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Redesigning Dutch Pension Funds

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Redesigning Dutch Pension Funds

by

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Abstract

Pension funds in the Dutch second pension pillar are under severe pressure, due to indistinct, inadequate pension contracts and an aging society. Rigorous changes in fund design are necessary in order to release tensions between stakeholders within pension funds. This thesis elaborates a pension fund redesign that makes collective pension contracts explicit, complete, fair and sustainable, overcoming important flaws in current pension fund design. The core of the redesign is that pension rights within the fund are split up in different risk categories, called tranches. Plan members allocate their pension wealth over these tranches, dependent on their age in line with life-cycle preferences. Shocks in funding level are absorbed by the tranches according to a 'waterfall construction', in which the most risky tranche absorbs the first shock. More secure tranches are only affected when risky tranches are wiped out or filled up completely. To arrive at a fair fund design from the perspective of all stakeholders, we use options that make the embedded risk sharing within the pension fund explicit. Subsequent analyses gain intuition about the working of the redesigned pension fund. The thoughts and analyses in this thesis can serve as basis for factual redesign of second-pillar pension funds.

KEYWORDS: pension fund, redesign, option pricing, risk tranches, risk sharing.

Contents

Chapter 1

Introduction

1.1 Problem Motivation

The Dutch pension system is under serious pressure. Clear indication of this pressure is the appearance of several government reports, covering problems within the pension system. These reports conclude that Dutch pension funds lack on a large scale attention for risks and risk management (Frijns, 2010), and more radically, that the Dutch pension system as a whole is not sustainable for the future (Goudswaard, 2010).

Important reason for this concern is that over the last decade it has come to light that pension contracts in Dutch pension funds are highly curious. Pension wealth in such funds is probably the biggest pile of non-public money ever accumulated, with such unclear rules and with so much tension building[1] (Kocken, 2006).

Although a small degree of obscurity has always been present within pension funds, this has long been concealed by the relative stability of the funds. In early years, risks were primarily borne by the employer and the contribution base of beneficiaries was solid, giving pension funds an effective risk-bearing capacity to get through bad economic weather. Hence, although the distribution within the fund was fuzzy, the indistinctness about ownership of pension wealth was not that big of a problem. Unfortunately, times have changed.

Due to the process of aging and unforeseen increased longevity pension funds have become unstable institutions. New accounting rules made the increased vulnerability for financial risks explicit. Many employers were eager to get out as risk-bearers in the pension funds, leaving behind the beneficiaries with an unclear pension contract. Somehow, the risks in the fund must be distributed among plan members. These developments have led to retrenchment of the pension benefits, but not to structural solutions

[1] Pension wealth in the Dutch second pension pillar amounted to €701 billion in the first quarter of 2010, according to DNB [23].

for the indistinctness within, and instability of pension funds.

After all, there is still no clear ownership of pension wealth within the funds, an issue which especially becomes tense in states of underfunding. This is exactly the status in which many funds find themselves in the aftermath of the financial crisis. Moreover, it is unclear which beneficiaries are responsible for bearing which specific risks in the fund. Furthermore, pension contracts are not differentiated for different types of beneficiaries. Investment strategy and steering instruments like conditional indexation are namely the same for all plan members. This makes the pension contract unsuitable with the life-cycle preferences of many beneficiaries.

The points above are however essential to run a collective pension fund in which risks are shared solely among plan participants. It is essential that the fairness and adequacy of the pension contract are ensured, so that conflicts of interest between different groups of fund beneficiaries are mitigated.

If no drastic measures are taken, it is just a matter of time before some stakeholders step out of the pension fund, when they notice that their pension contract is falling apart into a detrimental, unfair deal for them. The most extreme - but realistic - form of this unfairness holds that entire generations could be left without an old-age pension while having paid generously for it (Kocken and Potters, 2010). The exit of stakeholders could cause a downgrade of the Dutch pension fund to individual Defined Contribution (DC) arrangements, which are broadly considered to be inferior to collective pension plans. The message that changes are necessary to make Dutch pension funds sustainable for the future, is a clear one.

1.2 Research Goal

This thesis will elaborate a proposal to redesign second-pillar pension funds in which risks are borne solely by plan members. The ultimate aim is to resolve the current problems present within Dutch collective pension funds, while important strengths of these funds are maintained. More precisely, such a design gets rid of indistinct pension contracts as it will be defined explicitly how funding risks are absorbed by, and shared among beneficiaries. The redesigned pension fund is in addition fair in the sense that no ex-ante redistributions occur, but also in the sense that the reward for carrying risks is fair in terms of market value. Moreover, the new pension contracts are differentiated, so that they meet the life-cycle preferences of different plan members. Still, the contract maintains the advantages of collective pension plans. Taking these points together, the design is sustainable as the political tension between different stakeholders (and generations) is limited. The emphasis in the pension fund redesign will thus be on risk sharing among beneficiaries in a fair, explicit and hence sustainable way.

The most important task is to show how the combination of the redesign characteristics above can be implemented, worked out and analyzed in a stylized pension fund setting. Gaining intuition and grip on how 'things work' in the redesigned pension fund plays a central role. One should think of dealing with risk sharing agreements, inflow of new participants and decisions regarding risk taking by the pension fund. The elaborated stylized setting could subsequently serve as a basis for real-life pension fund redesign.

1.3 Research Approach

The elaboration of the redesign proposal consists of the following steps. Chapter 3 identifies flaws as well as strengths in the current design of pension funds, based on academic literature. The flaws are the points that need to be resolved and the strengths on the other hand, ought to be preserved for the redesigned pension fund.

Building on this analysis, Chapter 4 bundles the key features of the redesigned pension fund and works out the crux of the fund redesign. This can be summarized as follows: in the redesigned fund, pension rights are divided into multiple categories. These categories are called tranches and each tranche has its own risk profile. Plan members allocate their pension wealth over the tranches dependent on their age. Subsequently, shocks in funding level are processed via a 'waterfall construction', similar to the way shocks in Collateralized Debt Obligations (CDO) are processed. The most risky tranche, which is named 'equity', absorbs the first downward or upward shock in funding level. Here, absorption of a shock becomes explicit via a fluctuation in the value of accrued pension rights. The most secure tranche, which is called 'senior debt', only absorbs upward or downward shocks when the more risky tranches are completely wiped out or filled up. The essence of the waterfall construction is that it makes the pension contract explicit, complete and differentiated. This provides a set up in which the pension contract can be designed such that it is in addition fair for all plan members.

To arrive at a fair design, Chapters 5 and 6 elaborate a model for the stylized redesigned fund, which uses option theory. An important aspect of this model is that it serves to analyze and gain intuition about, how the redesigned pension fund works.

Sensitivity analysis with respect to the value of the pension contract is performed in Chapter 7. This analysis provides insight in the risk sharing agreements embedded in the waterfall construction. Subsequently, Chapter 8 examines how the stylized fund develops through time and what participants can expect in terms of pension payouts. Finally, Chapter 9 focuses on the inflow of new participants into the fund. Building on these analyses we expect that principles elaborated in this thesis could serve as a basis for redesign of Dutch pension funds.

Chapter 2

Preliminaries

This chapter provides information that is relevant for understanding of the material in coming chapters. The information concerns a short description of the Dutch pension system, the working of policy ladders within Dutch pension funds and a short discussion of traditional life-cycle theory. Readers already in the possession of this knowledge could proceed with Chapter 3.

2.1 The Dutch Pension System

The Dutch pension system is organized in three pension pillars. In order to clarify the positioning of this research it is useful to briefly discuss these three pension pillars. The first pillar is a public pension scheme of which the primary goal is poverty alleviation for the elderly. It is financed on a Pay-As-You-Go (PAYG) basis and it offers a basic flat-rate pension to all residents. This flat-rate pension is related to the legal minimum wage (Ponds and Van Riel, 2007). The second and the third pillar are funded systems. Together they form pension provisions complementary to the first pillar. The pension provisions in the second pillar are aimed at old-age insurance for workers, in order to maintain standard of living when old (Ponds and van Riel, 2007). Participation in this pillar is related to (former) occupation and pension plans are typically organized on a collective basis. Finally, the third pillar consists of voluntary individual savings. Savings in the third pillar are important for people who lack access to second pillar occupational pension schemes. One can think of self-employed people or employees of companies that do not adjoin occupational pension schemes. Moreover, third-pillar savings can also be used to tailor the old-age pension to individual preferences irrespective of participation in the second pillar (Bovenberg and Nijman, 2009).

This thesis analyzes the Dutch pension system purely from the perspective of second-pillar pension funds. These funds cover more than 90% of the Dutch labor force, which means that over 8.5 million people receive pension payments or are accumulating future pension claims via such funds (Ponds and van Riel, 2007; Goudswaard, 2010). For an extensive discussion of the structure and developments of the Dutch pension system see Ponds and van Riel (2007).

2.2 Policy Ladders

To create (some) distinctness regarding which plan members absorb which shocks within the fund, pension funds employ policy ladders. Absorption of shocks occurs via variations in contribution rates and via cuts in indexation of accrued pension rights. The contribution and indexation rates depend on the nominal funding ratio of the fund. A typical example of a policy ladder is depicted in Figure 2.1.

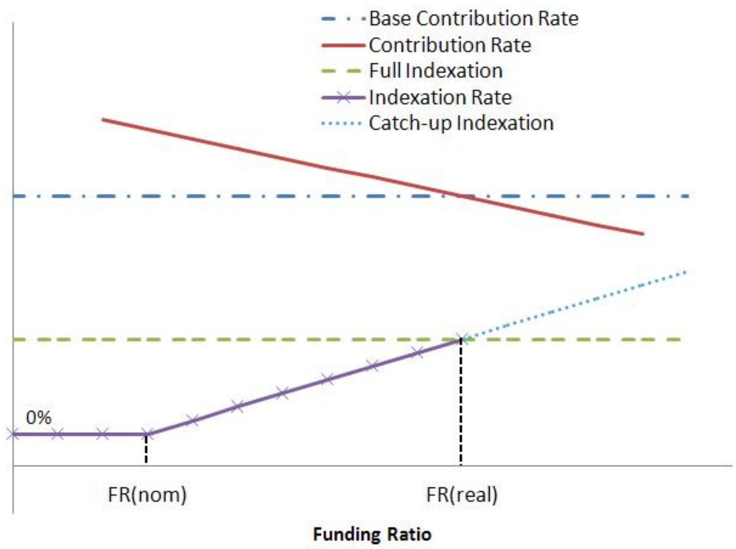

Figure 2.1: Typical Policy Ladder of Dutch Pension Funds (adjusted from Ponds and Van Riel, 2007).

The horizontal axis measures the funding ratio of the pension fund. The vertical axis measures both the extent to which indexation takes place and the contribution rate in the fund. When the funding ratio is below 100% nominal (i.e $FR(nom)$), recovery contributions are demanded and pension rights are not indexed at all. In case of higher funding ratios, the contribution rate decreases towards to base rate and indexation is partially granted. At 100% real funding ratio (i.e. $FR(real)$) full indexation is granted and the contribution rate equals the base rate. In case the funding ratio is above 100% real, the fund's board can decide to grant catch-up indexation to make up for missed indexation in the past (Bovenberg and Nijman, 2009).

Figure 2.1 makes clear that, in principle, nominal pension rights are guaranteed and indexation to inflation is contingent on the funding ratio. So, in fact one can view the system as a hybrid system of nominal guarantees and contingent real ambitions (Bovenberg and Nijman, 2009). This structure however, features indistinctness about which plan members bear shocks in case of poor funding levels. After all, the contribution rate is not specified for funding levels below 100% nominal; moreover, cuts in indexation are already maximal.

The policy ladder is an important element of the typical current pension contract. Therefore, it was useful to discuss it briefly. For a more comprehensive description of policy ladders see Bovenberg and Nijman (2009) and Ponds and van Riel (2007).

2.3 Traditional Life-Cycle Theory

Life-cycle theory engages in the optimization of individuals' utility with respect to consumption, saving and financial-risk-taking over the life cycle. According to 'traditional' life-cycle research (e.g. Bodie, Merton and Samuelson (1992), Samuelson (1969)), individuals should simply hold a constant fraction of their total wealth in risky assets. In Bodie et al. (1992), a person's total wealth is defined as the sum of financial wealth and human capital. Here, human capital stands for the present value of the stream of wages to be received over the remainder of the person's working life; it is assumed to be risk-free. Early in life, individuals only have human wealth, which is gradually transformed into financial wealth. At the moment of retirement, the individual has depleted its human wealth completely. This pattern is shown in Figure 2.2.

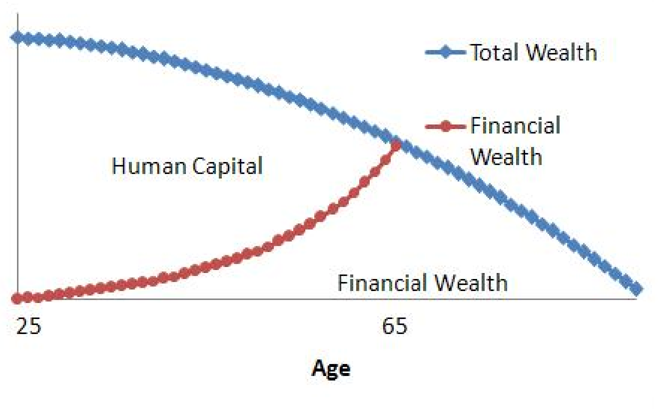

Figure 2.2: Sketch of the development of financial wealth, human wealth and total wealth over the life-cycle.

In order to arrive at a constant fraction of total wealth invested in risky assets, the fraction of financial wealth invested in risky assets is decreasing over the life-cycle. This result is even enforced by taking into account that younger individuals in addition have more flexibility left to absorb adverse shocks in financial wealth. They still have a lot of human capital and typically a long horizon to adjust their number of hours worked or their retirement date. Old people lack this ability and they should thus be exposed to less risk. A sketch of the fraction of financial wealth that should be invested in risky assets is depicted in Figure 2.3.

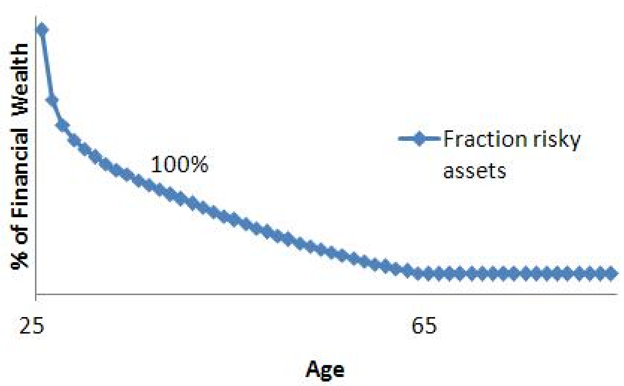

Figure 2.3: Sketch of the fraction of financial wealth invested in risky assets according to traditional life-cycle theory.

This thesis will not use these exact results, but primarily takes along the result that young workers should run more risks and older workers or retired people will prefer to take less risk in financial markets (via the pension fund). The results of traditional life-cycle theory are used as these are broadly accepted in academic literature[1].

[1] Some recent surveys put forward different results; Benzoni, Collin-Dufresne and Goldstein (2007) find that allocation to risky assets should be more 'hump-shaped' over the life cycle. The claim that in the long run labor income and capital income are highly cointegrated is crucial for this result. The pension fund redesign as elaborated in this thesis can in principle be adjusted to fit different life-cycle theories.

Chapter 3

Strengths and Flaws of Second-Pillar Pension Funds

In order to work out a pension fund redesign it is essential to analyze current features of pension funds that are causing tension between stakeholders in the funds. These are exactly the design flaws that need to be resolved to arrive at pension contracts, that are explicitly defined, fair and adequate from the perspective of all plan members. For each flaw a possible solution will be put forward based on academic literature. These solutions will be taken along in subsequent chapters to work out the redesigned pension fund.

Although Dutch collective pension funds display several flaws, they are also characterized by important strengths. These strengths are seen as rationales for collective pension funds. It is useful to discuss these strengths since it is desirable to maintain them in a pension fund redesign. Most of the strengths of Dutch second-pillar pension funds must be seen in relation to individual DC pension arrangements; this clarifies that a transition from collective pension funds to individual DC schemes would be detrimental.

First, the relevant strengths of pension funds are discussed and subsequently the flaws are treated.

3.1 Strengths of Dutch Second-Pillar Pension Funds

The strengths of Dutch collective pension funds relevant with respect to fund redesign are:

- *Pension funds complete financial markets.*
- *Pension funds are a stage for intergenerational risk sharing and certain other solidarities.*

It will appear that these two points are strongly related. Nevertheless, they need to be discussed separately.

3.1.1 Pension funds complete financial markets

One of the most important rationales for collective pension funds is that they make certain financial products available, which individuals cannot attain by themselves on financial markets, or only at excessive costs. Hence, pension funds play a role in completing financial markets. This property can be seen as a clear advantage of collective pension funds over individual DC schemes.

First of all, the market imperfection of adverse selection in longevity insurance can be overcome, thanks to mandatory participation in pension funds. This market imperfection stands in the way of full development of the market for longevity risks and in particular annuity markets (see Finkelstein and Poterba, 2004).

Second, pension funds offer (deferred) annuities - or products resembling annuities - that are linked to domestic price or wage inflation. Offering such products on a large scale is of vital importance for retirees as they form an old-age income which maintains purchasing power as long as the retiree is alive. In fact, such products can be seen as long-term options on inflation, products that are illiquid in financial markets (Kocken, 2006).

A third market incompleteness can be found in the fact that young agents face a liquidity constraint. The total wealth of young agents consists for a large part of human capital and yet only for a small part of financial wealth. Since it is not possible for the young to borrow financial capital against their human capital, they are constrained to participate in risk taking at financial markets (Beetsma and Bovenberg, 2007). However, following traditional literature on life-cycle investing (e.g Bodie et al., 1992), it would be optimal for young agents to invest large fractions of their financial wealth in risky assets. So, participation of young workers in pension fund allows them to gain (more) exposure to financial market risk[1].

Creation of these missing products is possible, because shocks in funding level are shared within and between different generations. Since these products add value for plan members, it is desirable to maintain the ‘completion of markets’ property in the pension fund redesign.

3.1.2 Intergenerational risk sharing and solidarity

Intergenerational risk sharing in pension funds - also called intergenerational solidarity - can be defined as the ability of current and future pension plan members to share shocks in asset returns and labor income via the pension fund (Hoevenaars and Ponds, 2008). Surpluses and deficits in the fund

[1] In Bovenberg and Nijman (2009) this embedded structure is described in an excellent way: “In a scheme that indexes benefits to prices and carries mismatch risk because of investments in risk-bearing assets, the active participants in effect borrow from the older, retired members by issuing price-indexed bonds to these older members and use the funds to invest in the risk-bearing assets”.

are shared among old, young, and future generations by adjusting pension contributions and/or pension benefits of specific generations. This leads to intergenerational wealth transfers. Hereby, generations smooth shocks over and even beyond the lifespan of any single generation. In other words, shocks are shared as broadly as possible.

Cui, de Jong and Ponds (2005) find that risk sharing among multiple generations is welfare improving compared to individual pension schemes. Even in funds with a deficit, the presence of well-designed intergenerational risk sharing can ex ante be welfare improving for new young entrants. Also Teulings and de Vries (2006) argue that intergenerational risk sharing is valuable within pension funds, and that this solidarity is extended in practice to risk sharing with future, non-overlapping generations. The fact that intergenerational risk sharing is welfare-enhancing is an important rationale for Dutch collective pension funds. This indicates that intergenerational risk sharing should be maintained in pension fund redesign. A reverse side however, is that risk-sharing agreements within current pension funds are complex and fuzzy. Badly structured risk-sharing agreements easily result in unwanted wealth transfers between stakeholders (Hoevenaars, Kocken and Ponds, 2009). So, careful attention must be paid to the design of this risk sharing to ensure the sustainability within pension funds[2].

3.1.3 Additional strengths

Two additional economic rationales for collective pension funds over individual DC schemes can be given. These have to be mentioned, forming additional arguments to prevent the evolution of pension fund towards in individual DC schemes.

First, pension funds operate at significantly lower 'client' expenses than commercial insurance companies[3] do. As saving for a pension is spread out over decades, these costs seriously erode the value of retirement wealth in comparison to a cheaper alternative, even if the annual cost difference is low. See Bikker and De Dreu (2009) for an extensive examination of cost aspect of pension funds versus insurance companies.

Second, pension funds help individuals to make complex financial decisions and they oblige people to save for their pension. From various surveys in the field of behavioral finance, it appears that financial illiteracy and inadequate saving behavior is widespread. See Lusardi and Mitchel (2005) or Els, van Rooij and Schuit (2006), of which the latter gives a clear overview of the importance of pension paternalism and mandatory retirement provisions in relation to behavioral aspects. Individual DC schemes taking care of behavioral failures are not yet common practice.

[2]Section 3.2 will return on this in detail.

[3]Typically, insurance companies offer individual DC-type pension products.

3.2 Flaws of Dutch Second-Pillar Pension Funds

From a broad perspective, problems of Dutch pension funds are found in the fact that current pension contracts are indistinct and unsustainable from the perspective of different fund beneficiaries. More concretely, the following aspects of pension funds and current pension contracts funds are responsible for these problems:

- *Current pension contract is curious and incomplete.*
- *Pension fund beneficiaries face unfair risk-reward trade-offs.*
- *Pension funds are in a split between nominal solvency regulations and real pension ambitions.*
- *Pension funds show a decreasing capacity for risk bearing.*
- *Current pension contract does not fit the life-cycle preferences of different groups of beneficiaries.*

3.2.1 Current pension contract is curious and incomplete

From the moment that employers (partly) stepped out of pension funds as risk bearer, there has been indistinctness regarding which of the fund's remaining stakeholders[4] are responsible for bearing which risks in the solvency level of the fund. Introduction of policy ladders has been an important step in this respect. Nevertheless, it is still unclear which participants have to absorb which shocks in cases of nominal under- or real overfunding. In these situations large negative or positive unowned buffers are borne, and tensions between the fund's participants can rise. Especially in the underfunding scenario this is the case, since there is not enough capital to fully pay out the pension rights. As it is unclear who bears which shocks in funding level, it is also unclear who should consequently be rewarded in the upside for bearing these risks. Another consequence is indistinctness about whose interest the pension fund's board should serve in terms of risk taking by the fund.

In principle, pension fund boards have the final discretionary power to decide how a particular shock is distributed among beneficiaries. This makes them very sensitive for political pressure from the fund's stakeholders (Bovenberg and Nijman, 2009). When discretionary decisions turn out to be detrimental for certain generations, it is just a matter of time until one of those generations steps out and breaks down the collective set up of pension funds.

[4]Besides current participants also future plan participants belong to the stakeholders of the fund.

A necessary step is to make pension contracts more complete by pointing out at each funding level, the stakeholders responsible for bearing risks[5]. Also property rights in the pension fund must be defined, ruling out the existence of unowned buffers. Making pension contracts more complete has several advantages as lined up by Bovenberg and Nijman (2009).

First of all, completion of contracts makes it possible to design the contract such that it is fair. Absorption of downside risks is then compensated with proportionate claims on the fund's upside in good scenarios. Moreover, complete contracts would give participants a better insight in the features of the pension contract. Risk sharing and the benefits of it are made explicit. This may lead to a better understanding of the relationship between different generations in a pension fund and thus to a broader support for sharing risks. Also, the interests of the different stakeholders become explicit[6]. In this way the pension fund board has more grip to weigh these different interests. This alleviates the political pressure on the pension fund board, as many decisions are already incorporated in the contract.

Bovenberg and Nijman (2009) also argue however, that some degree of discretion remains significant for the sustainability of pension contracts. Some contingencies just cannot be foreseen and especially for these situations it would be valuable to have flexibility in the way shocks are allocated to different stakeholders within the fund.

3.2.2 Unfair risk-reward trade-offs

The fact that risk sharing and property rights within the pension fund are vaguely defined makes it difficult to guard the fairness of the pension contract. As it is often not clear who bears certain risks within the fund, situations will arise in which participants face or have faced unfair risk-reward trade-offs: participants who turn out to absorb negative shocks would probably not have received compensation if the scenario had been prosperous.

Probably, the most well-known unfair aspect of the current pension contract is the uniform contribution and accrual system. Plan members younger than 46 pay too much for the pension rights they receive in return, while members older than 46 pay too little. In an increasingly dynamic labor market this system leads to several unfair solidarities (Boeijen, Jansen, Kortleve and Tamerus, 2006). Problems regarding unfairness within pension funds can again be solved by making the pension contract more complete and explicit, so that the contract can be designed such that it is fair.

Next, the fact that negative shocks are partly moved forward in time can lead to unfairness between generations. Due to vague agreements regarding risk sharing and the presence of fund deficits, future and young generations

[5]In line with Frijns (2010), Goudswaard (2010), Bovenberg and Nijman (2009).

[6]For example interests regarding investment strategy or implementing risk management by the pension fund.

face the risk that current generations eat too much out of the pension fund's wealth. Note that this risk is related to the current underfunding state of many pension funds. Young and future generations have incentives not to participate in pension funds as they know in advance that they are degraded.

This constraint on fund deficits forms an argument to reduce risk sharing between *non-overlapping* generations in the redesigned pension fund, in order to improve sustainability[7]. Gollier (2008) indicates that intergenerational risk sharing under realistic sustainability and solvency constraints - although less effective - is still welfare improving compared to individual pension plans. For the fund redesign in this thesis this means that risk sharing between *overlapping* generations is explicitly maintained, whereas pushing forward of buffers to *non-overlapping* generations is abolished (i.e. no 'buffer-solidarity').

3.2.3 Split between nominal solvency regulations and real pension ambitions

The aspired level of pension benefits in Dutch pension funds is defined in real terms. For example, the ambitioned yearly pension benefits correspond to 70% of one's career average salary corrected for inflation. Payout of this ambition ensures maintenance of purchasing power during retirement. Solvency requirements for pension funds, and in particular the funding ratio, are however primarily defined in nominal terms. For instance, nominal pension payouts are guaranteed to beneficiaries ('hard rights') and minimum funding ratios are set at 105% nominal funding ratio.

The short-term nominal emphasis causes misalignment with attaining the long-term ambition of paying out real pensions (Frijns, 2010 and van Ewijk, Janssen, Kortleve and Westerhout, 2009). After all, many pension funds fit out their policies to meet with the nominal requirements. This implies that pension funds aim at stabilizing the nominal funding ratio and at managing nominal risks. Regrettably, conditional real pension rights are often left out as a result. Moreover, the nominal focus can influence the development of the real funding ratio negatively (Frijns 2010).

Commission Frijns has already made recommendations to shift attention to a real framework with a well-defined real funding ratio as most important solvency measure. In that way solvency measures would be aligned with the only thing that matters for participants: disposition to a pension with preservation of purchasing power.

In addition, (nominal) pension guarantees should be released. They lead to a 'false idea of security' for beneficiaries. This security is false since the presence of inflation depreciates the value of a nominal pension considerably over time. Besides, the guarantees oblige pension funds to shy away

[7]In line with Bovenberg (2007), who also proposes certain limitation regarding intergenerational risk sharing.

from appropriate risk taking in situations of poor funding because then the guarantees are endangered (Goudswaard, 2010). In addition, the presence of guarantees makes it hard to distribute adverse shocks fairly and clearly among current and future stakeholders. After all, guarantees to current generations are simply fixed and given (Boeijen, Tamerus and Kortleve, 2010).

The tension between nominal and real is in the redesigned pension fund overcome by working completely in a real framework. Hence, this means that there will be no such steering instrument as conditional indexation[8]. In the redesigned fund, pension rights are defined purely in real terms. Also unconditional guarantees are abolished.

Finally, the fact that in the redesigned pension fund pension rights are explicitly more uncertain, may be frightening from the perspective of the participants. After all, the Dutch pension system has always been aimed at certainty. However, accepting more uncertainty and abolishing guarantees enables funds to focus on their real pension ambition (Goudswaard, 2010). Moreover, adjusting the value of pension rights is already common practice due to the conditional indexation instrument. Keeping this in mind, the change to let go of (false) security is not of high impact. The relevant difference is however that the uncertainty will become explicit.

3.2.4 Decreasing capacity for risk bearing

Despite the increased risk taking of pension funds over the last two decades, their ability to absorb these risks has decreased. The contribution base has shrunk compared to the amount of retired participants, making the contribution instrument ineffective. Active participants nowadays pay more than ever for their pensions and it seems that pension contributions have reached their maximum concerning the labor market distortions they cause (Goudswaard, 2010). Pension funds already have to rely considerably on the indexation instrument to absorb shocks in pension wealth.

Bovenberg (2007) recommends to extend the capacity to absorb risks by making more use of adjustments in the amount of pension rights depending on the solvency level of the fund. In this way, financial shocks are transmitted into ‘paper’ pension rights rather than directly into cash flows for workers via recovery contributions. Moreover, funding deficits or surpluses remain limited as the fund’s liabilities (i.e. beneficiaries’ pension rights) move together with the fund’s assets. Such an approach directly limits the scope for intergenerational risk sharing being in line with the arguments in Section 3.2.2.

[8]The split between nominal and real pension rights is also a large disadvantage of age- or return-dependent indexation as proposed by Molenaar, Munsters and Ponds (2008).

3.2.5 Misalignment with life-cycle preferences

Ideally, pension contracts connect to life-cycle preferences of plan members, as explained in Section 2.3. The current pension contract however, has uniform characteristics that are not in line with traditional life-cycle theory. For example, the indexation instrument is uniform in the sense that every participant receives the same indexation percentage. The contribution instrument is uniform in the sense that all active participants face the same increases or decreases in contribution.

These points lead to conflicts of interest between stakeholders about the amount of risk that is taken by the fund. Young plan members have the interest to increase risks, aiming at attaining their long-term real pension ambitions via relatively cheap financing. For retired plan members however, the nominal guarantees are of great importance, giving them incentives to reduce the risk taken by the pension fund to secure their nominal pension[9]. The fact that life-cycle preferences are not met and that conflicts of interest between stakeholders are present within pension funds, indicates that the uniform aspects of current contracts are not adequate.

To overcome these problems, the pension contract in the redesigned fund is differentiated for different types of beneficiaries. This is made explicit via differentiated fluctuations in pension wealth when shocks hit the fund[10]. In such differentiated contracts the pension wealth of young participants should fluctuate more strongly, and hence be less safe than the pension wealth of older participants. Pension rights are then gradually converted over the lifetime from being more risky ('soft') to being safer ('hard'). Next chapter clarifies how these ideas are embedded in the redesigned pension fund.

In short, this chapter shows that Dutch pension funds feature sizable flaws, leading to an increasing pressure on the current pension contract. This pressure can have detrimental consequences, such as the unraveling of collective pension funds to individual DC schemes. On the other hand, pension funds display several important strengths, which show that it is worthwhile to maintain the collective character of Dutch pension funds. Redesign of pension funds should therefore be searched for in the direction of a collective set up. Finally, for aspects not covered in this analysis of Dutch pension funds, see for example Goudswaard (2010) and Bovenberg (2007).

[9] Note that this corresponds again to the trade-off between short-term solvency and long-term pension ambitions.

[10] In line with ideas of Bovenberg and Nijman (2009), Goudswaard (2010).

Chapter 4

The Way Towards Redesign

This chapter introduces the set up of the redesigned pension fund. This is done according to the following three steps. First, the key features of the redesigned pension fund are put together, building on the discussion of the strengths and (solutions to the) flaws in Chapter 3. Second, a mechanism for distributing funding shocks among the plan members is introduced. This step is innovative and it forms the core of the redesign concept. The third step is to discuss the payoff profile of the redesigned pension contract resulting from this mechanism.

4.1 Key Features of the Redesigned Pension Fund

The following seven aspects form the key features of pension contracts within the redesigned pension fund:

1. **The contract is fair**, i.e. there are no ex-ante solidarities in the fund. Contributions are market-based and all stakeholders have fair risk-reward relationships with respect to downside risks and upside claims within the fund. (*fairness condition*)

2. **The contract is complete and explicit** in the sense that at every funding level there is distinctness about which member bears shocks in funding level. (*completeness condition*)

3. **Intergenerational risk sharing (IRS) is present but in a limited way**. In this way the redesigned fund maintains benefits of collective risk sharing, but mitigates the political uncertainty between generations[1]. (*IRS condition*)

[1] This implies that intergenerational risk sharing between overlapping generations is maintained (i.e. risk sharing between old and young), but risk sharing with non-overlapping generations is abolished. This implies that current generations do not move any buffers forward in time to future generations.

4. **The pension contract is defined in real terms** instead of being defined via a combination of nominal and real terms. (*real terms condition*)

5. **The pension contract is differentiated** corresponding to the life-cycle preferences of different participants. (*life-cycle condition*)

6. Shocks in funding level are absorbed via **fluctuations in the value of participants' pension rights**. Pension rights thus breathe along with the funding level of the pension fund. (*shock absorption condition*)

7. **The Pension Ambition is explicitly defined**. (*explicit ambition condition*)

Next section shows the set up of the redesigned pension fund. Here, it will become clear how the redesign features points are reflected in the fund set up.

4.2 From Redesign principles to Pension Fund Set Up

The crux of the pension redesign is that pension rights within the fund are split up into multiple risk levels. The risk levels can be seen as multiple layers of pension rights. We call these layers 'tranches', where the most risky tranche is called 'equity' and the most secure one is called 'senior debt'. The risk tranches play a central role in the way shocks at fund level are distributed within the pension fund. In principle, the fund could have more than two tranches[2]. However, from Section 4.4 onwards, the number of tranches is limited to two in order to keep the elaboration of the redesigned pension fund simple.

Shocks in pension wealth are distributed among the different tranches according to a 'waterfall construction'. This construction is contingent on the funding level. This contingency is defined for all possible values of the funding level, so that the *completeness condition* is satisfied. The waterfall mechanism is equivalent to the way shocks in funding level are processed within a Collateralized Debt Obligation (CDO)[3]: the equity tranche is responsible for the absorption of the first losses on the underlying portfolio. In pension fund context, the first loss is defined as the first shock downwards from a 100% real funding level. When in a negative scenario downside shocks have wiped out the equity tranche completely (i.e. value of pension rights

[2]When only one tranche is present in the pension fund, shocks are simply distributed uniformly among plan members

[3]CDO's are a type of asset-backed securities. The value of the CDO depends on the value of a portfolio of underlying debt assets.

in the equity tranche has reduced to zero), the tranche below equity will bear additional downward shocks in funding ratio. Finally, when all the above tranches are wiped out, the value of senior pension rights is exposed to shocks in funding level (cf. *shock absorption condition*). So, shocks in funding level are attributed to tranches in order of seniority.

Similarly for upside scenarios, the equity tranche absorbs the first positive shocks until equity has filled up to a specified threshold. Additional positive shocks are subsequently absorbed by less risky tranches. The specified upside thresholds have to be determined such that the value of upside claims on the fund is in line with the downside risks that each tranche faces (cf. *fairness condition*).

By use of the waterfall construction, more risky tranches (i.e. more risky pension rights) form a shield for the more secure tranches (i.e. more secure pension rights). Hence, the value of pension rights in senior debt is stable. The probability that the full amount of senior pension rights will be paid out is large. Pension rights in the equity tranche are on the other hand more sensitive as well for downside as for upside shocks.

A key aspect is that participants allocate their pension contributions over different tranches. This allocation is age-specific so that life-cycle preferences of different participants can be met (cf. *life-cycle condition*). Older members allocate a large part of their pension wealth to senior pension rights, whereas younger members have a larger part of their pension wealth in equity claims. Middle-aged workers apply an intermediate allocation between senior and equity. The exact allocations have to be determined by the pension fund. By using age-specific allocations, pension contracts are differentiated and can be tailor-made. Over the life-cycle more risky pension rights can then be converted into more secure pension rights.

Balance Sheet Example:
Before the fund design is developed further on, a stylized example is provided. This example makes intuitively clear how the waterfall construction works in case the redesigned pension fund suffers from a shock in funding level. Next to the redesigned CDC[4] pension fund, a CDC pension fund without risk sharing is analyzed. This basic fund design serves as a benchmark; it's structure will be discussed soon.

Both pension designs are analyzed on a fixed horizon (T years). The funds are closed, so that inflow of new participants is not an issue. Moreover, in both pension funds the pension ambition of each participant is explicitly defined (cf. *explicit ambition condition*). In order to keep the explanation of the waterfall construction simple, we abstract from (real) interest rates in this example. In this way, the level of pension contributions corresponds to the level of pension ambition at time T.

[4] Collective Defined Contribution.

The pension designs need to be analyzed on participant level. Therefore, three groups of participants are distinguished within both funds. These groups are Retirees, Old Actives and Young Actives. For each group the fund keeps up the accrued pension rights over fund horizon, so that ownership is explicitly defined (cf. Teuling and de Vries, 2006). Retirees have contributed 6000, Old Actives 2500 and Young Actives 1500, making the total pension wealth in the fund equal to 10,000.

Enough information is now available to display the initial balance sheet of the basic CDC pension fund, which is done in Table 4.1. In the basic CDC fund design each participant invests his money according to a collective investment strategy; shocks in funding level are distributed proportionally to each participant. Note that at $t = 0$ the fund is fully funded. Since all risks in this fund are borne by the participant, the liabilities of the fund will at all times be equal to the value of the fund's assets. Hence, shocks are fully absorbed by fluctuations in pension rights and as a result the (real) funding ratio will always equal 100%.

Balance Sheet Basic CDC Fund (time 0)

Assets		**Liabilities**	
Pension Wealth	10000	Pension Rights	
		Retirees	6000
		Old Actives	2500
		Young Actives	1500
Total	10000	Total	10000

Table 4.1: Balance sheet at $t = 0$ for the basic CDC fund.

To set up the initial balance sheet of the redesigned pension fund, the allocation of each group's contributions over the tranches is needed in addition. In this example the redesigned fund is set up with three tranches: Senior Debt, Equity and an intermediate tranche, called Junior Debt. The use of three tranches clarifies also at once the structure of the pension fund in case more than two tranches are used. The allocation for each group of participants over the three tranches is given in Table 4.2. We choose these

(%)	**Senior Debt**	**Junior Debt**	**Equity**
Retirees	90	10	0
Old Actives	70	20	10
Young Actives	50	30	20

Table 4.2: Members' allocation of pension rights over the tranches (%).

particular allocations as they are approximately in line with the life-cycle preferences of the different groups in the fund[5]. Also the redesigned fund starts from a fully funded situation. These allocations lead to the initial balance sheet of the redesigned pension fund as displayed in Table 4.3.

Balance Sheet Redesigned CDC Fund (time 0)

Assets		**Liabilities**	
Pension Wealth	10000	Senior Debt:	
		Retirees	5400
		Old Actives	1750
		Young Actives	750
		Total	7900
		Junior Debt:	
		Retirees	600
		Old Actives	500
		Young Actives	450
		Total	1550
		Equity:	
		Retirees	0
		Old Actives	250
		Young Actives	300
		Total	550
Total	10000	**Total**	10000

Table 4.3: Balance sheet at $t = 0$ for the redesigned CDC fund with risk tranches.

Now consider the scenario in which the fund's assets have made a (real) return of -15% over the horizon of T years. This corresponds to a shock of -1500 in absolute terms (cf. *real terms condition*). This -15% real shock is absorbed by adjustment in the value of pension rights. This gives rise to the following balance sheets for the different types of pension funds at time T:

[5]In a going-concern fund, these allocations could be obtained via utility optimization.

Balance Sheet Basic CDC Fund (time T)

Assets		**Liabilities**	
Pension Wealth	8500	Pension Rights	
		Retirees	5100
		Old Actives	2125
		Young Actives	1275
Total	8500	Total	8500

Table 4.4: Balance sheet at time T for the basic CDC fund (after the shock).

Balance Sheet Basic CDC Fund (time T)

Assets		**Liabilities**		
Pension Wealth	8500	Senior Debt:	(-0)	
		Retirees		5400
		Old Actives		1750
		Young Actives		750
		Total		7900
		Junior Debt:	(-950)	
		Retirees		232
		Old Actives		194
		Young Actives		174
		Total		600
		Equity:	(-550)	
		Retirees		0
		Old Actives		0
		Young Actives		0
		Total		0
Total	8500	**Total**		8500

Table 4.5: Balance sheet at time T for the redesigned CDC fund with tranches (after the shock).

For the basic CDC fund, one can see that shocks are distributed proportionally over the participants. Hence, all participants have suffered equally from the decline in funding level (namely -15%). For the redesigned pension fund however, the shock is distributed over the tranches according to the waterfall. The equity tranche absorbs the first 550 of the shock and as a consequence it is totally wiped out. The additional losses of 950 are borne by the junior tranche. Note that within each tranche, shocks are borne proportionally, i.e. the pension rights in a particular tranche make for each group the same return[6]. The senior pension rights have been fully sheltered by the equity and junior tranche. However, if also the junior tranche was depreciated due to the shock in funding level, also the senior tranche would have been affected. The result of this fund shock for the different (groups of) participants in the redesigned fund can be seen below:

	Fund	**Retirees**	**Old Actives**	**Young Actives**
Shock	-1500	-368	-556	-576
Return (%)	-15	-6.1	-22.2	-38.4

Table 4.6: Shock in pension rights for each group of participants due to the shock in funding level.

From Table 4.6 one can clearly see that different allocations over the tranches causes the shock to be differentiated between the plan members. So, the shock is shared among the different groups (i.e. generations) of participants[7] (cf. *IRS condition*). In the basic CDC fund on the other hand, every participant faces a return of -15% due to the shock in funding level.

For the opposite economic scenario, i.e. a shock of + 15%, the distribution of shocks could be designed in a similar way. If one would exactly stick to the waterfall construction, the equity tranche fills up to a specified limit, and the remainder of the shock is allocated to junior debt. When junior debt also fills up to its maximum, further positive shocks are attributed to the senior tranche. However, also other designs for upside sharing can be used that still resemble the waterfall construction. This will also be done in this redesign proposal: in the redesigned pension fund, the equity tranche will fully absorb the first positive shock up to a specified threshold. Additional positive shocks will again be absorbed proportionally by the tranches. This specified threshold should be determined such that the risk-return relationship is fair in each tranche. This is treated extensively in upcoming chapters.

[6]Pension rights in the equity tranche for each group make a return of -100%, pension rights in the junior tranche for each group make a return of -61% and pension rights in senior tranche make a return of 0% for each group of plan members.

[7]The resulting shocks also depend on the size and wealth allocations of the separate groups of plan members within the fund.

4.3 Core Function of the Waterfall

The core function of the waterfall construction is, that *it makes (intergenerational) risk within a collective pension fund explicit, complete, fair and sustainable*. This is a vital improvement over the current implicit design of pension contracts, which is marked by indistinctness. Moreover, a well-defined waterfall construction solves several flaws of the current pension design at one stroke.

To see how the waterfall maintains the strengths of current pension design, more attention is paid to the equivalence to shock absorption within Collateralized Debt Obligations. CDO's were originally designed to create more bond-like products with triple A credit rating from a large portfolio of debt obligations with lower credit ratings[8]. Risky tranches within CDO's on the other hand, are interesting for investors who are willing to run great risks in exchange for the outlook of considerable returns.

The use of tranches within the pension fund context however, serves to create senior pension rights from the fund's collective asset portfolio. Senior pension rights refer to annuities linked to domestic wage inflation, i.e. products not available at financial markets. Stemming from traditional life-cycle theory, young participants on the other hand prefer to run more risk in their pension contract, so that this group is suited to have allocations weighted towards more risky tranches[9]. These points make explicit the completing markets property of collective pension funds as elaborated in Section 3.1.1. Remember that these are exactly the points where the redesigned pension fund adds value in comparison to individual DC.

Finally, note that although the waterfall construction is equivalent to waterfall constructions used in CDO's, the developed pension contract in this thesis must definitely *not* be confused with other product features of CDO's or with the negative connotation these product still have after the financial crisis.

4.4 From Pension Fund Set up to Payoff Profile

Now that the contours of the waterfall construction are introduced, next step is to make pension contracts in the redesigned fund explicit via a payoff profile that clarifies which tranche bears what shocks at what funding level.

The payoff profile of the pension contract is defined at time T. Here, time T stands for the time of payout of pension benefits, or for the moment in time in which pension rights are transferred from one tranche to another. Next chapter discusses the horizon of the pension contract more detailed.

[8]I.e. similarly to the completing-markets property as explained in Section 3.1.1, CDO's in principle help to complete financial markets by creating more triple A rated bonds.

[9]Remember that in this way young agents overcome borrowing constraints as elaborated in Section 3.1.1.

The payoff of the pension contract is defined via the waterfall construction. Hence, the payoff depends on the funding level, i.e. on the extent to which the pension fund can meet its pension ambitions. In the balance sheet example however, we saw that in case of a shock, the value of pension rights is adjusted in such a way that the (real) funding ratio equals 100% at all times. Therefore, the funding ratio does not reflect anymore the extent to which the future pension ambitions can be met. Hence, it is necessary to introduce a new solvency measure which does reflect this. We call the new solvency measure the 'Ambition Ratio'. This measure is similar to the funding ratio in a traditional DB scheme, with unconditional pension liabilities. The Ambition Ratio can be obtained via the funding ratio by replacing the market value of the liabilities in the funding ratio by the market value of the pension ambition (in real terms).
The market value of the pension ambition at a particular time $t < T$ is for each participant k defined as:

$$RPA_{k,t} = \frac{NPA_{k,t_{0k}} \prod_{j=1}^{t}(1+i_j)}{(1+ryc_{T-t,t})^{T-t}}.$$

Here, $NPA_{k,t0k}$ represents the nominal pension ambition as made on the time t_{0k} of participant k's pension contribution. Next, i_j stands for the price inflation in year j and $ryc_{T-t,t}$ denotes the $(T-t)$ years real interest rate at time t. The formula can be explained as follows: in order to know the (real) market value of the nominal pension ambition at $t = 0$, $NPA_{k,t0k}$ needs to be discounted via real interest rates. However, in order to know its value at a time $t > 0$, the attributed inflation over past years should be taken along, which is done in the nominator. The pension ambitions for the fund as a whole are given by summing over the participants:

$$RPA_t = \sum_{k=1}^{n} RPA_{k,t}.$$

Subsequently, the Ambition Ratio is defined by:

$$AR_t = \frac{A_t}{RPA_t}, \tag{4.1}$$

where A_t denotes the value of the fund's assets at time t. So, the Ambition Ratio at time t reflects to what extent the pension fund is able to meet its current and future pension ambitions at a particular time t. When in the remainder we make reference to the funding level of the pension fund, we mean the Ambition Ratio.

4.4.1 Payoff Profile of the Pension Contract

Based on the waterfall construction, the shape of the payoff from the pension contract in the different tranches is given by Figure 4.1, dependent on the

Ambition Ratio. Here, the pension fund is set up with just *two* tranches. As announced this will also be the set up for the remainder of this thesis. The payoff profile is expressed as percentage of the Real Pension Ambition at time T, depending on the Ambition Ratio AR_T. The 45 degrees line represents the payoff in the basic CDC fund; this line serves as a benchmark for the payoff of the contracts in the redesigned fund.

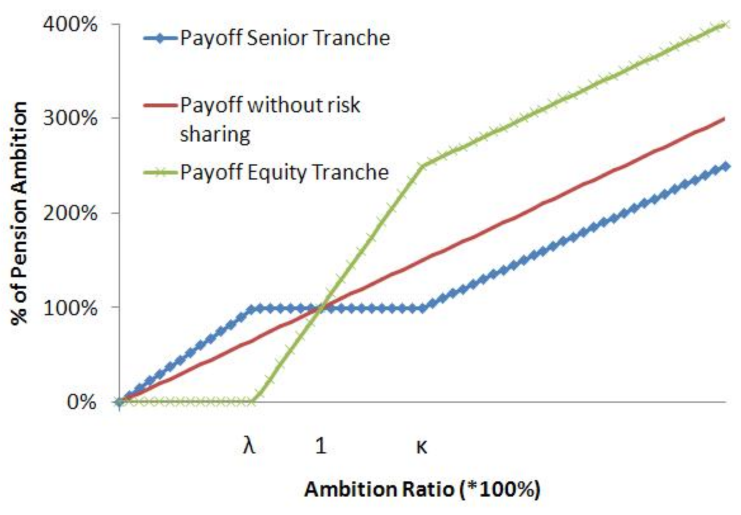

Figure 4.1: Payoff profile for the senior and equity tranche contingent on the Ambition Ratio.

In the payoff profile of the senior tranche one can clearly see that the senior tranche is sheltered from shocks for Ambition Ratios around 100%. In this area still 100% of the real pension ambition is paid out. Logically, the payoff profile for the equity tranche runs steeply in this area as the equity tranche absorbs all shocks in funding level. So, in fact the equity tranche is highly leveraged[10]. Below an Ambition Ratio of λ, the equity tranche is wiped out. Therefore, its payoff equals zero. In that case, shocks in funding level have to be borne by the senior tranche as can be seen from the senior payoff profile. Finally, in the extreme case of $AR = 0$, also the senior tranche would be fully depleted. So, the slope of the senior payoff is given by λ^{-1} on the interval of $AR = [0, \lambda]$.

The funding level κ denotes the threshold at which the senior tranche starts in upside sharing. For Ambition Ratios larger than κ, shocks in funding level are shared again proportionally among the senior and the equity tranche. This explains why both the payoff profile of the equity and the senior tranche, run parallel to the payoff profile of the Ambition Ratio.

[10] We will return on the amount of this leverage later on.

The payoff profile for a random plan participant will however look different from Figure 4.1, since each participant allocates his pension wealth over the tranches. Therefore, Appendix B.1 provides the payoff profile of the pension contract for a person who has allocated his wealth over the two tranches. More details concerning the payoff profiles in the pension fund are discussed in chapter.

The thresholds λ and κ, are very important for the design of the waterfall. They determine when each tranche bears shocks in funding level. These thresholds have to be set such that the downside risks borne within each tranche, are fair with respect to the upside claims each tranche receives on the funding level. After all, the pension fund redesign has to be fair from the perspective of all plan members. Therefore, next chapter continues with the analysis of the payoff profile. Subsequently, a model for the redesigned pension fund is set up, so that the fair thresholds in the waterfall construction can be determined.

Chapter 5

Modeling the Fund in a Closed Setting

To determine the fair values of the thresholds within the waterfall construction, this chapter elaborates a model for the redesigned pension fund. First, model settings are discussed in order to set up a stylized pension fund. Second, it is shown that the payoff profile of the redesigned pension contract can be replicated with the use of options and a stake in the Ambition Ratio. This allows us to apply option theory to determine the fair thresholds within the waterfall construction. Therefore, the model settings for the stylized pension fund are extended. The resulting model enables us to analyze the redesigned pension fund formally.

5.1 Model Settings (1)

The pension fund is set up at time $t = 0$ and will be analyzed on a fixed horizon of T=10 years. This horizon is chosen as it corresponds to the average time to payout of senior pension rights. It can moreover be interpreted as the average time before pension rights in equity are converted into more secure pension rights. This setting abstracts from the presence of an annuity stream for retirees. However, the slightly different outcomes resulting from this assumption do not change the conclusions regarding the analysis of the pension fund, in line with Kocken (2006).

Moreover, the fund is analyzed in a closed setting, which means that there is no inflow of new pension rights or meanwhile exit from the fund. Such a setting is appropriate to explain the basic risk sharing between the tranches embedded in the waterfall construction. Besides, a closed setting is very much linked to the current setting of maturing pension fund (Kocken, 2006).

Since participants' pension contributions are spread out in various ways over the tranches, remaining chapters will analyze the pension contract primarily from the perspective of the tranches, i.e. from the senior tranche or from the equity tranche. This way of looking to the pension contract is convenient, since the waterfall construction applies to the tranches. Subsequently, it is not a big step to show what the pension contract looks like for a combination of stakes in the Senior and Equity tranche.

So, the pension fund is set up by participants making a one-off contribution to the fund at time $t = 0$. Subsequently, the fund is closed for new participants or contributions. Analysis runs until time $T = 10$ on which pensions are paid out[1].

5.2 Decomposing the Pension Contract

The payoff profiles of the Senior and Equity Tranche as presented in Figure 4.1, can be decomposed into what we call a 'Uniform Stake' and a portfolio of European option contracts, written on the Ambition Ratio of the fund. These will be explained successively in the following paragraphs.

The Uniform Stake
The Uniform Stake US_t represents the stake that every plan member has in the Ambition Ratio. It simply refers to the level of pension rights[2] that a participant has in a basic CDC pension fund without any form of risk sharing. The payoff from the Uniform Stake for a plan member k depends on a member's real pension ambition and the fund's Ambition Ratio. It is given by:

$$US_{k,T} = AR_T \cdot RPA_{k,T}.$$

This payoff profile is also shown in Figure 5.1. Here, pension rights are expressed as percentage of the real pension ambition at time T, as defined in formula (4.1). The Uniform Stake serves as the basis of the pension contract; to arrive at the payoffs of the different tranches, options on the Ambition Ratio are added to the Uniform Stake.

[1] For participants in the equity tranche pensions are not factually paid out, but the resulting capital after this period serves as pension capital at that moment in time. This capital can be used to continue saving for retirement, e.g. by reinvesting it in more secure tranches within the pension fund.

[2] Pension rights expressed as percentage of the market value of the fully ambitioned pension rights.

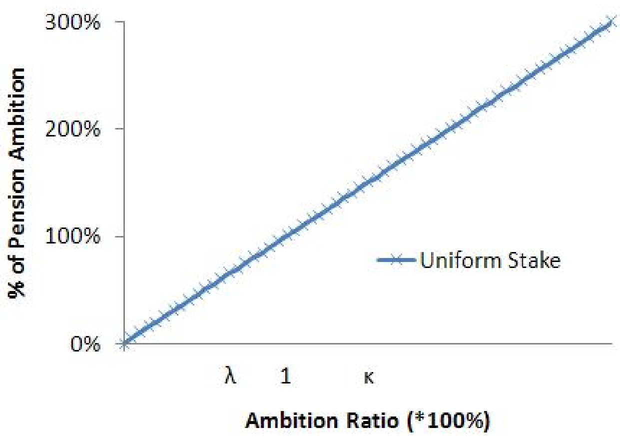

Figure 5.1: Payoff profile from the Uniform Stake as percentage of Ambitioned Pension Benefits.

Portfolio of Options

For replication of the payoff profiles of the senior and equity tranche, we use four European option contracts. These are written on the Ambition Ratio of the fund. The two tranches trade these options with each other, which means that options bought by the senior tranche are written by the equity tranche and vice versa. The options make explicit the risk sharing between plan members in the fund. In the coming paragraphs the four options will be looked at from the perspective of the senior tranche. This is convenient as the size of this tranche determines the specifications of these option contracts. The option payoffs for the equity tranche are obtained by multiplying the stated payoff by -1.

1) The Guarantee Put (GP)

This put option, which is long for senior, ensures that when the Ambition Ratio is below 100%, the senior tranche has a claim on the pension fund equal to 100% of the senior pension ambition. The strike level of the option lies at $AR_T = 100\%$. The intrinsic value of the option is given in the Figure 5.2.

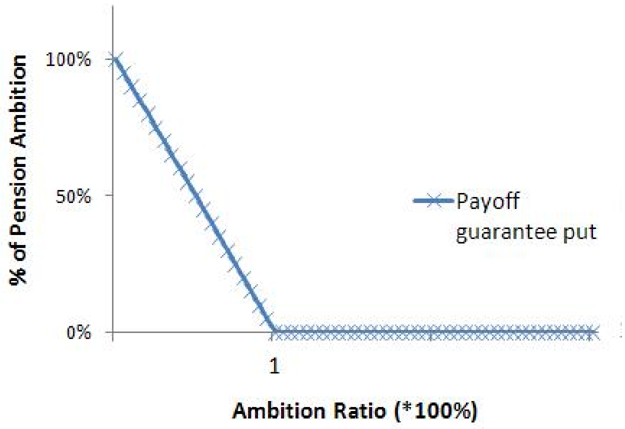

Figure 5.2: Payoff from the Guarantee Put as percentage of the Senior Pension Ambition.

The payoff of the Guarantee Put can be given by[3]:

$$GP_{Sen,T} = RPA_{Sen,T} \cdot \max(1 - AR_T, 0).$$

Finally, note that this option is written by the equity tranche, so that the equity tranche has to meet this financial obligation in case of low funding levels.

2) The Guarantee Call (GC)

The Guarantee Call is a short call option, written by senior debt to equity. This option ensures that the upside of the funding level (i.e. of the Uniform Stake) is transferred to the equity tranche. Hence, senior debt gives away the upside of its uniform stake via this option. The strike of the Guarantee Call is set at $AR_T = 100\%$ and its payoff function is given by:

$$GC_{Sen,T} = RPA_{Sen,T} \cdot \min(1 - AR_T, 0).$$

This payoff profile is in addition displayed in Figure 5.3.

[3]The subscript Sen denotes the senior tranche. Subscript Eq refers to the equity tranche.

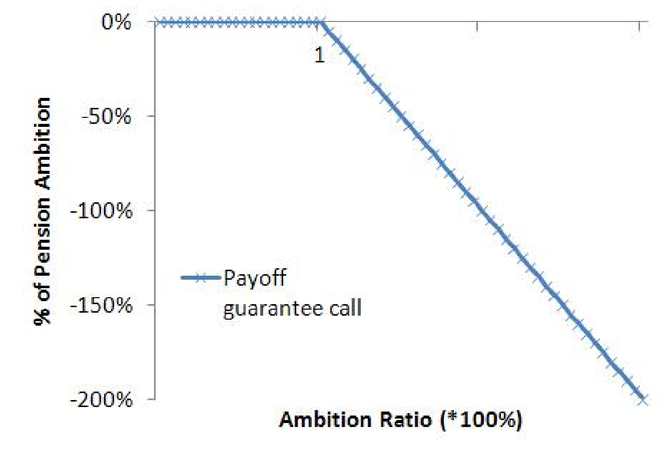

Figure 5.3: Payoff from the Guarantee Call as percentage of the Senior Pension Ambition.

So far, the Uniform Stake combined with the Guarantee Put and Guarantee Call ensures implies that for every value of the Ambition Ratio the senior tranche has a claim on 100% of its real pension ambition. This would simply be a guarantee for the senior tranche. This is however not feasible from the perspective of the equity tranche. After all, the assumption is made that equity would bear only the first downside shock in funding level until it has completely run down at a particular Ambition Ratio. If the Ambition Ratio experiences a further decline, senior debt would also have to be depreciated. One could say that the equity tranche 'defaults' on the Guarantee Put written to senior. The next option represents this and therefore we call it the 'Default Put'[4].

3) The Default Put (DP)
The intrinsic value of the default put can be found in Figure 5.4.

[4]So, this is literally a 'default put'; Note that no actual cashflows take place as a result of the default put between the senior tranche and equity tranche. The point is that cash flows actually *had to be present* according to the guarantee put, but do not occur due to the wipe out of the equity tranche. This can hence be seen as an implicit payoff for the equity tranche and that is why the Default Put is a long position for the equity tranche and a short position for senior debt.

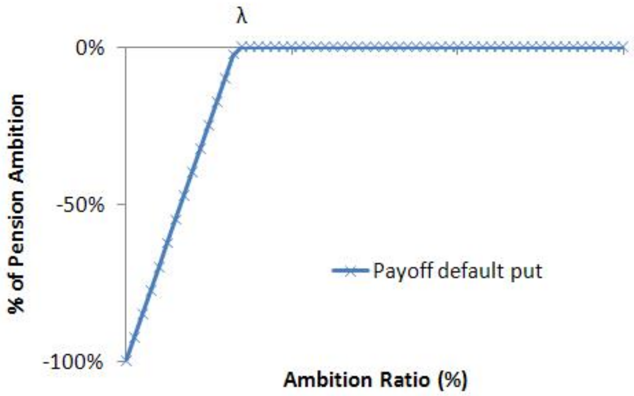

Figure 5.4: Payoff from the Default Put as percentage of the senior Pension Ambition.

The senior tranche has written this option to the equity tranche. The corresponding payoff function is:

$$DP_{Sen,T} = \frac{1}{\lambda} \cdot RPA_{Sen,T} \cdot \min(AR_T - \lambda, 0),$$

where λ is defined by:

$$\lambda \equiv \frac{RPA_{Sen,T}}{RPA_{Sen,T} + RPA_{Eq,T}}.$$

λ stands for the ratio between the amount senior pension ambitions and the total amount of pension ambitions in the fund. So, one could say that λ represents the proportion of senior pension rights to equity pension rights in the closed pension fund. Therefore, we say that λ refers to 'seniority of the fund'. Assuming that older 'senior' participants allocate a larger part of their wealth to the senior tranche and younger participants more to equity, the term seniority could also be linked to the maturity of the pension fund[5].

If the Ambition Ratio falls below λ, the equity tranche cannot meet the obligation to pay out the Guarantee Put anymore. The term $\frac{1}{\lambda}$ in (5.2) ensures that the payoff from the Default Put exactly offsets the amount of money on which the Equity tranche defaults. As a consequence, the senior tranche will also be depleted completely at $AR_T = 0$.

[5]Note that this is not necessarily the case as senior and equity pension rights can also be used, merely to discriminate explicitly between 'hard' and 'soft' pension rights. Soft, equity pension rights then serve as an explicit buffer for the hard, senior pension rights. Distinction between hard and soft pension rights is in line with Goudswaard (2010) and Bovenberg and Nijman (2009).

4) The Upside Call (UC)
Replication of the payoff profile in the upside of the Ambition Ratio is accomplished via the 'Upside Call'. This is a call option on the Ambition Ratio, written by equity (i.e. long position for senior). Its intrinsic value is denoted in Figure 5.5.

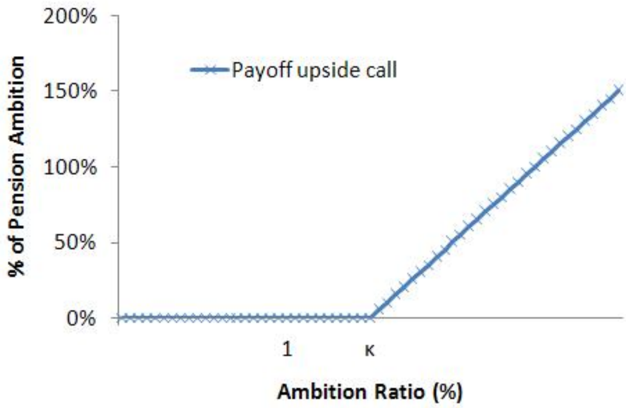

Figure 5.5: Payoff from the Upside Call as percentage of the Senior Pension Ambition.

The payoff function of the Upside Call is given by:

$$UC_{Sen,T} = RPA_{Sen,T} \cdot \max(AR_T - \kappa, 0).$$

The Upside Call is struck at κ, which is the upper threshold of the waterfall construction. From this point onwards the senior tranche has a claim on the upside of the funding level. Whereas the strike level λ of the Default Put follows from the size of the tranches in the fund, the strike level of the Upside Call has to be determined such that the waterfall construction is fair from the perspective of each tranche. As we will see in next chapter, κ can indeed be determined such that the construction of the four options has zero initial costs, when the Ambition Ratio equals 100%.

The Total Picture
Summing the payoffs from the four options and from the Uniform Stake leads to the following graphs for respectively the senior tranche and the equity tranche.

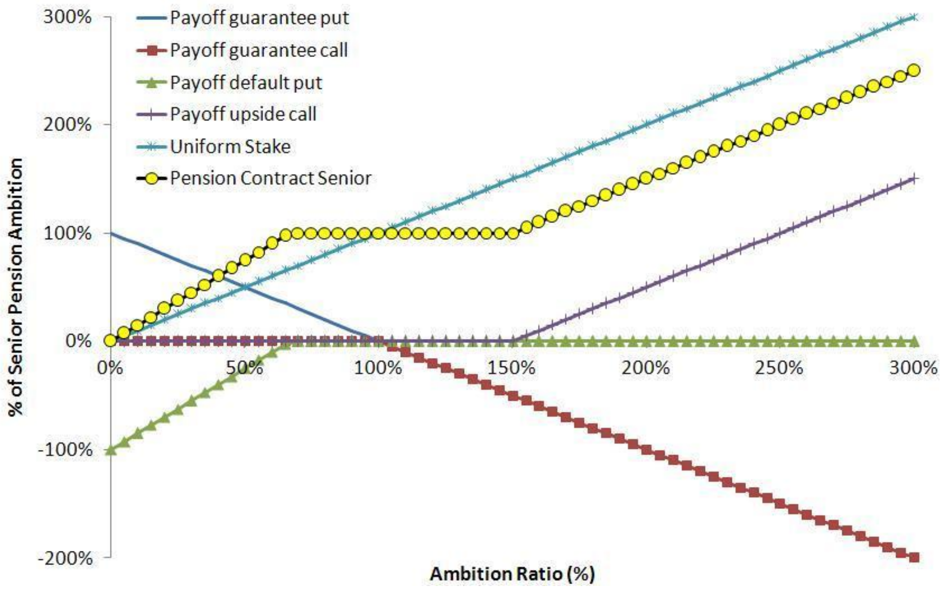

Figure 5.6: Payoff from the senior tranche as percentage of the Senior Pension Ambition. λ is assumed to equal 66.7%.

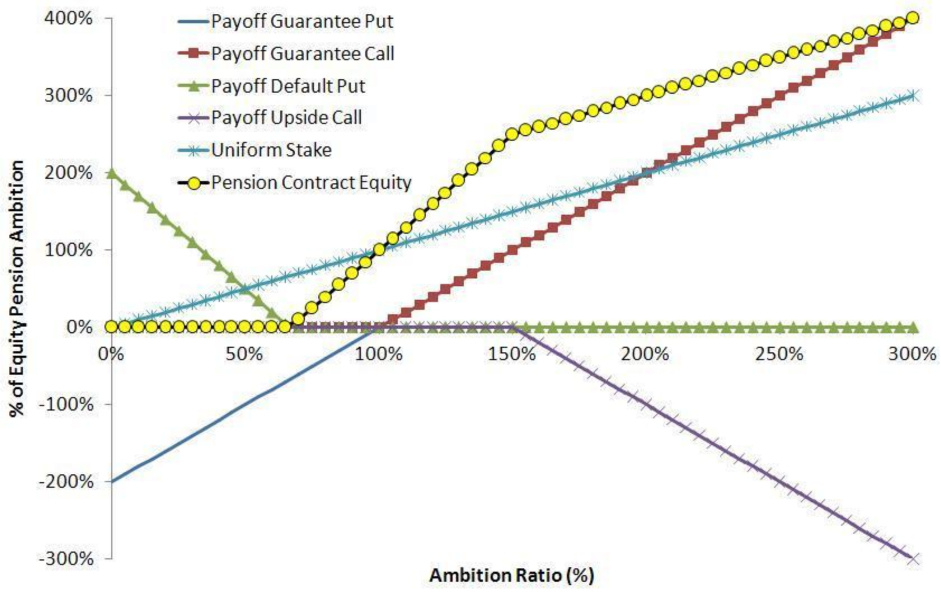

Figure 5.7: Payoff from the equity tranche as percentage of the equity Pension Ambition. λ is assumed to equal 66.7%.

In this pension fund, λ is assumed to equal 66.7%, which implies that the full pension ambition of the senior tranche is twice the pension ambition of the equity tranche. For both figures it holds that the graph of the pension contract equals the sum of the remaining graphs being the Uniform Stake plus the option payoffs. The shapes of the 'pension contract' graphs indeed equal the payoff profiles in Figure 4.1. Note that at $AR_T = 100\%$ the net payoff from the option construction equals 0, as the payoff from the pension contract equals the payoff from the Uniform Stake. However, in order to really have this profile, the strike κ of the Upside Call needs to be determined such that the net value of the options indeed equals zero at $AR = 100\%$.

So, the payoff from the senior pension contract $PC_{Sen,T}$ is given by:

$$\begin{aligned} PC_{Sen,T} &= US_{Sen,T} + GP_T + GC_T + DP_T + UC_T \\ &= US_{Sen,T} + OC_T, \end{aligned}$$

where $OC_{Sen,T}$ stands for the payoff from the option construction. For the equity tranche the payoff is given by:

$$\begin{aligned} PC_{Eq,T} &= US_{Eq,T} - GP_T - GC_T - DP_T - UC_T \\ &= US_{Eq,T} - OC_T. \end{aligned}$$

5.3 Model Settings (2)

In order to fairly design the pension contract, we set up a model that values the options within the fund. This model must be capable of clarifying the structure of the pension fund. Therefore we model the fund in a Black-Scholes (BS) setting, in such a way the crux of a pension fund is reflected, while maintaining simplicity and intuition about how things work within the redesigned fund. This setting is explained below.

An important aspect of modeling a pension fund is that pension funds take a certain amount of mismatch risk[6]. Here, the assumption is made that for the stylized pension fund, risks come purely from its investments. The fund's pension ambitions are assumed to be deterministic via constant real interest rates and inflation. The (mismatch) risk in the fund becomes explicit in a stochastic Ambition Ratio. The development of the Ambition Ratio is modeled via a Geometric Brownian Motion. Under the real-world probability measure $\mathbb{P}$ its dynamics are given by:

$$dAR_t = \mu AR_t dt + \sigma AR_t dW_t. \tag{5.1}$$

Here, W_t denotes a standard Wiener process under the probability measure $\mathbb{P}$. The parameter μ stands for the drift and σ for the (relative) volatility

[6]This risk arises because pension liabilities cannot be exactly matched due to the fact that certain risks cannot be exactly hedged. Moreover, pension funds often take a certain amount of risk via risky investments in order to reduce long-term expected contribution rates (Kocken, 2006).

of the Ambition Ratio. Both parameters are assumed to be constant. In principle, μ and σ follow from the investment strategy of the fund. Modeling the Ambition Ratio in this way implies that the Ambition Ratio is exogenous (via the investments of the pension fund). I.e. the fund's Ambition Ratio is not steered via e.g. recovery premiums[7]. From a solvency perspective, such steering is unnecessary, since the funding ratio always equals 100% thanks to the adjustments in pension rights. More important, exogenous Ambition Ratios allow to make a distinction between changes in option values due to deliberate actions, like changes in investment strategy or to external circumstances like movement of financial markets (Kocken, 2006). The assumption of exogenous Ambition Ratios is in addition appropriate, as the emphasis lies on the fund design and less on the underlying dynamics.

Besides the Ambition Ratio, a risk-free bond is present in the economy. The bond B_t has the following dynamics:

$$dB_t = rB_t dt, \tag{5.2}$$

where r stands for the constant risk-free interest rate.

Equations (5.1) and (5.2) define the standard Black-Scholes (BS) setting along with the assumptions of complete markets, absence of transaction costs, continuous trading, perfectly divisible securities, absence of dividends, and absence of arbitrage possibilities within the economy. See Hull (2006) for a discussion of the nature of these assumptions. In this context, the tranches within the pension fund can be seen as economic agents in an (internal) economy in which two assets are available; the fund's stochastic Ambition Ratio, and a risk free bond.

As a change to the standard settings, the assumption is made that the Ambition Ratio (as an asset) has zero cost of carry[8]. This is a natural assumption for the Ambition Ratio as a plan member does not face opportunity costs by having a stake in the Ambition Ratio. The assumption of zero carrying costs is realized by assuming that the Ambition Ratio pays a continuous yield q equal to the interest rate r in the economy (Carr, Ellis and Gupta 1998). This makes the model setting equal to the Black-Scholes setting for valuing options on a stock paying a continuous dividend yield q as elaborated by Merton (1973)[9].

[7]Compare Blake (1998) in which the funding level of the pension fund is steered only via the optimal liability matching asset portfolio, and not by meanwhile adjusting contributions.

[8]The 'cost of carry' measures possible storage costs on a particular asset plus the interest that is paid to finance the asset minus the income earned on it (separately from changes in the price of the asset). See Hull (2006).

[9]In this setting, dividend can be defined as the "reduction of the stock price on the ex-dividend date arising from any dividends declared" (Hull, 2006).

Chapter 6

Option Valuation and Fair Design

Using the model introduced in Chapter 5, the next step towards a fair design is calculating the value for the upside threshold in the waterfall construction of the pension fund, such that the option portfolio has zero initial value.

At first, valuation of options within the BS framework is discussed. Next, it is shown that the put-call symmetry for European options serves as an elegant rule to determine the threshold within the waterfall structure. Subsequently, conditions for the fair design of the fund can be deduced. Besides a fair design, these conditions allow us to express the value of the pension contract as a percentage of a member's real pension ambition, at each point in time. This creates more insight in the working of the pension fund.

6.1 Option Valuation and Put-Call Symmetry

In the Black-Scholes setting, the time t price of a derivative C_t, that pays off at time $T > t$, can be calculated using the risk neutral pricing formula, which is given by:

$$C_t = N_t \cdot E_t^{\mathbb{Q}_N} \left[\frac{C_T}{N_T} \right].$$

Here $\mathbb{Q}_N$ stands for the equivalent martingale measure that corresponds to taking the asset N_t as numéraire[1]. Risk-neutral pricing uses the result that under the assumption of completeness and absence of arbitrage, asset price processes relative to the numéraire must be martingales under the unique equivalent martingale measure $\mathbb{Q}_N$ (see Schumacher, 2008).

The BS pricing formulas for European call and put options on the pen-

[1] The price of an asset must always be positive in order to serve as a numéraire.

sion fund's Ambition Ratio are respectively given by[2]:

$$\begin{aligned} C_t &= AR_t e^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\ P_t &= Ke^{-rT}\Phi(-d2) - AR_t e^{-qT}\Phi(-d_1), \end{aligned}$$

where d_1 and d_2 are given by[3]:

$$\begin{aligned} d_1 &= \frac{log(\frac{AR_t}{K}) + (r - q + 0.5\sigma^2)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{log(\frac{AR_t}{K}) + (r - q - 0.5\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \end{aligned}$$

In these formulas, K stands for the strike of the option and AR_t for the spot level of the Ambition Ratio. $\Phi(.)$ stands for the CDF of the standard normal distribution. These formulas are used to value the options within the pension contract at a particular time $t < T$.

In order to make the pension contracts fair at initiation of the pension fund, the strike level of the Upside Call is determined in such a way that the total value of the option portfolio equals zero (cf. Kocken, 2006). To achieve this in an elegant way, the put-call symmetry (PCS) is used. This result is - among others - elaborated by Carr, Ellis and Gupta (1998) and Detemple (1999) and formally it states the following:

Theorem 1 *Put-Call Symmetry (based on Detemple, 1999)*

Consider a European put and call option with identical characteristics, i.e. strike level K, and maturity T written on an asset with price S, where the dynamics of price S under the equivalent martingale measure $\mathbb{Q}_B$ are given by:

$$dS_t = S_t(r - q)dt + \sigma d\tilde{W}_t, \quad \text{with } t \in [0, T], \; S_0 \text{ given.}$$

Here, the parameter r denotes again the risk free interest rate, q the continuous dividend yield paid on the stock, σ is the volatility of the stock and $\tilde{W}_t$ is a Brownian motion under the equivalent martingale measure $\mathbb{Q}_N$.
Let $P(S,\ K,\ r,\ q,\ \sigma,\ T - t)$ and $C(S,\ K,\ r,\ q,\ \sigma,\ T - t)$ denote the price functions of this put and call respectively. Then it holds that:

$$P(S_t,\ K,\ r,\ q,\ \sigma,\ T - t) = C(K,\ S_t,\ q,\ r,\ \sigma,\ T - t). \tag{6.1}$$

Proof. For the proof of this relationship see Appendix A.1. ■

The Put-Call Symmetry basically states that the value of a put in the financial market under consideration (i.e. with interest rate r and dividend

[2]Merton (1973) derived these formulas for options on dividend paying stocks. For precise derivation of these formulas see Schumacher (2008), pp. 58 - 65.

[3]The term log(a) stands for the natural logarithm $\ln(a)$.

yield q) is equal to the value of a call option on a stock with spot price K and strike level S on an auxiliary financial market with interest rate q and dividend yield r. Based on Haug (1999) this result can be extended to the following relationship, which is useful in the context of the pension fund model under consideration:

$$\lambda^{-1} P(S_t\ , \lambda S_t\ , r\ , q\ , \sigma\ , T-t\ ) = C(S_t\ , \lambda^{-1} S_t\ , q\ , r\ , \sigma\ , T-t), \quad (6.2)$$

with $\lambda \in [0,1]$. The derivation of (6.2) is provided in Appendix A.2. This result states that λ^{-1} put options on a stock with strike λ times the spot price, is equal to the value of a call option on the stock with spot price S with strike level of λ^{-1} times the spot price. Hence, the put and call show a geometric symmetry. In the pension fund model, the assumption of zero carrying costs ($r = q$) of the Ambition Ratio now is convenient, as it makes the presence of the auxiliary financial market superfluous[4]. A nice feature is moreover that in the lognormal model, formula (6.2) holds irrespective of the volatility or interest rates within the economy[5].

6.2 Fair Design and Value of the Pension Contract

The put-call symmetry enables to easily set up a construction of multiple options with strike levels on different sides of the spot price, that has zero initial value. In the pension fund the spot price is given by $AR = 100\%$. For the combination of the Guarantee Put and the Guarantee Call this has the following implication:

$$GP_t(AR_t,\ 1,\ r,\ q,\ \sigma,\ T-t) = GC_t(AR_t,\ 1,\ r,\ q,\ \sigma,\ T-t) \text{ if } AR_t = 1. \quad (6.3)$$

So, the value of the Guarantee Put equals the value of the Guarantee Call at the Ambition Ratio of 100%. For the combination of the Default Put and the Upside Call result 6.2 implies the following:

$$DP_t(AR_t,\ \lambda,\ r,\ \sigma,\ T-t) = UC_t(AR_t,\ \lambda^{-1},\ r,\ \sigma,\ T-t) \text{ if } AR_t = 1. \quad (6.4)$$

So, the strike level κ of the Upside Call can be set at λ^{-1} to equal the Default Put in value at an ambition ratio of 100% at time $t < T$. Hence, in order to set up the waterfall construction at $t = 0$, no money transfers within the pension fund are required, provided the Ambition Ratio equals 100%. A nice feature is that these relationships at AR_t =100% will hold independently of the fund volatility, interest rates and time to maturity.

When the Ambition Ratio moves away from 100% however, the net value of the option construction will change. For one tranche the option portfolio

[4]Note that the put-call symmetry is primarily used in valuing options on forwards or futures, which always have zero carrying costs.

[5]Carr et al. (1998) extend these results for the PCS also to more general settings.

increases in value while the other tranche suffers a loss in value. The left graph in Figure 6.1 gives the option values at time 0 for the senior tranche, depending on the values of the Ambition Ratio. These results are for a medium volatility pension fund with σ equal to 0.1 [6].

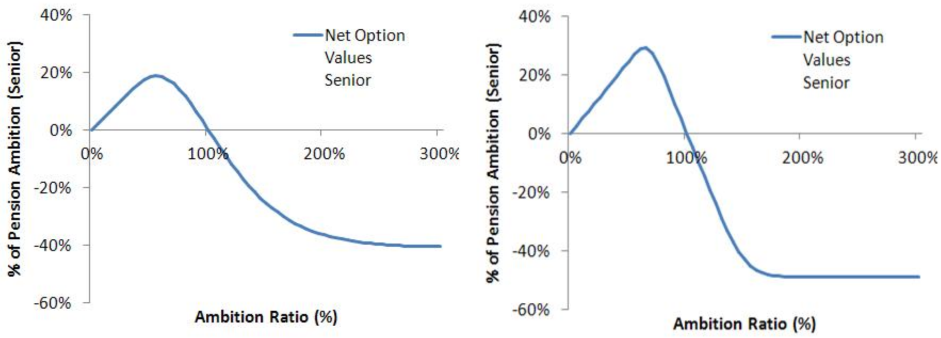

Figure 6.1: Net value options at time 0 (left) and time = 9 (right) for the senior tranche; parameter values: $\sigma = 0.1$, $r = 0.02$ and $\lambda = 66.7\%$.

What immediately stands out is that for certain levels of Ambition Ratio, the option values take in a large percentage of the pension ambition. Given σ, r and λ, the option construction takes in up to -40% and +20% of senior pension ambition at time 0. Naturally, these maxima vary for different parameter values and times to maturity[7]. For Ambition Ratios close to 100% however, the option construction has limited value.

In general for the senior tranche, maximum in option value is reached approximately at $AR = \lambda$, the strike level of the Default Put. This is exactly the area in which the senior tranche profits maximally from the value of the Guarantee Put, given the presence of the Default Put (see Figures 5.2 and 5.4). The net option value reduces to zero when the Ambition Ratio goes to zero. Intuitively, this happens because the value of the Default Put becomes more negative for the senior tranche. When time to maturity becomes smaller, the option values will convert to their intrinsic value. Therefore, one can expect that he maximum value of the option construction as % of pension ambition corresponds intuitively to $(100 - \lambda)\%$, i.e. the maximum payoff from the option construction to the senior tranche. This can be seen in the right graph of Figure 6.1.

The minimum of the option values is approached in the upside of the fund, a bit at the right from the strike level λ^{-1} of the Upside Call. That is the area in which the equity tranche has profits maximally from the value

[6] $\sigma = 0.1$ corresponds approximately to an asset mix of 40% equity, 60% bonds investment strategy as appears from scenario construction using the ALM application ALS. This is a software application developed by Ortec to simulate scenarios for ALM purposes.

[7] A sensitivity analysis is provided in Section 6.3 and Chapter 7.

of the Guarantee Call. For short times to maturity the minimum value is in general given by $(\lambda^{-1} - 1)$ in line with the payoff profile. This is the maximum value given away by the senior tranche in the upside of the funding level of the pension fund. For additional positive shocks in Ambition Ratio, the net option value remains constant. Reasons for this are that additional positive shocks are shared again proportionally by the two tranches (i.e. $45\,^\circ$ payoff lines for both tranches) and the fact that all options in the construction are deep in or out of the money.

In addition, the options values allow to display the value of the contract at time $t < T$, expressed as % of the pension ambition depending on the Ambition Ratio[8]. This is done in Figure 6.2.

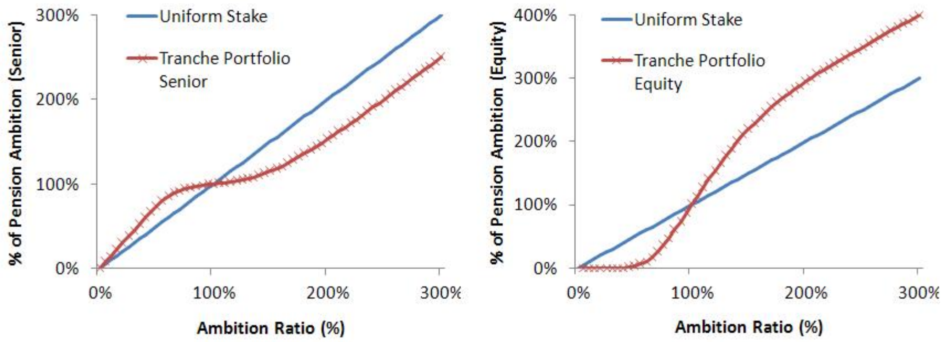

Figure 6.2: Value of pension contract for senior (left) and for equity (right) as percentage of real pension ambition; $t = 0$, $\lambda = 66.7\%$, $\sigma = 0.1$.

In both graphs the Uniform Stake serves as a benchmark representing the pension contract in the basic CDC fund without risk sharing. Compared to the basic CDC fund, the value of senior debt is clearly protected for downside losses. At time 0, this protection is not yet perfect, as the probability that the options will move out of the money is still considerable. The contract in the equity tranche has an opposite profile. Its value is volatile around Ambition Ratios of 100%. Downside protection is absent, but as a compensation the equity tranche has large claims on the fund's upside.

Figure 6.2 is quite valuable as it makes explicit how shocks in funding level are distributed to different participants (at time 0). Contrary to the current contracts, there is at each value of the Ambition Ratio distinctness about the value of the pension contract, and about the size of the shocks absorbed by specific plan members. Note that the graphs displaying the values of the pension contracts already resemble the final payoff profiles.

[8]Technically, the value of the pension contract at time 0 is projected to time T so that it can be expressed as fraction of the real pension ambition at T.

Appendix B.1 provides similar figures for different times to maturity and varying levels of volatility. These figures give an idea of the 'shape' of the pension contract for funds with different risk-profiles at different points in time.

6.3 Fair Design and Seniority of the Fund

The parameter λ reflects the seniority of the fund, i.e. the ratio between senior pension rights and the total pension rights in the pension fund[9]. So, when λ is large (e.g. $\lambda = 80\%$), the amount of senior rights as percentage of total pension rights in the fund is large. This implies that the equity tranche provides only small downside protection to the senior tranche. In other words, the default risk, i.e. the probability that the Default Put moves in-the-money is large.

The pension fund should manage this greater risk for senior debt via the total amount of risk taken by the fund. The total amount of risk should be less when the seniority of the fund is large[10]. Regardless of the total amount of risk, the larger downside risks faced by the senior tranche should be compensated via claims on the upside of the funding level. The result from the put-call symmetry aligns with this principle as the upside thresholds is also closer to $AR = 100\%$, when λ is closer to 1, as $\kappa = \lambda^{-1}$. So, the senior tranche starts with sharing in the fund's upside at lower Ambition Ratios. This situation presented in Figure 6.3. The left graph displays the payoff

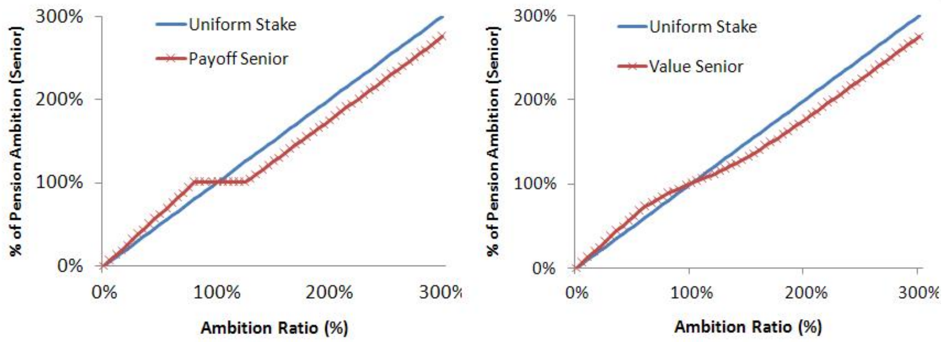

Figure 6.3: 'High seniority' fund. Payoff profile (left), value pension contract at time 0 (right); $\lambda = 80\%$; $\sigma = 0.1$.

profile, while the right graph displays the value of the pension contract at $t = 0$. Compare these profiles with Figure 6.2 to see the differences.

[9]Recall the introduction of λ in Section 5.2 (p. 34)

[10]Compare this with the advice of traditional life-cycle theory since senior pension rights belong primarily to older participants. High seniority would then simply imply a less risky investment strategy.

On the other hand, when the seniority of the fund is low, e.g. $\lambda = 50\%$, the default risk is consequently also lower for the senior tranche. This means that senior debt is very well protected, i.e. down to a funding ratio of 50% and correspondingly the upside threshold within the waterfall should be larger. This situation is depicted in Figure 6.4.

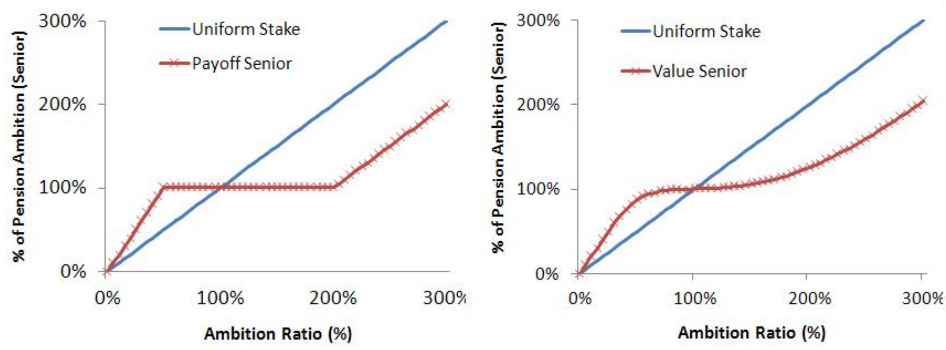

Figure 6.4: 'Low seniority' fund. Payoff profile (left) and value pension contract at time 0 (right); $\lambda = 50\%$; $\sigma = 0.1$.

So, Figures 6.2, 6.3 and 6.4 make clear the relationship between downside risk bearing and claims of the upside of the fund for the pension contract. The senior tranche should receive larger upside claims when the downside protection provided by the equity tranche is only small.

6.4 Remarks on Fair Design

In short, this chapter shows that the pension contract can be designed fairly using the conditions (6.3) and (6.4). This allows to examine the design and to observe the value of the pension contract at any moment in time in a closed setting.

Although the closed aspect of the pension fund looks restrictive at first sight, this setting is actually quite general under the assumption of a constant fund composition[11]. Constant composition implies that yearly inflow in the pension fund equals yearly outflow. Also transition of pension rights from the equity tranche to the senior tranche is constant under this assumption. Although the options of the individual beneficiaries in the fund mature as time passes, the aggregate option constructions on fund level remain more or less the same with respect to time to maturity. This makes the analysis

[11] Demographic changes in fund composition do not occur very rapidly and the pension fund board can reckon with changes by properly influencing the design parameters, such as the seniority.

in this and coming chapters valuable outside the scope of the closed setting as well.

Whereas we assume lognormality for the Ambition Ratio, in practice distributions of asset prices are not lognormal. They display statistical deviations such as heavy tails and gain-loss asymmetry (Cont, 2001). Therefore, market prices of (equity) options with strikes below the spot price, are usually larger than predicted by the lognormal model. One says that the options trade as if the underlying would have had a larger volatility in the lognormal BS model. The reverse holds for options with strike levels larger than the spot price. This volatility is called the implied volatility, and for equity options the corresponding pattern is called a volatility skew. A typical volatility skew is depicted in Figure 6.5.

Figure 6.5: Typical pattern for the implied volatility for equity options.

Presence of a volatility skew for the embedded options within the redesigned pension fund would imply a deviation from the fair thresholds as calculated in the lognormal (BS) model. Downside risk borne by the senior tranche (expressed via the Default Put) would thus have a larger market price. Qualitatively, this means that the strike of the Upside Call would be closer to $AR = 100\%$ than the value determined by condition (6.4). Therefore, in order to stay in a fair design, the senior tranche should obtain claims on the upside of the funding level already at lower Ambition Ratios[12].

Next chapter provides a sensitivity analysis of the risk sharing agreements within the pension fund, by analyzing the greeks of the options embedded in the waterfall construction. These sensitivities create more insight in the structure of the redesigned pension contract.

[12]The Guarantee Put and Guarantee Call have the same strike level and therefore the implied volatility for these options is the same. Only the Default Put versus the Upside Call is relevant regarding the volatility skew.

Chapter 7

Analysis of the Greeks

In standard option analysis, the Greek letters stand for the sensitivity of the value of an option with respect to market parameters, such as volatility or the price of the underlying. These sensitivities can be used to measure the risks stemming from these quantities involved in option contracts. In this pension fund context, they help to understand the risk sharing agreement between the two tranches within the fund: the Greeks clarify how the value of the pension contract changes for different participants, due to changes in Ambition Ratio or fund volatility. Throughout this chapter, the Greeks are determined at time $t = 0$ for a medium volatile pension fund ($\sigma = 0.1$) with seniority of $\lambda = 66.7\%$, and risk free rates of 2%.

7.1 Delta Analysis

Delta stands for the sensitivity of the value of an option with respect to the value of the underlying, i.e. the Ambition Ratio in this case. Hence, for the senior pension contract delta is given by the first-order derivative of $PC_{Sen,t}$ with respect to the Ambition Ratio AR. For the senior tranche this is given by:

$$\Delta(PC_{Sen,t}) = \frac{\partial(US_{Sen,t} + GP_{Sen,t} + GC_{Sen,t} + DP_{Sen,t} + UC_{Sen,t})}{\partial AR}.$$

The results for the delta analysis are depicted in Figure 7.1, expressed as percentage of real pension ambition.

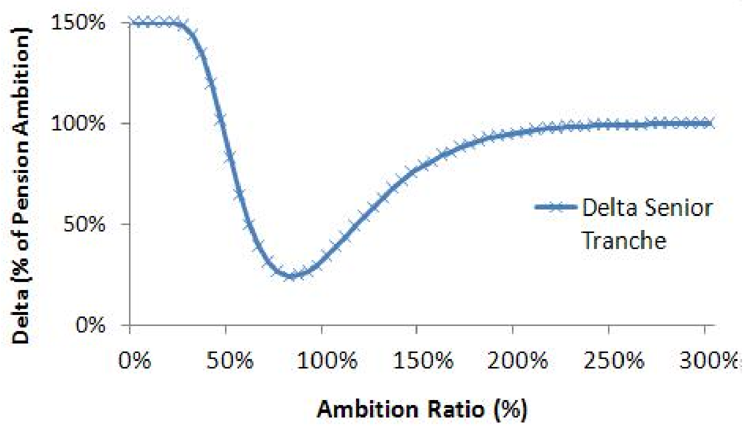

Figure 7.1: Delta for senior tranche as % of senior pension ambition at time 0.

The fact that delta is positive at all Ambition Ratios implies that the value of the pension contract always increases when the ambition ratio rises. Crucial for this result is the positive dependence of the Uniform Stake from the Ambition Ratio. This implies - not surprisingly - that at every moment in time, increases in Ambition Ratio are beneficial for the senior tranche. Looking purely from the perspective of option value however, the options will have positive value at low Ambition Ratios. The delta of the separate option construction in the senior tranche is obtained by subtracting 1 from the graph in Figure 7.1.

If we look at the area of $AR = 0\%$ up to approximately 30%, the delta of the contract has a constant value of 150%. This can be explained by the fact that the Default Put (short) and the Guarantee Put (long) are moving from both being deeply into the money towards their strike levels. The first 50% (= 150% − 100%) corresponds exactly to the difference in slope of the option payoffs from the Default Put and the Guarantee Put. In general, this relationship is given by: $\lambda^{-1} - 1$.

More towards $AR = 100\%$, the effect of the Default Put (being struck at $\lambda = 66.67\%$) diminishes. The effect of negative change in value of the Guarantee Put however increases. This pushes the delta down. As the AR increases further on to levels far above 100%, the senior tranche gets closer to the threshold at which it shares proportionally in the upside of the fund. The positive change for the Upside Call is responsible for this and as a result the delta is again pushed towards 1.

Figure 7.2 displays the pension contract delta again but now for a smaller time to maturity.

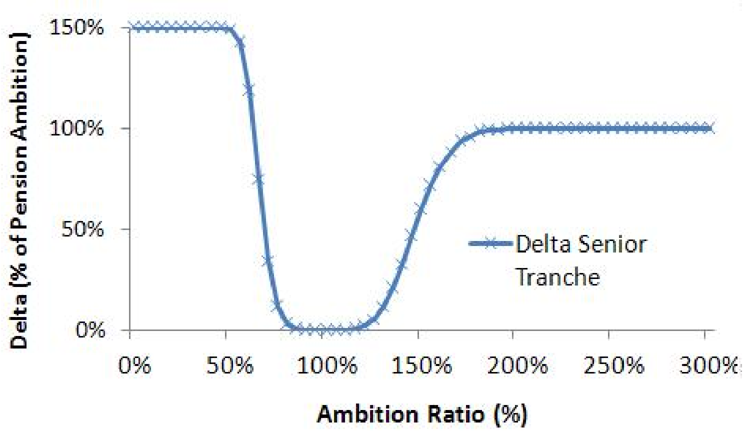

Figure 7.2: Delta for senior tranche as % of senior pension ambition at time 9.

Here, delta is approximately zero for values of the AR between λ and λ^{-1}. This is caused by the combination of the Guarantee Put (long) and the Guarantee Call (short) around $AR = 100\%$, whereas the Default Put and the Upside Call are out of the money[1]. Zero delta in that area implies that the pension contract is insensitive to changes in the Ambition Ratio. In fact, the horizontal part in the payoff profile of the senior contract already indicates this. Figure 7.2 indicates once again that the senior contract is sensitive for changes in AR when AR is smaller than λ or when AR is larger than $\lambda - 1$.

Similar analysis can be applied to the pension contract in the equity tranche. The corresponding delta profile is given in Figure 7.3. The delta profile of the equity tranche is expressed as percentage of the equity pension ambition, which is smaller than that of the senior tranche as $\lambda \in (0.5, 1]$. This explains why delta for equity is large compared to delta of the senior tranche. At very low Ambition Ratios the delta is zero as the equity tranche is empty in this region. Around AR of 100% however, the delta becomes very large, since in these area the equity tranche claims all positive shocks in funding level. Close to maturity, the maximum of delta equals 300%, which can be seen in Figure 7.4.

[1]To be precise: the Guarantee Put is in the money below $AR = 100$ % and the Guarantee Call is in the money above $AR = 100\%$. The fact that the Default Put and the Upside Call are both out of the money in the interval (λ, λ^{-1}) adds to this.

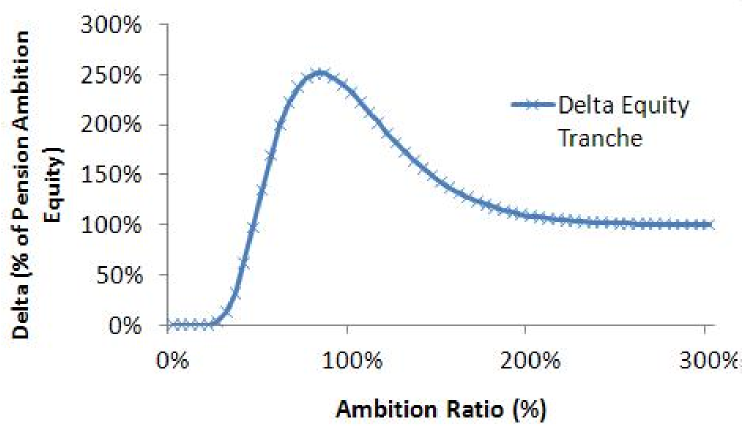

Figure 7.3: Delta for equity tranche as % of *equity* pension ambition at time 0.

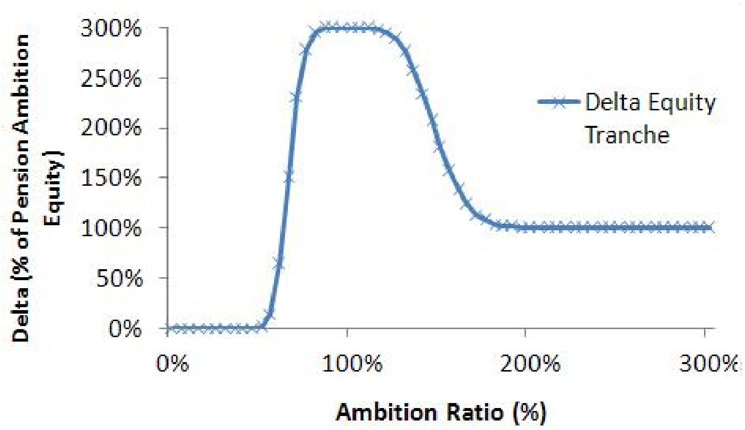

Figure 7.4: Delta for equity tranche as % of *equity* pension ambition at time 9.

In general, this maximum is given by $(1-\lambda)^{-1}$. This value exactly equals the amount of leverage the equity tranche faces for Ambition Ratios within $[\lambda, \lambda^{-1}]$, given current fund characteristics. As the Ambition Ratio increases further, delta of the equity tranche converges to 1, following delta of the senior tranche; this occurs because shocks are distributed proportionally to the tranches, for Ambition Ratios larger than $\lambda^{-1} = 150\%$.

Again in line with the senior tranche, the delta of the equity tranche is always larger than or equal to zero. This implies that increases in Ambition Ratios have positive effect on the value of the equity pension contract. Finally, the delta profile for equity clarifies that around $AR = 100\%$, the equity tranche is highly leveraged relative to the fund portfolio. This is exactly what adds value for younger participants in the pension fund[2].

7.2 Gamma Analysis

The gamma of the pension contract represents the sensitivity of the delta of the option construction with respect to a change in Ambition Ratio. A large positive or negative gamma implies that the delta of the pension contract is highly sensitive to changes in AR. For the senior tranche gamma is given by[3]:

$$\Gamma(PC_{Sen,t}) = \frac{\partial(GP_{Sen,t} + GC_{Sen,t} + DP_{Sen,t} + UC_{Sen,t})}{\partial AR^2}.$$

The gamma for the pension contract in the senior tranche is given in Figure 7.5. Also from the gamma profile it appears that the pension contract

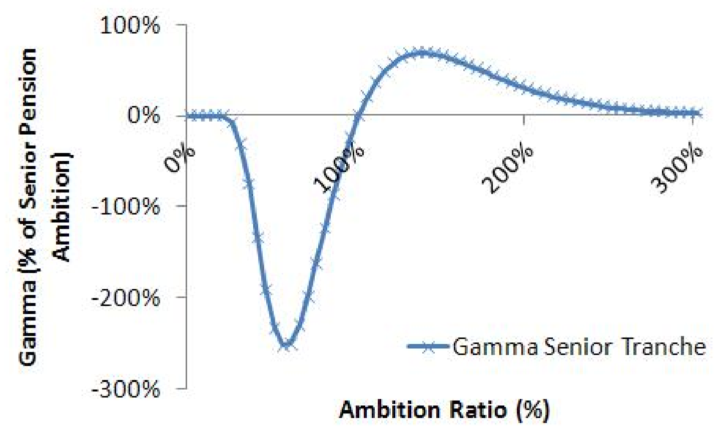

Figure 7.5: Gamma for senior tranche as % of senior pension ambition at time 0.

is most sensitive round the strike levels of the Default Put and the Upside Call. This is in line with standard option theory: gamma is large when an option is at-the-money. In those areas delta changes most rapidly due to changes in Ambition Ratio. This pattern becomes even more evident when the options are approaching maturity. However, the Guarantee Put and the Guarantee Call are both struck at $AR = 100\%$, whereas gamma is only small. In this region, the separate gammas of both options offset each other so that the gamma of the total option construction is close zero. Similar information can be given regarding the gamma profile of the equity tranche. So, we suffice to just display the gamma profile.

[2]Compare the overcoming of borrowing constraints via the pension contract in Section 3.1.1.

[3]Note that the Uniform Stake is dropped from this notation as its second derivative with respect to the Ambition Ratio is zero.

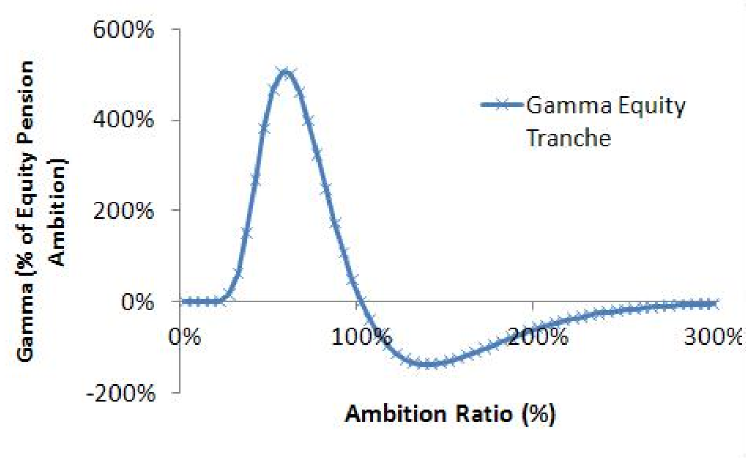

Figure 7.6: Gamma for equity tranche as % of *equity* pension ambition at time 0.

In standard option theory, gamma tells when it is dangerous to leave a delta-neutral (option) portfolio unhedged. In pension fund context, the gamma profile indicates at what Ambition Ratios, and at which moments in time, additional risk management decisions could be desirable for the separate tranches.

7.3 Vega Analysis

The vega of the pension contract represents the sensitivity of the option values with respect to the volatility of the pension fund, i.e. to the amount of risks faced by the fund. Although the volatility is assumed to be constant in the standard Black-Scholes setting, it is reasonable to compute the vega. From Hull and White (1987) it appears that calculations of vega in the BS model yield very similar results as for models with stochastic volatility.

The vega of the pension contract for the senior tranche is given by:

$$\mathcal{V}(PC_{Sen,t}) = \frac{\partial(GP_{Sen,t} + GC_{Sen,t} + DP_{Sen,t} + UC_{Sen,t})}{\partial\sigma}.$$

Figure 7.7 displays the vega profile for the senior contract.

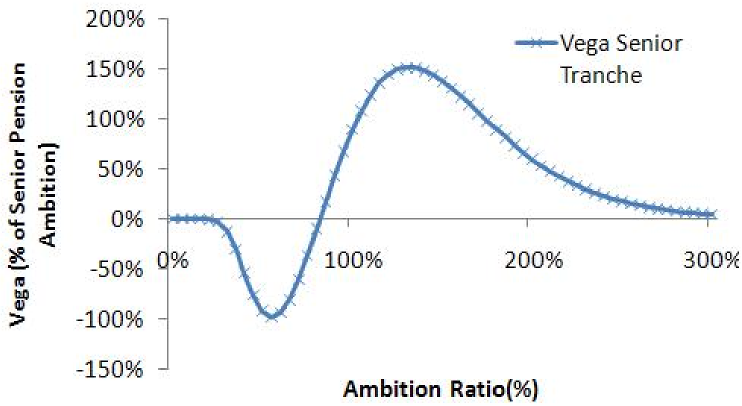

Figure 7.7: Vega for senior tranche as % of senior pension ambition at time 0.

The vega approximately reaches its minimum when the Default Put is at-the-money; its maximum values are reached when the Upside Call is at-the-money. Remarkable is that the vega profile shows larger absolute values for higher levels of AR than for lower ones. This can be explained from the geometric brownian motion assumed for the Ambition Ratio. These dynamics feature a larger absolute volatility for larger values of the Ambition Ratio. Hence, the sensitivity to changes in σ is also larger in that region.

From standard option theory it is known that short position options have a negative vega, and long positions a positive one. So, below Ambition Ratios of approximately 90, the vega of the Default Put (short) apparently dominates the vega of the Guarantee Put (long). The senior tranche would benefit from reducing volatility to secure its pension rights. After all, senior rights are not protected anymore by the equity tranche.

At very low Ambition Ratios, the probability that the Default Put remains in the money is very large and therefore the fund volatility has no impact on the option values. Hence vega is equal to zero. For higher Ambition Ratios the vega is positive. So, at higher funding levels, the senior tranche benefits from increases in volatility, which implies that the Upside Call (long) has a strong impact on the vega profile. Senior pension rights are indeed very secure in that region as low Ambition Ratios are far away; increases in volatility lead to a considerable increase in the probability that the Upside Call (long) will end up in-the-money. Keep in mind however, that for participants who have allocated their pension contributions over the tranches, the sensitivities to changes in volatility are somewhat smaller than for the separate tranches.

The vega profile of the equity tranche is opposite to that of the senior tranche as can be seen from Figure 7.8. The consequence of the mutual risk sharing agreements between equity and senior debt is, that both tranches

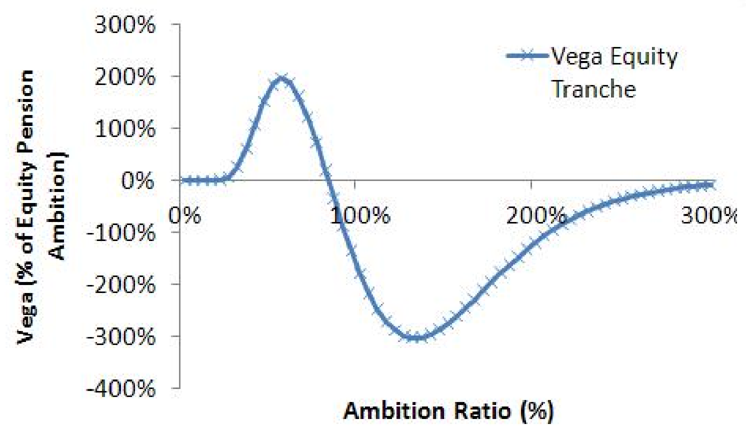

Figure 7.8: Vega for equity tranche as % of equity pension ambition at time 0.

have different interests with respect to changes in the fund volatility. These changes are however purely from the perspective of the option values within the fund. They should thus not be confused with the life-cycle preferences of the plan members. In theory, young plan members should have more a more risky pension contract and old members a more secure one. This is accomplished, exactly by the risk sharing agreements between the two tranches. Therefore, this volatility analysis is incomplete if one reckons with the fact that only the option values are involved. In order to draw a more robust conclusion about conflicts of interests regarding fund volatility also scenario analysis of the pension contract over time in combination with utility analysis should be added.

So, the fact that one tranche benefits from a rise in volatility according to the vega analysis should not imply that the volatility should indeed be increased. Nevertheless, vega is very useful to quantify direct value redistributions between the tranches, resulting from changes in volatility. The pension fund board should take this into account when for example the investment strategy of the fund is altered.

In short, this chapter discusses sensitivity of the value of the pension contract to changes in the Ambition Ratio and fund volatility. This analysis creates insight in the risk-sharing agreements embedded in the waterfall construction within the pension fund. Moreover, sensitivities serve as a tool for the pension fund board to keep track of value transfers between tranches within the fund. Next chapter analyzes what the pension redesign means for the plan members in terms of pension payouts, as the fund moves through time.

Chapter 8

Development of the Fund over Time

For pension fund participants it is essential to know how the pension contract behaves through time. Therefore, this chapter clarifies the development of the different pension contracts within the fund over time. Also a discussion is provided of the distribution of the pension benefits at end of the time horizon. This analysis makes use of simulated scenarios of the fund's Ambition Ratio. In line with previous chapters, intuition about how 'things work' within the fund is crucial. The analysis starts with elaborating the set up of the scenario analysis of the pension fund.

8.1 Fund Settings over Time

The aim of the scenario analysis is to see how the pension contract develops for different participants, given the parameter settings used to generate fund scenarios. Therefore keep in mind, that in line with Kocken (2006), no attempt is done to find the right parameter values, e.g. the optimal investment strategy for the fund.

The closed pension fund will again be analyzed on the fixed time horizon of 10 years. For the development of the fund, 5000 scenarios are used for the Ambition Ratio, using its dynamics as stated in Section 5.1. The parameters μ and σ are calibrated to ALS scenarios[1] for the real funding ratio of a standard Dutch pension fund[2]. This results in $\mu = 2.0\%$ and $\sigma = 8.1\%$.

[1]ALS is a software application developed by Ortec to simulate scenarios for ALM purposes.

[2]This pension fund is characterized by: a real duration of liabilities equal to 16; asset mix: 30% equity, 60% bonds, 10% real estate. Furthermore, the scenarios use the assumption that the pension fund does not steer the Ambition Ratio via contributions and that full indexation is granted. In this way the real funding ratio reflects the Ambition Ratio, being more exogenous.

8.2 Analysis of a Fund Scenario

In order to clarify the development of the pension contracts over time, this section zooms in on one particular scenario for the funding level of the pension fund. The development of pension contracts of the different tranches in this scenario is depicted in Figure 8.1.

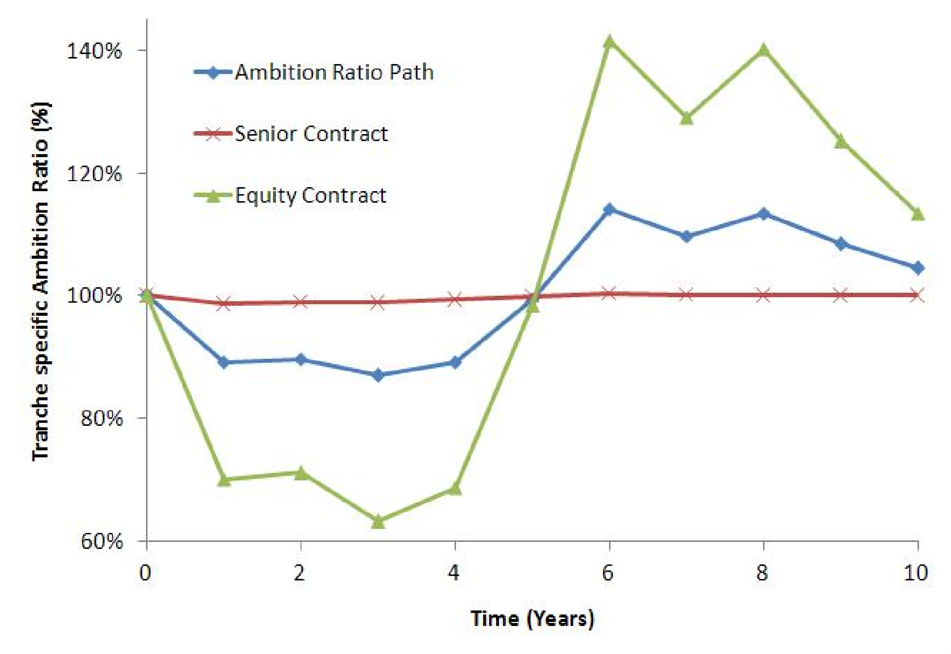

Figure 8.1: Development of pension contracts over time.

The graphs denote the values of the different pension contracts, expressed in terms of a tranche-specific Ambition Ratio. Each graph thus displays to what extent the pension contract can meet its corresponding (future) pension ambition. The blue graph marked with diamonds displays the value of the contract in the basic CDC fund, serving as benchmark to illustrate the effect of the waterfall construction. This graph simply coincides with the path of the Ambition Ratio of the fund.

The fund starts from a fully funded situation as the fund's Ambition Ratio equals 100% at time $t = 0$. The option construction is designed fairly and consequently, its value equals zero at $AR = 100\%$. Consequently, both the senior and the equity contract feature a tranche-specific Ambition Ratio of 100%.

Figure 8.1 shows that the value of the senior pension contract develops very stable through time. Shocks in funding level are diminished via the option construction. The protection in this scenario is close to perfect since deviations of the contract value from the full pension ambition are very small. Still, it will always hold that the value of the senior contract is between the graph of the Ambition Ratio and 100%, when the Ambition Ratio is above 100%. Similarly, at Ambition Ratios below 100%, the value of the contract is larger than the value of the Ambition Ratio, but smaller than 100%.

For the equity tranche, shocks in the Ambition Ratio are clearly enlarged as equity bears the first shocks around $AR = 100\%$. For Ambition Ratios deviating from 100%, the value of the equity tranche always diverges more strongly from 100%. So, participants bearing downside risks are also compensated, in upside scenarios.

In general, participants' contributions are distributed among the senior and equity tranche. Therefore, the same fund scenario is once again displayed but now for two fund members: an older and a younger member, who have each allocated their contributions over the two tranches[3]. Again the trajectory of the Ambition Ratio is added to serve as a benchmark.

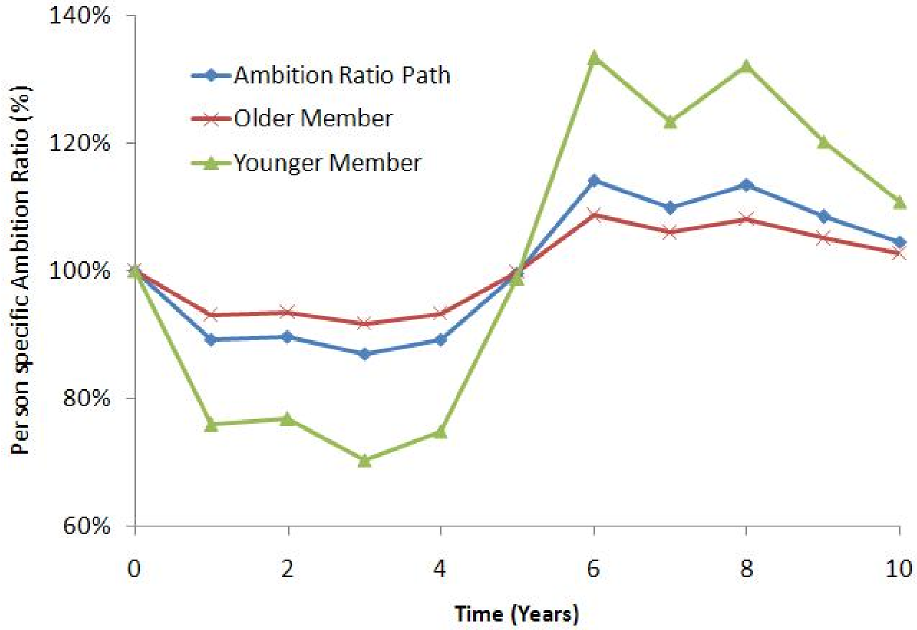

Figure 8.2: Development of pension contracts of different plan members.

The older member has 80% of his contributions allocated to senior debt and 20% to the equity tranche. The value of his pension contract is still quite

[3]Note that these members cannot be the only members present in this pension fund, unless they have equal levels of pension ambition. In that case, their stakes in both tranches could add up to the total pension capital in the fund.

stable as shocks in the fund's Ambition Ratio are softened. The younger person has allocated 20% of his pension wealth to the senior tranche and 80% into Equity. The development of the contract of the younger person is similar to that of the equity tranche, had it not that shocks in both upside and downside are less enlarged. By setting the weights in senior and equity, a tailor-made pension contract is created for each participant, given the settings of the fund as a whole[4].

Finally, Figures 8.1 and 8.2 above clarify that the value of the pension contract is explicitly defined at each moment in time. This creates insight for both the fund's board and the plan members regarding (fluctuations in) the value of pension contracts. Participants should use this information to steer the size of their pension via extra (or less) savings or via adjustment of their retirement date[5]. Also in order to transfer a member's accrued pension savings to another pension fund (e.g. due to switching of jobs), the value of the pension contract needs to be explicit at every point in time.

8.3 Probability Distributions of Pension Payouts

Probability distributions of pension payouts indicate the level of pension payouts plan members can expect in the redesigned fund. These distributions are determined at time T and they are calculated on the basis of the scenarios for the Ambition Ratio. In the probability distributions the effect of the waterfall construction becomes once again clear. Figures 8.3, 8.4 and 8.5 provide the probability density functions (pdf) and cumulative density functions (cdf) for respectively the senior tranche, the equity tranche and the basic CDC fund (i.e. the Ambition Ratio). Summary statistics of these distributions are provided in Table 8.1 (p. 63).

[4] Of course the seniority of the fund, i.e. the fund demographics, are restrictive for this, and seniority plays an important role in the fund as we saw in Section 6.3.

[5] In fact, the pension fund should also fulfill this role, overcoming (possible) behavioral failures of individual plan members in saving, dependent on current pension accumulation.

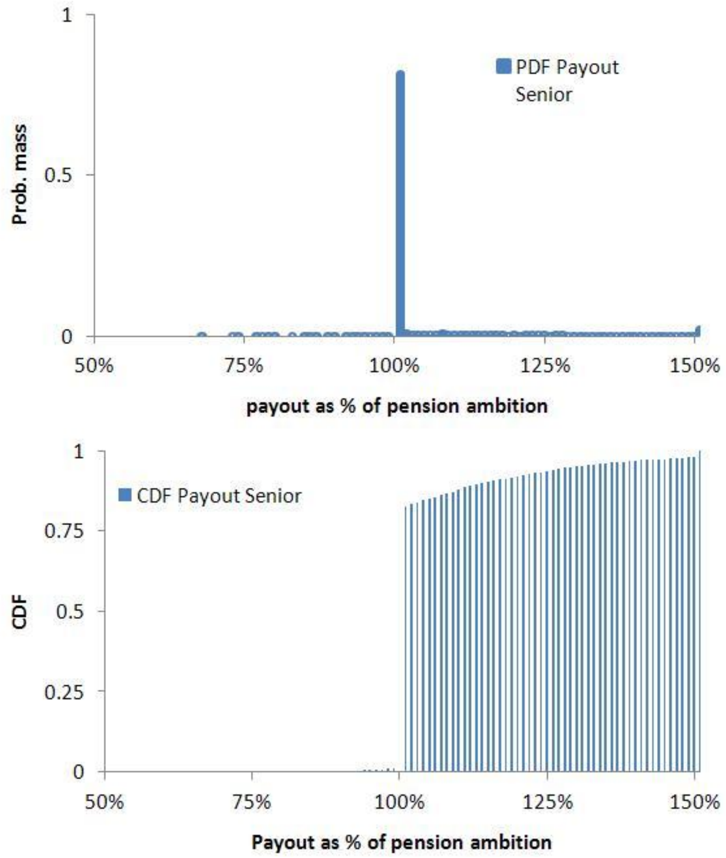

Figure 8.3: Probability Distributions of pension payouts in the senior tranche.

Figure 8.3 shows that the senior pension rights are very secure. Only in 1.2% of the scenarios the payout in the senior tranche is lower than aspired. Conditional on falling short of the full pension ambitions, still 89.3% of the senior pension ambition is paid out in expectation. In 81% of the scenarios senior receives its full pension ambition. In these cases the Ambition Ratio ends up between the thresholds $\lambda = 66.7\%$ and $\kappa = 150\%$ $(= \lambda^{-1})$.

The equity tranche shows more dispersed probability distributions as depicted in Figure 8.4. This becomes especially clear if one compares Figure 8.5, which displays the distributions of the basic CDC pension fund (p. 62). The dispersion is also indicated by a standard deviation of 77.7% and a negative kurtosis (-84.4%). Larger dispersion is due to the leverage of the equity tranche with respect to the fund portfolio for Ambition Ratios between 66.7% and 150%. The probability that the equity tranche receives less than 100% of the pension ambition, equals the probability that the Am-

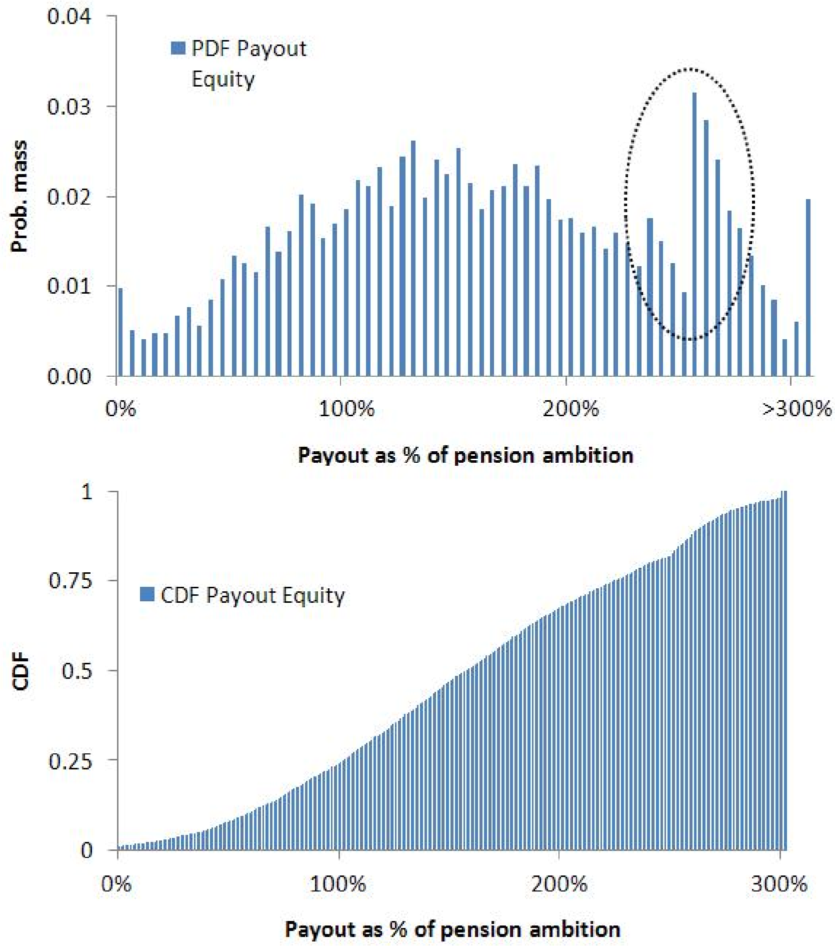

Figure 8.4: Probability Distributions of pension payouts in the equity tranche.

bition Ratio ends up below 100%, namely 24.3%[6]. Conditional on shortfall, the expected payoff in the equity tranche equals 58.6% of equity pension ambitions. In 1.2% of the scenarios, the equity tranche ends up empty.

On the other hand, the expected payoff from the equity contract equals 160% versus 104% for the senior contract and 123% for the basic CDC contract (i.e. the Ambition Ratio). This implies that in expectation the equity tranche gets compensated for the downside risk-bearing. This expected compensation can be interpreted as a risk premium for bearing downside pension risks. The level of this risk premium follows from the drift term in the dynamics of the Ambition Ratio, the time horizon and the seniority of the fund. So, in principle the investment strategy of the fund can be chosen to alter this risk premium (and correspondingly the risks assumed within the fund), dependent on the preferences of the fund members. In this analysis however, this risk premium is treated as given; it is outside the scope of this thesis to examine the optimal amount of risk taken by the fund.

A striking aspect of the pdf of the equity tranche is the peak in probability mass at 250% of equity's pension ambition, indicated by the dotted line in Figure 8.4. This 250% of the pension ambition is the amount of wealth present in the equity tranche, at the moment the ambition ratio passes $\kappa = 150\%$, i.e. the upside threshold in the waterfall construction. From that point onwards additional upside shocks are not anymore absorbed solely by the equity tranche, but they are allocated proportionally over the tranches. In other words, the equity tranche is not leveraged anymore with respect to additional positive shocks in Ambition Ratio. So, additional shocks lead just to proportionate increases in pension payout of equity. Therefore, the probability mass is more dense from 250% onwards.

The exact level $P(\kappa)$ of pension payouts of the equity tranche at $AR = \kappa$ can be calculated as follows:

$$P(\kappa) = 1 + (\kappa - 1) \cdot \frac{RPA_{Equity,T} + RPA_{Senior,T}}{RPA_{Equity,T}}. \tag{8.1}$$

The latter term in (8.1) equals 3 in current fund settings. The transition in the waterfall construction also becomes clear from the hump starting at 250% in the cdf of the equity tranche. The exact results for the probability distributions depend on characteristics of the fund such as the seniority λ. However, the rough shape of the distributions is similar for different values of λ. To provide in addition insight in the distribution of the pension contract within the time horizon of 10 years, Appendix B.1 provides figures displaying percentiles for the value of the pension contract. These figures also show the effect of the waterfall construction over time.

[6]Naturally, the length of the time horizon in combination with the drift plays an important role for the absolute level of this probability.

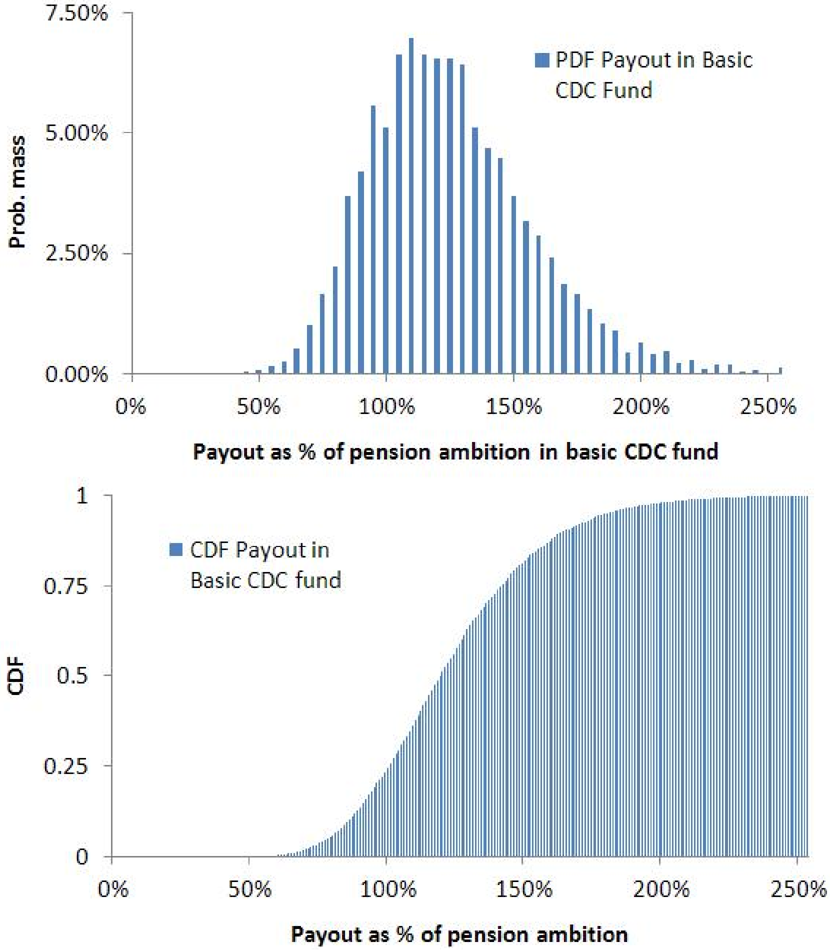

Figure 8.5: Probability Distributions of pension payouts in the basic CDC fund without risk sharing.

Payoffs denoted by X	**Basic CDC**	**Senior Debt**	**Equity**
$E[X]$	122.8	104.0	160.3
Std$[X]$	31.4	12.6	77.7
Skew$[X]$	72.3	408.0	1.5
Kurt$[X]$	84.4	2070.8	-84.4
P$[X < 100\%]$	24.3	1.2	24.3
E$[X \mid X < 100\%]$	86.2	91.0	59.6
Percentiles			
1%	65.6	98.4	0
5%	78.6	100.0	35.7
10%	85.9	100.0	57.7
50%	119.2	100.0	157.5
90%	179.2	114.0	264.0

Table 8.1: Summary statistics for the probability distributions of the pension payouts. All numbers are expressed in percentages of full real pension ambition.

In short, this chapter elaborated on how the redesigned pension contract develops through time and discussed the probability distributions of pension payouts. Next chapter extends the closed pension fund setting to an open one as inflow of new participants into the redesigned pension fund is examined.

Chapter 9

Inflow of New Pension Rights

Inflow of new participants into a pension fund can lead to considerable wealth redistributions, particularly at high or low funding levels[1]. Therefore, this chapter analyzes the entrance of new participants[2] at participant level, by extending the closed fund setting as introduced in Chapter 5 to an open one. Special attention is paid to inflow in case the pension fund's Ambition Ratio diverges from 100%.

9.1 Inflow From Perspective of a New Plan Member

In order to analyze 'what happens' from the perspective of a new plan member joining the fund, a setting for occurrence of new inflow needs to be introduced. We continue to use the pension fund setting as discussed in previous chapters. So, the horizon on which we analyze the fund is retained at 10 years, starting from time $t = 0$. In principle, new inflow can take place at any time between 0 and time 10. We assume that the new member enters the fund at $t_I = \epsilon$, after the pension fund has experienced an instantaneous shock in funding level on the interval $(0, \epsilon)$. So, ϵ is taken arbitrarily small. For the analysis in this chapter the time of inflow is not relevant. However, by assuming that the new member joins the fund at time ϵ, he also has a time horizon of 10 years, in line with previous chapters. In that case, the contract of the new member can be analyzed as if it were $t = 0$.

As a stepping stone to the analysis of inflow in the redesigned fund, inflow in the basic CDC fund is examined[3]. Consider the situation in which

[1] Also in the current pension design, a member contributing to the fund while funding level is low, faces considerable reductions of purchasing power of his pension savings due to cuts in indexation.

[2] A 'new plan member' can also refer to an existing member who contributes again to the fund to buy new pension rights.

[3] Recall that this fund design was introduced in Section 4.2 and it served as benchmark for the redesigned fund throughout previous chapters.

the basic CDC pension fund has experienced an instantaneous shock of -20% in the time interval $(0, \epsilon)$, so that the Ambition Ratio currently equals 80%. At this moment, a new member joins the fund by contributing money. The example below discusses how the inflow in the fund unwraps adversely for the new member.

Example: *Inflow in the basic CDC pension fund*
The pension contract of beneficiaries consists in this fund setting of a Uniform Stake, which depends on the fund's Ambition Ratio:

$$US_{k,t} = AR_t \cdot RPA_{k,t}.$$

By joining the fund at an AR of 80%, the new participant directly receives a pension contract with value of 80% of the current value of his pension ambition, whereas he pays for 100% of this value[4]. So, the new member subsidizes the early participants[5] as 20% of his wealth is spread out over the pension wealth of other members in the fund. This situation can be seen in Figure 9.1.

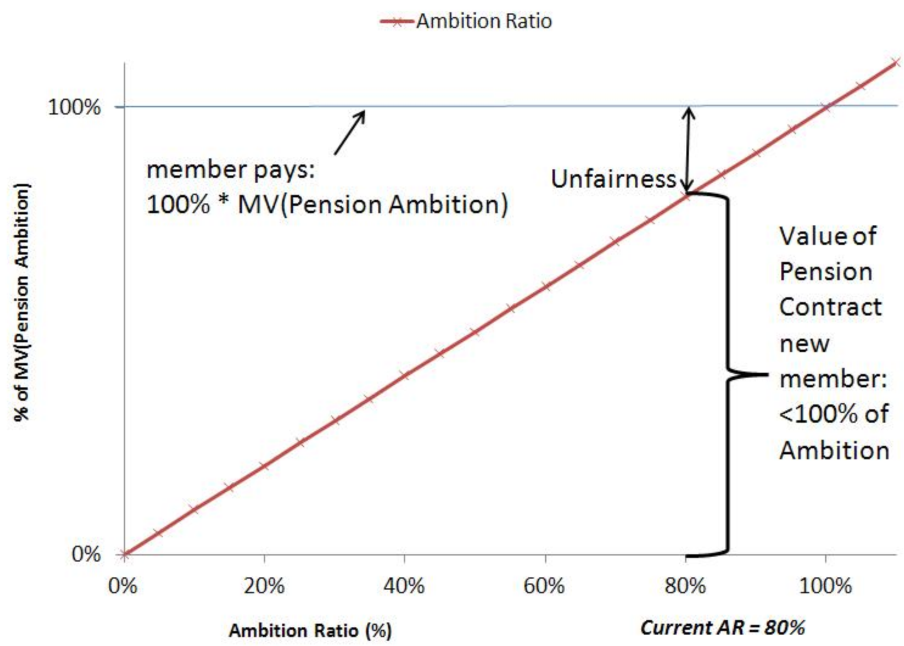

Figure 9.1: Unfairness for new member in the basic CDC fund.

[4]The assumption is made that the size of the new inflow is small compared to the total capital in fund, so that the Ambition Ratio of the whole fund stays practically constant.
[5]With 'early participants' reference is made to participants who entered the pension fund already at time $t = 0$.

In fact, the new participant 'shares' in the risk that the early participants have run from time $t = 0$ onwards. So, the inflow occurs in an unfair way from the perspective of the new participant. Next section discusses an elegant way to arrange the entrance of new members in such a way that it is fair from the perspective of the new member. This method will subsequently be applied for the slightly more complex situation of inflow within the redesigned pension fund.

9.2 How to Deal with Inflow

In line with ideas of Kocken (2006), a possibility to make entrance fair for the new plan participant, is to charge him the fair contribution for the pension contract he receives. In such an approach the fair price for the pension contract should be determined, given current Ambition Ratio. A second possibility is to award the new member more pension rights relative to the contribution he makes (and was planning to make). In this approach the fair amount of 'extra' pension rights needs to be determined.

The first approach can be summarized via the following condition:

$$C_{k,t_I} = (PC_{k,t_I} |\ AR_{t_I},\ RPA_T^{Planned}), \tag{9.1}$$

where C_{k,t_I} denotes the pension contribution of the new member k. The right hand side (RHS) of (9.1) stands for the market value of the pension contract PC_{k,t_I}, given the planned pension ambition and current Ambition Ratio. Condition (9.1) says that inflow is fair when the pension contributions (LHS) are such that they equal the current value of the pension contract (RHS). The second approach is summarized by the following condition:

$$(PC_{k,t_I}(RPA_T^{Adjusted}) |\ AR_{t_I}) = C_{k,t_I}^{Planned}, \tag{9.2}$$

where $C_{i,t_I}^{Planned}$ stands for the pension contributions the new member was planning to make. The left hand side denotes the value of the pension contract, given the current Ambition Ratio and the *adjusted* size of the pension ambition $RPA_T^{Adjusted}$. So, this adjusted pension ambition has to be determined so that condition (9.2) is met.

To make these approaches intuitively clear, the inflow example from previous section is extended. Remember that a new member joins the basic CDC fund while the Ambition Ratio equals 80%. Following the first approach, i.e. condition (9.1), the new member pays 80% of the market value of his planned pension ambition $RPA_{i,t}^{Planned}$. Following condition (9.2) the new member just pays his planned pension contribution $C_{i,t}^{Planned}$. In exchange he receives 80% of the market value of an *adjusted* pension ambition, which has to be determined. In this simple example the adjusted pension ambition should be equal to 125% of the original pension ambition

$(= AR_t^{-1} = (100\%/80\%) = 125\%)$. Another way to look at this 125% is the following: 125% of the planned pension ambition would be the amount that the new member would have had to contribute to the fund, if he would have joined the fund at time $t = 0$, in order to have a pension contract with value of 100% of this planned pension ambition at current time $t_I = \epsilon$ (i.e. after the shock).

Both for the first and the second approach, it holds that the new member 'blends in' with the early members of the pension fund. After all, the value of the pension contract for the early participants is also given by $AR_{t_I} \cdot RPA_{t_I}$. Moreover, note that the Ambition Ratio of the fund is not altered by the inflow in the fund, whereas this was the case in the first part of the example. This feature will prove to be valuable for the redesigned pension fund.

To the ensure that inflow in the redesigned pension fund is fair, the same principle as in condition (9.2) is applied. Due to the presence of the options however, the method will be slightly more complex, as will appear in next section.

9.3 Inflow in the Redesigned Pension Fund

9.3.1 Presence of options

Intuitively, it is clear that the presence of the options in the redesigned pension fund plays an important role when it comes to fair inflow. The value of the pension contract consists for a considerable part of these option values, when the Ambition Ratio deviates from 100%. Moreover, new options have to be written when new members enter the fund. This can bring along an administrative burden for the pension fund. To restrict this burden, it would be desirable for the option construction of new members to have the same characteristics as the options of the early participants. In addition, it would be convenient not to 'disturb' the existing option contracts, i.e. the parameters of the existing options should remain untouched. Also, the value of the underlying, i.e. the Ambition Ratio, has to remain unchanged, since it would otherwise impact the value of the existing options (option delta $\neq$ 0 in general[6]).

Despite these complicating factors it is well possible to use condition (9.2) to develop a method that deals with inflow. This method can be designed such that inflow is fair, and such that the administrative burden of the fund is minimized. As will appear, the new option construction features the same characteristics as the options of the early members[7].

[6] See Section 7.1 for analysis of the delta of the pension contract.

[7] More concrete, the strike level of the new Guarantee Put and the new Guarantee Call are also set at AR =100%. Also for the Default Put and the Upside Call the strikes are respectively equal to λ and λ^{-1}, i.e. the same as for the early participants.

9.3.2 Analysis of new inflow

The analysis of inflow in the redesigned fund is focused at new inflow into the senior tranche. The analysis for the equity tranche can be done in a similar way. To initiate the analysis of new inflow, the assumption is made that the seniority of the fund is constant, i.e. constant λ. This comes down to assuming that the pension fund is of such size that new inflow has no effect on the proportion of senior pension rights in the fund[8].

Consider the situation in which the new member wants to join the redesigned fund at time $t = \epsilon$, while the Ambition Ratio of the fund equals 80%. The seniority of the fund, λ, is given by 60% and the volatility of the fund is set at $\sigma = 0.1$. If the new member would just pay his planned pension contribution in exchange for the pension contract based on his planned pension ambition, Figure 9.2 shows that this is an unfair deal for him (compare Figure 9.1).

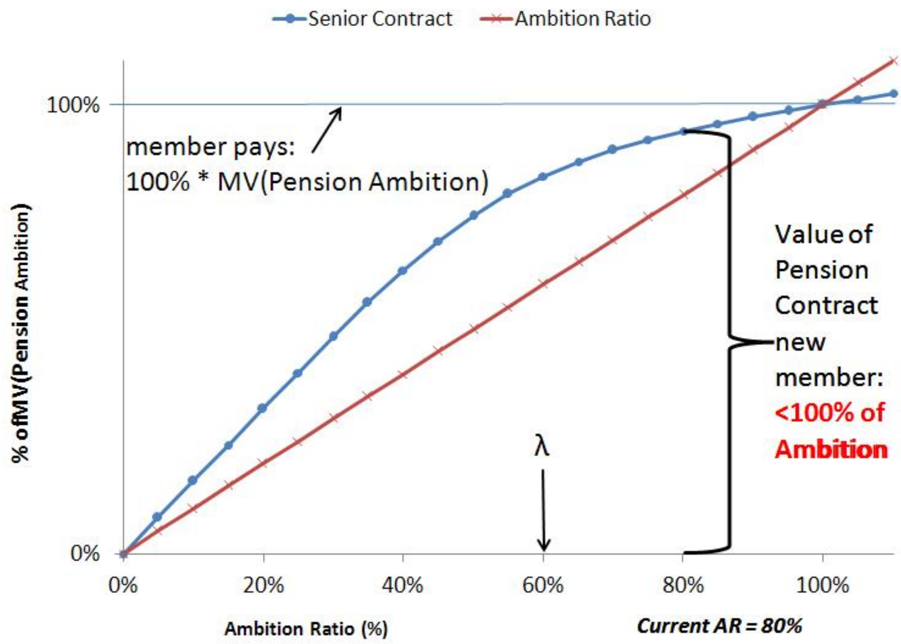

Figure 9.2: Unfairness for the new member in the redesigned fund.

The curved graph displays the value of the pension contract of the new member, dependent on the Ambition Ratio, in case no adjustments would be made. Just as for the early participants this contract would contain,

[8]Or that inflow into the pension fund equals outflow. These assumptions are not very strong.

besides the option construction[9], a Uniform Stake with a value dependent of the current Ambition Ratio. At $AR = 80\%$ however, the value of the pension contract is below the contribution, displayed by the horizontal line. This is due to the fact that the new member runs more 'default risk' at this funding level. After all, the Default Put is closer to the point of default λ.

On the other hand, if the Ambition Ratio would be above 100%, the value of the pension contract would be more than what the new member pays for. In this situation, the new member should pay more for his pension contract or receive pension rights corresponding to a smaller pension ambition.

Following condition (9.2) it is possible to determine the adjusted pension ambition, such that the value of the pension contract equals the contribution made by the new participant. This approach is depicted in Figure 9.3, in which several steps can be distinguished. These are discussed below.

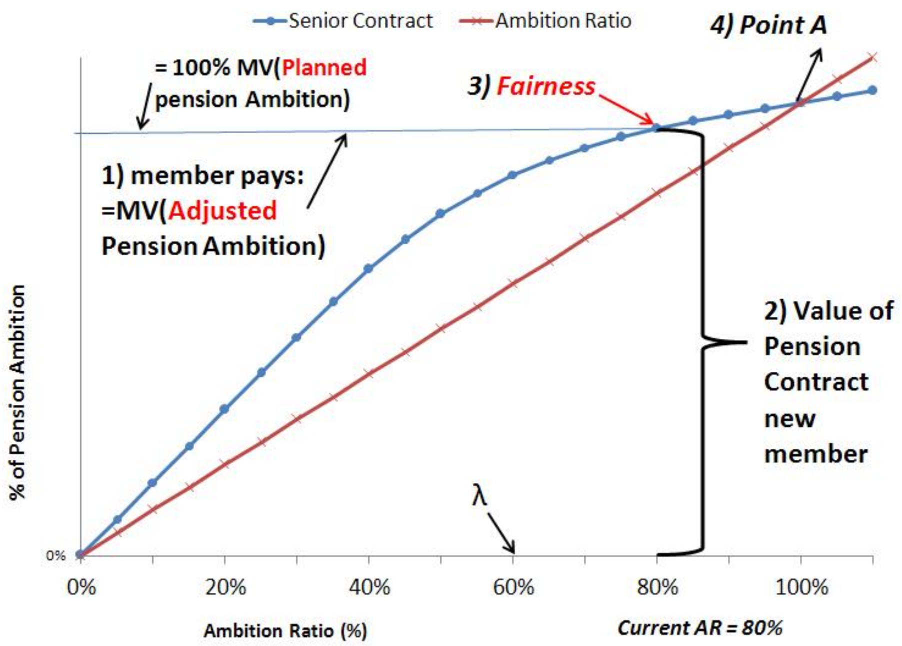

Figure 9.3: Fair inflow for the new member by determining the adjusted pension ambition.

The new member pays his planned pension contribution[10] in step 1. In exchange he receives a pension contract with equal value, which forms step 2. Hence, the pension contract has a fair price as can be seen in step 3. However, the specifications of the contract are based on the adjusted pension

[9]I.e. a Guarantee Put and a Guarantee Call both struck at $AR = 100\%$, a Default Put struck at λ and an Upside Call struck at λ^{-1}.

[10]I.e. 100% of the market value of his planned pension ambition.

ambition, which is given by point A in Figure 9.3. Point A needs to be determined to deduce the exact specifications of the pension contract, which forms step 4. This allows the fund's board to make the pension contract explicit by specifying the Uniform Stake and option construction for the new member. This needs some more attention.

The adjusted pension ambition for the new member k, expressed as percentage of the planned pension ambition, can be determined by the following formula :

$$\frac{RPA_{k,T}^{Adjusted}}{RPA_{k,T}^{Planned}} = \frac{1}{AR_{t_I} + OC_k(AR_{t_I}, T)}. \tag{9.3}$$

This formula is derived in Appendix A.3 taking condition (9.2) as a starting point. With the current fund settings (i.e. $\lambda = 60\%$ and $\sigma = 0.1$), the adjusted pension ambitions as percentage of planned pension ambition, are depicted in Figure 9.4 dependent on the Ambition Ratio.

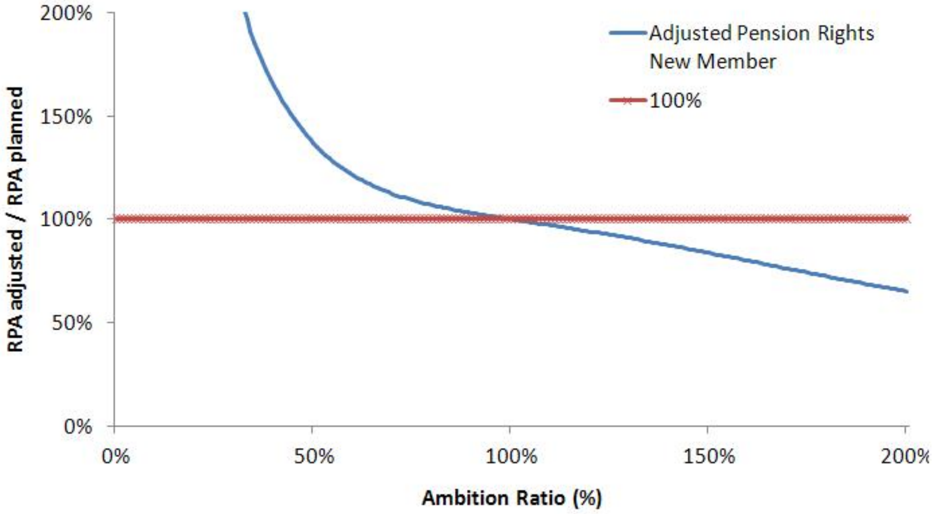

Figure 9.4: Adjusted pension ambition as percentage of planned pension ambition; $\lambda = 60\%$, $\sigma = 0.1$.

As one could expect, the adjusted pension ambition is close to 100% for Ambition Ratios near 100%. At an AR of exactly 100%, the value of the option construction is zero, so that no adjustment in pension ambition is needed. For very low values of the Ambition Ratio, the required extra pension rights increase rapidly; when new members join the senior tranche at poor funding levels, they have a lot of downside risk whereas claims on the upside of the fund are very far away. Compensation is hence needed via adjustments in the pension ambition. Remember that in the example, the fund's Ambition Ratio had dropped to 80%. From Figure 9.4, one can see that the new member claims the adjusted pension ambition equal to 106.9% of his planned pension ambition, to assure that inflow occurs without redistribution of wealth.

In line with the option analysis in Chapter 6, the adjusted pension rights also depend on the volatility σ, and on the seniority of the fund λ. As one expects, a higher volatility leads to larger adjustments in the pension ambition, since the probability of larger deviations from 100% Ambition Ratio increases hereby. A lower volatility on the other hand, leads to a considerable range of Ambition Ratios around 100%, in which the adjustments for the pension ambitions are only small. Both patterns can be seen in Figure 9.5.

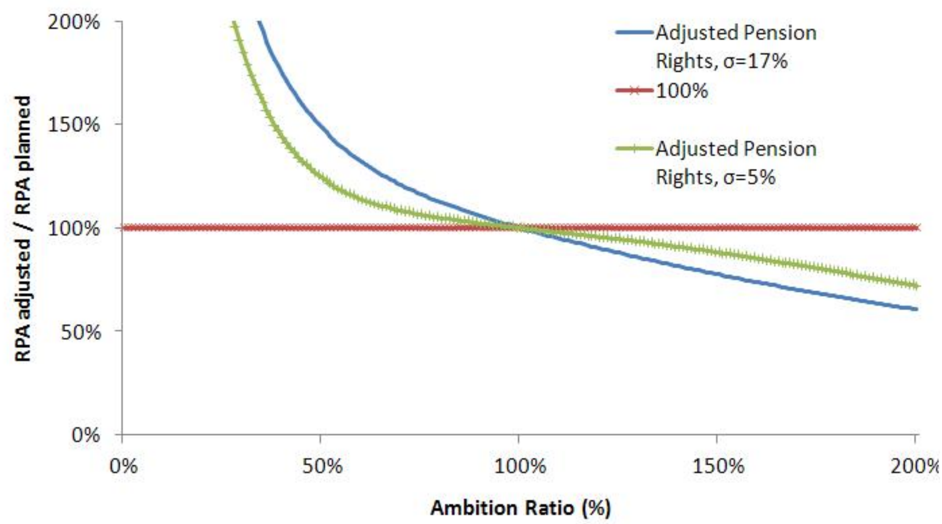

Figure 9.5: Adjusted pension ambition as percentage of Planned pension ambition; for the high volatility fund $\sigma = 0.17$ and for the low volatility fund $\sigma = 0.05$. Both fund have $\lambda = 60\%$.

Changing the seniority of the fund yields similar effects as for the fund volatility. Increases in λ lead to larger required adjustments. After all, for large λ the protection offered by the equity tranche is small, implying larger increases in the probability of default when the Ambition Ratio falls[11]. A lower value for λ has the opposite effect as can be seen in Figure 9.6.

[11]Naturally, same reasoning applies for the upside of the fund: also the probability is larger that the Upside Call will end up in the money.

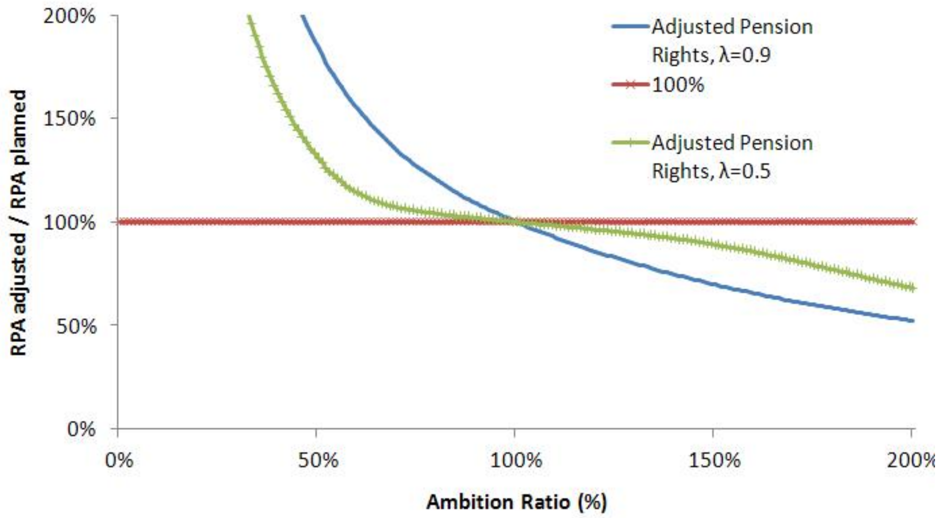

Figure 9.6: Adjusted pension ambition as percentage of planned pension ambition; for the high seniority fund $\lambda = 0.9$, for the low seniority fund $\lambda = 0.5$; $\sigma = 0.10$ for both.

The results regarding the seniority of the fund imply in addition, that the wealth redistribution for new participants can be considerable in pension funds that only have a small risk-bearing capacity (i.e. a small equity tranche or large λ). Although current funds have a somewhat different structure, they are indeed facing a decreasing risk-bearing capacity. So, Figure 9.6 forms an additional argument to take into account wealth redistribution at inflow, by compensating the concerning plan members accordingly.

9.4 The Pension Contract of the New Plan Member

To conclude the analysis of inflow in the redesigned pension fund, the exact composition of the pension contract of the new member is examined. Remember that the pension contract is set up following the four steps in Figure 9.3. The structure of the new pension contract is shown in Figure 9.7. Like for the early members the contract consists firstly of a Uniform Stake, which has the value:

$$\begin{aligned} US_{k,t_I} &= AR_{t_I} \cdot RPA_{k,t_I}^{Adjusted} \\ &= 80\% \cdot 103.6\% RPA_{k,t_I}^{Planned} = 82.9\% \cdot RPA_{k,t_I}^{Planned}. \end{aligned}$$

Secondly, it consists of the portfolio of options. This option construction has the same characteristics as that of early participants (i.e. Guarantee Put and Guarantee Call struck at 100% and the Default Put and Upside Call struck at respectively $\lambda = 60\%$ and $\lambda^{-1} = 150\%$. The value of the option construction is given by:

$$RPA_{k,t_I}^{Adjusted} \cdot OC(AR_{t_I}, T) = 17.1\% \cdot RPA_{k,t_I}^{Planned},$$

so that the value of the Uniform Stake and the option construction add up to the contribution $C_{k,t_I}^{Planned} = 100\% \cdot RPA_{k,t_I}^{Planned}$, paid by the new member.

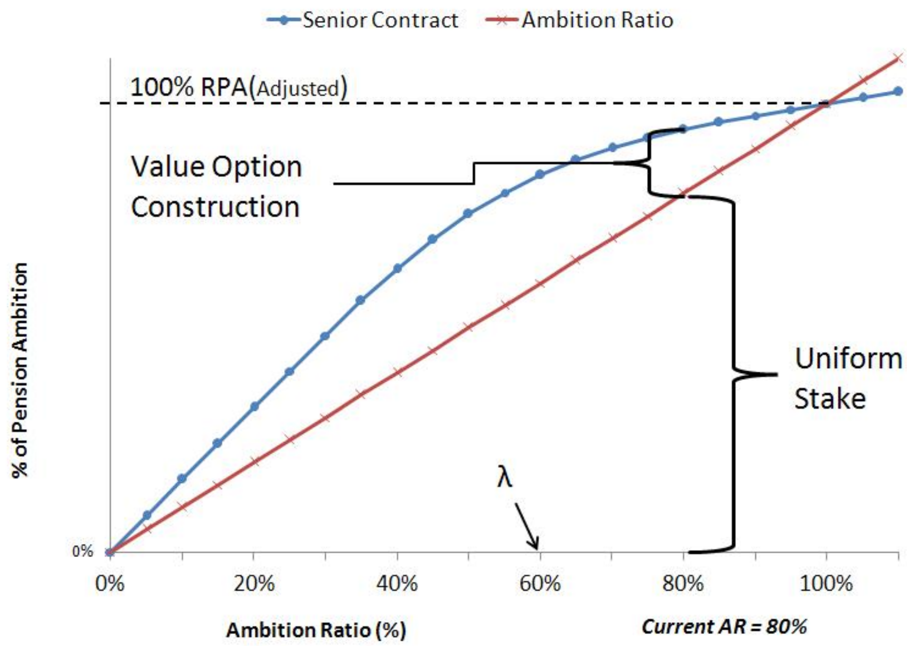

Figure 9.7: Pension Contract for the new member.

The option contracts, having positive value for the new (senior) member, have to be set up by trading with the equity tranche. So, part of the contribution of the new member is implicitly transferred to the equity tranche, which in exchange supplies the options contracts to the new senior member. In case the options have negative value for the new member (i.e. when $AR_t > 100\%$), the transfer takes place in opposite direction. Then, the equity tranche transfers cash to the senior tranche in exchange for the options which have the same positive value for the equity tranche[12]. So, also for the equity tranche wealth redistribution is absent. Nevertheless, the issuance of extra options leads inevitably to a change in the shape of the payoff profile

[12]The equity tranche always has enough wealth transfer money from equity to the senior tranche, since this only occurs in situations in which $AR_t > 100\%$.

for other values of AR than the current value AR_t [13]. This change however, will be very small since we assumed the pension fund is of considerable size.

9.5 Remarks on Fair Inflow

This chapter showed how the redesigned pension fund can deal with inflow of new members or new pension rights in the fund. The analyses complement the closed pension fund setting elaborated in previous chapters. The matter of fair inflow is considered to be troublesome when mutual risk sharing is present within pension funds (Kocken, 2006). The approach elaborated in this chapter regulates inflow in a fair way, viewed from the perspective of a new member. Moreover, this method prevents the origination of an administrative burden for the fund as new options have the same characteristics as the existing options. These points contribute to the sustainability of the fund design.

Fair inflow also opens up the possibility for (younger) members to steer the level of their pension via extra contributions (i.e. use the contribution instrument). In this case extra contributions do not fade away in a collective buffer. Recovery contributions then really serve to steer one's own pension instead of transferring (a large part) of them to other fund members[14].

[13] Technically, the leverage of the equity tranche relative to the asset portfolio of the fund increases a bit.

[14] Analogous to Goudswaard (2010), Box 8.1.

Chapter 10

Conclusions and Recommendations

In order to release pressure on Dutch second-pillar pension funds, rigorous changes in fund design are necessary. This thesis elaborates a pension fund redesign, making the pension contract explicit, complete, fair and sustainable. The core of the redesign is that pension rights within the fund are split up in tranches, each tranche with a different risk profile (e.g. senior debt and equity). Plan members distribute their pension wealth over these tranches. Shocks in funding level are transferred to the tranches according to a waterfall construction, in which risky tranches absorb the first shocks; pension rights in secure tranches are only affected when the risky tranches are completely wiped out or filled up. So, shocks are absorbed in order of seniority of the tranches. The essence of the waterfall is that is makes (intergenerational) risk sharing within a pension fund explicit, complete, fair and sustainable. Explicitness is a vital improvement over the current pension design, which is marked by indistinct pension contracts. The approach is moreover in line with recommendations of Bovenberg and Nijman (2009), Goudswaard (2010), Frijns (2010) en Boeijen et al. (2010).

The waterfall construction can be replicated via option contracts. The underlying of these options is the funding level of the pension fund. Valuing the options in a Black-Scholes settings allows for calculation of the fair upside threshold in the waterfall construction, so that the pension contract can be made fair for all plan members. Subsequent analyses serve to gain intuition about the redesigned pension fund: it appears that the seniority of the pension fund plays an important role regarding the characteristics of the pension contracts in the tranches. The fair waterfall construction ensures that, regardless of the composition of the fund, no plan member has to bear disproportionate risks in favor of other beneficiaries. Probability distributions for the final pension payout show that the waterfall construction has the desired effect; senior pension rights are very secure, whereas the distribution of the value of pension rights in the equity tranche is dispersed, compared to the pension rights within a basic CDC pension fund. Sensitivity analysis reveals the interest of different participants with respect to changes in funding level or fund volatility. The pension fund board can employ these sensitivities as a tool for taking particular decisions, such as carrying out a change in the investment strategy. Furthermore, inflow of new participants in a pension fund can lead to considerable wealth redistributions at high or low funding levels. Inflow into the redesigned fund however, takes place in a fair way. To achieve this, the amount of pension rights attributed to a new member are adjusted proportionally, dependent on the funding level.

All in all, the stylized fund redesign deals with important flaws in current pension design. The thoughts and analyses in this thesis can serve as a basis for factual redesign of Dutch second-pillar pension funds.

Finally, some limitations and recommendations regarding the pension fund redesign are mentioned. Concerning implementation of this redesign, questions arise about communication to plan members. Participants may have difficulties in understanding the pension contract. Therefore, advanced pension communication is needed to explain the ideas underlying the redesign in an effective way. Pension communication is however not just an issue in this particular pension redesign. Regarding the current, indistinct pension contract, improved pension communication is even more essential. Improved pension communication and changes in the way participants esteem their pension are necessary anyhow.

Abandoning the current pension design is not an easy task since sizeable transitional problems will arise. It seems impossible to make the transition to new systems without harming the existing pension rights of certain stakeholders within pension funds. Moreover, some legal changes may be required to allow for differentiated pension contracts (Goudswaard, 2010). However, such changes are necessary anyhow. Therefore, pension funds and pension regulators still face a huge and urgent challenge to make the pension system sustainable for the future.

In order to elaborate and clarify the redesigned pension fund, it has been essential to make some modeling assumptions. Future research could relax some of these assumptions, e.g. by dropping the assumption of deterministic liabilities. In that case the options within the waterfall construction transform into options to exchange the risky pension wealth at an exercise price indexed to the uncertain value of the liabilities. The options could be valued using arbitrage-free scenarios in order to design the waterfall construction fairly. Another possibility is to examine the pension fund redesign in a going-concern pension fund. This can be done by integrating inflow and outflow into the closed setting, that is primarily used in this research. We expect however, that conclusions regarding the working of this pension fund redesign will not be very different.

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Appendix A

This appendix contains derivations which are considered too long to appear in the main text.

A.1 Proof of the Put-Call Symmetry

The put-call symmetry (PCS) stems from the more general put-call transformation and is used for valuation and static hedging of certain exotic options on the basis of plain vanilla options (Haug, 1999). It is a useful property as it can reduce computational burden in implementation of option models. The PCS is also related to the well-known put-call parity.

In order to proof the Put-Call Symmetry within the Black-Scholes(BS) framework the BS pricing formulas for a European call and put option on a dividend paying stock are given. In line with the notation in Theorem 1, these values of a call and put option on stock S, with strike K, for volatility σ, dividend yield q and maturity $T-t$, in a economy with interest rate r are respectively given by:

$$\begin{aligned} C_t(S_t,K,r,q,\sigma,T-t) &= S_t e^{-q(T-t)}\Phi(d(S_t,K,r,q,\sigma,T-t)) \\ &\quad -Ke^{-r(T-t)}\Phi(d(S_t,K,r,q,\sigma,T-t)-\sigma\sqrt{T-t}), \\ P_t(S_t,K,r,q,\sigma,T-t) &= Ke^{-r(T-t)}\Phi(-d(S_t,K,r,q,\sigma,T-t)+\sigma\sqrt{T-t}) \\ &\quad -S_t e^{-q(T-t)}\Phi(-d(S_t,K,r,q,\sigma,T-t)), \end{aligned}$$

where $d(S_t,K,r,q,\sigma,T-t)$ is defined as:

$$d(S_t,K,r,q,\sigma,T-t) = \frac{\log(\frac{S_t}{K}) + (r-q+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}.$$

The first step to prove Theorem A is to show that:

$$\begin{aligned} d(K,S_t,q,r,\sigma,T-t) &= \frac{\log(\frac{K}{S_t}) + (q-r+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\ &= -\frac{\log(\frac{S_t}{K}) + (r-q+0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} + \sigma\sqrt{T-t} \\ &= -d(S_t,K,r,q,\sigma,T-t) + \sigma\sqrt{T-t}. \end{aligned}$$

Plugging in this result twice into $\Phi(.)$ in (A.1) for a call option in the auxiliary economy[1] gives the Put-Call symmetry as introduced in equation (6.1):

$$\begin{aligned} C_t(K, S_t, q, r, \sigma, T-t) &= Ke^{-r(T-t)}\Phi(d(K, S_t, q, r, \sigma, T-t)) \qquad \text{(A.1)} \\ &\quad -S_t e^{-q(T-t)}\Phi(d(K, S_t, q, r, \sigma, T-t) - \sigma\sqrt{T-t}) \\ &= Ke^{-r(T-t)}\Phi(-d(S_t, K, r, q, \sigma, T-t) + \sigma\sqrt{T-t}) \\ &\quad -S_t e^{-q(T-t)}\Phi(-d(S_t, K, r, q, \sigma, T-t)) \\ &= P(S_t, K, r, q, \sigma, T-t). \end{aligned}$$

A.2 Rewriting the Put-Call Symmetry

The aim of this section is to obtain the following result starting from the basic PCS from equation (A.1):

$$\lambda^{-1}P(S_t, \lambda S_t, r, \sigma, T-t) = C(S_t, \lambda^{-1}S_t, r, \sigma, T-t).$$

The steps in the following elaborations build on Haug (1999) and Detemple (1999). In line with the material in Section 5.3 the assumption is made that dividend yield q equals interest rate r so that q can be dropped out of notation for the prices for the call and put C and P. Now, rewriting of the payoff function for the European put with $\frac{K}{S_t}$ gives the following sequence:

$$\max(K - S_t, 0) = \frac{K}{S_t}\max(S_t - \frac{S_t^2}{K}, 0).$$

Hence, applying the Put-Call symmetry from (A.1) leads to the following expressions:

$$\begin{aligned} P(S_t, K, r, \sigma, T-t) &= \frac{K}{S_t}P(\frac{S_t^2}{K}, S_t, r, \sigma, T-t) \\ &= \frac{K}{S_t}C(S_t, \frac{S_t^2}{K}, r, \sigma, T-t). \end{aligned}$$

By setting the strike K equal to:

$$K = \lambda S_t, \text{with} \lambda \in [0, 1],$$

one can write the following:

$$\begin{aligned} P(S, \lambda S_t, r, \sigma, T-t) &= \frac{\lambda S_t}{S_t}C(S_t, \frac{S_t^2}{\lambda S_t}, r, \sigma, T-t) \\ &= \lambda C(S_t, \lambda^{-1}S_t, r, \sigma, T-t) \\ &= \lambda C(S_t, \lambda^{-1}, r, \sigma, T-t). \end{aligned}$$

[1] I.e. the economy with interest rate q and dividend yield r.

And by reshuffling, the result is obtained:

$$\lambda^{-1}P(S_t,\lambda,r,\sigma,T-t)=C(S_t,\lambda^{-1}S_t,r,\sigma,T-t). \tag{A.2}$$

Finally some intuition on this result as elaborated by Carr et al. (1998) is given. Note that the price of the call in (A.2) is larger than that of the put as $0 \leq \lambda \leq 1$. The reason the call has greater value, even though it is arithmetically further out of the money, is that when prices are high, the diffusion proces of the stock has greater absolute volatility. Since call and put values are determined by the arithmetic distance between S_T and strike price K, the higher absolute volatility at higher prices leads to larger values for the call option.

A.3 Inflow: Adjusted Pension Ambition

This section derives formula (9.3), stated on p. 70. The goal is to determine the extra pension rights a new fund member should receive or pay when he joins the fund in a state of under- or overfunding. This comes down to calculating the adjusted pension ambition $RPA_{k,T}^{Adjusted}$ as percentage of the Planned pension ambition. On this quantity the contract of a new member is based when he joins the fund at a certain Ambition Ratio AR_t. In order to achieve this, condition (9.2) from Section 9.2 serves as a starting point:

$$\begin{aligned} C_{k,t}^{Planned} &= (PC_{k,t}(RPA_{k,T}^{Adjusted}) \mid AR_t) \\ &= AR_t \cdot RPA_{k,t}^{Adjusted} + RPA_{k,T}^{Adjusted} \cdot OC_{k,t}(AR_t,T). \end{aligned} \tag{A.3}$$

Remember that $PC_{k,t}$ denotes the market value of the pension contract at time t and $OC_k(AR_t(AR_t,T)$ the value of the option construction for member k at time t. Moreover, note that the contribution $C_{k,t}$ is market based, so that we have:

$$C_{k,t}^{Planned} = RPA_{k,t}^{Planned} = \frac{NPA_k^{Planned} \cdot \prod_{j=1}^{t}(1+i_j)}{(1+ryc_{T-t,t})^{T-t}}. \tag{A.4}$$

In order to reshuffle the terms in (A.3), it is convenient to project the value of the pension contract to time T and substitute (A.4) for $C_{k,t}^{Planned}$. This leads to the following equation:

$$RPA_{k,T}^{Planned} = RPA_{k,T}^{Adjusted} \cdot AR_t + RPA_{k,T}^{Adjusted} \cdot OC_{k,T}(AR_t,t). \tag{A.5}$$

Rewriting (A.5) leads to the adjusted pension ambition as percentage of planned pension ambition:

$$\frac{RPA_{k,T}^{Adjusted}}{RPA_{k,T}^{Planned}} = \frac{1}{AR_t + OC(AR_t,T)}. \tag{A.6}$$

Appendix B

B.1 Additional Figures

Additional figures regarding the pension contract. Figure B.1 corresponds to Section 4.4.1:

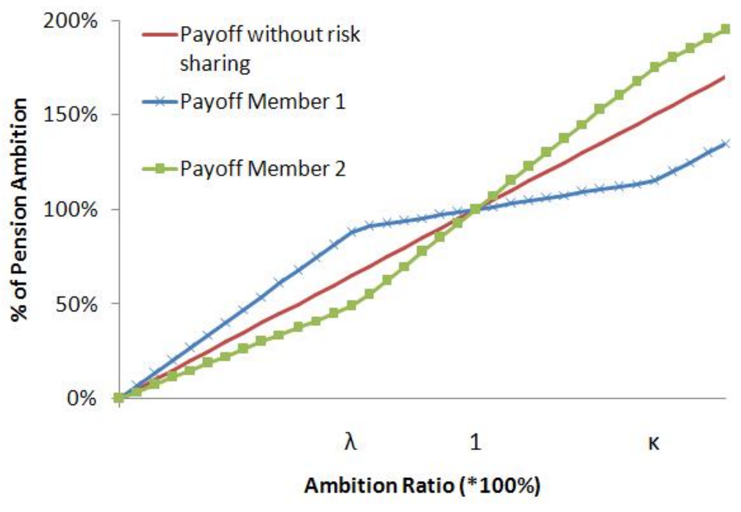

Figure B.1: Payoff profiles for two members who have allocated their wealth over the equity and senior tranche. Member 1 has allocated 90% to senior, 10% to equity; Member 2 has allocated 50% to senior and 50% to equity.

The following figures correspond to Section 6.2. They provide insight regarding the shape of the pension contract for different fund volatility at start of the fund horizon and when time to maturity is 1 year. Distinction is made between a low volatility fund ($\sigma = 0.05$), and a high volatility fund ($\sigma = 0.17$). The medium volatility case ($\sigma = 0.1$) is already treated in the main text.

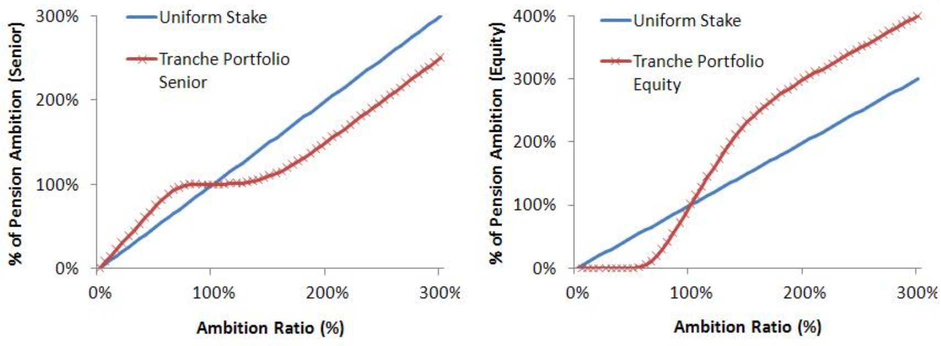

Figure B.2: For a low volatility pension fund ($\sigma = 0.05$): the value of the pension contract in both tranches at time 0, expressed as % of pension ambition.

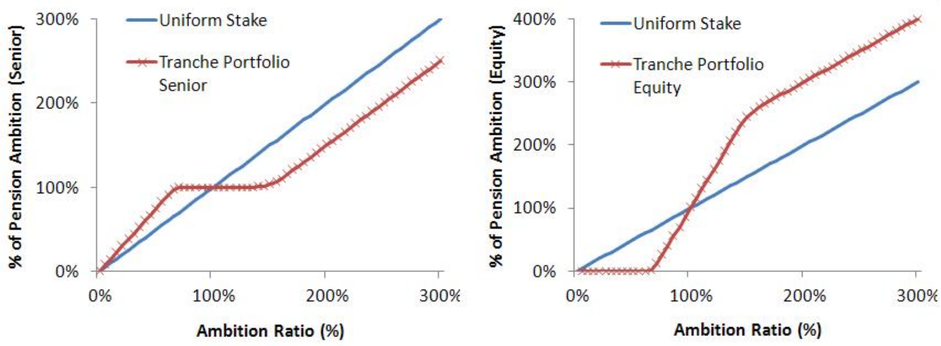

Figure B.3: For a low volatility pension fund ($\sigma = 0.05$): the value of the pension contract in both tranches at time 9, expressed as % of pension ambition.

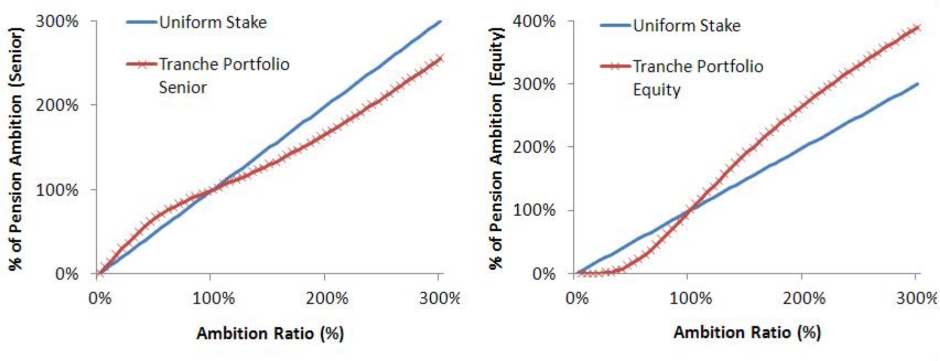

Figure B.4: For a high volatility pension fund ($\sigma = 0.17$): the value of the pension contract in both tranches at time 0, expressed as % of pension ambition.

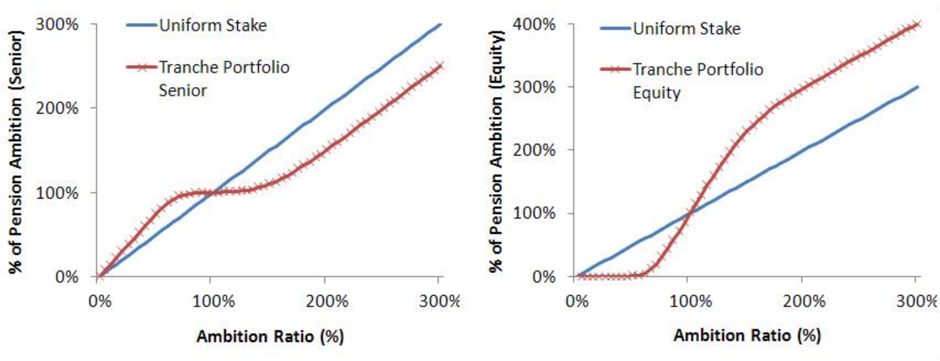

Figure B.5: For a high volatility pension fund ($\sigma = 0.17$): the value of the pension contract in both tranches at time 9, expressed as % of pension ambition.

Following figures correspond to Chapter 8. They show the percentiles of the value of the pension contract in the different tranches over time.

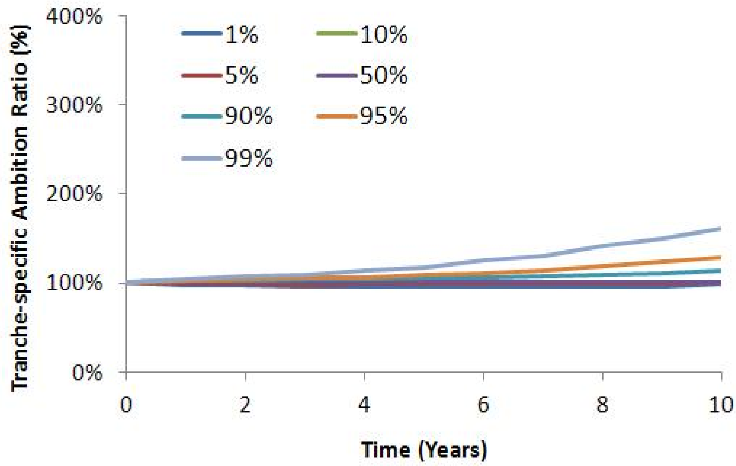

Figure B.6: Percentiles for the value of the pension contract in the senior tranche.

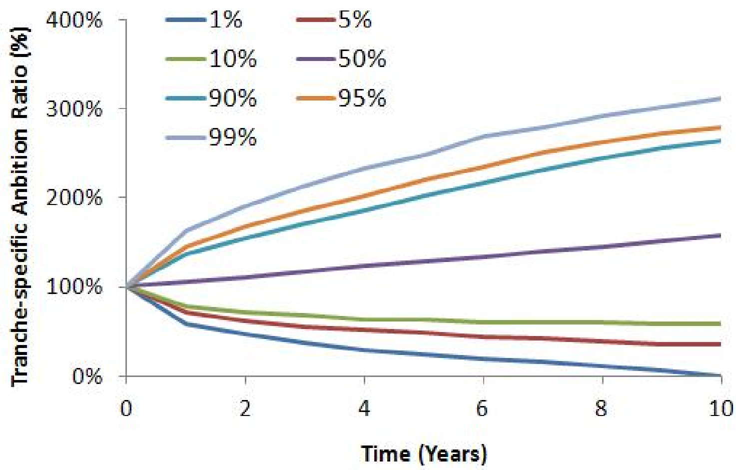

Figure B.7: Percentiles for the value of the pension contract in the equity tranche.

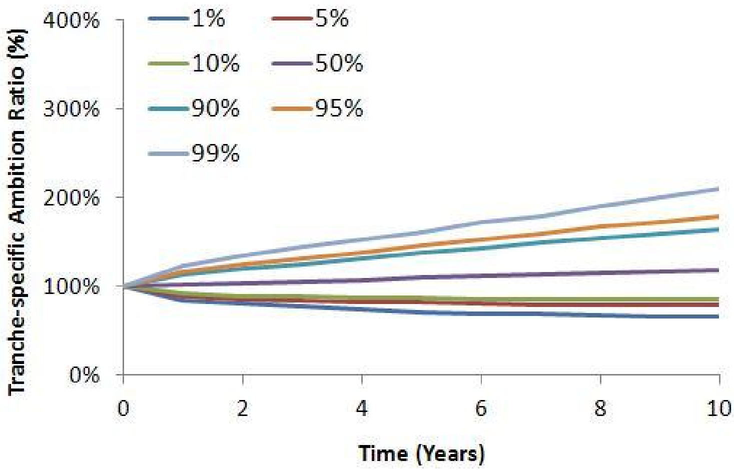

Figure B.8: Percentiles for the value of the pension contract in the basic CDC pension fund.