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Abstract

In this paper we analyze the consequences of intergenerational risk sharing in a generational DB pension fund. Each generation is subject to discretionary investment, indexation and contribution policies, thereby losing intergenerational diversification gains. Intergenerational risk sharing is repaired by introducing contingent claims on the generational surplus. We find that in some circumstances the values of these options can be substantial.

1 Introduction

In this paper we propose a generational pension plan design in which some degree of individualization can be realized at the level of a single generation, while at the same time advantages of economies of scale are present. The economies of scale can be achieved by pooling investment and operational strategies from the different generations. Compared with a pure collective defined benefit plan without any generational discretion, the new generational design provides only limited intergenerational risk sharing. As intergenerational risk sharing is an important feature of pension designs we explore whether risk sharing among generations can be improved by introducing explicit contingent claims contracts.

A generational pension plan consists of multiple generational funds. Each generational fund serves one particular generation regarding contribution, indexation and investment policies. Each generational fund is a self-financed fund in the sense that no intrinsic transfers are present. When the assets of a fund assets are lower than its liabilities, fund participants run the risk of having to accept lower benefits during retirement. When the fund has a surplus after paying all promised benefits, it has to forgo this terminal surplus with no reward as the fund is to be closed. These are two possible consequences of the absence of risk sharing among generations. To avoid such situations, we propose two solutions. Firstly, buying a benefit guarantee or put options on benefit payments can ensure participants in a generational fund from pan-generation risks that can not be diversified within a generation. Secondly, selling a call option on the surplus of its terminal assets can allow this generation to share its upside pension results with other generations. In this paper we use the setup of a generational conditional DB fund to evaluate both these contingent claims.

Allowing for contingent claims to be traded among generational funds greatly improves the possibilities for intergenerational risk sharing. In a way the existence of these claims makes the collection of individual plans resemble a traditional collective plan. An important difference between the generational plan and the traditional collective plan is that in the latter plan all generations adopt the same contribution, investment and indexation policies, while in the generational plan these policies still differ across generations.[1]

[1] From this perspective a traditional collective plan can be seen as a special case of a generational plan.

In order to study the magnitude of risk sharing and contract designs, we present the balance sheets for a generation fund of a DB and of a DC deal. To this end, we apply the value-based ALM approach in **?** that values of contributions and benefits are discounted by their corresponding risks. In this way the amount of risk sharing and the net transfer can be quantified a priori to judge the fairness of a pension design. Our approach also differs from **?** in that they use a 20-year time window to show the value transfers for each generation within a collective plan. We zoom in on one generation and analyze the value transfers over the entire life of this generation. Of course, this approach can also be applied to any other generation according to their specific pension deal.

To facilitate intergenerational risk sharing via contingent claims, the contracts need to be traded among generational funds within a composite parent plan. The claims can not be traded with counterparties outside the composite fund. We assume a complete market in the sense that all types of payoffs, especially the liabilities, can be replicated within this internal market.[2] Within a complete market we can apply traditional valuation methods for the guarantees.

The benefit guarantees in our setup differ from the guarantees discussed in **?**, **?**, and **?** in two aspects. Firstly, the benefit guarantee is a series of compound options. The payoff of the later options depends on the exercise of previous options. Secondly, both the underlying asset and the strike price of the guarantees are time-varying and dependent on the particular contribution, indexation and investment policies of the fund. Therefore we use a Monte Carlo simulation to value the options. In addition, we analyze three types of benefit guarantees that respectively provide nominal, accrued rights and real benefits as a minimum.

We start with two base cases that considers respectively a deterministic and a stochastic labor income, when the generational fund faces only the uncertainty from the financial market. We find that with a contribution rate aiming to cover real liabilities the nominal and accrued rights guarantee are relatively cheap, but the real benefit guarantee still costs more than half of the annual salary. The value of the call option on terminal surpluses is about half of the annual salary. We perform a sensitivity analysis when the fund is uncertain about the stock market volatility and

[2] Our assumption of a complete market is somewhat restrictive. In particular, labor income and longevity risks are difficult to hedge within the composite generational plan. However, in this paper we abstract from an incomplete market setting.

the life expectancy of the generation. The two risks increase the prices of the guarantees more considerably than the value of the call. The price of the options depends very much on the choice of the base contribution rate. We also find the break-even base contribution rate that makes a conditional DB fund a fair deal under three types of benefit guarantees.

The rest of the paper is organized as follows. First, in Section **??** we briefly describe a generational plan, and introduce the design of the guarantees and the call option. With these option arrangements, we make generational accounts for different types of generational funds to clarify how intergenerational risk sharing is achieved and how value transfer could occur. Section **??** presents the payoff structure for guarantees and the call option. In this section the generational DB design is also introduced in more detail. Section **??** presents our data and valuation techniques. In Section **??** the prices under various scenarios are presented and compared. Section **??** concludes.

2 Generational pension plan and risk sharing

2.1 Generational pension plan

A generational plan is a plan organized at a generation level. A generational plan has multiple generational funds, and each fund serves one particular generation. Once a person starts working in a company or an industry, he/she will enter one generational fund operated by the pension plan of this company or this industry. This generational fund can be one of the existing funds or a new fund just established for the people in his/her generation. This person will stay with this fund throughout the remaining time of his life if he/she does not change job[3]. It is assumed that people in the same generation share many commonalities such as time to retirement and risk attitudes[4]. Therefore they can set the contribution, indexation and investment policies according to their own preferences. This distinguishes the generational plan from the traditional collective plan where participants across all living generations share uniform policies and are highly susceptible to differential treatment. For example, when there is a high proportion of retirees and senior working

[3]If he/she changes job, they can bring the amount they have accumulated to an generational fund in another company/industry's pension plan. In this study we abstract from this pension portability issue for the moment.

[4]**?** show that individuals' experiences of different macro-economic situations affect people's risk attitudes.

participants, this collective plan tends to adopt a conservative investment policy so that the retirees and near-retirement participants can receive a stable benefit. For young working participants, however, this conservative investment may not be the optimal investment strategy. They then have to bear the consequence of paying a high contribution and accepting a low indexation.

Each generation is independent in that they can choose their preferred pension deal regarding policies on contributions, benefits and investment. If they choose a DC type, the participants in this fund pay fixed contributions and receive variable benefits depending on the investment performance. If they choose a hybrid type, the benefits are aimed at a target level, and the contributions are variable. When running short of assets to pay the targeted benefits, the fund can apply a negative indexation to pay less benefits. If a generation need some certainty in their pension benefits, they can choose a DB type where minimum benefits are guaranteed.

2.2 Option design

The uncertainty of nominal benefits received by a participant comes mainly from three sources: investment risk, mortality risk and labor income risk. The real benefit payment bears additionally the inflation risk. For a generational fund choosing a DB deal, it has to handle these risks to make the promised benefit payments. The fund can adjust its investment policy to reduce investment risk. However, there are pan-generation risks such as war outbreak or long-running economic depression that may not be diversified away within the generation itself. Regarding the mortality risk rising from uncertain death time of individuals, the idiosyncratic part of this risk can be diversified away within a generation. For example, some individuals die earlier while others die later than expected. The benefit payouts saved from the early death can supplement the payment to the late death. This is the intra-generational risk sharing in a generational fund. However, the systematic part of this risk, also referred as longevity risk, can not be diversified away within a generation. Longevity risk refers to the uncertainty in the life expectancy of a generation. The life expectancy is an important assumption in setting the contribution rate. If life expectancy of a generation is longer than the assumption, a generational fund runs short of assets to pay for the extra living years. The fund may buy a longevity linked bond in the open market but such securities are scarcely available currently[5]. The labor income risk refers to the mismatch risk between assets and liabilities that arises from the stochastic future income. The stochastic income influences the cash inflow of contributions and is also am important factor to determine the defined benefits for a given replacement rate. Unfortunately the current market does not provide income-linked securities. Thus the hedge against this risk also needs to be provided by other generations.

To protect the benefits from the mentioned risks, a generational fund that prefers a DB deal can purchase a benefit guarantee from other generational funds. The guarantee functions like a series of put options that generate payoffs each time when assets are not sufficient to pay for the retirement benefits.

Benefit guarantee provides a channel to eliminate the downside risk of pension assets. The mentioned risks can also lead to surplus to a generational DB fund. If desirable, the upside risk can also be shared or sold. In a DB deal for example, there is a possibility that a generational fund has experienced favorable economic conditions during its lifetime, or the cohort lives shorter than expected, or the average lifetime income rises higher than expected. In these case, after paying all its obligations, the fund will end in surplus. A generational fund is designed to serve one birth cohort. Once all its participants die, the fund will be automatically dissolved. This means the fund has to give up its terminal assets (possibly to the plan). This is a waste to the fund because it is giving away assets without any compensation. A generational fund with a DB deal can sell a call option on the surplus of its terminal assets to other generational funds.

Trading benefit guarantees and call options between generational funds is essentially a realization of intergenerational risk sharing. This is beneficial to both sides as long as different generations have a different appetite for risks. Option buyers can pay a price to eliminate unwanted risks and option writers can earn a premium for taking the risks. As each generational fund is financially independent, pricing the guarantee and the surplus option of terminal asset in a fair way is essential in executing the intended risk sharing. The counter party of the option is not pinpointed. If the price is fair, we assume there always exists an counter party. A suitable price can be the market-consistent price where the value of options are derived from the asset prices used by the financial

[5]See **?** for a discussion of a limited range of current longevity-linked securities.

market. This paper aims to provide a framework to price these options.

Any design of an insurance-like contract has to handle moral hazard and adverse selection problems. The potential moral hazard problem makes it difficult for generational funds to buy benefit guarantees in the open market. For example, a generational fund after purchasing a benefit guarantee, may take excessive risks to maximize the upside potential while ignoring the downside losses, which is at the expense of the guarantee provider. The best solution thus is to have this guarantee provided by the other generational funds under the same umbrella plan. Within the plan all generational funds are operated centrally although each fund has their own policies. All the information and activities are administered and regulated at the plan level by a group of representative trustees from each generational funds. The motive to take advantage of each other is thus minimal and the moral hazard problem can be greatly mitigated.

The best time to sign the contract on guarantee provision is at the establishment of a generational fund as soon as it has decided its contribution, indexation and investment policies. If a fund can choose when to buy guarantees, they may want to time the market and in the end this may lead to a collapse of risk sharing. For example, if a fund has experienced a period of good investment returns, the benefit guarantee on this fund will be cheaper due to less possibility that the guarantee will be effected. The fund also finds it is easy to buy a guarantee as it has accumulated much assets. Such funds that are least in need of guarantee can either easily buy the guarantees or may not buy them at all. On the contrary, if a fund has experienced a period of disappointing investment returns, the benefit guarantee on this fund will become expensive as there is a higher chance that the guarantee will be effected. The fund also finds it is difficult to buy a guarantee as its assets are not less than satisfactory. Such funds that are most in need of guarantees may not be able to buy the guarantees. In equilibrium, the fund that can afford the guarantees finds it unnecessary to buy, and the fund that needs the guarantees are not able to afford them. There will exist no guarantees. Therefore to facilitate this risk sharing mechanism, the decision and the contract to purchase the guarantees should be determined and signed at the fund establishment.

To enable this arrangement and contracting between non-overlap funds that prefer a DB deal, the parent plan can serve as an financial intermediary. When all options are traded within a plan at their fair prices, the sum of this activities will be 0. It means the internal market formed by all funds clears. The next section will show that the aggregate net transfers from the perspective of the parent plan is 0, making a generational plan with option arrangement a sustainable system. The zero-sum makes the generational plan resembles the collective plan that they are both a self-contained system. What distinguishes a generational plan from a collective one is that the transfers between generations are explicitly priced in the generational plan. Such prices quantify intergenerational risk sharing and contribute to the sustainability of a pension plan. In a collective plan there exist transfers between generations, but they are implicit and can lead to differential treatment to different generations. As a consequent some generational may opt out of the plan.

2.3 Intergenerational risk sharing

Before we price the options, we present the balance sheet of a generational fund to show how intergenerational risk sharing is achieved via the option arrangement. We are inspired by the idea of generational accounts applied in **?**, and make an account for different types of generational funds from the perspective of participants. In general, accrued rights (AR) and the net transfer (X) compose the asset side, reflecting what participants receive from a pension fund. The liability side consists of contributions and the investment results ($C+R$), reflecting what participants accumulate in the fund. The net transfer is the residual that makes the account in balance. We call it a transfer because it will be transferred to other funds when this generational account is closed.

We present the balance sheet at two dates. The first is at the time of fund closure. By then everything is realized and known. The balance sheet at this time is self-evident to show the ex post actual transfers between this fund and other funds. The second is at the contracting time when a generational fund decides its deal concerning contribution, indexation and investment policies, and accordingly the value of options related to this fund are also determined. The balance sheet at this date can demonstrate ex ante transfers and risk sharing.

We follow the spirit of the value-based ALM in **?** that the value of assets and liabilities of a pension fund should account for their respective risks in order to detect any a priori transfers. We use a "V" operator in the sequel to show the economic value of the items adjusted by their risks.

Readers can think of this value as the present value. The balance sheet expressed in economic values can show whether there exist a prior transfers and can signify whether the fund strikes a fair deal.

2.3.1 Account of a DC-type generational fund

If a generation prefers a DC type, contributions are fixed, accrued rights will vary to the contributions and their investment results. The DC deal prescribes that benefits are paid only out of the asset pool participants have accumulated, namely $AR = C + R$, and ex post actual net transfer (X) is 0. In sum the balance sheet (BS) of a DC fund at closure looks like:

BS of a DC fund at fund closure	
AR	$C + R$
X	

At the establishment of the fund its balance sheet is

BS of a DC fund at fund establishment		
	$V(AR)$	$V(C+R)$
	$V(X)$	

At the contracting time, the risk adjusted value of accrued rights equals the risk adjusted value of contributions. For example, if contributions are invested in risky assets, benefits accrued based on the resulting high returns will become volatile, and thus benefit amount will be discounted at a higher rate. The relationship can also be viewed as a budget constraint that benefits are financed by contributions. Namely, we have

$$V(AR) = V(C + R) \Rightarrow V(X) = 0$$

The value of net transfer is 0. The zero net transfers ex ante reflects that a DC deal is a fair design. Participants get all that they themselves have accumulated and there occurs no transfer between them and other generations. This signifies no intergenerational risk sharing for a DC fund.

2.3.2 Account of a DB-type fund with implicit guarantees and calls

When a generation prefers a DB type, the pension benefits are deterministically defined by a formula as a function of an accrual rate, life-time average salary and years of employment. Contributions vary due to uncertain investment results deviating from expected returns. The adjustment of contributions, however, is limited in accumulating sufficient assets to match liabilities[6]. To honor the benefits specified by the formula a DB deal implicitly owns a contingent claim, which generates payoffs (PO) when accrued rights backed up by the contribution and investment returns are lower than the defined benefits. With the benefits specified by a formula independent from investment results, there is also a possibility of some assets left at the fund closure, and such assets will be automatically transferred to other funds. Implicitly a DB fund is writing a call option on this terminal surplus. The obligation to give up any positive terminal assets (POC) is part of its liabilities. In sum, its account at the fund closure looks like

BS of a DB fund with implicit options at fund closure

AR	$C+R$
PO	POC
X	

PO is the payoff of benefit guarantees when assets are insufficient to pay for the defined benefits. AR is the accrued rights that is afforded by the accumulated assets $C+R$. $PO+AR$ is the defined benefits, or the promise made by the fund, actually received by participants. At the fund closure, when the contribution and investment surpass the defined benefits, the rest is counted as POC and transferred to other generations. In the option term, POC is the payoff of a surplus call when there are positive terminal assets left at the fund closure. $C + R + POC$ together represents the total amount of assets generated by contributions and investments. The ex post actual net transfer (X) is calculated as $C + R + POC - AR - PO$.

At the establishment of the fund, its balance sheet is

BS of a DB fund with implicit options at fund establishment

[6]For example, during the retirement phase the contribution is regulated at 0. If there occurs a bad investment return, the assets becomes lower than expected and not enough to cover the promised benefit payment.

$V(AR)$	$V(C+R)$
$V(PO)$	$V(POC)$
$V(X)$	

At the contracting time, with the same reasoning as in the DC deal, the economic value of contributions and investments equals the economic value of the accrued rights, namely $V(AR) = V(C + R)$. What makes a DB deal different from a DC deal is the existence of the options. The guarantee and the call option are implicitly respectively owned and issued by participants. The economic value of their payoffs is simply their respective market prices, $V(PO) = P_{gua}$ and $V(POC) = P_{call}$. However, in this case, both are not explicitly priced and paid or earned by participants. Accordingly the net transfer ex ante is

$$V(X) = V(C + R) - V(AR) + V(POC) - V(PO) = P_{call} - P_{gua}$$

If the price of the guarantee is larger than the price of the call, then the fund has a negative net transfer, meaning the fund is a net receiver. If the value of the guarantee is smaller than the value of the call, then the fund has a positive net transfer, meaning the fund is a net giver. Whether the net transfer is 0 determines whether the DB deal is a fair contract. This also highlights the importance of explicitly pricing the options in designing a fair plan, as put forward by **?**. The existence of the guarantee and the call shows there exists a mechanism for intergenerational risk sharing, because the deficiency in assets is made up by guarantee provider and the surplus of terminal assets is left to other funds. The magnitude of this risk sharing is $P_{call} + P_{gua}$.

2.3.3 Account of a DB-type fund with priced guarantees and calls

If a DB fund explicitly pays a price for the benefit guarantee it owns and explicitly earns a premium for the surplus call option it sells, then its account at the establishment looks like:

BS of a DB fund with guarantees and calls at the fund establishment

$V(AR)$	$V(C+R)$
$V(PO)$	P_{gua}
P_{call}	$V(POC)$
X	

P_{gua} is the price of the guarantee. It is a cost to the fund and placed on the lability side. P_{call} is the price of the call. It is earned by the fund and placed on the asset side.

If prices are the economic values of the option, namely

$$\begin{aligned} V(PO) &= P_{gua} \\ V(POC) &= P_{call} \end{aligned}$$

then the net transfer is

$$V(X) = V(C+R) - V(AR) + P_{gua} - V(G) + V(CA) - P_{call} = 0$$

It says a DB plan with properly priced guarantees and calls is a fair design. This paper tries to find out the economic values of the payoffs or the market price of the options. The existence of the guarantee and the call reflects there exists a mechanism for intergenerational risk sharing, and this risk sharing is priced. The magnitude of this risk sharing is $P_{call} + P_{gua}$.

The comparison between the balance sheet of a DB deal and that of a DC deal also reflects that a DB deal is essentially a DC deal plus a benefit guarantee and minus a surplus call.

The above account is only for one generational fund. At the pension plan level, the consolidated account is simply the aggregation of all individual accounts for each generational fund. Since options are traded within the pension plan, the total net transfer ($\sum X$) will be 0.

Our analysis of the generational account combines ideas from previous studies, but also distinguishes itself in the following aspects. **?** detects implicit options embedded in pension contracts but does not assign them to different generations. **?** introduce the generational account to study the transfers, but their focus is the change of the value of an generational account due to a change of the pension deal. In addition, they study the value changes for a generational account for a time window of 20 years, while we study the value transfers over the entire life of a generation. **?** discuss the value transfers between generations within a collective plan in the context that all generations apply the uniform policies in contribution, investment, and indexation. We show the value transfer from one particular generation, and this transfer varies to the particular pension deal chosen by the generation.

3 Option payoff structures

This section describes the development of the assets and liabilities of a generational conditional DB fund. We use a conditional DB setup to make the generational fund resemble the DB plan in reality. It differs from the traditional DB plan in that the indexation of accrued rights to inflation is dependent on funding status. With this setup we elaborate how the values of guarantees and calls are determined. Guarantees to pension benefits can take two forms: minimum rate of return guarantee and minimum benefit guarantee as summarized in **?**. In our setup, the guarantees are of the second form and can differ by providing three types of benefits. One is to guarantee the nominal pension benefits that are defined by a formula based on a life-time average salary and a replacement rate. One is to guarantee the earned pension rights that include the indexation granted in all the previous years. The other is to guarantee the real pension benefits that are fully indexed to the inflation rates all along.

3.1 A conditional DB generational fund with no guarantees or call options

We use a representative individual for a generation to model the development of a generational fund. Assume a participant enters the labor market at T_0, retires at T_r, and dies at T_d. He earns a flat salary (S_0).

The assets (A_t) grow due to premium collections (p_t) and investment returns ($r_{A,t}$).

$$A_t = A_{t-1} * (1 + r_{A,t}) + p_t * S_0 * (1 + s)^t \tag{1}$$

where s is time-independent salary growth rate of the generation, and is assumed 0.

During the working years the liabilities change due to the change in accrued rights (AR_t) and the corresponding yield curve.

$$L_t = \sum_{n=0}^{T_d-T_r-1} \frac{AR_t}{(1+y_{T_r-t+n,t})^{T_r-t+n}} \tag{2}$$

where $y_{T_r-t+n,t}$ is the yield of maturity $T_r - t + n$ at time t.

AR_t refers to accrued rights at time t, it is the future annual benefits that is entitled to participants since their retirement. In year 1 the accrued rights are $AR_1 = NAR * (1 + ind_1)$. From year 2 onwards until retirement the accrued rights are given by $AR_{t+1} = (AR_t + NAR) * (1 + ind_{t+1})$. ind_t is the indexation rate. NAR is the newly accrued rights for one year of service and time independent. If RR is the replacement rate, then for every year of service NAR is $\frac{RR}{T_r-T_0} * S_0$. The indexation rate in this study is bounded within [0,100%], so the accrued rights is always between the nominal rights and real rights.

Upon retirement ($t = T_r$), the fund starts to pay out benefits as defined in AR_t and still invests the rest. So the assets on the one hand decrease with the benefit payments, while on the other hand increase with the investment returns, and it follows

$$A_t = A_{t-1} * (1 + r_{A,t}) - AR_t \tag{3}$$

No new rights are accrued as of retirement, $NAR = 0$. The total accrued rights only increase with indexation $AR_{t+1} = AR_t * (1 + ind_{t+1})$. The value of the fund liabilities is the discounted value of the benefit payments with a declining amount of years.

$$L_t = \sum_{n=0}^{T_d-t-1} \frac{AR_t}{(1+y_{n,t})^n} \tag{4}$$

At the end of each year, the funding ratio (FR_t) is computed as A_t/L_t. Every year the contribution rate p_t and the indexation rate ind_t are determined according to a policy ladder, which stipulates the values for p_t and ind_t for the next year according to current funding ratio. This policy ladder is often set by the fund at its establishment and fixed for the whole life of the fund.

The dependency of indexation rate on the funding status reflects the conditional feature of this DB fund.

3.2 Provision of various guarantees

This section presents the payoff structure of various benefit guarantees for a conditional DB generational fund. A benefit guarantee can be seen as a series of put options, which are exercised at the time of each benefit payment. The underlying asset of the options is the pension assets. The strike is the value of guaranteed benefits.

We still use a representative individual to model the fund. During the working phase of its participants, the fund's assets and liabilities follow the same path as described in Equation (**??**) and (**??**). Upon retirement, the value of assets determines whether a put option is exercised, then whether an option is excised determines the starting value of assets for the next period.

At the time before each benefit payment, if the assets accumulated till that moment is lower than the guarantee, the guarantee put option is exercised. The fund receives the difference between the assets available and the guarantee. This amount is also the payoff of the guarantee option. Then the asset value for the next period becomes 0. If the asset value is not lower than the guarantee, the fund assets will continue as specified in Equation (**??**). The following lists the payoff of the guarantees, the development of the asset value and the accrued rights during the payout phase.

When a nominal pension benefit guarantee is provided, the minimum benefits participants receive are the nominal benefits (AR^{nom})[7]. Dependent on the assets at the time relative to the nominal benefits, the payoff of each put option entailed by the nominal guarantee[8] (PO_t) is given by

$$PO_t = \begin{cases} 0, & \text{for } A_t(1+r_{A,t}) \geq AR_t \\ 0, & \text{for } AR^{nom} \leq A_t(1+r_{A,t}) < AR_t \\ AR^{nom} - A_t(1+r_{A,t}), & \text{for } A_t(1+r_{A,t}) < AR^{nom} \end{cases}$$

[7] It equals $RR * S_0$.

[8] Benefits are paid at the end of the period, so at the time of option exercise the assets have already earned the investment return.

With the existence of guarantees, the actual benefits received by participants depends on the value of assets. Because the indexation rate is bounded within [0,100%], the accrued rights are between nominal benefits and real benefits. When the value of assets is lower than the nominal benefits, nominal guarantee is effected and participants receive the nominal benefits. When the value of assets is higher than the nominal benefits, but lower than the accrued rights, guarantee is not effected, participants receive all the assets. When the value of assets is higher than the accrued rights, nominal guarantee is effected and participants receive the accrued rights. Specifically they are

$$AR_t^{updated} = \begin{cases} AR_t, & \text{for } A_t(1+r_{A,t}) \geq AR_t \\ A_t(1+r_{A,t}), & \text{for } AR^{nom} \leq A_t(1+r_{A,t}) < AR_t \\ AR^{nom}, & \text{for } A_t(1+r_{A,t}) < AR^{nom} \end{cases}$$

Accordingly the value of assets for the next period depends on how much benefits are actually paid. When the assets are less than the nominal benefits, the fund pays the nominal benefits by using the money received from the guarantee. The assets for the next period is 0. In sum they are given by

$$A_{t+1} = A_t(1+r_{A,t}) - AR_t^{updated} \quad = \begin{cases} A_t(1+r_{A,t}) - AR_t, & \text{for } A_t(1+r_{A,t}) \geq AR_t \\ 0, & \text{for } AR^{nom} \leq A_t(1+r_{A,t}) < AR_t \\ 0, & \text{for } A_t(1+r_{A,t}) < AR^{nom} \end{cases}$$

When an accrued pension rights guarantee is provided, the indexation won't be negative and the accrued pension rights (AR_t) are protected from any current and future cutdown. Participants receive what they have accrued, namely $AR_t^{updated} = AR_t$. The payoff of this guarantee is

$$PO_t = \begin{cases} 0, & \text{for } A_t(1+r_{A,t}) \geq AR_t \\ AR_t - A_t(1+r_{A,t}), & \text{for } A_t(1+r_{A,t}) < AR_t \end{cases}$$

The assets for the next period becomes

$$A_{t+1} = A_t(1 + r_{A,t}) - AR_t^{updated} \qquad = \begin{cases} A_t(1 + r_{A,t}) - AR_t, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\ 0, & \text{for } A_t(1 + r_{A,t}) < AR_t \end{cases}$$

When a real pension benefit guarantee is provided, participants always receive the real benefits, $AR_t^{updated} = AR_t^{real}$, as the real benefits are the highest possible benefits participants can receive[9]. The real benefits are fully indexed to all the inflation rates over time, $AR_{t+1}^{real} = (AR_t^{real} + NAR)(1 + \pi_{t+1})$ and π_t is the inflation rate. The payoff of this guarantee only depends on the comparison between the assets and the real benefits. It is given as

$$PO_t = \begin{cases} 0, & \text{for } A_t(1 + r_{A,t}) \geq AR_t^{real} \\ AR_t^{real} - A_t(1 + r_{A,t}), & \text{for } A_t(1 + r_{A,t}) < AR_t^{real} \end{cases}$$

The assets for the next period is

$$A_{t+1} = A_t(1 + r_{A,t}) - AR_t^{updated} \qquad = \begin{cases} A_t(1 + r_{A,t}) - ARreal_t, & \text{for } A_t(1 + r_{A,t}) \geq AR_t^{real} \\ 0, & \text{for } A_t(1 + r_{A,t}) < AR_t^{real} \end{cases}$$

We give a numerical example. Suppose now the fund is at the point to pay out the first benefits to its participants. Suppose the assets accumulated till this moment is €40,000. Suppose the nominal and real pension rights are respectively €21,000 and €80,000, and the accrued rights till this moment is €60,000. At this point, the first put option contained in a guarantee is to be expired. When the fund has a nominal guarantee, because the value of assets is larger than the minimum guarantee of nominal benefits, this first put option is not exercised, payoff of the nominal guarantee at this moment is 0. Participants receive whatever the fund can pay, which is €40,000. The value of assets for next period is 0. When the fund has an accrued rights guarantee, because the value of assets is smaller than the minimum guarantee of accrued rights, the first put option contained in this guarantee is exercised, payoff of the accrued rights guarantee at this moment is

[9] As indexation rate is not larger than 100%, AR_t is not higher than AR_t^{real}.

€(60, 000 − 40, 000). Participants receive accrued rights, which is €60, 000. The value of assets for next period is 0. When the fund has a real benefit guarantee, because the value of assets is smaller than the minimum guarantee of real rights, the first put option contained in this guarantee is exercised, payoff of the real guarantee at this moment is €(80, 000 − 40, 000). Participants receive accrued rights, which is €80, 000. The value of assets for next period is 0.

The value of liabilities is calculated in the same way as described in Equation (**??**).

To pay for the costs of guarantees, a generational fund can charge its participants a higher contribution rate or generate other incomes by selling a call option on its terminal fund surplus.

3.3 Selling the fund surplus

As mentioned, the generational fund is an independent fund. At the fund disclosure when all participants die, the fund is dissolved automatically. If there are any assets left, then they would be automatically transferred to the other funds or to the plan level for central planning. From the perspective of the generational fund, this potential surplus is given away without compensation. To avoid wasting resources, the fund can sell a call option on its terminal surplus. This call option entitles the buyer any positive assets left with the fund at its closure. Using the previous setup the payoff of this call option (POC) is

$$POC = max(0, A_{T_d}(1 + r_{A,T_d}) - AR_{T_d}^{updated})$$

As the value of assets varies to the guarantee arrangement, the value of the call also differs for various guarantee arrangements.

4 Valuation techniques and data

Benefit guarantees and surplus options are traded among generational funds within a same umbrella plan, which forms an internal market. We assume a complete market that all states of liabilities can be replicated within this internal market. We apply the risk neutral technique introduced by **?**. It says that in the absence of arbitrage opportunities, there exists a risk neutral probability such that

$$V(t,T_r,T_d) = e^{-rf_t(T-t)}\tilde{E}_t[\sum_{T=T_r}^{T=T_d} PO(T)] \tag{5}$$

The left hand side is the value of the guarantees at time t and the guarantees are effective from the retirement date (T_r) until the death date (T_d). The guarantee functions like a number of $T_d - T_r$ put options. rf_t is the risk free rate at t. $\tilde{E}_t$ is the expectation taken at time t and under risk-neutral measure. In its bracket is the sum of options which each has a payoff at the time of each benefit payment. These payoffs are specified in the previous section. In the same way, the value of the surplus call at time t and expires at T_d is

$$VC(t,T_d) = e^{-rf_t(T_d-t)}\tilde{E}_t[POC(T_d)] \tag{6}$$

The payoffs are generated in a risk neutral world when all investments are earning a risk free rate. We specify the following stochastic process for stock returns, bond returns, yield curve and inflation in the risk-neutral world where risk premium is 0. We use the Vasicek model (1977) to model the risk free rate. We choose this approach for yield curve modeling because it is based on no-arbitrage principal and suitable for pricing purpose. As the Vasicek model does not preclude a negative rate, we apply this model to the real risk free rate. Then according to Fisher's hypothesis, the nominal short rate is the sum of the real rate and expected inflation rate. This two-factor approach for the nominal rate is also applied by **?**. Specifically, the real rate (r_t) and expected inflation (π_t) respectively follow:

$$\begin{aligned} r_{t+1} &= \bar{r} + \phi(r_t - \bar{r}) + \epsilon^r_{t+1} \\ \pi_{t+1} &= \bar{\pi} + \kappa(\pi_t - \bar{\pi}) + \epsilon^\pi_{t+1} \end{aligned}$$

$\bar{r}$ and $\bar{\pi}$ are the long term means of the real rate and expected inflation. ϕ and κ are mean reversion coefficients of the real rate and expected inflation. The risk free rate is $rf_t = r_t + \pi_t$.

According to the expectation theory, a long term rate is the average of expected future short rates. Therefore we have the yield for H periods at t as

$$y_{H,t} = \bar{r} + \bar{\pi} + \frac{1-\phi^H}{H(1-\phi)}(r_t - \bar{r}) + \frac{1-\kappa^H}{H(1-\kappa)}(\pi_t - \bar{\pi})$$

Accordingly the return of a zero-coupon bond with m years to maturity follows

$$bond_{m,t} = \frac{price_{m-1,t+1}}{price_{m,t}} - 1 = exp(m * y_{m,t} - (m-1) * y_{m-1,t+1}) - 1$$

We assume pension funds invest in 10-year zero bonds. Assuming stock prices follow a lognormal distribution, then the annual stock return follows

$$stock_t = exp(rf_t - \sigma^2/2 + \sigma\epsilon_t^s)$$

ϵ_{t+1}^r, ϵ_{t+1}^π, ϵ_t^s are shocks to the real rate, expected inflation and stock return respectively. Each follows a standard normal distribution and is independent from each other.

Because the payoffs of the guarantees and the surplus call option are path-dependent on the development of assets and liabilities, which are mediated by the contribution, indexation and investment polices set in the pre-specified policy ladder, it is hard to derive a closed-form solution for the values. We make a numeric valuation with a Monte Carlo simulation. Such simulation approach has been applied in various pension contexts in different countries[10].

Specifically, we simulate 1000 paths for stock returns and yield curves through the lifetime of a generational fund. From the simulated yield curves we derive bond returns, together with the simulated stock returns, we get the value for assets. Discounting accrued rights with the simulated yield curves, we get the value for liabilities. Accordingly we obtain 1000 scenarios of option payoffs. Discounting the payoffs at the risk free rates we get the present value of the options.

The values of all options will be determined at the time of its establishment as soon as the fund has announced its contribution, indexation and investment policy. A fund can only trade such options at this time by signing a contract with counter parties. However, they can fulfill the

[10]Consult **?** for public and private pension funds guarantee in Uruguay and Chile, **?** for DC plan guarantee in the US, **?** for DC to DB conversion, **?** for option valuation in pension contracts.

payment during the whole working phase. Of course the price will be higher the later they buy due to the time value of money.

We collect all the data in an annual frequency over the period between year 1954 and 2007[11]. We take the 3-month US T-bill rate for the short nominal rate. The annual inflation rate is calculated from US Consumer Price Index-All Urban Consumers. This realized inflation rate is used to back out the expected inflation assuming an AR(1) process. Then the real rate is obtained as the difference between the nominal rate and the expected inflation rate according to the Fisher hypothesis. From the Center for Research and Security Prices (CRSP) of the University of Chicago we get the stock return (including dividends), which is for a value-weighted portfolio including all stocks traded on the NYSE, NASDAQ, and AMEX. The parameter estimates are shown in Table **??**.

5 Prices of guarantees and the call option

The options are designed to handle the investment risk, longevity risks, and labor income risks that lead to the mismatch between assets and liabilities. Firstly we will present two sets of prices in a baseline case when only diversifiable financial risks are considered. One set of prices is under the assumption of a deterministic labor income, and the other is under a stochastic labor income. Then starting from the base prices under deterministic labor income, we go on to show the sensitivities of these prices when there is a generation-long shock to the financial market reflected by a higher market volatility of the stock market; and the sensitivity when there is a longevity risk reflected by a stochastic life expectancy.

5.1 Base prices under a deterministic labor income

In the baseline case, the uncertainty of the assets and liabilities of a generational fund comes only from the financial market risks, which are represented by the yield curve and the stock market described in the previous section. The assumptions on the fund and participants are specified in Table **??**. The entire life of a generational DB fund is 55 years. The first 40 years are the working

[11]This is the period when all the data are available.

phase when the fund collects contributions, and the later 15 years are the retirement phase where the fund pays out retirement benefits. The average of the participants earn a flat salary (S_0) over the working years at €30,000. All cash flows occur at the end of each year, namely participants get salary, pay contributions and receive pension benefits all at the end of the year. The policy ladder concerning contribution and indexation policy is defined in Figure **??**. The contribution rate is adjusted within a range of [-5%, 5%] around the base rate according to the funding ratio. In the beginning when the funding ratio is above 6, the contribution rate is 5% lower than the base rate, and is 5% higher when the funding ratio is lower than 1. Over time the upper bound of the funding ratio linearly declines to 2 at the time of retirement. The upper bound is set initially high at 6 in the consideration of the mechanically low starting liabilities which otherwise lead to no contribution requirement. Investment policy is set constant at 50% stocks and 50% bonds during the working and retirement phase. The base contribution rate is determined in a actuarially fair way. It is solved from $\sum_{t=1}^{T_d} pS_0(1+r_t)^{-t} = \sum_{t=T_r}^{T_d} S_0 RR(1+r_t)^{-t}$, which says the lifetime individual pension entitlements equals to lifetime individual pension contributions. Based on the historical mean real rate of 1.27% as the discount rate (r_t)[12] and 70% replacement rate (RR), the base contribution rate (p) is set at 18.38%. The dynamics of the fund over time is shown in Figure **??** depicting the average simulated contribution rate, indexation rate and various types of pension rights.

Panel A in Table **??** shows the prices of the three types of benefit guarantees and the surplus call under the three types of guarantees at the time of first contribution collection. When only the nominal pension benefit is guaranteed, namely participants can get at least 70% of the annual salary after retirement, the guarantee costs 4.17% of the annual salary when purchased at age 26, the time of the first contribution payment. If paid in annual installment over the whole working years, it is only 0.1%[13] of the salary. Compared to the base contribution rate of 18%, this nominal guarantee is only a marginal addition to the pension cost. The average paid benefits are €65,510, far over the nominal guarantee of €21,000. It reflects the fact that the nominal guarantee is not effected in most of the time , therefore leading to a low cost of the nominal guarantee. This low cost

[12] Pointed out in **?**, the discount rate is a central and contentious issue in setting contribution rate. In general there are three possible choices for the discount rate. They are market rate of return, riskless interest rate and fiscally sustainable rate. Here we choose the real riskfree rate to make a conservative calculation of contribution rate.

[13] =4.17%/40.

is expected because the real rate 1.27% is used as the discount rate[14] to set the base contribution rate to afford the real benefits.

When the accrued pension rights are guaranteed, namely, participants at least maintain the level of the previous year's benefits, this guarantee costs 12.11% of the annual salary. Such guarantee can provide an average annual benefit of €68,219, about 86% of the real benefits.

When the real pension benefit is guaranteed, where participants get inflation-proof benefits, it costs considerably 60% of the salary. If paid in annual installment over 40 years, this guarantee costs an annual amount of 1.5% of the salary.

Selling a call option on the surplus of the fund terminal assets, a generational fund can avoid wasting the upside potential of its assets to generate an extra income to pay for the benefit guarantees. The value of this call option on the fund surplus goes against the value of the benefit guarantee. When the fund assets become larger, the value of the guarantees decreases while the value of the call increases.

When a fund is provided with a nominal guarantee, the call option it can issue at the fund establishment is worthy of 49.31% of the annual salary. For a fund provided with an accrued rights guarantee, the call option is worth the same value. This is because the payoff of a call option only counts the upside potential, and this upside potential is the same for a fund with a nominal guarantee and a fund with an accrued rights guarantee. The conditional DB plan defines that participants get a high indexation when the value of assets are high. Therefore when the assets develop to its up state, participants receive the same granted indexation when the fund is provided with either a nominal or an accrued rights guarantee. The assets accordingly follow the same path in the up states, leading to the same value of the surplus call option.

For a fund provided with a real guarantee, the call option is worth 47.31%, not much lower than the call under the other two guarantee types. This is because the base contribution rate is set to aim for real benefits, which most of the time enables the fund to grant a high indexation. This leads to little difference between the surplus under a real guarantee and the surplus under the other guarantees.

[14] If pension assets can earn a return higher than this, then pension assets will be higher than the expected liabilities.

5.2 Base prices under a stochastic labor income

The previous case assumes a deterministic labor income. Now we incorporate an exogenous labor income risk[15]. The shocks in the labor income can influence the contributions and the asset accumulation, accordingly will influence the accrued rights and liabilities.

We apply a simple process for the labor income as $S_t = E(S) * (1 + s + \xi_t)$[16]. S_t is the income flow at year t. s is the expected income growth rate, and ξ_t follows $NIID(0, \sigma_\xi^2)$. For the simulation and comparison purpose we set $S_0 = 30,000$, $s = 0$ and $\sigma_\xi = 0.1$[17].

The second set of prices in Panel A of Table **??** shows that the prices do not change much from the base case with a deterministic labor income. This is because the average income is the same in both cases and after 40 years of accumulation the distribution of assets and liabilities are comparable at the time of retirement in both cases. In addition, the labor income risk does not influence the fund dynamics after retirement when the guarantee starts to be effective. Therefore a random income shock from a standard normal distribution has negligible impact on the option prices.

5.3 Sensitivities to non-diversifiable investment and longevity risks

The base prices of the guarantees and the call are based on the assumptions summarized in Table **??**. This section relaxes some of the assumptions and considers two non-diversifiable risks that the generation has to share with other generations, namely the uncertainty on the financial market and the life expectancy.

There is a possibility that one generation could suffer from a life-long shock from the financial market so that the generation cannot diversify such risk away within itself. We apply an alternative parameter for the stock market volatility to reflect this risk. We calculate the volatility of stock returns within a 20-year window during our sample period and find the highest volatility is 18.33%,

[15] Here we abstract from the endogenicity that labor supply can vary with labor income shocks and the correlation between the labor income shock and the investment return shock.

[16] Often used in literature is that labor income follows log normal distribution such as in **?**and **?** that $S_{t+1} = S_t exp(s + \xi_{t+1})$. But in order to make it comparable to the baseline case we assume the labor income normal distributed rather than lognormally distributed because the later assumption will lead to an increase in the expected labor income even when the error term has a mean of 0.

[17] This is taken from **?**.

which is a 22% increase from 15.06% for the base case. The investment risk directly influences the volatility of assets both during the working phase and the post-retirement phase. This causes considerable increases in the prices of the guarantees and the call, ranging from a 14% to a 43% increase. Figure **??** compares the payoffs of three types of guarantees under this scenario with the payoffs under the base case. There is an increase in the payoffs irrespective of the guarantee type. Relatively the percentage increase in the price of the nominal benefit guarantee is the highest, and the percentage increase in the price of the real benefit guarantee is the lowest.

The average annual pension benefits received by the participants under the macro-investment risk, seen in the last two columns of Table **??**, are lower than the base case because of the no catch-up indexation in our pension design. More volatile assets under the investment risk will miss some indexation which will not be made up later even when the value of assets picks up.

The base contribution rate is set based on the life expectancy of the generation. The uncertainty in the life expectancy is a non-negligible risk. We incorporate this risk by making the life expectancy a random variable from a normal distribution with a mean of 80 years and a standard deviation of 2 years.

When the generation has a lower life expectancy than the expected, the extra surplus assets are transferred to other funds who bought a surplus call from this fund. When the generation has a longer life expectancy than the expected, the fund policy is set as follows. For the years after the expected life expectancy, no investment is made and no indexation is given. Under the nominal guarantee, the fund aims to pay what it can afford, with the order of firstly the accrued rights, secondly the less indexed rights, and lastly the nominal benefits. Under the accrued rights guarantee, the fund always pay the accrued rights as participants have accrued till the last year of their expected life expectancy. Under the real guarantee, the fund still pays inflation-indexed benefits, including the inflation rate for these extra years.

Panel B in Table **??** shows that this longevity risk increases the prices of the guarantees considerably. Figure **??** compares the average payoffs of three types of guarantees under this scenario with the payoffs under the base case. In the base case, the payoffs stop at the expected death age of 80. In the longevity case, the payoffs before the expected death age are lower than the base case, but they extend to a longer age till 84. The combining effect is that guarantees become more valuable when there exists uncertainty on the life expectancy.

The impact of the longevity risk on the call option is negligible. This is because the surplus call only concerns the fund assets at the end when all participants die. The possibility of both out-living and under-living the expected age leads to a non-significant change in the average value of the terminal assets.

5.4 Net Transfers

A generational fund accommodates the needs of a particular generation. Buying a pension rights guarantee protects its participants from non-diversifiable generation-specific shocks. Selling a call option on the fund surplus avoids under-consumption. Recalling the balance sheet of a DB deal with implicit guarantees and the surplus call option, the nest transfers a prior is determined by the difference between the value of the guarantee and the value of the call. A conditional DB deal in our numerical example is a net giver when it is provided with either a nominal or an accrued rights guarantee, meaning this fund transfers net positive values to other funds because the call it sells has a higher value than the guarantee it receives. For a conditional DB deal with a real benefit guarantee, the fund is a net receiver, as the real guarantee it receives is more valuable than the call it sells. The above results remains when the additional investment risk and longevity risk are considered.

The value of the options to be traded among generational funds are contingent on the base contribution rate. If a contribution rate is higher, the value of the benefit guarantee will be lower but the value of the surplus call will be higher. Table **??** reports the break-even base contribution rate that equalize the value of the guarantee and the value of the surplus call when the underlying fund has the characteristics defined in Table **??**. With this contribution rate, a generational fund with a conditional DB deal does not have to pay a cash outflow explicitly for the guarantee and receive cash inflow explicitly for the call. The break-even base contribution under nominal guarantee is 10.16%. It means if a generational fund whose characteristics are defined as in the assumptions in Table **??** adopts a conditional DB deal with a nominal guarantee, then this DB deal is a fair deal if it sets its base contribution rate at 10.16%. It will be net giver(receiver) if its base contribution rate is higher(lower) than 10.16%. For a fund with an accrued rights and a real benefit guarantee, the break-even base contribution rate is respectively 13.09% and 19.28%. If we look at the values of the guarantee and the call under three types of guarantees, the values of the guarantee and the call are highest in the case of a real guarantee provision. This shows that the risk sharing among generational funds is the highest when a real benefit guarantee is traded, and is the lowest when only nominal benefit guarantee is traded.

6 Conclusion and discussion

A generational pension fund enables customized fund management in contribution, indexation and investment policies, facilitates risk management of a pension fund geared to the preference of a particular generation, and frees pension sponsors from unwanted risks. To further improve the welfare of participants, guaranteeing pension benefits can be desirable. As a generational fund is financially independent, the upside potential of its terminal assets can also be sold. The arrangement of such guarantees and the call option provides a mechanism for intergenerational risk sharing, and this paper tries to quantify this risk sharing by pricing the options. In the collective plan, such options are implicitly embedded in the contract and can hardly be quantified. Our design of option trading and their valuation in a generational plan makes intergenerational risk sharing more explicit and transparent.

Applying risk-neutral and simulation techniques, we show that with a base contribution rate determined by using a real rate as the discount rate the nominal and accrued rights guarantees are relatively cheap. The nominal benefits guarantee costs 4%, the accrued rights guarantee costs 12% and the real rights guarantee costs 60% of the average annual salary. A call option written on the surplus of the terminal assets is worth about 50% of the average annual salary for all types of the benefit guarantee. We consider another base case with a stochastic labor income that has the same expected annual salary as in the deterministic labor income case. This has negligible impact on the option prices for the labor income shock only influences assets till the retirement.

We also show the sensitivity of the option prices when some of the non-diversifiable risks are incorporated. We reflect the generation-long shock in investment by increasing the volatility of the stock returns to the historic highest in our sample period. This investment risk directly influences the volatility of the fund asset accumulation both during the working phase and the post-retirement phase. This causes considerable increases in the prices of all guarantees and the surplus call. Measured by a percentage increase, the nominal guarantee is influenced the most.

We reflect the longevity risk by making the life expectancy a random variable from a normal distribution. The uncertainty of the life expectancy leads to only a marginal change in the value of the surplus call due to the two-side possibilities of realized life-time. However it increases the prices of all benefit guarantees considerably. Measured by a percentage increase, the accrued rights guarantee is influenced the most.

The value of options traded among generational funds depends considerably on the base contribution rate. A higher contribution rate leads to a lower value of guarantees and a higher value of calls. We find that respectively at a break-even base contribution rate of 10.16%, 13.09% and 19.28%, a generational fund featured in our base case with a deterministic labor income does not have to make explicit cash flows for the options under nominal, accrued rights and real guarantee provisions. At these rates, a conditional DB deal is a fair deal. The value of guarantees and the call also reflects that the intergenerational risk sharing is highest when a real benefit guarantee is traded among generational funds.

An important practical issue concerning the option arrangement is that it can easily induce moral hazard that options buyers tend to take more risks as outsiders cannot fully force the buyers to stick to the stipulated rules/policy ladder, as discussed in **?** and **?**. Solutions could be guaranteeing standardized portfolio or taxing excess return or subsidize the shortfalls of non-standardized portfolios as proposed by **?**. In our case case, this problem can be mitigated if such options are to be traded among different generations within one parent pension plan. In that case, managers execute the policy ladders centrally, and the incentive to deviate from the rules is minimal. In addition, to avoid adverse selection that a fund might time the option trading which leads to the collapse of the risk sharing mechanism, we enforce the only time to trade the options is at the establishment of a fund when the details of its deal are revealed.

The valuation of the options traded among generational funds is done under the complete market assumption, allowing us to use risk-neutral techniques. However, the replication of the payoffs is hard to implement in practice. Hence our current estimation only provides the first proximate of their market values. The future research will be in the direction of the valuation of such options under an incomplete market.

Table 1: Parameter estimates

	Real world	Risk neutral world	σ
Real rate	$\bar{r}$=1.27%	$\bar{r}$=1.27%	1.50 %
	ϕ=0.56	ϕ=0.56	
	r_T=1.43%(initial value for simulation)		
Expected inflation	$\bar{\pi}$=4.16%	$\bar{\pi}$=4.16%	1.83%
	κ=0.78	κ=0.78	
	π_T=4.12%(initial value for simulation)		
Stock return	10.81%	5.43%	15.06%

$\bar{r}$ and $\bar{\pi}$ are long term means, ϕ and κ are mean reversion coefficients of real rate and expected inflation.

Table 2: Assumptions for baseline scenarios

Parameters	Values
Age of labor entry	25
Retirement age	65
Death age	80
Starting annual salary	€30,000
Annual salary growth rate	0
Replacement rate	70%
Investment policy	constant 50% in stocks and 50% in bonds
Risk free rate	5.43%
Base contribution rate	18.38%

Table 3: Value of the guarantees and call across different scenarios

Gua. type	Guarantee	Call	Ann. B.	% of real B.
Panel A				
	Base case with a deterministic labor income			
Nominal guarantee	4.17	49.31	65,510	83.45
Accursed rights gua.	12.11	49.31	68,219	85.84
Real guarantee	59.34	47.31	83,652	100
	Base case with a stochastic labor income			
Nominal guarantee	4.17	49.39	65,517	83.45
Accursed rights gua.	12.10	49.39	68,225	85.84
Real guarantee	59.32	47.32	83,652	100
Panel B				
	Sensitivity to investment risk			
Nominal guarantee	5.98	60.70	63,709	81.32
	(43.47%)	(23.09%)		
Accursed rights gua.	15.99	60.70	67,028	84.35
	(32.11%)	(23.09%)		
Real guarantee	67.69	57.99	83,652	100
	(14.08%)	(22.58%)		
	Sensitivity to longevity risk			
Nominal guarantee	5.42	49.64	63,981	81.69
	(29.95%)	(0.66%)		
Accursed rights gua.	17.65	49.64	67,170	84.4
	(37.55%)	(0.66%)		
Real guarantee	66.92	44.49	84,423	100
	(12.78%)	(-5.96%)		

The table reports the prices of the nominal benefit, accrued rights, and real benefit guarantees and the surplus call option by simulating a generational conditional DB fund. Panel A reports the prices for the base cases respectively with a deterministic and a stochastic labor income. The assumptions for the base case are specified in Table **??** and Figure **??**. Panel B reports the prices when the uncertainties in the volatility of the stock market and the life expectancy are considered. Specifically, in considering the investment risk we increase the volatility of the stock market to the highest 20-year volatility in the sample period. In considering the longevity risk, we make the life expectancy a random variable with a nominal distribution that has a mean of 80 years and a standard deviation of 2 years. "Guarantee" and "Call" are the one-time price of the benefit guarantee and the call respectively at the time of first contrition collection, expressed as as a percentage of the average annual salary. "Ann. B" displays the average annual benefits expressed in €that participants receive since their retirements, and its average percentage of the fully indexed benefits/real benefits is shown in the last column. Numbers in brackets are the change in value when compared with the base case with a deterministic labor income

Table 4: Break-even base contribution rate

	Contribution rate	Value of guarantee	Value of call
Nominal guarantee	10.16%	10.87%	10.86%
Accrued rights gua.	13.09%	19.97%	19.98%
Real guarantee	19.28%	54.41%	54.43%

The table reports the break-even base contribution rate that equalizes the value the guarantee and the value of the call under DB deal with a nominal benefit, accrued rights, and real benefit guarantee respectively. The last two columns report the resulting values of the guarantee and the call, which should be the same.