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Evaluating UK Defined-Benefit Pension Policies from the Dutch Perspective

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To Tingting and Tanya

Abstract

This thesis evaluates the UK defined benefit (DB) pension policies from the Dutch perspective using the holistic balance sheet (HBS) approach. The UK DB pension system differs from the Dutch one in terms of the steering tools and adjustment mechanisms. In addition to the sponsor guarantee, the UK system has the protection from the Pension Protection Fund (PPF) that guarantees DB pension schemes' funding shortfalls if the sponsors of the schemes are insolvent. The thesis first derives an analytical formula to value the embedded option implied by the PPF guarantee. Then a one-period model is developed to demonstrate how to build an HBS with the PPF guarantee option. To include more real-life features, we further use a multi-period model called value-based ALM to value the embedded options implied by UK pension policies and build the HBS. The HBS allows us to have a holistic view of the real and contingent assets and liabilities of a pension scheme and evaluate the impact of introducing a new policy on the stakeholders of the pension scheme. Finally, we compare the results of a typical UK policy with the HBS of a typical Dutch policy. The comparison suggests the UK policy is better for participants but worse for the sponsor compared to the Dutch policy. The UK policy is more generous in indexation and participants do not have the burden to contribute to the funding recovery of the pension scheme. The PPF provides protection of the benefits up to a certain level if the sponsor is insolvent, thus participants in a scheme with a UK pension policy are exposed to limited downside risk. On the other hand, the sponsor of the pension scheme with the UK policy shoulders a heavier burden to contribute to the recovery of the pension funding shortfalls than that of the pension scheme with the Dutch policy. Although this thesis does not address whether the PPF itself will be sustainable given its current policies, we find in our multi-period model that the *ex post* value of the PPF guarantee option provided by the PPF to the pension scheme is considerably less than the *ex post* value of the levy option that PPF charges from the pension scheme.

Contents

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1 Introduction

1.1 Research background

The second pillar pension plans in Europe vary considerably in size, pension promises, finance method, etc. The regulation and supervision of the pension funds also differ greatly. This diversity makes it difficult to compare plan designs and regulation burdens across countries fairly. To harmonise the framework of quantitative requirement for European pension funds, the European Insurance and Occupational Pension Authority (EIOPA), the European pension regulator, introduced a "holistic balance sheet" approach.

Unlike the traditional balance sheet (TBS) that states only the real assets and liabilities, holistic balance sheet (HBS) also takes into account contingent assets and liabilities implied by pension policies. These contingent assets and liabilities are called embedded options, as their values depend on the market conditions, like derivatives. Therefore, the derivative asset pricing approach can be employed to value the embedded options. The HBS presents the values of both real and contingent assets and liabilities, providing a "holistic" view of the pension fund status.

The development of the quantitative framework using the HBS approach is still in its early phase. EIOPA (2013) recently published the preliminary results of the Quantitative Impact Study (QIS), which conducted the HBS studies for the defined-benefit (DB) pension funds across eight European countries. The results shed some light on the impact of different pension policies across the countries. But there is also criticism of the methodology the study employed. However, the intention of this research is not to judge the methodology employed in the QIS, but to study how to build a HBS for some certain pension policies and examine their impact on the stakeholders.

The HBS approach is based on the literature on framing pension funds in terms of embedded options. An embedded option is an option or guarantee contained in a financial product. For example, the surplus sharing option may allow participants to enjoy extra benefits when there is a funding surplus resulting from better investment returns. Since the classic paper of Sharpe (1976), the contingent claim analysis has long been applied to real-life problem in the field of pension and insurance (Blake, 1998; Kocken, 2006; Kortleve et al., 2006).

Janssen (2012) implemented the HBS for the policies available to the Dutch DB pension funds. These policies include conditional indexation, recovery premium, sustainability cut, recovery plan, surplus sharing and employer guarantee. The embedded options is valued through a tool called value-based ALM model. The HBSs of pension funds are compared by adding a policy instrument once at a time. The study validated the HBS as a useful tool for the policy-makers to evaluate different pension deals.

As mentioned, pension policies vary across countries. This warrants studying specific features of the pension fund in a new country and implementing the HBS for the relevant policies. This thesis stresses this issue and tries to evaluate UK pension policies from a Dutch perspective using the HBS approach[1].

[1] The pension policies in this thesis refer to the policies available to occupational defined-benefit pension schemes only.

1.2 Features of UK pension policies

According to EIOPA (2013), UK has the largest private sector DB occupational pension scheme market in Europe with around £1 trillion of pension scheme assets. There are around 11.7 million members in total, including 2.1 million active members, in the DB pension schemes.

Pension Protection Fund One special feature of the DB pension schemes in UK is that almost all schemes in this group are eligible for the UK Pension Protection Fund (PPF) protection. The PPF was established in 2005 to protect members of private sector DB pension schemes in the event of pension scheme underfunding when the sponsor(s) of the pension scheme become(s) insolvent. The PPF guarantees full amount of the pensions in payment and 90% of the deferred pensions[2]. The total amount is also subjected to a cap set by the PPF annually. Once the sponsor(s) declares insolvency, the trustees of the pension scheme start to apply for the PPF protection and trigger an assessment period. During the assessment period, the PPF acts as a creditor to the insolvent sponsor of the pension scheme, and tries to retrieve some assets to mitigate the pension deficit. Meanwhile, the pension scheme in concern tries to quote life insurance contracts that pay each member at least their accrued PPF guaranteed benefits. If the remaining assets from the pension scheme can afford such contracts, the life insurance will be bought for each participant and the PPF stops to be involved. Otherwise, both the assets and the liabilities of the pension scheme and its participants will be transferred to the PPF. From then on, the PPF will pay the guaranteed amount of accrued benefits to participants when they retire.

The PPF charges an annual premium (called the levy) from each eligible pension scheme. The size of this premium depends on the size of the scheme and the level of risk in the scheme, including the pension funding shortfall, the credit risk of the sponsor, and the investment strategy of the pension scheme.

The premiums, the assets from the taken-over schemes, the assets retrieved from the insolvent sponsors, and together with investment returns form the assets of the PPF. Recent turbulent financial markets and high-profile insolvency events have put the PPF under the spotlight. Large pension scheme underfunding and the high number of insolvent sponsors increase the liabilities of the PPF. Low investment returns worsen the situation and the PPF has to increase the premiums it charges eligible schemes to achieve its long term self-sufficient funding target (PPF, 2012a). This thesis addresses the issue of how the PPF policy impacts an individual pension fund, thus we leave out the discussion of the viability of this pension guarantee mechanism.

Indexation Another particular feature of the UK pension system is that benefits must receive statutory pension increases in payment. The indexation to increase benefits each year used to link to the retail price index (RPI) and switched to the consumer price index (CPI) in recent years. The indexation is capped at 5% for the pensions in payment and 2.5% for the deferred pensions. A number of pension schemes also provide guaranteed pension increases in addition to the statutory requirements. It should be noted that indexation for the benefits paid out from the PPF is capped at 2.5%.

[2] The pensions in payment mean benefits paid to the pensioners and the deferred pensions refer to the accrued benefits that will start to be paid to participants when they retire.

Recovery plan UK DB schemes are subject to the Pension Act 2004, stating that "every scheme is subject to the statutory funding objective" to "have sufficient and appropriate assets to cover its 'technical provisions' ". The technical provision is the assets that are required to make provision for the pensions in payment and deferred pensions. If there is any funding shortfall at the effective date of each actuarial valuation, a so-called recovery plan must be prepared and agreed between the trustees and the sponsor(s) of the pension scheme. The recovery plan aims for any shortfall to be eliminated as quickly as the sponsor can reasonably afford. It consists of streams of annual cash flows that can span over many years. A typical recovery plan in UK lasts between 5 and 10 years with a median of 8 years. Some pension schemes have a recovery plan length of more than 17 years and some have a recovery plan length of shorter than one year.

As one can see, the only steering instrument available to the pension trustees in UK is the recovery contribution from the sponsor(s). The PPF does provide securities to participants of the pension scheme, but only after the scheme is liquidated. These policies have very different characteristics from the instruments available to the pension funds in the Netherlands.

1.3 Features of Dutch pension policies

The private sector DB occupational pension system in the Netherlands is ranked highly in the world. The system consists of around 80 industry pension funds, 300 pension schemes of individual companies, and 12 pension funds for certain professionals like medical doctors. Instead of having "sufficient and appropriate assets to cover" the "technical provisions", Dutch pension regulator requires pension schemes to have an asset buffer so that the probability of underfunding in the next period will be smaller than 2.5%. Several steering tools and adjustment mechanisms are available to the trustees of the pension scheme so that they can maintain the funding ratio at a healthy level. The typical Dutch policy analysed in this thesis includes the following features.

Conditional indexation Dutch pension schemes give indexations that are linked to the wage growth. Instead of full indexation, the actual indexation given depends on the funding position of the pension scheme. The trustees of the scheme set up a floor and a ceiling for the funding ratio, such that full indexation is given if the funding ratio is above the ceiling and no indexation is given if the funding ratio is below the floor. If the funding ratio is between the floor and the ceiling, the indexation is a proportion of the full indexation. The proportion equals the ratio of the difference of the actual funding ratio and the floor to the difference of the ceiling and the floor.

Sponsor support Like its counterpart in UK, the Dutch policy also requires sponsors of the pension scheme to contribute to the recovery of funding shortfalls. How much the sponsor contributes depends on the actual funding ratio and the target funding ratio set out in the pension contracts for the sponsor to support. The sponsor pays the pension scheme such an amount that the actual funding ratio can reach the target level, given the sponsor can afford this amount.

Sustainability cut An important difference of the Dutch policy from the UK policy is that participants also have the obligation to contribute to the recovery of the pension scheme's funding position. If the funding ratio is below the floor level required by the regulator, participants may have to bear benefit cuts so that the funding position can recover. To avoid unnecessary big losses, the recovery plan is set up for multiple years. Each year, there is a milestone for the funding ratio to recover to. If a milestone is missed in a year, participants will receive a negative indexation of their accrued benefits, so that the funding ratio can reach the milestone level. In this way, the funding ratio should reach the floor level at the end of the recovery plan.

All the three policies mentioned above depends on which part of the 'ladder' the actual funding ratio is. There are also other steering tools a Dutch pension scheme may apply, namely surplus sharing, catch-up indexation, and employee contribution. One can see that the trustees of a Dutch pension scheme have more flexibility than their counterparts of a UK scheme in terms of the selection of steering tools and adjustment mechanisms. We summarise the main features of the occupational DB pension schemes for both countries in Table 1.

UK	**Netherlands**
Full with cap	Conditional
Sponsor guarantee	Sponsor support
PPF guarantee	Sustainability cut
	Employee contribution
	Catch-up indexation
	Surplus sharing

Table 1: Features of occupational DB pension schemes in UK and the Netherlands.

1.4 Thesis structure

This thesis aims to value the embedded options in the UK policies, and uses the HBS approach to examine the impact if the Dutch pension system switches to adopt the UK policies. Therefore, we will use the Dutch demographic data, mortality and morbidity rates, and the way how pension liabilities are calculated[3].

Traditional analysis on pension fund focuses on the solvency of the fund itself. The credit risk of the employer is not taken into account when evaluating policy instruments that requires additional contributions from the employer. There are situations that the sponsor cannot afford to contribute the amount required for the recovery of the pension fund or even the solvency of the sponsor itself is under threat. Valuation of both the recovery plan and the PPF protection needs to take the credit risk into account, for this will affect the schedule of recovery plan, the annual levy a pension fund pays to the PPF and the insolvency event that triggers the takeover of the pension scheme by the PPF.

In Section 2, we introduce an analytical formula to value the PPF guarantee option in a Black-Scholes model. Then, in Section 3, we set up a stylised HBS based on a one-period framework. From a traditional balance sheet that contains assets and liabilities only, we add in the value of the recovery plan option and the PPF guarantee option to study the behaviour of the HBS. After these introductions, we extend our analysis to a multi-period model that closely represents real-life features of a Dutch pension fund. Section 4 introduces the approach, pension fund characteristics, the core model, and valuation method of this multi-period model. In section 5, we study the impact of the UK policies in the multi-period model and try to draw

[3]Dutch pension schemes switched to the market-consistent method and use a term-structure of risk-free rate to calculate the present value of pension liabilities. UK pension schemes use a much higher discount rate (∼6%). However the PPF adopts a rate that is based on market rates (details see the annual report of the PPF (2012b)).

some conclusions of what these policies mean to the stakeholders. Section 6 presents the results of the HBS for typical Dutch pension policies, after which we will compare the HBS results of UK and Dutch policies in Section 7. We discuss in Section 8 issues that are not covered in previous sections and finally draw our conclusions in Section 9.

2 Valuing the PPF guarantee option in a Black-Scholes model

In this section, we try to derive an analytical formula to calculate the value of the guarantee the PPF provides to a pension scheme. The size of the payoff depends on the values of the assets and liabilities the pension scheme has when the PPF takes over. In addition, the payoff is triggered by the insolvency event of the sponsor(s) only. If we perceive the PPF guarantee as a contingent claim, whose value depends on the values of the pension scheme's assets and liabilities and the status of the sponsor, we can use the asset pricing theories to value the PPF guarantee. Before we start to derive the analytical formula, we need to make some necessary assumptions.

2.1 Assumptions

Pension liabilities A pension scheme receives contributions and invests its assets to generate returns and hopefully the total assets will be enough to cover the benefits entitled to the scheme's participants when participants retire. In this section, we assume a closed pension scheme that does not receive contributions from its members or allow members to accrue new benefits. In addition, we view the pension scheme from the prospective of a representative participant who retires at time T. The participant will receive a single payment of L_T at T. L_T can be regarded as the present value (at T) of the total pension payments the participant receives each year after retirement, i.e.

$$L_T = \sum_{j=T}^{\infty} p_j e^{-r(j-T)} b_j, \tag{1}$$

where p_j is the survival probability of the beneficiary upto the year j, r represents the continuously compounded interest rate, and b_j the benefit paid at year j.

Pension assets Before T, the pension assets are invested in the financial markets in a self-financing way, i.e. no cash inflow or outflow between the asset portfolio and external. We assume that the pension asset portfolio consists of bonds and stocks only. The bond B is a risk-free asset and it has the dynamics

$$B_t = B_0 e^{rt} \Leftrightarrow dB_t = B_t r dt. \tag{2}$$

where r is the continuous annual risk-free rate. The equation to the left of the arrow in (2) is the solution of the differential equation to the right of the arrow.

The stock A is a risky asset. We assume that the price process of the stock follows a geometric Brownian motion under the risk-neutral measure $\mathbb{Q}$. Thus we can write the dynamics of the stock value under $\mathbb{Q}$ measure as

$$dA_t = A_t(rdt + \sigma_A dW_{1t}^{\mathbb{Q}}), \tag{3}$$

where σ_A represents the volatility of the instant return of the stock, $W_{1t}^{\mathbb{Q}}$ represents a Brownian motion under $\mathbb{Q}$ measure. By Ito's Lemma we can derive the solution to the stochastic differential equation as

$$A_t = A_0 \cdot exp\{(r - \frac{1}{2}\sigma_A^2)t + \sigma_A W_{1t}^{\mathbb{Q}}\}. \tag{4}$$

If the pension scheme invests a proportion θ ($\theta \in [0,1]$) of its asset portfolio in stocks and $1-\theta$ in bonds, we can write the price process of the asset portfolio P by a combination of the price processes of the bond and the stock as:

$$P_t = (1-\theta)B_t + \theta A_t. \tag{5}$$

Replacing the prices of the bond and the stock in (5) by (2) and (4), we obtain the dynamics of the price process of the pension asset portfolio under $\mathbb{Q}$ measure as:

$$P_t = P_0 \cdot exp\{(r - \frac{1}{2}\theta^2\sigma_A^2)t + \theta\sigma_A W_{1t}^{\mathbb{Q}}\} \Leftrightarrow dP_t = P_t(rdt + \theta\sigma_A dW_{1t}^{\mathbb{Q}}). \tag{6}$$

If $\theta = 0$ the asset portfolio consists of risk-free bonds only and $\theta = 1$ the stocks only.

PPF-guaranteed liabilities Although the PPF provides protection to participants when the sponsor is insolvent and there is a pension funding shortfall, the guaranteed liabilities are only part of the benefits that participants are entitled to. 100% of pensions in payment and 90% of deferred pensions will be guaranteed by the PPF. In addition, the PPF sets a cap to the entitled benefits before the percentage is taken. We can express the amount of liabilities that the PPF guarantees as

$$\eta L_T = 100\% \cdot min\{L_T^{pip}, C\} + 90\% \cdot min\{L_T^{dp}, C\}, \, \eta \in [0,1] \tag{7}$$

where L_T^{pip} is the pensions in payment, L_T^{dp} denotes the deferred pensions, and C represents the cap. Hence the total amount of liabilities PPF guarantees is a proportion of the original benefits that participants are entitled to. To simplify the analytical formula, we rewrite the PPF-guaranteed liabilities as ηL_T, where $\eta \in [0,1]$. This exempts us from considering the demographic composition of the pension scheme or modelling how the PPF would set up the cap. One can adjust the value of η to approximate the actual data.

The Merton model We model the insolvency event based on the Merton model of credit risk. The company's capital structure consists of debt and equity. The model assumes that the company does not pay coupons on its debt or dividends on its equity. The Merton model treats a company's equity as a European call option on its assets with the strike price of the face value of the company's debt and the maturity of the time the debt is due. If the total asset at the maturity date has a value less than the debt, the option is out of the money - equity holders does not get anything from the company. We define the insolvency event in our model as the total value of the sponsor's assets less than the total value of the sponsor's debts. This allows us to calculate the insolvent probability and simulate insolvency events.

Sponsor's debts Although there are pension schemes with multiple sponsors, we consider the pension scheme with one sponsor only. The sponsor's capital structure consists of debts and equities. We map the sponsor's debts to one zero-coupon bond that matures at time T. This zero-coupon bond pays a single cash flow of D_T when it matures. If the sponsor's assets S at time T are not enough to cover the zero-coupon bond payment, the sponsor becomes insolvent. We can write the insolvent condition as

$$S_T < D_T.$$

Sponsor's assets How much assets the sponsor will have at time T is uncertain. We need to model the evolution of the sponsor's assets. Like stocks, the sponsor company assets S are assumed to follow a geometric Brownian motion under the risk-neutral measure $\mathbb{Q}$. In practice, the sponsor's asset values are observed to be correlated with the value of stocks. To reflect this in our analytical solution, we assume that there is a correlation coefficient of ρ between the instant return of the sponsor's asset and that of the stock. A common practice to write the dynamics of a geometric Brownian motion process correlated with another is to disintegrate the Brownian motion into the Brownian motion driving the correlated process and another Brownian motion independent with the first one. Following this, we express the stochastic differential equation of the sponsor's asset under the risk-neutral measure $\mathbb{Q}$ as

$$dS_t = S_t(rdt + \rho\sigma_S dW_{1t}^{\mathbb{Q}} + \sqrt{1-\rho^2} \cdot \sigma_S dW_{2t}^{\mathbb{Q}}), \tag{8}$$

where σ_S is the volatility of the instant return of the sponsor's assets, $W_{1t}^{\mathbb{Q}}$ is the same Brownian process that drives the stock price process in (3), and $W_{2t}^{\mathbb{Q}}$ is a Brownian process under the risk-neutral measure $\mathbb{Q}$ and is independent of $W_{1t}^{\mathbb{Q}}$. This stochastic differential equation has the solution

$$S_t = S_0 \cdot exp\{(r - \frac{1}{2}\sigma_S^2)t + \rho\sigma_S W_{1t}^{\mathbb{Q}} + \sqrt{1-\rho^2} \cdot \sigma_S W_{2t}^{\mathbb{Q}}\}, \tag{9}$$

where S_0 is the initial value of the sponsor assets.

To aid the derivation in the following steps, we introduce a third geometric Brownian process $W_{3t}^{\mathbb{Q}}$ under the $\mathbb{Q}$ measure, such that

$$dW_{3t}^{\mathbb{Q}} = \rho dW_{1t}^{\mathbb{Q}} + \sqrt{1-\rho^2} \cdot dW_{2t}^{\mathbb{Q}}, \tag{10}$$
$$W_{3t}^{\mathbb{Q}} = \rho W_{1t}^{\mathbb{Q}} + \sqrt{1-\rho^2} \cdot W_{2t}^{\mathbb{Q}}. \tag{11}$$

We can show that the correlation between $W_{1t}^{\mathbb{Q}}$ and $W_{3t}^{\mathbb{Q}}$ is ρ. This enables us to rewrite the stochastic differential equation of the company asset (8) and its solution (9) as

$$dS_t = S_t(rdt + \sigma_S dW_{3t}^{\mathbb{Q}}), \tag{12}$$
$$S_t = S_0 \cdot exp\{(r - \tfrac{1}{2}\sigma_S^2)t + \sigma_S W_{3t}^{\mathbb{Q}}\}. \tag{13}$$

2.2 The analytical formula

Based on the above assumptions, we now start to derive the analytical formula to value the PPF guarantee option. Since we assume that all payments such as the benefits and the debts occur in the same time at T, the valuation formula is greatly simplified.

The payoff of the PPF guarantee is the shortfall of the pension assets and the PPF-guaranteed liabilities, i.e. $\eta L_T - P_T$. If there is no shortfall the payoff will be zero. Taking both situations into account, we can write the payoff as $max\{\eta L_T - P_T, 0\}$.

However, the above payoff only occurs when the sponsor becomes insolvent at time T. We mapped the sponsor's debts to a zero coupon bond with maturity at T and defined the insolvency as the sponsor's assets not enough to cover this zero-coupon bond's payment. By adding an indicator function of the insolvency event, we can express the PPF guarantee payoff as

$$\mathbf{1}_{\{S_T < D_T\}} \cdot max\{\eta L_T - P_T, 0\} \tag{14}$$

where $\mathbf{1}_{\{A\}}$ is the indicator function that equals 1 if event A happens and 0 otherwise.

The fundamental theory of asset pricing states that if there is a risk-neutral measure $\mathbb{Q}$, the present value of a derivative can be obtained by taking the expectation of the payoff discounted to present time under the risk-neutral measure. Therefore, we can write the present value of the PPF guarantee option as

$$\mathbb{E}^{\mathbb{Q}}[e^{-rT} \cdot \mathbf{1}_{\{S_T \leq D_T\}} \cdot max\{\eta L_T - P_T, 0\}] \tag{15}$$

The $max\{\eta L_T - P_T, 0\}$ function in the above equation is equivalent to the product of the indicator function $\mathbf{1}_{\{P_T < \eta L_T\}}$ and the payoff $\eta L_T - P_T$. Therefore (15) can be rewritten as

$$\begin{aligned} & \mathbb{E}^{\mathbb{Q}}[e^{-rT} \cdot \mathbf{1}_{\{S_T \leq D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}} \cdot (\eta L_T - P_T)] \\ = \;& e^{-rT} \eta L_T \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T \leq D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}}] - e^{-rT} \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T \leq D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}} \cdot P_T] \\ \equiv \;& V_1 - V_2 \end{aligned} \tag{16}$$

The first equation comes from the linearity of expectation and e^{-rT} and ηL_T both being deterministic. Now the problem remains to solve V_1 and V_2.

The expectation of an event indicator function simply equals the probability that the event happens, i.e. $\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{A\}}] = \mathbb{Q}(A)$. Since we have assumed that both the sponsor's assets and the pension scheme's assets follow the geometric Brownian motion under the risk-neutral measure $\mathbb{Q}$, we can calculate V_1 as

$$\begin{aligned} V_1 & \equiv e^{-rT} \eta L_T \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T < D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}}] \\ & = e^{-rT} \eta L_T \cdot \mathbb{Q}(S_T < D_T, P_T < \eta L_T) \end{aligned} \tag{17}$$

From (4) and (13), we know

$$\begin{aligned} & S_T < D_T \\ \Leftrightarrow \;& S_0 \cdot exp\{(r - \frac{1}{2}\sigma_S^2)T + \sigma_S W_{3T}^{\mathbb{Q}}\} < D_T \\ \Leftrightarrow \;& \frac{W_{3T}^{\mathbb{Q}}}{\sqrt{T}} < \frac{1}{\sigma_S \sqrt{T}}(ln(\frac{D_T}{S_0}) - (r - \frac{1}{2}\sigma_S^2)T) \end{aligned} \tag{18}$$

and

$$\begin{aligned} & P_T < \eta L_T \\ \Leftrightarrow \;& P_0 \cdot exp\{(r - \frac{1}{2}\theta^2 \sigma_A^2)T + \theta \sigma_A W_{1T}^{\mathbb{Q}}\} < \eta L_T \\ \Leftrightarrow \;& \frac{W_{1T}^{\mathbb{Q}}}{\sqrt{T}} < \frac{1}{\theta \sigma_A \sqrt{T}}(ln(\frac{\eta L_T}{P_0}) - (r - \frac{1}{2}\theta^2 \sigma_A^2)T) \end{aligned} \tag{19}$$

By definition, $W_{3T}^{\mathbb{Q}}/\sqrt{T}$ and $W_{1T}^{\mathbb{Q}}/\sqrt{T}$ follow the standard normal distribution. Since we assume that they have the correlation coefficient of ρ, the joint distribution of $W_{3T}^{\mathbb{Q}}/\sqrt{T}$ and $W_{1T}^{\mathbb{Q}}/\sqrt{T}$ is a standard bivariate normal distribution with the correlation coefficient ρ. If we denote

$$\begin{aligned} d_1 & = \frac{1}{\sigma_S \sqrt{T}}(ln(\frac{D_T}{S_0}) - (r - \frac{1}{2}\sigma_S^2)T), \\ d_2 & = \frac{1}{\theta \sigma_A \sqrt{T}}(ln(\frac{\eta L_T}{P_0}) - (r - \frac{1}{2}\theta^2 \sigma_A^2)T), \end{aligned}$$

then (17) is equal to

$$\begin{aligned} V_1 &= e^{-rT}\eta L_T \cdot \mathbb{Q}(\frac{W_{3T}^{\mathbb{Q}}}{\sqrt{T}} < d_1, \frac{W_{1T}^{\mathbb{Q}}}{\sqrt{T}} < d_2) \\ &= e^{-rT}\eta L_T \cdot \mathbf{BVN}(d_1, d_2; \rho) \end{aligned} \quad (20)$$

where **BVN** represents a standard bivariate normal distribution.

The expression of V_2 in (16) contains the stochastic term P_T in the expectation, therefore we can not apply the technique as in the derivation of V_1 directly. One technique to aid the calculation of this type of expectation is by changing the *numéraire* from the risk-free bond to P_t, which also changes the probability measure. The expectation under the new measure will contain only the indicator functions, thus we can calculate the expectation in a similar way as we did with V_1. To change the measure, we use the Girsanov theorem. Define

$$\begin{aligned} \xi_t &= \frac{P_t}{P_0} \cdot \frac{1}{e^{rt}} \\ &= \frac{P_0 \cdot e^{(r-\frac{1}{2}\theta^2\sigma_A^2)t+\theta\sigma_A W_{1t}^{\mathbb{Q}}}}{P_0} \cdot \frac{1}{e^{rt}} \\ &= e^{-\frac{1}{2}\theta^2\sigma_A^2 t+\theta\sigma_A W_{1t}^{Q}}. \end{aligned}$$

It can be shown that ξ_t is strictly positive and that $\mathbb{E}^{\mathbb{Q}}[\xi_t] = 1$. A process with these properties is called a Radon-Nikodym derivative. A new measure $\mathbb{R}$ can be defined by $\xi_t = d\mathbb{R}/d\mathbb{Q}$. With this definition, one can prove that the following equation is satisfied

$$\mathbb{E}^{\mathbb{R}}[\mathbf{1}_{\{A\}}] = \mathbb{E}^{\mathbb{Q}}[\xi_t \mathbf{1}_{\{A\}}]. \quad (21)$$

Using the above argument, we calculate V_2 by changing the measure from $\mathbb{Q}$ to a new measure $\mathbb{R}$, and then proceed in the same manner as in (20):

$$\begin{aligned} V_2 &\equiv e^{-rT}\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T \le D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}} \cdot P_T] \\ &= \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T \le D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}} \cdot \frac{P_T}{P_0} \cdot e^{-rT}] \cdot P_0 \\ &= \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{S_T \le D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}} \cdot \xi_T] \cdot P_0 \\ &= \mathbb{E}^{\mathbb{R}}[\mathbf{1}_{\{S_T \le D_T\}} \cdot \mathbf{1}_{\{P_T < \eta L_T\}}] \cdot P_0 \\ &= \mathbb{R}(S_T \le D_T, P_T < \eta L_T) \cdot P_0. \end{aligned} \quad (22)$$

We define $W_{1t}^{\mathbb{R}}$ a Brownian process under $\mathbb{R}$ and $dW_{1t}^{\mathbb{R}} = dW_{1t}^{\mathbb{Q}} - \theta\sigma_A dt$. We can also derive two other Brownian processes $W_{2t}^{\mathbb{R}}$ and $W_{3t}^{\mathbb{R}}$ such that

$$dW_{2t}^{\mathbb{R}} = dW_{2t}^{\mathbb{Q}}, \quad (23)$$

$$dW_{3t}^{\mathbb{R}} = dW_{3t}^{\mathbb{Q}} - \rho\theta\sigma_A dt. \quad (24)$$

Note that $W_{2t}^{\mathbb{R}}$ is independent of $W_{1t}^{\mathbb{R}}$, and the correlation coefficient between $W_{1t}^{\mathbb{R}}$ and $W_{3t}^{\mathbb{R}}$ is ρ.

Hence the dynamics of the sponsor's assets and pension scheme's assets under the $\mathbb{R}$ measure become

$$dS_t = S_t[(r + \rho\theta\sigma_A\sigma_S)dt + \sigma_S dW_{3t}^{\mathbb{R}}], \quad (25)$$

$$dP_t = P_t[(r + \theta^2\sigma_A^2)dt + \theta\sigma_A dW_{1t}^{\mathbb{R}}], \quad (26)$$

with solutions

$$S_t = S_0 exp\{(r + \rho\theta\sigma_A\sigma_S - \frac{1}{2}\sigma_S^2)t + \sigma_S W_{3t}^{\mathbb{R}}\}, \quad (27)$$

$$P_t = P_0 exp\{(r + \theta^2\sigma_A^2 - \frac{1}{2}\theta^2\sigma_A^2)t + \theta\sigma_A W_{1t}^{\mathbb{R}}\}. \quad (28)$$

Plug (27) and (28) into (22), we can show that

$$\begin{aligned} V_2 &= \mathbb{R}(S_T \leq D_T, P_T < \eta L_T) \cdot P_0 \\ &= \mathbb{R}(\frac{W_{3T}^{\mathbb{R}}}{\sqrt{T}} \leq d_3, \frac{W_{1T}^{\mathbb{R}}}{\sqrt{T}} < d_4) \cdot P_0 \\ &= \mathbf{BVN}(d_3, d_4; \rho) \cdot P_0, \end{aligned} \quad (29)$$

where

$$d_3 = \frac{1}{\sigma_S\sqrt{T}}(ln\frac{D_T}{S_0} - (r + \rho\theta\sigma_A\sigma_S - \frac{1}{2}\sigma_S^2)T),$$

$$d_4 = \frac{1}{\theta\sigma_A\sqrt{T}}(ln\frac{\eta L_T}{P_0} - (r + \theta^2\sigma_A^2 - \frac{1}{2}\theta^2\sigma_A^2)T).$$

The last equation in (29) comes from the fact that the joint distribution of $W_{3T}^{\mathbb{R}}/\sqrt{T}$ and $W_{1T}^{\mathbb{R}}/\sqrt{T}$ is the standard bivariate normal distribution with a correlation coefficient of ρ under the measure $\mathbb{R}$.

If we can solve the cumulative bivariate normal distributions in (20) and (29), given the values of various parameters, we can calculate the value of the PPF guarantee option as

$$V_{PPF} = e^{-rT}\eta L_T \cdot \mathbf{BVN}(d_1, d_2; \rho) - P_0 \cdot \mathbf{BVN}(d_3, d_4; \rho). \quad (30)$$

Example Now we present an example using the valuation formula (30) to calculate the value of the PPF guarantee option for a simple one-period model. In the model, the pension scheme pays benefits to participants which are equal to 100 at year 1. The pension scheme's asset portfolio has a present value of 90 and 50% of the assets are invested in stocks and 50% in bonds. The instant stock return has the volatility of 0.15 under the risk neutral measure. The assets of the sponsor have a value of 100 at present. The debts of the sponsor are mapped to a zero coupon bond at year 1 with a value of 70. The instant return of the sponsor's assets has a volatility of 0.2 and it correlates with the instant return of stocks with a correlation coefficient of 0.5. If the sponsor defaults in year 1 and the PPF needs to take over the pension scheme, the percentage of the benefits guaranteed by PPF is 95%. In summary, the assumptions are:

$$T = 1, r = 0.02, \sigma_A = 0.15, L_T = 100, \eta = 95\%, P_0 = 90,$$
$$\theta = 0.5, \sigma_S = 0.2, S_0 = 100, D_T = 70, \rho = 0.5.$$

We use the `mvncdf( )` order in Matlab to solve the cumulative standard bivariate normal distribution in (30). The calculation returns the value of the PPF guarantee option as 0.3851 under the above assumptions.

Levy The PPF (2012c) provides a levy formula to calculate how much levy the PPF charges each pension scheme. The calculation is complicated and we explain the procedure briefly in Appendix A. Using the values of the variables in the previous example, we calculate the levy for the pension scheme in the above example according to the levy formula[4] and obtain

$$\begin{aligned} & Levy \\ = \;& 100e^{-0.02} \times 0.000056 + min\{(100e^{-0.02} - 90) \times 0.04 \times 0.73, 100e^{-0.02} \times 0.0075\} \\ = \;& 0.0005 + min\{0.2342, 0.7351\} \\ = \;& 0.2347. \end{aligned}$$

This value is 40% less than the value of the PPF guarantee option obtained by (30).

2.3 Sensitivity analysis

To gain an insight into how changes in different parameters affect the valuation of the PPF guarantee option, we conduct a sensitivity analysis in this section. We modify one of the variables in (30) and keep the rest unchanged. The relationships of the valuation and the variable under concern are presented in graphs, providing a clear view of how 'sensitive' the value of the PPF guarantee option is to the change in variables. We choose the example from the previous section as the starting point. The values of the variables are given here again for convenience.

$$T = 1, r = 0.02, \sigma_A = 0.15, L_T = 100, \eta = 95\%, P_0 = 90,$$
$$\theta = 0.5, \sigma_S = 0.2, S_0 = 100, D_T = 70, \rho = 0.5.$$

Figure 1 presents the results of the sensitivity analysis against the present value of the pension assets and the liabilities of the pension scheme due at year 1. The relationship is very intuitive: more pension assets or less pension liabilities reduces the probability of funding shortfall at year 1, thus reducing the amount the PPF needs to pay if it takes over the scheme, resulting in a lower value of the PPF guarantee option.

Figure 2 presents the results of the sensitivity analysis against the sponsor's debt level and the volatility of the instant sponsor's asset return. In our model, increasing either the debt level or the asset volatility results in a higher insolvent probability, hence a higher chance that the PPF needs to take over the pension scheme. Therefore we see the upward curves in Figure 2. When the debt level is below 60, the value of the PPF guarantee option is low and insensitive. As the debt level keeps increasing to above 60, the value of the PPF guarantee option increases dramatically. The relationship of the value of the option and the volatility of the asset return appears to be linear and gradual.

The size of the PPF guarantee depends on how large the funding shortfall of the pension scheme is. Both the pension asset portfolio composition and the volatility of the instant return of stocks impact how much assets the pension scheme will have and thus affecting the value of the PPF guarantee option. Figure 3 presents the sensitivity of the option value to these two

[4] The PPF categorises companies to ten risk bands according to the companies' failure scores rated by an external company (see Attachment A). Each band is assigned with an insolvency risk factor. The riskier the band, the higher the insolvency risk factor is. Due to the difference of the credit risk model used by the PPF and us, we do not know which risk band we should categorise the sponsor in our model to. Because of the high leverage ratio in our assumption (70%), we categorise the sponsor to the riskiest band with the highest insolvency risk factor of 0.04.

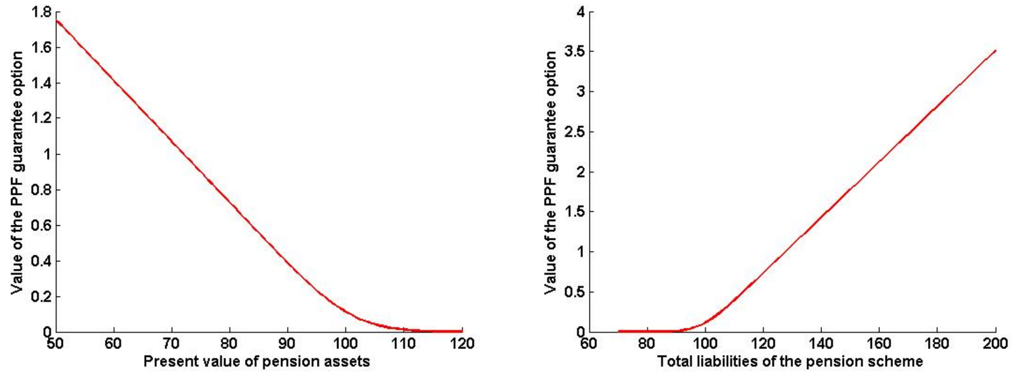

Figure 1: Sensitivity analysis of the value of the PPF guarantee option to the present value of pension assets (left) and the liabilities due at year 1 (right).

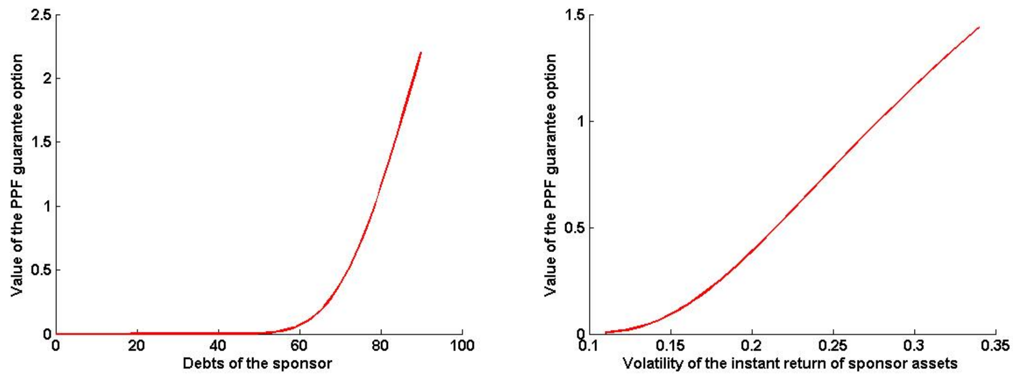

Figure 2: Sensitivity analysis of the value of the PPF guarantee option to the sponsor's debt level (left) and the volatility of the instant return of the sponsor's assets (right).

variables. An increase in the proportion of pension assets invested in stocks or a more volatile stock return means that the pension assets become riskier. Both the probability and the size of funding shortfall increases. Therefore we see the curves in Figure 3 sloping upwards.

Finally, we analyse how the correlation between the stock return and the sponsor asset return affects the value of the PPF guarantee option. The results are presented in Figure 4. The curve slopes upwards as a convex. If the correlation coefficient is negative, the stock return is likely to be high when the sponsor asset return is low, or vice versa. This reduces the occurrences of the PPF takeover as both sponsor insolvency and pension funding shortfall are the prerequisite conditions. Therefore the value of the PPF option is lower as the correlation coefficient is more negative. On the contrary, when the correlation coefficient increases, the chance that sponsor insolvency and pension funding shortfall occur simultaneously is high. The PPF is more likely required to take over the pension scheme, thus increasing the value of the PPF guarantee option.

2.4 Summary

In this section, we derived an analytical formula to calculate the value of the protection from the PPF. The PPF guarantees part of the funding shortfalls if the sponsor of the pension scheme becomes insolvent. The guarantee can be viewed as a contingent claim whose payoff

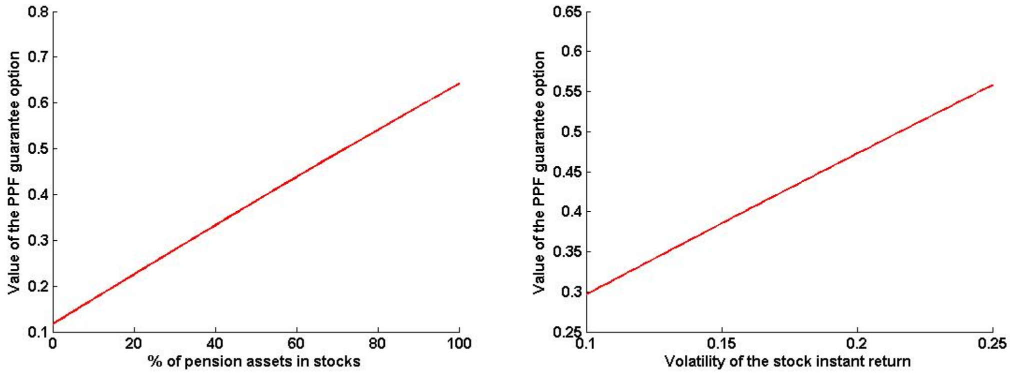

Figure 3: Sensitivity analysis of the value of the PPF guarantee option to the pension asset portfolio composition (left) and the volatility of the instant pension asset return (right).

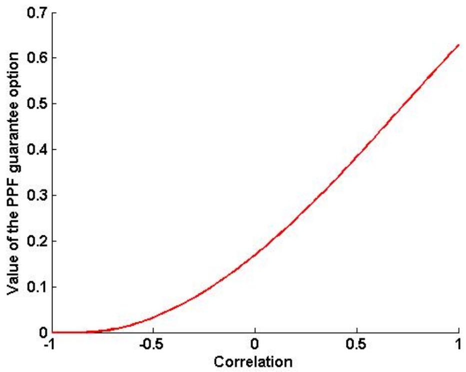

Figure 4: Sensitivity analysis of the value of the PPF guarantee option to the correlation coefficient between instant stock return and sponsor asset return.

depends on the development of pension assets and liabilities, and the sponsor's assets and debts. By making necessary assumptions of the dynamics of these variables and also the assumption of a complete market, we can value the PPF guarantee option using the asset pricing theory. Taking the expectation of the discounted payoff under the risk neutral measure returns the value of this option.

Several variables influence the value of the PPF guarantee option. As shown in the sensitivity analysis, variables that increase the insolvent probability or the size of the funding shortfall lead to a higher value of the option. A more positive correlation coefficient of the pension asset return and the sponsor asset return also increases the value. The directions of the change in the value of the PPF guarantee option in response to an increase in the value of one of the variables are summarised in Table 2.

With the PPF guarantee valuation formula, the trustees of a pension scheme can have an idea of the value of this contingent asset. To fully understand the impact of the PPF protection to the pension scheme, one needs to take into account all the stakeholders, such as the sponsor and participants. Participants' benefits will be cut by the PPF if the PPF takes over the scheme. The PPF is only the secondary guarantor if a sponsor contribution policy is also in place. The HBS approach is adapted to answer these issues, as the transactions between the

	$A_0 \uparrow$	$L_T \uparrow$	$D_T \uparrow$	$\sigma_S \uparrow$	$\alpha \uparrow$	$\sigma_A \uparrow$	$\rho \uparrow$
V_{PPF}	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 2: Direction of change in the value of the PPF guarantee option (V_{PPF}) when one of the variables increases in value. A_0 is the initial pension assets, L_T is the pension liabilities at T, D_T is the sponsor's debt level at T, σ_S and σ_A denote the volatility of the instant return of the pension assets and the sponsor's assets, respectively, α is the percentage of pension assets invested in stocks, and ρ is the correlation coefficient of the instant asset returns between the pension scheme and the sponsor.

stakeholders are valued as options and presented on the HBS as contingent assets and liabilities, thus providing a 'holistic' view of the pension policy. In the next section, we introduce how an HBS can be built for a simple one-period model.

3 Valuing the PPF guarantee option in a one-period model

In this section, we introduce the HBS for a simple one-period model. We construct examples with the PPF protection policy. Apart from the PPF protection, a pension scheme with funding shortfalls also receives contributions from the sponsor. The PPF is the secondary guarantor and guarantees the funding shortfalls only if the sponsor defaults. Therefore, we also value the sponsor guarantee in the HBS and demonstrate the interaction of the protections from the two sources.

In addition, the PPF protection impacts the HBS of a pension scheme from three aspects. It provides the protection if the sponsor defaults, charges the pension scheme annual premiums, and cuts the entitled benefits if it takes over the scheme. The premium charges and the benefit cuts depend on many factors and are dealt with later in a multi-period model. This section aims to explain the impact of the PPF guarantee on the HBS and thus we present the results with the PPF guarantee option only.

We first set up a simple baseline model without any asset risk or any protection contributions and later introduce more assumptions step by step.

3.1 The baseline model

The first example is a very simple model. This basic model, without any credit risk of the sponsor and asset risk of the pension scheme, provides a useful 'baseline' result. This allows us to examine how the increase in the complexity of the model effects on the valuation of the PPF guarantee option.

The following assumptions are made:

1. The pension scheme has a portfolio of assets worth 90 today;
2. The pension scheme's assets consist of risk-free bonds only;
3. The pension scheme's liability is a single cash flow of 100 that is fixed and is due at year 1;
4. the sponsor has no credit risk and it does not commit to cover the deficit of the pension scheme when the pension liability is due;

5. The 1-year risk-free interest rate is 2% (continuously compounded).

The present value of the pension liability 98.02 is obtained by discounting the liability 100 due at year 1 using the continuous annual risk-free interest rate 2%. Since there is no sponsor covenant and no credit risk of the sponsor, the residue is certain to be the difference of the present values of assets and liabilities, i.e. -8.02.

If we assume that the sponsor company of the pension scheme is not obliged to cover the pension deficit at year 1, we can view the residue as an option that participants write to the pension scheme. Participants bear the risk of pension underfunding and the option has a value of 8.02 to the pension scheme.

The HBS is presented in Table 3.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
		Residue	-8.02
Total	90.00	**Total**	90.00

Table 3: Baseline one-period model: no credit risk of the sponsor and no asset risk of the pension scheme.

If we introduce asset risk to the example, the status of the HBS at year 1 becomes uncertain. The residue level at year 1 varies according to the resulting asset level of the pension scheme. However, the HBS at the present time remains the same in Table 3, because present values of pension assets and liabilities are certain.

3.2 PPF guarantee without pension asset risk

In this example we introduce the additional assumptions

1. the sponsor of the pension defaults at year 1;
2. The PPF guarantees to pay the pension deficit;
3. There is no pension asset risk.

All the other assumptions remain the same as in the baseline example.

Since there is no pension asset risk, i.e. all the assets are invested in risk-free bonds, the pension assets will certainly grow to 91.8 At year 1. at year 1, the pension scheme will be in a deficit of $100 - 91.82 = 8.18$. The PPF guarantees to cover this funding shortfall when the sponsor becomes insolvent. The guarantee is like an option contract that the PPF writes to participants such that the PPF will cover any pension deficit. This PPF guarantee option has a present value of 8.02.

Note that this PPF guarantee option balances the residue of the HBS from the baseline example. The new HBS has a residue of 0, with pension assets and liabilities balanced.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
PPF guarantee	8.02		
		Residue	0
Total	98.02	**Total**	98.02

Table 4: Model with PPF guarantee: the sponsor insolvent at year 1 and no asset risk of the pension scheme.

3.3 Sponsor guarantee without credit risk

In the baseline example, we assumed that participants bear the risk of underfunding. The negative residue on the HBS was viewed as the value of the option that participants take a cut of their benefits in the case of underfunding. In reality, the sponsor company of the pension scheme may be obliged to cover this pension deficit.

In this model, we assume that the sponsor does not have credit risk, and is able to pay the pension scheme any deficit it incurs. This is like an option contract that the sponsor writes to participants such that any pension deficit will be guaranteed by the sponsor.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	8.02		
		Residue	0
Total	98.02	**Total**	98.02

Table 5: One-period model with the sponsor's guarantee: no credit risk of the sponsor and no asset risk of the pension scheme.

As shown in Table 5, the value of the sponsor guarantee option appears on the asset side on the HBS and is equal to the present value of the pension deficit (8.02). Therefore total assets and total liabilities are balanced. The risk of pension underfunding is shifted from participants in the baseline example to the sponsor.

3.4 Sponsor guarantee with credit risk

In this example, we introduce credit risk of the sponsor. To see the impact of credit risk on the value of the sponsor guarantee option, we assume that there is no PPF guarantee if the sponsor defaults. The assumptions that are used to model the sponsor insolvency event are:

1. The sponsor insolvency is triggered when the sponsor assets become less than the sponsor debts;

2. The total value of the sponsor's assets, denoted by S, has the dynamics of a geometric Brownian motion under the risk neutral measure. The sponsor's assets at year 0 have a value of 100. This value is not necessarily in absolute term but is only used to model the probability of the sponsor insolvency within this one period. The volatility of the sponsor assets return, σ_S, is assumed to be equal to 0.2;

3. The total value of the sponsor's debts, denoted by D, is a single cash flow of 70 that needs to be paid out at year 1;
4. The sponsor covers all pension deficits if the sponsor is still solvent at year 1;
5. The pension scheme ranks lower than all the other creditors of the sponsor and therefore the pension scheme cannot recover any assets from its defaulted sponsor.

Besides, the assumptions about pension assets and liabilities remain the same as in the baseline example.

Based on the above assumptions, the probability of the sponsor insolvency becomes the probability that the entire sponsor assets are less than the entire sponsor debts. If the total value of the sponsor assets follows a geometric Brownian motion under the risk neutral measure, then the value of the assets at time t is given by:

$$S_t = S_0 \cdot exp((r - \frac{1}{2}\sigma_S^2)t + \sigma_S W_t), \tag{31}$$

where W_t is the Brownian process under the risk neutral measure. Then, the insolvent probability at year 1 can be calculated as

$$\begin{aligned} PD &= \mathbf{P}(S_1 < D_1) \\ &= \mathbf{P}(S_0 \cdot e^{r - \frac{1}{2}\sigma^2 + \sigma W_1} < D_1) \\ &= \mathbf{P}(W_1 < \frac{1}{\sigma}(ln(\frac{D_1}{S_0}) - r + \frac{1}{2}\sigma^2)) \\ &= 0.0373, \end{aligned}$$

where PD stands for the probability of default, subscripts of S and D indicate the year, r is the continuous compounded interest rate, σ_S denotes the volatility of the sponsor asset return, and W_1 is the standard Brownian motion that drives the dynamics of the value of the pension asset.

From previous examples we know that at year 1 the sponsor guarantee option has a value of 8.02 if the sponsor becomes insolvent and 0 otherwise.

Therefore the value of the sponsor guarantee option becomes:

$$PD \times 0 + (1 - PD) \times 8.02 = (1 - 0.0373) \times 8.02 = 7.72.$$

Note that introducing credit risk lowers the value of the sponsor guarantee option. This is intuitive as there are situations in which the sponsor defaults and cannot fulfil its obligation to cover the pension deficit. The lower value of the sponsor guarantee option results in a negative residue on the HBS. We can assume that participants bear the risk of underfunding if the sponsor defaults. As in the baseline example, we view the residue as the value of an option that participants write to the pension scheme to allow their benefits to be cut when the risk materialises. The resulting HBS is given in Table 6.

3.5 Sponsor and PPF guarantees with credit risk

In this example, we re-introduce the PPF guarantee, and see how this will impact the option values of the HBS. We assume that the solvent sponsor will cover the pension deficit and the PPF pays any pension deficit if the sponsor becomes insolvent at year 1. All the assumptions about the sponsor and pension scheme remain the same as in the previous example.

Assets		Liabilities	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	7.72		
		Residue	-0.30
Total	97.72	**Total**	97.72

Table 6: Model with the sponsor guarantee with credit risk.

The sponsor and the PPF guarantees work as if two option contracts are written to participants. One has the payoff of the pension deficit paid by the sponsor when the sponsor is still solvent; one has the payoff of the pension deficit paid by the PPF if the sponsor is insolvent. These two contracts guarantee that participants will receive their full amount of benefits no matter whether the sponsor is solvent or not.

From previous examples, we know the value of the PPF guarantee option given the sponsor is default at year 1 is 8.02 and the probability that the sponsor is default at year 1 is 0.0373. We can calculate the value of the PPF guarantee option as follows

$$PD \times 8.02 + (1 - PD) \times 0 = 0.0373 \times 8.02 = 0.30.$$

Adding this value to the HBS balances the asset side and liability side. This is not surprising, as the purpose of the PPF is to take over the risk of pension underfunding from participants when the sponsor defaults.

The results are presented in Table 7.

Assets		Liabilities	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	7.72		
PPF guarantee	0.30		
		Residue	0
Total	98.02	**Total**	98.02

Table 7: Model with the sponsor and PPF guarantee with credit risk.

3.6 Sponsor guarantee with pension asset risk and without credit risk

To see the impact of introducing pension asset risk, we assume in this example that there is no credit risk of the sponsor, i.e. the sponsor will guarantee any pension deficit. In addition, the total value of the pension assets follows the dynamics of a geometric Brownian motion under the risk-neutral measure. The volatility of the instant pension asset return is equal to 15%.

The total amount of pension assets at year 0 is 90 and is uncertain at year 1. This leads to an uncertain sponsor guarantee at year 1. If at year 1, the pension assets are equal to or above the pension liabilities of 100, the sponsor will not pay anything. If at year 1, the pension assets are less than the pension liabilities, the sponsor needs to cover the pension deficit.

The above sponsor guarantee payoff describes a put option on an asset with current price at 90 and strike price at 100 at year 1. Given the assumption, the value of the sponsor guarantee option can be obtained by the Black-Scholes formula:

$$
\begin{aligned}
P(S_t, t) &= N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t, \qquad (32)\\
\text{where } d_1 &= \frac{1}{\sigma\sqrt{T-t}}[ln(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})],\\
\text{and } d_2 &= \frac{1}{\sigma\sqrt{T-t}}[ln(\frac{S_t}{K}) + (r - \frac{\sigma^2}{2})].
\end{aligned}
$$

P is the price of the put option at time t, S_t is the price of the asset at time t, σ is the volatility of the instant return of the asset, K denotes the strike price of the option, T is the mature time, r is the risk-free rate, and $N()$ represents the standard normal distribution. In our example, the values of these parameters are

$$S = 90, K = 100, \sigma = 0.15, t = 0, T = 1, r = 0.02.$$

According to (32), the present value of the sponsor guarantee option is equal to 10.53, higher than the value (7.72) in the example from Section 3.4.

The results are presented in Table 8. Note we have a positive value of the residue option of 2.51. This is exactly the value of a call option on the pension assets with a strike price of 100 at year 1. We did not state to whom this positive value belongs. We can also view it as the call option written by the sponsor to the pension scheme such that the sponsor can retrieve this amount if the fund is in surplus to compensate its promise to guarantee the pension scheme deficit. If the sponsor cannot receive a surplus of the pension scheme, we can view the positive residue as the surplus sharing that participants can enjoy when the funding level is higher than 100%.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	10.53		
		Residue	2.51
Total	100.53	**Total**	100.53

Table 8: Model with the sponsor guarantee: asset risk exists in the pension scheme and the sponsor is solvent at year 1.

3.7 PPF guarantee with pension asset risk

In this example, we value the PPF guarantee option when there is pension asset risk. Again we assume that the sponsor will default at year 1 and PPF will guarantee any pension deficit. The assumptions about the pension assets are the same as in Section 3.6, i.e. the value of pension assets follows the dynamics of a geometric Brownian motion under the risk-neutral measure. The volatility of the pension assets return is equal to 15%.

The PPF guarantee option has exactly the same payoff as the sponsor guarantee option in Section 3.6. The analysis of the valuation problem remains the same. Thus the PPF guarantee option has a value of 10.53 as presented in Table 9.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
PPF guarantee	10.53		
		Residue	2.51
Total	100.53	**Total**	100.53

Table 9: Model with PPF guarantee: asset risk exists in the pension scheme and the sponsor is insolvent at year 1.

Figure 5 shows how the market-consistent value of the PPF guarantee varies as a function of the initial pension assets with or without asset risk. The same can be said about the value of the sponsor guarantee options in Section 3.6. The blue line in the graph is the value of the put option on the risky asset, and the red line shows the difference between pension assets and liabilities discounted to the present.

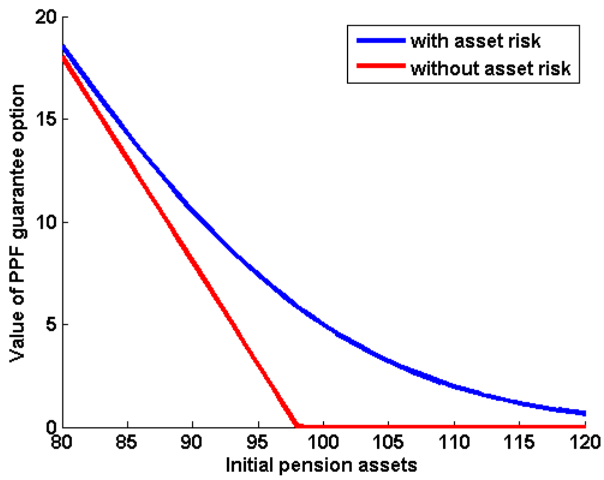

Figure 5: Impact of initial pension asset on PPF guarantee option value.

Introducing pension asset risk increases the value of the PPF guarantee option as there is a higher chance that the PPF needs to step in after the sponsor defaults to cover the funding shortfall. The value of the PPF guarantee option increases as the pension deficit increases.

An interesting result in Figure 5 is that introducing pension asset risk has the most significant impact on the PPF guarantee option when the pension scheme's asset is close to its liability. When the pension scheme is in large surplus or deficit, the impact of pension asset risk on the value of the PPF guarantee option diminishes. This is because of the formulas we use to calculate the price of the PPF guarantee option. Without asset risk, if the pension scheme is in surplus, the PPF guarantee option is certain to be 'out of the money' for the pension scheme at year 1 and thus has a value of 0. If there is asset risk, when the pension scheme is in large surplus, i.e. S is large, the ratio S/K becomes big and d_1 and d_2 in (32) will be large. $N(-d_2)$ and $N(-d_1)$ are then close to 0, thus the price of the PPF guarantee option $P(S,t)$ in (32) is close to 0. On the other hand, if the pension scheme is in large deficit, S/K is small and hence d_1 and d_2 are small. $N(-d_2)$ and $N(-d_1)$ will be close to 1. As a result, $P(S,t)$ in (32) is approximately equal to $Ke^{-r(T-t)} - S$, which is the formula to calculate the value of the PPF guarantee option without asset risk.

3.8 Default risk and no corrlation between asset returns

We have valued the Sponsor guarantee option given the sponsor is still solvent at year 1 and the PPF guarantee option given that the sponsor will be insolvent at year 1. Now we combine the two guarantees into one example and introduce both pension asset risk and sponsor credit risk.

The assumptions of the insolvency event and the sponsor assets and liabilities are the same as in Section 3.4. The assumptions about the pension assets and liabilities remain the same as in Section 3.6. We also assume that the correlation coefficient between the returns on pension assets and the sponsor assets is equal to 0.

From previous examples, we know that the probability of the sponsor becoming insolvent at year 1 is 0.0373, and the PPF guarantee option has a value of 10.53 given the sponsor defaults at year 1. Therefore the value of the PPF guarantee option becomes:

$$PD \times 10.53 + (1 - PD) \times 0 = 0.0373 \times 10.53 = 0.39.$$

Similarly, we can use the sponsor default probability and the value of the sponsor guarantee option given no default risk to calculate the value of the sponsor guarantee option

$$(1 - PD) \times 10.53 + PD \times 0 = (1 - 0.0373) \times 10.53 = 10.13.$$

Introducing credit risk lowers the value of the sponsor guarantee option, as there are situations that the sponsor cannot fulfil its responsibility to cover the pension deficit.

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	10.13		
PPF guarantee	0.39		
		Residue	2.50
Total	100.52	**Total**	100.52

Table 10: Model with the sponsor and PPF guarantee: both credit risk and asset risk exist in this model.

The results are presented in Table 10. The sponsor and PPF guarantees are option contracts written to cover any funding shortfalls. The sum of the guarantees would have the same value as when the guarantee is from the sponsor or PPF alone in the previous two examples. The difference in the numbers on the HBS are from the rounding errors.

3.9 Default risk with correlation between assets

In reality, low return of pension assets is correlated with low return of sponsor assets. This will affect the value of the PPF guarantee option in two ways, namely the size of the pension deficit and the default probability of the sponsor. In this example, we examine how the correlation between the pension and the sponsor asset returns impacts the option values on the HBS.

To obtain the values of the sponsor and the PPF guarantee options, we assume that the pension assets (A) and the sponsor assets (S) have the following dynamics under the risk neutral measure,

$$A_t = A_0 \cdot exp((r - \frac{1}{2}\sigma_A^2)t + \sigma_A W_{1t}) \quad (33)$$

$$S_t = S_0 \cdot exp((r - \frac{1}{2}\sigma_S^2)t + \sigma_S \rho W_{1t} + \sqrt{1-\rho^2}\sigma_S W_{2t}), \quad (34)$$

where r is the risk-free continuous interest rate, σ_A is the volatility of the pension asset return, σ_S is the volatility of the sponsor asset return, ρ is the correlation coefficient between pension and sponsor asset returns, and W_{1t} and W_{2t} are the Brownian processes under the risk-neutral measure. W_{1t} and W_{2t} are independent. In this way, the instant returns of the pension assets and the sponsor assets have the correlation coefficient of ρ and the volatility of the instant return of the sponsor assets is σ_S. In addition, we assume that the pension liabilities and the sponsor debts are mapped to fixed single cash flows, respectively, as in the example of Section 3.8.

We keep the values in equation (33) and (34) the same as in the previous examples, i.e. $r = 0.02$, $A_0 = 90$, $S_0 = 100$, $\sigma_A = 0.15$, $\sigma_S = 0.2$, and simulate 500,000 pairs of W_{11} and W_{21} (using the randn() Matlab function). Choosing a ρ value between -1 and 1, we can calculate A_1 and S_1, pension assets and the sponsor assets at year 1, respectively.

If the sponsor insolvency is triggered when the sponsor assets at year 1 become less than the sponsor debts, PPF will cover the pension deficit if there is any. This payoff can be written as

$$\mathbf{1}\{S_1 < D_1\} \cdot max\{L_1 - A_1, 0\}, \quad (35)$$

where $\mathbf{1}\{\}$ is the indicator function. Discounting the payoff of all 500,000 simulations to present and taking the average returns the value of the PPF guarantee option.

Similarly, the sponsor will cover any pension deficit given the sponsor is still solvent. The payoff of the sponsor guarantee can be written as

$$\mathbf{1}\{S_1 \geq D_1\} \cdot max\{L_1 - A_1, 0\}. \quad (36)$$

The sponsor guarantee option can then be valued the same way as the PPF guarantee option.

The value of the PPF guarantee option as a function of the correlation coefficient between pension asset return and sponsor asset return is presented in Figure 6. The high correlation suggests the sponsor insolvency risk increases when pension asset return is low. This means that the pension scheme is more likely to be in deficit and the average size of the deficit tends to be large when the sponsor becomes insolvent. Therefore, the curve in Figure 6 is a convex sloping upwards, i.e. the value of the PPF guarantee option increases rapidly as the correlation between pension and the sponsor asset returns becomes stronger.

Generally, the pension asset return and the sponsor asset return would be positively correlated. An assumption of 0.5 is typically used in models studying equity index and credit risk of a specific company. The value of the PPF guarantee option at a correlation coefficient of 0.5 is equal to 0.82, which is more than double the value under a correlation coefficient of zero. The value of the sponsor guarantee option is 9.7. The HBS with the correlation coefficient of 0.5 between asset returns of the pension scheme and the sponsor is presented in Table 11.

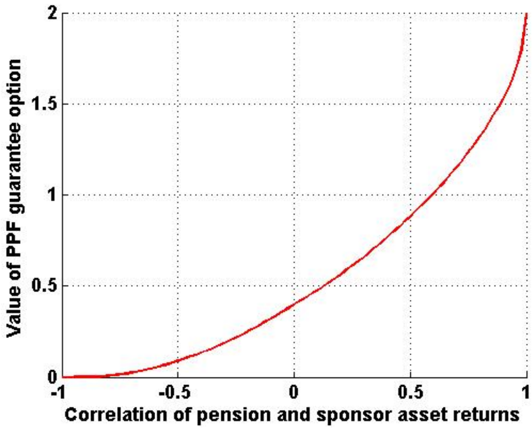

Figure 6: Value of the PPF guarantee option as a function of the correlation between the returns of pension assets and the sponsor assets

Assets		**Liabilities**	
Pension assets	90.00	Pension liabilities	98.02
Sponsor guarantee	9.70		
PPF guarantee	0.82		
		Residue	2.50
Total	100.52	**Total**	100.52

Table 11: HBS with both credit risk and asset risk. The correlation coefficient of asset returns of the pension scheme and the sponsor is equal to 0.5.

4 The multi-period model

4.1 Approach

In the previous sections, we developed a stylised one-period model to introduce the valuation of the PPF guarantee option. It gives insight into how the PPF guarantee option affects the HBS of a pension fund. We also derived a numerical solution to value the PPF guarantee option for the one-period example. In a complex model, when more real-life features are included and multiple time steps of cash flows are present, it is hard to find an accurate numerical solution.

Pension fund receives contributions and pays out benefits. The rate of the contributions and the indexation of the benefits depends on the pension contract that the pension scheme sets up with participants and the sponsor. There may be other policy features such as conditional indexation, recovery contributions and etc., which depend on the future economic development. Therefore the cash flows a pension scheme faces in the future are highly uncertain. In addition, to value the guarantee options we need to take into account the credit risk of the sponsor. In the one-period model, we considered a sponsor that can only default in year 1. However in a multi-period model, the time a sponsor becomes insolvent is uncertain. The evolutions of the sponsor's assets and debts are also path-dependent. These complexities make it hard to develop a structural formula to value the options embedded in pension policies. Therefore, we introduce a multi-period model based on the Monte Carlo simulation, which is well-suited to solve this type of complex path-dependent valuation problems.

Value-based ALM The model we use to value the embedded options implied by pension policies is called the value-based ALM model. It is an extension of the asset-liability management (ALM) model that financial institutes employ to manage the risks arising from the mismatch between assets and liabilities. ALM uses risk models to generate many scenarios of the possible evolutions of pension assets and liabilities, which allows the pension scheme to calculate the probability of underfunding and gives insight into how policies, such as indexation, benefit cut, or recovery contribution impact on the pension scheme.

ALM is a powerful tool to show possible outcomes of a pension scheme and enable policy makers to make well-informed decisions. But it lacks the ability to value the embedded options implied by different policy contracts. To overcome these shortages, the ALM model is extended to introduce the value-based ALM approach. Value-based ALM uses the option pricing theory to value embedded options. The contingent claims, whose payoff depends on market development, can be priced by using the deflator approach (De Jong, 2004) or risk neutral scenarios. In this thesis, we use the risk neutral scenarios approach to price option values. The values are added on the holistic balance sheet, thus giving a full picture of the pension fund status. Value-based ALM was previously used to study the value transfer between different generations of pension scheme participants (Lekniute, 2011). Janssen (2012) employed the value-based ALM to value the embedded options within Dutch pension policies and built the HBS. We extend their work and use the value-based ALM to compare UK and Dutch pension policies.

In the following subsections, the building blocks of the model, the valuation technique, assumptions required to model the sponsor's credit risk are explained in details.

4.2 Pension fund characteristics

In this multi-period model, we set up a pension scheme that pays out annual benefits to the pensioners several years into the future. Many UK DB pension schemes have stopped enrolling new members, whereas most Dutch ones are still open to new participants. As our aim is to value the guarantee options provided by the sponsor and the PPF and set up the HBS for UK DB pension policies, we simplify our analysis to a closed pension scheme. Therefore, participants do not pay contributions or accrue benefits from the beginning of the first year. One can easily modify the model and adjust it to an open fund setting.

To evaluate the embedded options within the pension contracts, several variants of the pension policy are set up. Some base assumptions of the pension scheme hold for these variants and are set up as follows:

1. The pension scheme is an average wage defined benefit scheme;
2. The annual accrual rate is 2% of the member's annual salary;
3. Each member earns equal salary;
4. The pension fund has a single sponsor company;
5. The benefits received by the pension members are indexed to price level. Caps or conditional indexation applies according to individual pension policy;
6. An individual participant is assumed to enter the pension fund at the age of 25, retire at the age of 65, and can survive only up to the age of 99;

7. The pension fund asset portfolio consists of 50% bonds and 50% stocks, and is rebalanced at each time period.

The pension scheme covers the whole Dutch population. Therefore, we use the demographic data provided by CBS (*Centraal Bureau voor de Statistick*). This gender specific data set includes the population size and the survival probabilities for each cohort and the projection of both for the upcoming years. The model uses the initial population data and generates the size of the population for each cohort in the future using the survival probabilities. For example, the size of male population of age x at time t is calculated as

$$MalePop_t^x = MalePop_{t-1}^{x-1} \times p_{t-1,x-1}^{male}, \tag{37}$$

where $p_{t-1,x-1}^{male}$ is the one-year survival probability of a male of age $x-1$ at year $t-1$.

4.3 Core model

The model starts with the determination of the initial pension assets and liabilities. The initial pension assets (A_0) are calculated as the product of the initial pension liabilities (L_0) and initial funding ratio (FR_0). The pension liabilities is the present value of the total accrued benefit claims. The initial funding ratio is predetermined before running the model. The calculation of the initial pension assets can be expressed as

$$A_0 = L_0 \times FR_0. \tag{38}$$

The value-based ALM model uses a scenario set generated by a risk model that will be explained later. The scenario set contains 5000 scenarios representing possible future economic developments. We assume that full indexations have been granted up till the beginning of the model. Therefore the accrued benefits for a certain age groups are the same for each of the 5000 scenarios. Since we assume that all the members earns equal annual salaries and the benefits are accrued at 2% of the annual salary, the total accrued benefits for a member would be the annual salary multiplied by the years the member has worked and the 2% accrual rate. We can express the accrued benefit matrix as

$$B_0 = \begin{bmatrix} 0 & 0.02 & 0.04 & \cdots & 0.8 & \cdots & 0.8 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0.02 & 0.04 & \cdots & 0.8 & \cdots & 0.8 \end{bmatrix} \ (5000 \times 75). \tag{39}$$

Each row of the matrix is one of the 5000 scenarios and the columns represent age groups from 25 (no accrued benefits yet) to 99 (accrued to 80% of the average salary). Multiply this matrix with the average wage level and the population for each cohort will give us the total accrued benefits of each age group in each scenario.

To calculate the initial liabilities, each element in the benefit matrix is multiplied by an appropriate discount factor. The discount factor is gender-, cohort-, and scenario-specific. It takes into account the survival probability and discount all the future benefit payments according to the present term structure generated from the scenario set. For example, in a certain scenario s, the discount factor for a male member aged x at time t can be calculated as

$$D_{t,s}^x = \sum_{i=max(65-x,0)}^{99-x} p_x^m(i|t)(1+R_{t,s}^{(i)})^{-i}, \tag{40}$$

where $p_x^m(i|t)$ is the probability that the male aged x at time t will survive to year i, and $R_{t,s}^{(i)}$ denotes the rate with maturity of i from the nominal term structure at time t in the scenario s. We can view this equation as to calculate a deferred annuity product that pays out benefits until a person deceases. We form the discount factor matrix with elements (40). Each row is one of the 5000 scenarios and each column represents a age group. The discount factor matrix thus has the form

$$D_t = \begin{bmatrix} D_{t,1}^{25} & \cdots & D_{t,1}^{99} \\ \vdots & \ddots & \vdots \\ D_{t,5000}^{25} & \cdots & D_{t,5000}^{99} \end{bmatrix} \quad (5000 \times 75) \tag{41}$$

The matrix (39) and the matrix (41) are multiplied element-wise, and then each row of the results is summed up to obtain the total liabilities of the pension scheme in a certain scenario. With the initial liabilities, the initial pension assets are calculated according to equation (38).

After determining the initial assets, the model starts running. The benefits are paid out to the pensioners at the beginning of each year, and the total amount of the benefits is subtracted from the assets. The remaining assets are then invested according to the predetermined investment strategy and at the end of the year the assets are updated with investment returns generated from the scenario set.

At the end of the year, a new term structure is determined. The pension scheme's liabilities are updated by multiplying the accrued benefit matrix, which takes into account the demographic change, and the discount matrix. The funding ratio at the end of the year is calculated as the ratio of total assets and total liabilities. Given the funding ratio, the indexation of the accrued benefits will be adjusted according to the pension contract and the price levels generated from the scenario set. The accrued benefit matrix will be adjusted accordingly.

To value the sponsor guarantee and the PPF guarantee options, we also need to consider the evolution of a sponsor's assets and debts, and simulate the insolvency event. In the model, we assume that the sponsor has an asset portfolio that consists of a different asset mix from the pension scheme's asset portfolio. Sponsor's assets evolve in a similar way as the pension scheme's assets: the assets at the end of each year is the sum of the assets at the beginning of the year and the investment returns. The initial sponsor's assets before the model starts running is predetermined. The size of the initial assets determines a sponsor's ability to pay contributions to the pension scheme when there is a pension funding shortfall. It also influences the insolvent probability as we define the insolvency is triggered when the level of the sponsor's assets hits the level of its debts. To strike a balance, we choose the level of the sponsor's assets to be 1.5 times of the level of the pension scheme's assets.

At the end of the year, the model examines whether the sponsor is still solvent. If the sponsor is still solvent, recovery contributions may be paid by the sponsor to the pension scheme according to the funding ratios and the pension contract. If the sponsor is insolvent, the pension scheme will be liquidated. The pension scheme can buy insurance contracts for its participants or the PPF may take over of both the pension assets and liabilities. How the pension scheme is liquidated depends on the funding position of the scheme and if there is the PPF protection in the pension policy. The details are explained when we introduce the pension policies and demonstrate the results. The model uses indicator matrices to record the statuses of the sponsor and the pension scheme, and adjusts the indexation rules, accrued benefits accordingly.

The above process is repeated for 15 years. In each time loop, the cash flows, the assets and liability levels, and the indexations are recorded so that we can value the embedded options in the pension policies.

4.4 Valuation

The pension policies can be viewed as the financial contracts between the stakeholders of the pension scheme. Participants in a conditional indexation contract receive less than full indexation when the funding ratio is low. The sponsor needs to contribute to the recovery of the pension scheme when the financial market performs badly and there is a pension funding shortfall. The PPF has to step in to guarantee part of the pension deficit when the sponsor defaults. The payoff of the pension contracts depends on the market contingency. Therefore we can view these contracts as a combination of contingent claims and value the contracts using the fundamental theory of asset pricing (FTAP), introduced by Black and Scholes (1973).

The price of a financial asset has many possible outcomes in the future under the real probability measure $\mathbb{P}$. We define the asset whose price process is always positive definitive as the *numéraire*. Assuming the asset portfolio is self-financing, i.e. returns are re-invested and no inflow of money from external source, the FTAP states that if there is no arbitrage in the market and there exists a *numéraire*, the process of the relative price (relative to the *numéraire*) of a financial asset is a martingale process under an equivalent risk-neutral measure $\mathbb{Q}$. Furthermore, if the market is complete, meaning that any contingent payoff can be replicated by a combination of financial assets, the measure $\mathbb{Q}$ is unique.

For example, we choose a risk-free bond with continuous interest rate r as the *numéraire*. If the bond has a price of 1 at time 0, the bond price at time t will be e^{rt}. According to the FTAP, the relative price process of an asset X_t has the relation

$$\frac{X_t}{e^{rt}} = E^{\mathbb{Q}}[\frac{X_T}{e^{rT}}]. \tag{42}$$

Therefore, the present value X_0 is $E^{\mathbb{Q}}[\frac{X_T}{e^{rT}}]$. Note one can select any assets with a positive definitive price process as the *numéraire* and the growth rate of the asset is not necessarily constant.

To value the embedded option within the pension policies, the value-based ALM model generates scenarios under the risk-neutral measure $\mathbb{Q}$. The contingent payoffs of the contracts are recorded and discounted with the appropriate risk-free rates. Taking the average of the discounted payoffs will give the value of the particular embedded option.

4.5 Scenario set

A risk model is used to generate economic scenarios. The risk factors, such as stock returns and bond returns, term structure of interest rates, price levels from the generated scenario set are used as the input variables in the value-based ALM model, so that the outcomes of the pension scheme's assets and liabilities can be simulated. The risk model deserves a thesis on its own and here we only introduce some basic information.

The ALM model is based on Monte Carlo simulation of 5000 possible future economic development (Rob et al., 2011). The state variables are modelled with a vector auto-regressive model with jumps and time-varying volatilities:

$$\mathbf{x}_{t+1} = \begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ xs_{t+1} \\ dy_{t+1} \\ cs_{t+1} \\ mp_{t+1} \end{bmatrix} = \mathbf{c}_t + \boldsymbol{\Gamma}\mathbf{x}_t + J_{t+1} + \boldsymbol{\Sigma}\mathbf{S}_t^{1/2}\zeta_{t+1}, \tag{43}$$

$$\mathbf{c}_t = (\mathbf{I}_6 - \boldsymbol{\Gamma})(\mu_0 + \mu_{\bar{\pi}}\bar{\pi}) - \mathbf{p}\nu, \tag{44}$$

$$\zeta_{t+1} \sim \mathbb{N}(\mathbf{0}, \mathbf{I}_6). \tag{45}$$

π_{t+1} is the log of the annual inflation in the Euro zone, y_{t+1} the Euribor three month rate, xs_{t+1} the excess return on stocks, dy_{t+1} the dividend yield, cs_{t+1} the credit spread and mp_{t+1} the maturity preference. $\bar{\pi}_t$ stands for deterministic inflation target. Jumps are modelled by the jump indicator J_{t+1} with probability $\mathbf{p}$ and size ν, and the time varying volatility is obtained by diagonal matrix $\mathbf{S}_t$.

Before the recent financial crisis, most models regard the 2008 event as highly unlikely. The scenario generator we use improve on this by introducing the jumps into the model. The model assumes a constant probability of a jump that stands for a sudden drop in the confidence in the market, accompanied by the plunge in stock market, lowering interest rate and high credit spreads.

The term structure is obtained from an affine term structure model. The parameters are calibrated to the historical data. The derived rates are used to value the liabilities and cash flows in the scenario set so that everything is valued in a market-consistent manner. For more explanation of the model, one can refer to Van den Goorbergh, Molenaar, Steenbeek and Vlaar (2011).

The value-based ALM model aims to value embedded options using risk-neutral asset pricing technique. For this purpose, the real world scenarios are transformed to the risk neutral measure (Lin and Vlaar, 2011). The ALM model for the pension fund also contains an additional variable, the average wage growth. Since wage growth is not traded in the market, a regression model for the wage growth w_{t+1} with lagged wage growth w_t, inflation π_{t+1} and lagged short term interest rate y_t is estimated under $\mathbb{P}$ measure to generate scenarios:

$$w_{t+1} = \alpha + \beta_w w_t + \beta_\pi \pi_{t+1} + \beta_y y_{t+1} + \epsilon_{t+1}. \tag{46}$$

The parameters from (46), together with the dynamics of π and y under $\mathbb{Q}$ measure, are used to generate the dynamics of wage growth under $\mathbb{Q}$ measure:

$$w^{\mathbb{Q}}_{t+1} = \alpha + \beta_w w^{\mathbb{Q}}_t + \beta_\pi \pi^{\mathbb{Q}}_{t+1} + \beta_y y^{\mathbb{Q}}_{t+1} + \epsilon_{t+1}. \tag{47}$$

5 Valuing the embedded options implied by UK pension policies in the multi-period model

5.1 Policies

As in Section 3 for the simple one-period model, we start from building an HBS for a basic pension policy, then gradually adding more complexity to the example. This way, we can easily track how each new feature of a pension policy affects the HBS. We call the first policy the baseline policy, as it does not have any steering tools or adjustment mechanisms that are available to the trustees of the pension scheme. The policies are summarised in Table 5.1. The details of each policy will be introduced when we discuss the results in the following subsections.

	Price indexation	Recovery plan	Credit risk	PPF guarantee
Policy 1	full			
Policy 2	UK			
Policy 3	UK	Recovery plan		
Policy 4	UK	Recovery plan	Credit risk	
Policy 5	PPF	Recovery plan	Credit risk	PPF guarantee

Table 12: The pension policies used to build the HBS. The indexation is linked to the price level. "UK" means that the indexation will be capped following UK DB pension indexation rules. "PPF" is a different cap rules for the indexation.

5.1.1 The HBS for Policy 1 (baseline policy)

The HBS for the baseline policy, Policy 1, is presented in Table 13. The pension assets on the HBS have the value of the pension asset portfolio and the pension liabilities equal the present value of the total benefits. These are the same as the assets and the liabilities in the traditional balance sheet. The pension scheme has a funding ratio of 1.175, which results from the pension assets divided by the pension liabilities. This is the initial funding ratio we set in our value-based ALM model. We choose 1.175 as the initial funding ratio for the UK pension policies because we want to compare the HBS of the UK pension policies with the Dutch ones, and 1.175 is the solvency funding ratio required by the Dutch regulator. In later sections, we will change the initial funding to different levels in the sensitivity analysis.

Policy 1 gives full indexation to the benefits and the indexation is linked to the price level. The amount of indexation is path dependent. From the risk-neutral scenario set, we know the actual indexation materialised, thus we can calculate the resulting increased benefits. Taking average of all these increased benefits and discounting to the present using the path dependent risk-free rate will return the value of this indexation option. The calculation can be expressed as

$$V_0^{ind} = E_0^{\mathbb{Q}}[\sum_{t=1}^{15}(ind_t \cdot B_t \cdot \prod_{k=0}^{t} \frac{1}{1+r_{f,k}})], \tag{48}$$

where V_0^{ind} is the value of the indexation option at time 0, ind_t is the indexation given at time t, B_t is the total benefits participants are entitled to at time t, and $r_{f,k}$ is the annual risk free rate in year k.

From Table 13[5], we can see full indexation is a very expensive policy for the pension scheme. It increases the total liability by 32.28 and will worsen the funding position of the pension scheme in the future.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
		Full indexation		32.28
		Residue		-14.44
		Surplus option	16.31	
		Deficit option	-30.74	
Total	117.50	**Total**		117.85

Table 13: The HBS for policy 1. Full indexation linked to the price level is guaranteed.

The last option on the HBS for Policy 1 is the residue option. Residue is the difference between assets and liabilities of the pension fund at the end of each scenario path. In this closed fund setting, the residue represents the value of surplus/deficit that the pension scheme has at the end of our investigation (15 years). Thus the deficit option is a bad result for participants, i.e. there are not enough assets to cover the promised benefits.

The value of the residue option is calculated the same way as the indexation option. The pension surplus/deficit for each scenario at year 15 is discounted to present using the risk-free interest rates of each scenario path, then the average is taken to give the residue option value. The expression to calculate the residue option can be written as

$$V_0^{RO} = E_0^{\mathbb{Q}}[(A_T - L_T) \cdot \prod_{k=0}^{T} \frac{1}{1 + r_{f,k}}], \tag{49}$$

where V_0^{RO} is the value of the residue option, T is the time the pension scheme is liquidated, A_T and L_T are the pension assets and liabilities at T, respectively. We can also take the average of the surplus and the deficit respectively, thus disintegrating the residue option into a surplus option and a deficit option. This way, we can have an idea of how the residue option is divided between surplus and deficit, and track the impact when we introduce new pension policies. Policy 1's deficit option has a value almost double the value of its surplus option, leaving the total residue option of a value of -14.44. This is not a good news for participants, as there are chances that the promised benefits may not be fulfilled by the pension scheme when it is liquidated.

[5] The value-based ALM model returns the values of assets and liabilities as well as the options in Euro amount. To aid comparison between policies and help understanding the results, each value on the HBS is normalised to the value of the pension liabilities, and then times 100. For example, if the pension assets have a value of €150 million and the pension liabilities have a value of €120 million, the recalculated values of the pension assets and liabilities on the HBS will be 125 and 100, respectively. One can read from the HBS quickly that the pension scheme has a funding ratio of 1.25. Similarly, an indexation option with the value of €30 million will be shown as 25 on the HBS. Since the value of the pension liabilities is the present value of future benefits, it remains the same for all different policies. Therefore, the relation between the values on different HBSs holds the same in Euro amount.

The whole set of pension options can be seen as a zero sum game. The value transferred between stakeholders through one option will appear up in other options transferred in the opposite direction. The high value of the indexation option due to the full indexation guarantee is counterbalanced by the negative value of residue option, bringing both sides of the HBS in balance. Note that the total assets and the sum of the total liabilities and residue options are not exactly the same. This is because of the rounding errors in the model.

5.1.2 The HBS for Policy 2 (UK indexation rules)

Now the pension scheme switched from Policy 1 to Policy 2. The indexation follows the UK indexation rules. The indexation is linked to the price level but capped at 5% for the pensions in payment and 2.5% for the deferred pensions. Policy 2 is worse for participants than Policy 1, as it gives less benefits than Policy 1 if the price level is higher than the capped level. This is reflected in the HBS. The indexation option now has the value of 21.95 as compared to 32.28 in Policy 1. The funding position is improved with the restrain on indexation as can be shown with a less negative value of the residue option. The less negative residue option can be explained by a more positive surplus option and a less negative deficit option compared to Policy 1, with the reduction in deficit contributing the most to the improvement of the residue option value.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
		UK indexation		21.95
		Residue		-4.10
		Surplus option	18.73	
		Deficit option	-22.83	
Total	117.50	**Total**		117.85

Table 14: The HBS for UK Policy 2. The indexation is linked to the price level and capped according UK pension indexation rules.

5.1.3 The HBS for policy 3 (recovery plan)

The UK Pension Act 2004 states that the pension scheme is subject to the statutory funding objective that the assets of the scheme are sufficient to meet its liabilities. If there is a pension funding shortfall, a recovery plan will be prepared and agreed upon between the sponsor company and the trustees of the scheme. The recovery plan can be such that the sponsor contributes a certain amount of assets to the pension scheme in a span of several years so that by the end of the planned period, the pension funding shortfall will be eliminated. We assume that the recovery contribution should not jeopardize the financial position of the sponsor.

To extend our model and include the recovery plan with the above mentioned features, we make the following assumptions. At the end of each year, after the indexation has been calculated, the pension scheme's funding position will be examined. If there is a funding shortfall, the model initiates a recovery plan. The amount of contributions and the length of the period of the plan will depend on both the size of the pension funding shortfall and the financial position of the sponsor company. If the pension funding shortfall is less or equal to the sponsor's equity value, the difference between the sponsor's assets value and debts value, a part of the sponsor's assets that equals the pension funding shortfall will be transferred to the pension scheme. In this way, the financial position of the sponsor will not deteriorate severely and the pension funding shortfall can be eliminated quickly.

If the size of the pension funding shortfall is larger than one quarter of the sponsor's equity value, but less than full equity value, the recovery plan will be set up in eight annual instalments with the first instalment equal to 1/8 of the pension funding shortfall. We choose 8 years as the length of the recovery plan because this is the medium length of years of the recovery plan in UK private DB pension schemes. Another situation is that the pension funding shortfall is larger than the full equity value of the sponsor, then the first instalment will be 1/8 of the sponsor equity. These setting allows pension fund to reduce its funding shortfall gradually and reach its "statutory target" in the future but does not put the sponsor into a stressed situation.

One should note that the recovery plan will be reviewed at each end of the year, i.e. if the conditions change, the recovery plan will be revised accordingly. We express the recovery contribution at a certain time t as

$$C_t^{rec} = \begin{cases} L_t - A_t & \text{if } 0 < L_t - A_t \leq \frac{1}{4}(S_t - D_t) \\ \frac{1}{8}(L_t - A_t) & \text{if } 0 < \frac{1}{4}(S_t - D_t) < L_t - A_t \leq (S_t - D_t) \\ \frac{1}{8}(S_t - D_t) & \text{if } 0 < S_t - D_t \leq L_t - A_t \\ 0 & \text{otherwise,} \end{cases} \tag{50}$$

where C_t^{rec} is the recovery contribution, A_t is the pension assets, L_t is the pension liabilities, S_t is the sponsor assets, and D_t is the sponsor debts. The model records the actual amount of the contributions the sponsor pays each year and then calculate the value of the sponsor guarantee option in this way:

$$V_0^{rec} = E_0^{\mathbb{Q}}[\sum_{t=0}^{15}(C_t^{rec}\prod_{k=0}^{t}(\frac{1}{1+r_{f,k}}))]. \tag{51}$$

In the one period model, we simply assume that the sponsor can afford any funding shortfall if there is no credit risk of the sponsor. The sponsor's assets and debts are modelled in a relative term but not in Euro amount. In the value-based ALM model, part of the assets value is transferred from the sponsor to the pension scheme. Therefore we need to know the sponsor's assets and debts in absolute Euro amount. As the purpose is to show how this recovery contribution affects the HBS, for now, we assume that the initial sponsor's assets is 1.5 times of the initial pension assets and the level of the sponsor's debts is 30% of the level of the sponsor's assets. In addition, there will be scenarios that the sponsor's asset level is below its debts level. If we don't allow the sponsor to become insolvent (for example, a public organization), we can assume the sponsor is still running as a going concern but cannot afford to contribute to the recovery plan until its asset level returns to above its debt level.

Another difference from the one period model is how we simulate the insolvency event. In the one period model, we assumed the value of the sponsor's assets S_t follows a geometric Brownian motion in the risk-neutral measure $\mathbb{Q}$. The value grows at the risk-free rate r with the volatility σ_S. In the value-based ALM model, the options are valued in the risk neutral measure $\mathbb{Q}$. The model involves time-variant volatilities and shocks that represent sudden financial market collapses such as the 2008 event. It generates scenarios that gives the values of relevant risk factors such as stock returns, bond returns, inflations and etc. The evolution of the pension assets and liabilities can then be simulated.

We simulate the evolution of the sponsor assets and debts in a similar manner as the pension assets and liabilities. We assume that the sponsor's asset portfolio consists of stocks and bonds which is different in proportion from the pension asset portfolio. If a portfolio of 100% stocks represents the market risk, to reflect the idiosyncratic risk inherent in the individual company, the sponsor asset portfolio is assumed to have 110% in stocks and -10% in bonds. We assume that the sponsor debts evolves in the same way as a risk-free asset, thus it grows according to the risk-free rates in the scenario set, which are the Euribor rates. The evolution of the sponsor assets S_t and debts D_t at the beginning of the year t can be written as

$$S_t = \alpha_S S_{t-1}(1+r_t^S) + (1-\alpha_S)S_{t-1}(1+r_t^B), \quad (52)$$
$$D_t = D_{t-1}(1+r_t) \quad (53)$$

where α_S is the proportion of the sponsor asset portfolio which is invested in stocks (110% in this example), r_t^S is the annual stock return, r_t^B is the annual bond return, and r_t is the risk-free rate.

The HBS for Policy 3 is shown in Table 15. To keep it consistent with the previous sections, we call the option that the sponsor pays recovery contributions the sponsor guarantee option. This option is the additional assets the pension scheme will receive from its sponsor in case of the pension funding shortfall. Therefore it appears on the assets side of the HBS and increases the total assets by 14.86.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor guarantee	14.86	UK indexation		21.95
		Residue		10.77
		Surplus option	23.23	
		Deficit option	-12.46	
Total	132.36	**Total**		132.72

Table 15: The HBS for Policy 3. The UK indexation rules apply. Sponsor contributes to the recovery plan.

The residue option now has a positive value and it balances both sides of the HBS. One can also see that the surplus option has a higher positive value and the deficit option has a lower negative value compared to Policy 2. We do not assume who will receive the pension funding surplus when the pension scheme is liquidated at the end of year 15. One may suggest that part of the surplus is returned to the sponsor to compensate its contribution to the recovery of pension funding shortfalls.

Another issue regarding to the residue option is that when the pension is liquidated after 15 years, if the pension scheme has a funding shortfall, the sponsor may not pay the recovery contribution that equals the full size of the shortfall according to equation (50). If the sponsor will eventually guarantee all the funding shortfall after the scheme is liquidated, the residue option will have a higher positive value. Pension participants will end up with only good situations (potential surplus and no downside risk).

5.1.4 The HBS for Policy 4 (credit risk)

Based on Policy 3, we introduce the credit risk to the model and form Policy 4 for the pension scheme. The assumptions about the evolution of the sponsor assets and debts are the same as in Policy 3. But if the sponsor's asset level becomes less than its debt level, the sponsor will be insolvent and remains so afterwards. For the pension scheme, this means that the sponsor can only contribute to the recovery plan as long as it is solvent. If the sponsor becomes insolvent, the pension scheme will operate independently until it is liquidated at year 15.

The HBS for Policy 4 is presented in Table 16. As expected, the value of the sponsor guarantee option is lower than in Policy 3. This is because the insolvent sponsor no longer contributes to the recovery plan. This reduces the value of this conditional asset. Correspondingly the residue option has a less positive value than in Policy 3. Participants have a worse contract than policy 3.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor guarantee	14.36	UK indexation		21.95
		Residue		10.27
		Surplus option	23.17	
		Deficit option	-12.90	
Total	131.86	**Total**		132.22

Table 16: The HBS for Policy 4. The UK indexation rules apply. Sponsor contributes to the recovery plan. Credit risk exists.

One may notice that the reduction in the value of the sponsor guarantee option by introducing the credit risk is very small (14.36 in Policy 4 as compared to 14.86 in Policy 3). This is due to our assumption of how the sponsor pays the recovery contributions and the choice of the asset portfolios of the pension scheme and the sponsor company. In Policy 3, the sponsor can have a negative equity value and remains a going concern. We assume that the sponsor with a negative equity will not pay any contributions to the recovery plan. In addition, the pension scheme has 50% assets in stock and the sponsor has 110% assets, thus the pension scheme and the sponsor company tend to be in deficit at the same time in our scenarios. For these reasons, the sponsor without credit risk and the sponsor with credit risk have very similar contribution cash flows in each scenario path. The sponsor without credit risk has a limited number of extra cash flows when the sponsor recovers from negative equity value. Thus the credit risk in Policy 4 reduces only a small amount of recovery contribution cash flows, and has a very limited effect on the value of the sponsor guarantee option.

5.1.5 The HBS for Policy 5 (PPF)

Finally, Policy 5 introduces the PPF protection to the pension scheme. The PPF takes over the pension scheme's assets and liabilities if the sponsor becomes insolvent and the pension scheme's assets can not cover the liabilities guaranteed by the PPF. The PPF policy affects the stakeholders of a pension scheme in multiple ways. The PPF only guarantees part of the liabilities, 100% of the benefits of the pensioners and 90% of the accrued benefits of the active members, subjected to a cap. In addition, the pension scheme needs to pay annual premiums to the PPF until the sponsor becomes insolvent in exchange for the protection.

Like a sponsor, the PPF covers the pension funding shortfall, but only up to a limit. The PPF guarantee option can be perceived as a contract between the PPF and the pension scheme's participants. Participants benefit from the contract when the sponsor becomes insolvent and there is a pension funding shortfall. The size of the payoff depends on the extent of the pension funding shortfall. The PPF guarantee option will have the highest payoff in the worst case scenario, i.e. the insolvent sponsor cannot contribute and the pension fund has a large funding shortfall. The PPF guarantee option appears on the asset side of the HBS as it is a contingent asset. If we denote the liabilities that the PPF guarantees to be L^{PPF}, the payoff the pension scheme will receive when it is taken over by the PPF is $G_t^{PPF} = L_t^{PPF} - A_t$, where $L_t^{PPF} > A_t$. The calculation of the PPF guarantee option can then be expressed as

$$V_0^{PPF} = E_0^{\mathbb{Q}}[\sum_{t=0}^{15}(G_t^{PPF}\prod_{k=0}^{t}(\frac{1}{1+r_{f,k}}))]. \tag{54}$$

On the other hand, participants' entitled benefits are cut once the PPF takes over the pension scheme. The extent of the cut depends on the status of the individual participant and the cap set by the PPF. There is also another situation that when the sponsor becomes insolvent, the pension assets are larger than the liabilities guaranteed by the PPF. In this case, life insurance contracts are bought which pays off benefits equal to the amount that the remaining pension assets can cover. This is also a cut as participants can not receive their entitled benefits in full amount. Although the PPF won't take over the pension scheme in this case, the cut is incurred because of the introduction of the PPF protection into the policy. Therefore we also consider this kind of cut as part of the PPF cut option. One more situation is that there may be more than enough assets in the pension scheme to cover the entitled benefits when the sponsor is insolvent. Instead of considering this surplus as a "negative cut", we categorize it as a residue and include it when we calculate the residue option. The PPF cut can be expressed as

$$Cut_t^{PPF} = \begin{cases} 0 & \text{if } A_t > L_t; \\ A_t - L_t & \text{if } L_t^{PPF} < A_t < L_t; \\ A_t - L_t^{PPF} & \text{if } A_t < L_t^{PPF} < L_t. \end{cases} \tag{55}$$

This cut option is a contract between the PPF and participants and reduces the entitled benefits of participants. Therefore the value of the PPF cut option appears as a negative value on the liability side of the HBS. The value of the PPF cut option is calculated in the following way:

$$V_0^{cut} = E_0^{\mathbb{Q}}[\sum_{t=0}^{15}(Cut_t^{PPF}\prod_{k=0}^{t}(\frac{1}{1+r_{f,k}}))]. \tag{56}$$

Participants' losses also include the reduction in indexation. Once in the PPF, the benefits are indexed to the price level capped at 2.5%. This does not affect deferred pensions but the pensions in payment will be less than the UK indexation rules if the price level is above 2.5%.

Like in an insurance contract, the pension scheme also pays premium, called levy, to the PPF annually up till the moment the sponsor becomes insolvent. The amount of the levy depends on multiple factors: the size of the pension scheme's liabilities, the pension scheme's funding position, the credit risk of the sponsor, and the funding position of the PPF itself. Given the above factors, one can calculate the levy according to the formula provided by the PPF (PPF Determination 2013). To avoid diverging from our main results, we present the calculation in Appendix A.

The levy charge presents a liability for the pension scheme. The future values of the levy cash flows depend on market conditions. Therefore, we consider the levy as a conditional liability and put it as an option on the liability side of the HBS. One can view the levy option as an exotic call option the PPF writes to the pension scheme's trustee. The payoff depends on the variables in levy valuation formula and the status of the sponsor company. However the levy charges don't affect the entitled benefits of the pension participants. The value of the levy option is calculated as

$$V_0^{Levy} = E_0^{\mathbb{Q}}[\sum_{t=0}^{15}(Levy_t^{PPF}\prod_{k=0}^{t}(\frac{1}{1+r_{f,k}}))]. \tag{57}$$

Table 17 presents the HBS for Policy 5. From the HBS, introducing the PPF protection to the pension policy appears to be a good deal for participants. The PPF guarantee option has a value of 7.9. The PPF takes the position of the sponsor after the sponsor becomes insolvent, and guarantees most of the benefits participants are entitled to. The losses participants need to bear due to the reduction in indexation is very limited: only a reduction of 0.45 in the indexation option value compared with Policy 4. The PPF cut option has a value of -1.13, a relative small amount compared with the PPF guarantee option.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor guarantee	14.49	PPF indexation		21.50
PPF guarantee	7.9			
		PPF levy		0.52
		PPF cut		-1.13
		Residue		19.12
		Surplus option	22.94	
		Deficit option	-3.82	
Total	139.88	**Total**		140.05

Table 17: The HBS for Policy 5. The UK indexation rules apply. Sponsor contributes to the recovery plan. Credit risk exists. The PPF guarantees pension funding shortfalls if sponsor becomes insolvent

A surprising result is the low value of the PPF levy option on the HBS. One would expect the premium to be at a similar level of the expectation of the payoff from an insurance contract. The big difference between the values of the PPF guarantee option and the PPF levy option appears to be a good contract for the pension scheme and its participants. This result maybe due to the scenario set we are using in the model. The economic scenarios are generated from financial data in 2011, when low investment returns complicated with high volatility were observed. This leads to a high frequency of sponsor insolvency and low and volatile asset returns in our model, which in turn increases the PPF guarantee option value and reduces the levy needs to be paid.

Overall, introducing the PPF protection increases the residue option value, mainly resulting from reduced value of the deficit option. We can conclude that the PPF protection is a good pension deal for participants as it eliminates some of the downside risk if the sponsor becomes insolvent and the pension scheme has a funding shortfall.

5.1.6 Summary of the HBS for UK pension policies

Now we finish the construction of the HBS for the stylised pension scheme that applies UK DB pension policies. We summarise the results in Table 18 so one can review the impact of each new feature on the value of embedded options.

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
Pension Assets	117.5	117.5	117.5	117.5	117.5
Sponsor guarantee			14.86	14.36	14.49
PPF guarantee					7.9
Total	117.5	117.5	132.36	131.86	139.88
Pension Liabilities	100	100	100	100	100
Indexation	32.28	21.95	21.95	21.95	21.5
PPF levy					0.52
PPF cut					-1.13
Residue	-14.44	-4.1	10.77	10.27	19.12
Surplus option	16.31	18.73	23.23	23.17	22.94
Deficit option	-30.74	-22.83	-12.46	-12.9	-3.82
Total	117.85	117.85	132.72	132.22	140.05

Table 18: Summary of the HBS for the UK pension policies

5.2 Sensitivity analysis

The HBS for the UK pension policies are built under various assumptions including the pension scheme's initial funding position, the investment strategy of the pension scheme, the sponsor's size and leverage ratio, etc. Changing the initial assumptions will affect the values of the contingent claims on the HBS. Therefore it is crucial to understand how the HBS behaves once the assumptions are adjusted to various situations. In this section, we present a sensitivity analysis of the HBS for the UK policy. By modifying one of the assumptions and keeping the others unchanged, we can have a general picture of how sensitive the option values are to the specific variable modified.

Policy 5 includes the sponsor guarantee, sponsor's credit risk, and the PPF protection. This represents most of the instruments available to a UK DB pension scheme. Therefore we choose Policy 5 to conduct the sensitivity analysis. We call Policy 5 the benchmark and the assumptions are

1. The sponsor has an initial leverage ratio of 30%, i.e. the total debts is 30% of the total assets.
2. The sponsor's initial asset level is 1.5 times of the pension scheme's.
3. The pension scheme has the initial funding ratio of 1.175.
4. The pension scheme invests 50% of its assets in stocks and 50% in bonds.
5. The sponsor invests 110% of its assets in stocks and -10% in bonds.

In the following subsections, we modify each one of the assumptions and keep the rest unchanged. The results of the HBS under the new assumptions are compared for the sensitivity analysis.

5.2.1 Sponsor's initial leverage ratio

In the value-based ALM model, we assume that the sponsor will be insolvent once its asset level hits its debt level. Therefore the higher the initial leverage ratio is, the more likely the sponsor will become insolvent as both assets and debts evolve. We choose the initial leverage ratios of 30%, 50% and 70% to represent sponsors ranging from a lowly leveraged company to a highly leveraged one. There is also a sponsor with zero debt. One can deem it as a sponsor with no credit risk, as its asset level will never become lower than 0.

Before we run the sensitivity analysis, we need to make an adjustment of how we calculate the PPF levy. In the value-based ALM model, we calculate the annual PPF levy using the formula provided by the PPF. One factor in the formula is the so called insolvency risk factor (see Appendix A). It is a multiplier factor determined by the credit risk of the sponsor. The PPF evaluates individual sponsor and assigns it to one of 10 levy bands. Each levy band has a corresponding insolvency risk factor and its value ranges from 0.0018 for the highest rated sponsor to 0.04 for the lowest rated sponsor.

In our stylised model, we simulated the evolution of the sponsor's assets and debts but did not obtain a credit rating for the sponsor. It was hard to assign a levy band to the sponsor. We chose 0.04 as the multiplier for the sponsor with an initial leverage ratio of 30%. This gave us a prudent view of how much PPF levy as a burden the pension scheme may shoulder.

In the sensitivity analysis, it is obvious that a sponsor with a lower leverage ratio, thus lower credit risk in our model, should have an insolvency risk factor of less than 0.04. Therefore, we assign the sponsor with a certain initial leverage ratio to a levy band in a way that the corresponding levy band will be the leverage ratio multiplied by 10. For example, a sponsor with 30% leverage ratio will be in levy band 3. This is not ideal, but given the uncertainty of how the PPF will calculate the PPF levy, this method at least gives an intuitive result: the higher the leverage, the higher the value of the PPF levy option. The insolvency risk factor for sponsors with different initial leverage ratio are summarised in Table 19.

The results of the HBS are presented in Table 20. The value of the sponsor guarantee option decreases by more than 75% when the initial leverage ratio increases from 30% to

D_0/S_0	0%	30%	50%	70%
Levy band	0	3	5	7
IR	0	0.0044	0.011	0.0201

Table 19: The levy rate used in the sensitivity analysis of the initial leverage ratio. D_0/S_0 stands for the initial leverage ratio. IR is the insolvency risk factor in the levy formula in Appendix A.

70%. The decrease in value is very intuitive. High leverage limits the sponsor's ability to contribute to recover the pension funding shortfalls. It also increases the insolvent probability. The insolvent sponsor doesn't have to pay any contribution to recover the pension deficit. In contrast, a sponsor with no credit risk, as shown with a zero leverage ratio, will shoulder a lot of recovery contributions, thus the sponsor guarantee option is high in value (23.01) on the HBS of the pension scheme.

Initial leverage ratio (D_0/S_0)	0	0.3	0.5	0.7
Pension Assets	117.50	117.50	117.50	117.50
Sponsor guarantee	23.01	14.43	8.28	3.37
PPF guarantee	0.00	7.81	8.90	6.84
Total	140.51	139.74	134.68	127.71
Pension Liabilities	100.00	100.00	100.00	100.00
UK indexation	21.95	21.54	20.95	20.67
PPF levy	0.07	0.26	0.31	0.15
PPF cut	0.00	-1.13	-2.03	-2.58
Residue	18.81	19.23	17.60	15.21
Surplus option	25.58	22.97	20.87	18.27
Deficit option	-6.77	-3.74	-3.28	-3.06
Total	140.84	139.89	136.83	133.44

Table 20: Sensitivity of the HBS to the initial sponsor leverage ratio ($\frac{D_0}{S_0}$). UK Policy 5 is used. The baseline case has the initial leverage ratio of 30%.

The PPF guarantee option changes in value in a complicated way. The value first increases from 7.81 to 8.9 when the initial leverage ratio increases from 30% to 50%, and then falls down to 6.84 when the initial leverage ratio increases to 70%. The increase in the value of the PPF guarantee option when the leverage ratio is raised from 30% to 50% is due to the higher probability of insolvency events. The payoff of the PPF guarantee option also depends on the size of the pension deficit. When the sponsor has an initial leverage ratio of 70%, a relative small economic downturn in the model may trigger the insolvency. However, the effect of the economic downturn may have limited effect on the pension scheme's asset so that the pension scheme has a relatively small deficit or is still even in surplus. This reduces the expected payoff of this PPF guarantee option, thus giving a smaller value on the HBS.

When the PPF takes over the pension scheme, the benefits participants are entitled to are cut. This contingent claim is valued as the PPF cut option, and presented as a negative value on the liability side of the HBS. The higher initial leverage ratio of the sponsor leads to a higher absolute value of the PPF cut option. This is due to the high probability of sponsor default when the sponsor has a higher leverage.

The PPF levy option shows a similar pattern as the PPF guarantee option. The value first increases when the initial leverage ratio is raised from 0 to 0.5. A pension scheme with the riskier sponsor does pay more levy than a pension scheme with the less risky sponsor. When the initial leverage ratio increases from 0.5 to 0.7, the value of the levy option decreases by 50%. The decrease is due to the higher probability of the sponsor insolvency at the initial leverage ratio of 0.7, thus the pension scheme has a shorter life time to pay the PPF levy.

In all, high sponsor leverage is not a good news for the pension scheme's participants, as it reduces the value of the sponsor guarantee option and increases the benefit cut option once the PPF takes over the pension scheme. These effects of increasing initial leverage ratio are also reflected in the residue options. A decrease in the value of the surplus option and a minor increase in the value of the deficit option, resulting in the total residue becoming less with an increasing leverage ratio.

5.2.2 Relative size of initial sponsor assets to the pension assets

As we can see from the previous analysis, the initial leverage ratio of the sponsor determines the sponsor's ability to contribute to the recovery of the pension funding shortfalls and the insolvent probability. These in turn affect the value of the contingent claims on the HBS. In reality, the sponsors vary in size. There are big companies with large market capitalizations and medium to small size companies with pension schemes having assets in various sizes. The relative size of the sponsor's assets to the pension scheme's assets also plays an important role in determining whether a sponsor can afford to fulfil its "statutory responsibility".

We now modify the ratio of the initial sponsor assets to the initial pension assets. A ratio of 1.5 is our benchmark. We add a lower ratio of 0.8 representing a relatively weak sponsor, and a higher ratio of 2 representing a strong sponsor. The results of the HBS are presented in Table 21.

As expected, a higher asset ratio enhances the sponsor's ability to contribute to the recovery plan. A strong sponsor with the asset ratio of 2 results in 70% increase in the value of the sponsor guarantee option compared to a weak sponsor with the asset ratio of 0.8. This in turn results in a lower value of the PPF guarantee option with a high sponsor/pension asset ratio, as there is a lower chance of sponsor insolvency and less pension funding shortfalls.

The reduction in the pension funding shortfall results in a lower PPF levy, and this is reflected from the lower value in the PPF levy option with the asset ratio of 2. On the other hand, the PPF cut option has a less negative value with the strong sponsor. A strong sponsor with high equity value can afford to contribute more than a weak sponsor. The pension funding shortfall can thus be reduced quickly. Even if the sponsor becomes insolvent, the pension scheme may end up with enough assets to cover its liabilities and the PPF does not need to step in to cut the benefits. When the relative sponsor assets increase, the indexation option also increases in value for the same reason, although to a small extent.

S_0/A_0	0.8	1.5	2
Pension Assets	117.50	117.50	117.50
Sponsor guarantee	9.77	14.49	16.58
PPF guarantee	9.42	7.90	7.17
Total	136.69	139.88	141.25
Pension Liabilities	100.00	100.00	100.00
UK indexation	21.51	21.54	21.55
PPF levy	0.66	0.52	0.47
PPF cut	-1.18	-1.13	-1.10
Residue	15.82	19.12	20.50
Surplus option	21.32	22.94	23.75
Deficit option	-5.51	-3.82	-3.25
Total	136.80	140.05	141.42

Table 21: Sensitivity of the HBS to the relative size of the sponsor's assets and the pension scheme's assets. S_0/A_0 is the ratio of the sponsor assets to the pension assets at time 0. UK Policy 5 is used. In the benchmark, the total assets of the sponsor is 1.5 times that of the pension scheme.

The pension funding shortfall is another important factor in the PPF levy formula. The pension funding shortfall can be more quickly recovered with a sponsor with larger assets. As a result, the value of the PPF levy option decreases when the sponsor's assets increase in size.

The above analysis is validated by the residue option as the surplus option increases in value and the deficit option decreases in value if the relative sponsor's assets increase in value, both contributing to a higher value of the total residue option. Therefore, we can conclude that a sponsor with relatively large asset value will benefit the pension scheme's participant and reduce the burden on the PPF.

5.2.3 Sponsor asset portfolio

In the value-based ALM model, we constructed a sponsor asset portfolio that consists of 110% stocks and -10% bonds. The high percentage in stocks takes into account the idiosyncratic risk inherent in an individual company. In reality, different sponsors have different risk profiles. Therefore we construct the sponsor asset portfolios with various proportions invested in stocks and conduct the sensitivity analysis. The percentage of the sponsor asset portfolio invested in stocks ranges from 0% to 150%, representing companies with a stable business to companies involved in a risky businesses with potential high returns.

As in the case of the sponsor leverage ratio, a sponsor's investment strategy influences its risk profile. One should assign a higher insolvency risk factor to a company involved in a risky business than a company in a stable business, to calculate the PPF levy the corresponding pension scheme should pay. Again, we designate the sponsor to one of the insolvency levy bands defined by the PPF approximately. A sponsor investing bonds only will be assigned to the levy band 1 and a sponsor investing 150% of assets in stocks to the levy band 10. The sponsors with the asset portfolios in between will be assigned to the bands between 1 and 10 approximately[6]. The results of the HBS are presented in Table 22.

Sponsor assets in stocks (%)	0	50	100	110	150
Pension Assets	117.50	117.50	117.50	117.50	117.50
Sponsor guarantee	29.60	23.65	16.02	14.49	9.30
PPF guarantee	0.00	1.31	6.91	7.90	8.94
Total	147.10	142.46	140.43	139.88	135.75
Pension Liabilities	100.00	100.00	100.00	100.00	100.00
UK indexation	21.95	21.93	21.65	21.54	21.05
PPF levy	0.15	0.70	0.58	0.52	0.11
PPF cut	0.00	-0.11	-0.89	-1.13	-1.93
Residue	25.35	19.90	19.02	19.12	18.26
Surplus option	27.64	25.44	23.34	22.94	21.41
Deficit option	-2.29	-5.54	-4.32	-3.82	-3.15
Total	147.45	142.42	140.36	140.05	137.48

Table 22: Sensitivity of the HBS to the sponsor's asset portfolio. UK Policy 5 is used. In the baseline case the sponsor invests 110% of its asset portfolio in stocks.

When the sponsor increases the risky asset proportion, its insolvent probability becomes higher. This translates into a lower value of the sponsor guarantee option. The value of the sponsor guarantee option reduces by 35% once the proportion invested in stocks increases from the benchmark of 110% to 150%. In contrast, a sponsor with low asset risk will shoulder much of the obligation to recover the pension funding shortfall. The value of the sponsor guarantee option increases to as high as 29.6 for the sponsor with 0% of assets in stocks.

The higher insolvent probability implied by the riskier sponsor asset portfolio also means that the PPF is more likely to take over the pension scheme. Thus the value of the PPF guarantee option increases when the sponsor invests more in stocks. In general, the increase in the value of the PPF guarantee option is less than the decrease in the value of the sponsor guarantee option. The results suggest that the PPF will have much less burden from the pension scheme with a sponsor in a stable business than in a risky business.

One may question why the value of the PPF levy option decreases when the sponsor invests more in risky assets. This results from the same reason as in the sensitivity analysis of the sponsor leverage ratio. A sponsor with a higher proportion of assets in risky stocks has a higher probability to be insolvent. Once the sponsor becomes insolvent the pension scheme stops to pay the PPF levy afterwards. Hence we see a lowering value of the PPF levy option

[6] The insolvency risk factors are 0.0018 (0), 0.011 (50%), 0.0201 (100%), 0.026 (110%) and 0.04 (150%). The figure in the brackets are the proportion of the sponsor's assets invested in stocks

when the sponsor invests a higher percentage of assets in stocks. The pension scheme with a risky sponsor appears to get away with the lowest expected levy charges in total. In addition, it is this very scheme that benefits the most from the PPF guarantee of the pension funding shortfall. Due to the uncertainty in the levy calculation as we explained earlier, we can not comment on whether the levy the PPF charges is fair. But the results give us a hint of how the PPF guarantee option and the PPF levy option may inversely link to each other.

The value of the PPF cut option becomes more negative if the sponsor invests more in stocks. This is intuitive as more insolvency events occur when the sponsor's assets become riskier. Thus pension participants need to bear the PPF cut more frequently in our scenarios. For the same reason, the indexation option decreases in value when the sponsor becomes riskier, as the PPF imposes a lower cap on the indexation for the pensions in payment than the UK indexation rules.

The overall effect of the sponsor's asset portfolio can be read from the residue options. When the sponsor's asset portfolio becomes riskier, the value of the surplus option decreases while the value of deficit option becomes more negative. Both contribute to a lower total residue option value. We therefore can conclude that a risky sponsor portfolio will worsen the participant's situation as they end up with less sponsor guarantee but face higher possibility of potential benefits cut. The PPF provides the protection to participants and cushions the impact of a sponsor becoming riskier.

5.2.4 Pension scheme's initial funding ratio

Now we examine how the initial funding ratio of the pension scheme will affect the HBS. In the benchmark case, Policy 5, the initial funding ratio is 1.175. We include the initial funding ratios of 0.8 and of 1.5 to represent a bad and a good initial pension funding positions, respectively.

The results are presented in Table 23. As one would expect, an improved initial funding ratio reduces the chance and the size of the pension funding shortfall, resulting in less burden on the sponsor or the PPF in the case of sponsor insolvency. For this reason, the value of the sponsor guarantee option decreases by 60% when the initial funding ratio increases from 1.175 to 1.5. In contrast, the value of the sponsor guarantee option increases by 100% when the initial funding ratio worsens from 1.175 to 0.8. The value of the PPF guarantee option changes in a similar fashion. Therefore the sponsor and the PPF will benefit from a better initial funding position.

The PPF levy depends on the funding shortfall of the pension scheme. The larger the shortfall the higher the levy is. An improved initial pension funding ratio reduces the levy charges. Therefore the PPF levy option has a lower value when the initial funding ratio becomes higher. A better funded pension scheme will benefit from a lower PPF levy charge.

A higher initial pension funding position is also a good news for participants. Their benefits are less likely to be cut by the PPF and the cut will be to a smaller extent. Thus the PPF cut option has a less negative value when the initial funding ratio increases. In addition, the cap for the indexation of the pensions in payment is higher for participants in the pension scheme than in the PPF.

initial funding ratio	0.8	1.175	1.5
Pension Assets	80	117.5	150
Sponsor guarantee	30.62	14.43	5.69
PPF guarantee	14.78	7.81	4.93
Total	125.40	139.74	160.62
Pension Liabilities	100.00	100.00	100.00
UK indexation	21.36	21.54	21.65
PPF levy	1.65	0.26	0.27
PPF cut	-1.45	-1.13	-0.89
Residue	2.29	19.23	40.02
Surplus option	9.77	22.97	42.53
Deficit option	-7.48	-3.74	-2.51
Total	123.85	139.89	161.05

Table 23: Sensitivity of the HBS to the initial pension funding ratio. UK Policy 5 is used. In the benchmark case the sponsor has an initial funding ratio of 1.175.

The aggregate effects result in a higher surplus option value and less negative deficit option value in the case of the higher initial funding ratio. This increases the total residue option value and all the stakeholders benefit from the high initial funding ratio.

5.2.5 Pension asset portfolio

The returns on stocks and bonds are different and varies each year. The stock returns are highly volatile whereas the bond returns are less so. How a pension scheme allocates its assets between stocks and bonds will directly impact the evolution of the pension scheme's assets, thus affecting the results of the HBS. In our model, we assume that the benchmark Policy 5 invests 50% of the pension assets in stocks and 50% in bonds, with assets readjusted to this proportion each year. Now we include two more cases with one invests all the pension assets in stocks and the other invests all in bonds. The results of the HBS are presented in Table 24.

The sponsor needs to pay contributions to recover pension funding shortfalls. There is a higher probability of pension funding shortfall when the pension scheme has a higher percentage of assets in stocks, placing more burdens on the sponsor. This is reflected in the value of the sponsor guarantee option on the HBS. When the pension scheme increases the proportion of its assets invested in risky stocks from 0% to 50%, the benchmark policy, the value of the sponsor guarantee option increases by 80%. The sponsor bears partially the pension funding shortfall resulting from pool performance of the risky assets. When the pension asset portfolio consists of the stocks only, the sponsor guarantee option increases by a further 43% in value.

The value of the PPF guarantee option has a similar pattern as the value of the sponsor guarantee option. The PPF, like the sponsor, shoulders the investment losses from the risky pension asset portfolio. When 100% of the pension assets are invested in stocks, the value

Pension assets in stocks (%)	0	50	100
Pension Assets	117.5	117.5	117.5
Sponsor guarantee	10.16	14.43	20.61
PPF guarantee	1.67	7.81	14.73
Total	129.32	139.74	152.84
Pension Liabilities	100.00	100.00	100.00
UK indexation	21.78	21.54	21.47
PPF levy	0.24	0.26	1.02
PPF cut	-0.68	-1.13	-1.22
Residue	8.53	19.23	32.44
Surplus option	10.10	22.97	39.50
Deficit option	-1.57	-3.74	-7.06
Total	129.87	139.89	153.71

Table 24: Sensitivity of the HBS to the pension scheme's asset portfolio. UK Policy 5 is used. In the benchmark case the pension scheme invests 50% assets in stocks and 50% assets in bonds.

of the PPF guarantee option increases by 88% compared to the benchmark case. The high percentage of increase may be due to the fact that big pension funding shortfalls from the risky pension asset portfolio deplete the sponsor's equity quickly thus increasing the insolvent probability. The burden shifts from the sponsor to the PPF, resulting in a large increase in the value of the PPF guarantee option.

The changes in the value of the PPF levy option and the PPF cut option are in line with the increase in the proportion of the pension assets invested in stocks. The riskier the pension scheme invests its assets, the higher the PPF levy the pension scheme has to pay. Pension scheme's participants also bears more risk of benefit cut as there is a higher probability of sponsor insolvency and PPF taking over of the scheme.

Although the increase in the pension asset risk is not a good policy for the sponsor and the PPF, it may benefit the pension scheme's participants. The stock returns have large volatilities, therefore a pension asset portfolio with a high proportion in stocks can generate either good returns or severe losses. Since the sponsor and the PPF commit to recover the pension funding shortfall (partially in the PPF case), participants only suffer limited loss, i.e. the PPF cut and the lower indexation cap for pensioners. However, participants will enjoy the funding surplus when the risky investments turn out to generate good returns.

The above analysis is also reflected in the residue options. Both surplus and deficit options increase in absolute value when the pension scheme invests more in stocks. Since the increase in the surplus option value is larger than the decrease in the deficit option value, the residue option value becomes larger.

5.2.6 Summary of the sensitivity analysis

We have presented the sensitivity analysis of the HBS by modifying one of the assumptions predetermined in the model and keeping the rest unchanged. Table 25 provides a review of how each of the modified variables affects the value of the embedded options implied by the pension policy.

	$D_0/S_0 \uparrow$	$S_0/A_0 \uparrow$	$\alpha_S \uparrow$	ifr $\uparrow$	$\alpha_A \uparrow$
Pension Assets	-	-	-	↑	-
Sponsor guarantee	↓	↑	↓	↓	↑
PPF guarantee	mix	↓	↑	↓	↑
Total	↓	↑	↓	↑	↑
Pension Liabilities	-	-	-	-	-
UK indexation	↓	↑	↓	↑	↓
PPF levy	mix	↓	mix	↓	↑
PPF cut*	↓	↑	↓	↑	↓
Residue	mix	↑	↓	↑	↑
Surplus option	↓	↑	↓	↑	↑
Deficit option*	↑	↑	↓	↑	↓
Total	↓	↑	↓	↑	↑

Table 25: Summary of the sensitivity analysis. The table presents the directions of change in the value of embedded options after an assumption is modified. D_0/S_0 is the initial leverage ratio, S_0/A_0 is the relative size of the initial sponsor's assets and the initial pension's assets, α_S is the proportion of the sponsor's assets invested in stocks, ifr represents the initial funding ratio of the pension scheme and α_A is the proportion of the pension's assets invested in stocks. * The PPF cut and the deficit option have negative values. Up means the values become less negative and down more negative.

6 Valuing the embedded option implied by Dutch pension policies in the multi-period model

In the previous section, we valued the embedded option of pension contracts and constructed the holistic balance sheet. The HBS allows us to study the value transferred between the stakeholders of the pension scheme as a result of introducing a new policy feature. We can use the HBS approach as a quantitative tool to compare different pension policies. In this section, we first build an HBS for the pension policies with typical Dutch steering tools and adjustment mechanisms, namely the sponsor guarantee contribution, the conditional indexation, and recovery contribution. Like for the UK policies, we add each one of the features and build the HBS step by step and conduct the sensitivity analysis.

The aim is to compare policies from UK and the Netherlands, thus we keep the assumptions about the pension scheme, the demographic composition, and the survival probabilities the same as in the previous section. The only difference is what kind of tools are available to the scheme's trustees.

6.1 Policies

The Dutch policies we will build the HBS for are summarised in Table 6.1. We keep Policy 1 from the previous section as our baseline policy. The characteristic of each new steering tools and adjustment mechanism will be explained in details when we present the results.

	Price indexation	Sponsor support	Credit risk	Employee contribution
Policy 1	full			
Policy 6	Conditional			
Policy 7	Conditional	Sponsor support		
Policy 8	Conditional	Sponsor support	Credit risk	
Policy 9	Conditional	Sponsor support	Credit risk	Employee contribution

Table 26: The pension policies with Dutch tools.

For the ease of comparison, we present the results of the HBS for Policy 1 in Table 27. Note that Dutch DB pension schemes link indexation to average wage growth, whereas in the baseline policy, we employ the UK indexation linked to the price level. We will keep to link the indexation to the price level for the Dutch policies in this section, so that our comparison of UK and Dutch policies are not influenced by the differences of price and wage levels.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
		Full indexation		32.28
		Residue		-14.44
		Surplus option	16.31	
		Deficit option	-30.74	
Total	117.50	**Total**		117.85

Table 27: The HBS for Policy 1. Full indexation linked to price level is given

6.1.1 The HBS for Policy 6 (conditional indexation)

To maintain a healthy funding ratio, Dutch pension schemes can employ a conditional indexation policy. How much indexation is given to participants depends on the actual funding ratio. If the funding ratio is above the predetermined cap, the full indexation is given. There is also a floor under which if the funding ratio falls, no indexation will be given. If the funding ratio is between the floor and the cap, only a proportion of the full indexation is given. The proportion equals the ratio of the difference between the funding ratio and the floor to the difference between the cap and the floor. We can express the conditional indexation as:

$$I_t^c = \begin{cases} I_t^f & \text{if } C < FR_t; \\ \frac{FR_t - F}{C - F} \cdot I_t^f & \text{if } F < FR_t < C; \\ 0 & \text{if } FR_t < F, \end{cases} \tag{58}$$

where C and F are the cap and floor, respectively, I_t^f is the full indexation at time t, I_t^c the conditional indexation, and FR_t the funding ratio. In the model, the floor and the cap are set at 1.05 and 1.30, respectively. By giving conditional indexation, the pension scheme avoids being too generous and can recover from a low funding ratio quickly.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
		Conditional indexation		10.74
		Residue		7.13
		Surplus option	19.27	
		Deficit option	-12.14	
Total	117.50	**Total**		117.87

Table 28: The HBS for Dutch Policy 2. Conditional indexation depending on the pension funding ratio is given.

The results of the HBS for Policy 6 are presented in Table 28. The value of the conditional indexation option is only 1/3 of that of the full indexation option in the HBS of Policy 1. This greatly improves the funding status of the pension scheme, as the residue option value increases from -14.44 to 7.13. This is partly due to the increase in the value of the surplus option but mostly because of the reduction in the negative value of the deficit option. From participants' perspective, the conditional indexation is a worse contract than the full indexation. However, just because of the sacrifice participants make when the funding ratio is low, there is a higher probability that a funding surplus is left when the scheme is liquidated.

6.1.2 The HBS for Policy 7 (sponsor support)

Now we introduce the sponsor support to form Policy 7. The sponsor support in a Dutch pension policy can be compared with the recovery plan in a UK pension policy. In both cases, the sponsor pays contributions to improve the pension scheme's funding position. Instead of paying to diminish the pension funding shortfall over a span of years, the sponsor of a Dutch pension scheme is obliged to make up the difference as soon as the funding ratio is below a certain level. We call this critical level the guaranteed funding ratio FR^g. To model the sponsor support contributions, we need to take into account if the sponsor can afford the payment. There is a chance that the amount of the contribution to recover the funding ratio to the guaranteed level is so big, that the sponsor can not afford to pay or will be in a stressed situation after the payment. To avoid these situations, we set up rules so that the amount a sponsor contributes in a year can never exceeds one quarter of its equity. We express the contribution as

$$C_t^{sup} = \begin{cases} (FR^g - FR_t) \cdot B_t & \text{if } 0 < (FR^g - FR_t) \cdot B_t \leq \frac{1}{4}(S_t - D_t) \\ \frac{1}{4}(S_t - D_t) & \text{if } 0 < \frac{1}{4}(S_t - D_t) < (FR^g - FR_t) \cdot B_t \\ 0 & \text{otherwise,} \end{cases} \tag{59}$$

where C_t^{sup} is the contribution payment of the sponsor support, B_t the total benefits, S_t the sponsor's assets and D_t the sponsor's debts. Our model assumes FR^g to be 0.96. Like for Policy 3, the model for Policy 7 assumes no credit risk of the sponsor. The sponsor is still running as a going concern with a negative equity value and it will only contribute to the sponsor support when its equity is in the positive territory.

Assets		**Liabilities**		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor support	10.67	Conditional indexation		11.59
		Residue		16.98
		Surplus option	22.86	
		Deficit option	-5.88	
Total	128.17	**Total**		128.57

Table 29: The HBS for Policy 7. Conditional indexation depending on the pension funding ratio is given. Sponsor support is required to ensure the funding ratio to be at least 0.96.

The HBS is presented in Table 29. The sponsor support option is a conditional asset and is presented on the asset side of the HBS. The sponsor support option has a value of 10.67, improving the HBS. This contract is a good deal for participants not only for there is additional contributions from the sponsor when the funding ratio is low, and also for it increases the value of the indexation option. When the sponsor guarantees part of the funding shortfalls, the funding ratio can recover more quickly than without such a support. As a result, participants are more likely to receive higher indexation. Intuitively, the sponsor support policy decreases the value of the deficit option compared with Policy 6, i.e. participants are partially protected from pension underfunding. Thus the value of the total residue option increased by 130% to 16.98.

6.1.3 The HBS for Policy 8 (credit risk)

Policy 8 introduces the credit risk of the sponsor. No contributions can be received any longer once the sponsor becomes insolvent. The HBS is presented in Table 30. As one can imagine, the assumption of credit risk will reduce the value of the sponsor support option. However the reduction is limited to a small extent ($< 4\%$). The reason is the same as in Policy 4 when we introduced the credit risk to the UK policy. Given our assumptions of how the sponsor insolvency is modelled, the cash flows of the sponsor support when there is credit risk differs very slightly from that in the case of no credit risk. For the same reason, the values of the conditional indexation and the residue options do not change much.

6.1.4 The HBS for Policy 9 (sustainability cut)

In UK, the PPF plays an important role in protecting most part of the benefits participants are entitled to if the sponsor becomes insolvent. On the other hand, the indexation of participants are guaranteed up to a certain cap depending on the status of the individuals. If these contracts can be fulfilled, the biggest loss participants will bear is the cut of benefits imposed by the PPF if the PPF takes over the scheme.

In comparison, participants in Dutch schemes do not enjoy the protection if the sponsor is insolvent and there is a pension funding shortfall. In addition, the indexation can be conditional on the funding ratio. Some contract even requires the participant to sacrifice so that the funding

Assets		Liabilities		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor guarantee	10.27	Conditional indexation		11.58
		Residue		16.60
		Surplus option	22.78	
		Deficit option	-6.19	
Total	127.77	**Total**		128.18

Table 30: The HBS for Dutch Policy 8. Conditional indexation depending on the pension funding ratio is given. Sponsor guarantee when the funding ratio to minimum 0.96. There is the sponsor credit risk.

ratio will return a certain level. Policy 9 will address this issue and introduce the so-called sustainability cut contract.

As the name suggests, the sustainability cut allows the benefits to be cut to ensure the sustainability of the scheme. The cut is through a negative indexation of the benefits. Once the funding ratio falls below a critical level, the floor, the sustainability cut contract is initiated and a plan to recover the funding ratio to be at least at the floor level is set up. To avoid abrupt losses to participants and the resulting unfairness across generations[7], the sustainability cut is executed across a few years. There is an equal step of increase in the funding ratio in each year which the pension scheme needs to achieve. If not, the benefits will be cut so that funding ratio will reach the required milestone in that particular year. After all the cuts and the asset returns are taken into account, the funding ratio should be at least at the floor level at the end of the planned period.

Assets		Liabilities		
Pension Assets	117.50	Pension Liabilities		100.00
Sponsor guarantee	7.60	Conditional indexation		12.35
		Sustainability cut		-12.40
		Residue		25.56
		Surplus option	25.73	
		Deficit option	-0.17	
Total	125.10	**Total**		125.51

Table 31: The HBS for Dutch Policy 9. Conditional indexation depending on the pension funding ratio is given. Sponsor guarantees to contribute so that the funding ratio is at least 0.96. There is the sponsor credit risk. The pension scheme employs the sustainability cut policy.

Policy 9 sets the floor at 1.05 and the length of the sustainability cut to be 3 years. We give

[7]The pensioners suffer from less income. Younger generations has less accrued benefits and they have longer time to wait and may gain back the loss finally if there is the catch-up indexation policy. For older generations, the chance to gain back through such policy will be lower.

an example of how the plan can be set up. Suppose the funding ratio is 0.99 now and it is the first time that the funding ratio is below 1.05. According to the rules above mentioned, each year the funding ratio needs to increase at least $(1.05 - 0.99)/3 = 0.02$. Thus the milestones for the funding ratio to recover to in the next three years will be 1.01, 1.03, and 1.05. If the funding ratio at the end of the first year turns out to be 1, the benefits will be cut. A negative indexation is then given as $1/1.01 - 1 = -0.0099$. Multiply this indexation by the total benefits will give how much cut participants have to bear at the end of the first year. The same procedure is practised each year till the end of the plan.

One more assumption we make in Policy 9 is that the sponsor support is exercised before the sustainability cut. For example, if the funding ratio falls down to below 0.96, the level sponsor will guarantee upto, the sponsor will pay the contribution first so that the funding ratio recovers to 0.96. Then the sustainability cut is initiated, and a plan with milestones of 0.99, 1.02, and 1.05 for each of the next three years is set up.

The HBS is presented in Table 31. The value of the sustainability support is calculated the same way as the indexation option. Since it is a cut of the benefits, the sustainability cut appears a negative figure on the liability side of the HBS. In our results, this option has a value of -12.40. Introducing the sustainability cut also reduce the conditional assets. The sustainability cut contract alleviates the burden on the sponsor, as the sponsor guarantee option has a value 26% less than that in Policy 8. Furthermore, the value of the deficit option is close to 0, resulting in a higher total residue option value.

The value of the sustainability cut option and that of the conditional indexation option have the same magnitude. One should notice that they have different economic values to the pension scheme. The sustainability cut is employed in the bad scenarios. The negative indexation is vital to the solvency of the pension scheme. On the contrary, the conditional indexation is given when the funding ratio is above the critical floor level. Therefore, the sustainability cut has a more economic value to the pension scheme than the conditional indexation.

6.1.5 Summary of the HBS for Dutch pension policies

Now we summarise the above results in Table 32 so one can review the impact of each new feature on the value of embedded options. From this table, we can see that the sponsor support benefits participants and improves the funding position. Participants sacrifice through the sustainability cut, which reduces part of the sponsor's obligation.

6.2 Sensitivity analysis

In this section, we present the sensitivity analysis for Policy 9, the benchmark policy with typical policy instruments of a Dutch DB pension scheme. We make the following assumptions for Policy 9:

1. The sponsor has an initial leverage ratio of 30%, i.e. the total debts are 30% of the total assets.
2. The initial sponsor's asset level is 1.5 times of that of the pension scheme's.
3. The pension scheme has the initial funding ratio of 1.175.
4. The pension scheme invests 50% of its assets in stocks and 50% in bonds.

	Policy 1	Policy 6	Policy 7	Policy 8	Policy 9
Pension Assets	117.50	117.50	117.50	117.50	117.50
Sponsor support			10.67	10.27	7.60
Total	117.50	117.50	128.17	127.77	125.10
Pension Liabilities	100.00	100.00	100.00	100.00	100.00
Indexation	32.28	10.74	11.59	11.58	12.35
Sustainability cut					-12.40
Residue	-14.44	7.13	16.98	16.60	25.56
Surplus option	16.31	19.27	22.86	22.78	25.73
Deficit option	-30.74	-12.14	-5.88	-6.19	-0.17
Total	117.85	117.87	128.57	128.18	125.51

Table 32: Summary of the HBS for the Dutch pension policies

5. The sponsor invests 110% of its assets in stocks and -10% in bonds.

We modify each one of the assumptions and keep the rest unchanged. The effect on the HBS is then examined and discussed in the following sections.

6.2.1 Sponsor's initial leverage ratio

As we explained in the sensitivity analysis for Policy 5, a typical UK policy, the initial leverage ratio influences the insolvent probability of the sponsor in our model. The higher the leverage ratio, the more likely the sponsor becomes insolvent. The results are presented in Table 33.

D_0/S_0	0	0.3	0.5	0.7
Pension Assets	117.50	117.50	117.50	117.50
Sponsor support	11.89	7.60	4.12	1.53
Total	129.39	125.10	121.62	119.03
Pension Liabilities	100.00	100.00	100.00	100.00
Conditional indexation	12.51	12.35	12.17	12.01
Sustainability cut	-9.11	-12.40	-14.99	-16.87
Residue	26.38	25.56	24.89	24.31
Surplus option	26.50	25.73	25.07	24.51
Deficit option	-0.12	-0.17	-0.18	-0.20
Total	129.78	125.51	122.06	119.45

Table 33: Sensitivity of the HBS to the initial sponsor leverage ratio ($\frac{D_0}{S_0}$). Policy 9 is used as the benchmark policy and has the initial leverage ratio of 30%.

When the initial leverage ratio increases from 0.3 to 0.7, the value of the sponsor support option decreases by 80%. The high leverage ratio limits the sponsor's ability to support the guarantee funding ratio and the high frequency of insolvency in the scenario set reduces sponsor's payments directly. Both factors contribute to the low value of the sponsor support option at high initial leverage ratio.

On the other hand, when the initial leverage ratio increases, the benefits are more likely to be reduced according to the sustainability cut. The cut at the leverage ratio of 0.7 is 36% higher compared to the benchmark. When the sponsor is not able to pay due to low equity or insolvency, participants are required to contribute more to recover the funding ratio to a healthy level.

The impact on the conditional indexation option and the residue options are relatively small compared to the sponsor support and the sustainability cut. Overall the high leverage ratio is not a good deal for participants, as they are left with less chance of sponsor's contribution and are more likely to sacrifice the indexation.

6.2.2 Relative size of initial sponsor assets to the pension assets

The relative size of the sponsor's assets compared to the pension scheme's also influences the sponsor's ability to contribute to the recovery of the funding ratio. Sensitivity of the HBS to the relative size is shown in Table 34.

S_0/A_0	0.8	1.5	2
Pension Assets	117.50	117.50	117.50
Sponsor support	5.56	7.60	8.42
Total	123.06	125.10	125.92
Pension Liabilities	100.00	100.00	100.00
Conditional indexation	12.25	12.35	12.39
Sustainability cut	-13.93	-12.40	-11.79
Residue	25.16	25.56	25.73
Surplus option	25.34	25.73	25.89
Deficit option	-0.18	-0.17	-0.17
Total	123.48	125.51	126.33

Table 34: Sensitivity of the HBS to the relative size of the sponsor's assets and the pension scheme's assets. Policy 9 is used as the benchmark and the assets of the sponsor is 1.5 times of that of the pension scheme.

Reducing the ratio of the initial sponsor's assets to the initial pension scheme's assets from 1.5 for the benchmark case to 0.8 leads to a 27% decrease in the value of the sponsor support option. When the ratio increases from 1.5 to 2, the value of the sponsor support option increases by 10%. The impact of changing the asset ratio on the value of the rest options are relatively small. Overall, the larger amount of the initial assets the sponsor has, the higher value the conditional indexation option has and the less negative the value of the sustainability cut option is. A sponsor with more assets benefits participants.

6.2.3 Sponsor asset portfolio

Sensitivity analysis of the HBS to the sponsor asset portfolio is analysed and the results are presented in Table 35.

Sponsor assets in stocks (%)	0	50	100	110	150
Pension Assets	117.50	117.50	117.50	117.50	117.50
Sponsor support	13.59	12.08	8.41	7.60	4.70
Total	131.09	129.58	125.91	125.10	122.20
Pension Liabilities	100.00	100.00	100.00	100.00	100.00
Conditional indexation	12.56	12.52	12.39	12.35	12.21
Sustainability cut	-7.80	-8.97	-11.77	-12.40	-14.59
Residue	26.71	26.42	25.71	25.56	25.02
Surplus option	26.81	26.54	25.87	25.73	25.21
Deficit option	-0.09	-0.12	-0.16	-0.17	-0.19
Total	131.47	129.96	126.33	125.51	122.64

Table 35: Sensitivity of the HBS to the sponsor's asset portfolio. Policy 9 is the benchmark. In the benchmark the sponsor invests 110% of its asset portfolio in stocks.

The insolvent probability of the sponsor increases when the sponsor's asset portfolio becomes riskier. Thus the impact of a riskier sponsor asset portfolio on the HBS should be in line with a sponsor with a higher leverage, or with a relative small amount of assets compared with the pension assets. Indeed, the value of the sponsor support option decreases when the sponsor invests a higher proportion of its assets in stocks. The value decreases by 38% when the percentage of sponsor assets in stocks increases from 110% to 150%. The reduction in sponsor support option value is reflected by the increase in the absolute value of the sustainability cut option on the liability side of the HBS. The impacts on the conditional indexation option and the residue options are comparatively small.

Now we have analysed the sensitivity of the HBS to the factors that directly impact the sponsor's ability to contribute and the insolvent probability, which are the initial leverage ratio, relative asset size, and the asset portfolio of the sponsor. All the three results have one thing in common: a decrease in the value of the sponsor option is accompanied with an increase in the value of the sustainability cut to a similar extent. In Policy 9, both the sponsor and participants share the burden of maintaining the funding ratio above a healthy level. When the sponsor's ability to contribute is lowered by the change in one of the assumptions, the burden on the sponsor shifts to participants, and participants are more likely to bear the benefit cuts.

The conditional indexation option is less sensitive to the adjustment of one of the three assumptions. The overall effect is that the value of the residue option is more or less stable around the benchmark level.

6.2.4 Pension scheme's initial funding ratio

From now on, we start to modify the assumptions about the pension scheme and examine the effect on the results of the HBS. The first assumption we adjust is the initial funding ratio. A lower initial funding ratio worsens the pension funding position, both the sponsor and participants are more likely to contribute to the recovery of the pension scheme. On the contrary, a higher initial funding ratio improves the pension funding position, the sponsor is less likely to contribute to the recovery of the pension scheme, while participants benefit in the form of reduced sustainability cut and higher conditional indexation. The results are shown in Table 36.

Initial funding ratio	0.8	1.175	1.5
Pension Assets	80.00	117.50	150.00
Sponsor support	21.00	7.60	3.13
Total	100.99	125.10	153.13
Pension Liabilities	100.00	100.00	100.00
Conditional indexation	6.88	12.35	19.11
Sustainability cut	-21.87	-12.40	-7.95
Residue	16.38	25.56	42.46
Surplus option	16.54	25.73	42.62
Deficit option	-0.16	-0.17	-0.17
Total	101.39	125.51	153.61

Table 36: Sensitivity of the HBS to the initial pension funding ratio. Policy 9 is the benchmark. In the benchmark case the sponsor has an initial funding ratio of 1.175.

The value of the sponsor support option increases by 176% when the initial funding ratio decreases from 1.175 to 0.8, and the value decreases by 59% when the ratio increases from 1.175 to 1.5. The changes in the absolute value of the sustainability cut option are 76% and 36%, respectively. The value of the conditional indexation option reduces by 44% at the ratio of 0.8 and increases by 55% at the ratio of 1.5, compared to the benchmark. The overall effect is increased value of the residue option when the initial funding ratio increases, mainly because of the increase in the value of the surplus option.

6.2.5 Pension asset portfolio

The last assumption we adjust is the pension scheme's asset portfolio. The asset portfolio consists of stocks and bonds. The higher proportion of assets in stocks, the riskier the asset portfolio is. Risky pension assets increase the uncertainty of future funding position, both the sponsor and participants are subjected to increased probability of larger contributions in the future. The results are presented in Table 37.

As compared to the benchmark, an all bond portfolio reduces the value of the sponsor support by 81% and the absolute value of the sustainability cut option by 54%. On the contrary, an all stock portfolio increases the (absolute) values by 111% and 72%, respectively. The effect of the portfolio composition on the value of the conditional indexation is relatively small.

Pension assets in stocks (%)	0	50	100
Pension Assets	117.50	117.50	117.50
Sponsor support	1.41	7.60	16.06
Total	118.91	125.10	133.55
Pension Liabilities	100.00	100.00	100.00
Conditional indexation	12.96	12.35	11.81
Sustainability cut	-5.69	-12.40	-21.30
Residue	11.68	25.56	44.42
Surplus option	11.70	25.73	44.77
Deficit option	-0.02	-0.17	-0.35
Total	118.95	125.51	134.93

Table 37: Sensitivity of the HBS to the pension scheme's asset portfolio. Policy 9 is the benchmark. In the benchmark case the pension scheme invests 50% assets in stocks and 50% assets in bonds.

The total residue option increases in value when the proportion of pension assets invested in stocks increases, which is mainly due to larger value of the surplus option. The value of the deficit option reduces by less than 0.5.

6.2.6 Summary of the sensitivity analysis

The results of the sensitivity analysis are summarised in Table 38. From the table, we can see that the adjustment of each one of the assumptions mainly impacts the value of the sponsor support option and the sustainability cut option. The Dutch policy focuses on maintaining a healthy funding position. The burden of keeping the funding ratio above a target minimum level is born on both the sponsor and participants. Any adjustment in the assumptions that harms the sponsor's ability to contribute will result in an increase in the benefit cuts participants need to sacrifice. Both the sponsor and participants will benefit from an improved funding ratio such as an increase in the initial funding ratio, so that the sponsor support contributions and benefit cuts will be reduced (in absolute term).

We also notice that changes in most of the assumptions, apart from the initial funding ratio, have limited effect on the value of the conditional indexation and the deficit option. The conditional indexation is linked to the funding ratio and it gives no indexation when the funding ratio is lower than the minimum target and gives full indexation only when the funding ratio is above a certain high level. The downside risk of low funding ratio has already been taken by the sponsor and participants through the sponsor support and the sustainability cut. None of these assumptions, apart from the initial funding ratio, has an influence on the funding ratio above the minimum target level. Hence we observe limited impact of the changes in these assumption on the value of the conditional indexation option. For a similar reason, the impact on the value of the deficit option is also limited.

	D_0/S_0 ↑	S_0/A_0 ↑	α_S ↑	ifr ↑	α_A ↑
Pension Assets	-	-	-	↑	-
Sponsor support	↓	↑	↓	↓	↑
Total	↓	↑	↓	↑	↑
Pension Liabilities	-	-	-	-	-
Conditional indexation	↓ (limited)	↑ (limited)	↓ (limited)	↑	↓ (limited)
Sustainability cut*	↓	↑ (limited)	↓	↑	↓
Residue	↓ (limited)	↑ (limited)	↓ (limited)	↑	↑
Surplus option	↓ (limited)	↑ (limited)	↓ (limited)	↑	↑
Deficit option*	↓ (limited)	↑ (limited)	↓ (limited)	↑	↓ (limited)
Total	↓	↑	↓	↑	↑

Table 38: Summary of the sensitivity analysis. The table presents the directions of change in the value of embedded options after an assumption is modified. D_0/S_0 is the initial leverage ratio, S_0/A_0 is the relative size of the initial sponsor's assets and the initial pension's assets, α_S is the proportion of the sponsor's assets invested in stocks, ifr represents the initial funding ratio of the pension scheme and α_A is the proportion of the pension's assets invested in stocks. * The sustainability cut and the deficit option have negative values. Up means the values become less negative and down more negative.

7 Comparison of Policy 5 and Policy 9

Policy 5 and Policy 9 include typical tools available to DB pension schemes in UK and the Netherlands, respectively. Now we present results of Policy 5 and Policy 9 side by side, so that the policy instruments from UK and the Netherlands can be compared.

The first impression of Table 39 is that Policy 5 has higher values (12% more in total) of the conditional assets and the conditional liabilities. Both policies provide recovery contributions from the sponsor, whereas the value of the sponsor guarantee option in Policy 5 almost doubles the value of the sponsor support in Policy 9. This is partly because Policy 5 sets up a goal of full recovery (funding ratio of 1), whereas Policy 9 targets a funding ratio of 0.96, and partly because contributions from participants through the sustainability cut in Policy 9 help reduce sponsor's obligations.

In addition, Policy 5 contains an extra protection instrument for participants, the PPF guarantee, which insures a big proportion of the benefits if the sponsor becomes insolvent. All of these contribute to a higher level of total assets on the HBS of Policy 5.

Besides, Policy 5 has a more generous indexation policy than Policy 9, 74% higher in the value of the indexation option. The indexation in Policy 5 is the minimum of the price level and 2.5% (5% for the pensions in payment) but the indexation in Policy 9 depends on the funding ratio. The impact on participants of Policy 9 is further complicated by the negative

	Policy 5	Policy 9
Pension Assets	117.50	117.50
Sponsor guarantee/support	14.49	7.60
PPF guarantee	7.90	
Total	139.88	125.10
Pension Liabilities	100.00	100.00
Indexation	21.50	12.35
Sustainability cut		-12.40
PPF levy	0.52	
PPF cut	-1.13	
Residue	19.12	25.56
Surplus option	22.94	25.73
Deficit option	-3.82	-0.17
Total	140.05	125.51

Table 39: The HBS of Policy 5 (UK) and Policy 9 (NL).

indexation through the sustainability cut contract. The aggregate value of the both positive and negative indexation options is close to 0 for Policy 9 versus 21.5 in Policy 5.

In exchange of the PPF protection, the pension scheme in Policy 5 bears the cost of the PPF levy, and participants endures benefit cut. However, the values of the PPF levy option and the PPF cut option are considerably less than the value of the PPF guarantee option. The PPF protection is a good deal for the pension scheme.

The steering tools available to both Policy 5 and Policy 9 result in high surplus option values and low deficit option values in the absolute term. Thus the total values of the residue options of both policy are high, 19.12 for Policy 5 and 25.56 for Policy 9.

However, the figure itself does not tell us whether a policy is a good or bad deal for the stakeholders of the pension scheme, as manifested by the residue option. If we consider the pension deals a zero sum game, one's gain will be at the expense of the others. Therefore the judgement of a policy should be from the prospective of a particular interest group.

Participants Based on our analysis, Policy 5 is a better deal for participants than Policy 9. In Policy 5, participants receive a higher indexation, and the pension funding shortfall is guaranteed between the sponsor and the PPF. The benefits are never cut unless the sponsor becomes insolvent and the PPF takes over the scheme. Even so, the losses participants bear are to a limited extent. On the contrary, in Policy 9, participants receive less indexation conditional on the funding position, bear benefit cuts to contribute to the scheme's recovery, and do not enjoy any protection if the sponsor becomes insolvent.

Sponsor As seen in Table 39, the sponsor of Policy 5 clearly bears a heavier burden than the sponsor of Policy 9. The sponsor of Policy 5 is the only one who contributes to the recovery of funding shortfalls given it is still solvent, whereas the sponsor of Policy 9 shares the obligation with participants. The sponsor guarantee, as a liability, worsens the sponsor's financial situation and increases the insolvent probability. Thus Policy 5 is a worse contract from the sponsor's perspective than Policy 9.

PPF Since this section compares Policy 5 and Policy 9 and there is no PPF protection deal in Policy 9, we leave our comments on the PPF in the section of general discussion.

The main difference between Policy 5 and Policy 9 maybe the different priorities of the pension schemes. Both schemes aim to have enough assets to cover the liabilities. Policy 5 focuses on the guarantee of participants benefits. The indexation is more generous and there is never a benefit cut if there is a funding shortfall. The recovery of the funding position solely relies on the sponsor's ability to contribute. This imposes a stress on the sponsor's financial situation. On the other hand, Policy 9 targets to maintain a healthy funding ratio. The obligation is split between the sponsor and participants, with the sponsor guarantees up to a level and participants bear the rest. Participants of Policy 9 do not enjoy as high indexation as those of Policy 5, and their final benefits also depend on the sponsor's ability to support.

In addition, Policy 5 has a unique protection of the benefits if the sponsor becomes insolvent. The PPF can be perceived to take place of the sponsor's position to guarantee part of the funding position. Only then participants will sacrifice part of their benefits. This is a very good deal compared to Policy 9, as Policy 9 has no such mechanism to protect participants if the sponsor defaults.

Before we draw some conclusions, one should note that there are some other adjustment tools available to Dutch pension schemes, such as the catch up indexation. Some of the tools are used to make sure participants can recover some of their losses when the funding position improves in a better economic condition. These instruments are in line with the priority of the Dutch schemes to maintain a sustainable funding ratio and give the scheme's trustees more flexibility in the trade off between the indexation and the funding position.

In conclusion, Policy 5 is a better deal than Policy 9 for participants, as it is more generous and there is an external protection mechanism in place. Although both policies impose an obligation on the sponsor to contribute to the recovery of the funding shortfalls, from the sponsor's perspective, Policy 9 is better, as part of the obligation is shared with participants through the sustainability cut and conditional indexation.

8 Discussion

This thesis evaluates the UK DB pension policies from a Dutch prospective. A special feature of the UK DB pension policy is the PPF protection, which guarantees a certain level of the benefits if the sponsor of the pension scheme becomes insolvent and there is a pension funding shortfall. In the thesis, we first derived an analytical formula to value the PPF guarantee option in a one-period model, then we demonstrated how an HBS can be built for the one-period model. Both the analytical formula and the one-period model require assumptions which reduce many real-life features of a pension scheme. Thus we implemented a multi-period model based on the value-based ALM to value the embedded options implied by each pension contracts. This multi-period model is based on a stylised pension scheme with Dutch pension characteristics. With the results, we built the HBS for both UK and Dutch policies and compared the impact of the pension deals on different stakeholders.

We discussed the results in previous sections. To avoid repetition, we discuss issues that have not been mentioned before in this section.

Insolvency One prerequisite condition for the PPF guarantee is the sponsor insolvency event. In our thesis, we define insolvency as the sponsor's assets being less than its debts at year 1. This way of modelling insolvency is based on the Merton's framework of credit risk. In the Merton model, a company's capital consists of debt and equity. The model views the equity value of a company as the payoff of a call option on the company's asset with the strike price of the debt value at the maturity date. By assuming the dynamics of the asset value, we can derive the insolvent probability based on the Black-Scholes formula.

Additional assumptions are required. We assume that it is feasible to map all the debts to a zero-coupon bond with a certain maturity. Besides, the sponsor can only default at the maturity date but not earlier. These assumptions restrict the application of the analytical formula derived in our thesis. Extensions of the model can be developed, which looses the assumption of the debt maturity and allows early defaults.

The Merton model is one of the structural models, as this class of models focus on the financial structure of a company. Apart from the structural models, there are other approaches to model the insolvency event. One approach is the intensity-based models that treat insolvency as exogenous events driven by a stochastic process, such as a Poisson jump process. Another approach is to derive the insolvent probability from the spread between the yield curve of the company's bonds and the risk free bonds. This approach requires estimation of the recovery rate of a company's debt, i.e. the percentage of principal the debt-holders can receive in the event of company default. With the recovery rate, we can derive the insolvent probability implied by the yield spread between the company's bond and risk-free bond in the market.

The PPF uses the intensity-based models in their internal model to simulate the sponsor insolvency events (Charmaille et al., 2013). In their model, the credit rating of the sponsor transits each year between eight different levels ranging from AA to D, where D constitutes a default. The probability of transition depends on the sponsor's current rating, its industry sector, the current state of the economy and the company's own idiosyncratic risk.

HBS The EIOPA proposed to use the HBS as a quantitative tool to compare various pension systems across Europe. The HBS is an extension of the traditional balance sheet, which includes the values of the contingent assets and liabilities implied by the pension contracts. Different pension policies can thus be compared quantitatively.

When comparing the results of the HBS of different pension contracts, an important factor to consider is that the valuations are sensitive to the risk model and the scenario set. Therefore, the regulator may need to impose a risk model and a scenario set for the HBS analysis, so that different pension policies can be compared fairly.

PPF The thesis aims to evaluate the UK pension policies, especially the PPF protection, from the perspective of a Dutch pension scheme. In earlier sections, we concluded that the UK policy is more generous to participants but imposes a heavier burden on the sponsor, compared with the Dutch policy. Another important stakeholder we did not discuss is the PPF. The PPF collects levies, recovers assets from insolvent sponsors, takes over assets from schemes transferred to the PPF, and invests in financial markets to fund its existing and potential future benefit claims. The PPF, as a pubic corporation, is partly like a credit insurance business that would underwrite policies insuring the insolvency risk of the sponsors of DB pension schemes, and partly like an annuity business that would take on the assets and liabilities of the claimant schemes. Thus the PPF is exposed to the credit risk of the sponsors of eligible schemes and the risk arising from the asset-liability mismatch.

From the analytical formula and the HBS for the one-period model, we see that a high correlation of the sponsor asset return and the pension asset return results in a high value of the PPF guarantee option. When the sponsor's assets perform poorly, the pension scheme's assets tend to perform poorly too. The coincidence of insolvency event and pension funding shortfall suggests that the PPF is more likely to expect future claimants.

The asset return of the PPF may be also correlated with the sponsor asset return and the pension asset return. When the PPF faces a large surge in the number of new claimants in a bad economic condition, its own funding may also deteriorate due to the poor investment performance. The aggregate effect worsens asset-liability mismatch further on the balance sheet (current liabilities) and off the balance sheet (future claimants).

From the HBS of Policy 5, the PPF seems to charge too little levy for the promise it makes to the pension scheme. One should note that the value of the levy option in our model is calculated *ex post*, while the annual levy charges determined by the levy formula are calculated *ex ante*.

The mismatch between the values of the guarantee option and the levy option in the HBS may be due to the contradicting nature of the guarantee payoff and levy payments. The pension schemes that benefit most from the PPF protection are the ones with sponsors in riskier business. These schemes also 'benefit' from less aggregate levy payment because their lifetime before the PPF takeover is likely to be shorter (than the schemes with sponsors in less risky business), even though their annual levy payment may be higher.

The amount of levy a pension scheme has to pay is designed to reflect the risk inherent in the scheme and its sponsor (Appendix A). The riskier the sponsor, or the bigger the funding shortfalls, the higher levies the scheme has to pay. On the other hand, the higher levies worsen the funding position of the scheme, which in turn stresses the sponsor's financial situation through the higher sponsor guarantee contributions. This increases the insolvent probability and the chance that the PPF needs to takeover the pension scheme. A fair question is who will bear the cost if the levy structure and the guarantee promise appear to be unsustainable for the PPF.

Most DB schemes in UK are closed to new participants or new accruals. As more schemes are liquidated or taken over by the PPF due to sponsor's insolvency, the universe of the eligible schemes under the PPF protection is shrinking. If in the future there is a large funding shortfall, the remaining independent pension schemes will shoulder heavier burdens of the levy payment.

In addition, the PPF is run by a Board that is independent of UK government. Powers are given to the Board allowing it to manage levies and set investment strategies. This is "to ensure that the PPF would ... not have to be underwritten by the Government and ultimately taxpayers". Given the great social importance of the PPF, and the history of financial crisis, the PPF is an organization that is 'too big to fail' and its future losses may have to be underwritten by the taxpayer.

To make a well-informed judgement of the PPF protection, one should take into account many factors. Is the PPF sustainable? Will the society bear the cost if it turns out not to be so? Can it create moral hazards? Do the levy charges put a weak scheme into a weaker situation? How to minimise the cross-subsidies between schemes? All these questions are not easy to answer. In addition, the takeover process is not as easy as a single click in a computer model. The recent drama between the PPF and bankrupt Eastman Kodak highlights the difficulties and complexity surrounding the issue. For this thesis only evaluates the impact of the PPF protection on an individual pension scheme using the HBS approach, we can only conclude that the PPF protection is good for participants from a pension scheme with a weak sponsor.

9 Conclusion

The holistic balance sheet, as a quantitative tool, does allow us to evaluate pension policies with different features. Comparing the HBS for typical private defined benefit pension policies from both UK and the Netherlands, we conclude that the UK policy is a better deal for participants, but a worse deal for the sponsor, compared with the Dutch policy.

The UK policy is more generous in giving indexation of the benefits. Potential funding shortfalls are guaranteed by the sponsor. Additionally, the Pension Protection Fund underwrites the credit risk of the sponsor. Therefore, participants of a pension scheme with UK policy bear limited risks of losses of their entitled benefits.

In contrast, the Dutch policy gives indexation that depends on the actual funding ratio of the pension scheme. Participants and the sponsor share the burden of recovery contributions. There is no protection of the benefits if the sponsor is insolvent.

For the sponsor, the UK policy imposes a heavier burden than the Dutch one. In addition, the UK policy is less flexible for the scheme's trustees to choose the steering tools and adjustment mechanisms.

To fully evaluate the impact of adopting the UK policy, one should also take into account the PPF. Whether the PPF is sustainable? Is the structure of the levy charges fair? How to minimise the cross-subsidies between schemes? These are important questions to be addressed.

A The PPF levy formula

The PPF publishes how the levy charges are calculated in its annual publications (PPF, 2012c). The calculations are based on both the funding position of a pension scheme and the insolvency risk of its sponsor or sponsors. The PPF sets up a target of the total levies it collect during the years for three years and determine the parameters used in the levy formula accordingly. This means that the parameters vary from year to year. Therefore it is difficult to estimate the parameters beyond three years. To avoid the difficulties of the calculation, we assume that the parameters remain unchanged after three years.

The levy is the sum of two parts: the scheme-based pension protection levy (SBL) and the risk-based pension protection levy (RBL). The formula to calculate the SBL is:

$$UL \times SLM, \tag{60}$$

where UL is unstressed liabilities and SLM denotes 'scheme-based levy multiplier' that equals 0.000056 for 2013/14. Since we do not distinguish stressed and unstressed liabilities in our model, we use the present value of the pension liabilities as UL.

RBL is calculated as

$$U \times IR \times LSF \tag{61}$$

where U is the pension underfunding, IR is the measure of the insolvency risk, and LSF is the 'risk-based levy scaling factor' that equals 0.73 for 2013/14.

The PPF employs Dun & Bradstreet to provide monthly failure score of the sponsors of the pension schemes. According to the failure score each sponsor is categorised to one of ten risk bands. There is a LSF value assigned to each of the risk bands. The LSF ranges from 0.0018 for the least risky band to 0.04 for the band with highest risk. The failure score of a sponsor depends on its location, industry and other factors. The LSF for each risk band is in Table 40.

Risk band	1	2	3	4	5	6	7	8	9	10
LSF	0.0018	0.0028	0.0044	0.0069	0.011	0.016	0.0201	0.026	0.0306	0.04
Score*	100-99	98-96	95-92	91-87	86-73	72-66	65-46	45-38	37-30	29-1

Table 40: Table of LSF corresponding to the risk band. Note this is set for the year 2013/14. Future values probably will change. *Score is the Employer Failure Score the PPF uses to assign the pension scheme to a certain risk band

In our model, we only assume the assets and debts of the sponsor. Therefore we cannot assign the sponsor in our model to a specific risk band. To be prudent, we choose the LSF for the highest risky band for our calculation of RBL, which is 0.04.

In addition, to avoid the levy charges impose too much stress on the pension scheme's funding position, the PPF set up a cap for the RBL. The cap is

$$UL \times K, \tag{62}$$

where UL is the unstressed liabilities of the scheme, and K is the 'RBL cap' set by the PPF, which equals 0.0075 for 2013/14.

In summary, we calculate the annual PPF levy in our model as

$$Levy = SBL + RBL = UL \times SLM + min\{U \times IR \times LSF, UL \times K\}. \tag{63}$$

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