

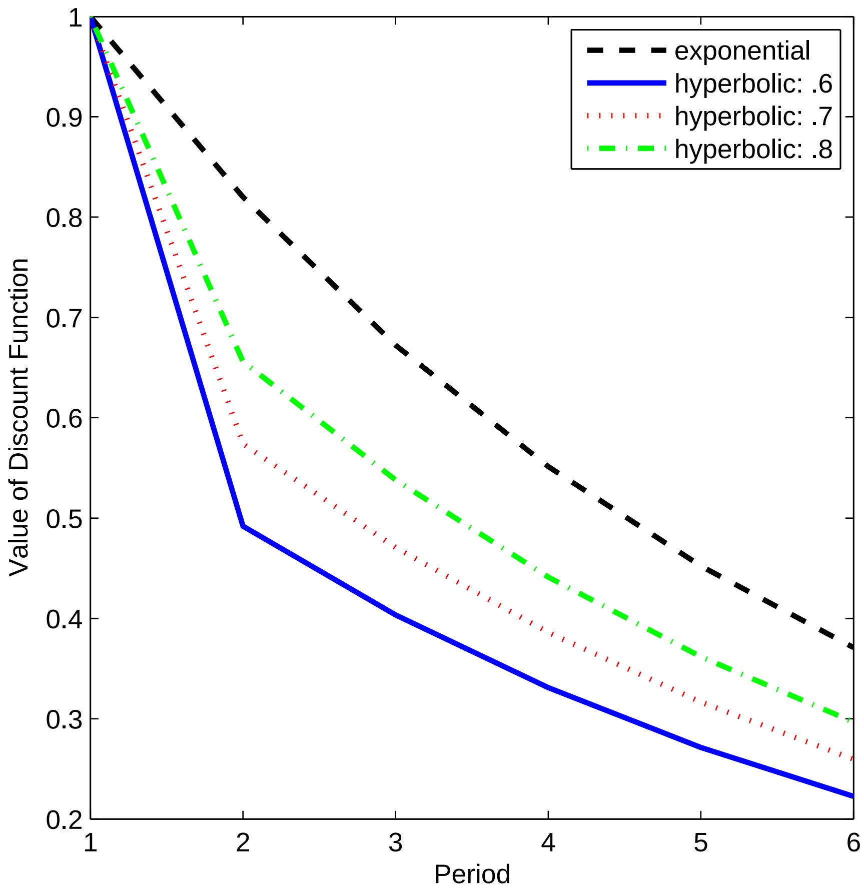


Figure 13: 6-Period Life-Cycle Model, Hyperbolic, PAYG

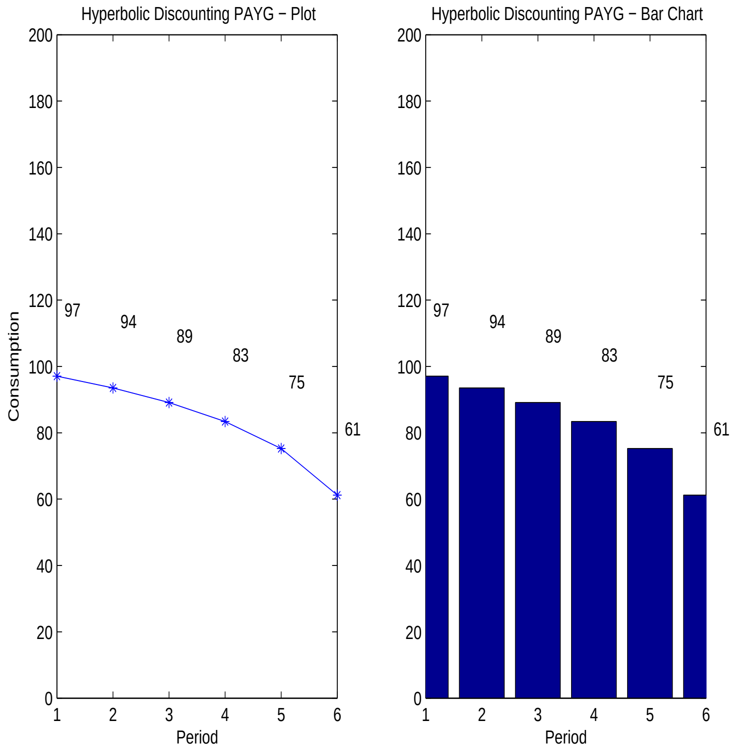


Figure 14: 6-Period Life-Cycle Model, Exponential, PAYG

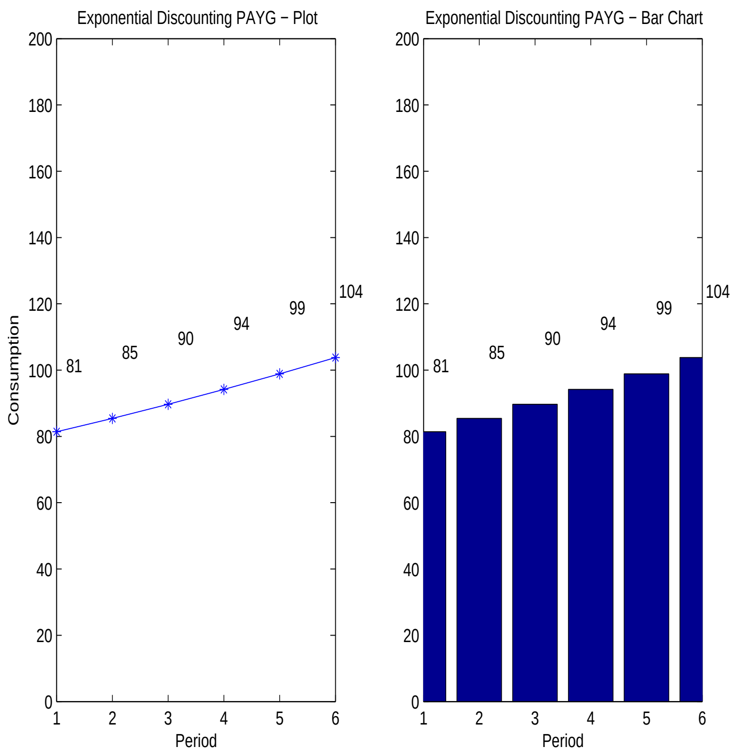


Figure 15: 6-Period Life-Cycle Model, Hyperbolic, FF

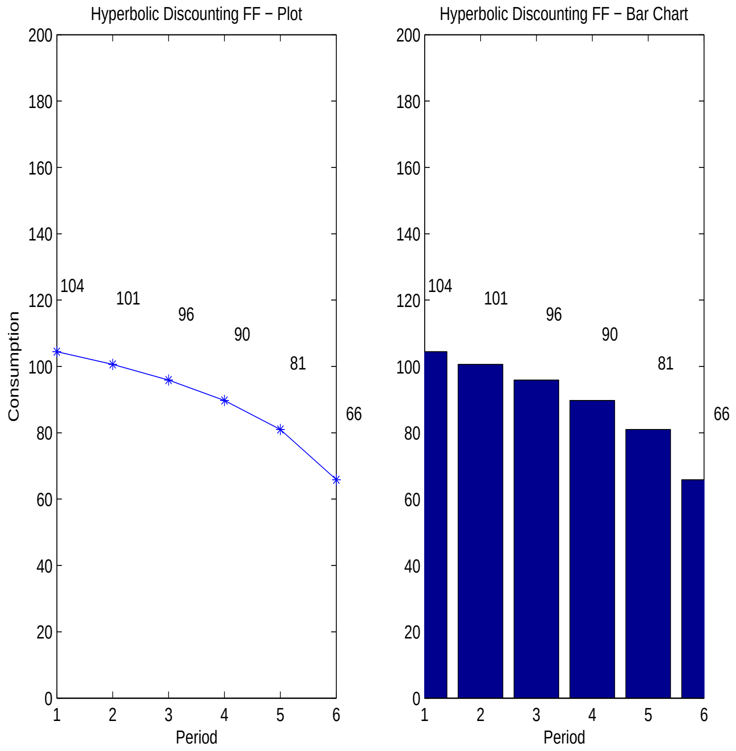
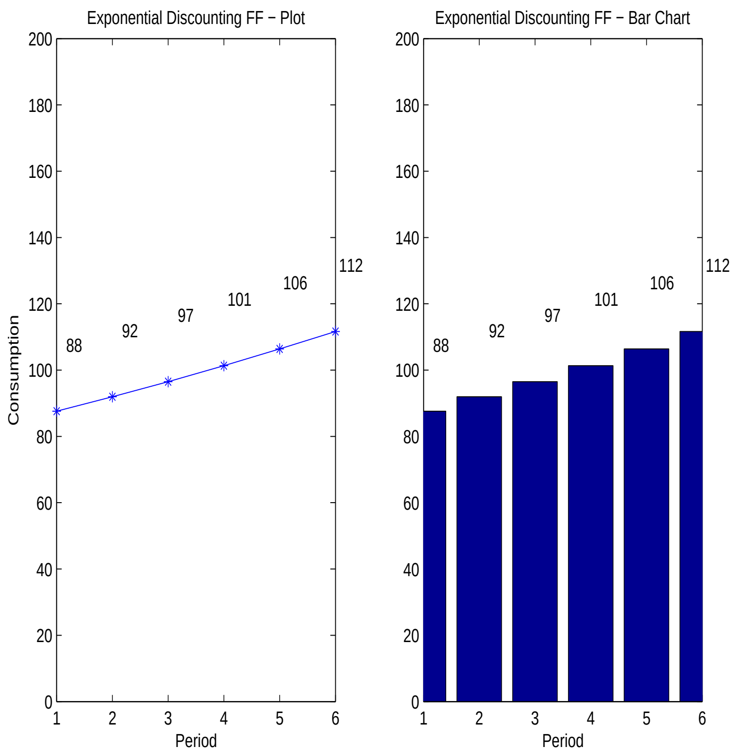


Figure 16: 6-Period Life-Cycle Model, Exponential, FF



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Stefan Zimmermann
Berlin, August 17th, 2010