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# The Eligibility and Optimization of the Life-Cycle Approach for Long Investment Horizons

A Case Discussion Based on the Nuernberger  
Life-Cycle Product

MAASTRICHT UNIVERSITY

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***The Eligibility and Optimization of the Life-Cycle Approach  
for Long Investment Horizons***

***- A Case Discussion Based on the Nuernberger Life-Cycle Product -***

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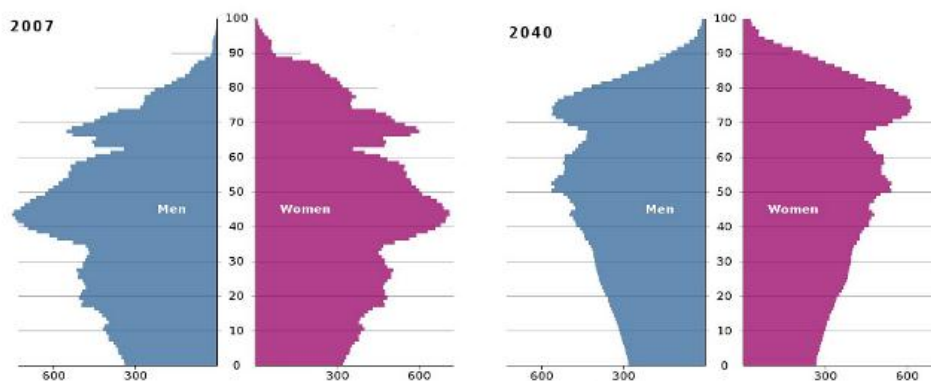
## Abstract

This thesis intends to answer the question if and how term-structure and human capital idea should be combined within a long-term investment portfolio strategy. With the help of a case study on an existing life-cycle investment product (Nuernberger Beteiligungs-Aktiengesellschaft), this paper shows why it might be beneficial to use an alternative approach to the traditional Markowitz portfolio optimization for long holding periods. The results show that, following the current product prospectus, the life-cycle portfolio tends to be preferable to the Markowitz portfolio alternative over the analyzed time frame. In addition, this paper states that the specific life-cycle product uses the theoretical foundation of term-structures in asset risks and returns. Extending the analysis by a human capital discussion based on statistical labor market data gives reason to assume that the product outcome might be improved by a hump shape equity position.

## 1 Introduction

In Germany, there exists a three-pillar system of retirement provisions (Lehmann, 2011). The first and traditional one is the pillar provided by the German state. Employees pay a percentage of their gross income into the retirement system each month during their whole work lives. With that money, former employees, which are already retired, are paid a monthly retirement annuity depending on their former labor income. With that payment, the retirees cover their living expenses during retirement. Since one generation always pays for the older one and does not save for its own retirement, the German retirement model is an *unfunded* system (Munnell, 2006).

**Figure 1: Demographic Change in Germany**



Source: Statistisches Bundesamt (2007)

As many other industrial countries, Germany is facing the problem of *demographic change*, or an aging population (Figure 1). The first problem is that the birth rates in Germany decreased significantly over the last 50 years and are currently at about 1.4 children per women (Statistisches Bundesamt, 2009). Consequently, there is a decreasing number of employable people paying into the system. In addition, retired people are becoming older due to better medical care and nutrition. As a result, the gap between contributions and expenses increases (Geissler & Meyer, 2006; Indenso, 2011). For current and future employees, that means that they receive much less money after retirement than during work life, in particular when considering inflation. As a result, they are most likely not able to keep their living standards.

The consequence of the development described above is that the second and the third pillar of the German retirement system, namely occupational and private pension schemes, become more important. Although also applicable for the occupational pension system, this paper focuses on the challenges and opportunities of efficient private retirement arrangements. Whereas it is obligatory to participate in the federal retirement system, saving privately is the employees' own authority. Investors have to decide on how to set up their savings plans and how to combine the investments they make in the most efficient way. The average private investor is, however, overstrained by constructing an optimal investment portfolio. For many private investors, it suggests itself to decide on a portfolio composition once based on the recent market developments and personal circumstances and keep that portfolio over long investment horizons. This way of an optimal asset allocation is already well known in the literature. A famous scientist in the field of portfolio optimization theory is Markowitz. In his paper, *Portfolio Selection*, published in 1952, he developed a model for optimal asset allocations. The Markowitz model is a one-period optimization model building on constant expected returns and annualized variances. Markowitz assumes that the characteristics of single assets and their combination do not change over time. Consequently, investors following this approach simply choose an asset allocation based on short-term performance data and keep it over the whole holding period. This is, however, assumed to be insufficient. The single assets as well as their combinations show varying characteristics over time. On top, the investors' life situations, including health, job status, and labor income change over time. Consequently, making investment decisions on a regular basis is more appropriate, but time-consuming, often costly, and requires market knowledge. "Most individuals lack the experience or training to make wise saving, investment, and withdrawal decisions" (Munnell, 2006, p. 377).

Although not explicitly developed for retirement arrangements, the *life-cycle* model offers a potential solution to the difficulties named above, by building on the combination of individuality and standardization for long-term investment decisions. Life-cycle funds are time-varying investment portfolios, which take into account varying demands and circumstances over the investments' life-cycle influencing the optimal portfolio structure. The composition of life-cycle funds follows an automated and predetermined investment schedule. All life-cycle funds start with an initial strong tilt towards stocks, which shifts automatically to a more conservative bond portfolio once the investment approximates its maturity date. In sum, it offers investors a cost- and time-efficient one-stop solution without requiring further market knowledge, and considers simultaneously specific characteristics like the investors' age and investment horizon, respectively, as well as major optimization theories for long-term investments.

Theoreticians agree on the fact that investment portfolios have to be structured differently when set up for a long investment horizon than those for the short term. They also agree on a strong tilt towards equities for portfolios with long maturities (Bodie, 1995; Campbell & Viceira, 2005; Malkiel, 1996; Viceira, 2008). "The longer the time period over which you can hold on to your investments, the greater should be the share of common stocks in your portfolio" (Malkiel 1996, pp. 404–405). The theoretical foundations of these opinions, however, differ. The first theory is the idea of *term-structures* in asset variances, returns, and correlations. It argues that Markowitz uses the insufficient assumption of constant expected returns and annualized variances. Markowitz assumes for his one-period model *independent and identically distributed* asset returns, meaning that all return variables are mutually independent and equally probable, so that expectations on returns cannot change over time (Campbell & Viceira, 2002, p. 27). These *unconditional* expectations based on short-term data usually lead to the result that stocks are riskier than bonds. Campbell and Viceira (2005), however, conduct a regression analysis of real return variables on their lagged values and find strong persistency. Persistency in variables implies that they are to some extent predictable. Campbell and Viceira (2005) find that using *conditional* expectations shows that the risk of stocks decreases and the risk of bonds increases in the long term. Consequently, they recommend a strong tilt towards equities for portfolios with long maturities.

Another perspective taken for long-term investments is the *human capital* idea. It says that an investor should not optimize his *financial wealth* portfolio, but his *total wealth* portfolio. This

total wealth portfolio consists of financial wealth and human wealth. Human wealth, or human capital, describes the discounted expected future labor income (Viceira, 2008). Its value depends on several factors like education, job choice, and health care. Human wealth is highest at the beginning of the employees' career when all labor income payments are still ahead. Over time, it reduces with wage payments until retirement. Since human capital provides regular and secure income streams, it seems to be very similar to the coupon payments of a bond position. Theoreticians state that due to this bond-like position within the total wealth portfolio, the investor can afford to tilt the financial portfolio towards equities. Over time, when the human capital is reduced, the financial portfolio should be shifted to a more conservative one.

The main goal of this paper is to answer the question if and how the term-structure and the human capital idea should be combined within a long-term investment portfolio strategy. Although the two arguments lead to similar portfolio compositions, they build on completely different theoretical foundations, so that the detailed implications for the portfolio strategy can vary significantly. This divergence in the literature results in imprecise recommendations for providers of long-term investment products. It is not clear, which theory holds or if they can even be combined or, stated differently, which theoretical aspects are actually valid. It might be that providers decide to consider only the obvious and least complex factors for their long-term investment products. This can, however, lead to suboptimal portfolio compositions and strategies.

To answer the problem statement of this paper appropriately, the thesis works through a series of sub-questions. As a starting point, this paper describes in detail the assumptions, on which the Markowitz approach builds, and why they are not appropriate for long investment horizons. This critique of the Markowitz approach requires alternative solution statements. Consequently, this paper provides insights into two main theoretical portfolio optimization approaches for long investment horizons, namely the term-structure and the human capital approach. The second one, the idea of human capital, goes along with the basic concept of life-cycle investments. Therefore, this paper discusses to what extent the basic life-cycle idea might be a sufficient alternative to the Markowitz approach. The thesis, however, also includes a case study part. In this part, the paper answers the question how the theoretical idea is applied in practice with the help of a product launched by the German insurance company Nuernberger Beteiligungs-Aktiengesellschaft (hereafter: Nuernberger). It is clarified on which theoretical argument the product is actually based and if there is any eligibility for this prod-



uct as it is in comparison to the Markowitz approach. Finally, this paper provides hints how the product might be improved based on statistical data.

This thesis intends to bring together two divergent theoretical solution statements on the same problem. It tries to clarify, how and if the two arguments can be combined or if they exclude each other. By doing that, the paper's goal is to develop one theoretical foundation, which is robust to both perspectives. The paper, however, does not only provide an academic contribution. It intends additionally to provide clear and applicable recommendations to providers of life-cycle funds on how to structure their products, so that they meet their clients' requirements.

The remainder of the paper is divided into five parts. The following part is an overview of the theories already available in the literature. It starts with a discussion on the traditional portfolio optimization theory developed by Markowitz. In the first subsection, it is explained how the concept actually works and how the model builds on well-known theoretical foundations. In the second subsection, it is explained why the Markowitz is insufficient for long-term investment horizon based on the figures used in the model. In the second part of the first chapter, two different approaches to deal with long investment horizons are introduced. In the first subsection, the idea of a term-structure in asset returns is explained and discussed based on a paper by Campbell and Viceira (2005). In the second subsection, the perspective is changed by introducing the idea of human capital as non-tradable. This section explains the basic characteristics of human capital and why it should be treated as an asset. This idea is adopted by the basic life-cycle investment model, which is illustrated in the third part of this chapter. This part starts with an explanation of how the idea of human capital leads to the assumption of a total wealth portfolio, which should be optimized instead of the financial portfolio alone. In addition, it is explained in detail what that means for the composition of a basic life-cycle fund. In the following subsections, it is discussed how the model contributes to the challenges of long-term investments, but also which aspects of the basic idea are already criticized by other authors. To provide a first bridge between the two perspectives of long-term investing, an extra section with comparing thoughts is provided. With the third chapter, the paper enters a case study discussion based on the life-cycle model of the German insurance company Nuernberger. The following three chapters are all broadly separated into two main analyses. The first one compares investment performance of the Markowitz and the Nuernberger life-cycle approach based on the same investment funds and time period. The second one adds an analy-

sis of human capital based on recent statistical data. In the first part of the third chapter, the individual characteristics of the Nuernberger life-cycle are described and compared to the theoretical life-cycle idea. The second and the third part intend to provide a description of how the analyses are conducted. The fourth chapter provides the results of the analyses, which are then interpreted in the fifth chapter. The paper concludes with a summary of the paper and recommendations for an improved life-cycle model. In addition, the conclusion refers to limitations of the analyses and hints for further research.

## 2 Literature Review

### 2.1 The Markowitz Approach Under Criticism

The reason, why this paper discusses life-cycle funds as potential long-term investment, is that the traditional and well known portfolio optimization approach by Markowitz is said to be inappropriate for long holding periods. To better understand that critique and to learn more about the methodology used for the analytic part of this chapter, this section provides a detailed description of the Markowitz model. Before introducing long-term investment approaches, it is also explained why the Markowitz approach might be insufficient.

#### 2.1.1 The Idea of Markowitz Portfolio Optimization

When investors decide on a capital asset, they usually look at the key performance indicators, past return and variance. From that, they make assumptions on the asset's future development. If investors have to choose among two investment alternatives, the *risk / return-principle* says that investors should choose the investment, which shows either for the same or lower risk a higher expected return or for the same or higher return a lower risk (Elton, Gruber, Brown, and Goetzmann, 2003, p. 44).

Markowitz (1952) builds on the same idea, but extends the case to portfolios of single assets. In his paper, *Portfolio Selection* (1952), he intends to find *mean-variance efficient* combination of assets so that the investors' utility is maximized. Depending on the investors' goals, the utility is maximized by finding either the highest return possible for a given risk or the lowest level of risk for a given amount of return. The reason to invest in portfolios instead of single assets is the potential to find investments, which are more mean-variance efficient than single assets due to *diversification* effects. Diversification describes the potential of reducing the risk of portfolios by combining assets, which are not perfectly correlated and which do not react to movements in the market in the same way respectively (Levy & Sarnat, 1970, p. 668). Asset classes are defined as groups of single assets, which are highly correlated, whereas assets from different asset classes are rather uncorrelated (Brealey, Myers, Marcus, Maynes, and Mitra, 2006, p. 251). As a consequence, assets from diverse asset classes tend to react differently to market movements. By investing in portfolios, investors can benefit from the fact that usually at least part of the assets included performs well regardless of how the market moves. Since the overall portfolio does not react to market movements as strongly as collections of the same single assets do, the overall risk is reduced while keeping the same level of return.

The explanation above implies that the performances of portfolios and their risk patterns depend mainly on the way the included assets move together. The return of a portfolio ( $\mu_p$ ) is the weighted sum of the single asset returns ( $\mu_i$ ).

$$\mu_p = \sum_{i=1}^N w_i \mu_i \tag{2.1.1.1}$$

The effect of diversification becomes obvious when calculating the risk, or the variance, of the portfolio  $\sigma_p^2$ . The risk of a portfolio is not only the average of the risk on individual assets. It depends on whether the returns on single assets tend to move together. There is a risk reduction from holding a portfolio of assets if the assets do not move in perfect unison (Elton et al., 2003, p. 44). The variance of a portfolio depends on the weights of the  $n$  single assets the portfolio invests in, but mainly on the covariances between these single assets. This can be seen with the help of a *variance-covariance-matrix* (Table 1), a matrix consisting of the covariances between single assets of a portfolio. Since the covariance between two identical assets is calculated as well and  $Cov(i, i) = Var(i)$ , the main diagonal of the matrix consists of the single assets' variances.

**Table 1: Variance-Covariance-Matrix**

Security weights in portfolio	$w_1$	$w_2$	$w_3$	...	$w_n$
$w_1$	$w_1^2 \sigma_1^2$	$w_2 w_1 \sigma_{21}$	$w_3 w_1 \sigma_{31}$		$w_n w_1 \sigma_{n1}$
$w_2$	$w_1 w_2 \sigma_{12}$	$w_2^2 \sigma_2^2$	$w_3 w_2 \sigma_{32}$		$w_n w_2 \sigma_{n2}$
$w_3$	$w_1 w_3 \sigma_{13}$	$w_2 w_3 \sigma_{23}$	$w_3^2 \sigma_3^2$		$w_n w_3 \sigma_{n3}$
...					
$w_n$	$w_1 w_n \sigma_{1n}$	$w_2 w_n \sigma_{2n}$	$w_3 w_n \sigma_{3n}$		$w_n^2 \sigma_n^2$

Source: Own Table, adapted from Bodie, Kane, and Marcus (2001, p. 211)

The sum of the individual terms of the variance-covariance-matrix can also be expressed by the following formula written in single summand or matrix notation

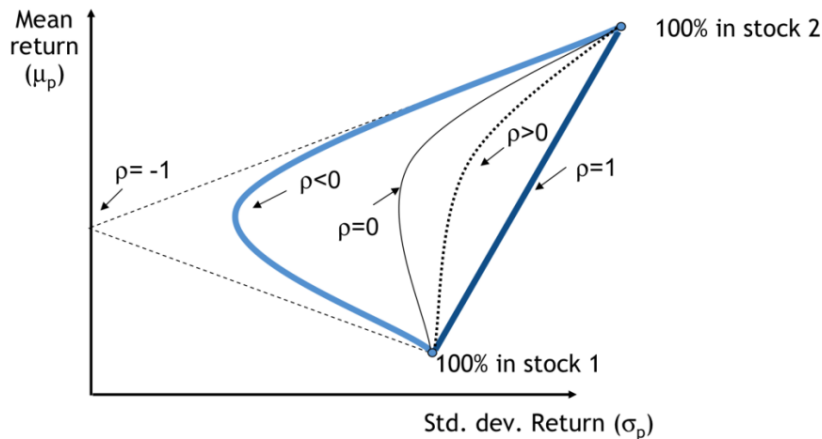
$$\begin{aligned}
\sigma_P^2 &= w_1 \sum_{j=1}^n w_j \sigma_{1,j} + w_2 \sum_{j=1}^n w_j \sigma_{2,j} + \dots + w_n \sum_{j=1}^n w_j \sigma_{n,j} \\
&= \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{i,j} = w' \Sigma w
\end{aligned}
\tag{2.1.1.2}$$

where  $\sigma_P^2$  is the portfolio's variance,  $w_i$  and  $w_j$  stand for the security weights in the portfolio illustrated in the  $i$ -th column or  $j$ -th row of the variance-covariance matrix respectively,  $\sigma_{i,j}$  describes the covariance between the  $i$ -th and the  $j$ -th asset,  $w$  determines the  $n$ -dimensional vector of asset weights in the portfolio,  $w'$  is the transposed vector of weights and  $\Sigma$  is the  $n \times n$  dimensional variance-covariance matrix. The second part of the formula denotes the calculation of a portfolio's variance with the help of a multiplication of matrices. The covariances of the single assets described by  $\Sigma$  are weighted by the portion they have within the portfolio and can alternatively be illustrated by single summands, like it is done in the first part of the equation. Due to the rules of matrices multiplication, the outcome is a single number, namely the variance of the portfolio.

To show the effect of imperfect correlation between single assets on the risk of a portfolio, the case of a two-assets-portfolio is presented.

$$\begin{aligned}
\sigma_P^2 &= \sum_{i=1}^2 \sum_{j=1}^2 w_1 w_2 \sigma_{1,2} = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{1,2} \\
&= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}
\end{aligned}
\tag{2.1.1.3}$$

where  $\rho_{1,2}$  is the correlation between asset 1 and asset 2. Combining two identical assets, which inherently show a perfectly positive correlation, means that the portfolio's variance is at its maximum. The combination of two assets, which are positively but not perfectly correlated, still contributes some risk to the portfolio, but has diversification effects in comparison to a pure one-asset portfolio. The more the assets are correlated, the riskier the portfolio becomes. If the two assets are negatively correlated to a certain extent, the diversification effect becomes even more obvious.

**Figure 2: Diversification Effects of Combining two Assets**

Source: Adapted from Bodie et al. (2001, p. 225)

Figure 2 shows how the correlation between two assets contributes to the performance of a portfolio. Compared to case of positive correlation, in the case of negative correlation a higher return can be generated for the same risk. Alternatively, the same return can be generated for a much lower risk. *Efficient frontiers* describe all mean-variance efficient portfolios given the performance data of the underlying single assets, meaning portfolios with the highest return possible for a given level of risk or portfolios with the lowest risk possible for a given return. Portfolios lying below the efficient frontier are inefficient in the way that they could generate higher returns for the same risk or the same returns for a lower risk. No portfolio can lie above the frontier, since it would offer a more efficient combination of risk and return than the efficient frontier does. The efficient portfolio with the lowest risk possible is called *global minimum-variance portfolio*.

To derive analytically the frontier of mean-variance efficient portfolios according to the Markowitz (1952) idea, the variance ( $\sigma_p^2$ ) must be minimized for given portfolio returns ( $\mu_p$ ). For the derivation of efficient frontiers, the variance formula (2.1.1.2) is used in its matrix notation.

$$\min_w \frac{1}{2} w' \Sigma w \quad (2.1.1.4)$$

$$\text{so that } 1'w = 1$$

$$\bar{\mu}'w = \mu_p$$

The constraints state that the weights need to sum to one and that the minimization of the variance of the portfolio is done with respect to the given return  $\mu_P$ . Optimization with constraints works with the Lagrangian condition

$$L(w, \lambda, \gamma) = \frac{1}{2} w' \Sigma w - \lambda (1' w - 1) - \gamma (\bar{\mu}' w - \mu_P) \quad (2.1.1.5)$$

For the derivation of a minimum variance, the first order conditions of the Lagrangian formula need to be calculated.

$$\frac{\partial L}{\partial w} = \Sigma w - \lambda 1 - \gamma \bar{\mu} = 0 \quad (2.1.1.6a)$$

$$\frac{\partial L}{\partial \gamma} = 1' w - 1 = 0 \quad (2.1.1.6b)$$

$$\frac{\partial L}{\partial \lambda} = \bar{\mu}' w - \mu_P = 0 \quad (2.1.1.6c)$$

The optimality set ( $w_i^*$ ) can be derived from the first of the first order conditions (2.1.1.6a) and equals the formula

$$w_i^* = \lambda \Sigma^{-1} 1 + \gamma \Sigma^{-1} \bar{\mu}_i \quad (2.1.1.7)$$

The weights of the different assets within the portfolio can be calculated by entering the respective mean return data over the observed time period ( $\bar{\mu}$ ). By that, for each asset its optimal fraction within the portfolio for a minimized portfolio can be calculated.  $\lambda$  and  $\gamma$  can be derived from the constraints so that

$$\lambda = \frac{C - \bar{\mu} B}{D} \text{ and } \gamma = \frac{\bar{\mu} A - B}{D} \quad (2.1.1.8)$$

with  $A = 1' \Sigma^{-1} 1$ ,  $B = 1' \Sigma^{-1} \bar{\mu}$ ,  $C = \bar{\mu}' \Sigma^{-1} \bar{\mu}$ , and  $D = AC - B^2$ .

To obtain the respective values for the efficient frontier analysis, the minimum variance for given portfolio returns must be determined. This can be done by the formula

$$\begin{aligned} \sigma_P^2 &= w' \Sigma w = w' \Sigma (\lambda \Sigma^{-1} 1 + \gamma \Sigma^{-1} \bar{\mu}) = \lambda w' 1 + \gamma w' \bar{\mu} = \lambda + \gamma \mu_P \\ &= \frac{A \mu_P^2 - 2B \mu_P + C}{D} \end{aligned} \quad (2.1.1.9)$$

For the plotting of efficient frontiers, usually the standard deviation  $\sigma_p = \sqrt{\sigma_p^2}$  is calculated from the variance data. (Ingersoll, 1987, pp. 82-85)

### 2.1.2 The Markowitz Approach and Long Investment Horizons

The detailed description of the Markowitz idea provided in the previous section detects the two main problems of using that approach for long-term investments. The first one puts the mathematical set-up of the model into question by stating that Markowitz as a static one-period model might provide insufficient results for long-investment horizons. The second one criticizes the Markowitz perspective as an only financial one, whereas it states that a sufficient wealth perspective should include human wealth as well.

There are several problems of transferring the short-term oriented Markowitz model to long-term optimization problems, which are related to the mathematical set-up of the model. The main components feeding into the model are the expected returns of the assets involved and their variances. The Markowitz approach uses a rather short observation period to form expectations on the average returns and their variances (Markowitz, 1952, p. 77). When extending the investment horizon, the expected returns and annualized variances stay constant and equal to the short-term figures. This implies that the Markowitz approach works with unconditional expectations on returns and variances, which are independent of each other and of the investment horizon. As a consequence, Markowitz assumes that the correlation structure of asset classes within a portfolio stays constant with changing holding periods as well. Assuming that, the long-term optimized portfolio is equivalent to the short-term one. Campbell and Viceira (2005, p. 35), however, state that expectations might change conditionally upon predictable changing state variables along the investment horizon. Changing expected returns and conditional variances change imply that the correlation structure between assets changes with the investment horizon as well, which results in varying optimal portfolio compositions. A “static portfolio analysis is not only inappropriate in theory, but can be seriously misleading in practice, when investment opportunities are time-varying and investors have long time horizons” (Campbell & Viceira, 2002, p. 67). Another problematic assumption related to long-term asset characteristics is the risk-free rate, which is used in Tobin’s (1958) extension of the Markowitz approach. Tobin (1958) suggests that all investors should hold the same risky portfolio and combine it with the risk-free cash asset depending on the level of risk aversion. Scientists like Viceira (2008, p. 7) argue that the Tobin approach is not valid for long-term in-



vestments, since the risk-free rate is actually far from being risk free over long investment horizons. Working with a risk-free rate for long-term investments might underestimate reinvestment risks. As a consequence, suggesting a universally valid market portfolio to the long-term investors is incorrect, since the individual level of risk-aversion needs to be included directly in the portfolio set-up.

The second aspect, which has to be taken into consideration, is that the Markowitz approach only deals with the financial wealth of investors. That means that the Markowitz as a one-period model takes the investor's budget as given and provides a static way of allocating this budget. Due to Viceira (2008), one has to consider human wealth in addition to financial wealth when setting up portfolios. Financial wealth is the budget the Markowitz approach works with. Human wealth is the expected money earned in the future, which depends on several factors illustrated in section 2.2.2. Viceira (2008) argues that one has to optimize the total wealth portfolio, including financial and human wealth, instead of only the financial wealth portfolio. In addition, financial wealth is not constant over time, since the money earned can be invested again. Consequently, human wealth is transferred into financial wealth over long investment horizons, which results in the fact long-term investors can benefit from *cost average effects*. These are based on the idea that investors can benefit from intermediate fluctuations of the investment's market price when investing on a regular basis instead of a lump-sum. The reason is that investors buy many units of the asset when prices are low and only a few units when prices are high. In sum, "the average purchasing price of the asset is below the average security price of the investment period, so that the investor generates positive returns" (Steinberger, personal communication, June 14, 2011).

## **2.2 The Driving Forces of Long Investment Horizons**

This section illustrates two potential driving forces of long-term investments, namely term-structures in asset risks and returns, and human capital as additional wealth component. The first one is criticized by Pástor and Stambaugh (2011), who state that the term-structure argument provided by Campbell and Viceira (2005) ignores uncertainty risks. The basic human capital idea is criticized and improved in section 3. All ideas, however, lead with very different arguments to a tilt towards equities for long term investments. In section 4 and 5, this paper analyzes if and to what extent the arguments are considered in a practical example and how the model could account for them in the best way possible.

### 2.2.1 Term-Structure of Asset Risks and Returns

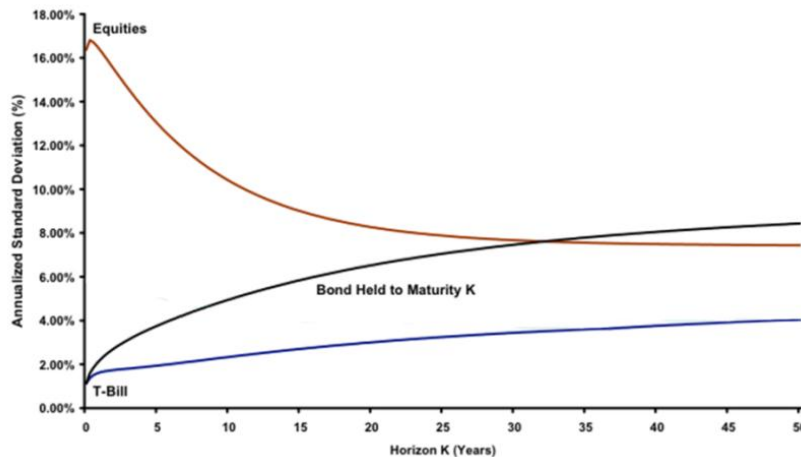
As already mentioned in section 2.1.2, there are reasons to assume that expected returns, annualized variances, and correlations between major asset classes change with the investment horizon. Alternatively, it is argued that the asset classes' risks, returns, and correlations follow a term-structure. The underlying reasons for this argument are illustrated in more detail for stocks, bonds, and the risk-free rate in this section.

The term-structure idea is known particularly from interest rates and describes the maturity-dependent level of yields. "The term-structure of interest rates measures the relationship among the yields on default-free securities that differ only in their term to maturity" (Cox, Ingersoll, and Ross, 1985, p. 385). Campbell and Viceira (2005) introduce this term for annualized standard deviations and returns of traditional asset classes, which differ only in their holding periods as well. What both of these applications have in common is that they assume predictable and time-varying changes in the level of the analyzed figure. While a yield curve describes the predicted yield for varying bond maturities, Campbell and Viceira (2005) assume that "expected excess returns on bonds and stocks, real interest rates, and risk shift over time in predictable ways" (p. 34). It is crucial to observe persistency in the return variables, since otherwise variations in expected returns cannot be captured (Ghysels, Santa-Clara, and Valkanov, 2005, pp. 512-513).

Assets' risks and returns are interconnected. It is well known from the Markowitz optimization model that risky assets tend to have higher expected returns than certain ones, since return works as a compensation for risk taken. According to Campbell (1987) "it appears that time variation in the conditional covariance matrix of bill, bond and stock returns is an important part of the explanation for time-varying risk premia on these assets" (p. 390). Vice versa, return can also have an impact on risk. Due to the predictability in returns, the time-varying risk of assets is might be affected as well, as it becomes obvious in the case of long-term equity investments. As a result of this interconnection, Campbell and Viceira (2005) speak of a "term-structure of the risk-return trade-off" (p. 34). One cannot only observe patterns in single assets' expected returns and annualized variances, but also in the correlation between them. As a consequence, it becomes possible to adjust the portfolio to the planned investment horizon and find the optimal buy-and-hold asset allocation strategy for each maturity.

The returns of stocks and bonds and their variances, respectively, are driven by different forces. As a consequence, these asset classes are affected by an increasing holding period in very different ways. To better understand the return and risk drivers and how they are affected from varying maturities, this paper presents the asset classes in detail.

**Figure 3: Annualized Percent Standard Deviations of Real Returns**



Source: Adapted from Campbell & Viceira (2005)

From Figure 3, one can see that the risk of equities, measured by the annualized standard deviations of real returns, tends to decrease with an increasing investment horizon. According to Campbell and Viceira (2005), the “strong decline in volatility is the result of mean-reverting behavior in stock returns induced by the predictability of stock returns from the dividend yield” (p. 38).

The total shareholder return is defined by the formula

$$TSR = \frac{P_1 - P_0 + D}{P_0} = \frac{P_1 - P_0}{P_0} + \frac{D}{P_0} \quad (3.2.5)$$

where  $P_1$  represents the price of the stock at the end of the period,  $P_0$  is the stock price at the beginning of the period and  $D$  illustrates the dividends paid for the period between 0 and 1 (Schaefer, 2002, p. 229). The first term,  $\frac{P_1 - P_0}{P_0}$ , is also called *capital gains*, whereas the last term,  $\frac{D}{P_0}$ , is defined as *dividend yield*. According to Campbell and Viceira (2005), it is this dividend yield which determines the predictability in stock returns.

After a dividend is paid to the shareholders, the share price tends to decrease by the same amount. The dividend payment, however, is usually made due to strong fundamental values. The signal of a healthy company implies that the stock price will increase again in the future. Mathematically speaking, “the large negative correlation of shocks to the dividend yield and unexpected stock returns, and the positive significant coefficient of the log dividend yield in the stock return forecasting equation imply that low dividend yields tend to coincide with high current stock returns, and forecast poor future stock returns” (Campbell & Viceira, 2005, p. 38). This *mean-reverting* behavior of is the reason for predictability assumption in stock returns. Short-term fluctuations become less important the longer the holding period is, since stock returns tend to go back to their mean. As a consequence, the standard deviation of real stock returns decreases with an increasing investment horizon.

Figure 3 shows mean aversion in the real returns of bonds held to maturity dependent on the length of the investment horizon, meaning that the real return variance tends to increase over time. Equivalently to the stock return presentation, the inherent reason for the bonds’ term-structure lies in the return components. Whereas in the short run not that relevant, in the long run “the risk of this bond is the risk of cumulative inflation over the investment horizon” (Campbell & Viceira, 2005, p. 38). According to Fabozzi (2006), Treasury bonds are U.S. government bonds with the longest maturity after Treasury bills and Treasury notes, with maturities of greater than 10 years. Treasury bonds are generally issued as coupon bonds, but to illustrate the influence of a long-term investment horizon it is reasonable to focus on zero-coupon government bonds generated by *coupon stripping* (p. 8). Accordingly, Campbell and Viceira (2005) use a “zero-coupon nominal Treasury bond with  $k$  years to maturity that is held to maturity” (p. 34). Since all Treasury securities are backed by the credit of the government, it is assumed that these products suffer only from inflation rate risk.

$$R_B = \frac{P_{B,1} - P_{B,0}}{P_{B,0}} = \frac{M \left[ \frac{1}{(1+i)^{k-1}} \right] - M \left[ \frac{1}{(1+i)^k} \right]}{M \left[ \frac{1}{(1+i)^k} \right]} \quad (3.2.5)$$

where  $R_B$  describes the bond’s return,  $P_B$  is the price of the bond,  $M$  represents the maturity value,  $k$  is equal to the number of periods, and  $i$  means the nominal interest rate with  $i = r + \pi^e$ , where  $r$  means the real interest rate and  $\pi^e$  the expected inflation rate (Fabozzi, 2006, p. 78).

With an increasing  $k$ , it becomes obvious that the predetermined maturity value suffers from inflation risk. The value of the bond in each period is determined by discounting the final value with the nominal interest rate  $i$ . The longer the investment horizon is, the more difficult it becomes to estimate inflation rates correctly. In contrast to bonds, equities are even weakly positively correlated with inflation in the long run (Campbell & Viceira, 2005, p. 39).

T-bills are traditionally taken as the risk-free asset when conducting portfolio optimizations following Markowitz and Tobin. According to Tobin (1958), investors should hold the same risky portfolio, optimized by the model of Markowitz (1952). Depending on the level of risk aversion, they ought to combine that portfolio with the risk-less T-bill or cash asset. T-bills are zero-coupon Treasury securities with maturities of less than one year (Fabozzi, 2006, p. 8). In the traditional view, it is reasonable to use these securities as risk-free asset, since with such short maturities and the credit of the U.S. government the Treasuries do practically not suffer from any risk. How these assets suffer from long investment horizons can again be seen best in the return formula.

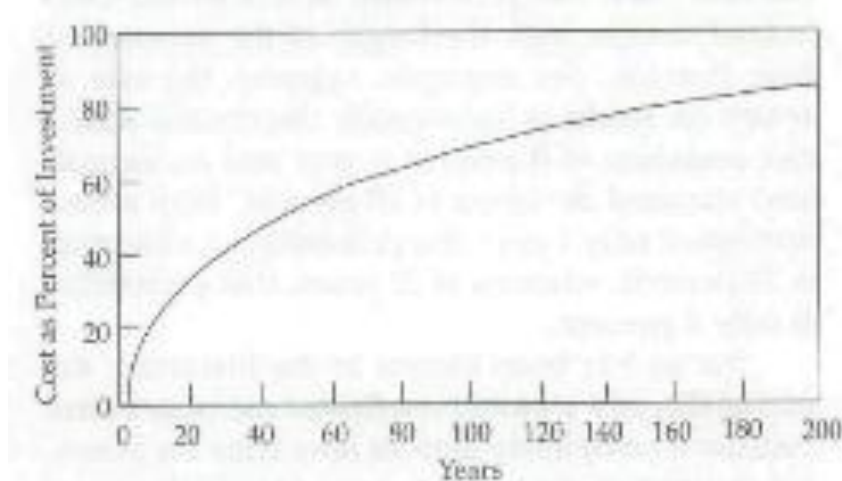
$$R_B = \frac{P_{B,1} - P_{B,0}}{P_{B,0}} = \frac{\sum_{l=j+1}^k M_l \left[ \frac{1}{(1+i_l)^l} \right] - \sum_{j=1}^k M_j \left[ \frac{1}{(1+i_j)^j} \right]}{\sum_{j=1}^k M_j \left[ \frac{1}{(1+i_j)^j} \right]} \quad (3.2.5)$$

where  $P_B$  describes the cumulative price of the bond,  $j$  describes the  $j$ th out of  $k$  periods,  $M$  represents the maturity value, and  $i$  means the nominal interest rate with  $i = r + \pi^e$ , where  $r$  means the real interest rate and  $\pi^e$  the expected inflation rate (adapted from Fabozzi, 2006, p. 78). It is assumed that the price of the Treasury position over long investment horizons is equal to the sum of the discounted maturity values of the single T-bills, similar to a coupon bond. Due to the short investment periods, expected inflation is not as important as it is for bonds held to maturity. Over long investment horizons, however, T-bills need to be reinvested. Since interest rates change on a regular basis, Treasury bills suffer from reinvestment risk or real interest rate risk on a long-term basis (Figure 3). “Uncertainty about short-term nominal interest rates [...] is important in pricing both Treasury bills and long term assets” (Campbell, 1987, p. 390).

With the help of a detailed view on stocks and bonds, one can explain how the asset classes' risks change with the investment horizon. The time-varying expected returns and variances imply changing correlation patterns between asset classes, which suggest that the time horizon might affect the optimal asset allocation within portfolios as well. "When expected returns are time varying [...] and the term-structure is not flat, efficient frontiers may be different at different horizons" (Campbell & Viceira, 2005, pp. 39-40). Since the risk of stocks tends to decrease with an increasing investment horizon, whereas the risk of bonds tends to increase, it is reasonable that an optimal long-term buy-and-hold portfolio shows a strong tilt towards equities.

Although the presented results seem to be reasonable in the first place, there are several authors stating that the term-structure argument might be insufficient and inaccurate, in particular related to equities. According to them, the discussion on mean-reversion in stock returns does not capture all driving forces. They name *estimation risk* as a force, which can lead to stock returns being even more volatile than in the short run (Baberis, 2000; Bodie, 1995; Pastor and Stambaugh, 2011) "We have seen that despite the fact that the probability that stocks will earn less than the risk-free rate of interest decreases with the length of the time horizon, the cost of insuring against this eventuality increase" (Bodie, 1995, p. 20) (Figure 4).

**Figure 4: Cost of Shortfall Insurance as a Function of Time Horizon**



Source: Bodie (1995)

Bodie (1995), however, does not generally deny the sense of investing heavily in stocks when having a long investment horizon ahead. He suggests that it still makes sense to tilt portfolios

towards equities due to the value of human capital (Bodie, 1995, pp. 20-21). This issue is discussed in the next section.

Pástor and Stambaugh (2011) even strengthen Bodie's critique of mean-reversion as the only driving force of long-term equity risk. The authors contrast the negative impact of mean-reversion on stock return variance with the positive impact of distribution uncertainty, future expected returns uncertainty, current expected return uncertainty, and estimation risk (p. 2). They differentiate the risk discussion into empirical results and the investors' perspective. Although they agree that there is mean-reversion observable in empirical results, they state that from the perspective of investors, uncertainty and estimation risks increase with the time horizon. They present an investor-oriented variance measure, the *predictive variance*, in addition to the *true variance*. In contrast to the latter one, the predictive variance includes an additional estimate on the expected true variance, namely various uncertainty and estimation risk measures. Consequently, the predictive variance is generally higher than the true one and increases the longer the investment horizon becomes. Pástor and Stambaugh (2011) suggest a downward sloping glide path of stock weights within a portfolio depending on the investment horizon. They argue that investors, who have to decide today on a fixed future asset allocation schedule will decide on "lower stock allocations at longer horizons, simply because they view single-period stock returns as more volatile at longer horizons" (Pástor & Stambaugh, 2011, p. 29).

### 2.2.2 Human Capital as Non-Tradable Asset

Another main approach of dealing with long-term investments is the wealth perspective. Markowitz detects the optimal asset allocation based on the current financial wealth. Financial wealth is the budget left after consumption, which can be used for investments. Besides financial wealth, there are authors like Bodie (1995) and Viceira (2008), who introduce human capital as additional source of wealth. Human capital is defined as discounted future labor income (Becker, 1994, p. 15). Each investor can influence the level and risk of his human capital to a certain extent by the level of education, the level of medical care, and the choice of job. In addition, the human capital characteristics can also be influenced during work life due to on-the-job trainings (Becker, 1994, p. 29). Basically, however, all investors have in common that the percentage of human wealth out of total wealth is rather high compared to financial wealth in the beginning of the career, since all the labor income streams are still

ahead. Over time, the labor income streams are paid regularly to the employee, so that the sum of expected cash flows decreases. It is important to note that financial wealth and human wealth are not only components of the investors' total wealth, but they are also interrelated. The savings, which are left after consumption, can be used for investments on capital markets again. "Investors cannot sell claims against their future labor earnings, but they can extract value from their human capital through the earnings it produces over time, which they can then use either to finance their current spending, or to save and thus increase their financial wealth" (Viceira, 2008, p. 18). Vice versa, the initial level of human capital depends on the existing financial wealth, since the budget available often influences medical care and educational opportunities. To sum up, one can state that financial wealth increases and human capital decreases over time.

According to the literature (Viceira, 2008), human capital has to be interpreted as a non-tradable risk-free asset. Since it generally provides secure and regular income streams to the employee, one can interpret human capital as a bond-like investment. It is argued that investors should optimize their total wealth portfolio including financial and human wealth, instead of focusing only on the financial portfolio. In the next section, it is explained how an optimized total wealth portfolio should be structured over long investment horizons, following the concept of life-cycle investments.

### **2.3 The Life-Cycle Model as Long-Term Investment Approach**

This section describes the basic idea of life-cycle investment products according to Viceira (2008). The first part explains how the life-cycle model builds on the idea of human capital. The second part describes how the basic life-cycle approach contributes to the development of long-term oriented investment products. Although this form of investment has several advantages, it suffers from several shortcomings and insufficiencies. Consequently, this section closes with several improvements of the basic idea, which are already known in the literature.

#### **2.3.1 The Idea of Total Wealth Portfolios**

Life-cycle funds are non-static, long-term investment funds, whose composition is based on the investors' age, "or the notion that investors should allocate a larger share of their long-term savings to stocks when they are young and have long retirement horizons, and decrease



this allocation as they approach retirement” (Viceira, 2008, p. 4). The asset allocation within life-cycle funds is not static as in the Markowitz model, but contains age-dependent shifts from a risky equity portfolio towards a rather conservative bond or cash fund. Although the life-cycle model allows for regular rebalances, they provide a one-stop solution to investors, meaning that the portfolio composition and the time and extent of rebalances are determined at the initial investment date.

The reasoning for the age-dependent shifts in the financial asset mix lies in the total wealth portfolio. The total wealth portfolio implies an additional component besides the monetary budget, namely human wealth, which represents a certain value. Although human capital is non-tradable, there are still two main reasons why it should be included in the investment planning. The first reason is the interpretation of human capital as an asset. The human capital asset can be acquired, for instance, by attending high quality schools and universities. The labor income stream during work life is an alternative type of return, namely the return of investment in education and effort (Becker, 1994, p. 59). In the basic case, Viceira (2008) defines labor income as steady, secure, and regular income stream comparable to coupon payments, so that human capital is interpreted as a bond-like position. All these aspects lead to the assumption that the investors have to consider the total wealth instead of the financial wealth portfolio and that the human capital asset should be used as an additional diversification component. A second reason for considering human wealth for the overall portfolio optimization is the interconnection of financial and human wealth. The savings left after consumption feed into the financial budget, which can then again be used for investments on capital markets. It is, thus, insufficient to interpret financial wealth as static.

In the very basic case, one assumes that, once the initial level of human wealth is determined, its amount depends solely on age and on the labor income already received, respectively. Young investors already have a strong bond position, because they have many years of receiving labor income ahead. In the light of optimal diversification, it is, thus, reasonable to tilt the financial wealth portfolio towards risky equities. The human wealth then decreases steadily with age or with approximating the retirement date, respectively, since the employees have already received part of their total labor income. The bond position, represented by the human capital asset, decreases smoothly as well. To keep an optimally diversified total wealth portfolio, it is necessary to decrease the stock position for the benefit of tradable bonds. “As the investor ages, the value of his human wealth declines (he has fewer years left to earn labor

income) while his financial wealth grows. Thus the bond investment represented by his human wealth becomes less important relative to his total wealth, and the investor will want to attenuate the tilt toward risky assets in his financial portfolio” (Viceira, 2008, p. 18). Once the investor approaches the retirement date, he should be mainly invested in bonds.

### **2.3.2 Contributions of the Basic Life-Cycle Idea**

When investors intend to invest over a long investment horizon, like when they save privately for retirement, they should, in the optimal case, rebalance their portfolios continuously depending on their individual risk aversion, human capital circumstances, and overall market conditions. This, however, is rather time-consuming, challenging, and costly. Private investors are usually not experienced in making the right investment decisions, which may result in poor investment performance and major losses (Ashcroft, 2009; Munnell, 2006, p. 368). In a study by Brown, Liang, and Weisbenner (2007), it is analyzed how employees deal with an increasing amount of investment opportunities for their investment portfolios. The authors find that the employees tend to follow a “naïve diversification strategy” (p. 1993), meaning that they simply allocate their money equally among the investment opportunities without considering the implied correlation pattern and, therefore, the real diversification potential. In addition, they are usually not able or willing to decide on a regular basis on rebalances of their portfolio. And even if the investors themselves or professional investment managers are able to do that, this might become very costly. In a phone call with the German insurance company Nuernberger (Bauer, personal communication, June 14, 2011), it was confirmed that the decrease in investment return due to the high costs of daily rebalances is usually not worth the optimization benefit during normal market conditions.

Life-cycle investment funds are a compromise of individuality and standardization. Generally, they are available for different maturities and risk levels, so that the investors can choose the products, which fit the current age and level of risk aversion. Such a product cannot be as perfect as an individually optimized one, since it offers only broad categories and lacks in considering specific human capital characteristics. Life-cycle funds, however, offer a one-stop solution to the investors, since they are based on the idea of maximizing the retirement wealth at the initial investment day. That means that by investing in a life-cycle product, the whole investment schedule is automatically determined. As a consequence, transaction costs are more balanced and investors do not need to concern themselves with the investment decisions

anymore. In the light of the behavioral issues presented before, life-cycle funds seem to be a reasonable alternative.

### **2.3.3 Model Problems and Potential Improvements**

This section illustrates the shortcomings of the basic human capital idea, which are already documented by the literature. The section helps to understand the status-quo of the theoretical life-cycle argument. The list of shortcomings can be separated into problems related to financial wealth, human capital, individuality, and age-based rebalances.

Young investors do usually not have a strong financial wealth, in particular after they invested high amounts of money in education. Viceira (2008, pp. 17-18) agrees that young investors tend to have a high degree of human capital and very little savings. In addition, he describes that human capital “gets depleted as it is transformed into consumption and savings” (Viceira, 2008, p. 18), so that the financial wealth generally increases over time. Labor income follows a hump shape, meaning that employees earn most of their labor income in the intermediate term, when they have already gained some experience and improved their occupational position. In particular well educated people tend to have a high level of human capital, so that the respective stock position within the financial portfolio should be high as well. This high level of education, however, has its price, so that young investors might not be able to invest adequate amounts in stocks. The alternative is to start with a relatively low position in equity that is increased over time to a peak when financial wealth is high and human capital is at its peak. It is then decreased again due to the decreasing human capital. As a consequence, however, the total wealth portfolio cannot be diversified in the optimal way. Thus, authors like Ayres and Nalebuff (2008) suggest that it might make sense to borrow money in the beginning to invest in stocks and pay the money back once human capital is transformed into savings. The first argument against that view is that most young investors are not able to borrow sufficient amounts of money. “Because of moral hazard issues, many investors face borrowing constraints that prevent them from capitalizing future labor income” (Cocco, Gomes, Maenhout, 2005, p. 492). Furthermore, authors like Benzoni and Chyruk (2009, p. 3) find that even the richest households do not invest major parts in stocks in early years. Bodie and Treussard (2007, p. 6) argue that this might be because of the high correlation of human capital with the stock market, when the investors are young. This argument goes hand in hand with the next critique, namely the insufficient characterization of the human capital asset.

The initial idea of life-cycle investment funds is based on the assumption that labor earnings are not too volatile and rather uncorrelated with the stock market (Viceira, 2008, pp. 20-21). Many authors, however, advance the view that most individuals face substantial human capital risk and high dependence of their jobs on stock market developments in their early work life. When beginning the occupational career, employees usually have no experience or superior knowledge, but limited contracts. During market downturns, many business sectors tend to suffer as well, for example due to weak order situations. If the employers are forced to lay off part of the workforce, the junior level is the most exchangeable one. From this worst-case scenario it seems reasonable that the human capital risk tends to be correlated with the stock market and rather stock- than bond-like in early years. Consequently, most authors agree on starting out with a relatively low fraction of the investment portfolio in stocks and increasing it over time (Bodie, 1995, p. 21; Cocco et al., 2005).

Another insufficiency of the basic life-cycle model is its stand-alone assumption. The life-cycle approach intends to model an optimal time-varying asset allocation by taking into account changes in human capital. The result is an optimally diversified total wealth portfolio. Most investors, however, hold additional investment products and savings agreements. These additional products represent further parts of the total wealth portfolio, which are ignored by the basic life-cycle idea. The consequence might be that the total wealth portfolio is incorrectly diversified. This argument leads directly to the problem of standardization. The basic life-cycle idea optimizes the asset allocations based only on the investors' age and the time remaining until maturity, respectively. Differences among the investors concerning their additional savings or their human capital risk might, however, lead to significant divergences in the optimal life-cycle investment strategy.

A final critique of the basic life-cycle idea is the missing link to short-term market movements. Even Viceira (2008, p. 16) states that one should not ignore the short-term market movements. "It is logically inconsistent to count on reduced long-term risk while ignoring the variation in returns that produces it" (Viceira, 2008, p. 17). Life-cycle portfolios, however, are optimized at the starting date taking all predetermined rebalances into account but ignoring short-term market conditions. Although short-term movements might be balanced over a long holding period, ignoring market movements can lead to sub-optimal results when approximating the maturity date.

## 2.4 Additional Thoughts on Term-Structure and Human Capital Approach

The previous section illustrates potential driving forces of long-term investments, namely a term-structure in asset risks and returns, and human capital as additional wealth component. There are two additional differences in the approaches' assumptions which are rather a side-note but worth noting.

The first aspect is the one of predictability. As already mentioned in section 2.2.1, the term-structure idea builds on predictability in asset returns due to their persistency characteristics. Without discussing the issue any further, it should be noted that the human capital idea works without that assumption. "To the extent that the level and risk of the labor income stream change over the life cycle, and to the extent that portfolio choice depends on these factors, the presence of labor income can provide a rationale for age-varying investment strategies, without relying on predictability in asset returns" (Cocco et al., 2005, p. 492).

An additional thought related to the different investment approaches is the risk perspective. There are two different ways of understanding a tilt towards equities in the beginning of a long-term investment and a later shift towards secure assets from a risk perspective. The first one is directly related to the human capital argument. Young investors seem to be exposed to a very risky portfolio, since they invest a major part in stocks following the theory of life-cycle investments. Taking into account the amount of human wealth, however, they end up with being strongly invested in a bond-like asset. In terms of the total wealth portfolio, the level of risk remains constant over time by rebalancing the financial portfolio according to the changes in human capital. The risk argument related to the term-structure perspective considers only the financial portfolio. It states that the position in stocks should be large up to a few years before retirement. During that time, fluctuations are canceled out and the investors have the possibility to receive high average returns by allowing for a higher short-term level of risk when being young. A few years before the investors retire, the readiness to take risk decreases so that the investors rebalance their portfolios towards bonds and, thus, lock in the returns generated in the years before. This view mirrors the increasing estimation uncertainty argument by Pástor and Stambaugh (2011) and is quite in line with the practical case presented in sections 3 and 4. Both arguments have in common that the level of risk aversion related to financial wealth changes with age. Consequently, the financial portfolio becomes more conservative the closer the maturity date is. The first argument, however, assumes a constant lev-

el of risk-aversion when considering total wealth, whereas the latter one does not directly include the human capital aspect.

So far, this paper answered the question why Markowitz might be inappropriate over long investment horizons and how the long-term portfolio can be improved by applying the ideas of term-structure and human capital. The basic approaches are complemented by several critiques. All the different suggestions lead to a similar decreasing tilt towards equities over long investment horizon. The following sections analyze if and to what extent the ideas are included in the Nuernberger life-cycle model and if there is any eligibility for further improvements.

### 3 Case Study: The Nuernberger Life-Cycle Model

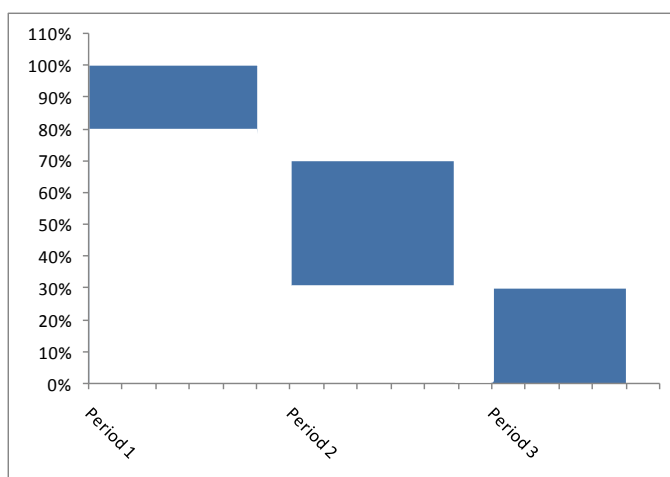
This and the following section discuss the life-cycle model used by the German insurance company Nuernberger. The first part introduces the model with its similarities and differences to the theoretical life-cycle idea. It follows a comparison of its historical performance with the one resulting from the Markowitz approach. It becomes obvious that the Nuernberger model is quite different to the basic life-cycle idea of human capital. As a consequence, it is analyzed in which way and to what extent human capital might improve the model.

#### 3.1 The Nuernberger Life-Cycle Product as Term-Structure Model

Nuernberger has its major business in the field of life insurance products and is one of the leading insurance companies in Germany. In addition, the company introduced funds-linked life-insurance products in Germany and, thus, has a lot of experience in this business area (Appendix; Nuernberger, 2011). In particular, Nuernberger offers a life-cycle model for its funds-linked insurance products characterized by a three-stage asset allocation path. Due to the company's strong market knowledge, this paper uses the Nuernberger life-cycle product for the following analyses.

Similar to the basic idea illustrated in the second section, the model is characterized by a tilt towards equities in the early stage and a shift towards safety-oriented products in the later part. The investor is free in choosing the funds he wants to invest in, but he is obliged to fulfill the contractual conditions concerning the maximum or minimum asset weights. It is important to note that the investor decides in advance on the asset weights for each sub-period.

**Figure 5: Possible Equity Weights Within the Boundaries of the Nuernberger Contract**



Source: Own Figure, adapted from Nuernberger (2010)

The first stage of the model covers the main part of the investment horizon and ends two years before the last third of the maturity starts. During this time, the investor is invested with at least 80% in an equity fund. The model allows for an equity percentage range from 80% to 100% depending on the investor's willingness to take risk. In the second stage, the capital invested is shifted towards a still return-oriented deposit with a maximum equity share of 70%. Depending on the investor's level of risk aversion, he is again free in choosing from a range between 30% and 70%. The money invested stays in this stage until three years before the maturity date. In the last stage, which covers always the last three years, there is another shift towards a safety-oriented deposit with an equity share of 0% to 30%. During that period, the return, which was generated in the previous two stages, is secured, so that fluctuations in market prices only have a minor influence.

The Nuernberger life-cycle model is in several major aspects very similar to the theoretical life-cycle idea. The first important characteristic both models have in common is the tilt towards equities in the beginning of the contract, which is reduced over time. By that, both ideas account for the fact that it is not reasonable to invest in a short-term optimized and static portfolio when intending a long holding period. Another important similarity is the fact that both models work with predetermined asset weights. Both models have an automated and standardized rebalancing schedule, and the investors are obliged to fulfill those contractual conditions. Therefore, both models represent a one-stop solution to the investors, who are not challenged any more with the choice of stock and bond weights. Finally, none of the models includes adjustments to current market conditions at any point. Consequently, the investment schedules of both models are transferable to any time period.

There are, however, several differences to the basic life-cycle idea, which are worth mentioning. First of all, the investors can make regular payments in equity funds instead of a single stock. Consequently, they benefit from diversification, and active quality management within the funds. By that, the investments become more risk / return efficient and provide higher average returns for the same level of risk. Secondly, the investors can decide at each rebalancing date how the stock and bond deposit ought to be combined. Although it is determined in the beginning how the weights of stocks and bonds should change over time, the composition of stock part and bond part can be changed at every rebalancing step.



There are three further differences, which provide a first hint on the academic idea the Nuernberger life-cycle model is based on. First of all, the rebalances do not follow a smooth path. The Nuernberger life-cycle model offers a separation of the investment horizon into three sub-periods. This is an individual characteristic of the model and is not necessarily implied by the basic idea. Rebalances usually come along with transaction costs, which lower the final investment return. By reducing the number of rebalances, one reaches a compromise of optimal asset allocation and reasonable level of transaction costs. The fact, that the whole investment horizon is separated into periods implies that the model is actually based on a series of static buy-and-hold optimizations. The contractual deposit weights of each stage illustrate the asset allocation that is in line with the respective period length. This observation is a first allusion to the use of the term-structure idea explained earlier. This view is supported by the fact that the contractual asset weights within the Nuernberger life-cycle model depend only on the remaining time until maturity. The model does not make a difference between young investors, who are at the beginning of their career, and older investors, who are close to retirement. This leads to the assumption, that human capital does not play a major role in the Nuernberger life-cycle model. Finally, the rebalances are all carried out in the late part of the investment horizon, instead of being allocated evenly. This fact supports the impression that the model works mainly with the term-structure argument. Following the very basic term-structure idea by Campbell and Viceira (2005), it is enough to optimize the buy-and-hold portfolio for the whole investment horizon without any rebalances. Accounting for the increased uncertainty and estimation risks described by Pástor and Stambaugh (2011), however, the model adds two other periods in the later part of the investment horizon to avoid any potential losses due to unexpected market downturns. The description of the Nuernberger life-cycle approach leads to the assumption that the model is mainly based on the term-structure idea. The only indication for human capital is that the model allows for regular payments into the contract. Although it is also possible to simply separate the initial budget into single portions to benefit from the cost average effect, it seems to be unlikely that young investors have such a high financial wealth. By allowing for regular payments, it is possible to invest the savings from labor income, so that one can state that part of the human capital is feeding into the contract. All other ideas related to human wealth, which are presented in section 2.3, like differences in human capital level or risk, are obviously not considered. The following sections show if the Nuernberger life-cycle model is preferable to the Markowitz approach as it is. Afterwards, it is analyzed if the model can be improved by a more in-depth consideration of the human capital idea.

## 3.2 Eligibility of the Nuernberger Life-Cycle Product

### 3.2.1 Data

For the reconstruction of the Nuernberger life-cycle model and the Markowitz alternative, there are four investment funds used; two for the equity and two for the bond deposit. Following the prospectus published by Nuernberger (2010), investors are free in choosing stock and bond investment funds. Moreover, they can even choose a deposit of several investment funds and their composition. For the analysis, it is important that there are data available for several years so that is possible to reconstruct a potential life cycle investment schedule. In addition, the funds need to be well managed and well known, so that it is probable that they represent a potential investment. Finally, the funds should have a German origin allowing for international investments, since those reflect the most likely local investors' choice (Bae, Stulz, and Tan, 2008, p. 582). The funds chosen on the equity side are *DWS Vermoegensbildungsfonds 1* (ISIN: DE0008476524), which was emitted in 1970, and *DWS Investa* (ISIN: DE0008474008), which was emitted in 1956. The former fund invests in international equity, whereas the latter one invests only in German stocks. On the bond side, the *Allianz PIMCO Internationaler Rentenfonds A* (ISIN: DE0008475054), emitted in 1969 and investing in international bonds, and *DWS Euroland Strategie (Renten)* (ISIN: DE0008474032), which was emitted in 1966 and invests in European bonds, are chosen. The choice of funds represents an international investment opportunity set with a tilt towards German and European assets, respectively. The reason for choosing two funds for each asset class is that the paper intends to show how optimal portfolio compositions can change over time and depending on the underlying model. The thesis, however, does not intend to provide a full diversification picture, but a general statement about the structural benefits of the Nuernberger life-cycle model compared to the Markowitz alternative. The restriction to two funds on each side does not affect that message. From the funds named above, monthly closing prices are extracted from the business database Bloomberg (2011). Unfortunately, for some funds performance data are available from 1985 onwards only. These monthly data from January 31, 1985, until December 31, 2010, still provide 312x4 funds prices to the analysis of this paper.

A personal communication with Nuernberger (Bauer, personal communication, June 14, 2011) clarifies that it is possible to make regular payments into the contract. To consider cost average effects, both the Markowitz and the life-cycle model analysis are conducted under the assumption of a monthly payment of €10. It should be noted that Markowitz does actually not

assume increases in the financial budget due to savings of labor income. The reason, why the model is still analyzed including regular payments, is that it might be possible to separate the initial budget into single portions. The probability, however, that young investors have an initial budget with the same size as the sum of all the regular payments is quite low.

### 3.2.2 Markowitz Portfolio Optimization

As mentioned earlier, Markowitz assumes that expected returns and “annualized variances are independent of the time horizon” (Campbell & Viceira, 2005, p. 35), so that these figures stay constant over varying holding periods. It follows that, according to the Markowitz idea, the optimal asset weights determined for a short-term portfolio remain constant for long investment horizons. To illustrate this idea, the paper conducts a short-term optimization of the four available investment funds’ weights based on data of the two most recent years available, 2009 and 2010. As described earlier, the goal of the Markowitz optimization is to set the asset weights in a way that the portfolio return is maximized and the variance is minimized, respectively. These two approaches do not necessarily lead to the same result, since they are related to different portfolios on the efficient frontier. Although both solutions are risk / return efficient, the most optimal asset is found by maximizing the *Sharpe ratio*.

$$SR = \frac{\overline{\mu}_P - r_f}{\sigma_P} \quad (3.2.2.1)$$

The Sharpe ratio, or the “reward-to-variability ratio” (Sharpe, 1994) describes the ratio of excess return and risk or the extra return compensation for each unit of risk. Its maximization provides the most optimal portfolio among all the risk / return efficient portfolios. Making use of the *Microsoft Office Excel* function *Solver*, it is possible to maximize the Sharpe ratio under the constraints of no short-selling and a closed portfolio, meaning that the budget is invested completely in the four investment funds. The constraint of no short-selling accounts for the fact that, in reality, most private investors are not allowed to hold short positions in their portfolios. The second constraint represents the assumption that the only portfolio the investors invest in is the analyzed one. This is not necessarily the case, but it enables the reader to have a stand-alone view on the model itself. After finding the two-year optimal asset weights, the portfolio can be set up. The portfolio prices over time are then the weighted investment funds’ prices. The next step is to analyze, how the short-term optimal and static portfolio per-

forms over the long investment horizon of 26 years. Assuming that the investors save €10 each month, it is calculated how many units of the portfolio can be bought for the respective portfolio price. At the maturity date, the investors sell all units bought over the investment horizon for the last trading day's price. Ignoring inflation and transaction costs, the gain from this investment is the value of the units bought over time in excess to the total money invested.

### 3.2.3 Nuernberger Portfolio Optimization

The portfolio set-up for the Nuernberger life-cycle model is in many points similar to the Markowitz approach, but more complex due to its implied rebalances. According to Nuernberger (Bauer, personal communication, June 14, 2011), investors can decide freely on the funds in their deposits and their weights at every rebalancing step as long as the contractual requirements are met. Consequently, it is assumed that the investors choose the optimal combination of several equity funds for the stock part and the optimal combination of several bond funds for the bond part for each period. Once the optimal stock portfolio and the optimal bond portfolio are found for each stage, they are weighted by the contractual requirements of the respective stage to find the total deposit composition. For the analysis, that means that the paper conducts three buy-and-hold optimizations for the stock side and for the bond side, respectively. At each stage the optimal combination of *DWS Vermoegensbildungsfonds 1* and *DWS Investa* representing equity funds and *Allianz PIMCO Internationaler Rentenfonds A* and *DWS Euroland Strategie (Renten)* representing bond funds are determined by maximizing the Sharpe ratio. This time, the optimizations are based on return and variance data for the full time period, to which the respective investment stage belongs, instead of building on short-term data. The investors are allowed to invest in a bunch funds in period 1 and invest in a completely different one in period 2 as long as the contractual weights of each period are met. By taking a stable pool of two funds for each asset class side, it is still shown that the optimal composition of the total deposit components can vary with the rebalances. After determining the weights of the two funds for each group and each period, the prices of equity and component can be calculated as weighted sum of the time-varying weights of single funds. The results are two portfolios, one stock and one bond portfolio, whose compositions are optimized for each contractual life-cycle period. The final combination of these two portfolios depends on the investors' initial choices and their levels of risk aversion, respectively. In this paper,

the final portfolios are created for the most conservative and the most risky investment possible within the boundaries of the Nuernberger life-cycle contract (Table 2).

**Table 2: Extreme Case Equity Weights**

	<b>Most Conservative Investment</b>	<b>Most Risky Investment</b>
<b>Period 1</b>	80% equity funds	100% equity funds
<b>Period 2</b>	31% equity funds	70% equity funds
<b>Period 3</b>	0% equity funds	30% equity funds

Source: Adapted from Nuernberger (2010)

To sum up, for each risk group a final investment portfolio is created, which changes at every rebalancing date, firstly, in the combination of equity and debt and, secondly, in the composition of equity and debt component.

The second major part of the analysis is the performance of the two life-cycle funds based on the same historical data as used for the Markowitz approach. It is assumed again that monthly payments are possible. So planning that €10 are saved each month, this paper calculates how many units of the portfolio are bought each month for the respective portfolio price. However, the analysis is more complex due to the three separate stages within the investment horizon.

**Table 3: Nuernberger Life-Cycle Funds Return Calculation**

<b>Total Maturity: 26 years</b>			
	<b>Maturity</b>	<b>Investment Product</b>	<b>Result</b>
1. Period	15 years	Portfolio 1	Value Period 1 (resulting from regular payments)
2. Period	8 years	Portfolio 2	Value Period 1 (resulting from lump-sum investment) + Regular Payments in Period 2 = Value Period 2
3. Period	3 years	Portfolio 3	Value Period 2 (resulting from lump-sum investment) + Regular Payments in Period 3 = Value Period 3 = Total Investment Value

Source: Nuernberger (Bauer, personal communication, June 14, 2011)

As shown in Table 3, the process is equal to the one for the Markowitz approach until the first rebalancing date at the end of the year 1999. At that date, the investors sell all units bought so far for the last trading day's price. The sales value is invested in the portfolio of the second period as a lump sum, which yields interest by the mean return of the second period. In addition to that, the ongoing monthly payments are invested in the second portfolio as well. At the

end of the second period, the investors sell the units bought in the second period at the period's last trading day's price. This sales value is added to the value received from the invested lump sum from the first value. This total value is again invested as a lump sum in the portfolio determined for the third period and yields interest by the mean return of this period. The investors continue to pay €10 per month into the contract, from which they buy units of the third portfolio. At the maturity date, the units bought in the third period are sold at the last trading day's price. This value plus the value of the lump sum investment is the final sales value of the life-cycle investment. This value is then set in relation to the money paid into the contract to receive the overall return of the approach.

### 3.3 The Role of Human Capital

So far, two steps have been made. The first one is the assumption that the Nuernberger life-cycle model reminds of an advanced term-structure model. The second one is the analysis of the eligibility, the Nuernberger life-cycle model might have as it is. This section introduces, if and how a stronger consideration of human capital might improve the model.

#### 3.3.1 Data

For the analysis of the human capital characteristics, this paper focuses on human capital risk and the correlation of human capital risk with the stock market, observable from unemployment rates. The goal is to discover the driving forces of unemployment risk as the main source and a proxy for human capital risk (Cocco et al., 2005, p. 498). For that, the paper uses statistical data from *eurostat*, available from 1992 until 2010, and a study from 2006 published by the German statistical institution, *Statistisches Bundesamt*. "Eurostat is the statistical office of the European Union situated in Luxembourg. Its task is to provide the European Union with statistics at European level that enable comparisons between countries and regions" (eurostat, 2011). With the help of this database, the thesis conducts three out of four parts of the human capital analysis. The first information used is a statistic on yearly unemployment rates in Germany sorted by age and by educational level. Concerning age, four different subgroups are created: Group 1 is for those between 25 and 39, Group 2 contains all employable people from 40 to 44, Group 3 the ones between 45 and 49, and Group 4 covers the group close to retirement with an age between 50 and 64 years. The educational background is separated into three subgroups, namely low, medium, and high education depending on the type of

educational institution visited. The second part of the analysis discusses a statistic of German unemployment rates among major job categories over time by combining existing statistics on absolute employment and unemployment figures. The third part of the analysis is about the relation of education and job choice and is a statistic offered directly by eurostat. The last part of the analysis intends to gain some knowledge on the divergent labor income developments for different job categories. To have some exemplarily numbers related to the German market to work with, this part uses a study from 2006 published by the Statistisches Bundesamt.

### 3.3.2 Description of Analysis

The fact that the Nuernberger model stresses on the term-structure idea, gives rise to the question why this is actually the case. There are two main possible answers, which are analyzed in more detail within the following chapters. The first possibility is that the human capital idea is correct but simply ignored due to unawareness or due to complexity. This implies that there is potential to improve the practical application of the life-cycle idea. Another answer might be that the human capital idea is not or only partly correct and does not show the characteristics assigned by theoreticians. If that is the case, it might be better to concentrate on the term-structure approach to optimize long-term investment portfolios. To understand the differences between theoretical foundation and practical application, this paper intends to clarify the statistically observable characteristics of human capital. Viceira (2008) states that the basic assumption of a bond-like human capital asset is low the correlation of human capital risk with the stock market and a low volatility (pp. 20-21). This theory was already adjusted in a way that there might be differences of the human capital characteristics depending on the age of the investor. It is assumed that young employees, due to lack of experience and status, face larger risks of becoming unemployed and losing the value of their human capital position than older ones. This implies that companies, which have economic problems due to market downturns, lay off the more exchangeable employees in the first place. Older investors, who already climbed part of the social ladder, are too valuable to lose their job that easily so that they face practically no human capital risk due to stock market movements. If this theory holds in practice, the better solution is to keep the level of equities low for young investors, increase it over time, and decrease it again once the investor approaches retirement, so that the portfolio is characterized by a hump-shaped equity position.

To understand how and if human capital should be considered for long-term investments, there are two questions, this paper tries to answer:

1. Based on statistical data, can human capital be interpreted as bond- or stock-like and does that change with age or education and job?
2. Does human capital risk differ among age groups or among education groups? What is the driving force of human capital risk?

Similar to Viceira (2001), this paper separates the human capital analysis into a discussion on the correlation of human capital risk with the stock market and the driving forces of human capital risk in general. This is done to provide better recommendations for an improved life-cycle model. “If labor income is risky but uncorrelated with risky financial assets, then riskless asset holdings are still crowded out, but less strongly; the portfolio tilt toward risky assets is reduced. If labor income is positively correlated with risky financial assets, then risky assets can actually be crowded out, tilting the portfolio toward safe financial assets“ (Campbell, Cocco, Gomes, Maenhout, 2001, p. 441). For this analysis, the equity fund deposit represents the stock market with the weights determined in the course of the life-cycle analysis. This is done, because taking the equity position as stock market reflects the real correlation exposure within the investors’ portfolios more precisely than taking another stock index. This stock market exposure is set in relation to several statistics from eurostat. First of all, the stock market development is compared to unemployment rate developments, sorted by the age group and level of education. The paper includes the level of education, because it provides indications for the initial level of human wealth. By investing in the career, it is likely to enter a well-paid job, so that the human capital, as discounted future labor income, tends to increase. Since the labor market usually reacts to stock market movements with a time lag, the unemployment rates were lagged by about two years to see directly the correlation picture. The analysis is also conducted for different job categories. Unfortunately, there was no sorting by age and job category available. Consequently, the relationship between education and job choice is statistically analyzed to bring the two previous parts of the analysis together. Afterwards, the paper provides an extreme-case example of two different labor income developments. Section 4 and 5 present and interpret the results of the four analyses and their impact on life-cycle portfolio constructions. To sum up, the goal of this analysis is to find any complete or partial eligibility of the human capital argument in statistical data and to construct potential improvements for the Nuernberger life-cycle model.



## 4 Case Study: Results

This section provides the results of the analyses described in section 3. Equivalently to the previous period, the first part presents the comparison of the Nuernberger life-cycle product and the Markowitz alternative, while the second part focuses on the statistically observable human capital characteristics. Any interpretations and recommendations are left to section 5 and the conclusion.

### 4.1 Eligibility of the Nuernberger Life-Cycle Product

#### 4.1.1 Markowitz Portfolio Optimization

As already explained in the model description part, the optimal weights of the portfolio are calculated based on the asset data of two most recent years, 2009 and 2010.

**Table 4: Summary Statistics for 2009-2010 and 1985-2010**

Variance-Covariance Matrix 2009-2010					Variance-Covariance Matrix 1985-2010				
	A	B	C	D		A	B	C	D
A	<b>0.00193</b>	0.00272	-0.000537	5.5E-05	A	<b>0.00247</b>	0.002702	0.000456	8.42E-05
B	0.00272	<b>0.00515</b>	-0.001082	0.000146	B	0.002702	<b>0.00394</b>	0.000117	0.000144
C	-0.000537	-0.001082	<b>0.00085</b>	9.2E-06	C	0.000456	0.000117	<b>0.00395</b>	0.000238
D	5.5E-05	0.000146	9.2E-06	<b>0.0005</b>	D	8.42E-05	0.000144	0.000238	<b>0.00031</b>
$\bar{\mu}$	1.24%	2.04%	0.63%	0.37%	$\bar{\mu}$	0.59%	0.70%	0.30%	0.02%
Annualized Variance	0.10%	0.26%	0.04%	0.03%	Annualized Variances	0.01%	0.02%	0.02%	0.00%
Annualized Std	3.11%	5.08%	2.07%	1.59%	Annualized Std	0.98%	1.23%	1.23%	0.34%
Annualized Sharpe Ratio	<b>40.07%</b>	<b>40.18%</b>	<b>30.66%</b>	<b>23.31%</b>	Annualized Sharpe Ratio	<b>60.24%</b>	<b>56.46%</b>	<b>24.15%</b>	<b>5.15%</b>

A	DWS VERMOEGENSBILDUNGSFONDS I	A	DWS VERMOEGENSBILDUNGSFONDS I
B	DWS Investa	B	DWS Investa
C	ALLIANZ PIMCO INTERNATIONALER RENTENFONDS	C	ALLIANZ PIMCO INTERNATIONALER RENTENFONDS
D	DWS EUROLAND STRATEGIE (RENTEN)	D	DWS EUROLAND STRATEGIE (RENTEN)

Source: Own Table

Markowitz assumes constant expected returns and constant annualized variances. Alternatively, one can state that Markowitz assumes stable annualized Sharpe ratios. One can already see from the direct comparison of the two-year sample and the full observation period (Table 4) that this assumption does not hold. When comparing the annualized Sharpe ratios of the two holding periods, one can see that the equity funds' Sharpe ratios increase from about 40% in the two-year case to about 60% in the long holding period of 26 years. In contrast to that, the Sharpe ratios of the bond funds decrease from 30.66% and 23.31% to 24.15% and 5.15%. Although the Sharpe ratios are higher for equity funds in both cases, this analysis shows that the reward for liability is extremely beneficial over long investment horizons in the case of equity funds, and vice versa in the case of bond funds.

This observation is mirrored in the optimal portfolio composition (Table 5). Maximizing the portfolio's Sharpe ratio of the two-year optimal portfolio leads to a strong tilt towards bonds.

**Table 5: Optimal Buy-and-Hold Asset Weights**

Observation Period	Stocks		Bonds	
	A	B	C	D
2009-2010	8.29%	19.83%	54.88%	17.00%
<b>Σ</b>	<b>28.12%</b>		<b>71.88%</b>	
1985-2010	52.24%	27.36%	20.40%	0.00%
<b>Σ</b>	<b>79.60%</b>		<b>20.40%</b>	

Source: Own Table

The portfolio is invested with 71.88% in bonds, whereas stocks are represented by only 28.12%. Comparing that to the optimal buy-and-hold weights over the full 26 years shows that investors put more weight in stocks. The optimal buy-and-hold weights are 79.6% in stocks and 20.4% in bonds.

To show the differences of the Markowitz approach to the Nuernberger life-cycle product to their full extent, the analysis continues with the short-term optimized asset weights. In the Appendix, there is Table 15 illustrating the portfolio's performance assuming monthly payments of €10. First, the portfolio prices over time are calculated from the weighted sum of the single funds' prices. Afterwards, it is calculated how many units of the portfolio are bought each month for the €10 invested. At the maturity date, the investor has bought 91.29 units of the portfolio. These are sold for the last trading day's portfolio price, namely €55.55 per unit. As a consequence, the investor generates a sales value of €5,071.11. Subtracting the payments made over time, €3,120, leads to a profit of €1,951.11 or a return of 62.54%.

#### **4.1.2 Nuernberger Portfolio Optimization**

As mentioned in section 3.1, investors have the opportunity to decide on the combination of single funds within the deposit at every rebalancing step, whereas the asset class weights are determined at the beginning of the contract. To account for that, the composition of the equity and debt security funds is optimized at every rebalancing step. The weights of equity and bond part within the portfolio are then fixed at the contractual weights. The paper conducts analyses for the most risk averse and the most risk taking investor possible according to the model description (Table 2).

In the first step, the optimal combinations of the equity funds and the bond funds are determined for each stage of the investment horizon independent of the level of risk aversion. Since this step describes a buy-and-hold optimization strategy, it accounts for time-varying expected returns and variances. In particular for the first period, which covers almost two thirds of the total investment horizon, it becomes possible to capture term-structure characteristics of the single asset classes. The optimal combinations of the two funds within the equity and debt security deposits are again found by maximizing the Sharpe ratio (Table 6; Table 7).

**Table 6: Optimal Allocation of Chosen Equity Funds Over Three Investment Stages**

Optimal Allocation of Chosen Equity Funds Over Three Investment Stages					
<b>Period 1 (1985-1999)</b>					
	Variance-Covariance Matrix (Cov(i,j))			Mean Returns	
	A	B		A	B
Weights	<b>56.6203%</b>	<b>43.3797%</b>	Total Weight	<b>1.0219%</b>	<b>1.1577%</b>
A	0.002621963	0.002541147	100%	Riskfree	0.0000%
B	0.002541147	0.003438605		Portfolio Return	1.0808%
	Cov(i,j)*w(i)*w(j)			Variance	0.2736%
	A	B		<b>Sharpe Ratio</b>	<b>20.6634%</b>
A	0.000840563	0.000624149			
B	0.000624149	0.000647077			
<b>Period 2 (2000-2007)</b>					
	Variance-Covariance Matrix (Cov(i,j))			Mean Returns	
	A	B		A	B
Weights	<b>0.0000%</b>	<b>100.0000%</b>	Total Weight	<b>0.0628%</b>	<b>0.1660%</b>
A	0.002057542	0.002525187	100%	Riskfree	0.0000%
B	0.002525187	0.003851785		Portfolio Return	0.1660%
	Cov(i,j)*w(i)*w(j)			Variance	0.3852%
	A	B		<b>Sharpe Ratio</b>	<b>2.6747%</b>
A	0	0			
B	0	0.003851785			
<b>Period 3 (2008-2010)</b>					
	Variance-Covariance Matrix (Cov(i,j))			Mean Returns	
	A	B		A	B
Weights	<b>0.0000%</b>	<b>100.0000%</b>	Total Weight	<b>-0.1731%</b>	<b>-0.1913%</b>
A	0.002755338	0.003902984	100%	Riskfree	0.0000%
B	0.003902984	0.006729034		Portfolio Return	-0.1913%
	Cov(i,j)*w(i)*w(j)			Variance	0.6729%
	A	B		<b>Sharpe Ratio</b>	<b>-2.3322%</b>
A	0	0			
B	0	0.006729034			

A DWS VERMOEGENSBILDUNGSFONDS I  
 B DWS Investa

Source: Own Table

In the first period, the stock component consists with 56.62% of *DWS Vermoögensbildungs-fonds I* and with 43.38% of *DWS Investa*. In the second and third period, the equity portfolio is fully invested in *DWS Investa*.

**Table 7: Optimal Allocation of Chosen Bond Funds Over Three Investment Stages**

Optimal Allocation of Chosen Bond Funds Over Three Investment Stages					
<b>Period 1 (1985-1999)</b>					
	<b>Variance-Covariance Matrix (Cov(i,j))</b>		<b>Mean Returns</b>	<b>C</b>	<b>D</b>
	<b>C</b>	<b>D</b>	<b>Total Weight</b>	<b>0.4512%</b>	<b>0.0300%</b>
<b>Weights</b>	<b>76.3608%</b>	<b>23.6392%</b>	100%	Riskfree	0.0000%
<b>C</b>	0.006461709	0.000333626		Portfolio Return	0.3516%
<b>D</b>	0.000333626	0.000334644		Variance	0.3907%
	<b>Cov(i,j)*w(i)*w(j)</b>			<b>Sharpe Ratio</b>	<b>5.6254%</b>
	<b>C</b>	<b>D</b>			
<b>C</b>	0.003767806	6.02231E-05			
<b>D</b>	6.02231E-05	1.87003E-05			
<b>Period 2 (2000-2007)</b>					
	<b>Variance-Covariance Matrix (Cov(i,j))</b>		<b>Mean Returns</b>	<b>C</b>	<b>D</b>
	<b>C</b>	<b>D</b>	<b>Total Weight</b>	<b>-0.1783%</b>	<b>0.0161%</b>
<b>Weights</b>	<b>0.0000%</b>	<b>100.0000%</b>	100%	Riskfree	0.0000%
<b>C</b>	0.000357475	0.000174312		Portfolio Return	0.0161%
<b>D</b>	0.000174312	0.000187094		Variance	0.0187%
	<b>Cov(i,j)*w(i)*w(j)</b>			<b>Sharpe Ratio</b>	<b>1.1770%</b>
	<b>C</b>	<b>D</b>			
<b>C</b>	0	0			
<b>D</b>	0	0.000187094			
<b>Period 3 (2008-2010)</b>					
	<b>Variance-Covariance Matrix (Cov(i,j))</b>		<b>Mean Returns</b>	<b>C</b>	<b>D</b>
	<b>C</b>	<b>D</b>	<b>Total Weight</b>	<b>0.8027%</b>	<b>-0.0397%</b>
<b>Weights</b>	<b>97.6698%</b>	<b>2.3302%</b>	100%	Riskfree	0.0000%
<b>C</b>	0.001034667	-6.305E-05		Portfolio Return	0.7831%
<b>D</b>	-6.305E-05	0.000501955		Variance	0.0984%
	<b>Cov(i,j)*w(i)*w(j)</b>			<b>Sharpe Ratio</b>	<b>24.9592%</b>
	<b>C</b>	<b>D</b>			
<b>C</b>	0.000987008	-1.43498E-06			
<b>D</b>	-1.43498E-06	2.72564E-07			

C ALLIANZ PIMCO INTERNATIONALER RENTENFONDS  
 D DWS EUROLAND STRATEGIE (RENTEN)

Source: Own Table

In the first period, the bond component is a combination of a 76.36% position in *Allianz PIMCO Internationaler Rentenfonds A* and a 23.64% position in *DWS Euroland Strategie (Renten)*. In the second period, the bond component invests completely in *DWS Euroland Strategie (Renten)*. In the last period, the bond portfolio is invested in *Allianz PIMCO Internationaler Rentenfonds A* with 97.67% and in *DWS Euroland Strategie (Renten)* with 2.33%.

In a next step, the sub-portfolios for stock and bond part are combined by the contractually determined weights, so that one receives the monthly prices of the life-cycle fund. This is done for the two most extreme investors possible. With these time series of life-cycle funds prices, the paper calculates the performances for the conservative and risky investment including monthly payments and interest-bearing lump sum investments as explained in the descriptive part in section 3.2.3. A detailed calculation is provided in the Appendix (Table 16). A summarizing table with the sales values per period is given here (Table 8).

**Table 8: Summary of Life-Cycle Portfolio Performance 1985-2010**

Summary of Life-Cycle Portfolio Performance 1985-2010						
<u>Conservative Investment</u>						
	Savings	Units	Lump Sum Investment Value	Sales	Total Sales Profit	Return
Period 1	1800.00	58.40		5722.03	5722.03	
Period 2	960.00	18.21	5724.52	1137.56	6862.08	
Period 3	360.00	10.08	6909.25	406.40	7315.65	
$\Sigma$	3120.00	86.69			7315.65	4195.65 <b>134.48%</b>
<u>Risky Investment</u>						
	Savings	Units	Lump Sum Investment Value	Sales	Total Sales Profit	Return
Period 1	1800.00	55.81		6320.48	6320.48	
Period 2	960.00	13.62	6328.27	1305.93	7634.20	
Period 3	360.00	7.23	7657.55	421.95	8079.50	
$\Sigma$	3120.00	76.66			8079.50	4959.50 <b>158.96%</b>

All values are in €  
Source: Own Table

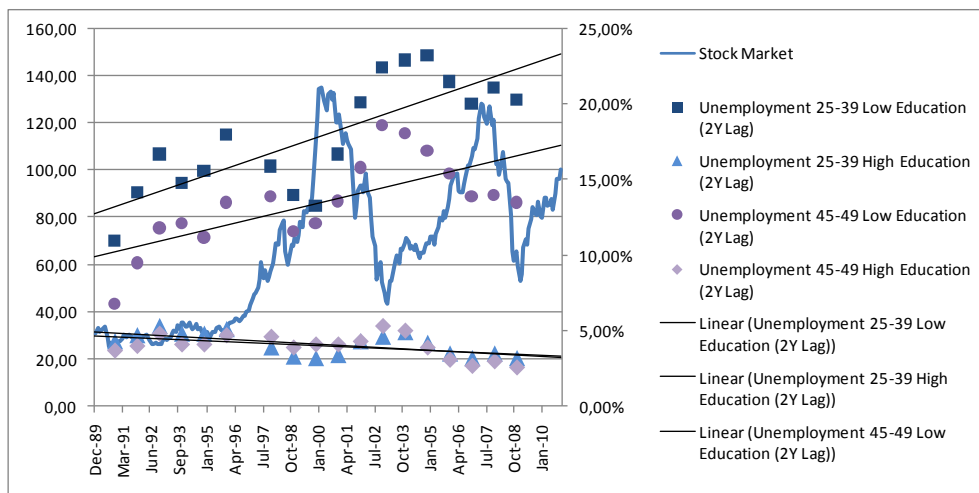
In the first period, the sales value of the conservative investment is €5,722.03. This value is invested as a lump sum in the portfolio of the second period. In that period, the monthly payments are invested in the second portfolio and its sales value is €1,137.56. Simultaneously, the sales value of the previous period brings 0.04%, so that its value has reached €5,724.52. In sum, this leads to a total sales value in the second period of €6,862.08. This value is invested as a lump sum in the third portfolio and yields an interest of 0.69%. Besides that, monthly payments are again invested in the third portfolio. The sales value of the regular payments at maturity is €406.40, while the lump sum has reached a value of €6,909.25. When comparing this total sales value of €7,315 to the payments made, a total investment return of 134.48% has been generated. The same analysis is conducted for the most risky investment possible within the boundaries of the Nuernberger contract. The results of the single periods are reproduced in the table equivalently to the conservative portfolio. The overall return of the investment for the historic analysis period is 158.96%.

## 4.2 The Role of Human Capital

This section provides the results of the human capital analysis with focus on unemployment risk as major human capital risk. The goal is to discover the driving forces of unemployment risk and, consequently, to learn more about human capital characteristics observable in reality.

The first part of the analysis describes the relationship between unemployment rates and age and education, respectively. For that, the paper combines the stock part of the life-cycle portfolio with the unemployment rates developments from 1992 to 2010, sorted by age group and educational level (Figure 6). The first observation is that the impact of stock market movements on unemployment rates seems to take some time. To better see causalities, the unemployment rate process is lagged by two years.

**Figure 6: Co-Movement of Lagged Labor Market With Stock Market (Age / Education)**



Source: Own Figure, based on data derived from eurostat (2011)

It can be observed that the general level of unemployment seems to be much higher for those people, who have a low level of education, than for those with a high one. From the data, it seems like there is no difference observable in the level of unemployment for people with a high education between the age group 25-39 years and the age group 45-49 years. For those people, who have a low level of education, the average unemployment rate tends to be a bit lower for the older interest group than for the younger one.

From Figure 6, one can see that all curves tend to move in the opposite direction to the stock market. When there is an upward trend in the stock market, there tends to be a downward trend in unemployment rates and vice versa for each age group or education level. Furthermore, it seems like the unemployment rates fluctuate much less for the high education groups than for the low education groups. This first glance analysis, however, might provide a misleading picture due to two reasons. The unemployment rates for high education groups are moving on a much lower level than the ones of low education groups. Consequently, small percentage point changes in the high education groups might still imply a high rate of change.

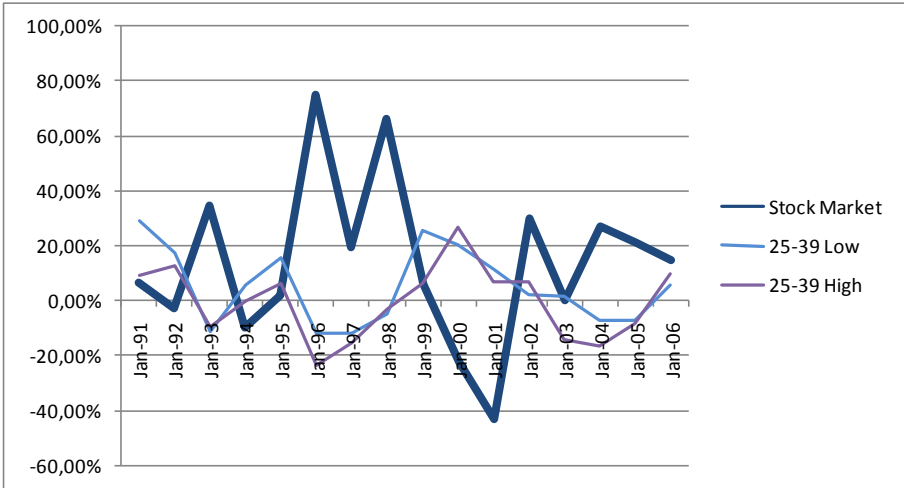
In addition, there are trends observable in both the market and the unemployment rate developments, which do not reflect short-term market up- or downward movements and reactions in unemployment rates, respectively. By simply taking the mean and the variance of the movements, these trends would be ignored. To get a better view on how the unemployment rates change when the market changes and on how extreme these reactions are, market returns and rates of change in unemployment rates are calculated (Table 9).

**Table 9: Rates of Change in Unemployment Rates (Age / Education)**

Rates of Change in Unemployment Rates					
Year	Stock Market	25-39 Low	25-39 High	45-49 Low	45-49 High
Dec-91	6.46%	29.09%	9.30%	24.21%	20.00%
Dec-92	-3.21%	17.61%	12.77%	2.54%	-14.58%
Dec-93	34.26%	-11.38%	-9.43%	-7.44%	0.00%
Dec-94	-9.85%	5.41%	0.00%	20.54%	14.63%
Dec-95	1.75%	15.38%	6.25%	2.96%	-2.13%
Dec-96	74.86%	-11.67%	-23.53%	-16.55%	-15.22%
Dec-97	19.27%	-11.95%	-15.38%	4.31%	5.13%
Dec-98	66.10%	-5.00%	-3.03%	12.40%	0.00%
Dec-99	5.91%	25.56%	6.25%	16.18%	4.88%
Dec-00	-22.02%	20.36%	26.47%	17.72%	23.26%
Dec-01	-43.34%	11.44%	6.98%	-2.69%	-5.66%
Dec-02	30.08%	2.23%	6.52%	-6.63%	-22.00%
Dec-03	0.15%	1.31%	-14.29%	-8.88%	-20.51%
Dec-04	27.07%	-7.33%	-16.67%	-9.74%	-12.90%
Dec-05	20.85%	-6.98%	-8.57%	0.72%	11.11%
Dec-06	14.65%	5.50%	9.38%	-3.57%	-13.33%
<b>Average</b>		<b>4.97%</b>	<b>-0.44%</b>	<b>2.88%</b>	<b>-1.71%</b>
<b>Stdev</b>		<b>13.66%</b>	<b>13.29%</b>	<b>12.13%</b>	<b>14.17%</b>

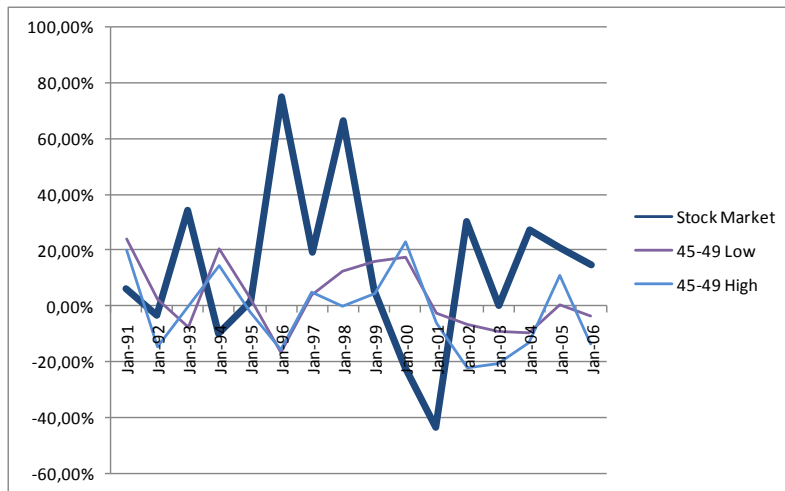
Source: Own Table

**Figure 7: Rates of Change in Unemployment Rates (Young Age Group)**



Source: Own Figure

**Figure 8: Rates of Change in Unemployment Rates (Old Age Group)**



Source: Own Figure

**Table 10: Correlation Calculations (Age / Education)**

		Correlation Calculation				
Age	Education	1992-1994	1995-1996	1996-2000	2001-2002	Total
25-39	Low	-0.8404	-1.0000	-0.7516	-1.0000	-0.6270
	High	-0.7316	-1.0000	-0.7942	-1.0000	-0.6079
45-49	Low	-0.8539	-1.0000	-0.7005	-1.0000	-0.3630
	High	-0.1407	-1.0000	-0.9066	-1.0000	-0.2924

Source: Own Table

One can see from Figures 7 and 8 that after taking changes in rates and, consequently, eliminating the trends, there are still contrarian movements observable between stock market and unemployment rates. In periods of about two years, the market changes its direction from positive to negative rates of change, while unemployment rates tend to change from negative to positive ones and vice versa. This observation is analyzed in more detail by calculating correlations for the short two-year-periods and for the whole analyzed time frame (Table 10). It shows that for short periods, the correlations between stock and labor market tend to be very high. This result is valid for both analyzed age and education groups. Over the whole time period, the correlation decreases for all age groups, whereas the effect is much stronger for the older age groups. The correlations are calculated with a two-year lag for the young age group and a three-year lag for the old age group, since then the time it takes the stock market developments to feed into the labor market is considered most precisely.

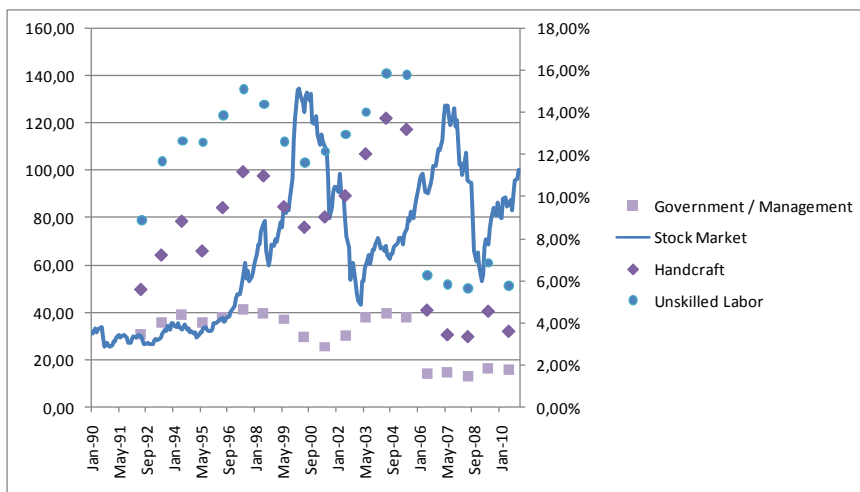
To better understand the size of the fluctuations, the paper calculates the average rates of change (Table 9). The average rates of change for the high education groups are between -0.44% and -1.71%, whereas the rates for low education groups are between 2.88% and



4.97%. The observation of positive averages for the high education groups and negative averages for low education groups is independent of the age group. This result explains the positive trends in unemployment rates for low education groups and the slightly negative ones for high education groups (Figure 6).

To better understand the impact of education on human capital risk, further analyses are conducted. According to the literature, investment in education increases the initial level of human capital. If and why this is the case is analyzed by conducting a similar analysis as before by combining stock market movements with different job categories. For that, the same equity fund combination is taken as before. Its up- and downturns are compared to the changes in unemployment rates within the job categories *government / management*, *handcraft*, and *unskilled labor* (Figure 9).

**Figure 9: Co-Movement of Lagged Labor Market With Stock Market (Job Category)**



Source: Own Figure, based on data derived from eurostat (2011)

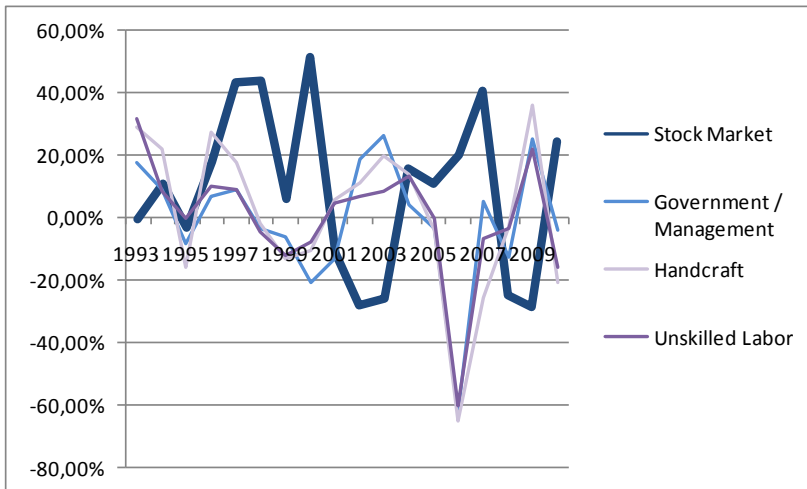
From the first impression, it can be noticed that there seem to be reactions of the labor market to the stock market in all job categories. Similar to the high education groups compared to the low education groups, the initial level of unemployment rates is much lower for the government / management group than for the handcraft and the unskilled labor group.

**Table 11: Rates of Change in Unemployment Rates (Job Category)**

Rates of Change in Unemployment Rates					
Year	Stock Market	Government / Management	Handcraft	Unskilled Labor	
1993	-0.41%	17.36%	29.30%	31.75%	
1994	10.73%	8.86%	22.28%	8.29%	
1995	-3.03%	-8.61%	-16.02%	-0.40%	
1996	18.03%	6.77%	27.78%	10.10%	
1997	43.75%	8.64%	18.00%	8.95%	
1998	43.92%	-3.88%	-1.74%	-4.63%	
1999	6.31%	-6.52%	-13.37%	-12.51%	
2000	51.71%	-20.81%	-10.25%	-7.70%	
2001	-11.34%	-13.42%	5.73%	4.46%	
2002	-28.36%	18.70%	11.20%	6.62%	
2003	-25.74%	26.19%	19.83%	8.21%	
2004	15.82%	3.75%	14.07%	13.22%	
2005	10.75%	-3.68%	-3.88%	-0.41%	
2006	19.98%	-62.77%	-65.14%	-60.56%	
2007	40.62%	4.77%	-25.42%	-6.60%	
2008	-24.72%	-13.04%	-2.66%	-3.50%	
2009	-28.63%	25.16%	36.19%	21.77%	
2010	24.48%	-4.05%	-20.95%	-16.06%	
<b>Average</b>		<b>-0.92%</b>	<b>1.39%</b>	<b>0.05%</b>	
<b>stdev</b>		<b>20.40%</b>	<b>24.70%</b>	<b>19.18%</b>	

Source: Own Table

**Figure 10: Rates of Change in Unemployment Rates (Job Category)**



Source: Own Figure

**Table 12: Correlation Calculations (Job Category)**

Job Category	Correlation Calculation				
	1996-2000	2001-2003	2003-2007	2007-2009	Total
<b>Government / Management</b>	-0.8210	-0.9652	-0.8363	-0.4115	-0.5624
<b>Handcraft</b>	-0.5340	-0.9529	-0.8580	-0.7346	-0.2154
<b>Unskilled Labor</b>	-0.7825	-0.9797	-0.8226	-0.6178	-0.3562

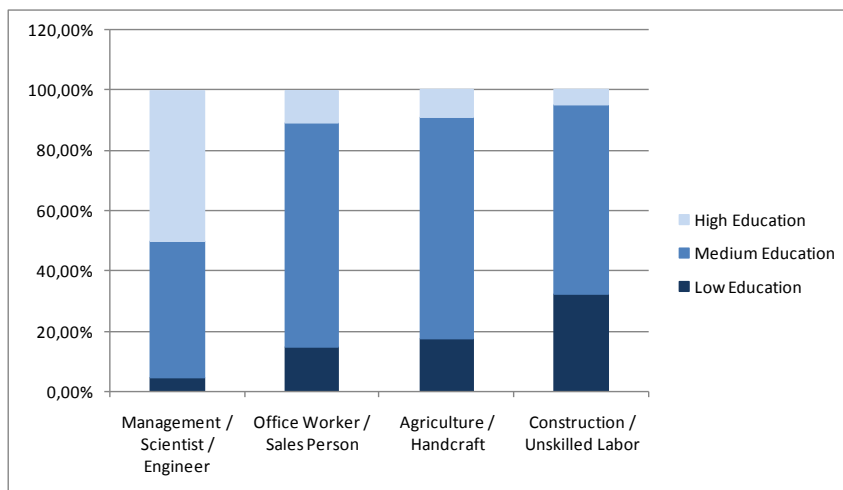
Source: Own Table

To receive a more detailed view on the changes of unemployment rates, the changes in rates are calculated (Table 11; Figure 10). The average rates of change show again that there are

differences in the way the unemployment rates react to market movements. For the government / management group the average change in rates is -0.92%. For the handcraft and the unskilled labor groups, the average change in rates is 1.39% and 0.05%, respectively. The analysis of the correlations between stock market and unemployment rates (Table 12) shows, similar to the previous analysis, that there seem to be strong negative correlations over short-term horizons. The total observation period is again separated into subperiods, whose lengths depend on the market movements. For these subperiods, the correlations vary around 80%. When extending the analysis to the whole observation period, the correlation decreases significantly.

To bring the two analyses together, the paper analyzes with the help of the statistical database eurostat how education and job choice are interconnected (Figure 11).

**Figure 11: Relation of Education and Job Choice**

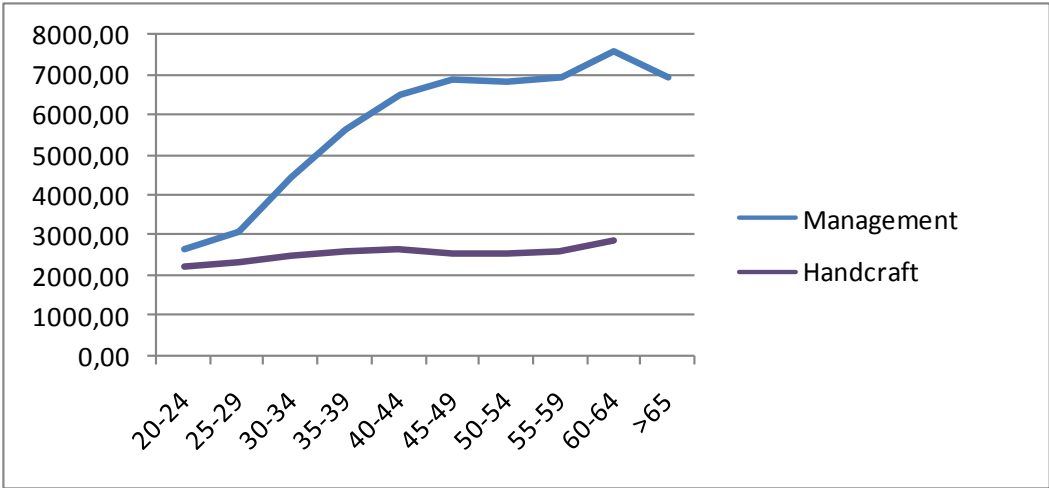


Source: Own Figure, based on data derived from eurostat (2011)

Figure 11 shows that employees in management tend to have a medium or high level of education. In contrast, workers in handcraft and unskilled labor are statistically not that well educated. Instead, this job group is characterized by a low or medium educational level.

After analyzing the connection between education and job choice, the final step is made by analyzing the differences in labor income developments over time depending on the two job categories *management* and *bricklayer* as an example of the job category *handcraft*. This is done with the help of a study by the German Statistisches Bundesamt published in 2006 (Figure 12).

Figure 12: Labor Income Development Examples



Source: Own Figure, based on data derived from Statistisches Bundesamt (2006)

From Figure 12, one can conclude that the average labor income develops extremely differently with age depending on the job category. By discounting these average income streams by the European average inflation rate of 3% (European Central Bank, 2011), the huge difference in the human capital level becomes obvious. Based on the study, the management group has an average human capital of €125,817.33, whereas the handcraft group has an average human capital of €62965.45 (Appendix, Table 17).

## 5 Case Study: Interpretation of the Results

In the last two sections, the single steps and results of the Nuernberger life-cycle analysis in the context of a potential Markowitz alternative and additional human capital consideration were presented. In this section, the paper tries to interpret these results. In the conclusion, the thesis then provides recommendations for an improved portfolio, given the basic assumptions of the Nuernberger life-cycle model.

### 5.1 Eligibility of the Nuernberger Life-Cycle Product

Section 4.1 describes in detail how the historical returns are generated for the two different investment approaches. Table 13 summarizes the historical performances of the Markowitz and the Nuernberger life cycle product and shows that, based on the sample period and on the investment funds used, the Nuernberger life-cycle product tends to outperform the Markowitz approach.

**Table 13: Performance Markowitz Versus Nuernberger Life-Cycle**

Performance: Markowitz vs. Life-Cycle			
	Markowitz	Life-Cycle	
		Conservative	Risky
<b>Return</b>	62.54%	134.48%	158.96%

Source: Own Table

When stating that the life-cycle product outperforms the Markowitz portfolio, the reader needs to remember the fact that this analysis and also its interpretation is done for one individual product from the real market, which is not in all points reflecting the structure of other life-cycle products or the theoretical life-cycle idea. The major differences are described in section 3.1. Although the comparison of the performances supports the Nuernberger life-cycle product, there is one important shortcoming independent of the human capital discussion. According to the Nuernberger (2010) contract, investors can choose the contractual weights dependent on their individual risk-aversion from the same range no matter how long the investment horizon is. The only condition is that the holding period is at least five years (Nuernberger, 2010). Therefore, the asset weights at each stage might be identical for an investor with a 30-year horizon and an investor with a five-year horizon. Considering the first stage, which has the longest buy-and-hold period, an investor chooses a minimum weight of stocks of 80% for 18 years and for one year, respectively. In the light of the term-structure idea, this cannot be ideal.

One research question is, however, if there is any eligibility for the Nuernberger life-cycle model as it is. This paper answers the question by stating that based on the funds and the time period used, the Nuernberger life-cycle product is preferable to the Markowitz approach. The initial critique against the Markowitz approach is its insufficient use over long investment horizons. As a consequence, as long as the Nuernberger life-cycle product is used for sufficiently long holding periods, it clearly outperforms the Markowitz product. Once the maturity becomes close to five years, the return gap should become smaller. In addition, the life-cycle product is justified by its basic consideration of human capital. By using the savings from labor income, the investor can benefit from cost average effects. Although it is assumed in the analysis, that the investor has enough initial financial wealth to separate it over the holding period, this is rather unlikely for young investors. To sum up, one can say that everything else equal, the Nuernberger life-cycle product as it is offered right now is preferable to a portfolio based on the Markowitz approach.

## 5.2 The Role of Human Capital

This section interprets the results of the human capital analysis of section 4.2 focusing on their potential impact on life-cycle investments. In the literature (Viceira, 2008), human capital is described as a bond-like asset due to the assumed safe and regular labor income streams. *Safe* means that the payments are independent of the market environment and rather uncorrelated with stock market indices. Following the definition provided in section 2, human capital can only belong to different asset classes than stocks when the correlation between stock market and human capital risk is low. In addition, for human capital to be bond-like it is important to be not too risky. There are many factors, which can be analyzed for potential human capital risk like mortality risk or the risk of occupational invalidity. This paper focuses on unemployment risk. All the risk factors named have in common that the respective person loses at least part of the planned future labor income. Human capital risk is, therefore, equivalent to the risk of losing part or all of the planned future labor income.

The description of the analyses' results related to the correlation of unemployment rates with the stock market provides the following major results:

1. All curves representing the development of unemployment rates over time show the same contrarian course compared to the stock market. The only observable impact of

age is that older groups tend to be affected by the stock market about one year later than younger ones.

2. Analyzing the rates of change in unemployment rates, all age, education, and job groups show a similar strong short-term correlation with the market. It is important to note that this strong negative correlation only holds as long as the market is moving in one direction. Over the whole sample period, the correlation decreases significantly, in particular for the older age group.

These observations require a more detailed interpretation and discussion. First of all, the correlation analysis suffers from the limited data availability. Unemployment rates, separated by age, education, and job choice are only available on a yearly basis and from 1992 onwards. Since the correlation analysis for the short periods is often based on only four observations, the results need to be interpreted very carefully. For further research, the analysis should be conducted with the help of a more detailed database. There are still tendencies in the results observable, which might be interesting for the future interpretation of human capital. One can see from Figures 7, 8, and 10, as well as from the Tables 10 and 12, that there are by trend strong contrarian short-term developments of the stock market and the unemployment rates. This leads to the assumption that no group characteristic (age, education, job) seems to have a significant impact on the strength of the short-term negative correlation of unemployment rates with the stock market. This is in particular striking for the different age groups, since it is argued in the literature that human capital risk is correlated with the stock market for young persons but becomes uncorrelated when the employees grow older (Bodie, 1995, p. 21; Cocco et al., 2005). This time-varying correlation assumption leads to the opinion that human capital is stock-like in the beginning of a career and becomes bond-like later. When accounting for the time lags between stock market and labor market effects, and when separating the time frame into single market direction periods, one finds strong correlations between stock market and human capital risk at all times. Consequently, this paper argues that in terms of correlation there is no reason to assume from the statistical data used that human capital becomes bond-like at any age or with any level of education or chosen job category. It rather suggests that human capital might be a stock-like asset independent from education and job and, in particular, from age. It can also be noted from the correlation analysis that the values decrease for the whole observation period. Whereas the correlation remains relatively high for the young age group, it decreases to about 30% for the older age group. This might be because the short-term correlations are only calculated for periods with clear contrarian movements. In the correlation calculation for the whole observation period, however, all disturbing and forc-

es are incorporated, so that the overall correlation is decreased. In the conclusion, this paper provides a few more thoughts related to that characteristic.

The focus of the analysis then switches to another layer of human capital risk. In addition to the correlation of human capital risk with the stock market, the risk itself depending on age, education, and job category is analyzed in more detail. It was already mentioned that there seem to be trends in the unemployment rates' developments, which differ among the age, education and job groups. There are positive average rates of change in unemployment rates for low education people and people working in the job categories 'unskilled labor' and 'handcraft'. In contrast, there are negative average rates for people with high education and employees of the job category 'government / management'. Stated differently, the upward reactions of unemployment rates to market downturns tend to exceed the downward reactions in good market times for low education groups and the job categories 'unskilled labor' and 'handcraft'. Vice versa, downward reactions to market upturns tend to be dominating for high education groups and the government / management category. Usually, only the positive unemployment risk, or the human capital downside risk, matters. One can, thus, interpret that highly educated people in academic job positions suffer much less from human capital risk than weakly educated people do. This tendency is very similar for both age groups. The downside risk, however, seems to be weaker for older age groups than for younger ones. The reason might be that employees can add value by gaining experience over working years. This observation implies that age might have an effect on unemployment risk. The driving force, which decides on the direction of the trend, is, however, education instead of age.

In another study it was shown how education and job choice are interrelated. One can interpret that people with a higher educational level rather end up in the government / management job category, whereas weakly educated people often work in handcraft or similar jobs or do any unskilled labor. A final question is what the investment in education and the resulting job choice actually mean for human capital. The study by Statistisches Bundesamt closes the gap by showing that the labor income raises much more steeply for management positions than for handcrafters. Summing up all the expected labor income streams leads to a significantly higher human capital for managers than for handcrafters. Combining the two analyses, one can conclude that a high investment in education leads, on average, to a high level of human capital.



Summing up all the single results, it is suggested that human capital ought to be treated as a stock-like asset at any time, which is bought with education and sold back partly over time as labor income or return on education. On top, it is found that education has a positive impact on the human capital level or the asset size, while it has a negative impact on its volatility. As a final thought, that might lead to the idea that the level of human capital decides on its volatility, so that one should not interpret level and risk as separate components any more.

## 6 Conclusion

In the near future, it becomes more and more difficult to build on the federal pension scheme. Due to the increasing gap between the number of contributors and receivers, current employees usually cannot keep their living standards during retirement when only relying on the retirement provisions by the state. Instead, it becomes crucial to initiate additional private retirement arrangements.

One savings possibility is to invest directly in capital market products, like in single stocks and bonds or more diversified investment funds. The question is then how to combine these capital market products to receive an optimal portfolio. It was found, that it is usually not sufficient to decide on asset weights within the investment portfolio once based on the current market and personal situation and keep them over a long holding period, as it is implicitly suggested by Markowitz (Campbell & Viceira, 2005). In the optimal case, the investor is required to rebalance his portfolio on a regular basis due to changes in personal circumstances, labor income situation, and market developments to receive a portfolio, which is risk / return efficient at any time. The average private investor is, however, not able to make such decision adequately, since he generally does not have the required time and financial knowledge.

One potential way to combine the characteristics of long-term holding periods and the requirements of private investors is life-cycle investing. Life-cycle funds are investment funds including an automated and predetermined rebalancing schedule from a strong tilt towards equities in the first part to a more conservative asset allocation when approximating the maturity date. Life-cycle funds have the advantage that their asset weights are rebalanced on a regular basis. Consequently, they account for the fact that expected returns and annualized variances are not static over time. In addition, they offer the investors a one-stop solution, since they need to decide on all future asset weights only once at the beginning of the contract. Afterwards, the investment process is automated and does not need to be managed by the investor anymore. The comparison of the life-cycle funds' theoretical foundation and its practical application leads to two different arguments for the initiation and structure of life-cycle investments. The theoretical one is the idea of human capital. It says that the discounted future labor income is comparable to a bond asset, since it is provided by rather secure and regular income streams as in the case of coupon payments. Considering total wealth instead of only the financial wealth portfolio, investors can afford tilting their financial portfolio towards equities in the beginning of the contract. Over the life-cycle, the human capital is reduced due

to labor income already paid, so that this has to be compensated by an increasing tilt towards bonds in the financial portfolio. To sum up, this argumentation leads to the roll-down schedule in equity weights the life-cycle funds idea builds on. This paper, however, also analyzes how the life-cycle funds idea is applied in practice. The prospectus of the Nuernberger life-cycle product leads to the assumption that it rather builds on the idea of term-structures in asset classes than on the human capital approach. The term-structure theory says that assets' expected returns and annualized variances vary with changing holding periods. Summarized, equity returns tend to show a predictable mean-reverting behavior so that the inherent risk reduces over time, whereas bonds tend to suffer from inflation risks when being held for a long time. In addition, the existence of a risk-free rate is very doubtful, since it suffers from reinvestment risk when extending the investment horizon. Since the asset class are exposed to unique risk patterns over time, the correlation structure between the assets changes as well. These observations bring scientists like Campbell and Viceira (2005) to recommend equity weights, which increase with the investment horizon. With its three-step buy-and-hold structure, the Nuernberger life-cycle model mainly builds on the theory of term-structures in asset classes.

Due to the gap between theory and practice, the main question of this paper is how to combine the two argumentations. Since the Nuernberger life-cycle model already uses the term-structure idea, the first question is if this model is already preferable to the traditional short-term optimization by Markowitz. For that, this paper conducts an analysis of the life-cycle model based on the prospectus published by Nuernberger and compares it with the results of a Markowitz portfolio optimization based on the same time period and data. This comparison shows that the Nuernberger life-cycle model seems to be preferable to the Markowitz approach based on the input used. Since this model, however, considers the human capital idea only in its very basic form, the paper continues with an analysis of the human capital characteristics using real statistical data. In a four-step analysis, this paper analyzes how age, educational level, and job choice influence the risk of becoming unemployed as a major human capital risk. It is found that independent of age, educational level or job choice, there might be strong short-term correlations between unemployment rates and stock market. As a consequence, this paper suggests treating human capital as a stock-like asset instead of a bond-like one. Concerning the human capital risk, however, there are trends observable in the average rates of change of unemployment rates. The results show that representatives of low education groups and workers of the fields 'unskilled labor' and 'handcraft' tend to be more exposed to

unemployment risk than highly educated employees and employees in management positions. The age seems to have a slightly weakening effect on unemployment risk, in particular for low education groups. The initial trend in average rates of change, however, is determined by the educational level. With the help of another statistic, this paper bridges the two analyses by showing that educational level and job choice are highly interrelated. The statistic says that employees in management positions tend to have a medium or high level of education, whereas handcrafters usually do not have such a strong educational background. A final analysis shows that the level of education tends to decide, via the chosen job category, on the size of human capital. In an extreme case comparison, it is shown how the chosen job category can decide on the development of labor income over working years. The analysis closes with the thought that human capital level and human capital risk might be directly interconnected, so that one should not see them as independent components of human wealth anymore.

Right now, the idea of human capital is, perhaps even unconsciously, implemented in the Nuernberger life-cycle product only in its very basic form. The model allows the savings from labor income to feed into the funds over time. From the series of single analysis, this paper intends to provide some recommendations for the practical application of the life-cycle idea, in general, and the improvement of the Nuernberger life-cycle model, in particular.

1. Human capital should be generally included in the investment planning over the whole investment horizon, since its individual characteristics might have divergent impacts on the individual investment behavior. That implies that the investors should optimize their total wealth portfolios instead of their financial wealth portfolio.
2. Since it is suggested treating human capital as stock-like asset independent from age, the equity weights within the life-cycle fund should be generally decreased. The extent to which the weights should be decreased depends on human capital risk. The idea is that an investor is more risk-taking when he is relatively sure of his labor income and can afford to gamble a bit on the stock market. When he faces a comparatively high risk of becoming unemployed, however, there is no balancing force so that the investor should be more risk-averse.
3. The risk of human capital is, at least from the analyses conducted within this paper, mainly driven by the education or the job choice. Since only downward risk really matters, it is recommended to take the education and the job choice, respectively, into account for the level of risk-aversion within the portfolio. For investors with a low

level of education or a related job, the stock weights should be lower over the whole horizon than for those with a high level of education, in particular when the investors are young. The downward risk of human capital for low education investors reduces only partly with age and stays always above the downward risk for high education investors.

4. The human capital feeds into the stock position of the total wealth portfolio, so that the financial stock position should be adjusted as mentioned above. Over time, however, the human capital asset becomes smaller due to the labor income already received. In addition, the risk of becoming unemployed decreases with age, in particular for weakly educated employees (cf. section 4.2). Consequently, the human capital stock becomes smaller and less exposed to unemployment risk with increasing age. As long as the life-cycle fund has a relatively long holding period, this paper suggests decreasing the stock weight in the first years more than in the mid-term, so that one ends up with a hump-shaped stock position.

There are a few conceptual limitations of both main analyses conducted for this paper. The first analysis is the performance comparison of the Markowitz and Nuernberger life-cycle approach. Following the two-stage approach by Markowitz, the optimal asset weights within an investment portfolio are based on historical data. “The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio” (Markowitz, 1952, p. 77). The Nuernberger life-cycle product allows the investors to decide on the composition of equity and debt part at every rebalancing date. The compositions of the sub-portfolios for the following stage of the contract, however, are also based on historical data. In addition, the weights of equity and bond component are also pre-determined, based on rather backward directed market experiences. The observation period, however, usually shows other characteristics than the actual investment period, since the market might develop in a completely unexpected way. The result might be a suboptimal choice of asset weights. As a second limitation of the analysis, the paper ignores transaction cost, opportunity costs or any asset-based fees. In the analyses of this paper, only total yields, not excess returns, are calculated to avoid biases. The life-cycle product requires using different risk-free rates, since the portfolios are optimized for different time periods. On top, the life-cycle product used by Nuernberger is actually a series of buy-and-hold portfolios with different lengths. In particular for the first period, which covers almost two thirds of the investment

horizon, the question arises if there is any risk-free rate at all. To avoid all these complexities, the total yield was calculated. In addition, the analysis ignores the transaction costs related to the investment funds. Table 14 indicates that including transaction costs might lead to very different results.

**Table 14: Transaction Costs of Investment Funds**

	Transaction Costs			
	A	B	C	D
<b>Asset-based fees</b>	5.00%	5.00%	3.00%	2.50%
<b>Management fee</b>	1.45%	1.40%	0.85%	0.70%
<b>Total expense ratio</b>	1.43%	1.40%	1.04%	0.71%
A	DWS Vermoegensbildungsfonds 1			
B	DWS Investa			
C	Allianz PIMCO Internationaler Rentenfonds A			
D	DWS Euroland Strategie (Renten)			

Source: Own Table, based on data derived from Bloomberg (2011)

For the comparison conducted within this paper, transaction costs are perhaps not that significant, because there are only two rebalances intended by the Nuernberger contract. Following the initial life-cycle idea and the recommendations given from this paper's analysis, the asset weights should be rebalanced on a regular basis due to the changes in human capital. This, however, might be insufficient in practice due to the high transaction costs.

With that aspect, this section provides already a first problem of the second main analysis, namely the complexity of human capital considerations. Besides that, it is important to highlight again that the human capital analysis was conducted only regarding unemployment risk. There are various other positive and negative human capital risks, like the risk of becoming incapable of working, career opportunities within a job category, and the employees' loyalty. Combining several human capital risk factors might support or weaken the statements of this paper. Finally, this paper comes up with a rather new suggestion, namely that human capital might be rather stock-like than bond-like independent of age due to the strong short-term correlations between stock and labor market. These short-term correlations are, however, calculated with a time lag between unemployment and market movements. Interestingly, this time lag is longer for the older age group than for the younger one. Benzoni, Collin-Dufresne and Goldstein (2007, p. 3) analyze the correlation between stock market and labor income, but they come to the conclusion that the time it takes the stock market developments to feed into the labor market might be crucial for the actual impact. The correlation analysis for the older

age group shows that the values tend to decrease significantly for the whole observation period due to disturbing movements in unemployment rates. Due to the comparatively long time lag between the unemployment rates and the stock market, it might be that the influence of the stock market's short-term developments is weakened by other forces. It is left for future research to analyze if the age-dependent time lags have any impact on the asset class, human capital should be assigned to.

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## Appendix

## Part A: Additional Tables

Table 15: Investment Performance Under Markowitz Portfolio Optimization

Investment Performance Under Markowitz Portfolio Optimization					
Date	Portfolio Prices	Units	Savings		
Jan-85	25.38	0.39	10	<b>Units</b>	91.29
Feb-85	24.24	0.41	10	<b>Sales</b>	5071.11
Mar-85	24.29	0.41	10	<b>Savings</b>	3120.00
Apr-85	24.89	0.40	10	<b>Profit</b>	1951.11
May-85	26.13	0.38	10	<b>Return</b>	62.54%
Jun-85	26.50	0.38	10		
Jul-85	26.08	0.38	10		
Aug-85	26.93	0.37	10		
Sep-85	27.03	0.37	10		
Oct-85	27.61	0.36	10		
Nov-85	26.84	0.37	10		
Dec-85	27.68	0.36	10		
Jan-86	27.31	0.37	10		
Feb-86	26.22	0.38	10		
Mar-86	27.60	0.36	10		
Apr-86	27.59	0.36	10		
May-86	27.28	0.37	10		
Jun-86	26.89	0.37	10		
Jul-86	26.28	0.38	10		
Aug-86	27.37	0.37	10		
Sep-86	26.68	0.37	10		
Oct-86	27.02	0.37	10		
Nov-86	26.38	0.38	10		
Dec-86	26.36	0.38	10		
Jan-87	25.26	0.40	10		
Feb-87	24.44	0.41	10		
Mar-87	24.95	0.40	10		
Apr-87	25.05	0.40	10		
May-87	25.22	0.40	10		
Jun-87	25.76	0.39	10		
Jul-87	26.27	0.38	10		
Aug-87	26.32	0.38	10		
Sep-87	26.15	0.38	10		
Oct-87	24.15	0.41	10		
Nov-87	22.87	0.44	10		
Dec-87	22.77	0.44	10		
Jan-88	22.85	0.44	10		
Feb-88	22.94	0.44	10		
Mar-88	23.01	0.43	10		
Apr-88	23.04	0.43	10		
May-88	23.40	0.43	10		
Jun-88	23.99	0.42	10		
Jul-88	24.41	0.41	10		
Aug-88	24.28	0.41	10		
Sep-88	24.95	0.40	10		
Oct-88	25.04	0.40	10		
Nov-88	24.03	0.42	10		
Dec-88	24.59	0.41	10		
Jan-89	25.24	0.40	10		
Feb-89	23.68	0.42	10		
Mar-89	24.23	0.41	10		
Apr-89	24.65	0.41	10		
May-89	25.29	0.40	10		
Jun-89	25.78	0.39	10		
Jul-89	26.17	0.38	10		
Aug-89	26.79	0.37	10		
Sep-89	26.52	0.38	10		
Oct-89	25.86	0.39	10		
Nov-89	25.43	0.39	10		
Dec-89	25.98	0.38	10		
Jan-90	26.25	0.38	10		
Feb-90	25.11	0.40	10		
Mar-90	26.01	0.38	10		
Apr-90	25.33	0.39	10		
May-90	25.82	0.39	10		
Jun-90	26.20	0.38	10		
Jul-90	26.52	0.38	10		
Aug-90	24.93	0.40	10		
Sep-90	23.68	0.42	10		
Oct-90	24.44	0.41	10		
Nov-90	23.19	0.43	10		
Dec-90	23.08	0.43	10		
Jan-91	23.33	0.43	10		
Feb-91	24.26	0.41	10		
Mar-91	24.67	0.41	10		
Apr-91	25.43	0.39	10		

May-91	26.00	0.38	10
Jun-91	25.78	0.39	10
Jul-91	25.81	0.39	10
Aug-91	26.19	0.38	10
Sep-91	26.03	0.38	10
Oct-91	26.08	0.38	10
Nov-91	23.99	0.42	10
Dec-91	23.88	0.42	10
Jan-92	24.83	0.40	10
Feb-92	25.35	0.39	10
Mar-92	25.25	0.40	10
Apr-92	25.50	0.39	10
May-92	25.93	0.39	10
Jun-92	25.34	0.39	10
Jul-92	24.77	0.40	10
Aug-92	24.04	0.42	10
Sep-92	23.94	0.42	10
Oct-92	24.74	0.40	10
Nov-92	23.41	0.43	10
Dec-92	23.64	0.42	10
Jan-93	23.81	0.42	10
Feb-93	24.82	0.40	10
Mar-93	25.16	0.40	10
Apr-93	24.94	0.40	10
May-93	25.30	0.40	10
Jun-93	26.00	0.38	10
Jul-93	26.80	0.37	10
Aug-93	27.33	0.37	10
Sep-93	27.02	0.37	10
Oct-93	28.37	0.35	10
Nov-93	26.90	0.37	10
Dec-93	28.09	0.36	10
Jan-94	28.15	0.36	10
Feb-94	27.30	0.37	10
Mar-94	26.88	0.37	10
Apr-94	27.43	0.36	10
May-94	26.61	0.38	10
Jun-94	25.83	0.39	10
Jul-94	26.35	0.38	10
Aug-94	26.68	0.37	10
Sep-94	25.68	0.39	10
Oct-94	25.77	0.39	10
Nov-94	24.48	0.41	10
Dec-94	24.68	0.41	10
Jan-95	24.29	0.41	10
Feb-95	24.51	0.41	10
Mar-95	23.58	0.42	10
Apr-95	24.22	0.41	10
May-95	25.01	0.40	10
Jun-95	25.03	0.40	10
Jul-95	25.68	0.39	10
Aug-95	26.38	0.38	10
Sep-95	26.03	0.38	10
Oct-95	25.98	0.38	10
Nov-95	25.34	0.39	10
Dec-95	25.66	0.39	10
Jan-96	26.91	0.37	10
Feb-96	26.71	0.37	10
Mar-96	26.95	0.37	10
Apr-96	27.39	0.37	10
May-96	27.78	0.36	10
Jun-96	27.88	0.36	10
Jul-96	27.40	0.36	10
Aug-96	27.94	0.36	10
Sep-96	28.90	0.35	10
Oct-96	29.07	0.34	10
Nov-96	28.96	0.35	10
Dec-96	29.22	0.34	10
Jan-97	30.22	0.33	10
Feb-97	31.50	0.32	10
Mar-97	32.03	0.31	10
Apr-97	32.32	0.31	10
May-97	33.08	0.30	10
Jun-97	34.70	0.29	10
Jul-97	37.88	0.26	10
Aug-97	35.18	0.28	10
Sep-97	36.50	0.27	10
Oct-97	34.70	0.29	10

Nov-97	34.17	0.29	10
Dec-97	35.35	0.28	10
Jan-98	36.67	0.27	10
Feb-98	38.09	0.26	10
Mar-98	39.95	0.25	10
Apr-98	39.81	0.25	10
May-98	42.13	0.24	10
Jun-98	43.50	0.23	10
Jul-98	43.84	0.23	10
Aug-98	39.55	0.25	10
Sep-98	37.58	0.27	10
Oct-98	38.38	0.26	10
Nov-98	39.39	0.25	10
Dec-98	39.07	0.26	10
Jan-99	50.16	0.20	10
Feb-99	49.28	0.20	10
Mar-99	49.96	0.20	10
Apr-99	52.61	0.19	10
May-99	51.26	0.20	10
Jun-99	52.98	0.19	10
Jul-99	51.88	0.19	10
Aug-99	52.89	0.19	10
Sep-99	52.17	0.19	10
Oct-99	53.76	0.19	10
Nov-99	54.60	0.18	10
Dec-99	59.94	0.17	10
Jan-00	59.45	0.17	10
Feb-00	62.98	0.16	10
Mar-00	63.69	0.16	10
Apr-00	63.86	0.16	10
May-00	62.28	0.16	10
Jun-00	61.68	0.16	10
Jul-00	63.58	0.16	10
Aug-00	65.15	0.15	10
Sep-00	64.40	0.16	10
Oct-00	65.42	0.15	10
Nov-00	60.52	0.17	10
Dec-00	59.99	0.17	10
Jan-01	61.22	0.16	10
Feb-01	59.33	0.17	10
Mar-01	58.57	0.17	10
Apr-01	59.74	0.17	10
May-01	59.94	0.17	10
Jun-01	59.71	0.17	10
Jul-01	58.76	0.17	10
Aug-01	55.86	0.18	10
Sep-01	51.24	0.20	10
Oct-01	53.01	0.19	10
Nov-01	53.68	0.19	10
Dec-01	53.95	0.19	10
Jan-02	54.29	0.18	10
Feb-02	53.80	0.19	10
Mar-02	55.26	0.18	10
Apr-02	53.36	0.19	10
May-02	52.53	0.19	10
Jun-02	49.91	0.20	10
Jul-02	48.20	0.21	10
Aug-02	47.79	0.21	10
Sep-02	44.38	0.23	10
Oct-02	45.67	0.22	10
Nov-02	45.08	0.22	10
Dec-02	42.92	0.23	10
Jan-03	41.39	0.24	10
Feb-03	40.98	0.24	10
Mar-03	40.26	0.25	10
Apr-03	42.52	0.24	10
May-03	42.60	0.23	10
Jun-03	44.30	0.23	10
Jul-03	44.90	0.22	10
Aug-03	46.17	0.22	10
Sep-03	45.07	0.22	10
Oct-03	46.38	0.22	10
Nov-03	44.92	0.22	10
Dec-03	45.41	0.22	10
Jan-04	46.51	0.22	10
Feb-04	46.24	0.22	10
Mar-04	46.31	0.22	10
Apr-04	46.05	0.22	10

May-04	45.26	0.22	10
Jun-04	46.02	0.22	10
Jul-04	45.18	0.22	10
Aug-04	45.24	0.22	10
Sep-04	45.62	0.22	10
Oct-04	45.72	0.22	10
Nov-04	45.36	0.22	10
Dec-04	45.49	0.22	10
Jan-05	46.45	0.22	10
Feb-05	46.71	0.21	10
Mar-05	46.82	0.21	10
Apr-05	46.46	0.22	10
May-05	48.27	0.21	10
Jun-05	49.35	0.20	10
Jul-05	50.25	0.20	10
Aug-05	49.98	0.20	10
Sep-05	51.10	0.20	10
Oct-05	49.93	0.20	10
Nov-05	50.28	0.20	10
Dec-05	51.34	0.19	10
Jan-06	52.57	0.19	10
Feb-06	53.45	0.19	10
Mar-06	53.10	0.19	10
Apr-06	52.89	0.19	10
May-06	50.78	0.20	10
Jun-06	50.59	0.20	10
Jul-06	50.89	0.20	10
Aug-06	51.81	0.19	10
Sep-06	52.89	0.19	10
Oct-06	54.00	0.19	10
Nov-06	52.93	0.19	10
Dec-06	53.59	0.19	10
Jan-07	54.30	0.18	10
Feb-07	54.20	0.18	10
Mar-07	55.05	0.18	10
Apr-07	56.76	0.18	10
May-07	57.99	0.17	10
Jun-07	57.81	0.17	10
Jul-07	56.89	0.18	10
Aug-07	56.45	0.18	10
Sep-07	57.20	0.17	10
Oct-07	57.71	0.17	10
Nov-07	55.18	0.18	10
Dec-07	55.46	0.18	10
Jan-08	51.86	0.19	10
Feb-08	51.98	0.19	10
Mar-08	50.26	0.20	10
Apr-08	51.44	0.19	10
May-08	52.24	0.19	10
Jun-08	49.09	0.20	10
Jul-08	48.98	0.20	10
Aug-08	49.97	0.20	10
Sep-08	46.90	0.21	10
Oct-08	44.80	0.22	10
Nov-08	43.22	0.23	10
Dec-08	43.40	0.23	10
Jan-09	43.14	0.23	10
Feb-09	41.14	0.24	10
Mar-09	41.08	0.24	10
Apr-09	43.73	0.23	10
May-09	44.30	0.23	10
Jun-09	44.43	0.23	10
Jul-09	46.67	0.21	10
Aug-09	48.01	0.21	10
Sep-09	49.15	0.20	10
Oct-09	48.42	0.21	10
Nov-09	47.95	0.21	10
Dec-09	49.38	0.20	10
Jan-10	48.78	0.21	10
Feb-10	49.31	0.20	10
Mar-10	51.66	0.19	10
Apr-10	52.12	0.19	10
May-10	52.49	0.19	10
Jun-10	53.03	0.19	10
Jul-10	52.96	0.19	10
Aug-10	53.05	0.19	10
Sep-10	53.47	0.19	10
Oct-10	54.56	0.18	10
Nov-10	54.65	0.18	10
Dec-10	55.55	0.18	10

Source: Own Table



Table 16: Investment Performance Under Life-Cycle Portfolio Optimization

Date	Investment Performance Under Nuernberger Life-Cycle Portfolio Optimization (Conservative / Risky)									
	Prices					Prices				
	Conservative	Returns	Savings	Units	Risky	Returns	Savings	Units		
Jan-85	22.60		10.00	0.44	21.53		10	0.46		
Feb-85	22.55	-0.19%	10.00	0.44	21.91	1.76%	10	0.46		
Mar-85	22.54	-0.08%	10.00	0.44	21.88	-0.11%	10	0.46		
Apr-85	22.93	1.77%	10.00	0.44	22.22	1.56%	10	0.45		
May-85	24.48	6.74%	10.00	0.41	23.92	7.65%	10	0.42		
Jun-85	25.00	2.13%	10.00	0.40	24.52	2.51%	10	0.41		
Jul-85	24.32	-2.73%	10.00	0.41	23.74	-3.20%	10	0.42		
Aug-85	25.42	4.54%	10.00	0.39	24.98	5.21%	10	0.40		
Sep-85	26.23	3.17%	10.00	0.38	26.09	4.47%	10	0.38		
Oct-85	28.03	6.86%	10.00	0.36	28.39	8.79%	10	0.35		
Nov-85	27.26	-2.73%	10.00	0.37	27.57	-2.89%	10	0.36		
Dec-85	28.50	4.53%	10.00	0.35	29.04	5.34%	10	0.34		
Jan-86	28.07	-1.49%	10.00	0.36	28.59	-1.52%	10	0.35		
Feb-86	27.50	-2.04%	10.00	0.36	28.24	-1.23%	10	0.35		
Mar-86	29.39	6.87%	10.00	0.34	30.36	7.50%	10	0.33		
Apr-86	29.70	1.03%	10.00	0.34	30.81	1.47%	10	0.32		
May-86	28.42	-4.29%	10.00	0.35	29.10	-5.53%	10	0.34		
Jun-86	28.18	-0.85%	10.00	0.35	28.91	-0.66%	10	0.35		
Jul-86	27.24	-3.36%	10.00	0.37	27.83	-3.74%	10	0.36		
Aug-86	29.74	9.19%	10.00	0.34	30.93	11.15%	10	0.32		
Sep-86	28.50	-4.17%	10.00	0.35	29.47	-4.73%	10	0.34		
Oct-86	28.84	1.20%	10.00	0.35	29.82	1.18%	10	0.34		
Nov-86	28.48	-1.23%	10.00	0.35	29.58	-0.79%	10	0.34		
Dec-86	28.13	-1.23%	10.00	0.36	29.10	-1.62%	10	0.34		
Jan-87	26.19	-6.90%	10.00	0.38	26.79	-7.94%	10	0.37		
Feb-87	25.48	-2.72%	10.00	0.39	26.10	-2.57%	10	0.38		
Mar-87	26.33	3.35%	10.00	0.38	27.09	3.79%	10	0.37		
Apr-87	26.49	0.59%	10.00	0.38	27.27	0.65%	10	0.37		
May-87	26.53	0.18%	10.00	0.38	27.26	-0.02%	10	0.37		
Jun-87	27.83	4.89%	10.00	0.36	28.87	5.91%	10	0.35		
Jul-87	28.99	4.18%	10.00	0.34	30.33	5.05%	10	0.33		
Aug-87	29.31	1.10%	10.00	0.34	30.76	1.41%	10	0.33		
Sep-87	28.91	-1.37%	10.00	0.35	30.25	-1.65%	10	0.33		
Oct-87	23.43	-18.95%	10.00	0.43	23.44	-22.53%	10	0.43		
Nov-87	20.94	-10.61%	10.00	0.48	20.42	-12.87%	10	0.49		
Dec-87	20.65	-1.39%	10.00	0.48	20.04	-1.85%	10	0.50		
Jan-88	20.13	-2.53%	10.00	0.50	19.26	-3.91%	10	0.52		
Feb-88	21.55	7.05%	10.00	0.46	21.23	10.24%	10	0.47		
Mar-88	21.43	-0.53%	10.00	0.47	21.05	-0.83%	10	0.48		
Apr-88	21.48	0.22%	10.00	0.47	21.10	0.23%	10	0.47		
May-88	21.81	1.52%	10.00	0.46	21.44	1.59%	10	0.47		
Jun-88	22.87	4.87%	10.00	0.44	22.68	5.79%	10	0.44		
Jul-88	23.54	2.92%	10.00	0.42	23.45	3.39%	10	0.43		
Aug-88	23.39	-0.63%	10.00	0.43	23.30	-0.64%	10	0.43		
Sep-88	24.41	4.35%	10.00	0.41	24.47	5.02%	10	0.41		
Oct-88	24.66	1.06%	10.00	0.41	24.82	1.45%	10	0.40		
Nov-88	22.99	-6.81%	10.00	0.44	22.87	-7.85%	10	0.44		
Dec-88	23.87	3.86%	10.00	0.42	23.90	4.49%	10	0.42		
Jan-89	24.88	4.19%	10.00	0.40	25.01	4.65%	10	0.40		
Feb-89	23.85	-4.11%	10.00	0.42	24.21	-3.20%	10	0.41		
Mar-89	24.43	2.42%	10.00	0.41	24.80	2.43%	10	0.40		
Apr-89	25.25	3.37%	10.00	0.40	25.80	4.03%	10	0.39		
May-89	26.18	3.69%	10.00	0.38	26.84	4.03%	10	0.37		
Jun-89	27.05	3.31%	10.00	0.37	27.89	3.89%	10	0.36		
Jul-89	27.97	3.38%	10.00	0.36	29.00	3.98%	10	0.34		
Aug-89	29.04	3.82%	10.00	0.34	30.25	4.31%	10	0.33		
Sep-89	28.91	-0.43%	10.00	0.35	30.21	-0.13%	10	0.33		
Oct-89	27.55	-4.71%	10.00	0.36	28.53	-5.55%	10	0.35		
Nov-89	27.53	-0.07%	10.00	0.36	28.70	0.59%	10	0.35		
Dec-89	29.08	5.61%	10.00	0.34	30.70	6.99%	10	0.33		
Jan-90	29.59	1.77%	10.00	0.34	31.37	2.16%	10	0.32		
Feb-90	29.09	-1.70%	10.00	0.34	31.11	-0.81%	10	0.32		
Mar-90	31.02	6.64%	10.00	0.32	33.51	7.70%	10	0.30		
Apr-90	29.59	-4.61%	10.00	0.34	31.73	-5.31%	10	0.32		
May-90	30.57	3.31%	10.00	0.33	32.86	3.56%	10	0.30		
Jun-90	31.02	1.46%	10.00	0.32	33.35	1.50%	10	0.30		
Jul-90	31.37	1.15%	10.00	0.32	33.76	1.22%	10	0.30		
Aug-90	27.78	-11.45%	10.00	0.36	29.31	-13.17%	10	0.34		
Sep-90	24.74	-10.95%	10.00	0.40	25.50	-12.99%	10	0.39		
Oct-90	26.23	6.05%	10.00	0.38	27.32	7.14%	10	0.37		
Nov-90	25.04	-4.55%	10.00	0.40	26.15	-4.31%	10	0.38		
Dec-90	24.76	-1.12%	10.00	0.40	25.76	-1.47%	10	0.39		
Jan-91	25.09	1.33%	10.00	0.40	26.13	1.40%	10	0.38		
Feb-91	26.64	6.19%	10.00	0.38	27.95	6.97%	10	0.36		
Mar-91	26.72	0.31%	10.00	0.37	27.87	-0.29%	10	0.36		
Apr-91	27.85	4.23%	10.00	0.36	29.18	4.73%	10	0.34		

May-91	29.01	4.15%	10.00	0.34	30.60	4.85%	10	0.33
Jun-91	28.26	-2.58%	10.00	0.35	29.62	-3.20%	10	0.34
Jul-91	28.40	0.48%	10.00	0.35	29.80	0.60%	10	0.34
Aug-91	28.88	1.71%	10.00	0.35	30.34	1.80%	10	0.33
Sep-91	28.37	-1.79%	10.00	0.35	29.68	-2.19%	10	0.34
Oct-91	28.15	-0.76%	10.00	0.36	29.35	-1.10%	10	0.34
Nov-91	26.18	-7.02%	10.00	0.38	27.39	-6.66%	10	0.37
Dec-91	26.18	0.00%	10.00	0.38	27.43	0.13%	10	0.36
Jan-92	27.56	5.30%	10.00	0.36	29.00	5.75%	10	0.34
Feb-92	28.40	3.05%	10.00	0.35	29.99	3.42%	10	0.33
Mar-92	28.14	-0.94%	10.00	0.36	29.66	-1.11%	10	0.34
Apr-92	28.37	0.83%	10.00	0.35	29.87	0.71%	10	0.33
May-92	29.10	2.58%	10.00	0.34	30.75	2.92%	10	0.33
Jun-92	28.21	-3.08%	10.00	0.35	29.72	-3.33%	10	0.34
Jul-92	26.90	-4.64%	10.00	0.37	28.09	-5.48%	10	0.36
Aug-92	25.83	-3.98%	10.00	0.39	26.87	-4.35%	10	0.37
Sep-92	25.49	-1.32%	10.00	0.39	26.41	-1.72%	10	0.38
Oct-92	26.16	2.63%	10.00	0.38	27.04	2.37%	10	0.37
Nov-92	25.32	-3.20%	10.00	0.39	26.37	-2.46%	10	0.38
Dec-92	25.52	0.79%	10.00	0.39	26.55	0.67%	10	0.38
Jan-93	25.71	0.77%	10.00	0.39	26.77	0.85%	10	0.37
Feb-93	27.10	5.40%	10.00	0.37	28.35	5.89%	10	0.35
Mar-93	27.47	1.37%	10.00	0.36	28.73	1.33%	10	0.35
Apr-93	27.17	-1.12%	10.00	0.37	28.36	-1.27%	10	0.35
May-93	27.63	1.73%	10.00	0.36	28.87	1.80%	10	0.35
Jun-93	28.36	2.62%	10.00	0.35	29.60	2.52%	10	0.34
Jul-93	29.64	4.51%	10.00	0.34	31.11	5.09%	10	0.32
Aug-93	30.80	3.93%	10.00	0.32	32.56	4.67%	10	0.31
Sep-93	30.25	-1.77%	10.00	0.33	31.92	-1.97%	10	0.31
Oct-93	32.34	6.90%	10.00	0.31	34.34	7.57%	10	0.29
Nov-93	30.86	-4.57%	10.00	0.32	32.83	-4.38%	10	0.30
Dec-93	33.19	7.55%	10.00	0.30	35.64	8.56%	10	0.28
Jan-94	33.12	-0.24%	10.00	0.30	35.47	-0.49%	10	0.28
Feb-94	31.92	-3.60%	10.00	0.31	34.12	-3.79%	10	0.29
Mar-94	31.66	-0.84%	10.00	0.32	33.97	-0.45%	10	0.29
Apr-94	32.94	4.04%	10.00	0.30	35.59	4.76%	10	0.28
May-94	31.55	-4.20%	10.00	0.32	33.93	-4.66%	10	0.29
Jun-94	30.52	-3.28%	10.00	0.33	32.78	-3.39%	10	0.31
Jul-94	31.60	3.54%	10.00	0.32	34.11	4.07%	10	0.29
Aug-94	32.35	2.39%	10.00	0.31	35.05	2.75%	10	0.29
Sep-94	30.46	-5.86%	10.00	0.33	32.75	-6.55%	10	0.31
Oct-94	30.79	1.09%	10.00	0.32	33.21	1.38%	10	0.30
Nov-94	29.31	-4.82%	10.00	0.34	31.61	-4.79%	10	0.32
Dec-94	29.71	1.40%	10.00	0.34	32.13	1.64%	10	0.31
Jan-95	28.89	-2.79%	10.00	0.35	31.12	-3.17%	10	0.32
Feb-95	29.28	1.35%	10.00	0.34	31.63	1.64%	10	0.32
Mar-95	27.52	-6.01%	10.00	0.36	29.46	-6.84%	10	0.34
Apr-95	28.61	3.98%	10.00	0.35	30.77	4.42%	10	0.33
May-95	29.51	3.14%	10.00	0.34	31.74	3.15%	10	0.32
Jun-95	29.56	0.16%	10.00	0.34	31.78	0.15%	10	0.31
Jul-95	30.87	4.43%	10.00	0.32	33.38	5.03%	10	0.30
Aug-95	31.44	1.86%	10.00	0.32	33.91	1.57%	10	0.29
Sep-95	30.67	-2.44%	10.00	0.33	32.98	-2.73%	10	0.30
Oct-95	30.15	-1.69%	10.00	0.33	32.28	-2.12%	10	0.31
Nov-95	29.99	-0.54%	10.00	0.33	32.28	0.00%	10	0.31
Dec-95	30.37	1.25%	10.00	0.33	32.69	1.28%	10	0.31
Jan-96	32.61	7.38%	10.00	0.31	35.38	8.22%	10	0.28
Feb-96	32.60	-0.04%	10.00	0.31	35.44	0.18%	10	0.28
Mar-96	32.92	0.99%	10.00	0.30	35.80	1.01%	10	0.28
Apr-96	33.47	1.68%	10.00	0.30	36.38	1.60%	10	0.27
May-96	34.19	2.13%	10.00	0.29	37.25	2.40%	10	0.27
Jun-96	34.40	0.62%	10.00	0.29	37.51	0.71%	10	0.27
Jul-96	33.17	-3.56%	10.00	0.30	36.00	-4.02%	10	0.28
Aug-96	34.03	2.58%	10.00	0.29	37.01	2.80%	10	0.27
Sep-96	35.32	3.80%	10.00	0.28	38.46	3.91%	10	0.26
Oct-96	35.32	0.00%	10.00	0.28	38.40	-0.15%	10	0.26
Nov-96	36.70	3.91%	10.00	0.27	40.38	5.16%	10	0.25
Dec-96	37.09	1.06%	10.00	0.27	40.82	1.10%	10	0.24
Jan-97	38.88	4.81%	10.00	0.26	42.94	5.18%	10	0.23
Feb-97	41.25	6.10%	10.00	0.24	45.80	6.66%	10	0.22
Mar-97	42.63	3.37%	10.00	0.23	47.59	3.92%	10	0.21
Apr-97	42.99	0.84%	10.00	0.23	47.95	0.76%	10	0.21
May-97	44.86	4.33%	10.00	0.22	50.26	4.81%	10	0.20
Jun-97	47.92	6.82%	10.00	0.21	53.93	7.29%	10	0.19
Jul-97	53.98	12.65%	10.00	0.19	61.32	13.70%	10	0.16
Aug-97	48.23	-10.64%	10.00	0.21	54.22	-11.58%	10	0.18
Sep-97	51.05	5.84%	10.00	0.20	57.72	6.47%	10	0.17
Oct-97	47.37	-7.21%	10.00	0.21	53.18	-7.88%	10	0.19

Nov-97	48.37	2.12%	10.00	0.21		54.77	3.00%	10	0.18
Dec-97	50.36	4.11%	10.00	0.20		57.17	4.37%	10	0.17
Jan-98	53.23	5.70%	10.00	0.19		60.65	6.10%	10	0.16
Feb-98	56.52	6.19%	10.00	0.18		64.74	6.74%	10	0.15
Mar-98	60.06	6.26%	10.00	0.17		69.10	6.73%	10	0.14
Apr-98	59.72	-0.57%	10.00	0.17		68.74	-0.52%	10	0.15
May-98	64.46	7.95%	10.00	0.16		74.65	8.59%	10	0.13
Jun-98	66.89	3.76%	10.00	0.15		77.61	3.96%	10	0.13
Jul-98	67.94	1.57%	10.00	0.15		78.94	1.72%	10	0.13
Aug-98	57.49	-15.38%	10.00	0.17		65.79	-16.67%	10	0.15
Sep-98	52.91	-7.98%	10.00	0.19		60.05	-8.72%	10	0.17
Oct-98	55.38	4.67%	10.00	0.18		63.14	5.15%	10	0.16
Nov-98	59.69	7.78%	10.00	0.17		68.78	8.93%	10	0.15
Dec-98	59.23	-0.77%	10.00	0.17		68.18	-0.86%	10	0.15
Jan-99	64.55	8.97%	10.00	0.15		71.20	4.42%	10	0.14
Feb-99	63.46	-1.68%	10.00	0.16		69.90	-1.82%	10	0.14
Mar-99	65.66	3.46%	10.00	0.15	#Units	72.51	3.74%	10	0.14
Apr-99	70.60	7.52%	10.00	0.14	58.40	78.51	8.28%	10	0.13
May-99	68.49	-2.99%	10.00	0.15	Savings	75.96	-3.25%	10	0.13
Jun-99	73.65	7.53%	10.00	0.14	1800.00	82.51	8.61%	10	0.12
Jul-99	73.32	-0.44%	10.00	0.14	Sales	82.24	-0.32%	10	0.12
Aug-99	75.38	2.80%	10.00	0.13	5722.03	84.74	3.03%	10	0.12
Sep-99	74.39	-1.31%	10.00	0.13		83.52	-1.43%	10	0.12
Oct-99	78.14	5.04%	10.00	0.13		88.21	5.62%	10	0.11
Nov-99	84.47	8.11%	10.00	0.12		96.40	9.28%	10	0.10
Dec-99	97.98	15.99%	10.00	0.10		113.25	17.48%	10	0.09
Jan-00	62.45		10.00	0.16		96.42		10	0.10
Feb-00	66.26	6.09%	10.00	0.15		104.72	8.61%	10	0.10
Mar-00	66.76	0.76%	10.00	0.15		105.30	0.55%	10	0.09
Apr-00	65.83	-1.40%	10.00	0.15		103.22	-1.97%	10	0.10
May-00	64.88	-1.44%	10.00	0.15		101.17	-1.99%	10	0.10
Jun-00	63.76	-1.72%	10.00	0.16		98.47	-2.66%	10	0.10
Jul-00	65.93	3.41%	10.00	0.15		103.20	4.81%	10	0.10
Aug-00	66.39	0.69%	10.00	0.15		104.20	0.97%	10	0.10
Sep-00	65.39	-1.51%	10.00	0.15		101.73	-2.37%	10	0.10
Oct-00	66.34	1.45%	10.00	0.15		103.74	1.98%	10	0.10
Nov-00	61.60	-7.14%	10.00	0.16		94.79	-8.63%	10	0.11
Dec-00	61.78	0.30%	10.00	0.16		94.66	-0.14%	10	0.11
Jan-01	63.18	2.26%	10.00	0.16		97.24	2.73%	10	0.10
Feb-01	60.87	-3.67%	10.00	0.16		91.75	-5.65%	10	0.11
Mar-01	59.70	-1.92%	10.00	0.17		88.66	-3.37%	10	0.11
Apr-01	60.98	2.15%	10.00	0.16		91.77	3.51%	10	0.11
May-01	60.22	-1.25%	10.00	0.17		89.87	-2.07%	10	0.11
Jun-01	59.95	-0.44%	10.00	0.17		88.79	-1.21%	10	0.11
Jul-01	59.44	-0.86%	10.00	0.17		87.33	-1.65%	10	0.11
Aug-01	56.47	-4.99%	10.00	0.18		80.24	-8.12%	10	0.12
Sep-01	50.46	-10.64%	10.00	0.20		67.10	-16.38%	10	0.15
Oct-01	52.32	3.68%	10.00	0.19		70.43	4.97%	10	0.14
Nov-01	53.06	1.42%	10.00	0.19		74.62	5.95%	10	0.13
Dec-01	53.49	0.81%	10.00	0.19		76.13	2.02%	10	0.13
Jan-02	53.52	0.07%	10.00	0.19		76.05	-0.11%	10	0.13
Feb-02	52.86	-1.24%	10.00	0.19		74.37	-2.20%	10	0.13
Mar-02	55.15	4.34%	10.00	0.18		79.85	7.37%	10	0.13
Apr-02	52.94	-4.02%	10.00	0.19		74.46	-6.76%	10	0.13
May-02	52.23	-1.34%	10.00	0.19		72.70	-2.36%	10	0.14
Jun-02	49.70	-4.84%	10.00	0.20		66.54	-8.47%	10	0.15
Jul-02	47.53	-4.35%	10.00	0.21		61.33	-7.84%	10	0.16
Aug-02	46.61	-1.95%	10.00	0.21		58.60	-4.45%	10	0.17
Sep-02	42.62	-8.55%	10.00	0.23		48.92	-16.52%	10	0.20
Oct-02	44.17	3.64%	10.00	0.23		52.86	8.06%	10	0.19
Nov-02	43.74	-0.97%	10.00	0.23		53.64	1.48%	10	0.19
Dec-02	41.63	-4.84%	10.00	0.24		48.06	-10.41%	10	0.21
Jan-03	40.31	-3.17%	10.00	0.25		44.61	-7.17%	10	0.22
Feb-03	39.60	-1.76%	10.00	0.25		42.64	-4.43%	10	0.23
Mar-03	39.05	-1.39%	10.00	0.26		41.59	-2.44%	10	0.24
Apr-03	42.24	8.16%	10.00	0.24		48.60	16.84%	10	0.21
May-03	42.83	1.42%	10.00	0.23		48.83	0.48%	10	0.20
Jun-03	44.56	4.04%	10.00	0.22		52.75	8.02%	10	0.19
Jul-03	45.43	1.95%	10.00	0.22		54.79	3.87%	10	0.18
Aug-03	46.11	1.50%	10.00	0.22		56.36	2.87%	10	0.18
Sep-03	45.38	-1.60%	10.00	0.22		54.08	-4.05%	10	0.18
Oct-03	46.95	3.48%	10.00	0.21		58.14	7.52%	10	0.17
Nov-03	45.67	-2.74%	10.00	0.22		57.37	-1.32%	10	0.17
Dec-03	46.72	2.31%	10.00	0.21		59.28	3.32%	10	0.17
Jan-04	47.73	2.15%	10.00	0.21		61.23	3.29%	10	0.16
Feb-04	47.47	-0.53%	10.00	0.21		60.12	-1.81%	10	0.17
Mar-04	46.74	-1.54%	10.00	0.21		58.16	-3.26%	10	0.17
Apr-04	46.84	0.21%	10.00	0.21		58.67	0.88%	10	0.17

May-04	46.35	-1.06%	10.00	0.22		57.50	-2.00%	10	0.17
Jun-04	47.07	1.57%	10.00	0.21		59.12	2.81%	10	0.17
Jul-04	46.10	-2.07%	10.00	0.22		56.54	-4.36%	10	0.18
Aug-04	45.91	-0.40%	10.00	0.22		55.57	-1.72%	10	0.18
Sep-04	46.70	1.72%	10.00	0.21		57.10	2.75%	10	0.18
Oct-04	46.93	0.48%	10.00	0.21		57.20	0.18%	10	0.17
Nov-04	46.85	-0.17%	10.00	0.21		58.49	2.26%	10	0.17
Dec-04	47.46	1.32%	10.00	0.21		59.66	2.00%	10	0.17
Jan-05	47.81	0.72%	10.00	0.21		59.97	0.51%	10	0.17
Feb-05	48.48	1.40%	10.00	0.21		61.62	2.76%	10	0.16
Mar-05	48.58	0.21%	10.00	0.21		61.67	0.07%	10	0.16
Apr-05	48.00	-1.19%	10.00	0.21		59.70	-3.19%	10	0.17
May-05	49.58	3.29%	10.00	0.20		62.88	5.33%	10	0.16
Jun-05	50.68	2.21%	10.00	0.20		64.84	3.11%	10	0.15
Jul-05	51.97	2.56%	10.00	0.19		67.93	4.77%	10	0.15
Aug-05	51.66	-0.61%	10.00	0.19		66.94	-1.46%	10	0.15
Sep-05	52.95	2.51%	10.00	0.19		69.92	4.46%	10	0.14
Oct-05	51.81	-2.15%	10.00	0.19		67.84	-2.99%	10	0.15
Nov-05	52.15	0.64%	10.00	0.19		70.48	3.90%	10	0.14
Dec-05	53.28	2.17%	10.00	0.19		72.75	3.22%	10	0.14
Jan-06	54.89	3.03%	10.00	0.18		76.70	5.42%	10	0.13
Feb-06	55.91	1.86%	10.00	0.18		78.96	2.95%	10	0.13
Mar-06	56.12	0.36%	10.00	0.18		79.83	1.11%	10	0.13
Apr-06	56.23	0.20%	10.00	0.18		80.36	0.66%	10	0.12
May-06	53.98	-3.99%	10.00	0.19		74.92	-6.76%	10	0.13
Jun-06	53.81	-0.33%	10.00	0.19		74.75	-0.23%	10	0.13
Jul-06	53.94	0.25%	10.00	0.19		74.65	-0.13%	10	0.13
Aug-06	55.31	2.53%	10.00	0.18		77.37	3.64%	10	0.13
Sep-06	56.32	1.82%	10.00	0.18		79.35	2.55%	10	0.13
Oct-06	57.81	2.66%	10.00	0.17		82.75	4.29%	10	0.12
Nov-06	57.19	-1.08%	10.00	0.17	Development of P2	82.70	-0.06%	10	0.12
Dec-06	58.13	1.64%	10.00	0.17	5724.52	85.20	3.02%	10	0.12
Jan-07	59.15	1.76%	10.00	0.17	Average Return	87.64	2.87%	10	0.11
Feb-07	59.19	0.08%	10.00	0.17	0.04%	87.37	-0.31%	10	0.11
Mar-07	60.48	2.17%	10.00	0.17	#Units	90.33	3.39%	10	0.11
Apr-07	63.10	4.33%	10.00	0.16	18.21	96.44	6.76%	10	0.10
May-07	64.82	2.73%	10.00	0.15	Savings	100.52	4.24%	10	0.10
Jun-07	64.70	-0.20%	10.00	0.15	960.00	100.35	-0.17%	10	0.10
Jul-07	63.36	-2.07%	10.00	0.16	Sales	96.86	-3.47%	10	0.10
Aug-07	62.69	-1.06%	10.00	0.16	1137.56	94.84	-2.09%	10	0.11
Sep-07	63.78	1.75%	10.00	0.16	Total Sales	97.52	2.83%	10	0.10
Oct-07	65.07	2.02%	10.00	0.15	6862.08	99.95	2.49%	10	0.10
Nov-07	61.88	-4.90%	10.00	0.16		94.13	-5.82%	10	0.11
Dec-07	62.47	0.96%	10.00	0.16		95.86	1.84%	10	0.10
Jan-08	32.23		10.00	0.31		53.42		10	0.19
Feb-08	32.26	0.08%	10.00	0.31		53.48	0.13%	10	0.19
Mar-08	31.81	-1.38%	10.00	0.31		51.67	-3.39%	10	0.19
Apr-08	31.13	-2.14%	10.00	0.32		52.96	2.50%	10	0.19
May-08	30.80	-1.06%	10.00	0.32		53.94	1.85%	10	0.19
Jun-08	30.53	-0.87%	10.00	0.33		50.23	-6.88%	10	0.20
Jul-08	30.68	0.47%	10.00	0.33		50.13	-0.20%	10	0.20
Aug-08	32.05	4.46%	10.00	0.31		50.90	1.54%	10	0.20
Sep-08	32.92	2.74%	10.00	0.30		47.39	-6.90%	10	0.21
Oct-08	36.35	10.41%	10.00	0.28		45.25	-4.52%	10	0.22
Nov-08	35.98	-1.01%	10.00	0.28		43.69	-3.43%	10	0.23
Dec-08	35.02	-2.65%	10.00	0.29		44.19	1.13%	10	0.23
Jan-09	37.09	5.89%	10.00	0.27		43.76	-0.96%	10	0.23
Feb-09	36.67	-1.11%	10.00	0.27		41.59	-4.96%	10	0.24
Mar-09	35.45	-3.35%	10.00	0.28		41.61	0.06%	10	0.24
Apr-09	35.09	-1.02%	10.00	0.29		44.86	7.82%	10	0.22
May-09	34.30	-2.24%	10.00	0.29		45.33	1.03%	10	0.22
Jun-09	34.79	1.42%	10.00	0.29		44.94	-0.85%	10	0.22
Jul-09	35.37	1.67%	10.00	0.28		47.45	5.59%	10	0.21
Aug-09	35.80	1.22%	10.00	0.28		48.94	3.15%	10	0.20
Sep-09	35.97	0.49%	10.00	0.28		50.47	3.11%	10	0.20
Oct-09	35.67	-0.83%	10.00	0.28		49.44	-2.04%	10	0.20
Nov-09	35.39	-0.80%	10.00	0.28	Development of P2	49.05	-0.78%	10	0.20
Dec-09	35.22	-0.47%	10.00	0.28	6909.25	50.62	3.19%	10	0.20
Jan-10	36.63	4.00%	10.00	0.27	Average Return	49.71	-1.78%	10	0.20
Feb-10	37.55	2.50%	10.00	0.27	0.69%	50.20	0.98%	10	0.20
Mar-10	37.53	-0.04%	10.00	0.27	#Units	52.82	5.23%	10	0.19
Apr-10	38.12	1.56%	10.00	0.26	10.08	53.28	0.87%	10	0.19
May-10	40.69	6.75%	10.00	0.25	Savings	53.93	1.21%	10	0.19
Jun-10	41.92	3.02%	10.00	0.24	360.00	54.97	1.94%	10	0.18
Jul-10	40.68	-2.95%	10.00	0.25	Sales	54.81	-0.31%	10	0.18
Aug-10	42.74	5.06%	10.00	0.23	406.40	54.96	0.28%	10	0.18
Sep-10	40.94	-4.22%	10.00	0.24	Total Sales	55.65	1.25%	10	0.18
Oct-10	40.42	-1.26%	10.00	0.25	7315.65	57.19	2.78%	10	0.17
Nov-10	40.88	1.13%	10.00	0.24		57.55	0.63%	10	0.17
Dec-10	40.32	-1.38%	10.00	0.25		58.40	1.48%	10	0.17
Total			Savings	#Units	Total Sales	Total	Savings	#Units	Total Sales
			3120.00	86.69	7315.65		3120.00	76.66	8079.50
			Profit	Return			Profit	Return	
				4195.65	134.48%		4959.50	158.96%	

All prices are in €

Source: Own Table

Table 17: Human Capital Calculation

Human Capital Calculation						
Inflation-Rate: 3%		Average Income		Discounted Income		
Age	Working years	Management	Handcraft	Management	Handcraft	
20	0	2646.79	2237.96	2646.79	2237.96	
21	1	2646.79	2237.96	2569.70	2172.78	
22	2	2646.79	2237.96	2494.86	2109.49	
23	3	2646.79	2237.96	2422.19	2048.05	
24	4	2646.79	2237.96	2351.64	1988.40	
25	5	3075.55	2364.22	2653.00	2039.40	
26	6	3075.55	2364.22	2575.73	1980.00	
27	7	3075.55	2364.22	2500.71	1922.33	
28	8	3075.55	2364.22	2427.87	1866.34	
29	9	3075.55	2364.22	2357.16	1811.98	
30	10	4457.45	2479.47	3316.76	1844.96	
31	11	4457.45	2479.47	3220.16	1791.22	
32	12	4457.45	2479.47	3126.36	1739.05	
33	13	4457.45	2479.47	3035.30	1688.40	
34	14	4457.45	2479.47	2946.90	1639.22	
35	15	5641.05	2606.22	3620.78	1672.83	
36	16	5641.05	2606.22	3515.32	1624.11	
37	17	5641.05	2606.22	3412.93	1576.80	
38	18	5641.05	2606.22	3313.52	1530.88	
39	19	5641.05	2606.22	3217.01	1486.29	
40	20	6459.80	2647.80	3576.64	1466.02	
41	21	6459.80	2647.80	3472.46	1423.32	
42	22	6459.80	2647.80	3371.32	1381.86	
43	23	6459.80	2647.80	3273.13	1341.62	
44	24	6459.80	2647.80	3177.80	1302.54	
45	25	6851.36	2566.54	3272.25	1225.79	
46	26	6851.36	2566.54	3176.94	1190.09	
47	27	6851.36	2566.54	3084.41	1155.43	
48	28	6851.36	2566.54	2994.57	1121.77	
49	29	6851.36	2566.54	2907.35	1089.10	
50	30	6807.26	2561.56	2804.50	1055.33	
51	31	6807.26	2561.56	2722.82	1024.59	
52	32	6807.26	2561.56	2643.51	994.75	
53	33	6807.26	2561.56	2566.52	965.78	
54	34	6807.26	2561.56	2491.76	937.65	
55	35	6929.19	2610.71	2462.52	927.80	
56	36	6929.19	2610.71	2390.80	900.78	
57	37	6929.19	2610.71	2321.16	874.54	
58	38	6929.19	2610.71	2253.55	849.07	
59	39	6929.19	2610.71	2187.92	824.34	
60	40	7565.90	2864.89	2319.38	878.25	
61	41	7565.90	2864.89	2251.82	852.67	
62	42	7565.90	2864.89	2186.24	827.84	
63	43	7565.90	2864.89	2122.56	803.72	
64	44	7565.90	2864.89	2060.74	780.31	
<b>Human Capital</b>				<b>125817.33</b>	<b>62965.45</b>	

Source: Own Table, based on data derived from Statistisches Bundesamt (2006)

## Part B: Nuernberger

### B.1 Company Information

The Nuernberger Beteiligungs-Aktiengesellschaft is, with €4.5bn in sales in 2010 and 28,000 employees, one of the major insurance companies in Germany. The holding company combines a series of legally separated entities, which all cover one part of the array of products offered by Nuernberger. With that, Nuernberger offers the insurance of property, life-insurance services, company-related insurance products, and capital investment services.

With €2.4bn sales volume in 2010, the life insurance products are the major business area of Nuernberger. In particular in this business, Nuernberger stands out due to ongoing product development processes. As one example, Nuernberger popularized funds-linked life-insurance products in Germany. Due to the long experience in this field and ongoing product development processes, Nuernberger belongs today to the market leaders. (Nuernberger, 2011)

### B.2 Life-Cycle Model Return Calculation

**Figure 13: Original Example of Nuernberger Life-Cycle Return Calculation**

Laufzeit 35 Jahre

1.Periode: Laufzeit 21 Jahre	→	angenommene Wertentwicklung 6 % p.a.	→	Wert Periode 1	}	Wert Periode 2
2.Periode: Laufzeit 11 Jahre	→	angenommene Wertentwicklung 5 % p.a.	→	Wert aus Periode 1 + laufender Beitrag		
3.Periode: Laufzeit 3 Jahre	→	angenommene Wertentwicklung 4 % p.a.	→	Wert Periode 2 + laufender Beitrag	}	Gesamtwert

Mit dem laufenden, monatlichen Beitrag, der Vertragslaufzeit und dem berechneten Gesamtwert kann die Gesamrendite berechnet werden.

Source: Nuernberger (2011)

### **B.3 Interview**

Telephone interview on June 14, 2011 with

**Sandra Bauer (Product Specialist) & Christoph Steinberger (Asset Manager)**

*Mrs. Bauer is working for the German insurance company Nuernberger AG in Nuernberg (Germany). The Nuernberger AG is mainly known for its life insurance business. Mrs. Bauer is working as product specialist within the life insurance department. Mr. Steinberger is working as an asset manager for FORUMFINANZ AG, a German financial services company with a strong network with banks and insurance companies. For the purpose of this thesis, he made the contact to Mrs. Bauer.*

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www.forumfinanz.de

**Why does the Nuernberger life-cycle contract only allow for two rebalances?**

*Bauer:* To provide an optimal asset allocation within the savings deposit at any time, it makes indeed sense to rebalance on a continuous basis. By planning only two rebalances, the portfolio can only capture major average movements and might even be against the market developments. Rebalances are, however, transactions which imply transaction costs. These transaction costs can decrease the investment return to a significant extent. We try to balance costs and benefit from rebalances by allowing only for two rebalances.

**Is it possible and reasonable to make regular payments into the contract?**

*Bauer:* The contract allows for regular payments and many investors make use of it. For most investors, this makes more sense than paying one singular amount, since most investors can rather afford many small payments than one big lump-sum investment.

*Steinberger:* Making regular payments makes more sense anyway because of the cost average effects. If investors invest a certain amount only once and keep it stable, they suffer from all the market movements directly. At maturity, they often did not earn very much. The more

reasonable alternative is to make regular payments. The advantage is that at low market prices, the investors can buy comparatively more units of the product than at high market prices, since these units are rather cheap. Consequently, the average purchasing price of the asset is below the average security price of the investment period, so that the investor generates positive returns.

**The equity and bond weights are fixed at the beginning of the contract. Does the investor have to predetermine all other details of the rebalances as well?**

*Bauer:* The investors only have to predetermine the equity and bond weights at each rebalancing date, or the overall risk level of the portfolio. They can, however, decide freely on the funds in their deposits and their weights at every rebalancing step as long as the contractual requirements are met. That means that an investor can decide on one portfolio for the first period and decide on completely new funds for the second period. What counts is that the overall equity and bond weights are as determined.