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Pension Funding and Human Capital

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Abstract

In this paper we analyze the consequences of pension funding in a general equilibrium model of both formal schooling decisions and on-the-job human capital formation à la Heckman, Lochner and Taber (1998). Our focus lies on the distortive and redistributive effects of a Bismarckian pension system as well as the macroeconomic and welfare consequences of its abolition.

We find that a Bismarckian PAYG style pension system like the German one strongly enhances on-the-job human capital formation and redistributes from the lower to the higher skilled, a result that, to the best of our knowledge, is new to the literature. Our reform simulations indicate that in a small open economy setting pension funding reduces the amount of human capital formed via on-the-job training by about 50 percent on average. In a closed economy setup however, the annual interest rate decreases by 2.6 percentage points which in turn boosts human capital accumulation. In the long run, we report a strong welfare gain of about 6.5 percent of initial resources. However, this gain comes along with short run losses up to nearly 5 percent for the middle aged generations, who still have to pay contributions in order to finance existing pension claims. Overall, pension funding comes at efficiency costs of about 2.2 percent in a closed economy setting.

JEL Classifications: J24, J26, H55

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1 Introduction

The reform of the pension system has been a major concern in the world wide public discussion for many years. On the one hand, with rising life expectancies and declining birth rates, the sustainability of pay-as-you-go (PAYG) social security regimes seems at risk in the near future. In Germany, the Riester and Rürup reforms, which basically paved the way for tax promoted old-age savings, as well as the increase in normal retirement age from 65 to 67 from 2012 onwards aim at addressing this sustainability issue. On the other hand, the variety of pension systems within OECD countries has given rise to a lot of discussion in the economic literature. Whereas countries like Germany, Italy and Spain have large, nearly perfectly earnings related schemes, the UK and the Netherlands only provide a minimum pension at retirement and build on private retirement savings.

In the economic literature, pension funding as well as the design of social security systems has been intensively discussed. Various arguments have been put forth in favor and against public pension schemes. Defenders pointed out that they might work as a substitute for missing annuity markets and as a commitment device for myopic agents. Critics, on the contrary, argued that pension contributions mainly distort labor supply and enforce liquidity constraints at younger ages, see Fehr, Habermann and Kindermann (2008) or Fehr and Kindermann (2010) for a discussion of both the literature and macroeconomic, welfare and efficiency effects of pension funding reforms. In terms of redistribution through the social security system, a common consensus is that more progressive pension arrangements provide more insurance against income shocks, but distort labor supply much stronger, see e.g. Fehr and Habermann (2008) for a quantitative analysis of different grades of progressivity in the pension system for Germany.

However, there is one fact usually neglected in the discussion about social security, namely how its size and design interacts with the human capital investment decision of households. This is where the present study wants to step in. We therefore construct an overlapping generations (OLG) model in the spirit of Heckman, Lochner and Taber (1998), in which agents choose both their schooling level and amount of time to invest in human capital formation on-the-job endogenously. The goal of this study is then to quantify the effects of a fully Bismarckian PAYG pension scheme on these decisions as well as on the macroeconomy and welfare.

In the literature, the interaction between human capital formation and social security has not completely been neglected. Docquier and Paddison (2003) as well as Le Garrec (2005) for example analyze the growth and inequality effects of different pension arrangements in theoretical OLG models in which schooling is the engine of growth. In the model of Docquier and Paddison (2003), households differ by the ability to transform time invested in education into labor efficiency. They find that funded pension systems only increase human capital investment and growth when benefits are linked to parts of the earnings history. PAYG schemes have an additional opposite effect via changes in the interest rate. As these pensions crowd out capital, the interest rate increases which makes human capital investments less valuable. Complementary numerical simulations show that the latter overcompensates the former. In addition to this, Le Garrec (2005) finds that Bismarckian pension systems always are to be favored over Beveridgean ones, when benefits are linked to the full earnings history of households. However, there is a pension scheme which consists of a flat part and a part that is only related to the last years of employment, which leads to the same growth rate but less equality than the pure Bismarckian system.

Another stream of the literature deals with the interaction between schooling, retirement and the pen-

sions system. Lau and Poutvaara (2006) therefore construct an analytically solvable model in which agents choose their amount of education and the timing of retirement. They find that increasing the link between pension benefits and contributions encourages human capital investment. Furthermore, actuarially adjusted arrangements like old-age benefits lead to later retirement compared to a retirement subsidy scheme and therefore prolong the period of yield for human capital investment. In consequence, schooling effort rises further, a result already found by Jensen, Lau and Poutvaara (2004) in a numerical study and confirmed by Montizaan, Cörvers and de Grip (2010) in a quantitative study. In addition, Jensen et al. (2004) surprisingly find that even low ability households might favor Bismarckian over Beveridgean systems, as the positive efficiency effects of higher education might outweigh the adverse redistributive consequences.

Finally, Cascarico and Devillanova (2008) explore the consequences of funding social security in a model of human capital investment and capital skill complementarity in production. They state that the privatization of social security comes along with a higher steady state level of physical and human capital. As the capital stock increases, the wage gap between the skilled and the unskilled widens. Therefore across-group wage and income inequality rises.

The present study aims at complementing the above analyses in several ways. We build a multi-period OLG general equilibrium model in which households make both formal schooling and on-the-job training decisions. We thereby build on the seminal works of Auerbach and Kotlikoff (1987) as well as Heckman et al. (1998), the latter of whom first used their model to study the evolution of wage inequality between unskilled and skilled labor in the US. In order to make the model suitable for analyzing pension reforms, we extend it in various directions. First, we introduce variable labor supply to account for distortions in the utilization of human capital induced by the pension system. As pointed out by Jacobs (2008), taking into account both skill formation and variable labor supply increases labor supply elasticities and might lead to very different conclusions than those derived from a model with inelastic labor supply. Next, we introduce labor income uncertainty into the model. In addition we assume, in line with García-Peñalosa and Wälde (2000), that at the end of their university phase, students have to take a final exam, which they only pass with a certain probability. This also makes college education a risky investment. Third, we extend the one-time schooling choice in Heckman et al. (1998) to a multi period drop-out decision model in line with Gallipoli, Meghir and Violante (2008). This allows us to on the one hand consider three different types of schooling levels, lower secondary, higher secondary and tertiary (college/ university) education. On the other hand, by also distinguishing agents according to socio-economics background at the beginning of the life cycle, i.e. schooling level of their parents, we are able to create an intergenerational transmission channel. Heineck and Riphahn (2009) e.g. show that the educational decision of children in Germany strongly depends on what schooling level their parents achieved. Therefore, non-pecuniary or psychological costs of formal schooling will depend on the socio-economic background of an agent in our model. In consequence, with the fraction of high-skilled households declining, non-pecuniary costs of education for future generations will increase, which works as external effect. Last but not least, as already pointed out by Cascarico and Devillanova (2008), the imperfect substitutability between labor of different skill types plays an important role in the analysis of pension systems. Therefore, we let the production sector of our model produce with three different types of labor, low skilled, middle skilled and high skilled. A distortion in schooling decisions via a pension reform therefore causes different reaction in wages for different skill types.

In opposite to Docquier and Paddison (2003) our model does not feature endogenous growth. However, Bouzahzah, de La Croix and Docquier (2002) showed that endogenous growth is not very im-

portant for the analysis of pension reforms in a model with human capital accumulation. Our model is calibrated carefully to the German economy in 2007, which features a fully earnings related PAYG pension system in which earnings points are earned on a yearly basis and don't pay interest over time.¹ We use SOEP data to estimate on-the-job human capital formation technology as well as income shock processes for different schooling types. Having done that, we simulate long-run effects of pension funding as well as a whole transition path from the initial equilibrium to a new one. Therefore we also are able to quantify the short-run effects of our reform, a feature usually neglected by most studies.

We find that a Bismackian PAYG style pension system like the one in Germany strongly enhances on-the-job human capital formation, a result that, to the best of our knowledge, is new to the literature. This is due to the fact that implicit taxes inherent in the social security contribution decrease over the working life of an agent. Hence, in times of investment in human capital agents face a lower net wage per hour worked than in the time of yield. In addition, since pension claims don't pay interest over time, labor income in later years factually gets more weight in the pension formula. Therefore, the German pension system redistributes from the lower skilled towards the high skilled, since the latter face the steepest increases in labor efficiency. We compute that the relation between the present value of pension payments received and contributions paid varies from 33.8 percent for the lower skilled up to nearly 40 percent for college graduates.

Our reform simulation indicate that in a small open economy setting pension funding reduces the amount of human capital formed via on-the-job training by about 50 percent on average. Due to different incentive from the progressive tax system, higher skilled reduce their on-the-job training effort less than the lower skilled. In addition, since the redistribution towards higher income earners is eliminated, the number of college students is reduced which results in a 10 percent increase in the college wage premium in the long run. Finally, in a closed economy setup, the annual interest rate decreases by 2.6 percentage points which in turn boosts human capital accumulation on the job. In the long run, we report a strong welfare gain of about 6.5 percent of initial resources. However, this gain comes along with short run losses up to nearly 5 percent for the middle aged generations, who still have to pay contributions in order to finance existing pension claims. Overall, pension funding comes at efficiency costs of about 2.2 percent in a closed economy setting, while in a small open economy, efficiency loss amounts to roughly 2.6 percent.

The remainder of the paper is arranged as follows: in the next section we discuss the distortionary effects of Bismarckian pension contributions and their effect on on-the-job human capital formation in a very simple analytical framework. In Section 3 we present the numerical general equilibrium OLG model, the calibration of which is discussed in Section 4. Section 5 describes simulation results and the last section offers some concluding remarks.

¹ This is different to so-called "Notional Defined Contribution" (NDC) systems, where pension contributions are noted on an individual account and accumulated with an internal rate of return that depends on mortality, as well as population and productivity growth. As internal rates of return will usually be lower than the capital market interest rate, our arguments will also hold at least partly for NDC schemes. Nevertheless, if we talk about PAYG Bismarckian pension systems throughout the rest of the paper, our focus lies on systems like the German one in which pension claims don't pay any interest.

2 How distortive are Bismarckian pension systems?

In this section we show in a simple analytical framework that Bismarckian PAYG regimes *ceteris paribus* enforce human capital investment, a result that to the best of our knowledge is new to the literature. For assessing this result we use a three period Ben-Porath (1967) style model.² We will also stress the role of implicit taxes in Bismarckian pension contributions and their redistributive role within the social security system.

Let there be a continuum of households distinguished by the ability A_i to translate studying time into human capital. For simplicity, we let A_i be distributed equally on the interval $[0, 1]$. Every household maximizes individual utility

$$u(c_1, c_2, c_3)$$

where $\frac{\partial u}{\partial c_i} > 0$ and $\frac{\partial^2 u}{\partial c_i^2} < 0$. c_i thereby is consumption in the three different periods of life, when households are young workers, old workers, and retired, respectively. When young, agents can invest a certain fraction of their full time endowment of 1 into skill formation. Labor efficiency³ in the second period evolves according to

$$h_2 = 1 + A_i e_1^\alpha, \quad \text{with } 0 < \alpha < 1,$$

where e_1 is the time invested in human capital formation. Human capital in the first period is normalized to $h_1 = 1$. In addition to human capital, households might invest in the capital market and earn the interest rate r . The intertemporal budget constraint therefore is

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = (1 - e_1)(1 - \tau)w + \frac{(1 + A_i e_1^\alpha)(1 - \tau)w}{1+r} + \frac{p_i}{(1+r)^2},$$

where p_i is a pension benefit that is related to agent's earnings history and τ is the pension contribution rate. We let the pension benefit be

$$p_i = \tau[(1 - e_1)w + (1 + A_i e_1^\alpha)w].$$

Obviously, this formulation insures that the pension system is balanced when we let it run on a PAYG basis and abstract from population growth.

Taking the first order condition with respect to e_1 , we find that the optimal schooling decision is

$$e_1 = \left[\frac{A_i \alpha}{1+r} \cdot \frac{1 - \tau \gamma_2}{1 - \tau \gamma_1} \right]^{\frac{1}{1-\alpha}},$$

where $\gamma_1 = 1 - \frac{1}{(1+r)^2}$ and $\gamma_2 = 1 - \frac{1}{1+r}$ measure the implicit tax share of the pension contribution in period 1 and 2, respectively. As already pointed out in Fehr and Kindermann (2010), the implicit tax share of PAYG pension contributions decrease over the life-cycle, as pension claims earned at a certain point in time, in opposite to the case of funded systems, don't pay any interest. With implicit taxes being high at the beginning and low at the end of the working period, during the time of investment in human capital households factually face lower net wages than during the time of yield. This leads

² The model of Ben-Porath (1967) constitutes the basis for on-the-job human capital formation in the Heckman et al. (1998) framework.

³ In the following we will use labor efficiency and human capital synonymously.

to a positive distortion of the education decision compared to the case without pension system where $e_1 = [A_i \alpha / (1+r)]^{\frac{1}{1-\alpha}}$.

The decrease in implicit taxes over the working phase also has a second effect. As the savings share in pension contributions increases with age, later incomes in life play a more important role in the calculation of pension benefits. As the more able, i.e. those with a higher A_i , will have higher wages at older ages than the less intelligent, Bismarckian PAYG pensions actually redistribute from the less able to the more able in the economy. In our model this can be seen from calculating the internal rate of return for different individuals. This rate can be calculated via comparing the present value of pension benefits and pension contributions

$$R_i = \frac{p_i / (1+r)^2}{\tau \left[(1-e_1)w + \frac{[(1+A_i e_1^\alpha)w]}{1+r} \right]} - 1.$$

Plugging in the definition of the pension benefit we obtain

$$R_i = \frac{\left[1 + \frac{1+A_i e_1^\alpha}{1-e_1} \right] / (1+r)^2}{1 + \frac{(1+A_i e_1^\alpha) / (1+r)}{1-e_1}} - 1 = \frac{1 + (1+r)g_i}{(1+g_i)(1+r)^2} - 1,$$

where $g_i = \frac{(1+A_i e_1^\alpha) / (1+r)}{1-e_1}$ is the rate of return of the individual human capital investment. Taking the derivative of R_i with respect to A_i we get

$$\frac{\partial R_i}{\partial A_i} = \frac{r}{(1+g_i)^2 (1+r)^2} \cdot \frac{\partial g_i}{\partial A_i} > 0$$

as $\frac{\partial g_i}{\partial A_i} > 0$. Consequently, the internal rate of return in the pension system increases with ability A_i . Hence, the system is not actuarially fair in terms of individual ability to form human capital.

3 The model economy

Taking this knowledge about the redistributiveness of PAYG Bismarckian pension systems, we will now form a large-scale OLG model that features much more details than the small analytical framework above and analyze to which extent our theoretical results still hold.

3.1 Demographics and intracohort heterogeneity

We consider an economy populated by overlapping generations of individuals, which may live up to a maximum possible lifespan of J periods. At the beginning of each period, a new generation is born where we assume a population growth rate of n . Since individuals face lifespan uncertainty, $\psi_j < 1$ denotes the time-invariant conditional survival probability from age $j-1$ to age j with $\psi_{J+1} = 0$.

Our model is solved recursively. Consequently, an age- j agent faces the state vector

$$z_j = (s_j, \zeta_j, s_p, a_j, ep_j, h_j, \eta_j)$$

where $s_j \in \mathcal{S} = \{1, \dots, S\}$ and $s_p \in \mathcal{S}$ are agent's current schooling level at age j and time-invariant socio-economic background, i.e. parent's schooling level. $\zeta_j \in \mathcal{I} = \{0, 1\}$ indicates whether the

individual is still in education or already has fully joined the labor force, see below. Finally, $a_j \in A = [0, \infty)$, $ep_j \in P = [0, \bar{ep}]$ and $h_j \in H = [0, \infty)$ denote assets held at the beginning of age j , accumulated earning points for public pension claims and stock of human capital, and $\eta_j \in \mathcal{E} = [0, \infty)$ is a shock to household's productivity. The productivity shock is assumed to follow a first-order Markov process described in more detail below. For the sake of simplification, we let $\mathcal{C} = A \times P \times H \times \mathcal{E}$ the set of continuous states.

According to the initial distribution at age $j = 1$, mortality rates, population growth, the Markov process and optimal household decisions, each age- j cohort is fragmented into subgroups $\xi(z_j)$. Let $X(z_j)$ be the corresponding cumulated measure to $\xi(z_j)$. Hence,

$$\int_{S \times I \times S \times H \times \mathcal{E}} dX(z_1) = 1 \quad \text{with} \quad z_1 = (s_1, \zeta_1, s_p, 0, 0, h_1, \eta_1)$$

must hold, since we have normalized the cohort size of newborns to be unity.

By assumption, parents are of age j_p when their children enter the economically relevant age. Accordingly, the initial distribution of agents across socio-economic backgrounds s_p depends on the number of agents of different schooling types s_{j_p} at age j_p . For example, assume that there are only two types of education, i.e. $S = 2$, and at age j_p 60 percent of agents hold a high-school and 40 percent a college degree. Hence, in the newborn cohort, there will be 60 percent of agent with socio-economic background $s_p = 1$ (i.e. high school) and 40 percent with $s_p = 2$.

In the following, we will omit the state indices z_j for every variable whenever possible. Agents are then only distinguished according to their age j .

3.2 The household decision problem

3.2.1 The decision about assets, leisure and on-the-job training

Our model assumes a preference structure that is represented by a time-separable, nested CES utility function. Extending the model of Heckman et al. (1998) by variable labor supply, we assume an agent at age j to solve the optimization problem

$$V(z_j) = \max_{c_j, \ell_j, e_j} \left\{ u(c_j, \ell_j) + \delta \psi_{j+1} E [V(z_{j+1})]^{1-\frac{1}{\gamma}} \right\}^{\frac{1}{1-\frac{1}{\gamma}}}, \quad (1)$$

where c_j , ℓ_j and e_j indicate consumption, leisure time and on-the-job human capital investment at age j , respectively, and γ is the intertemporal rate of substitution between consumption in different years. We thereby define the expectations operator as

$$E [V(z_{j+1})] = \left\{ \int_{\mathcal{E}} V(z_{j+1})^{1-\eta} dX(z_{j+1}) \right\}^{\frac{1}{1-\eta}},$$

where η defines the risk aversion of the individual, see Epstein and Zin (1991). The instantaneous utility function is defined as

$$u(c_j, \ell_j) = \left[(c_j)^{1-\frac{1}{\rho}} + \alpha (\ell_j)^{1-\frac{1}{\rho}} \right]^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}}, \quad (2)$$

where ρ denotes the intratemporal elasticity of substitution between consumption and leisure at each age j while α is the age-independent leisure preference parameter.

Households maximize (1) subject to the budget constraint

$$a_{j+1} = a_j(1+r) + w_j + p_j - \tau \min[w_j; 2\bar{w}] - T[y_j^w, y_j^r] + b_j + \kappa_j - (1 + \tau_c)c_j \quad (3)$$

with $a_1 = a_{J+1} = 0$ and $a_j \geq 0$. In addition to interest income from savings ra_j , households receive gross labor income w_j during their working period as well as public pensions p_j during retirement. If they have fully joined the labor force, i.e. $\varsigma_j = 0$, agents can devote their overall time endowment of 1 to leisure consumption, on-the-job human capital investment and work. If still in education, they are not allowed to additionally invest into human capital on-the-job and have to devote a fixed fraction ω of their time endowment to their studies. Hence,

$$w_j = \begin{cases} w^s(1 - \ell_j - e_j)h_j\eta_j, & \text{if } \varsigma_j = 0 \\ w^s(1 - \ell_j - \omega)h_j\eta_j, & \text{if } \varsigma_j = 1, \end{cases}$$

where w^s defines the wage rate for effective labor of schooling class s , h_j household's labor efficiency and η_j the current shock to labor income. At specific ages, households also receive accidental bequests b_j as well as inter vivos transfers κ_j . Contributions at a rate τ are paid to the public pension system up to a ceiling which amounts to the double of average income \bar{w} . Income taxes depend on taxable labor and capital income y_j^w and y_j^r and the tax schedule $T[\cdot, \cdot]$ which is explained below. Finally, the price of consumption goods c_j includes consumption taxes τ_c .

In order to manage their costs of living, children that are in education, i.e. $\varsigma_j = 1$, receive lump sum transfers κ_j which amount to a fixed fraction ζ of parents' income at j_p , i.e.

$$\kappa_j(z_j) = \zeta \int_{\mathcal{I} \times \mathcal{S} \times \mathcal{C}} w_{j_p} dX(z_{j_p}), \quad \text{with } z_j = (\cdot, \cdot, s_p, \cdot, \cdot, \cdot, \cdot) \text{ and } z_{j_p} = (s_p, \cdot, \cdot, \cdot, \cdot, \cdot).$$

Note that, if children are of socio-economic background s_p , their parents obviously have to be of schooling class $s_{j_p} = s_p$. As agents with higher educational degree usually have a higher income, the level of inter vivos transfers will rise with increasing socio-economic background. Finally, the necessary (negative) κ_{j_p} for the parent generations can be computed from the amount of transfers, cohort sizes and the fraction of children choosing a certain level of education.

Our model abstracts from private annuity markets. Consequently, private assets of all agents who died are aggregated and then distributed among all working age cohorts following an age-dependent distribution scheme Γ_j , i.e.

$$b_j(z_j) = \frac{\Gamma_j}{(1+n)} \sum_{k=1}^J (1 - \psi_{k+1}) \int_{\mathcal{I} \times \mathcal{S} \times \mathcal{C}} q_{k+1}(z_k) dX(z_k) \quad \forall j < j_r,$$

$$\text{with } z_j = (\cdot, \cdot, s_p, \cdot, \cdot, \cdot, \cdot) \text{ and } z_k = (s_p, \cdot, \cdot, \cdot, \cdot, \cdot),$$

where $q_{k+1}(z_k) = (1+r)a_{k+1}(z_k)$. The distribution of bequests is computed in every period, where we assume that children always inherit assets of their parents' generation j_p . Since bequest can only be received during employment, we adjust this rule at the beginning and the end of the employment phase. In order to account for agents' socio-economic background, within each cohort, the distribution is computed such that households with background s_p receive assets from agents of schooling

type $s_{jp} = s_p$ of the parental generation. Among individuals of a cohort and socio-economic background, bequests are then distributed equally. As the more educated usually are the more wealthy, households with richer parents will receive more bequests than those with poorer socio-economic background.

Accumulated earning points of the pension system depend on the relative income position w_j/\bar{w} of the worker at working age $j < j_r$. Since the contribution ceiling is fixed at the double of average income \bar{w} , maximum earning points collected per year are 2. Therefore, earning points accumulate according to

$$ep_{j+1} = ep_j + \mu \cdot \min[w_j/\bar{w}; 2], \quad (4)$$

where $ep_1 = 0$ and μ is an indicator for the size of the pension system, i.e. when $\mu = 1$, the pension system is in place, where we have completely privatized the system when $\mu = 0$.

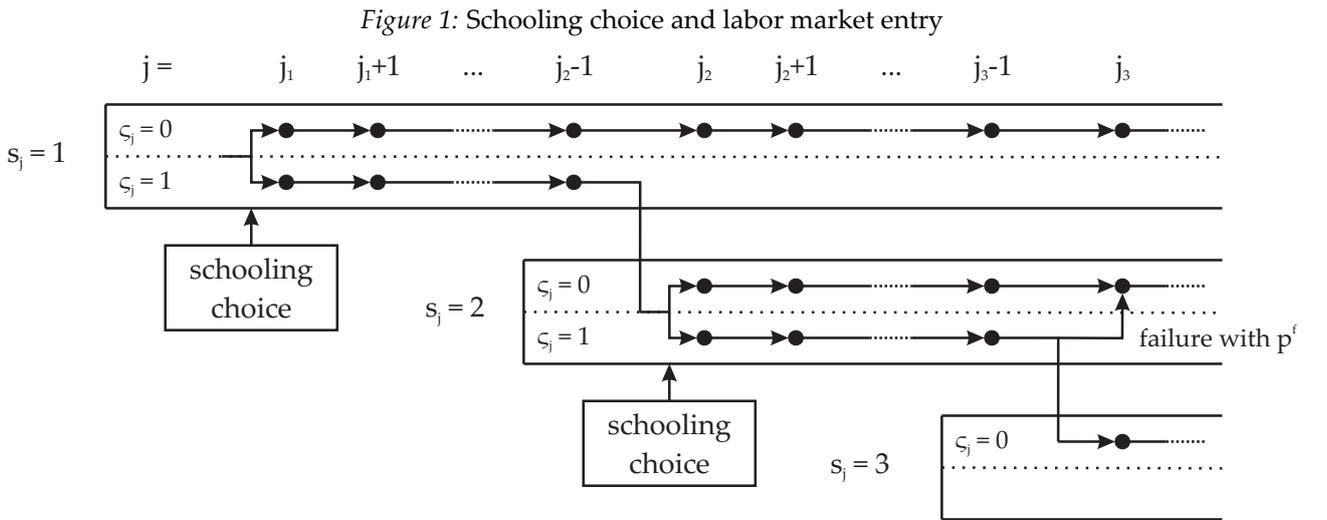
In addition to investing in capital, households can devote time to on-the-job human capital investment. Following Heckman et al. (1998) we assume that on-the-job training just needs time effort. Hence, human capital evolves according to

$$h_{j+1} = A_s e_j^{v_s} h_j^{\omega_s} + (1 - \delta_s^h) h_j, \quad (5)$$

where A_s is a production efficiency parameter, which indicates agent's ability to transform received education into labor productivity, v_s and ω_s are elasticities with respect to time and human capital input and δ_s^h is depreciation on actual human capital.

3.2.2 Schooling choice and inter vivos transfers

Figure 1 shows the dynamics of schooling choice in our model, where we have assumed $S = 3$. Following Gallipoli et al. (2008) we assume several schooling choices that take place at different stages in the life-cycle. Let's denote by j_s the date of labor-market entry for an agent who has successfully completed schooling level s . At the beginning of the life-cycle at j_1 , all agents are of the same initial



schooling type $s_1 = 1$. They now have to make their first educational decision. If they choose to drop

out of the schooling system, i.e. $\zeta_{j_1} = 0$, they enter the labor force and stay in schooling level 1 for the rest of their life. If they decide to stay in school, they remain in schooling level 1 until the next date of labor market entry j_2 . At j_2 they again have to make a drop out or stay decision, etc.

We model the highest possible level of education, i.e. $s_j = 3$, as a risky investment. At this level there is a certain chance of failing. If households decide to participate in the highest educational program, they have to take a final exam before labor market entry, which they either pass or fail with the exogenous probability p^f . Hence, at the beginning of j_3 , a fraction p^f of the agents that are still in the schooling system remains in schooling class 2, but changes to $\zeta_{j_3} = 0$, and a fraction $1 - p^f$ changes to $s_j = 3$. The expected college wage premium therefore can be defined as

$$\left(\frac{(1 - p^f)w_3 + p^f w_2}{w_2} - 1 \right) \times 100.$$

Note that there is no $\zeta_j = 1$ agent in the highest educational class, as households in this class already have run through the maximum number of years of education.

In line with Taber (2002), agents decide about their drop-out via a comparison of utilities. An agent entering schooling level s at time j_s will stay in school, if

$$V_{j_s} \left(z_{j_s}^1 \right) + \varepsilon_{s,s_p} \geq V_{j_s} \left(z_{j_s}^0 \right),$$

where $V_{j_s} \left(z_{j_s}^1 \right)$ and $V_{j_s} \left(z_{j_s}^0 \right)$ are the utilities agent receives from staying in school or dropping out, i.e. $z_{j_s}^1 = (s, 1, s_p, \cdot, \cdot, \cdot)$ and $z_{j_s}^0 = (s, 0, s_p, \cdot, \cdot, \cdot)$, respectively, and ε_{s,s_p} measures psychological or non-pecuniary costs of schooling. We hereby assume that ε_{s,s_p} is normally distributed with mean μ_{s,s_p} and variance σ^2 across every socio-economic background $s_p \in S$.

Assuming a large amount of people in every cohort, due to the law of large numbers,

$$P \left(\left\{ V_{j_s} \left(z_{j_s}^1 \right) + \varepsilon_{s,s_p} < V_{j_s} \left(z_{j_s}^0 \right) \right\} \right) = \Phi_{\mu_{s,s_p}, \sigma^2} \left[V_{j_s} \left(z_{j_s}^0 \right) - V_{j_s} \left(z_{j_s}^1 \right) \right]$$

is the fraction of agents that decide to drop out of the schooling system at age j_s , where $\Phi_{\mu_{s,s_p}, \sigma^2}$ is the cumulative normal distribution function with mean μ_{s,s_p} and variance σ^2 .

3.3 The production side

Firms in this economy use capital and labor of different types s to produce a single good according to a Cobb-Douglas production technology $Y = \rho K^\epsilon L^{1-\epsilon}$ where Y , K and L are aggregate output, capital and labor, respectively, ϵ is capital's share in production, and ρ defines a technology parameter. Labor is aggregated by a CES-technology

$$L = \left(\sum_{s=1}^S \lambda_s L_s^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}, \quad \text{with } \sum_{s=1}^S \lambda_s = 1.$$

Capital depreciates at a constant rate δ_k and firms have to pay corporate taxes

$$T_k = \tau_k \left[Y - \sum_{s=1}^S w^s L_s - \delta_k K \right],$$

where a corporate tax rate τ_k is applied to output net of labor costs and depreciation. Firms maximize profits renting capital and hiring labor from households, so that net marginal products equal r the interest rate for capital and w^s the wage rates for effective labor of different types.

3.4 The government sector

Our model distinguishes between the tax system and the pension system. In each period, government issues new debt nB_G and collects taxes from households and firms in order to finance general government expenditure G which is fixed per capita, educational spending G_s^4 on the different types of schooling systems which are held constant per student as well as interest payments on its debt, i.e.

$$nB_G + T_y + T_k + \tau_c C = G + \sum_{s=2}^S G_s + rB_G.$$

Revenues of income taxation are computed from

$$T_{y,t} = \sum_{j=1}^J \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} T \left[y_j^w(z_j), y_j^r(z_j) \right] dX(z_j)$$

and C defines aggregate consumption (see (8)).

We assume that contributions to public pensions are exempted from tax while benefits are fully taxed. Consequently, taxable labor income y_j^w is computed from gross labor income net of pension contributions, a flexible work related allowance $d_w(w_j)$, and – after retirement – public pensions. Interest income is taxed separately at a flat tax rate with a fixed allowance of d_r . Hence,

$$y_j^w = \max[w_j - \tau \min[w_j, 2\bar{w}] - d_w(w_j); 0] + p_j, \quad \text{and} \quad y_j^r = \max[ra_j - d_r; 0].$$

Given taxable income, we apply the progressive tax code of 2005 in Germany to labor income and a flat tax τ_r to capital income, i.e. $T \left[y_j^w(z_j), y_j^r(z_j) \right] = (1 + \tau_z) \left[T05(y_j^w) + \tau_r y_j^r \right]$, where τ_z is a solidarity surcharge. This corresponds to the flat capital gains tax system recently introduced in Germany. The governmental budget is closed period-by-period by adjusting the consumption tax rate.

In each period, the pension system pays old-age benefits and collects payroll contributions from wage income below the contribution ceiling of $2\bar{w}$. Individual pension benefits p_j of a retiree at age $j \geq j_r$ in a specific year are computed from the product of her earning points ep_j , she has accumulated at retirement and the actual pension amount (*APA*) per earning point

$$p_j = ep_j \times \text{APA}. \quad (6)$$

The budget of the pension system must be balanced periodically by adjusting the social security tax rate τ . Consequently, $\tau = \frac{PB}{PC}$, where

$$PB = \sum_{j=j_r}^J \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} p_j(z_j) dX_t(z_j), \quad \text{and}$$

$$PC = \sum_{j=1}^{j_r-1} \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} \min[w_j(z_j); 2\bar{w}] dX(z_j)$$

define aggregate pensions benefits and the contribution base.

⁴ Note that, as the fraction of people who complete schooling type 1 is always 1 by definition, we can incorporate G_1 in general government expenditure G .

3.5 Welfare calculation

The welfare criterion we use to assess the impact of policy reforms is ex-ante expected utility of an agent before his productivity level is revealed (i.e. looking upon her life behind the Rawlsian veil of ignorance). Expected utility of a newborn in period t is computed from

$$E [V(z_1, Z_t)] = \left\{ \int_{S \times \mathcal{I} \times S \times H \times \mathcal{E}} V(z_1, Z_t)^{1-\eta} dX_t(z_1) \right\}^{\frac{1}{1-\eta}}$$

with $z_1 = (s_1, \zeta_1, s_p, 0, 0, h_1, \eta_1)$,

where Z_t defines the aggregate state of the economy at time t . In order to compare welfare for a specific individual before and after a reform, we follow Auerbach and Kotlikoff (1987, 87) and compute the proportional increase (or decrease) in consumption and leisure ϕ which would make an agent in the initial equilibrium as well off as after the reform, i.e.

$$E [V(z_j, Z_t)] = E [V(z_j, Z_0, \phi)],$$

where

$$V(z_j, Z_0, \phi) = \left\{ u [c_j(1 + \phi), \ell_j(1 + \phi)] + \delta \psi_{j+1} E [V(z_{j+1}, Z_0, \phi)]^{1-\frac{1}{\gamma}} \right\}^{\frac{1}{1-\frac{1}{\gamma}}}.$$

We compare all existing cohorts in the reform year Z_1 and all newborn cohorts along the transition path with the respective cohorts in the initial equilibrium Z_0 . Due to the homogeneity of the utility function (1) and (2) we have $E [V(z_j, Z_0, \phi)] = (1 + \phi) E [V(z_j, Z_0)]$. Therefore, for all agents living in the initial equilibrium, the necessary increase (or decrease) in percent of resources is

$$\left[\frac{E [V(z_{j+1}, Z_1)]}{E [V(z_{j+1}, Z_0)]} - 1 \right] \times 100.$$

A value of 1.0 indicates that this agent would need one percent more resources in the initial long-run equilibrium to attain the expected utility level she receives after the policy reform. The other way round, the agent after the policy reform is 1 percent better off than the one in the initial equilibrium. Within each cohort, we aggregate for each schooling level the percentage changes across different individuals in order to derive average welfare changes. For newborn generations who enter the labor market during the transition we can only report the ex-ante welfare change for the whole cohort. Consequently, in order to compare the intra-cohort welfare consequences of agents born in the future, we also compute ex-post welfare changes after formal schooling decisions were made.

In order to assess aggregate efficiency consequences of different reforms, we introduce a Lump-Sum Redistribution Authority (LSRA) in the spirit of Auerbach and Kotlikoff (1987, 62f.) as well as Nishiyama and Smetters (2005), Fehr et al. (2008) and Fehr and Habermann (2008) in a separate simulation. The LSRA treats those cohorts already existing in the initial equilibrium and newborn cohorts differently. To already existing cohorts it pays a lump-sum transfer (or levies a lump-sum tax) $v_j(z_j, Z_1)$, $j > 1$, to bring their expected utility level after a reform back to the level of the initial equilibrium $E V(z_j, Z_0)$. Since utility depends on age and state, these transfers (or taxes) have to be computed for every agent in the first year of the transition. Consequently, after compensation, their relative welfare change is zero. Those who enter the labor market in period $t \geq 1$ of the transition receive a transfer $v_1(z_1, Z_t, V^*)$ which guarantees them an expected utility level V^* . Note that the transfers $v_1(z_1, Z_t, V^*)$ may differ among future cohorts but the expected utility level V^* is identical

for all. The value of the latter is chosen by requiring that the present value of all LSRA transfers is zero:⁵

$$\sum_{j=2}^J \int_{A \times R \times P \times E_j} v_j(z_j, Z_1) dX_1(z_j) + \sum_{t=1}^{\infty} v_1(z_1, Z_t, V^*) \prod_{s=1}^{t-1} (1+r_s)^{-1} = 0.$$

In the first period of the transition the LSRA builds up debt (or assets) from

$$(1+n)B_{RA,2} = \sum_{j=2}^J \int_{S^2 \times A \times P \times H \times E} v_j(z_j, Z_1) dX_1(z_j) + v_1(z_1, Z_1, V^*)$$

which has to be adjusted in each future period according to

$$(1+n)B_{RA,t+1} = (1+r_t)B_{RA,t} - v_1(z_1, Z_t, V^*).$$

Of course, LSRA assets are also included in the asset market equilibrium condition (9).

Given the compensated expected utility V^* of newborns, we compute the (compensated) relative change in initial resources which would be required in order to attain V^* . If the latter is positive (negative), all households in the reform year who lived in the previous period would be as well off as before the reform and all current and future newborn households would be strictly better (worse) off. Hence, the new policy is Pareto improving (inferior) after lump-sum redistributions.

3.6 Equilibrium conditions

Given the fiscal policy $\Psi_t = \{G, \{G_{s,t}\}_{s=2}^S, T[\cdot, \cdot], B_{G,t}, \tau_{c,t}, \tau_{r,t}, \tau_k, \tau_t\}$, a recursive equilibrium path is a set of value functions $\{V(z_j, Z_t)\}_{j=1}^J$, household decision rules $\{c_j(z_j, Z_t), \ell_j(z_j, Z_t), e_j(z_j, Z_t)\}_{j=1}^J$, distributions of unintended bequest $\{b_j(z_j, Z_t)\}_{j=1}^J$, measures of households $\{\zeta(z_j, Z_t)\}_{j=1}^J$ and relative prices of labor and capital $\{\{w_t^s\}_{s=1}^S, r_t\}$ for all t , so that the following conditions are satisfied:

1. Households' decision rules solve the household's decision problem (1) subject to the given constraints (3), (4), and (5).
2. Factor prices are competitive, i.e.

$$\begin{aligned} w_t^s &= (1-\epsilon)q \left(\frac{K_t}{L_t}\right)^\epsilon \frac{\partial L_t}{\partial L_{s,t}}, \\ r_t &= (1-\tau_k) \left[\epsilon q \left(\frac{L_t}{K_t}\right)^{1-\epsilon} - \delta_k \right]. \end{aligned} \quad (7)$$

3. In the closed economy aggregation holds,

$$L_{s,t} = \sum_{j=1}^J \int_{\mathcal{I} \times \mathcal{S} \times \mathcal{C}} w_j(z_j, Z_t) / w_t^s dX(z_j, Z_t)$$

⁵ In order to avoid that transfers have liquidity effects at young ages, they are actually given (with interest) to cohorts just before they retire. Further information on the computation of V^* is available upon request.

$$C_t = \sum_{j=1}^J \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} c_j(z_j, Z_t) dX(z_j, Z_t), \quad (8)$$

$$K_t = \sum_{j=1}^J \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} a_j(z_j, Z_t) dX(z_j, Z_t) - B_{G,t}, \quad (9)$$

while in the small open economy aggregate capital is derived from (7).

4. Unintended bequests satisfy

$$(1+n) \sum_{j=1}^{j_r-1} \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} b_j(z_j, Z_{t+1}) dX(z_j, Z_{t+1}) = \sum_{j=1}^J \int_{\mathcal{S} \times \mathcal{I} \times \mathcal{S} \times \mathcal{C}} q_{j+1}(z_j, Z_t) (1 - \psi_{j+1}) dX(z_j, Z_t).$$

5. The budgets of the government and the pension system are balanced.

6. The goods market clears, i.e.

$$\begin{aligned} Y_t &= C_t + (1+n)K_{t+1} - (1-\delta_k)K_t + G + \sum_{s=2}^S G_{s,t} && \text{(closed economy)} \\ Y_t &= C_t + (1+n)K_{t+1} - (1-\delta_k)K_t + G + \sum_{s=2}^S G_{s,t} + NX_t && \text{(open economy)} \end{aligned}$$

with NX_t as net exports.

3.7 The computational algorithm

3.7.1 Solving the household problem

In order to compute a solution of the complex household problem, we discretize the state space. The state of a household is determined by $z_j = (s_j, \varsigma_j, s_p, a_j, ep_j, h_j, \eta_j) \in \mathcal{S} \times \mathcal{I} \times \mathcal{S} \times A \times H \times P \times \mathcal{E}$ where $A = \{a^1, \dots, a^{n_A}\}$, $P = \{ep^1, \dots, ep^{n_P}\}$, $H = \{h^1, \dots, h^{n_H}\}$, and $\mathcal{E} = \{e_j^1, \dots, e_j^{n_E}\}$ are discrete sets. For all these possible states z_j we compute the optimal decision of households from (1). Since $u(c_j, \ell_j)$ is not differentiable in every (c_j, ℓ_j) and $V(z_{j+1})$ is only known in a discrete set of points $z_{j+1} \in \mathcal{S} \times \mathcal{I} \times \mathcal{S} \times A \times H \times P \times \mathcal{E}$, this maximization problem can not be solved analytically. Therefore we have to use the following numerical maximization and interpolation algorithms to compute households optimal decision:

1. Compute (1) in age J for all possible z_J . Note that $V(z_{J+1}) = 0$, households are not allowed to work anymore and they die for sure in the next period. Hence, they consume all their resources.
2. Find (1) for all possible z_j using Powell's algorithm (Press et al., 1996, 406ff.). Since this algorithm requires a continuous function, we have to interpolate $V(z_{j+1})$. Having computed the data $V(z_{j+1})$ for all $z_{j+1} \in \mathcal{S} \times \mathcal{I} \times \mathcal{S} \times A \times H \times P \times \mathcal{E}$ in the last step, we can now find a function sp_{j+1} which satisfies the interpolation conditions

$$sp_{j+1}(a_{j+1}^k, ep_{j+1}^l, h_{j+1}^m) = EV(z_{j+1}) \quad (10)$$

for all $k = 1, \dots, n_A$, $l = 1, \dots, n_P$ and $m = 1, \dots, n_H$. In this paper we use multidimensional spline interpolation, see Habermann and Kindermann (2007).

3.7.2 The macroeconomic computational algorithm

Our simulations start from an initial equilibrium that reflects the German macroeconomy and schooling system. The computation method follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for aggregate variables, bequest distribution and policy parameters. Then we compute factor prices, individual decision rules, and value functions. This involves a discretization of the state space which is explained in the previous section. Next we obtain the distribution of households and aggregate assets, labor supply and consumption as well as the social security tax rate and the consumption tax rate that balances government's budget. This information allows us to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values for capital, labor, bequests and endogenous taxes have sufficiently converged. Next we solve for transition path towards a new long-run equilibrium resulting from a change in the social security system and compare the results.

4 Calibration of the initial equilibrium

This section describes how we fit our model to German data. Parameters of the production function for human capital on-the-job and the autoregressive process for labor income shocks are estimated from German Socio-Economic Panel data (SOEP), a description of which can be found in Wagner, Frick and Schupp (2007). Finally, we calibrate the remaining parameters in order to match main macroeconomic variables observed in Germany in 2007.

The timing of the model is as follows: as each model period covers 5 years, agents start life at age 15 ($j = 1$), are forced to retire at age 60 ($j_r = 10$) and face a maximum possible life span of 100 years ($J = 17$). We assume three different educational classes: lower secondary, higher secondary and tertiary education. Ages of entry into the labor market are 15 ($j_1 = 1$), 20 ($j_2 = 2$) and 25 ($j_3 = 3$) respectively, i.e. every additional educational degree needs 5 additional years of studying.

4.1 Parameter estimation for on-the-job training

In order to estimate the parameters for on-the-job human capital formation and the autoregressive income process, we use inflated income data y_{its} of primary household earners from the German SOEP. Our unbalanced panel data covers full-time workers between ages 20 and 60 of the years 1984 to 2006 and was divided into different educational groups according to the International Standard Classification of Education (ISCED) of the UNESCO of 1997. In order to receive three groups, we merge levels 0 to 2 (primary and lower secondary education), levels 3 and 4 (higher secondary and post-secondary education) as well as levels 5 and 6 (tertiary education) to one group each. This approach leads us to a total of 81798 observations, where we have 11298, 54081 and 16419 observations in groups one to three, respectively.

Having extracted this data from the SOEP, we use a variant of the estimation technique proposed by Heckman et al. (1998). Specifically, we take the above household model and assume that there are no shocks to labor income, agents are not liquidity constraint and there is no leisure consumption. In order to make the model comparable to the above specification with leisure choice, we assume for the estimation process a maximum time endowment of 0.4, which amounts to a 40 hours workweek

length, see Auerbach and Kotlikoff (1987). Following Taber (2002), we then approximate the German tax schedule of 2005 using a second order polynomial T_a .

In this simplified model setup, we can separate consumption choice from human capital investment decisions. Hence, an agent's utility maximizing amount of on-the-job-training can be calculated from

$$PVE(h_j, ep_j) = \max_{e_j} \left\{ (0.4 - e_j)h_j(1 - \tau) + p_j - T_a [(0.4 - e_j)h_j w^s(1 - \tau) + p_j] + \frac{PVE(h_{j+1}, ep_{j+1})}{1 + r(1 - \tau^r)} \right\}, \quad (11)$$

where h_j and ep_j evolve according to (5) and (4), respectively. As we use a partial equilibrium model for the estimation procedure, we normalize the interest rate $r = 0.05$ and set the wages per efficiency unit of all three types of labor to 1. Next, we fix the pension contribution rate at $\tau = 0.195$ and choose an actual pension amount which is close to the one predicted from our general equilibrium model. Our model extends the estimation model of Taber (2002) by explicitly accounting for a PAYG pension system. As seen above, a PAYG estimation model and has a major influence on the estimated parameters. Unfortunately, due to the lack of data, we can't estimate ability parameters that depend on agent's educational background.

With the above model, we now estimate the parameters A_s, ν_s and ω_s in the following way via non-linear least squares. We first set depreciation rates δ_s exogenously, so that we obtain a good fit of the model to the data. Specifically, we assume no depreciation for the educational classes 1 and 2 in accordance with Heckman et al. (1998) and a slight depreciation of $\delta_3 = 0.005$ for class 3 in order to account for the falling labor efficiency at the end of working life in this group, compare Figure 2. Next, we start with some initial guesses of parameters A_s, ν_s, ω_s and compute the age gross income profiles $\hat{y}_{ts} = (0.4 - e_j)h_j$ for every educational background resulting from the above model (11). We then form log-residual sum of squares

$$RSS = \sum_i \sum_t \sum_s (\log(y_{its}) - \log(\hat{y}_{ts}))^2. \quad (12)$$

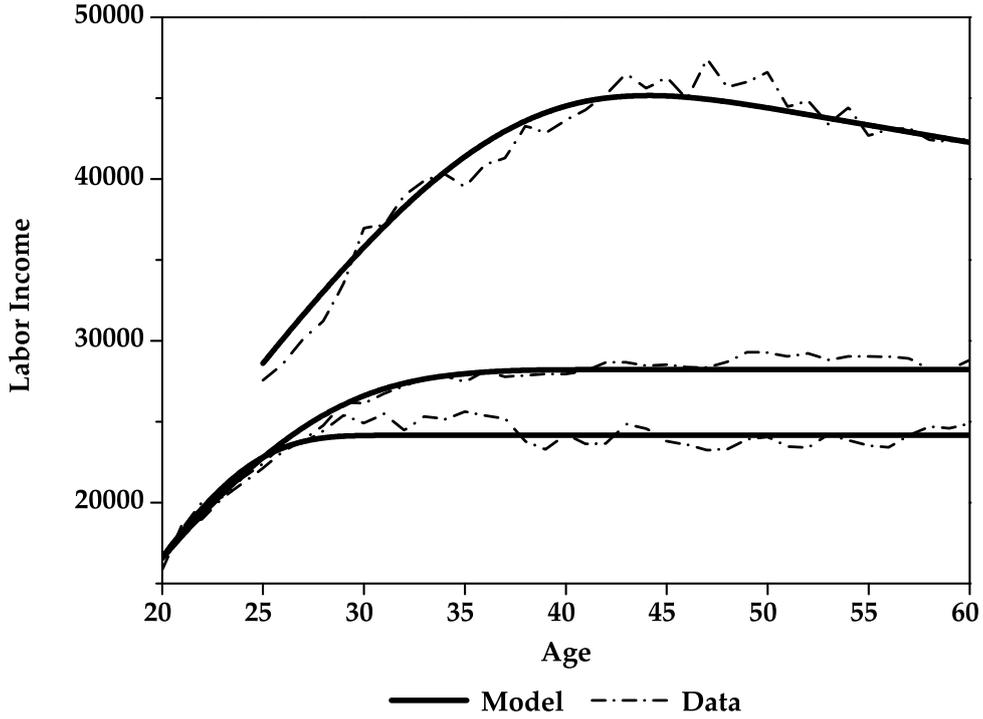
Our algorithm updates the parameter guesses in order to minimize RSS.

Table 1: Parameter estimates for human capital production functions

	Group 1	Group 2	Group 3
ability A_s	0.2469 (0.1940)	0.2874 (0.1311)	0.2628 (0.4897)
elasticity educational time ν_s	0.9838 (0.0065)	0.9448 (0.0035)	0.8784 (0.0132)
elasticity human capital ω_s	0.6723 (0.0052)	0.6138 (0.1874)	0.7294 (0.6340)
initial human capital h_{js}	9.8856 (0.0119)	10.4692 (0.0223)	16.7978 (0.1023)

The resulting parameters and the corresponding Huber-White type standard errors (in parentheses) are reported in Table 1. Estimated gross income profiles and the respective means computed from

Figure 2: Estimated and mean income profiles



the data are shown in Figure 2. Note that, due to the lack of data, we can only estimate the initial level of human capital at age 20 for the lowest educational group. Given the estimated parameters, this corresponds to a level of $h_1 = 7.5957$ at age 15, the age of labor market entry. As our estimation model features 80 periods, we have to adapt the productivity parameters A_s and depreciation rates δ_s in order to yield the same amount of human capital investment in the 17 period model. We therefore set $A_1 = 1.28$, $A_2 = 1.56$ and $A_3 = 1.45$ as well as $\delta_1 = \delta_2 = 0.0$ and $\delta_3 = 0.025$.

4.2 Estimating the autoregressive income process

Taking log residuals of our above parameter estimation, we can now estimate labor income risk processes. Following Love (2007), we assume an autoregressive structure

$$\pi_j = \rho\pi_{j-1} + \epsilon_j \quad , \quad \epsilon_j \sim N(0, \sigma_\epsilon^2) \quad \text{and} \quad \pi_0 = 0. \quad (13)$$

Concerning our data, we therefore estimate the equation

$$\log(y_{its}) - \log(\hat{y}_{ts}) = v_i + \pi_{it} \quad (14)$$

with an individual effect $v_i \sim N(0, \sigma_v^2)$ separately for any of the three educational groups s by means of GLS, assuming π to follow an AR(1) process as in (13). This approach leads us to the parameter estimates shown in Table 2 (standard errors are again reported in parenthesis).

There are two things to notice. First, we find a strong AR(1) correlation of around 0.75 for the error term, which lies in the range of typical values for these types of models, see e.g. Love (2007). Second, except for group 3, we see a small persistent variance, which means that our groups are strongly

Table 2: Parameter estimates for individual productivity

	Group 1	Group 2	Group 3
AR(1) correlation ρ	0.6999 (0.0111)	0.7807 (0.0043)	0.7679 (0.0085)
persistent variance σ_v^2	0.0211 (0.0041)	0.0340 (0.0033)	0.0938 (0.0078)
transitory variance σ_ϵ^2	0.0635 (0.0044)	0.0728 (0.0036)	0.0801 (0.0069)

homogeneous. In the highest educational group, however, there is a certain chance of climbing up into the area of extraordinary high salaries or failing and just getting a job for higher secondary earners. This makes the group somewhat more heterogeneous and explains a higher variance of the individual effect.

We then have to aggregate the process in order to obtain an AR(1) process that covers five years. This can easily be done via computing variance and correlations of the process

$$\pi_j^n = \sum_{i=1}^n \pi_{j+i-1}.$$

with $n = 5$. The respective aggregated process then has a correlation and variance of

$$\rho_n = \rho^n \quad \text{and} \quad \sigma_{\epsilon,n}^2 = \Gamma_n \cdot \sigma_\epsilon^2 \cdot \frac{1 - \rho^{2n}}{1 - \rho^2} \quad \text{with} \quad \Gamma_n = \frac{1}{n^2} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \rho^{|j-i|}.$$

For computational reasons, we finally approximate the shock π^n by a first order discrete Markov process with two nodes using a discretization algorithm as described in Tauchen (1986).

4.3 The schooling choice

In order to calibrate the schooling choice, we set the standard deviation of psychological costs at $\sigma = 0.00518$, which is in line with the estimates reported in Heckman et al. (1998).⁶ Next we set the expected values μ_{is} like in Table 3.

With this specification we can replicate the observed schooling transition matrix in Germany. Table 4 reports on the left hand side the schooling choices of agents of different educational backgrounds generated from our model. On the right hand side, we report estimates from Heineck and Riphahn (2009), who estimated transitional probabilities from German SOEP data.

Finally, we fix the probability of failing at $p^f = 0.2$ which is in line with the fraction of college dropouts reported in AB (2008). This leads to a college premium of 49.4 percent, which corresponds to the tertiary education wage premium of 48.4 estimated for Germany in Strauss and de La Maison-neuve (2007).

⁶ We adjusted the standard deviation in order to account for the fact that we let agents make their schooling choice via a comparison of utilities, not present values of income and have a period length of 5 years.

Table 3: Expected values of psychological costs

$s_p \setminus s$	1	2
1	-0.003545	-0.005078
2	0.001029	-0.002942
3	0.000842	0.000371

Table 4: Decision matrix

<i>Model</i>				<i>Data</i>			
$s_p \setminus s$	1	2	3	$s_p \setminus s$	1	2	3
1	52.97	35.64	11.39	1	53.86	35.14	11.00
2	15.37	51.03	33.60	2	16.05	50.60	33.35
3	8.03	33.50	58.47	3	8.95	33.71	57.34

4.4 Parameterizing the model

Table 5 reports the central remaining parameters of the model. The population growth rate is set at

Table 5: Parameter selection

<i>Demographic parameters</i>	<i>Individual parameters</i>	<i>Technology parameters</i>	<i>Government parameters</i>
$J = 17$	$\gamma = 0.5$	$\theta = 3.15$	$\tau_c = 0.17$
$j_r = 10$	$\eta = 2$	$\varepsilon = 0.35$	$B_G/Y = 0.6$
$j_1 = 1$	$\rho = 0.6$	$\delta_k = 0.295$	$d_s = 1800$
$j_2 = 2$	$\alpha = 0.5$	$\chi = 1.144$	$d(w_j) = 1200 + 0.04w_j$
$j_3 = 3$	$\delta = 0.938$	$\lambda_1 = 0.22$	$T(y^w, y^r)$ see text
$j_p = 5$	$\omega = 0.4$	$\lambda_2 = 0.37$	APA see text
$n = 0.0$	$\zeta = 0.16$	$\lambda_3 = 0.41$	$\mu = 1$
ψ_j :Bomsdorf (2003)			

$n = 0.0$, since population growth is close to zero in Germany. Conditional survival probabilities ψ_j are computed from the year 2000 Life Tables for Germany reported in Bomsdorf (2003).

With respect to the preference parameters, we set the intertemporal elasticity of substitution γ at 0.5, the risk aversion η at 2 and the intratemporal elasticity of substitution ρ at 0.6. This is within the range of commonly used values (see Auerbach and Kotlikoff, 1987, Fehr, 1999). Next, we calibrate the leisure preference parameter α in order to obtain an average labor time of 0.4 which corresponds to a 40 hours workweek length, see Auerbach and Kotlikoff (1987). In order to calibrate a realistic capital to output ratio, the discount factor is set at 0.938 which implies an annual discount rate of roughly 1 percent. Finally, we let the time students devote to studying be 40 hours per week, i.e.

$\omega = 0.4$ and the fraction of income transmitted from parents to their studying children 16 percent, which corresponds to the figures reported in AB (2008).

With respect to technology parameters we specify the general factor productivity $\theta = 3.15$ in order to normalize labor income and set the capital share in production ε at 0.35. We set the elasticity of substitution between different types of labor at 1.441 which is equal to the estimate of Heckman et al. (1998) for US data and calibrate the input shares λ_s , so that the marginal product of labor equals 1 for every type of education. The annual depreciation rate for capital is set at 5.3 percent which results in a rate of 0.295 for one period, the annual *APA* value is chosen in order to derive a replacement rate of net income of 60 percent, which yields a realistic contribution rate for Germany. As already explained, the taxation of gross income (from labor, capital and pensions) is close to the current German income tax code. We apply the marginal tax rate schedule *T05* which was introduced in 2005 to labor and pension income and a flat tax of 25 percent to interest payments. In addition, we consider a special allowance for labor income of $d(w_j)$ which combines a fixed amount of 1200 € and an additional deduction of 0.04 percent of labor income and a fixed allowance of 1800 € for capital income. Given taxable labor income y_j^w , the marginal tax rate rises linearly after the basic allowance of 7800 € from 15 percent to maximum of 42 percent when y_j passes 52.000 €. A solidarity surcharge of 5.5 percent is added to the total amount of taxes. In the initial long-run equilibrium, we assume a debt-to-output ratio of 60 percent, fix the consumption tax rate at 17 percent and compute *G* endogenously to balance the budget.

4.5 The initial equilibrium

Table 6 reports the calibrated benchmark equilibrium and the respective figures for Germany in 2007. As one can see, the initial equilibrium reflects quite realistically the current macroeconomic situation in Germany. The interest rate of 4.5 percent per year is close to the one we used in our estimation procedure. The amount of human capital produced via on-the-job training in the different educational classes is depicted as a fraction of initial human capital h_j .⁷ The educational participation rates are the number of people in different educational programs (according to the ISCED standard) as a fraction of the overall population that currently is in educational programs. The share of lower secondary education participants is relatively low, compared to the German data, as we assume a dropout age of 15, whereas in reality there are also people dropping out at 16.

5 Simulation Results

In this section we discuss the simulation results from the model described above. We thereby proceed as follows. Having calculated the initial equilibrium sketched in the previous section, we completely eliminate the PAYG pension system. However, we do this in a way such that older generations keep their existing pension claims, i.e. we set $\mu = 0$. A payroll tax is therefore levied in the first periods of transition in order to finance these existing claims. In the long-run, however, the social security contribution rate will go down to zero. We then report macroeconomic and welfare effects for the short and the long run. In order to disentangle all driving forces at work, we first consider a partial

⁷ A value of 1% consequently means that the agent increases his labor efficiency throughout the life-cycle by 1% of his initial human capital when entering the labor market.

Table 6: The initial equilibrium

	Model solution	Germany 2007
<i>Calibration targets</i>		
Capital-output ratio	3.0	2.9 ^a
Educational government spending (in % of GDP)	2.4	2.4 ^b
- secondary education	1.3	1.3 ^b
- tertiary education	1.1	1.1 ^b
Pension benefits (% of GDP)	12.2	11.5 ^a
Pension contribution rate (in %)	19.5	19.9 ^a
Tax revenues (in % of GDP)	21.8	23.8 ^a
<i>Other benchmark coefficients</i>		
Interest rate p.a. (in %)	4.5	–
Bequest (in % of GDP)	5.5	4.7-7.1 ^c
Human capital formed on-the-job (in %)		
- lower secondary education	23.3	–
- higher secondary education	12.8	–
- tertiary education	16.6	–
Educational participation (in %)		
- lower secondary education	58.4	65.7 ^b
- higher secondary education	25.9	20.3 ^b
- tertiary education	15.7	13.6 ^b
Wage premium on tertiary education (in %)	49.4	48.4 ^d

Source: ^aIdW (2009), ^bAB (2008), ^cBraun (2002),
^dStrauss and de La Maisonneuve (2007).

equilibrium version of our model where wages and interest rates don't react to changes in household behavior. We then proceed with the general equilibrium framework and compare the case of a small open and a closed economy.

5.1 Distortions and redistribution

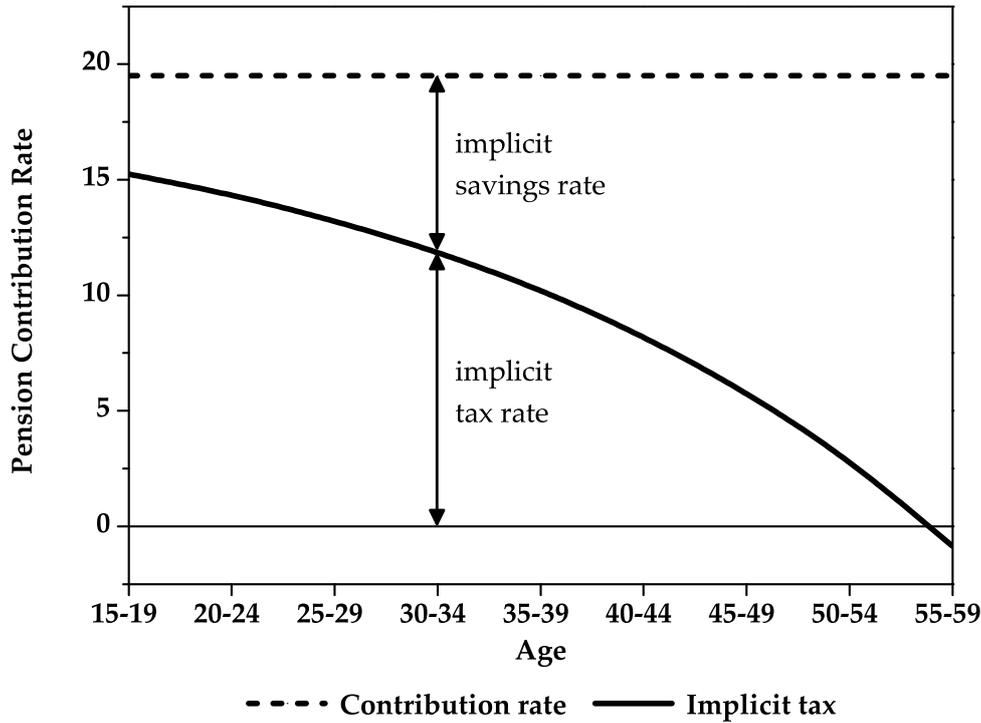
Before we discuss our simulation results, we want to bring into focus again the distortive and redistributive effect that are at work in our model. As already explained in Section 2, in a PAY pension system implicit taxes decrease over the working phase of a households, as accumulated pension claims or earnings points don't pay any interest. In the initial equilibrium, it is quite easy to calculate implicit tax and savings rate of the overall pension contribution from equations (4) and (6), as pensions are perfectly earnings related. Suppose an agent at age j earns the average income \bar{w} . He then pays a contribution of $\tau\bar{w}$ and earns one earnings point. Therefore, his pension payments throughout the retirement phase increase by APA . The present value as of age j of the additional pension payment consequently is $APA \cdot \sum_{i=j}^J (1+r)^{j-i}$. Hence, the household had to save exactly this amount in the capital market in order to receive a pension of the same size on a funded basis. Putting this present value in relation to \bar{w} yields the fraction of household's income he would actually have to

invest in the capital market

$$\tau_j^s = \frac{APA}{\bar{w}} \sum_{i=j_r}^J (1+r)^{j-i}.$$

τ_j^s corresponds to the implicit savings rate of the PAYG pension system. Consequently, $\tau - \tau_j^s$ constitutes the implicit tax share. Implicit savings and tax rates over working life in our model are depicted in Figure 3. As mentioned above, the implicit tax component decreases with time and at the latest

Figure 3: Savings and implicit tax component of pension contributions



working years even turns negative. However, as discussed in Section 2, the increasing savings share in pension contribution also leads to a redistribution from the less able to the more able. As the latter face the steepest increase in labor income over their working period and labor income of later periods is weighted much higher in the pension formula, the ratio between pension contributions and payments, i.e. the internal return, should increase with schooling level.⁸ Table 7 shows the ratio between contributions to and payments from the pension system for agents of different schooling levels. As expected, this ratio increases with ability or schooling level. Note, that on average only

Table 7: Ratio of pension contributions and payments

s	1	2	3
Ratio	33.8	35.5	39.7

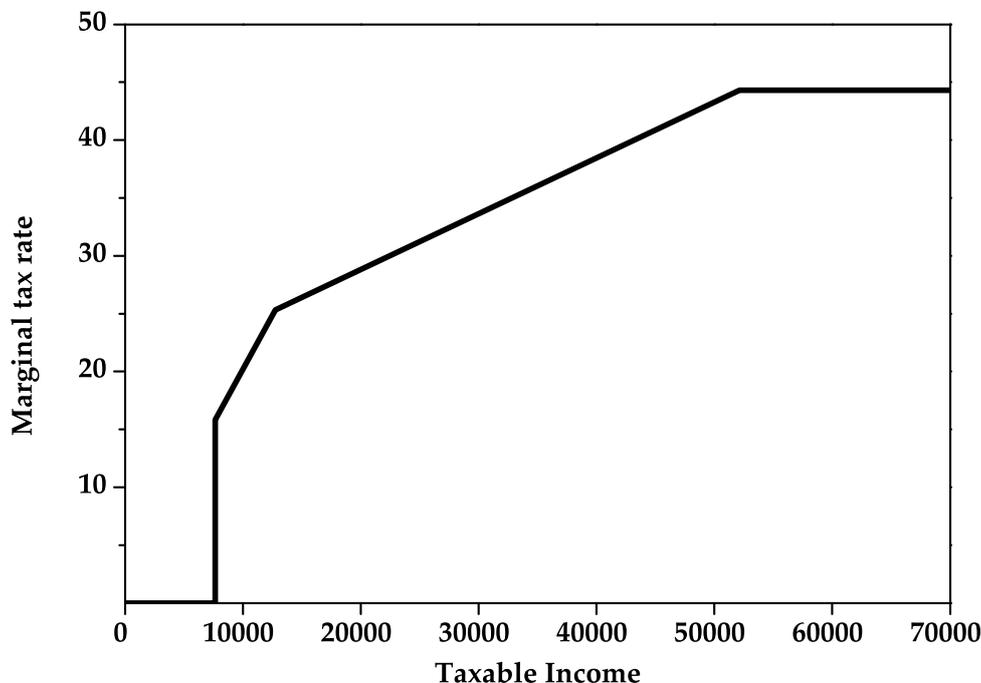
about 37 percent of pension contributions are actually savings, i.e. 63 percent are implicit taxes. This

⁸ This is a direct consequence of increasing savings shares in pension contributions.

is in line with the figures in Fehr and Kindermann (2010), who report an average savings rate of 44 percent with a net replacement rate of 75 percent of average income.

Next we want to turn to the tax system. Figure 4 shows the tax schedule T05 we use to calculate labor income taxes in our model. As described in the Section 4, above a threshold marginal tax

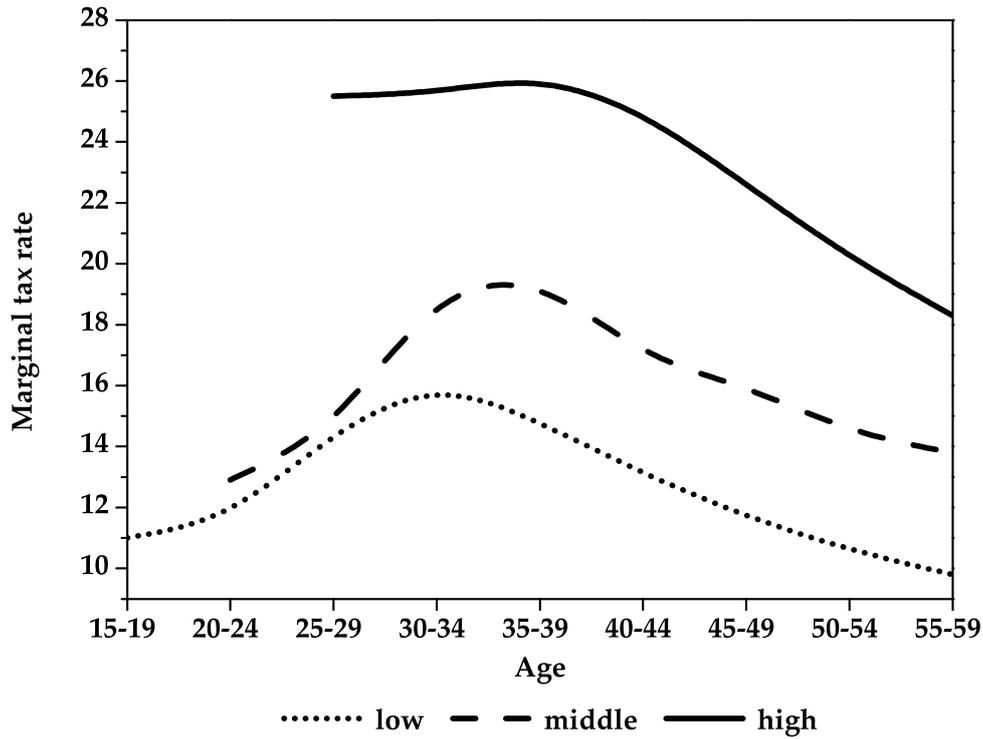
Figure 4: Tax schedule



rates increase up to an income of about 52000€ a year, where they stay constant afterwards. The progressivity in the tax schedule results in varying marginal tax rates across different ages and different schooling levels. Figure 5 depicts average marginal tax rates of the three schooling types over their whole working period. Obviously, the higher the schooling level the higher is labor income and therefore are marginal tax rates. However, there is one specialty for the highest schooling level. Those agents already face the highest marginal tax rates at the beginning of their working life, i.e. most of them are in the top bracket of the tax schedule already when they enter the labor force. For all other schooling types, marginal tax rates increase throughout the first years of employment when labor income rises. Towards later periods in life, marginal tax rates again decrease, since the wealthier agent, namely those with several positive shocks in the labor market start reducing their labor supply. For the accumulation of human capital on-the-job this means that during the time of investment, agents of lower schooling level experience relatively low marginal tax rates whereas university graduates have relatively high ones. Therefore we expect human capital accumulation on-the-job to be enforced by the tax system for college workers but to be discouraged for all other types of employees.

Last but not least, liquidity constraints play an important role in the accumulation of human capital via on-the-job training. If agents would like to borrow on the capital market in order to consume more in the present but are liquidity constraint, they might decrease human capital investment effort in order to increase their current consumption level at the cost of future labor income. This works like an implicit borrowing mechanism. Consequently, the more liquidity constraint will make less effort to increase their labor efficiency. Table 8 shows the fraction of agents of the different schooling levels

Figure 5: Marginal tax rates



that are liquidity constraint in the different years of their working life. We find that agents from lower

Table 8: Liquidity constraints and on-the-job training

<i>s</i>	1	2	3
15-19	50.0		
20-24	49.0	48.7	
25-29	13.6	31.2	50.0
30-34	1.0	3.7	12.1
35-39	0.0	9.1	17.0
44-44	0.0	0.0	6.0

schooling levels during the accumulation phase of human capital are much more liquidity constraint than agents from higher ones. This is due to the fact that on the one hand, agents of higher ability classes have more income to form precautionary saving to counteract bad labor market shocks. On the other hand, as higher educated households receive a higher return from the pension system, they don't have to save as much for retirement and therefore might again build more precautionary savings.

5.2 The macroeconomic effects of pension funding

With the information of the previous section in mind, we can now turn to the macroeconomic effects of a pension funding reform. We therefore first study a partial equilibrium model in which wages and the interest rate do not react to changes in household behavior. In order to point out the different distortive and redistributive effects, we start with a model version in which the schooling decision is fixed and the tax system is perfectly proportional, i.e. everyone pays as tax a fixed fraction of his labor income in a way that overall income tax revenues are the same as with the progressive tax system. We then successively relax these restrictive assumptions until we reach the setup described in the previous sections. Table 9 shows the respective simulation results.

When we privatize the pension system, agents have to save much more for their retirement in the capital market. Therefore, in Simulation (1), assets increase throughout the transition up to a level of about 1.55 times the value of the initial equilibrium. The resulting rise in capital income tax revenues makes the consumption tax rate decrease in the long-run. In the short run, however, with consumption especially for older workers falling, the consumption tax has to increase. The social security tax rate rises in the first period of transition, since labor supply declines but the amount of pension payments stays rather constant. In the following periods, however, where people don't accumulate any more pension claims, the rate gradually decreases by 19.5 percentage points to a value of zero. The interest rate as well as the wage premium of tertiary education stay constant, as we assumed a partial equilibrium model. The supply of labor supply declines for every schooling type in any period of the transition. In the short run, this due to the fact that agents still pay contributions to the social security system, however, don't accumulate anymore pension claims. Hence, the whole contribution acts as a tax which additionally distorts labor. In the long run, the distortive effect of the implicit tax in pension contributions is absent. However, as on-the-job training efforts decrease, so does labor efficiency and therefore labor supply of all households. Now lets turn to on-the-job human capital accumulation. As one expected, on-the-job training effort is reduced significantly in the long run, since the positive distortion induced by the Bismarckian pension system vanishes. The effect is less intensive for low-skilled workers who had the stronger binding liquidity constraints in the initial equilibrium. As the abolition of the pension system loosens liquidity constraints due to the increase in disposable income, human capital accumulation is enforced. In the short run, however, human capital formed on-the-job increases due to the time variability of the social security tax rate. As this rate falls over time in the first periods of transition and all pension contributions are pure taxes, on-the-job training is fostered.

In the second simulation, we now consider the progressive tax system described above. In such a system, pensions are taxed relatively lower than labor income compared to a proportional tax system. Hence, pensions constitute a much larger fraction of old-age income and from funding the social security system, assets have to increase stronger compared to Simulation (1). With assets and therefore capital income tax revenues increasing further, the consumption tax can decrease more. The social security tax rate on the other hand reacts in nearly the same manner as before, since labor supply changes are very similar. Labor supply again decreases due to the distortions arising from the social

Table 9: Macroeconomic effects of pension funding in partial equilibrium

Simulation	(1)					(2)					(3)					
	proportional					progressive					progressive					
Tax system	no					no					yes					
Schooling choice																
Period t	1	3	5	∞	1	3	5	∞	1	3	5	∞	1	3	5	∞
Assets ^a	0.0	5.4	13.6	55.0	0.0	8.3	21.4	75.7	0.0	8.4	21.8	74.5	0.0	8.4	21.8	74.5
Interest rate ^b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cons. tax ^b	1.0	0.6	-0.2	-2.9	0.8	0.0	-1.2	-5.6	0.8	-0.2	-1.4	-4.2	0.8	-0.2	-1.4	-4.2
SS tax rate ^b	0.3	-1.1	-4.3	-19.5	0.3	-1.1	-4.1	-19.5	0.3	-1.1	-4.2	-19.5	0.3	-1.1	-4.2	-19.5
Wage premium ^b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Labor supply ^a																
- $s = 1$	-1.8	-1.7	-2.1	-14.3	-1.3	-1.8	-2.6	-14.4	-1.1	-0.0	2.4	35.0	-1.1	-0.0	2.4	35.0
- $s = 2$	-2.1	-1.0	-0.7	-13.2	-1.8	-1.3	-1.9	-13.3	-1.8	-1.5	-1.9	-12.5	-1.8	-1.5	-1.9	-12.5
- $s = 3$	-1.5	-2.3	-3.6	-16.6	-0.0	0.5	-1.5	-15.4	-0.1	0.3	-4.1	-48.9	-0.1	0.3	-4.1	-48.9
Wages ^a																
- $s = 1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
- $s = 2$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
- $s = 3$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
On-the-job training ^b																
- $s = 1$	7.6	-3.5	-12.7	-28.2	-9.1	-22.4	-33.6	-53.9	-10.0	-23.2	-34.4	-55.2	-10.0	-23.2	-34.4	-55.2
- $s = 2$	6.9	-6.9	-18.6	-36.3	-5.4	-19.3	-27.8	-42.5	-6.9	-20.8	-30.1	-51.7	-6.9	-20.8	-30.1	-51.7
- $s = 3$	-3.7	-16.2	-25.9	-44.8	-1.2	-14.6	-27.4	-40.3	-2.4	-15.9	-28.7	-40.9	-2.4	-15.9	-28.7	-40.9
Schooling Choice ^b																
- $s = 1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	2.5	4.5	14.4	0.8	2.5	4.5	14.4
- $s = 2$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	1.3	1.0	0.0	0.8	1.3	1.0
- $s = 3$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.8	-3.3	-5.8	-15.4	-0.8	-3.3	-5.8	-15.4

Change in ^apercent over initial equilibrium, ^bpercentage points.

security payroll tax in the initial periods of the transition and the falling labor efficiency. Note that the changes are quite similar to the previous simulation although on-the-job training reacts quite differently. Note however that when households do not invest in human capital, they might devote a larger fraction of their available time to working at younger ages which counteracts the decreased labor productivity effect. Finally, we now find that on-the-job training declines much stronger for the less educated and slightly less for college workers. This is a direct consequence of the evolution of marginal tax rates over the life cycle for workers from different schooling levels, see Figure 5. As pensions were taxed and contributions were exempt from tax, with a decreasing social security contribution a larger fraction of income is taxed progressively. Therefore the distorting influence of the tax system rises, which means that the lower skilled reduce their on-the-job training effort while college workers increase it slightly.

In the last simulation of Table 9, we also allow schooling level to be chosen endogenously. This only has a slight effect on the accumulation of assets as well as the evolution of the payroll tax rate. With the share of high skilled agents in the population decreasing, labor income tax revenues decrease and therefore the consumption tax rate has to be higher than in simulation (2). Labor supply, however, is heavily affected since the composition of the work force changes. In terms of on-the-job training, we only see slight effects which is not very surprising since we didn't change the institutional framework but only the distribution of agents across skill levels. Finally, we take a look at schooling choice. We find that the number of college workers declines by about 15 percentage points. This is due to two facts. First, as shown in Table 7, the PAYG Bismarckian pension system in the initial equilibrium distributed from the unskilled towards the skilled. As this distribution is absent after the reform, choosing to stay unskilled becomes more favorable. Second, with the social security tax rate decreasing over time, the amount of labor income that is taxed progressively increases. As high-skilled have the highest average tax rates, they are hurt more than the lower skilled and therefore the gains of being skilled decline. Overall, this causes a reduction in the number of students. Note that this change in the number of students only takes place gradually, since the payroll tax rate is still greater than zero in the first period of transition and therefore the effects of progressive taxation only come into effect at later dates.

Up to now, we assumed that factor prices can't react to the decisions of households. This assumption is relaxed in the simulations in Table 10. We thereby assume in simulation (4) a small open economy in which wages of the different skill levels may change, but the interest rate stays at the world capital market level. As the interest rate does not react to changes in savings behavior, the change in aggregate assets is nearly unaltered. Since the decline in the number of highly educated workers now amounts to only 1.2 percentage points, income tax revenues don't fall as much as in Simulation (3). Consequently the consumption tax rate can again decrease by 5.4 percentage points similar to the simulation where schooling was fixed. Again, the social security tax rate reacts in the same way as existing pension claims were formed in the initial equilibrium and are unaltered by the assumptions made about the transition. In addition, aggregate labor supply behaves quite similar in the first years of transition compared to the previous simulation. However, we now see a rise in the college wage premium. This rise is due to the increase in college worker wages resulting from the decline in the number of students. We find no more rise in labor supply for the lower educated, since there is nearly no more change in the composition of the labor force. This is a direct result of the reaction in wages. With the number of college workers running short, firms will increase their wages and decrease those of the poor. This in turn heightens the college wage premium and enforces college attainment. The adaption of wages however only takes place gradually throughout the transition,

Table 10: Macroeconomic effects of pension funding in general equilibrium

Simulation	(4)				(5)			
	yes				no			
Smopec	1	3	5	∞	1	3	5	∞
Period t								
Assets ^a	0.0	8.5	21.8	74.5	0.0	2.8	10.0	53.04
Interest rate ^b	0.0	0.0	0.0	0.0	-0.2	-0.3	-0.6	-2.6
Cons. tax ^b	0.8	0.0	-1.3	-5.4	1.1	0.4	-1.2	-6.9
SS tax rate ^b	0.3	-1.1	-4.2	-19.5	0.3	-1.1	-4.2	-19.5
Wage premium ^b	-1.6	-1.2	0.7	9.5	-2.1	-0.5	2.3	6.4
Labor supply ^a								
- $s = 1$	-1.1	-1.1	-1.3	-8.3	-4.1	-3.6	-2.3	-0.1
- $s = 2$	-1.7	-1.3	-2.0	-12.1	-4.7	-3.0	-1.2	-3.6
- $s = 3$	-0.1	0.4	-2.1	-20.9	-1.6	0.0	0.1	-9.3
Wages ^a								
- $s = 1$	0.2	0.4	-0.4	-5.4	1.8	3.1	5.5	16.6
- $s = 2$	0.6	0.5	0.0	-2.6	1.9	2.7	4.6	19.5
- $s = 3$	-0.6	-0.6	0.2	4.8	0.1	0.5	3.7	24.8
On-the-job training ^b								
- $s = 1$	-12.9	-24.9	-34.4	-55.2	35.3	34.0	33.6	49.0
- $s = 2$	-6.9	-20.1	-29.3	-44.0	57.9	54.0	49.4	52.5
- $s = 3$	-0.6	-14.6	-27.4	-42.7	39.0	36.6	33.5	24.4
Schooling Choice ^b								
- $s = 1$	0.0	0.1	0.6	0.9	-0.3	0.2	0.3	0.2
- $s = 2$	0.0	0.0	0.2	0.3	-0.4	-0.2	-0.2	0.0
- $s = 3$	0.0	-0.1	-0.9	-1.2	0.7	0.0	-0.1	-0.1

Change in ^apercent over initial equilibrium, ^bpercentage points.

since on the one hand schooling choice only reacts slowly, on the other hand, young college workers only constitute a small fraction of the overall labor force. On-the-job training effort of households only underlies small changes since there is no real change in distortions. However, the reaction in schooling choice is significantly dampened, as the rise in the college wage premium now again distributes resources from the unskilled to the skilled and nearly compensates them for the rising tax payments and the missing redistribution arising from the pension system. This result is in line with the findings in Heckman et al. (1998) who state that partial equilibrium models usually overstate the reaction in formal schooling decisions.

In the last simulation, we assume a closed economy. Obviously, in this setup, declining interest rates cushion the accumulation of assets. On the other hand, the increase in wages and the resulting rise in labor income taxes overcompensate the short-fall in capital income tax revenues and therefore the consumption tax is lowered in the long-run equilibrium. Pension contribution rate and wage premium react in a very similar manner as before, however, labor supply reactions are significantly altered. This is a direct consequence of the increase in wages for all types of workers resulting from the strong increase in assets and the decline in interest rates as well as the fact that individuals from all types of educational backgrounds now invest significantly more in human capital. The latter again

results from the decrease in interest rates. This makes human capital investment as an alternative to the capital market more favorable and increases on-the-job training efforts of the whole population. Last, schooling choice again is nearly uninfluenced by the reform, since wage reactions compensate households from different schooling levels for their losses or gains from redistribution.

5.3 Welfare effects

In this section, we discuss the welfare effects of the above reform in the two general equilibrium setups. Table 11 shows average compensating variation à la Hicks for agents of different schooling levels for cohorts that already were economically active in the initial equilibrium. For cohorts starting their economically relevant life in the year of the reform and afterwards, we report ex ante welfare changes. The figures in brackets are ex-post welfare changes for different schooling levels.

Table 11: Welfare effects in general equilibrium

Simulation	(4)				(5)			
	without LSRA			with	without LSRA			with
	s = 1	s = 2	s = 3	LSRA	s = 1	s = 2	s = 3	LSRA
80-84	-0.44	-0.42	-0.37	0.00	-0.99	-0.93	-0.82	0.00
60-64	0.31	0.29	0.25	0.00	0.19	0.16	0.08	0.00
40-44	-3.44	-3.52	-3.54	0.00	-3.76	-3.92	-4.21	0.00
20-24	-2.25	-2.50	-2.27	0.00	-4.41	-4.95	-4.61	0.00
10-14	(-0.19)	-0.15	(-1.33)	-2.61	(-2.59)	-2.38	(-3.97)	-2.18
0- 4	(2.41)	2.19	(1.18)	-2.61	(0.38)	0.31	(-1.34)	-2.18
∞	(7.17)	6.42	(6.20)	-2.61	(7.21)	6.58	(5.26)	-2.18

Change in percent of initial resources.

In the case of a small open economy in the left part of Table 11, the oldest generation loses from the increase in consumption tax rates. However, since they expect a decline in consumption taxes, the younger retirees in row 60-64 gain slightly. The big losers of the reform are the middle aged generations. On the one hand, they still have to pay social security contributions, on the other hand, they can't accumulate any more pension claims. Hereby, the generation 40-44 loses most, since earning pension claims is most valuable in the periods close to retirement. This is due to the implicit tax and savings structure of pension contributions. As mentioned above, implicit savings rates are high at the end of the working phase and income of later periods in life is weighted more in the pension formula. The welfare consequences of redistribution in the pension system can be seen best from the young generations starting their economically relevant phase at the beginning of the transition. For those generations, the redistribution towards the skilled induced by the PAYG pension system is absent and there are only small changes in wages. Hence, the higher skilled generations are worse off than the lower skilled. In addition, the reform loosens liquidity constraints which acts more on the lower skilled. Last but not least, the higher skilled are not as well off as the lower skilled, as income is progressively taxed and more income is taxed during the working periods. In the long run, however, households gain due to the absence of a social security implicit tax. In addition, with assets also bequest rise which redistributes towards future generations. Note that the rising skill premium does not offset welfare losses from intragenerational redistribution for college workers. The columns

"with LSRA" reports efficiency effects of the reform. We find a strong loss in aggregate efficiency which is mainly due to the loss in longevity insurance and the increase in marginal tax rates. The latter is a consequence of the payroll tax rate going down to zero. Hence, no contribution to the pension system can be deducted from tax and therefore the full labor income is taxed progressively. Note that the positive liquidity effects induced by pension funding can't compensate these losses.

In the closed economy, older agents are worse off compared to Simulation (4), since consumption taxes are slightly higher at the beginning of the transition. Now, the younger working generations are significantly worse off, since the decline in interest rates enforces liquidity constraints especially in the first periods of transition, where social security taxes are still high. Nevertheless, the difference in welfare of about 1.2 percent between the low and the high skilled remains the same. In the long-run the negative effect of stronger binding liquidity constraints is compensated by the increase in wages. Hence, aggregate welfare is nearly the same as before. Note however, that the higher skilled are one percent worse off than in the small open economy, since the positive effects of a wage increase are damped by the progressive tax system. In the small open economy, the long-run welfare gain was mostly due to increases in bequests which are not taxed. In addition, the bequest level is the highest for college graduates on average, as their parents tend to be richer. In consequence of that, aggregate efficiency loss is also slightly smaller than in the small open economy setup.

6 Conclusion

In this paper we analyze the consequences of pension funding in a general equilibrium model of both formal schooling decisions and on-the-job human capital formation. Our focus lies on the distortive and redistributive effects of a Bismarckian pension system, as well as the macroeconomic and welfare consequences of its abolition.

We find that a Bismarckian PAYG style pension system like the German one strongly enhances on-the-job human capital formation, a result that, to the best of our knowledge, is new to the literature. This is due to the fact that implicit taxes inherent in the social security contribution decrease over the working life of an agent. Hence, in times of investment in human capital agents face a lower net wage per hour worked than in the time of yield. In addition, since pension claims don't pay interest over time, labor income in later years factually gets more weight in the pension formula. Therefore, the pension system redistributes from the lower skilled towards the high skilled, since the latter face the steepest increases in labor efficiency. We compute that the relation between the present value of pension payments received and contributions paid varies from 33.8 percent for the lower skilled up to nearly 40 percent for college graduates.

Our reform simulations indicate that in a small open economy setting pension funding reduces the amount of human capital formed via on-the-job training by about 50 percent on average. Due to different incentives from the progressive tax system, higher skilled reduce their on-the-job training effort less than the lower skilled. In addition, since the redistribution towards higher income earners is eliminated, the number of college students is reduced which results in a 10 percent increase in the college wage premium in the long run. Finally, in a closed economy setup, the annual interest rate decreases by 2.6 percentage points which in turn boosts human capital accumulation on the job. In the long run, we report a strong welfare gain of about 6.5 percent of initial resources. However, this gain comes along with short run losses up to nearly 5 percent for the middle aged generations, who still have to pay contributions in order to finance existing pension claims. Overall, pension funding

comes at efficiency costs of about 2.2 percent in a closed economy setting, while in a small open economy, efficiency loss amounts to roughly 2.6 percent.

Our results give rise to reconsidering the role and design of social security. On the one hand, the privatization of a PAYG Bismarckian pension system may lead to a strong decline in on-the-job training effort. In addition, when it comes to the question of progressivity in a pension system, the discussion of insurance versus labor supply distortion should be extended to the area of human capital accumulation. Furthermore, as a Bismarckian system pays an implicit skill premium, it might be interesting to analyze whether a complementary Beveridgean pillar could counteract this redistributive issue. A mixture system between the two extremes might therefore lead to more schooling without increasing inequality. Finally, the discussion about whether in Bismarckian systems only the last years of employment should be used to calculate pension benefits should also take into account the fact that such a feature already is inherent in an earnings related pension scheme, even if pension claims are calculated from the whole earnings history. Summing up, our analysis leaves room for more discussion.

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