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Netspar THESES

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Essays in Pension Economics and Intergenerational Risk Sharing

PhD Thesis 2012-017

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ISBN: 978-90-5335-591-6

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ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Universiteit van Amsterdam
op gezag van de Rector Magnificus
prof. dr. D.C. van den Boom
ten overstaan van een door het college voor promoties
ingestelde commissie,
in het openbaar te verdedigen in de Aula der Universiteit
op woensdag 7 november 2012, te 11:00 uur

door

Siert Jan Vos

geboren te Hilversum

Promotiecommissie

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Voor pa en ma

Acknowledgements

The work on this thesis started in october 2007, when I started as a PhD-student of Roel Beetsma as part of the Netspar research project 'The macroeconomics of pension reform'. I was very interested in the topic, but didn't think it was a particularly high-profile one. While I was writing this thesis, that changed a lot, and 'pensions' has become a topic over which debates can rage with quite some fervor in the popular press.

In the years since the start, I've been working steadily at completing my thesis, resulting in this book as you hold it in your hands. During this time, the guidance and assistance of Roel have been invaluable. This goes both for the direct guidance in writing, improving, further improving the chapters of this thesis (our highscore is approximately 80 versions of chapter 4) and for the many discussions on the topics of this thesis and their potential policy implications.

The second person to whom I owe a lot for the completion of this thesis is Ward Romp. Sharing an office, you taught me how to do some decent programming, how to use LaTeX, how to deal with being stuck in a project (all very useful), a lot about mountaineering and horses (interesting) and how to deal with very direct criticism of my work (occasionally painful).

I also want to extend my gratitude to Wouter den Haan. The time and effort you invested in the economics skills of the PhD-students of the reading group have been of enormous help to gain insight in the models and methods used in much of the macroeconomic literature. Honestly, every PhD-student should be able to be part of such a reading group.

Further, a big thanks goes to the researchers of the Netspar macroeconomics theme for the great interaction and discussions at conferences, theme meetings and Netspar days, and to my colleagues at the MInt research group at the UvA for the countless informal discussions and advice on research, teaching and football during lunch, coffee, dinner and drinks.

Chapter 5 of this thesis owes its existence to MN. For the first six months of 2011, I did a research project at MN aimed at quantifying the effects of a number of proposals

for a new occupational pension contract. The report generated in that research is the basis from which I started working on Chapter 5.

Ik wil dit voorwoord graag afsluiten door iedereen te bedanken die me over de afgelopen jaren voortdurend gesteund heeft tijdens het schrijven van dit proefschrift. Jullie weten wel wie jullie zijn: pa, ma, mijn zussies, familie, vrienden en vriendinnen, begane collega's, uit de grond van mijn hart: bedankt!

Siert Jan Vos

November 2012

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Chapter 1

Introduction and Overview

This thesis is inspired by the developments over the last decades in pension schemes worldwide and specifically in the Dutch pension system. In this introduction, I describe these developments as they have unfolded in the Netherlands, although the general trends in terms of demography and financial markets that will be discussed hold on a worldwide scale.

In the Netherlands, the pension system is organized in three different parts, or 'pillars' as they are usually referred to. The first pillar, the AOW, is state sponsored and is intended to prevent old-age poverty. The level of the first pillar pension income depends only on years of residence and on marital status and is such that an individual without any additional pension income will be above the subsistence level. The second pillar of pension income provision is the occupational pension. Ninety percent of the Dutch employees are covered by some sort of second pillar pension scheme, while participation in occupational pension schemes is obligatory for about 70% of the employees. Finally, the third pillar consists of private additional savings for retirement.

The first pillar is financed on a pay-as-you-go (PAYG) basis, in such a way that the currently active part of the population pays the contributions that are used to finance the retirement benefits of the retirees. The second and third pillars are organised on a funded basis. In the second pillar, employees and employers pay contributions to a pension fund or insurance company that executes the occupational pension arrangement offered by the employer to the employee. These contributions are part of the employment contract and can be considered to be deferred wage income. Third pillar pensions consist of pension savings to provide additional retirement income - such as a pension savings account, life insurance, etcetera - that individuals privately decide on.

Currently there is substantial debate around the design of the Dutch pension system. The first driving force behind this debate is the changing demographic composition of the population. Following the baby-boom just after the second World War, birth rates have been declining steadily. At the same time, life expectancy has increased substantially. Although the increasing longevity brings us much happiness, it can be problematic for a pension system that is not designed to deal with it (see King (2004) for some accessible illustrations of changes in life expectancy). The combination of these two trends implies that, for a given retirement age, the fraction of the population that is retired has been increasing relative to the fraction that is working. This increase in the so-called dependency ratio is foreseen to continue over the next twenty to thirty years. This has caused a debate on how to deal with these demographic changes: if the retirement age remains fixed at its current level of 65 years, in the first pillar those of working age will have to contribute an increasing share out of their labour income to pay for the retirement benefits, while in the second pillar the same generations have to increase their savings for retirement, or accept lower pension benefits. At the same time questions arise on how to deal with the rapidly increasing life expectancy of current retirees. During the period when they were saving for retirement, their contribution levels were based on an expected length of life that is substantially lower than actual length of life has turned out to be. Should benefits to these retirees be reduced or should contributions by current workers be increased to cover the funding gap that this unexpected longevity causes?

The second important issue is the impact of financial markets on the pension system. After World War II, the amount of the second-pillar occupational pension savings has been growing steadily. The occupational system has gradually transformed away from a system in which most participants are in the working phase of their lives and annual contributions paid into the system are relatively large compared to the total amount of assets. Slowly the system has evolved to a mature one, in which an increasing fraction of the participants are close to or in the retirement phase of their lives and the total amount of accumulated assets in the system is very large. Hence, annual contributions into the system have become small compared to total assets. This implies that shocks in the financial markets have increasingly large consequences for occupational pensions. This increased sensitivity of the funded pension pillar to financial market shocks has become very clear over the last 20 years, with periods of both fast growth in pension assets and periods of rapid declines. These took place in particular during the dot-com crisis of 2000-2002 and the financial crisis that started in 2008.

As a consequence of these developments, for some years a debate has been going on about whether the Dutch pension system needs to be redesigned. Two reports were commissioned by the Dutch government to analyse the situation and the sustainability of the second pension pillar. The Frijns committee (2010) brought out a report on the asset and risk management by pension funds, while the Goudswaard committee (2010) analysed the sustainability of the system of occupational pensions. Both reports convincingly show the need for structural reforms of the system. Subsequently, the social partners and the Dutch government started negotiating the restructuring of both the first and second pillar pensions. This resulted in an agreement on principles in the summer of 2010. However, working out the details of the agreement has proved an arduous process, and no definitive results have been reached yet.

Overview

In this thesis, I analyse the design of pension arrangements, paying particular attention to the intergenerational risk sharing aspects of pension design. Chapters 2 and 3 deal with the optimal design of multipillar pension arrangement when taking into account multiple sorts of shocks and distortions. In these chapters, participation in the pension system is taken for granted. Chapter 4 investigates the decision to participate in a pension system, and the impact this has on pension arrangement design when participation is not mandatory. Chapter 5 performs a detailed analysis of some of the options that have been considered in the redesign of the Dutch occupation pension contracts.

Chapter 2 deals with optimal pension system design when taking into account the labour supply decisions that individuals make. The central question of chapter 2 is: 'How should a two-pillar pension system - with a PAYG first pillar and a funded second pillar - be designed when taking into account endogenous labour supply decisions'? In this chapter, the sources of uncertainty are productivity and financial market risks. In principle, the pension system designer would like to create the pension system in such a way that these risks are shared optimally by the different generations. However, the pension system may have distortionary effects. Here, the distortion concerns the labour supply decision. Specifically, if the contribution to the pension system is linked to the wage an employee earns, this lowers his net wage. This may induce the employee to supply more or less labour than in the absence of the pension contribution. If all employees face this same incentive, the resulting suboptimal aggregate labour supply distorts wages, capital returns and national production and decreases welfare for all individuals in the economy. This chapter shows that if such a distorting link from individual pension contributions exists, the optimal response of the pension system designer is to find an alternative way for the market economy to attain the socially optimal allocation. The solution is to link the contributions to the second pillar to the aggregate wage sum rather than the individual wage rate. This in fact imposes a lump-sum contribution on the working generation, thereby evading the distortion, while the risks inherent in wages can still be shared with the retired generation.

Chapter 3 also explores optimal pension system design, but from a very different perspective. In that chapter, instead of looking at behavioural distortions, uncertainty about demographic developments is taken into account. Specifically, in addition to the shocks of chapter 2 we include uncertainty about fertility and life expectancy (the mortality rate). Hence, four fundamental sources of risk are present in the model. Demographic uncertainty affects all macroeconomic relations. They are determinants of the amount of labour supply, wages, capital returns, national income, private and pension savings, the size of bequests and the relative size of transfers through the PAYG first pillar. Even though the model is highly stylized, the presence of demographic uncertainty renders it impossible to find constant pension system parameters that produce the optimal degree risk sharing arrangement implemented by a social planner. Therefore, a detailed numerical analysis is performed to determine how much risk sharing can be achieved in a two-pillar pension system and how closely the market economy combined with the pension arrangement can approximate the social planner's solution. It turns out that although the social planner solution can not be replicated, an appropriately designed pension system with a defined-benefit second pillar results in a very small welfare loss compared to the social planner's solution. Obviously, an open question is whether this finding is generalisable to a less stylised setting.

Chapter 4 deals explicitly with a very important aspect of collective pension arrangements that was assumed to exist in chapters 2 and 3: obligatory participation in collective arrangements. The question that chapter 4 poses is whether or under what circumstances collective funded pension arrangements are sustainable when participation by new employees is not mandatory. It is well known that from an ex-ante perspective collective pension arrangements can result in large welfare gains to participants because of the risk-sharing they provide. However, new employees that are required to enter into a collective arrangement may find that at the time of their entry, the financial position of the arrangement is not very good. Thus, additional contributions may be asked of them without corresponding additional entitlements being awarded. This raises the question whether this particular generation would be better off not entering into the collective arrangement and, if this is the case, whether it is possible to design a pension arrangement such that it becomes attractive to this generation to enter, while preserving some of the risk-sharing benefits among participating generations. The set-up is an infinite horizon model with two overlapping generations, where the young generation can choose to join the existing pension arrangement or to break the existing arrangement by saving for retirement privately. Once a young generation decides not to participate, the pension arrangement breaks down forever. The challenge for the designer of the pension arrangement to design it in such a way that the expected utility of joining is always at least equal to the expected utility of staying out. Whether it is possible to design an arrangement that is attractive for new young generations depends both on the volatility of the shocks against which the collective arrangement offers some protection (financial market risk in this case) and on the risk aversion of the young generation. We demonstrate that the collective arrangement breaks down when the volatility of the financial market shocks and risk aversion are relatively low, while for intermediate values of these parameters the arrangement can only be maintained if it provides less risk sharing than is socially optimal. In these circumstances, optimal risk sharing can only be achieved by making participation in the pension arrangement mandatory.

Finally, Chapter 5 is of direct importance for the current public debate about the redesign of collective pension contracts in the Netherlands. As indicated before, several options for redesigning the pension contract have been contemplated. The proposal that initially garnered most support was the proposal of a 'combined contract'. Under such a contract, pension entitlements would be split into a 'hard' and a 'soft' part, where soft entitlements would form the flexible shell around hard entitlements that would - in theory at least - be almost surely guaranteed. Chapter 5 performs a detailed analysis of three different ways in which such a combined contract could be implemented and it compares these three variants with pension results under the current contract. In particular, the analysis provides insight into how shocks are distributed across the current and future generations participating in the pension fund.

Under the first variant, accumulated entitlements are initially soft, but are converted into hard entitlements after a fixed number of years. Under the second variant, a fixed share of newly accrued entitlements are hard, while the remainder are soft. Under the third variant, newly accumulated entitlements are soft. If the funding ratio of the pension fund is sufficiently high, soft entitlements are transformed into hard entitlements. Using an asset-liability management (ALM) model of the pension fund, we simulate funding ratios, the degree of indexation awarded to both soft and hard entitlements as well as reductions in hard and soft entitlements when funding ratios become too low. The results show that under these new contracts, indexation is higher and more readily awarded than under the current contract. Hence, under these proposals the pension fund's assets tend to shrink more rapidly and the currently-retired generations benefit at the expense of the current young and future generations. Of the three new proposals, only the variant in which hard and soft entitlements are accrued in a fixed proportion (the second variant) is able to effectively guarantee that 'hard' entitlements indeed need to be reduced only in very rare circumstances. Moreover, under the other two variants young generations hold almost all of the soft entitlements, so that they bear almost all of the risk associated with the pension fund. This may produce large intergenerational transfers. The results suggest that effective risk sharing among all participants in the pension arrangement requires either all entitlements to be of the same type, as is the case under the current contract, or all participants to have an equal share of both types of entitlements, as is the case under the second variant of the combined contract.

Chapter 2

Intergenerational risk sharing, pensions and endogenous labour supply in general equilibrium

There is a trend towards a greater degree of funding in pension systems in OECD countries – see OECD (2011). In a number of countries, such as Chile, Denmark, the Netherlands, the U.K. and the U.S., pension funds already play a prominent role in the social security system. Anticipating the rising future costs of pension provision caused by population aging, more countries are setting up or expanding their funded pension pillars, often with mandatory participation. Examples of countries that have recently moved towards more funding are Israel and Norway. This trend will have important implications for the distribution of economic risks in society.

In this chapter the optimal design of two-tier pension systems in an overlapping generations general equilibrium model with endogenous labor supply is explored. While the first tier allows for both systematic redistribution and risk sharing between the young and old generations, the second pillar only allows for potential intergenerational risk sharing, as it is fully funded. Funded pension benefits can be of the defined contribution (DC) type or of the defined benefit (DB) type. Under DC, the contribution rate is fixed and the pension benefit is uncertain, while under DB the contribution rate is stochastic and adjusts to guarantee a fixed benefit. Of the latter type, I shall explore a defined real benefit (DRB) system, where the pension benefit is ex ante determined

This chapter is joint work with Roel Beetsma and Ward Romp and is forthcoming in the Scandinavian Journal of Economics.

in real terms, and a defined wage-indexed benefit (DWB) system, where the benefit is linked to the realized wage rate. With a DC fund, no risk sharing is possible through the second pension pillar, because the entire value of the fund is paid out to the retired. Hence, the social optimum cannot generally be replicated. With a DRB fund, optimal risk sharing requires wage risks to be shared via the first pillar. However, this requires a distortionary pension premium to be levied on wages, which, in turn, distorts the labor supply. Hence, also with a DRB fund, the social optimum cannot generally be achieved. The only system that enables the market economy to replicate the social optimum is a properly designed DWB system. Such a system allows a complete separation between systematic redistribution, which is the task of the first pillar, and optimal risk sharing, which is the domain of the second pillar. This way, the labour supply is undistorted and the first best can be mimicked.

Finding funded arrangements that minimise distortions in the labour market is of particular relevance nowadays for countries that face substantial pension deficits, while at the same time their labour forces are shrinking. In these circumstances, the mentioned trend of moving from solely pay-as-you-go systems towards more funding is to be welcomed. However, these new funded arrangements are usually of the DC type, and in existing funded schemes there is a tendency to replace DB arrangements with DC arrangements. This happened on a large scale in the U.K. and is starting to happen in the Netherlands as well. Since our results suggest that this development is not optimal as far as the scope for intergenerational risk sharing is concerned, policymakers would do well to carefully consider the design of funding arrangements.

Related to this chapter is Beetsma and Bovenberg (2009). However, there the labour supply is exogenous and DRB and DWB both achieve the first best. Hence, relaxing the assumption that the labour supply is exogenous has substantial implications for the optimal design of the funded pension pillar. There is a growing literature studying intergenerational risk sharing via pension systems. For example, Wagener (2004) and Gottardi and Kubler (2011) focus on risk-sharing within PAYG systems. Matsen and Thøgersen (2004) explore the optimal division between PAYG and funding from a risk-sharing perspective. However, they do not consider funded systems of the DB type. Neither do Teulings and De Vries (2006), who study a funded system in which each generation builds up its own pension account. Moreover, in contrast to this chapter, they adopt a partial-equilibrium setting and assume that capital markets are complete such that individuals can already invest in equity before they are born. Cui et al. (2011) do study funded systems of the DB type, but, as all the other papers mentioned so far, they do not consider endogenous labour supply. Bonenkamp and Westerhout (2010) is one of the few papers that combines both funded DB pension systems and endogenous labour supply. They compare the welfare gains from intergenerational risk sharing with the losses due to the labour market distortions caused by the income-dependent contributions in such systems. Their model lacks the analytical tractability of the model employed here. More importantly, these authors are not concerned with the design of optimal pension arrangements like I am in this chapter.

A second paper that combines funded DB schemes and endogenous labour supply is Mehlkopf (2011). The author finds that in a sixty generations OLG model, the presence of labour supply distortions forces the pension fund to deviate from optimal consumption smoothing and absorb a relatively large fraction of shocks when they occur, in order to avoid an accumulation of shocks and thus very high distortionary welfare costs in the future. The focus of Mehlkopf's paper is different from this chapter. He employs a partial equilibrium model of a funded pension fund, which is modelled in detail and focuses on the question how, quantitatively, for different parameter settings of the model the pension fund should distribute shocks to pension fund assets over different participating generations. In contrast, this chapter features a general equilibrium model, where a PAYG first pillar is included besides the funded second pillar pension fund, and where shocks do not only affect pension fund savings, but also wages and the capital stock in the economy. The two-period set-up of the chapter allows for analytical results for the social planner's objective of optimal risk sharing of the shocks occurring in the economy.

The remainder of this chapter is structured as follows. Section 2.1 lays out the model and presents the social planner's (first-best) solution. Section 2.2 discusses the market economy with the different pension systems. Section 2.3 shows that only DWB achieves the first best. Finally, Section 2.4 discusses the main results.

2.1 The command economy

In this section I derive the conditions that characterise the social planner's solution in an economy with overlapping generations that cannot share their risks through direct trade in the financial markets. The planner's solution is presented as the benchmark that the market economy combined with a pension arrangement would ideally be able to achieve.

2.1.1 Individuals and preferences

I assume a closed economy that runs for two periods (0 and 1). Periods are denoted by subscripts. In period 0, a continuum of identical individuals of total mass 1 is born. This generation lives through periods 0 and 1 and is termed the "old generation". It is denoted by the supercript "o". Utility of each agent from this generation is

$$u\left(c_0^o\right) - z\left(n_0^o\right) + \beta \mathrm{E}_0\left[u\left(c_1^o\right)\right], \tag{2.1}$$

where c_0^o denotes its consumption in period 0 and c_1^o its consumption period 1. Further, n_0^o is the endogenous labour supply in period 0, $-z\left(.\right)$ is the disutility of work effort, β is the discount factor and $\mathrm{E}_0\left[.\right]$ denotes expectations conditional on information in period 0. I assume that $u' > 0$, $u'' < 0$, $z' > 0$ and $z'' > 0$.

In period 1, a new generation (the "young generation") is born that also consists of a continuum of identical individuals of total mass 1. It lives just for this period and during this period it overlaps with the other generation. Utility of each agent from this generation is

$$u\left(c_1^y\right) - z\left(n_1^y\right), \tag{2.2}$$

which is defined over consumption c_1^y and endogenous work effort n_1^y in period 1.

2.1.2 Investment and production

In period 0 each member of the old generation receives an initial non-stochastic endowment of capital k_0. With a mass 1 of old generation members, this implies an initial aggregate capital stock equal to $K_0 = k_0$. The aggregate capital stock in period 1 is

$$K_1 = (1 - \delta_0) K_0 + I_0, \tag{2.3}$$

where I_0 is aggregate investment in period 0 and δ_0 is the (non-stochastic) depreciation rate in period 0.

Production is endogenous in both periods. Given the aggregate labour supply N_0^o and N_1^y in periods 0 and 1, respectively, production in these periods is given by

$$Y_0 = A_0 F(K_0, N_0^o), \quad Y_1 = A_1 F(K_1, N_1^y). \tag{2.4}$$

Further, A_0 is the (non-stochastic) total factor productivity in period 0, while A_1 is the total factor productivity in period 1, which I assume to be stochastic. Finally, I assume that function F exhibits constant returns to scale.

Following Bohn (1999a) and Smetters (2006), depreciation risk is introduced to reduce the correlation between labour and capital income. A growing number of recent articles argue that depreciation shocks are an important source of economic fluctuations – see, for example, Barro (2006, 2009) and Liu et al. (2010). Such shocks can occur for a variety of reasons, such as natural disasters, armed conflicts and other violence causing harm to the capital stock. Barro (2006) documents evidence of these types of shocks and finds that they occur with a probability of roughly 2% per year and an impact ranging from a decrease of 15% to 64% of real GDP per capita. Other sources of depreciation risk are unexpected technological advances and the associated creative destruction that renders capital obsolete. Further, changes in environmental regulation and other regulatory standards (such as town planning) may affect the value of the existing capital stock.

2.1.3 The resource constraints

The period 0 and 1 resource constraints are, respectively,

$$C_0^o = A_0 F\left(K_0, N_0^o\right) + \left(1-\delta_0\right) K_0 - K_1, \tag{2.5}$$

$$C_1^y + C_1^o = A_1 F\left(K_1, N_1^y\right) + \left(1-\delta_1\right) K_1, \tag{2.6}$$

where the left-hand sides denote aggregate consumption. Further, $0 \leq \delta_1 \leq 1$ is the stochastic depreciation rate of the capital stock between periods 0 and 1. The right-hand side of (2.5) represents total production minus investment. Because the world ends after period 1, whatever capital is left after production in this period is used for consumption. Hence, the right-hand side of (2.6) is total production plus capital left after depreciation.

2.1.4 The social planner's solution

The vector $\xi_0 \equiv \{A_0, \delta_0\}$ is known at the start of period 0, while the vector of shocks for period 1, $\xi_1 \equiv \{A_1, \delta_1\}$, is unknown in period 0 and only becomes known before period 1 variables are determined. As a benchmark, I consider a utilitarian social planner who chooses an optimal state-contingent plan in period 0 to maximize the sum of the expected utilities of all individuals. The consumption levels and the labour supply in period 1 are functions of the shocks, which implies $c_1^o = c_1^o\left(\xi_1\right)$, $c_1^y = c_1^y\left(\xi_1\right)$ and $n_1^y = n_1^y\left(\xi_1\right)$. Since the masses of the old and the young generations are both unity, the planner realizes that $N_0^o = n_0^o$, $C_0^o = c_0^o$, $N_1^y = n_1^y$, $C_1^o = c_1^o$ and $C_1^y = c_1^y$. Using this the Lagrangian of the planner's problem can be written as:

$$\begin{aligned} \pounds &= \int \left[\begin{array}{c} \left[u\left(c_0^o\right) - z\left(n_0^o\right) + \beta u\left(c_1^o\left(\xi_1\right)\right)\right] + \beta\left[u\left(c_1^y\left(\xi_1\right)\right) - z\left(n_1^y\left(\xi_1\right)\right)\right] \\ +\beta\lambda_1\left(\xi_1\right)\left[A_1 F\left(K_1, n_1^y\left(\xi_1\right)\right) + \left(1-\delta_1\right)K_1 - c_1^y\left(\xi_1\right) - c_1^o\left(\xi_1\right)\right] \end{array} \right] f\left(\xi_1\right) d\xi_1 \\ &+ \quad \lambda_0\left[A_0 F\left(K_0, n_0^o\right) + \left(1-\delta_0\right)K_0 - K_1 - c_0^o\right]. \end{aligned}$$

Here, $f\left(\xi_1\right)$ stands for the probability density function of the vector of stochastic shocks ξ_1. The Lagrange multipliers on the resource constraints in periods 0 and 1 are denoted by λ_0 and $\lambda_1(\xi_1)$, respectively.

The optimality conditions are

$$\begin{aligned} c_1^y &= c_1^o, \forall\, \xi_1, & (2.7)\\ z'(n_0^o)/u'(c_0^o) &= A_0 F_{N_0}, & (2.8)\\ z'(n_1^y)/u'(c_1^y) &= A_1 F_{N_1}, \forall \xi_1, & (2.9)\\ u'(c_0^o) &= \beta \mathrm{E}_0 \left[\left(1+r_1^{kn}\right) u'(c_1^o)\right]. & (2.10) \end{aligned}$$

where $r_1^{kn} \equiv A_1 F_{K_1} - \delta_1$ is the "net-of-depreciation return on capital" in period 1 and F_{K_t} (F_{N_t}) is the marginal product of capital (labour) in period t. (I drop function arguments whenever this does not create ambiguities.) Condition (2.7) equalizes the marginal utilities of the two generations, (2.8) and (2.9) provide the optimal consumption - leisure trade-offs for the old, respectively young generation, while (2.10) determines the optimal intertemporal trade-off.

2.2 The decentralized economy

This section describes the decentralized market economy in which individuals and firms maximize their objective functions under the relevant constraints. A key question will be which pension system can replicate the planner's solution. (2.7) can be interpreted as the condition for ex-ante trade in risks between the young and old generations in complete financial markets. However, in a decentralized economy, the two generations cannot trade risk in financial markets, because the young generation is born only after the shock vector ξ_1 has materialized. Other institutions thus have to replace this missing market and this chapter shall explore to what extent the pension system can perform that role.

In the decentralized market economy events unfold as follows. In period 0, given their initial capital holdings k_0 and the known vector ξ_0, the members of the old generation take their investment, consumption and labour supply decisions, while firms take hiring and production decisions. At the beginning of period 1 the shock vector ξ_1 materializes. After this, firms take hiring and production decisions for period 1, while the members of the young generation choose their consumption - labour trade-off.

2.2.1 The pension systems

The market economy features a two-tier pension system, with a pay-as-you-go (PAYG) first tier and a fully-funded second tier. The former consists of each young in period 1 paying to the old generation a lump-sum transfer θ^l and a fraction θ^w of their wage income. Hence, given that the two generations are both of size 1, the PAYG transfer per old person is $\theta^l + \theta^w w_1 N_1^y$.

The second tier consists of a pension fund that in period 0 per old-generation member collects a fraction θ^s of their labour income as a mandatory contribution, so total payments to the fund equal $\theta^s w_0 N_0^o$. The fund invests aggregate amounts B_1^s and K_1^s in real bonds and capital, respectively, such that

$$\theta^s w_0 N_0^o = B_1^s + K_1^s. \tag{2.11}$$

The corresponding investments per individual contributor will be denoted by b_1^s and k_1^s. The total value of the fund in period 1 is

$$(1 + r_1^a)\theta^s w_0 N_0^o = \left(1 + r_1^f\right) B_1^s + \left(1 + r_1^{kn}\right) K_1^s, \tag{2.12}$$

where r_1^a is the average fund return and r_1^f is the non-stochastic real-debt return. r_1^s denotes the net return in period 1 to the old generation members on their pension fund investment. Depending on the type of benefit scheme, the value of the fund may differ from the value of the total pay-out $(1+r_1^s)\theta^s w_0 N_0^o$ to the old. The young are the fund's residual claimants and receive the difference $(r_1^a - r_1^s)\theta^s w_0 N_0^o$. The difference (positive or negative) is spread out over the young generation in a lump-sum fashion.

The possibility that r_1^s differs from r_1^a allows for potential intergenerational risk sharing. It is assumed that the second pillar is fully funded in utility terms in an *ex ante* sense, which means that an old individual is indifferent between paying an additional unit into the fund and consuming it now (or investing it privately). Hence,

$$u'(c_0^o) = \beta E_0[(1 + r_1^s)u'(c_1^o)]. \tag{2.13}$$

The members of the old generation pay their mandatory contribution $\theta^s w_0 n_0^o$ into the fund and this contribution generates a pay off $(1 + r_1^s)\theta^s w_0 n_0^o$. Hence, the total payout to each member of this generation depends on the individual contribution. The full funding condition is necessary to ensure that the old generation makes an optimal consumption-savings decision in the first period of their life and to prevent a distortion on the labour market via the wage dependent pension contribution. Hence, it is a necessary condition for a market equilibrium with a pension arrangement to replicate the first best.

The net flows between the generations can be summarized by the generational accounts:

$$\begin{aligned} g^o &= \theta^l + \theta^w w_1 N_1^y + (r_1^s - r_1^a)\theta^s w_0 n_0^o, \\ -g^y &= \theta^l + \theta^w w_1 n_1^y + (r_1^s - r_1^a)\theta^s w_0 N_0^o, \end{aligned} \tag{2.14}$$

where g^o and g^y are the accounts for each old, respectively young, generation member and where the assumption that each generation's size is 1 has been used. Terms involving aggregate labour supply variables are the result of lump-sum transfers and will be taken as given at the individual level, because each individual is too small to affect aggregate variables by changing its own labour supply. Of course, in equilibrium $n_1^y = N_1^y$ and $n_0^o = N_0^o$ and, hence, $G^o + G^y = 0$, where G^y and G^o are defined as the aggregate accounts of the young and old generations, respectively.

Defined contribution (DC)

If the second-pillar pension is of the DC type, the total payout is simply equal to the value of the fund, i.e. assets and liabilities are always equal. Hence, $r_1^s = r_1^a$ and the second pillar provides no additional intergenerational risk-sharing opportunities. Since

$r_1^s = r_1^a$, the generational account of a young generation member reduces to

$$-g^y = \theta^l + \theta^w w_1 n_1^y. \tag{2.15}$$

Defined real benefit (DRB)

With a DRB system, each old receives a safe real return on its contribution. Full funding excludes *ex ante* intergenerational redistribution and, hence, requires

$$r_1^s = r_1^f. \tag{2.16}$$

The young receive what is left over of the pension fund after the old have been paid their safe real benefit. Hence, the young absorb the mismatch risk of the pension fund and they receive $(r_1^a - r_1^f)\theta^s w_0 N_0^o = \left(r_1^{kn} - r_1^f\right) K_1^s$, where (2.11), (2.12) and (2.16) have been used. In this case, the generational account of a young generation member becomes:

$$-g^y = \theta^l + \theta^w w_1 n_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s. \tag{2.17}$$

Defined wage-indexed benefit (DWB)

Finally, with a DWB system, each old receives

$$\left(1 + r_1^s\right) \theta^s w_0 n_0^o = \theta^{dwb} N_1^y w_1, \tag{2.18}$$

where θ^{dwb} is the (non-stochastic) fraction of the aggregate wage sum in period 1. The pension benefit received by the old generation depends on the wage rate w_1 per unit of labour and is stochastic since w_1 is determined by market forces only after the shocks have materialised. The pension contribution is like an investment in a wage-linked bond, with a payout that depends on aggregate wage developments.

Combining the full-funding condition (2.13) with (2.18) yields[1]

$$\theta^s = \theta^{dwb} \frac{\beta \mathrm{E}_0 \left[N_1^y w_1 u'\left(c_1^o\right)\right]}{n_0^o w_0 u'\left(c_0^o\right)}. \tag{2.19}$$

[1]Multiply both sides of (2.18) by $u'\left(c_1^o\right)$ and take expectations $\mathrm{E}_0\left[.\right]$ on each side of the resulting equation. Combine the result with (2.13).

In the absence of a risk-free bond, $\theta^s w_0 n_0^o$ would simply be fully invested in capital. When risk-free bonds exist, as in our economy, the pension fund issues or accepts risk-free bonds to create a wedge between the collected contributions and its investment in physical capital. The generational account of a young generation member becomes

$$-g^y = \theta^l + w_1\left(n_1^y\theta^w + N_1^y\theta^{dwb}\right) - \left(1 + r_1^f\right)B_1^s - \left(1 + r_1^{kn}\right)K_1^s. \tag{2.20}$$

Because the payment $\theta^{dwb}N_1^y w_1$ is distributed in a lump-sum fashion over young generation individuals and a change in the labour supply of such an individual has a negligible effect on the aggregate wage sum, the presence of θ^{dwb} does not distort this individual's labour supply decision. Hence, the factor in front of θ^{dwb} in (2.20) should be the aggregate labour supply N_1^y of the young generation.

2.2.2 Individual budget constraints

With voluntary private investments b_1^p and k_1^p in real bonds and capital, period 0 consumption of each member of the old generation is

$$c_0^o = (1-\theta^s)\,n_0^o w_0 + \left(1 + r_0^{kn}\right)k_0 - \left(b_1^p + k_1^p\right), \tag{2.21}$$

while period 1 consumption of, respectively, each young and old generation member is:

$$c_1^y = w_1 n_1^y + g^y, \tag{2.22}$$

$$c_1^o = \left(1 + r_1^{kn}\right)k_1^p + \left(1 + r_1^f\right)b_1^p + \left(1 + r_1^a\right)\theta^s n_0^o w_0 + g^o. \tag{2.23}$$

2.2.3 Individual and firm optimization

The model is solved through backwards induction. Under all three pension schemes, the optimal consumption - leisure trade-off to the period 1 young is

$$z'\left(n_1^y\right)/u'\left(c_1^y\right) = (1-\theta^w)\,w_1. \tag{2.24}$$

In period 0, the old generation decides about its labour supply and the allocation of its savings over risk-free bonds b_1^p and risky capital k_1^p according to:

$$z'\left(n_0^o\right)/u'\left(c_0^o\right) = w_0 \tag{2.25}$$

$$\beta\left(1+r_1^f\right)\mathrm{E}_0\left[u'\left(c_1^o\right)\right] = u'\left(c_0^o\right), \tag{2.26}$$

$$\beta\mathrm{E}_0\left[\left(1+r_1^{kn}\right)u'\left(c_1^o\right)\right] = u'\left(c_0^o\right). \tag{2.27}$$

Appendix 2.B contains the details on the derivation of the first-order conditions.

Assuming perfectly competitive representative firms, the profit-maximization conditions in periods 0 and 1 for firms are:

$$A_tF_{N_t} = w_t, \quad A_tF_{K_t} - \delta_t = r_t^{kn}, \quad t = \{0,1\}. \tag{2.28}$$

2.2.4 Market equilibrium conditions

The model is completed with the labour, capital and bond market equilibrium conditions. The labour market equilibrium conditions are $N_1^y = n_1^y$ and $N_0^o = n_0^o$. The capital market equilibrium condition for period 0 is $K_0 = k_0$. In period 1, the total capital stock must be equal to total privately held capital plus total capital held by the pension fund, $K_1 = K_1^p + K_1^s$. Further, since the mass of the old generation is 1, $K_1 = k_1$, $K_1^p = k_1^p$ and $K_1^s = k_1^s$. The aggregate net supply of bonds must be zero, so that total private bond holdings and total pension fund bond holdings cancel out: $B_1^p + B_1^s = 0$. Finally, $B_1^p = b_1^p$ and $B_1^s = b_1^s$.

2.3 Optimality of pension systems

2.3.1 Pension fund optimality conditions

It is now explored whether and how the market economy can replicate the social optimum with an appropriate choice of the first and second pension pillars. It is easy to see that when a pension arrangement produces (2.7) - (2.9) for all possible realizations of the shock vector ξ_1, then the market equilibrium reproduces the socially-optimal allocation under this arrangement. From equations (2.25) and (2.28) note that the pension system always satisfies (2.8). It is assumed that the pension system parameters θ^l, θ^w, θ^s, θ^{dwb}, K_1^s and B_1^s are not shock-contingent, as would be required for a realistic arrangement.

The optimality condition (2.7) requires the right-hand sides of (2.22) and (2.23) to be equal for all possible shock vectors ξ_1. Because both generations are of unity mass and populated by representative agents, the aggregate versions of these expressions can be used. Hence, condition (2.7) requires the generational accounts to vary such that:

$$\tfrac{1}{2}A_1 F_{N_1} N_1^y - \tfrac{1}{2}\left(1 + r_1^{kn}\right) K_1 = -G^y, \tag{2.29}$$

where (2.12), (2.28), $B_1^p + B_1^s = 0$, $K_1 = K_1^p + K_1^s$ and $G^y + G^o = 0$ have been used. Hence, if profit income plus the scrap value of capital of the old generation in period 1, $\left(1 + r_1^{kn}\right) K_1$, exceeds the wage income of the young generation, $A_1 F_{N_1} N_1^y$, the old would have more per-capita resources for consumption in period 1 than the young. Intergenerational equality of period 1 consumption requires the generational accounts to offset these income differences.

Reproduction for all shocks of (2.9) by (2.24) is possible if and only if

$$\theta^w = 0. \tag{2.30}$$

In other words, replication of the social optimum requires (at least) the elimination of the wage-linked part of the first pillar.

2.3.2 Optimality of different pension systems

The main result of this chapter can now be stated:

Proposition 2.1. *(a) With a DC second pillar it is generally not possible to replicate the social optimum. (b) With a DRB second pillar it is generally not possible to replicate the social optimum. (c) With a DWB pillar it is possible to replicate the socially-optimal allocation for all possible shock combinations. The appropriate parameters of the pension arrangement are* $B_1^s = \theta^s N_0^o w_0 - K_1^s$, $\theta^l = \left(1 + r_1^f\right) B_1^s$, $\theta^w = 0$, $\theta^{dwb} = \frac{1}{2}$

and $K_1^s = K_1^p$, where θ^s follows from (2.19) with N_0^o substituting for n_0^o.

Proof. Part (a): Under a DC fund, $-G^y = \theta^l + \theta^w A_1 F_{N_1} N_1^y$. Substitution into (2.29) yields as a necessary condition for reproducing the social optimum that $\theta^l + \theta^w A_1 F_{N_1} N_1^y = \frac{1}{2} A_1 F_{N_1} N_1^y - \frac{1}{2}\left[A_1 F_{K_1} + (1-\delta_1)\right] K_1$. It is immediately obvious that this expression cannot hold for all possible shock realizations for a constant parameter combination $(\theta^l,\, \theta^w)$.

Part (b): Under a DRB scheme, $-G^y = \theta^l + \theta^w A_1 F_{N_1} N_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s$. Substitution into (2.29) yields as a necessary condition for reproduction of the social optimum:

$$\theta^l + \theta^w A_1 F_{N_1} N_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s = \frac{1}{2} A_1 F_{N_1} N_1^y - \frac{1}{2}\left[A_1 F_{K_1} + (1-\delta_1)\right] K_1.$$

There are three instruments $\left(\theta^l, \theta^w, K_1^s\right)$ to produce equality of the constant terms and the shock coefficients on both sides of this expression. The solution is $K_1^s = K_1^p$, $\theta^l = -\left(1 + r_1^f\right) K_1^s$ and $\theta^w = \frac{1}{2}$. The solution for θ^w contradicts (2.30).

Part (c): Under a DWB fund, $-G^y = \theta^l + A_1 F_{N_1} N_1^y \left(\theta^w + \theta^{dwb}\right) - \left(1 + r_1^f\right) B_1^s - \left(1 + r_1^{kn}\right) K_1^s$. Substitute this into (2.29). It is easy to check that the proposed solution ensures that the resulting expression holds for all possible shock combinations. Because $\theta^w = 0$ is part of the proposed solution, also (2.9) is fulfilled for all possible shock combinations. □

Intuitively, the first pillar can only be used to offset possible systematic transfers between generations via its lump-sum part. This pillar should not contain a wage-linked part since this would distort the young generation's labour supply decision. Hence, this pillar cannot be employed to share wage risks. Notice that the lump-sum component of the first pillar is a necessary part of the optimal arrangement, because making a lump-sum transfer out of the second-pillar fund would break the full-funding condition. Hence, inter-temporal optimisation would be distorted and the first-best would not be reached. Since a DC scheme does not allow for any risk sharing, it is obvious that a pension system consisting of a DC scheme and a lump-sum PAYG transfer cannot mimic the social planner's allocation. A lump-sum PAYG plus a DRB second pillar cannot achieve the first best either, contrary to the results in Beetsma and

Bovenberg (2009). In their paper, labour supply is exogenous, so a wage-linked PAYG part does not distort the labour supply decision. They use the first pillar to share wage risks optimally and the second pillar to share financial risks. Here the PAYG pillar does distort the labour supply, so the second pillar should share both risks. This is only possible under a DWB scheme.

2.4 Discussion

This chapter has shown that a two-tier pension arrangement with a DWB second tier is able to combine optimal intergenerational redistribution with optimal intergenerational risk sharing, without distorting the labour market. The appropriate DWB arrangement completely separates the roles of both pension system pillars, where that of the first pillar is to provide the right amount of systematic redistribution and that of the second pillar is to provide for optimal risk sharing. This contrasts with the DRB system where a distortionary pension premium is needed to share wage risks between the two generations via the first pension pillar.

Our results have clear implications for the design of pension arrangements. From the perspective of the sustainability of adequate future pension provision the trend towards more funding is to be welcomed. However, the design of new funding arrangements tends to be of the defined-contribution type, which implies that risk sharing through the second pillar of the pension system will be very limited or non-existent. Shifting the task of providing risk sharing to the PAYG first pillar creates distortions in the labour market. Hence, policymakers would do well to carefully consider the design of funded arrangements, since our results indicate that a properly designed funded DB arrangement improves welfare of participants.

An obvious extension of the present analysis would be to cast the analysis into an infinite horizon framework with endogenous labour supply and production in every period. In this infinite horizon model, the young and the pension fund must save for the new capital stock, whereas in the model in this chapter the world ends after period 1 and there is no need for this capital stock. Due to this additional complication it is no longer possible to exactly replicate the social planner's allocation in the infinite horizon model, even with a DWB system. The pension system must ensure that the two generations living at the same time have the same exposure to productivity and depreciation shocks and have the same level of consumption in a base scenario. The parameter constellation proposed in this chapter equalises exposure to economic shocks, but the old generation does not contribute to the new capital stock. Moreover, the optimal new capital stock will depend on previous technological and depreciation shocks. In the infinite horizon model, pension planners must make a trade-off between perfect risk sharing and equalisation of consumption. The optimal constrained allocation has features of both. However, I again find that to avoid labour market distortions, it is necessary that the first pillar is only used for redistribution and not for risk sharing. Although it is impossible to replicate the social planner's solution exactly, our main result still holds. A DWB system outperforms a DRB system with respect to risk sharing since any risk sharing allocation that is possible with a DRB system, is also possible with a DWB system, but without the distorting effect via the wage-linked part of the second pillar.

APPENDICES

2.A Derivation of the planner's solution

Maximization of the planner's program with respect to c_0^o, n_0^o, K_1, $c_1^y(\xi_1)$, $c_1^o(\xi_1)$ and $n_1^y(\xi_1)$ for all ξ_1 yields the following first-order conditions:

$$\begin{aligned} u'(c_0^o) &= \lambda_0, \\ z'(n_0^o) &= \lambda_0 A_0 F_{N_0}, \\ \lambda_0 &= \int \beta\lambda_1(\xi_1)\left(1+r_1^{kn}\right) f(\xi_1)\, d\xi_1, \\ u'(c_1^y(\xi_1)) &= \lambda_1(\xi_1), \forall \xi_1, \\ u'(c_1^o(\xi_1)) &= \lambda_1(\xi_1), \forall \xi_1, \\ z'(n_1^y(\xi_1)) &= \lambda_1(\xi_1) A_1 F_{N_1}, \forall \xi_1. \end{aligned}$$

By eliminating the Lagrange multipliers from these first-order conditons, we obtain

$$u'(c_1^y) = u'(c_1^o), \forall \xi_1,$$

and (2.8)-(2.10). This reduces to (2.7)-(2.10).

2.B Individual first-order conditions

2.B.1 Period 1 individual first-order conditions

The young generation solves:

$$\max_{c_1^y, n_1^y} \left\{u(c_1^y) - z(n_1^y)\right\},$$

subject to the following budget constraint, which differs according to the pension scheme that is in place:

$$
\begin{aligned}
\text{DC: } c_1^y &= (1-\theta^w)\, w_1 n_1^y - \theta^l, \\
\text{DRB: } c_1^y &= (1-\theta^w)\, w_1 n_1^y - \theta^l - \left(r_1^f - r_1^{kn}\right) K_1^s, \\
\text{DWB: } c_1^y &= (1-\theta^w)\, w_1 n_1^y - \theta^{dwb} w_1 N_1^y - \theta^l + \left(1+r_1^f\right) B_1^s + \left(1+r_1^{kn}\right) K_1^s.
\end{aligned}
$$

In all three cases, the first-order conditions for c_1^y and n_1^y are given by, respectively,

$$
\begin{aligned}
u'\left(c_1^y\right) &= \mu, \\
z'\left(n_1^y\right) &= \mu\,(1-\theta^w)\, w_1,
\end{aligned}
$$

where μ is the Lagrange multiplier on the budget constraint. The first-order conditions combine to (2.24).

2.B.2 Period 0 individual first-order conditions

We can write consumption per old individual in period 1 as:

$$
c_1^o = \left(1+r_1^{kn}\right) k_1^p + \left(1+r_1^f\right) b_1^p + (1+r_1^s)\,\theta^s w_0 n_0^o + \theta^l + \theta^w w_1 N_1^y. \tag{2.31}
$$

In period 0 a member of the old generation solves:

$$
\max_{c_o^o, c_1^o, n_0^o, k_1^p, b_1^p} \left\{ u\left(c_0^o\right) - z\left(n_0^o\right) + \beta \mathrm{E}_0\left[u\left(c_1^o\right)\right] \right\},
$$

subject to (2.21) and (2.31). The first-order conditions are (2.26), (2.27) and:

$$
\begin{aligned}
u'\left(c_0^o\right)(1-\theta^s)\, w_0 - z'\left(n_0^o\right) + \beta\theta^s w_0 \mathrm{E}_0\left[(1+r_1^s) u\left(c_1^o\right)\right] &= 0 \Leftrightarrow \\
u'\left(c_0^o\right) w_0 - z'\left(n_0^o\right) - \theta^s w_0 u'\left(c_0^o\right) + \theta^s w_0 \beta \mathrm{E}_0\left[(1+r_1^s) u\left(c_1^o\right)\right] &= 0 \Leftrightarrow \\
u'\left(c_0^o\right) w_0 - z'\left(n_0^o\right) &= 0,
\end{aligned}
$$

where we have used (2.13).

2.C Infinite horizon model

2.C.1 Notation

In the paper we use o and y superscripts to identify generations (so c_0^o is consumption in period 0 of the generation born in period 0 and c_1^o consumption of the same generation in period 1). This is not possible with an infinite number of generations, so in this appendix we use a subscript to identify the timing of the variable and a superscript to indicate whether this generation was born in the previous period or in this period (c_0^y is consumption in period 0 by the generation born in period 0, c_1^o is this generation's consumption in period 1). People only work when young, so individual labour supply does not need an age indicator (n_t is labour supply by someone born in period t).

2.C.2 Social Planner

Full Diamond-Samuelson OLG model for the central planner

$$\max_{c^o,c^y,n} \sum_{t=0}^{\infty} \beta^t E\Big[u(c_t^y) - z(n_t) + \beta u(c_{t+1}^o)\Big] + u(c_t^o) \tag{2.32}$$

$$\text{s.t.} \quad c_t^o + c_t^y + K_{t+1} = A_t F(K_t, n_t) + (1-\delta_t)K_t \quad \text{for each } t \geq 0 \tag{2.33}$$

A_0 and δ_0 are known (non-stochastic), future A_t and δ_t for $t \geq 1$ are stochastic.

The FOC's are

$$c_t^o = c_t^y \quad \forall\, t \geq 0 \tag{2.34}$$

$$\frac{z'(n_t)}{u'(c_t^y)} = A_t F_N(K_t, n_t) \quad \forall\, t \geq 0 \tag{2.35}$$

$$u'(c_t^y) = \beta E_t\Big[\Big(A_{t+1} F_K(K_{t+1}, n_{t+1}) + (1-\delta_{t+1})\Big) u'\left(c_{t+1}^o\right)\Big] \quad \forall\, t \geq 0 \tag{2.36}$$

These conditions are comparable to the conditions in the two-period model in the paper. The first equalises consumption of everybody living at the same time, the second describes the optimal trade-off between leisure and consumption, the third is the Euler equation, describing the optimal intertemporal trade-off.

2.C.3 Decentralised economy

The pension system is as in the paper. The relevant incentives are

- The member of the young generation pays a fraction θ^w per received euro wage income plus a fixed contribution θ^l. Paying to the first pillar has an incentive effect!
- The member of the old generation receives $\theta^l + \theta^w w_{t+1} N_{t+1}$, regardless of the individual work history. The first pillar's pension payouts have no incentive effect.
- Each member of the young generation pays θ^s to the pension fund. This contribution has an incentive effect. Per paid euro this participant receives a stochastic pension when old, so this pension also has an incentive effect. The full funding condition ensures that these two effects cancel out.
- The residual value of the pension fund (positive or negative) is equally spread over the young generation. This has no incentive effect.

The pension fund's budget constraint is

$$B^s_{t+1} + K^s_{t+1} = (1 + r^f_t) B^s_t + (1 + r^k_t) K^s_t + \theta^s_t w_t N_t - (1 + r^s_t)\theta^s_{t-1} w_{t-1} N_{t-1} \qquad (2.37)$$

The fully funded condition still holds, so the expected return on paid contributions must be equal to the expected return on other assets.

The individual budget constraint for each young generation is

$$c^y_t = w_t n_t - b^p_{t+1} - k^p_{t+1} - (\theta^l + \theta^w w_t n_t) - \theta^s_t w_t n_t \qquad (2.38)$$

And when old

$$c^o_{t+1} = (1 + r^f_{t+1}) b^p_{t+1} + (1 + r^k_{t+1}) k^p_{t+1} + (\theta^l + \theta^w w_{t+1} N_{t+1}) + (1 + r^s_{t+1})\theta^s_t w_t N_t \qquad (2.39)$$

Optimisation of individual utility, taking the factor payments as exogenous and using the pension fund's full funding condition gives (besides the two budget constraints)

$$\frac{z'(n_t)}{u'\left(c_t^y\right)} = (1-\theta^w)w_t \tag{2.40}$$

$$u'\left(c_t^y\right) = \beta E_t\Big[\Big(1+r_{t+1}^k\Big)u'\left(c_{t+1}^o\right)\Big] \tag{2.41}$$

$$u'\left(c_t^y\right) = \beta E_t\Big[\Big(1+r_{t+1}^f\Big)u'\left(c_{t+1}^o\right)\Big] \tag{2.42}$$

The possible distortionary effect of the second pillar is neutralised by the fully funded condition. These individual first order conditions are comparable to the ones in the paper. For equilibrium factor prices (so $w_t = A_t F_N(K_t, n_t)$ and $r_t = A_t F_K(K_t, n_t) - \delta_t$), markets ensure that the intertemporal trade-off is optimal. The first-pillar's wage component distorts the intratemporal trade-off and a necessary requirement to mimic the social planner's allocation is that this wage component is zero ($\theta^w = 0$).

As in the paper, the task of the pension system is to equalise consumption for the young and the old living at the same time. Substitution of the pension fund's budget identity into that of the young and using the equilibrium conditions on the market for real bonds ($B_{t+1}^p + B_{t+1}^s = 0$) and the capital market ($K_{t+1}^p + K_{t+1}^s = K_{t+1}$) gives for the consumption of the young

$$\begin{aligned} c_t^y = w_t n_t - K_{t+1} - (\theta^l + \theta^w w_t n_t) + (1+r_t^f)B_t^s \\ + (1+r_t^k)K_t^s - (1+r_t^s)\theta_{t-1}^s w_{t-1} N_{t-1} \end{aligned} \tag{2.43}$$

This equation differs from the paper in one crucial aspect: it includes K_{t+1}. In the paper, the economy ends after period 1 and there is no reason to save so $K_{t+1} = 0$. In this infinite horizon model, the young have to save for the new capital stock. Using the pension parameters proposed in the paper gives for the consumption of the old and young living at time t

$$c_t^y = \tfrac{1}{2}w_t n_t + \tfrac{1}{2}(1+r_t^k)K_t - K_{t+1} \tag{2.44}$$

$$c_t^o = \tfrac{1}{2}w_t n_t + \tfrac{1}{2}(1+r_t^k)K_t \tag{2.45}$$

The proposed parameters do give the old and the young the same exposure to economic shocks, but it will not equalise the level of consumption because the young also have to (and want to) save for the next period. They save k^p_{t+1} themselves and their pension fund saves K^s_{t+1}, but for the proposed parameters, it all comes from their consumption. Since the optimal new capital stock is non-linear in wages and interest rate shocks, and all pension parameters must be shock-independent, it is impossible to exactly mimic the social planner's allocation.

The pension system must ensure that the two generations living at the same time have the same exposure to productivity and depreciation shocks and have the same level of consumption in a base scenario. In the infinite horizon model, pension planners must make a trade-off between perfect risk sharing and equalisation of consumption. The optimal constrained allocation has features of both. However, we again find that to avoid labour market distortions, it is necessary that the first pillar is only used for redistribution and not for risk sharing. Although it is impossible to replicate the social planner's solution exactly, our main result still holds. A DWB system outperforms a DRB system with respect to risk sharing since any risk sharing allocation that is possible with a DRB system, is also possible with a DWB system, but without the distorting effect via the wage linked part of the second pillar.

Chapter 3

Sharing of Demographic Risks in a General Equilibrium Model with funded Pensions

In general, pension arrangements will affect the risks faced by both workers and retirees. One type of risk that one typically thinks of in this context is financial market risk. This risk is relatively large for funded pension systems in which workers accumulate assets for future retirement via their pension fund. Another important source of risk that is present in pension systems is demographic risk. Expected and unexpected developments in both survival probabilities and fertility rates have a direct impact on the distribution of burdens and benefits through pension systems. Taking them explicitly into account in the design of the pension arrangement may therefore lead to an improved allocation of these types of risks over the different generations of participants in the pension system.

In the literature, the impact of longevity or mortality risk has received a lot of attention. The paper by Andersen (2005) studies the impact of of longevity risk on PAYG systems, while Cocco and Gomes (2009) studies the effects on retirement savings. Also, the structural effect of increasing longevity and decreasing fertility (population aging) has been studied extensively, see for example Brsch-Supan et al. (2006) and Bovenberg and Knaap (2005).

In this chapter, the effects of the presence of demographic and financial shocks on optimal pension system design are investigated. Closest to our set-up are the papers by and Bovenberg and H.Uhlig (2006). Bohn (1999b) discusses the impact of demographic uncertainty on PAYG arrangements, while Bovenberg and H.Uhlig (2006) investigate the impact of demographic shocks in a model with capital and endogenous growth. Their set-up is an infinite horizon model with 2 overlappig generations, yielding generational conditions for optimal intergeneration risk sharing.

In this chapter, to keep the set-up as simple as possible, the model used is a two-period model with two generations whose lifes overlap only in the second period. This set-up allows for a very detailed numerical analysis of the effects of demographic shocks on the optimal set-up of the pension system. The young generation is born in the second period after all shocks have materialized. Hence, under laissez-faire – i.e., in the absence of a pension system – the young generation is unable to participate in financial markets and thus unable to share risks with the old generation. The pension systems under consideration consist of a pay-as-you-go (PAYG) first tier and a fully-funded second tier. Benefits in the second tier can be organized in different ways, as a defined-contribution (DC) or a defined-benefit (DB) scheme. Defined benefits can be defined either in real terms or relative to wages.

The two types of demographic shocks affect optimal defined-benefit pension arrangement in a highly nonlinear nature and are integral to both the total size of the economy and the distribution of resources over different generations. Specifically, fertility risk is important for both the wage level and the returns to capital (and thus aggregate production), and also for the relative size of transfers within the pension system. Mortality risk is important both for the returns on the funded part of the pension system and for the relative sizes of transfers within the pension system.

Optimal pension arrangements are explored under the assumption that the parameters of the arrangement are constant. In particular, they are not contingent on the stochastic shocks. Allowing the pension system parameters to be shock dependent would allow for more flexibility in distributing the various shocks over the pension system participants. However, frequent changes in the pension parameters may undermine the confidence in the pension system, while, moreover, each time that the system's parameter combinations are to be reset, an intergenerational conflict may be re-opened on how the parameters should exactly be set. Finally, it may be hard to fine-tune the parameters in response to shocks.

It is well known (see Beetsma and Bovenberg (2009)) that in the absence of demographic risks the decentralized pension system is able to replicate the social planner's solution by determining optimal redistribution via the first pillar and optimal risk sharing through the second pillar. When demographic risk is added to the model, this result disappears. Even if there is only one source of demographic risk (either fertility risk of mortality risk), it is no longer possible to replicate the social planner's solution anymore due to the complicated and non-linear effects each demographic shock has on the economy. Specifically, fertility risk affects both the wage level and the returns to capital (and thus aggregate production), as well as the relative size of transfers within the pension system. Mortality risk is important both for the returns on the funded part of the pension system and for the relative sizes of transfers within the pension system. Finding a solution that is as close as possible to the social planner's solution requires abandoning the separation of redistribution and risk sharing between the first and second pillar. Instead, in the presence of demographic risk, both pillars are involved in achieving an amount of risk sharing that is as close as possible to that under the social planner's solution.

With all four sources of risk present, it is socially optimal for the pension system to feature a funded second pillar of the defined-benefit type where the benefit is linked to the aggregate wage level. This setup, combined with a PAYG first pillar consisting of a lump-sum transfer and a transfer linked to individual wages, allows for an allocation of all sources of risks such that the resulting welfare gains (measured in terms of the percent increase in consumption in all periods and all states of the world) are very close to those obtained under the social planner's solution. Even with highly volatile demographic shocks, the short-fall under the wage-linked DB scheme from the social planner's solution is only about 0.01%, compared to welfare gains of over 5% relative to the laissez-faire economy. Further, the numerical analysis suggests that welfare gains from a defined real-benefit scheme are close to those under a scheme with defined wage-linked benefits. Hence, the precise format in which benefits are defined in a DB scheme may be less important than the scheme being defined benefit rather than defined contribution – the latter corresponds to the laissez-faire economy and implies that substantial gains from risk sharing may be foregone.

The setup of the remainder of this chapter is as follows. Section 2 presents the command economy and the social planner's solution. The decentralized economy in combination with the different pension schedules is introduced in Section 3. Section 4 presents the optimal pension scheme in the case of perfect foresight. Section 5 discusses the numerical procedure and the calibration, while Section 6 presents the numerical results. Section 7 investigates the robustness of the results for variations in the degree of risk aversion. Finally, Section 8 concludes the main text of the chapter.

3.1 The command economy

The model is deliberately kept as simple as possible, in order to describe the intuitions as clearly as possible.

3.1.1 Individuals and preferences

The model represents a closed economy. It incorporates two periods (0 and 1) and two generations. In period 0, a generation of mass N_0^o is born. This generation is referred to as the "old generation". The old generation consumes only in period 1 when it has become old. Therefore, this generation's utility function is given by:

$$\mathrm{E}_0\left[\psi u\left(c_o\right)\right], \tag{3.1}$$

where c_o represents consumption when the agent is old and ψ is the probability that an individual old person survives period 0 and enters period 1. Further, $\mathrm{E}_0\left[.\right]$ is the expectations operator conditional on information before any of the shocks have occurred (see below). It is assumed that function $u\left(.\right)$ is twice continuously differentiable, with $u' > 0$ and $u'' < 0$.

In period 1 a new generation of size N_1^y is born. This generation is referred to as the "young generation". The representative individual of the young generation features utility

$$u\left(c_y\right), \tag{3.2}$$

which is defined over per-young person consumption c_y in period 1.

3.1.2 Demographics

There are two sources of demographic risk in the model. The first source of risk is uncertainty about the survival probability ψ of the old generation. In the remainder of the chapter this type of risk is referred to as "mortality risk". The size of the old generation in period 0 is denoted by N_0^o and is nonstochastic. The size of the old generation in period 1 is denoted N_1^o and depends on the realization of ψ, i.e. $N_1^o = \psi N_0^o$. The second source of risk is fertility risk. The size of the young generation in period 1, N_1^y, is also uncertain.

3.1.3 Production

In period 0 each old generation member receives an exogenous non-storable initial endowment η. Production is endogenous only in period 1, when the two generations co-exist. Labour supply is exogenous in the model and amounts to 1 unit of labour per young person. Hence, production in period 1 is given by

$$Y = AF\left(K, N_1^y\right), \tag{3.3}$$

where A denotes (stochastic) total factor productivity, K represents the aggregate capital stock and N_1^y is the aggregate labor input. The production function exhibits constant returns to scale. In the closed economy setting in this chapter, the capital stock K in period 1 is the result of investment in the previous period, period 0.

3.1.4 Resource constraints

The resource constraints in periods 0 and 1 are given by, respectively,

$$N_0^o \eta = K, \tag{3.4}$$

$$AF\left(K, N_1^y\right) + (1-\delta) K = N_1^y c_y + N_1^o c_o, \tag{3.5}$$

where $0 \leq \delta \leq 1$ is the stochastic depreciation rate of the capital stock. The left-hand sides of (3.4) and (3.5) are the available resources in the two periods. Specifically, the left-hand side of (3.5) equals total production plus what is left over of the capital stock after taking depreciation into account. The right-hand sides of (3.4) and (3.5) describe the use of these resources. The entire endowment in period 0 is spent on investment in physical capital (equation (3.4)), while the right-hand side of (3.5) is total consumption in the economy.

3.1.5 The social planner's solution

The vector of the stochastic shocks hitting the command economy is $\xi \equiv \{A, \delta, N_1^y, \psi\}$. It is unknown in period 0, but becomes known before period 1 variables are determined. A (utilitarian) social planner who aims at maximising the sum of the discounted expected utilities of all individuals serves as a benchmark. In period 0, the planner commits to an optimal state-contingent plan. Hence, the consumption levels are functions of the shocks, so that $c_o = c_o(\xi)$ and $c_y = c_y(\xi)$. The planner's problem can be written as:

$$\pounds = \int \begin{bmatrix} N_1^o \left[u\left(c_o\left(\xi\right)\right)\right] + N_1^y \left[u\left(c_y\left(\xi\right)\right)\right] + \\ \lambda\left(\xi\right)\left[AF\left(K, N_1^y\right) + \left(1-\delta\right)K - N_1^y c_y\left(\xi\right) - N_1^o c_o\left(\xi\right)\right] \end{bmatrix} f\left(\xi\right) d\xi. \quad (3.6)$$

$$(3.7)$$

Here, $f(\xi)$ stands for the probability density function of the vector of stochastic shocks ξ. The Lagrange multiplier on the resource constraint in period 1 is denoted by λ. Maximization of the planner's program with respect to $c_y(\xi)$ and $c_o(\xi)$ for all ξ yields the following first-order conditions:

$$\frac{\partial \pounds}{\partial c_y} = 0 \Rightarrow u'\left(c_y\left(\xi\right)\right) = \lambda\left(\xi\right), \forall \xi, \quad (3.8)$$

$$\frac{\partial \pounds}{\partial c_o} = 0 \Rightarrow u'\left(c_o\left(\xi\right)\right) = \lambda\left(\xi\right), \forall \xi. \quad (3.9)$$

By eliminating the Lagrange multiplier from these first-order conditons, the following is obtained:

$$u'(c_y) = u'(c_o), \forall \xi, \tag{3.10}$$

which, owing to the identical utility functions reduces to:

$$c_y = c_o, \forall\ \xi, \tag{3.11}$$

Condition (3.10) equalises the marginal utilities of the two generations which implies that with identical utility functions for the two generations all shocks are perfectly spread over all individuals. If a decentralised equilibrium is to replicate the planner's solution, conditions (3.8) and (3.9) need to be met in addition to the resource constraints (3.4) and (3.5). This is also the set of necessary conditions. This case differs from Beetsma and Bovenberg (2009) in that the latter include a trade off between period 0 and period 1 consumption for the old generation. Hence, in their paper there is an extra optimality condition that needs to be fulfilled to replicate the social planner solution. However, the intertemporal consumption trade off of the old generation is dropped in this chapter as it does not substantially add to the insights that can be gained from the analysis below.

3.2 The decentralized economy

3.2.1 The pension arrangements

The pension system in the decentralized economy consists of a PAYG first pillar and a funded second pillar. The first pillar in the pension system is composed of a lump-sum part and a wage-indexed part. Each old person receives a (possibly negative) amount θ^p and a fraction θ^w of wage income of an individual young. Therefore, the systematic transfer per surviving old member via the first pillar is $\theta^p + \theta^w w$.

The second pillar of the pension system consists of a pension fund that collects contributions θ^f per old-generation member in period 0, invests these contributions and pays out benefits to the fraction ψ of surviving old-generation members in period 1. Denoting by r^f the rate of return that a surviving member of the old generation receives on his contributions, the payout to this individual is $(1+r^f)\theta^f$. Depending on whether the pension fund is of the DC, DRB or DWB type, the format of the pension benefit will be different (see below).

The pension fund can invest the old generation's contribution in price-indexed bonds or in physical capital:

$$N_0^o \theta^f = B^f + K^f, \tag{3.12}$$

where B^f and K^f are the fund's aggregate investments in price-indexed bonds and physical capital, respectively. The bonds provide a non-stochastic return of r and physical capital provides the stochastic net-of-depreciation rate of return $r^{kn} \equiv AF_K - \delta$, where F_K is the marginal product of capital. The average return on the assets held by the fund is denoted by r^a. Therefore, the total value of the fund before the payout is

$$(1+r^a)N_0^o\theta^f = (1+r)\,B^f + \left(1+r^{kn}\right)K^f.$$

Depending on the pension scheme and the fund's investment scheme, a difference may arise between the value of the fund and the payout equal to $\left[(1+r^a)\,N_0^o - \left(1+r^f\right)N_1^o\right]\theta^f$, where $\left(1+r^f\right)N_1^o\theta^f$ is the total pension benefit payout to the old generation. The young are the residual claimants of the fund and receive this difference. Denoting the generational account *per old person in period* 1 by G:

$$G = \theta^p + \theta^w w + \left[\left(1+r^f\right) - \frac{N_0^o}{N_1^o}\left(1+r^a\right)\right]\theta^f.$$

Defined contribution (DC)

The DC scheme is discussed purely as a benchmark in which risk sharing between the two cohorts is completely absent. For this reason, it is not discussed in the formal analysis below. In a DC system the pension fund invests the contributions of the old generation in period 0 and pays out whatever these investments turn out to be worth in period 1:

$$\left(1+r^f\right)\theta^f = \frac{N_0^o}{N_1^o}(1+r^a)\theta^f,$$

which can be rewritten as:

$$r^f = \frac{N_0^o}{N_1^o}(1+r^a) - 1. \tag{3.13}$$

Therefore, the payout and the value of the pension fund coincide in every state of the world and there is no residual claim left for the young generation. This implies that the generational account per old person in period 1 under a DC system simplifies to:

$$G = \theta^p + \theta^w w. \tag{3.14}$$

Note that the pension assets of the non-surviving members of the generation go to the survivors.

Defined real benefit (DRB)

Under a DRB system, the pension fund promises to provide each retired individual with a given real benefit of size p:

$$p = \left(1+r^f\right)\theta^f, \tag{3.15}$$

where r^f is the return to the pension fund contribution which will be determined by the so-called "actuarial fairness condition" to be discussed below.

In period 1 the pension fund will have accumulated assets equal to $(1+r)B^f + \left(1+r^{kn}\right)K^f$. Hence, the difference between the total benefit payout and the assets of the pension fund equals

$$\left(1+r^f\right)N_1^o\theta^f - (1+r)B^f - \left(1+r^{kn}\right)K^f.$$

The generational account per old person in period 1 is equal to:

$$G = \theta^p + \theta^w w + \left(1+r^f\right)\theta^f - \frac{1}{N_1^o}\left[(1+r)B^f + \left(1+r^{kn}\right)K^f\right]. \tag{3.16}$$

Defined wage-indexed benefit (DWB)

Under a DWB system, the pension fund benefit is indexed to the aggregate wage sum:

$$\left(1+r^f\right)\theta^f N_1^o = \theta^{dwb} N_1^y w, \tag{3.17}$$

where $N_1^y w$ is the total wage bill and parameter θ^{dwb} links the pension payout to the total wage bill. This parameter is also determined by the actuarial fairness condition discussed below.

The return to the old generation on their pension contribution is stochastic, because the size of the young generation is unknown when the contribution is made, while the wage rate is only determined by market forces after the shocks have materialised and depends on the a priori unknown size of the young generation and level of productivity. Hence, if the realised total wage bill exceeds the expected total wage bill, the payout of the pension fund is higher than expected. This structure is intended to capture indexation of occupational pension benefits to aggregate wage growth, which exists in some funded pension schemes in Germany, the Netherlands, Norway, Slovenia and Sweden (OECD, 2011).

The difference between the total pension benefit payout and total assets of the pension fund is:

$$\left[\left(1+r^f\right)N_1^o - \left(1+r^a\right)N_0^o\right]\theta^f = \theta^{dwb} N_1^y w - \left[\left(1+r\right)B^f + \left(1+r^{kn}\right)K^f\right]. \tag{3.18}$$

Hence, in this case the generational account per old person in period 1 is:

$$G = \theta^p + \theta^w w + \frac{N_1^y}{N_1^o}\theta^{dwb} w - \frac{1}{N_1^o}\left[\left(1+r\right)B^f + \left(1+r^{kn}\right)K^f\right]. \tag{3.19}$$

3.2.2 Individual budget constraints and generational accounts

In period 0, each individual old person receives his deterministic endowment η, pays the mandatory pension fund contribution θ^f, and invests the remainder of his income in bonds, $\frac{B^p}{N_0^o}$, and physical capital, $\frac{K^p}{N_0^o}$:

$$\eta = \frac{B^p}{N_0^o} + \frac{K^p}{N_0^o} + \theta^f, \tag{3.20}$$

where B^p and K^p are the aggregate private investments in real bonds and physical capital, respectively.

In period 1, the surviving members of the old generation receive the returns on their private investments plus their pension benefits, while the members of the young generation receive their wage plus the residual (potentially negative) value of the pension fund. In addition, the non-pension assets of those old generation members that do not survive until period 1 are collected by the government and distributed evenly across all individuals alive in period 1. Thus, in period 1 each individual alive receives a lumpsum "bequest" τ, such that

$$c_o = (1+r)\frac{B^p}{N_0^o} + \left(1+r^{kn}\right)\frac{K^p}{N_0^o} + \frac{N_0^o}{N_1^o}(1+r^a)\,\theta^f + \tau + G, \tag{3.21}$$

$$c_y = w + \tau - \frac{N_1^o}{N_1^y}G, \tag{3.22}$$

where τ is given by:

$$\tau = \frac{N_0^o - N_1^o}{N_1^o + N_1^y}\left[(1+r)\,B^p + \left(1+r^{kn}\right)K^p\right].$$

3.2.3 Individual and firm optimization

The model is solved by backward induction. A continuum of perfectly competitive representative firms, with mass normalized to unity, produce according to (3.3) and maximize period-1 profits $AF(K,L) - wL - r^kK$ over labour L and capital K, taking the wage rate and rental rate on capital as given. The first-order conditions are:

$$AF_L = w, \tag{3.23}$$

$$AF_K = r^k, \tag{3.24}$$

where the subscripts on F denote its first-order partial derivatives.

In period 0, the old generation decides on the allocation of its savings over the two assets. Subject to (3.20) they maximize (3.1) over $\frac{B^p}{N_0^o}$ and $\frac{K^p}{N_0^o}$, where c_o is given by (3.21). The first-order condition is:

$$(1+r)\,\mathrm{E}_0\left[\psi u'(c_o)\right] = \mathrm{E}_0\left[\psi\left(1+r^{kn}\right)u'(c_o)\right]. \tag{3.25}$$

Finally, the so-called "actuarial fairness condition" of the pension fund states that, in expectation, the pension fund should not redistribute resources between generations. This condition is based on the work of Yaari (1965) that shows that in a model with lifetime uncertainty, the actuarially fair rate of return on an annuity is higher than the market interest rate. The rate of return on an annuity contains a mark-up for mortality risk. The reason for this is the zero-profit condition for insurance companies offering the annuities. An insurance company that would try to sell annuities with a rate of return below the actuarial fair one would be driven out of the market by other insurance companies offering a rate of return closer to the actuarial fair rate.

In the setting in this chapter, the pension fund effectively provides an annuity, because the contribution paid to the pension fund is not part of the bequest if an individual dies and can be used by the pension fund to increase the benefits to surviving participants. Analoguous to Yaari's setup, the actuarial fairness condition is written as:

$$\theta^f\ \mathrm{E}_0\left[\left(1+r^f\right)N_1^o\right] = \theta^f\mathrm{E}_0\left[\left(1+r^a\right)N_0^o\right], \tag{3.26}$$

where the left-hand side represents the expected total pension fund payout and the right-hand side represents the value of the assets in the pension fund expected at the moment that the fund makes its investments.

For the DRB pension system, $p = \left(1+r^f\right)\theta^f$, which is deterministic, so that the actuarial fairness condition can be rewritten as:

$$\begin{aligned} & p\ \mathrm{E}_0\left[N_1^o\right] = N_0^o\theta^f\ \mathrm{E}_0\left(1+r^a\right), \\ \Rightarrow\quad & p = \theta^f\frac{N_0^o}{\mathrm{E}_0\left[N_1^o\right]}\ \mathrm{E}_0\left(1+r^a\right). \end{aligned} \tag{3.27}$$

For the DWB system, using (3.17) the actuarial fairness condition (3.26) can be written as:

$$\begin{aligned} \mathrm{E}_0\left[\left(1+r^f\right)N_1^o\right] &= N_0^o\ \mathrm{E}_0\left(1+r^a\right) \\ \Rightarrow\quad \theta^{dwb} &= \theta^f \frac{N_0^o}{\mathrm{E}_0\left[N_1^y w\right]}\ \mathrm{E}_0\left(1+r^a\right). \end{aligned} \tag{3.28}$$

3.2.4 Market equilibrium conditions

The goods market equilibrium conditions in periods 0 and 1 are (3.4) and (3.5), respectively. Equilibrium in the labour market requires that the amount of labour L demanded by firms is equal to N_1^y. Equilibrium in the capital market requires that

$$K = K^p + K^f, \tag{3.29}$$

while, with zero net outstanding debt, equilibrium in the market for risk-free debt in period 0 requires that:

$$B^p + B^f = 0. \tag{3.30}$$

Finally, the factor prices are determined as $w = AF_L\left(K, N_1^y\right)$ and $r^k = AF_K\left(K, N_1^y\right)$.

3.3 Optimal pension policy

This section investigates the optimal pension parameter settings and checks whether the pension system designer is able to mimick the social planner's solution. The assumed objective of the pension system designer (for example, the government) is a utilitarian maximisation of the welfare of the pension system participants. Hence, the pension system designer solves the following problem

$$\max_{\Pi}\left\{W \equiv \mathrm{E}_0\left[N_1^o u\left(c_o\right) + N_1^y u\left(c_y\right)\right]\right\}, \tag{3.31}$$

where Π is the set of instruments (pension system parameters) available to the pension system designer, subject to the resource constraints ((3.4) and (3.5)), the individual's and firm's optimality conditions ((3.23), (3.24) and (3.25)), the actuarial fairness condition (3.26), and the market equilibrium conditions (3.29) and (3.30). In its choice of the optimal pension parameters, the pension designer internalizes how individuals react to changes in those parameters (see Chamley, 1986; Chari et al., 1994, for a detailed treatment of this method). The set Π under a DRB, respectively a DWB system, consists of the following parameters:

$$\Pi^{drb} = \left\{\theta^p, \theta^w, K^f, p\right\},$$
$$\Pi^{dwb} = \left\{\theta^p, \theta^w, K^f, \theta^{dwb}\right\}.$$

Specific realisations of the shocks A and N_1^y and the solution for K yield w and r^k. Using the same specific realisations for A and N_1^y and specific realisations for δ and N_1^o, realisations of c_o and c_y are obtained via (3.21) and (3.22). Using all probability-weighted possible realisations of the shock vector ξ and the associated outcomes for c_o and c_y, W can be computed for the given set of pension system parameters. The set that generates the highest value for W yields the optimal pension system in this procedure.

3.3.1 The optimum under perfect demographic foresight

Under perfect foresight about the demographic parameters N_1^y and N_1^o of the young and elderly in period 1 the social planner solution can be achieved, as the following proposition states:

Proposition 3.1. *(i) Under a DRB system, the social planner solution can be replicated and only be replicated by setting the pension parameters as:*

$$\theta^p + p = (1+r)\frac{N_0^o + N_1^y}{N_0^o\left(N_1^y + N_1^o\right)}B^f,$$
$$K^f = \frac{N_1^y N_0^o}{N_1^y + N_0^o}\eta,\ \theta^w = \frac{N_1^y}{N_1^y + N_1^o}.$$

(ii) Under a DWB system, the social planner solution can be replicated and only be

replicated by setting the pension parameters as:

$$\theta^p = (1+r)\,\frac{N_0^o + N_1^y}{N_0^o\,(N_1^y + N_1^o)}B^f,\;\; K^f = \frac{N_1^y N_0^o}{N_1^y + N_0^o}\eta,$$
$$\textit{and } \theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb} = \frac{N_1^y}{N_1^y + N_1^o}.$$

Proof. See Appendix 3.B for the proof. □

In the special case in which demographic risks are absent, the pension parameters are known with certainty at the moment that the old generation takes its investment decisions. This is the case discussed in Beetsma and Bovenberg (2009). The intuition for the results closely follows that in Beetsma and Bovenberg (2009). However, there is a major difference in that their "full-funding condition" is replaced by the actuarial fairness condition. The full-funding condition says that the expected return on an additional euro contributed to the pension fund should be the same as the expected return on an additional euro invested in any asset traded in the financial market. Here, this condition is dropped in favour of the actuarial fairness condition, because the probability of prematurely dying before retirement results in an above-market return on the contribution to the pension fund made by those who survive into retirement.

The intuition for the parameter settings in Proposition 1 is as follows. Let us focus on the case of the DRB system. Risks associated with the pension fund's asset portfolio are effectively shifted to the young generation, because the old generation gets a fixed benefit, while the residual risk is carried by the young. Hence, each young generation member has an individual capital exposure of $\left(1+r^{kn}\right)\left(K^f/N_1^y\right)$, while each old generation member retains an individual capital exposure of $\left(1+r^{kn}\right)\left(K^p/N_0^o\right)$. Here, the capital exposure implicit in the accidental bequest τ is ignored, because the bequest is identical for young and old individuals. Identical capital exposures among all individuals are achieved by setting $K^f/N_1^y = K^p/N_0^o$, which can be written as $K^f = K^p\left(N_1^y/N_0^o\right)$. Hence, if $N_1^y > N_0^o$, then $K^f > K^p$ is needed in order to bestow the same amount of capital risk on young and old generation individuals. Using $K^p + K^f = K$, the equation $K^f = K^p\left(N_1^y/N_0^o\right)$ can be rewritten as $K^f = \left[N_1^y/\left(N_1^y + N_0^o\right)\right]K$, which can be written as the expression in the proposition by using the resource constraint in period 0.

The intuition for the expression for θ^w follows from the fact that for condition (3.11) to be fulfilled, exposure of all individuals to the realisation of the wage rate must be identical. Since each old person receives a fixed transfer of size $\theta^w w$, each young person has to pay a transfer of the size $\frac{N_1^o}{N_1^y}\theta^w w$. Identical exposure of each individual to wages then requires that $\left(1 - \frac{N_1^o}{N_1^y}\theta^w\right) w = \theta^w w$, which is equivalent to $\theta^w = \frac{N_1^y}{N_1^y + N_1^o}$. Thus, the combination $\theta^w = \frac{N_1^y}{N_1^y + N_1^o}$ and $K^f = \frac{N_1^y N_0^o}{N_1^y + N_0^o}\eta$ is necessary to simultaneously ensure identical exposures to w and r^{kn}.

Finally, after having ensured that both generations' consumption responds identically to movements in wages and the net return on capital, the expression for $\theta^p + p$ ensures that the lump-sum first-pillar transfer and the benefit from the funded pillar are set such that the period-1 consumption levels of the two generations are identical. Obviously, this expression depends on the sizes of both generations.

3.3.2 Demographic uncertainty

In the absence of perfect foresight about the future sizes of the two cohorts, it is not possible to design a pension arrangement in period 0 that reproduces the social planner's solution. While multiple combinations of the pension system parameters can reproduce the social planner's solution under perfect foresight, all these combinations require the realisations of N_1^y and N_1^o to be known in period 0. If this is not the case, then the social planner's solution is not attained.

It is no longer possible to solve analytically for the optimal pension system arrangement in the presence of demographic risk. In the remainder of the chapter a numerical analysis is performed to investigate how well the different pension systems perform in the presence of demographic uncertainty. As already motivated above, only arrangements that do not depend on the shocks are considered. This means that only arrangements for which the pension parameters are known in period 0 are considered.

3.4 Calibration

In the numerical analysis, the pension parameter combination that yields the highest welfare level W under the assumption that all elements of Π are independent of the shocks is determined. Under this constraint the optimal solution will in general not replicate the social planner's solution. Before turning to the analysis, functional forms for the utility and production functions need to be specified, as well as assumptions on the distributions of the various shocks in the model.

For the utility and production functions, commonly-used specifications in macroeconomic analysis are adopted. The utility function is of the constant relative risk aversion (CRRA) format:

$$u(c_i) = \frac{c_i^{1-\varphi}}{1-\varphi}, \quad i = y, o,$$

where φ is the relative risk aversion parameter. The baseline value is $\varphi = 2.5$, which is close to the median of the values found in the literature (see, for example Gertner, 1993; Beetsma and P.C.Schotman, 2001). However, later φ is varied to see how the results are affected by changes in risk attitude.

Production is given by the Cobb-Douglas function

$$AF(K, N_1^y) = AK^{\alpha} (N_1^y)^{1-\alpha},$$

where α is the share of national income accruing to the equity providers. The value assumed is $\alpha = 0.3$, a value commonly used in the literature. Hence, 70% of national income accrues to workers.

For each of the four shocks, a 2-point distribution is assumed, with probability $\frac{1}{2}$ for each of the two values that the shock can attain. Therefore, the mean of each stochastic variable is always the mid-point between the two possible values of that variable. The amount of uncertainty associated with a shock is measured by absolute value of the distance between the mean of the shock and its possible realisations. This measure is denoted by Δ. It can differ over the various types of shocks.

The mean size of the young generation N_1^y is normalized to 1, such that in expectation both generations are equally large at the moment of birth. The mean of the

Table 3.1: Parameters

parameter	mean	Δ
α	0.3	
η	1	
φ	2.5	
A	3	0.3
δ	0.5	0.1
N_1^y	1	0.2
N_1^o	0.8	0.1

mortality shock ψ is 0.8, so that on average 80% of the old generation members survive until period 1.[1] The mean of the depreciation shock δ is 0.5, which corresponds to an annual depreciation rate of around 2% over a 30-year period. The remaining parameters to be calibrated are η and A. The choice of η is only relevant in relation to the average value of A. These two parameters determine the scale of the economy and one of them can be fixed to unity. The choice made is to set $\eta = 1$ and set the mean of the productivity shock A to 3. This mean is chosen so that the numerical simulations yield a realistic net-of-depreciation return on capital and a realistic return on bonds. In the numerical experiments, mean-preserving increases in the variances of the shocks will be considered. The base calibration is summarised in Table 3.1, including the base values of Δ for each of the shocks.

3.5 Numerical results

3.5.1 Measures for welfare comparison

Two welfare comparisons are conducted. First, welfare is compared under the optimal pension fund arrangement to welfare under the laissez-faire situation. Second, welfare

[1] According to the 2010 tables of the Dutch Association of Actuarial Scientists (AG, 2010), the probability for a newborn to survive until the current mandatory retirement age of 65 is approximately 0.87, while the probability of surviving until 67 is roughly 0.84. Ideally, the calibration of ψ would serve two purposes. One is to capture the probability of surviving until retirement date, while the other is to replicate the old-age dependency ratio, defined as the number of retired divided by the number of people of working age. The choice for the value of ψ is made to replicate closely the former objective. The stylised two generation objective employed prevents the replication of both objectives simultaneously.

under the social planner solution is compared to welfare under the optimal pension fund arrangement. The welfare consequences of introducing a pension arrangement are measured by calculating the constant perunage increase Ω^{pf} in consumption of both generations in all possible states of the world that is required to raise social welfare under the laissez-faire economy to that under the economy with the optimal pension arrangement. The welfare difference between the economy with the optimal pension arrangement and the social-planner economy is measured in an analogous way by Ω^{sp}.

$$\begin{aligned} \mathrm{E}_0 \left\{ u\left[\left(1+\Omega^{pf}\right) c_o^{lf} \right] + u\left[\left(1+\Omega^{pf}\right) c_y^{lf} \right] \right\} &= \mathrm{E}_0 \left[u\left(c_o^{pf}\right) + u\left(c_y^{pf}\right) \right], \\ \mathrm{E}_0 \left\{ u\left[\left(1+\Omega^{sp}\right) c_o^{pf} \right] + u\left[\left(1+\Omega^{sp}\right) c_y^{pf} \right] \right\} &= \mathrm{E}_0 \left[u\left(c_o^{sp}\right) + u\left(c_y^{sp}\right) \right], \end{aligned}$$

where the superscripts lf, pf and sp on consumption indicate consumption in the laissez-faire economy, consumption under the optimal pension arrangement and consumption under the social planner solution, respectively.

3.5.2 No demographic uncertainty

To establish some intuition for the numerical results, first the case in which there is no demographic uncertainty is considered. In this case, the values for the demographic shocks are fixed at their means, $N_1^y = 1$, $N_1^o = 0.8$.

Table 3.2 reports the instrument settings for optimal pension arrangements. For the case of a DRB system the optimal arrangement with $p = 0$ is selected, while for the case of the DWB system the optimal arrangement with $\theta^w = 0$ is chosen. Obviously, for $p = 0$ to be consistent with the actuarial fairness condition, it is necessary that either $\theta^f = 0$ or $\mathrm{E}_0 \left(1 + r^a\right) = 0$. If θ^f is not zero, then $\mathrm{E}_0 \left(1 + r^a\right) = 0$. Table 2 presents a solution with $\mathrm{E}_0 \left(1 + r^a\right) = 0$. This solution allows the fund to take a short position in real debt that is slightly larger in absolute value than its long position in capital, the reason being that the expected net return on capital $\mathrm{E}_0 \left[r^{kn}\right]$ is slightly larger than r. Because K^f and r^{kn} are tied down by demographic variables and period-0 resources, equation $\mathrm{E}_0 \left(1 + r^a\right) = 0$ and the arbitrage equation (3.25) determine together B^f and

Table 3.2: Different pension systems, no demographic uncertainty

	DRB	DWB
θ^p	-0.76	0.24
θ^w	0.56	0
θ^{dwb}	-	0.45
p	0	-
θ^f	-0.01	0.65
K^f	0.5	0.5
K^p	0.5	0.5
K	1	1
B^f	-0.51	0.15
B^p	0.51	-0.15
r	0.37	0.37
$E\left(r^{kn}\right)$	0.40	0.40
$E\left(r^a\right)$	0	0.39
$E\left(w\right)$	2.10	2.10
$E\left(c_o\right)$	1.94	1.94
$E\left(c_y\right)$	1.94	1.94
Ω^{pf}	0.067	0.067
Ω^{sp}	0	0

r. Equation (3.12) then ties down θ^f. Individuals from both generations have an identical exposure to capital income, which implies that $K^p/N_0^o = K^f/N_1^y = \frac{1}{2}$, and to labour income in period 1. Hence, they have an identical exposure to both remaining sources of risk, productivity and depreciation, which are perfectly shared. Because the two generations are of different size in period 1, the pension system parameters correct for this to ensure identical exposures at the individual level. Hence, conform Proposition 1, $\theta^w = 1/1.8 \simeq 0.56$ and $\theta^{dwb} = 0.8/1.8 \simeq 0.44$.

3.5.3 Deterministic variation in demographic variables

Table 3.3 shows how under perfect foresight about the future demography the parameters of the optimal pension arrangement change when either the fertility rate N_1^y or the survival probability ψ differs from its mean value, while keeping the other demographic parameter at its mean. The outcomes can be partly obtained by applying Proposition 1. To compare them to the outcomes in Table 3.2, those optimal arrangements are selected in which $p = 0$ in the DRB system and $\theta^w = 0$ in the DWB system.

Table 3.3: Varying values for demographic shocks

	$N_1^y = 0.8$	$\psi = 0.8$	$N_1^y = 1.2$	$\psi = 0.8$	$N_1^y = 1$	$\psi = 0.6$	$N_1^y = 1$	$\psi = 1$
	DRB	DWB	DRB	DWB	DRB	DWB	DRB	DWB
θ^p	-0.64	0.37	-0.92	0.13	-0.88	0.10	-0.70	0.34
θ^w	0.50	0	0.60	0	0.63	0	0.50	0.00
θ^{dwb}	0	0.5	0	0.40	0	0.38	0	0.50
p	0	0	0	0	0	0	0	0
θ^f	-0.01	0.71	-0.01	0.62	-0.01	0.56	-0.01	0.75
K^f	0.44	0.44	0.55	0.55	0.50	0.50	0.50	0.50
K^p	0.56	0.56	0.45	0.45	0.50	0.50	0.50	0.50
K	1	1	1	1	1	1	1	1
B^f	-0.45	0.26	-0.56	0.08	-0.51	0.06	-0.51	0.25
B^p	0.45	-0.26	0.56	-0.08	0.51	-0.06	0.51	-0.25
r	0.25	0.25	0.49	0.49	0.37	0.37	0.37	0.37
$E\left(r^{kn}\right)$	0.27	0.27	0.52	0.52	0.40	0.40	0.40	0.40
$E\left(r^a\right)$	0	0.26	0	0.52	0	0.40	0	0.39
$E\left(w\right)$	2.25	2.25	1.99	1.99	2.10	2.10	2.10	2.10
$E\left(c_o\right)$	1.92	1.92	1.95	1.95	1.75	1.75	2.19	2.19
$E\left(c_y\right)$	1.92	1.92	1.95	1.95	1.75	1.75	2.19	2.19
Ω^{lf}	0.089	0.089	0.045	0.045	0.08	0.08	0.054	0.054
Ω^{sp}	0	0	0	0	0	0	0	0

Because the total capital stock K is fixed by the period-0 resource constraint, a higher fertility rate N_1^y produces a higher expected net return r^{kn} on capital and a lower expected wage rate w. Owing to the trade-off that individuals make between investing in capital and investing in real debt, the higher expected net return on capital requires a rise in the risk-free interest rate in order for individuals to be willing to invest in the risk-free asset. The increase in N_1^y also produces an increase in K^f. The young generation carries the residual risk associated with the pension fund's asset portfolio. To evenly distribute the exposure to capital risk between the two generations, K^f needs to make up a larger fraction of the total capital stock if the relative size of the young generation increases. Given that the volume of benefit payments is kept fixed, the increase in K^f requires a fall in B^f. Further, an increase in N_1^y implies a higher value of θ^w. For a given value of θ^w, an increase in N_1^y implies that, per young person, a smaller fraction of its wage is taken away by the government and given to the old generation, since θ^w is defined per old-generation member. Hence, for a fixed θ^w, the young's exposure to wage risk increases. Therefore, to ensure indentical exposure to wage risk for all individuals, an increase in θ^w is needed, thereby raising the exposure of the old generation members to wage risk. However, the increase in θ^w raises the average amount of period-1 resources transfered to the old. To compensate for this systematic increase in transfers to the old, θ^p is reduced.

The consequences of the increase in N_1^y for the optimal pension arrangement under DWB are qualitatively the same as under DRB. However, because θ^w is fixed to zero under DWB, the identical exposures to wage risk now have to be restored through a reduction in θ^{dwb}, which is defined as a share of the total wage sum. Finally, although a priori θ^f could go both up or down, for this specific parameter combination the effect is that it goes down.

Now, the effect of a foreseen increase in the survival probability of the old generation is discussed. As old generation members do not work in period 1, factor prices and the return on debt are unaffected. The increase in the survival probability implies an increase in the old-age dependency ratio. That is, it implies an increase in the number of retirees N_1^o over the number of people of working age N_1^y. As a result, under DRB and for given θ^w, a larger fraction of the young generation members wage income is transfered to the old via the PAYG pillar of the pension arrangement. To restore their exposure to wage risk, θ^w needs to go down. However, this implies a smaller average transfer from the young to the old. To undo this, θ^p increases.

Analogously, under the DWB system θ^{dwb} needs to increase and θ^p needs to decrease. Further, under this system θ^f rises. The reason is that there are more old generation members, so that the "survivor-dividend" of the pension fund is smaller. Hence, θ^f needs to rise to compensate for this decrease in the survivor dividend. Indeed, for the actuarial fairness condition to continue to hold in the face of an increase in θ^{dwb} and a fall in $\mathrm{E}_0(1+r^a)$, while the term $N_0^o/\mathrm{E}_0[N_1^y w]$ remains constant, θ^f needs to rise. The average return on the pension fund portfolio falls, because the increase in θ^f leads to a higher investment in the risk-free asset, while the investment in equity and the factor prices remain unaltered.

3.5.4 Introducing demographic uncertainty

In this section optimal pension arrangements in the presence of demographic risk are explored. This is done by introducing each one of the sources of demographic risk separately, while shutting the other source off. In the final step, both sources are introduced simultaneously. Two important observations need to be made.

First, in general the social planner solution can no longer be achieved. Second, the multiplicity of solutions that were obtained under foresight about the demography will vanish. All four pension system parameters have to be deployed to maximise social welfare.

Tables 3.4-3.11 display the results of introducing uncertainty about N_1^y and N_1^o. This is done by keeping the mean values of N_1^y and N_1^o at 1 and 0.8, respectively, but increasing the spread between the possible outcomes of the two-point distributions as indicated in the tables by ΔN_1^y and ΔN_1^o, respectively. When considering only one source of demographic risk, the limiting case in which the spread of the outcomes is set to zero is also reported. This corresponds to the situation without demographic uncertainty. One of the solutions in this situation is displayed in the column under $\Delta N_1^y = 0$ ($\Delta N_1^o = 0$). Specifically, in the case of the DRB system when varying ΔN_1^y, the solution is chosen such that $p_{|\Delta N_1^y=0} = p_{|\Delta N_1^y=0.1}$, while in the case of the DWB system the solution is chosen such that $\theta^{dwb}_{|\Delta N^y_1=0} = \theta^{dwb}_{|\Delta N^y_1=0.1}$. In other words, those solutions under demographic certainty are chosen that are "close" to the solutions when a minimum level of uncertainty is introduced.

Obviously, under uncertainty about N^y_1 and N^o_1, the solutions for the pension system parameters differ from those in Proposition 3.1. To obtain additional intuition for the solutions under demographic uncertainty, the deviations of the solutions under uncertainty from those when $\Delta N^y_1 = \Delta N^o_1 = 0$ are decomposed into two components. The first component is the change in the average value of the pension system parameters under each of the two possible realisations of the demographic parameter under consideration (N^y_1 or N^o_1) assuming that these realisations are foreseen with certainty when the pension parameters are decided. This component arises purely from the non-linearity of the solutions in Proposition 3.1. As an example, consider the case of uncertainty about N^y_1. Then, formally, for each pension system parameter $\pi \in \Pi^{\mathbf{drb}}$ or $\pi \in \Pi^{\mathbf{dwb}}$, the following is computed:

$$d_1 = \frac{\pi_{|N^y_1=\bar{N}^y_1-\Delta N^y_1} + \pi_{|N^y_1=\bar{N}^y_1+\Delta N^y_1}}{2} - \pi_{|N^y_1=\bar{N}^y_1},$$

where $\pi_{|N^y_1=\bar{N}^y_1}$, $\pi_{|N^y_1=\bar{N}^y_1-\Delta N^y_1}$ and $\pi_{|N^y_1=\bar{N}^y_1+\Delta N^y_1}$ are the solutions for parameter π under certainty (computed using Proposition 3.1) for the indicated value of N^y_1. Here, $\bar{N}^y_1$ denotes the mean of N^y_1.

The second component measures the change in the solution that is the result purely from the demographic uncertainty. Formally, for the case of uncertainty about N^y_1 it is given by

$$d_2 = \pi_{|N^y_1\sim U\left\{\bar{N}^y_1-\Delta N^y_1, \bar{N}^y_1+\Delta N^y_1\right\}} - \frac{\pi_{|N^y_1=\bar{N}^y_1-\Delta N^y_1} + \pi_{|N^y_1=\bar{N}^y_1+\Delta N^y_1}}{2},$$

where $\pi_{|N^y_1\sim U\left\{\bar{N}^y_1-\Delta N^y_1, \bar{N}^y_1+\Delta N^y_1\right\}}$ is the solution under uncertainty about N^y_1 given by the discrete uniform distribution with support elements $\bar{N}^y_1 - \Delta N^y_1$ and $\bar{N}^y_1 + \Delta N^y_1$.

Analytical insight into the signs of d_1 for the various pension parameters is obtained by differentiating the solutions in Proposition 3.1 with respect to N^y_1 or N^o_1. The results are reported in Appendix B. For DRB, the values of the first- and second-order derivatives for $\theta^p + p$ are ambiguous. For K^f, the derivative with respect to N^y_1 is positive and the second derivative is negative. This means that K^f is a concave function of N_1^y and, by Jensen's inequality, an increase in the spread of the possible values of N_1^y leads to a decrease in the expected value of K^f. The reason is the correlation between the realisation for N_1^y and the return on capital r^{kn}. If N_1^y is low, there are few young and the return on capital is low, implying a shortfall in the pension fund which has to be paid for by relatively few young individuals, which is costly in utility terms. Utility costs in the low state become increasingly higher as uncertainty goes up (there are fewer young individuals in the low state), so K^f decreases to make sure the utility loss in the low state does not become too large. The derivative of K^f with respect to N_1^o is zero, since only the size of the old generation in the initial period, N_0^o, matters for K^f.

Further, the first derivative of θ^w with respect to N_1^y is positive and the second derivative is negative, implying that θ^w is also a concave function of N_1^y and that its expected value falls when the spread in N_1^y rises. A low realisation of N_1^y implies relatively many old individuals each one of them receiving $\theta^w w$. Since the aggregate amount of resources transfered to the old generation has to be paid for by relatively few young individuals, this becomes increasingly costly in terms of young individuals' utility as the realisation of N_1^y in the low state becomes lower. Hence, θ^w decreases to compensate for the bad low state as uncertainty goes up.

Conversely, the first derivative of θ^w with respect to N_1^o is negative, while the second derivative is positive. At a higher realisation of N_1^o the optimal value of θ^w decreases by less for a given increase in N_1^o. The reason is that precisely because of the increased size of the old generation at a higher value for N_1^o a given reduction in θ^w becomes more effective in reducing the transfer per young individual. Thus, the average of the optimal values of θ^w goes up as N_1^o increases.

Analogous results are obtained for the case of DWB. The signs of the first- and second-order derivatives of θ^p with respect to both sources of demographic risk are ambiguous, while K^f is a concave function of N_1^y and does not depend on N_1^o. Finally, the expression from Proposition 3.1 involving the wage-related parameters, $\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb}$ is a concave function of N_1^y and a convex function of N_1^o.

Table 3.4: Varying fertility risk, no mortality risk

System	DRB				DWB			
ΔN_1^y	0	0.1	0.2	0.3	0	0.1	0.2	0.3
θ^p	-1.1468	-1.1376	-1.0851	-1.0011	-0.6207	-0.6160	-0.6043	-0.5841
θ^w	0.5556	0.5498	0.5326	0.5043	0.4776	0.4738	0.4640	0.4470
θ^{dwb}	-	-	-	-	0.0623	0.0623	0.0609	0.0583
p	3.3210	3.3210	3.0937	2.7319	-	-	-	-
θ^f	1.9246	1.9266	1.8005	1.5989	0.0933	0.0932	0.0910	0.0867
K^f	0.5000	0.4951	0.4806	0.4567	0.5000	0.4970	0.4877	0.4714
K^p	0.5000	0.5049	0.5194	0.5433	0.5000	0.5030	0.5123	0.5286
K	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B^f	1.4246	1.4316	1.3199	1.1421	-0.4067	-0.4038	-0.3968	-0.3847
B^p	-1.4246	-1.4316	-1.3199	-1.1421	0.4067	0.4038	0.3968	0.3847
r	0.3736	0.3719	0.3667	0.3572	0.3736	0.3719	0.3667	0.3571
$E\left(r^{kn}\right)$	0.4000	0.3991	0.3962	0.3913	0.4000	0.3991	0.3962	0.3913
$E\left(w\right)$	2.1000	2.1041	2.1168	2.1391	2.1	2.1041	2.1168	2.1391
$E\left(r^f\right)$	0.7256	0.7238	0.7182	0.7086	0.7542	0.7534	0.7512	0.7481
$E\left(c_o\right)$	1.9444	1.9421	1.9351	1.9226	1.9444	1.9422	1.9352	1.9229
$E\left(c_y\right)$	1.9444	1.9422	1.9353	1.9231	1.9444	1.9422	1.9353	1.9232
Ω^{pf}	0.0450	0.0465	0.0512	0.0589	0.0450	0.0466	0.0512	0.0589
Ω^{sp}	0	$3.92*10^{-6}$	$1.57*10^{-5}$	$3.55*10^{-5}$	0	$2.41*10^{-6}$	$9.91*10^{-6}$	$2.34*10^{-5}$

3.5.5 Fertility risk

Table 3.4 displays the results for the introduction of uncertainty about N_1^y, i.e. fertility risk. For a given DRB pension arrangement, a change in N_1^y has a variety of effects. First, if N_1^y is low, then w is high and r^{kn} is low, which is beneficial for the young. Second, the wage-linked transfer in the first pillar has to be paid for by fewer young, increasing the amount to be transfered per young. Third , the lumpsum transfer θ^p from the old to the young is divided over less young individuals, increasing the per young transfer as well. Finally, if N_1^y is low, then the amount in the pension fund is not sufficient to pay out the defined real benefit p, because of the lower-than-expected r^{kn}. This is bad news for the few young people, because of the guarantee they have given to the old generation.

As a result of these effects, it can be observed that as uncertainty about N_1^y goes up, θ^f, p, K^f and B^f all go down. This means that the size of the funded pillar becomes smaller and that the "leverage" in the system for young participants goes down. The reason is that as the uncertainty about N_1^y becomes larger, the outcomes for the young generation when it turns out to be small (N_1^y is low) can be particularly unfortunate, and, hence, utility can be particularly low, because of the cost of providing the guarantee p to each old generation member. Specifically, this is the case for a low realisation of productivity and a high realisation of the depreciation rate. Since the expected cost to the young generation of providing the guarantee to the old generation rises as the uncertainty about N_1^y becomes larger, the utilitarian pension planner lowers p, which also lowers θ^f because of the actuarial link between the two pension parameters. Since the guarantee that the young generation provides becomes more costly in utility terms as the uncertainty surrounding N_1^y goes up, the size of the guarantee goes down. This can be seen from the fact that r^f goes down for increasing uncertainty about N_1^y (remember that for the DRB case this is defined as $\frac{p}{\theta^f} - 1$ and is thus the guaranteed return per old generation member). This is corroborated by the results from Table 3.5, where for the DRB system it can be observed that d_2 ranges from -0.0037 to -0.0317 for the solution for θ^f and from -0.0036 to -0.0318 for the solution for K^f.

Moreover, for the first pillar, θ^w falls as uncertainty about N_1^y rises. This compensates for the nonlinearity in the movement in w. If N_1^y is low, the wage rate is very high, while for at high N_1^y the wage rate does not go decrease quite as fast when N_1^y rises due to the Cobb-Douglas specification of the production function. To prevent the wage-linked transfer from becoming very burdensome for the young agents alive when N_1^y is low, θ^w goes down slightly. Decreasing θ^w in this fashion causes redistribution from old to young, because the average wage linked transfer goes down. To offset this redistribution, θ^p rises, which implies that the lump-sum transfer from old to young goes down on average.

For the DWB system, an almost identical response to the introduction of N_1^y risk can be observed from Tables 3.4 and 3.5: the second-pillar parameters θ^{dwb}, θ^f and K^f all fall to prevent the occurrence of truly bad outcomes for the young generation. Parameter θ^w goes down, and to prevent systematic redistribution θ^p needs to be raised. Finally, both systems can handle the presence of fertility risk very well; the welfare gains compared to the laissez-faire situation continue to be sizable (an increase in the certainty-equivalent consumption of 4,5% to 6%), while the welfare loss compared to the social planner's solution remains very small (on the order of magnitude of 0,001%).

3.5.6 Mortality risk

The results for the case with mortality risk, ψ, as the only source of demographic uncertainty are displayed in Tables 3.6 and 3.7. For the DRB system, all mortality risk falls on the young generation, since both the first- and second-pillar benefits are fixed per old generation member. To ensure that the income of the young generation members does not fluctuate too much, the mortality risk the young generation runs through the second pillar is reversed. This is done by setting the pension fund contribution θ^f and the pension fund benefit p at negative values. To prevent distortions of risk sharing of the financial shocks, this is achieved by a large negative pension fund position in bonds. The move in the bond portfolio means that the old generation now borrows money from the pension fund, invests this privately in bonds and pays back a per-surviving old amount of p to the fund in period 1. This means that the risk the young generation runs is now reversed: if fewer than expected old generation members survive until period 1, there is a funding shortfall in the pension fund. This contrasts with the mortality risk transmitted through the first pillar, where the parameters only

Table 3.5: Decomposed solutions, fertility risk, no mortality risk

ΔN_1^y	0.1		0.2		0.3	
	d_1	d_2	d_1	d_2	d_1	d_2
DRB						
$\theta^p + p$	0.0087	0.0005	-0.1674	0.0018	-0.4470	0.0036
θ^w	-0.0014	-0.0044	-0.0056	-0.0174	-0.0127	-0.0386
θ^f	0.0057	-0.0037	-0.1098	-0.0143	-0.2940	-0.0317
K^f	-0.0013	-0.0036	-0.0051	-0.0143	-0.0115	-0.0318
DWB						
θ^p	0.0050	-0.0003	0.0174	-0.0010	0.0390	-0.0024
$\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb}$	-0.0013	-0.0025	-0.0055	-0.0099	-0.0126	-0.0230
θ^f	0.0016	-0.0017	0.0048	-0.0071	0.0105	-0.0171
K^f	-0.0013	-0.0017	-0.0051	-0.0072	-0.0115	-0.0171

Table 3.6: Varying mortality risk, no fertility risk

System	DRB					DWB				
ΔN_1^o	0	0.05	0.1	0.15	0.2	0	0.05	0.1	0.15	0.2
θ^p	-0.7333	-0.7262	-0.7055	-0.6733	-0.6323	-0.5144	-0.5119	-0.5084	-0.5036	-0.4985
θ^w	0.5556	0.5528	0.5450	0.5333	0.5193	0.4195	0.4179	0.4156	0.4122	0.4083
θ^{dwb}	-	-	-	-	-	0.1089	0.1089	0.1072	0.1044	0.1005
p	-0.4007	-0.4007	-0.4057	-0.4142	-0.4267	-	-	-	-	-
θ^f	-0.2430	-0.2420	-0.2421	-0.2422	-0.2423	0.1629	0.1629	0.1604	0.1562	0.1505
K^f	0.5000	0.4993	0.4975	0.4951	0.4926	0.5000	0.4997	0.4989	0.4977	0.4965
K^p	0.5000	0.5007	0.5025	0.5049	0.5074	0.5000	0.5003	0.5011	0.5023	0.5035
K	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B^f	-0.7430	-0.7414	-0.7396	-0.7373	-0.7350	-0.3371	-0.3368	-0.3385	-0.3415	-0.3460
B^p	0.7430	0.7414	0.7396	0.7373	0.7350	0.3371	0.3368	0.3385	0.3415	0.3460
r	0.3736	0.3736	0.3737	0.3738	0.3739	0.3736	0.3736	0.3736	0.3737	0.3737
$E\left(r^{kn}\right)$	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
$E\left(w\right)$	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
$E\left(r^f\right)$	0.6490	0.6558	0.6758	0.7102	0.7611	0.7542	0.7611	0.7820	0.8180	0.8710
$E\left(c_o\right)$	1.9444	1.9458	1.9498	1.9567	1.9665	1.9444	1.9459	1.9503	1.9578	1.9683
$E\left(c_y\right)$	1.9444	1.9460	1.9508	1.9587	1.9698	1.9444	1.9460	1.9505	1.9582	1.9690
Ω^{pf}	0.0450	0.0452	0.0456	0.0462	0.0471	0.0450	0.0452	0.0456	0.0462	0.0472
Ω^{sp}	0	$2.61*10^{-6}$	$9.80*10^{-6}$	$1.99*10^{-5}$	$3.08*10^{-5}$	0	$5.35*10^{-7}$	$2.04*10^{-6}$	$4.24*10^{-6}$	$6.78*10^{-6}$

change to compensate for the nonlinearity in wage income w. Here, a smaller-than-expected old generation implies a lower aggregate transfers. Hence, the first and second pillars hedge each other implying a more even distribution of mortality risk across the generations. The first-pillar settings of θ^w and θ^p move into the same direction as in the case of fertility risk: θ^w falls as uncertainty increases and θ^p becomes less negative to compensate for the redistribution effect of the change in θ^w.

Under the DWB system, mortality risk is more evenly distributed across generations through the set-up of the pension fund. In the first pillar, the young generation still bears all the mortality risk. However, in the second pillar, the total payout of the fund depends on the aggregate wage sum, which does not depend on the size of the old generation. Hence, the mortality risk associated with the second pillar falls on the old generation. Thus, by construction the DWB system distributes mortality risk over the different generations and there is no need – as there is in the DRB system – to hedge mortality risk by making second-pillar pension savings negative. Therefore, the pension designer can afford to pay more attention to the distribution of the financial shocks.

As in the case of fertility risk alone, θ^p becomes less negative and θ^w becomes smaller when uncertainty increases. Similarly, in order to limit the impact on consumption in the bad state in the second pillar, θ^f and θ^{dwb} decline as uncertainty rises. In terms of welfare, the DWB system deals slightly better with mortality risk than the DRB system, since its set-up is more amenable to evenly distributing the effects of a mortality shock. However, compared to the welfare gains that are obtained relative to the laissez-faire solution (between 4.5% to 4.7%), the difference between the two systems is very small.

3.5.7 Simultaneous presence of both types of demographic risk

Varying fertility risk with constant mortality risk

Finally, the combined effect of the presence of both sources of demographic uncertainty are analyzed. In Table 3.8 the results when varying fertility risk, while mortality risk is

Table 3.7: Decomposed solutions, mortality risk, no fertility risk

ΔN_1^o	0.05		0.1		0.15		0.2	
	d_1	d_2	d_1	d_2	d_1	d_2	d_1	d_2
DRB								
$\theta^p + p$	0.0016	0.0055	0.0016	0.0212	0.0007	0.0458	-0.0020	0.0770
θ^w	0.0004	-0.0032	0.0017	-0.0123	0.0038	-0.0261	0.0069	-0.0432
θ^f	0.0016	-0.0006	0.0034	-0.0025	0.0057	-0.0049	0.0080	-0.0073
K^f	0.0000	-0.0007	0.0000	-0.0025	0.0000	-0.0049	0.0000	-0.0074
DWB								
θ^p	0.0000	0.0025	-0.0038	0.0098	-0.0105	0.0213	-0.0203	0.0362
$\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb}$	0.0004	-0.0014	0.0017	-0.0055	0.0039	-0.0120	0.0069	-0.0202
θ^f	0.0003	-0.0003	-0.0014	-0.0011	-0.0044	-0.0023	-0.0089	-0.0035
K^f	0.0000	-0.0003	0.0000	-0.0011	0.0000	-0.0023	0.0000	-0.0035

Table 3.8: Varying fertility risk, mortality risk constant at $\Delta N_1^o = 0.1$

System	DRB				DWB			
ΔN_1^o	0	0.1	0.2	0.3	0	0.1	0.2	0.3
θ^p	-0.7055	-0.5933	-0.5105	-0.4661	-0.5084	-0.5855	-0.5918	-0.5736
θ^w	0.5450	0.4889	0.4446	0.4162	0.4156	0.4544	0.4539	0.4380
θ^{dwb}	-	-	-	-	0.1072	0.0847	0.0776	0.0731
p	-0.4057	-0.3963	-0.3998	-0.4153	-	-	-	-
θ^f	-0.2421	-0.2357	-0.2381	-0.2494	0.1604	0.1266	0.1158	0.1087
K^f	0.4975	0.4162	0.3583	0.3302	0.4989	0.5194	0.5154	0.5001
K^p	0.5025	0.5838	0.6417	0.6698	0.5011	0.4806	0.4846	0.4999
K	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B^f	-0.7396	-0.6519	-0.5964	-0.5796	-0.3385	-0.3928	-0.3997	-0.3915
B^p	0.7396	0.6519	0.5964	0.5796	0.3385	0.3928	0.3997	0.3915
r	0.3737	0.3719	0.3666	0.3567	0.3736	0.3719	0.3664	0.3563
$E\left(r^{kn}\right)$	0.4000	0.3991	0.3962	0.3913	0.4000	0.3991	0.3962	0.3913
$E\left(w\right)$	2.1000	2.1041	2.1168	2.1391	2.1000	2.1041	2.1168	2.1391
$E\left(r^f\right)$	0.6758	0.6814	0.6791	0.6652	0.7820	0.7820	0.7801	0.7772
$E\left(c_o\right)$	1.9498	1.9468	1.9395	1.9275	1.9503	1.9465	1.9388	1.9258
$E\left(c_y\right)$	1.9508	1.9485	1.9417	1.9298	1.9505	1.9495	1.9436	1.9330
Ω^{pf}	0.0456	0.0471	0.0517	0.0596	0.0456	0.0471	0.0518	0.0597
Ω^{sp}	$9.80*10^{-6}$	$6.45*10^{-5}$	$1.07*10^{-4}$	$1.33*10^{-4}$	$2.04*10^{-6}$	$2.03*10^{-5}$	$5.05*10^{-5}$	$1.02*10^{-4}$

kept constant at $\Delta N_1^o = 0.1$ are displayed. For the DRB arrangement, mortality risk is shared in the same way as in the case when mortality risk was the only source of risk, by decreasing θ^f and p to -0.25 and -0.44 respectively. Since this prohibits sharing of fertility risk the way this was done previously (through increases in θ^f, B^f and p), this has to be done in another way. Fertility risk is compensated for by having both θ^w and K^f going down. The decrease in θ^w ensures that in case of a bad fertility shock – the young generation turns out to be small – the burden on individual young generation members does not become too high. However, this also shifts more productivity risk to the young generation. To mitigate this effect, K^f is reduced and, thus, K^p is increased. This shifts some of the productivity as well as some of the depreciation risk to the old generation. As the variance of the fertility shock rises, these effects become more and more pronounced. To compensate, θ^w and K^f have to fall by more.

The DWB system turns out to be more robust to the presence of two types of demographic risks. Since for both the fertility shock and the mortality shock the changes in the pension fund parameters were qualitatively the same – a decrease in the size of the second pillar and an increase in the size of the wage-linked transfer in the first pillar, and the lump-sum first pillar transfer becoming negative to offset the redistributive impact of the increase in θ^w – the parameters are simply set as an average of the parameter settings under the separate demographic risks.

Not surprisingly, given that the DWB system is better suited for handling the presence of both types of demographic risks simultaneously, the welfare gains under the DWB system are slightly higher than under the DRB system. The presence of two sources of demographic risk causes the resulting allocation of consumption to both generations to be more remote from the social planner's solution than when only one source of demographic risk is present. Therefore, the shortfall from the welfare gain under the social planner has become slightly higher than in the cases with only one source of demographic risk.

Varying mortality risk with constant fertility risk

In Table 3.10 fertility risk is kept constant at $\Delta N_1^y = 0.2$, while the uncertainty surrounding the mortality shock is increased progressively. The same patterns are observed

Table 3.9: Decomposed solutions, varying fertility risk, constant mortality risk at $\Delta N_1^o = 0.1$

ΔN_1^y	0.1		0.2		0.3	
	d_1	d_2	d_1	d_2	d_1	d_2
DRB						
$\theta^p + p$	0.1141	0.0075	0.2015	-0.0006	0.2320	-0.0022
θ^w	0.0109	-0.0670	0.0067	-0.1071	-0.0004	-0.1284
θ^f	0.0889	-0.0825	0.1406	-0.1366	0.1510	-0.1583
K^f	0.0012	-0.0825	-0.0026	-0.1366	-0.0090	-0.1583
DWB						
θ^p	-0.0924	0.0153	-0.1011	0.0177	-0.0853	0.0201
$\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb}$	0.0042	0.0061	0.0000	0.0007	-0.0071	-0.0138
θ^f	-0.0545	0.0207	-0.0652	0.0206	-0.0634	0.0117
K^f	-0.0002	0.0207	-0.0040	0.0205	-0.0104	0.0116

Table 3.10: Varying mortality risk, fertility risk constant at $\Delta N_1^y = 0.2$

System	DRB					DWB				
ΔN_1^o	0	0.05	0.1	0.15	0.2	0	0.05	0.1	0.15	0.2
θ^p	-1.0851	-0.5417	-0.5105	-0.4806	-0.4442	-0.6043	-0.6018	-0.5918	-0.5736	-0.5497
θ^w	0.5326	0.4525	0.4446	0.4353	0.4243	0.4640	0.4614	0.4539	0.4424	0.4284
θ^{dwb}	-	-	-	-	-	0.0609	0.0670	0.0776	0.0846	0.0871
p	3.0937	-0.3069	-0.3998	-0.4250	-0.4434	-	-	-	-	-
θ^f	1.8005	-0.1868	-0.2381	-0.2479	-0.2513	0.0910	0.1001	0.1158	0.1262	0.1299
K^f	0.4806	0.3612	0.3583	0.3579	0.3579	0.4877	0.4984	0.5154	0.5237	0.5221
K^p	0.5194	0.6388	0.6417	0.6421	0.6421	0.5123	0.5016	0.4846	0.4763	0.4779
K	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B^f	1.3199	-0.5480	-0.5964	-0.6058	-0.6092	-0.3968	-0.3983	-0.3997	-0.3976	-0.3922
B^p	-1.3199	0.5480	0.5964	0.6058	0.6092	0.3968	0.3983	0.3997	0.3976	0.3922
r	0.3667	0.3667	0.3666	0.3664	0.3661	0.3667	0.3666	0.3664	0.3660	0.3656
$E\left(r^{kn}\right)$	0.3962	0.3962	0.3962	0.3962	0.3962	0.3962	0.3962	0.3962	0.3962	0.3962
$E\left(w\right)$	2.1168	2.1168	2.1168	2.1168	2.1168	2.1168	2.1168	2.1168	2.1168	2.1168
$E\left(r^f\right)$	0.7182	0.6429	0.6791	0.7144	0.7644	0.7512	0.7585	0.7801	0.8166	0.8699
$E\left(c_o\right)$	1.9351	1.9351	1.9395	1.9469	1.9575	1.9352	1.9359	1.9388	1.9450	1.9547
$E\left(c_y\right)$	1.9353	1.9368	1.9417	1.9498	1.9613	1.9353	1.9376	1.9436	1.9528	1.9650
Ω^{pf}	0.0512	0.0512	0.0517	0.0525	0.0536	0.0512	0.0513	0.0518	0.0526	0.0536
Ω^{sp}	$1.57*10^{-5}$	$9.66*10^{-5}$	$1.07*10^{-4}$	$1.20*10^{-4}$	$1.33*10^{-4}$	$9.91*10^{-6}$	$2.23*10^{-5}$	$5.05*10^{-5}$	$8.30*10^{-5}$	$1.14*10^{-4}$

as in the previous cases that included mortality risk. Under the DRB system, the first two columns provide an illustration of the reaction of the pension system parameters to the introduction of mortality risk. In the absence of mortality risk (the first column) the pension system parameters are set to distribute the fertility shock equally across generations as in Table 3.4. When mortality risk is introduced (the second column, where the realization of N_1^o is either 0.05 above or below its mean), the funded pillar is used as a hedge against mortality risk that the young generation runs through the first pillar by setting θ^f and p at negative values (-0.19 and -0.31, respectively). However, with these settings, young generation members run more fertility risk than the old generation members. The solution then is to shift some fertility risk towards the old generation by increasing their exposure to the equity return. This is accomplished by raising K^p and reducing K^f. However, the downside of this is that it also shifts more productivity and depreciation risk to the old generation. The various consequences are balanced by raising K^p up to the point at which the additional benefits of increased sharing of fertility risk no longer outweigh the utility costs of decreased risk sharing of productivity and depreciation risk. The larger the uncertainty about fertility risk becomes, the higher the benefits of sharing this risk and the larger (lower) is the optimal value of K^p (K^f). The presence of two sources of demographic risk increases the shortfall in welfare terms relative to that under the social planner. However, the increase in the shortfall is still very small compared to the gains as measured in terms of certainty-equivalent consumption relative to the laissez-faire solution.

For the DWB system, the same result is obtained as in the previous subsection: because the pension system parameters react in the same way to fertility and mortality risk, the pension system parameters simply are an average of the parameter settings that resulted in the presence of each type of demographic risk separately. As before, the welfare gains under DWB are slightly higher than under DRB, because the optimal instrument settings under DRB are forced to deviate more from those under the social planner. Nonetheless, both types of pension arrangement constitute a substantial improvement over the laissez-faire solution but lead to only small deviations from the social planner solution.

Table 3.11: Decomposed solutions, varying mortality risk, constant fertility risk at $\Delta N_1^y = 0.2$

ΔN_1^o	0.05		0.1		0.15		0.2	
	d_1	d_2	d_1	d_2	d_1	d_2	d_1	d_2
DRB								
$\theta^p + p$	-2.8425	-0.0147	-2.9183	-0.0006	-2.9362	0.0220	-2.9465	0.0503
θ^w	0.0178	-0.0979	0.0191	-0.1071	0.0213	-0.1186	0.0244	-0.1327
θ^f	-1.8536	-0.1337	-1.9020	-0.1366	-1.9114	-0.1370	-1.9148	-0.1370
K^f	0.0143	-0.1337	0.0143	-0.1366	0.0143	-0.1370	0.0143	-0.1370
DWB								
θ^p	-0.0017	0.0042	-0.0052	0.0177	-0.0043	0.0350	0.0007	0.0539
$\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb}$	0.0103	-0.0050	0.0116	0.0007	0.0138	-0.0019	0.0169	-0.0125
θ^f	0.0056	0.0035	0.0042	0.0206	0.0063	0.0289	0.0117	0.0272
K^f	0.0072	0.0035	0.0072	0.0205	0.0072	0.0288	0.0072	0.0272

3.6 Robustness: varying the degree of risk aversion

In the previous section, the effect of changes in the volatility of the shocks on the optimal pension system parameters was investigated. In this section, the robustness of the results for different values of the CRRA parameter ϕ (which was assumed to be equal to 2.5 so far) is checked. Both a lower degree of risk aversion $\phi = 1$ (i.e., log-utility preferences) and a higher degree of risk aversion, $\phi = 5$, are considered. Results are presented for the three baseline scenarios considered in the previous section: only fertility risk ($\Delta N_1^y = 0.2$), only mortality risk ($\Delta N_1^o = 0.1$), fertility and mortality risk at the same time ($\Delta N_1^y = 0.2$ and $\Delta N_1^y = 0.1$). They are found in Table 3.12.

The top panel of the table shows the case with only fertility risk. For both DRB and DWB, in general the impact of varying the degree of risk aversion on the values of the parameters is quite limited, indicating that the chosen degree of risk aversion is immaterial for the results reported so far. As risk aversion becomes lower (i.e., $\phi = 1$), slightly more wage risk is shifted to the old generation by increasing in θ^w and θ^{dwb}, while at the same time slightly more financial risk is shifted to the young generation by increasing K^f somewhat. The intuition can be extracted from Table 3.5. Here, it was seen for $\phi = 2.5$ that the solutions for both θ^w, θ^{dwb} and K^f under a pension scheme were slightly lower than the corresponding social planner's solutions under perfect foresight. This way young individuals were protected against low income and utility in the case of a bad combination of shocks. However, when risk aversion is lower, the penalty in terms of utility when income is low is less severe and, hence, the optimal pension parameters shift in the direction of the social planner's solution. The opposite happens when risk aversion rises. The risk-free interest rate is quite sensitive to the degree of risk aversion. As individuals become less risk averse, they require a return on the risk-free bonds that is closer to the expected return on capital, so that the risk-free rate goes up. Finally, the size of the welfare gains from introducing a pension arrangement depend quite strongly on the risk aversion parameter: as risk aversion falls from $\phi = 2.5$ to $\phi = 1$, the welfare gain relative to laissez-faire falls from around 5% to slightly less than 2% for both DRB and DWB, while raising risk aversion to $\phi = 5$ increases the welfare gain to almost 10.5%. This sensitivity is not surprising, because the advantage of having a pension arrangement is tightly linked to the risk-sharing it provides. If risk aversion becomes smaller the value of having a pension arrangement that allows for risk sharing goes down. For both lower and higher risk aversion, the welfare differences between the social planner and the decentralised solutions with pension arrangements are small. Moreover, the differences become smaller as risk aversion falls.

The results for mortality risk only and the simultaneous presence of both types of demographic risk are reported in the middle and bottom panels of Table 3.12. In these cases varying the degree of risk aversion has a rather small impact on the outcomes, comparable to the effects for the case of only fertility risk. The directions of the changes in the parameters is the same as in the case of fertility risk only. Again the quantitatively largest effects are observed for the risk-free rate and the welfare gains of introducing a defined-benefit pension arrangement.

3.7 Conclusion

In this chapter intergenerational risk sharing within two-tier pension systems in the presence of productivity, financial market and demographic risks has been explored. Compared to the laissez-faire situation, relatively large welfare gains from the presence of a two-tier pension system with a defined-benefit second pillar are found. The first, PAYG pillar takes care of appropriate redistribution between the young and the old generation while also allowing for some sharing of wage risks and demographic risks. The fully-funded second pillar allows for further risk sharing between the two generations. This is accomplished by the mismatch between the assets and the liabilities of the pension fund, where the young are the residual claimants to the pension fund. Obviously, the size of the welfare gains associated with the pension arrangements under consideration should not be over-emphasized, as the underlying economic model is necessarily very stylised.

The simulation results suggest that while having a defined-benefit second pension pillar yields large welfare gains, the exact form in which the benefits are defined is less relevant, as the welfare differences between the defined-benefit systems under con-

Table 3.12: Varying risk aversion

System	DRB			DWB		
ϕ	1	2.5	5	1	2.5	5
$\Delta N_1^y = 0.2$, $\Delta N_1^o = 0$						
θ^p	-1.0911	-1.0851	-1.0776	-0.6117	-0.6043	-0.5924
θ^w	0.5331	0.5326	0.5320	0.4639	0.4640	0.4640
θ^{dwb}	-	-	-	0.0614	0.0609	0.0603
p	3.1398	3.0937	3.0353	-	-	-
θ^f	1.8104	1.8005	1.7931	0.0918	0.091	0.0898
K^f	0.4808	0.4806	0.4808	0.4884	0.4877	0.4868
B^f	1.3296	1.3199	1.3124	-0.3966	-0.3968	-0.3971
r	0.3843	0.3667	0.3388	0.3843	0.3667	0.3387
Ω^{pf}	0.0197	0.0512	0.1043	0.0197	0.0512	0.1043
Ω^{sp}	$6.37 * 10^{-6}$	$1.57 * 10^{-5}$	$2.98 * 10^{-5}$	$3.93 * 10^{-6}$	$9.91 * 10^{-6}$	$1.98 * 10^{-5}$
$\Delta N_1^y = 0$, $\Delta N_1^o = 0.1$						
θ^p	-0.7077	-0.7055	-0.7023	-0.5153	-0.5084	-0.4977
θ^w	0.5472	0.5450	0.5417	0.4162	0.4156	0.4147
θ^{dwb}	-	-	-	0.1075	0.1072	0.1067
p	-0.4219	-0.4057	-0.3814	-	-	-
θ^f	-0.2429	-0.2421	-0.2416	0.1611	0.1604	0.1594
K^f	0.4973	0.4975	0.4981	0.4988	0.4989	0.4991
B^f	-0.7402	-0.7396	-0.7397	-0.3376	-0.3385	-0.3397
r	0.3894	0.3737	0.3487	0.3894	0.3736	0.3487
Ω^{pf}	0.0176	0.0456	0.0913	0.0176	0.0456	0.0913
Ω^{sp}	$4.20 * 10^{-6}$	$9.80 * 10^{-6}$	$1.58 * 10^{-5}$	$8.70 * 10^{-7}$	$2.04 * 10^{-6}$	$3.31 * 10^{-6}$
$\Delta N_1^y = 0.2$, $\Delta N_1^o = 0.1$						
θ^p	-0.5140	-0.5105	-0.5026	-0.6040	-0.5918	-0.5730
θ^w	0.4472	0.4446	0.4393	0.4565	0.4539	0.4501
θ^{dwb}	-	-	-	0.0776	0.0776	0.0775
p	-0.4107	-0.3998	-0.3828	-	-	-
θ^f	-0.2367	-0.2381	-0.241	0.1161	0.1158	0.1151
K^f	0.3567	0.3583	0.3611	0.5173	0.5154	0.5121
B^f	-0.5934	-0.5964	-0.6021	-0.4013	-0.3997	-0.397
r	0.3843	0.3666	0.3380	0.3842	0.3664	0.3377
Ω^{pf}	0.0197	0.0517	0.1065	0.0197	0.0518	0.1066
Ω^{sp}	$4.29 * 10^{-5}$	$1.07 * 10^{-4}$	$2.09 * 10^{-4}$	$2.06 * 10^{-5}$	$5.05 * 10^{-5}$	$9.52 * 10^{-5}$

sideration are very small, irrespective of the degree of uncertainty and the degree of risk aversion. Moreover, while it is not possible to perfectly replicate the social planner's solution in the presence of demographic risks, the defined-benefit arrangements under consideration yield welfare gains compared to the situation without a pension arrangement that are very close to the welfare gains realised under the social planner's solution. The DWB system performs slightly better in this regard.

An avenue for further research concerns the design of pension arrangements that are as robust as possible to foreseen and unforeseen changes in the environment. While changes in the underlying socio-economic environment tend to make adjustments to pension arrangements desirable, making those changes is often a difficult process because of the unavoidable conflicting interests. This has been made painfully clear by the arduous process of switching from the old to a new pension contract in the Netherlands. While it is widely recognised that the funded pension promises given under the old contract are unsustainable in view of past and expected future increases in life expectancy and the poor performance of the pension funds' assets during the current crisis, it has proven to be extremely difficult to agree on the terms of the new contract. In many instances when reform is contemplated the interests are split along generational lines. The difficulties in changing the arrangements increase the desirability of automatic adjustment rules, such as an automatic link between life expectancy and the retirement age or an explicit loss sharing rule in case the financial position of the pension fund deteriorates. However, while the game-theoretic aspects of resetting pension arrangements are highly relevant, they have been underexplored so far.

APPENDICES

3.A Description of solution of model

Given the set of pension system parameters Π^{drb} or Π^{dwb}, we solve the model as follows. The pension contribution θ^f is tied down by the actuarial fairness condition (3.27) for the DRB fund and (3.28) for the DWB fund. In particular, in the case of a DRB system for each combination θ^p and p that fulfills the condition in the proposition, an outcome for θ^f is found that fulfills the actuarial fairness condition. Similarly, for the DWB system for each combination θ^w and θ^{dwb} that fulfills the condition in the proposition, an outcome for θ^f is found that fulfills the actuarial fairness condition. Then, equation (3.12) pins down $B^f = N_0^o \theta^f - K^f$. Next, via (3.30) we obtain $B^p = -B^f$. Using the solutions for θ^f and B^p in (3.20) we obtain K^p and, hence, we also obtain the total capital stock $K = K^p + K^f$. Finally, substituting the relevant expression for G into (3.21) and substituting the resulting expression into (3.25), we are left with an equation that we can solve for the remaining unknown, r. In solving this equation, we make use of the firm's first-order conditions (3.23) and (3.24).

3.B Proof of Proposition 1

3.B.1 Part (i)

A necessary condition for replicating the social planner solution is $u'(c_o) = u'(c_y)$, which simplifies to $c_o = c_y$. The general expressions for consumption of the young and of the old are given by equations (3.21) and (3.22), respectively.

Under the DRB system, the generational account is given by equation (3.16). Substituting this into the consumption functions (3.21) and (3.22) yields:

$$\begin{aligned}
c_o &= (1+r)\frac{B^p}{N_0^o} + \left(1+r^{kn}\right)\frac{K^p}{N_0^o} + \frac{(1+r)B^f + \left(1+r^{kn}\right)K^f}{N_1^o} + \tau + \theta^p + \theta^w w \\
&+ p - \frac{(1+r)B^f + \left(1+r^{kn}\right)K^f}{N_1^o} \\
&= \left(1+r^{kn}\right)\frac{K^p}{N_0^o} + (1+r)\frac{B^p}{N_0^o} + \tau + \theta^p + \theta^w w + p, \\
c_y &= w + \tau - \frac{N_1^o}{N_1^y}\left[\theta^p + \theta^w w + p - \frac{(1+r)B^f + \left(1+r^{kn}\right)K^f}{N_1^o}\right].
\end{aligned}$$

Setting $c_o = c_y$, using that $K^p = K - K^f$, and rearranging, we obtain:

$$\begin{aligned}
&\left(1+r^{kn}\right)\left(\frac{K-K^f}{N_0^o} - \frac{K^f}{N_1^y}\right) + (1+r)\frac{B^p}{N_0^o} + \left(\frac{N_1^y + N_1^o}{N_1^y}\right)\left(\theta^p + \theta^w w + p\right) \\
&= w + (1+r)\frac{B^f}{N_1^y}
\end{aligned}$$

By setting $\theta^w = \frac{N_1^y}{N_1^y + N_1^o}$ we obtain identical exposure to w on both sides of the equation. The equation then reduces to:

$$\begin{aligned}
&\left(1+r^{kn}\right)\left(\frac{K-K^f}{N_0^o} - \frac{K^f}{N_1^y}\right) + (1+r)\frac{B^p}{N_0^o} + \left(\frac{N_1^y + N_1^o}{N_1^y}\right)\left(\theta^p + p\right) \\
&= (1+r)\frac{B^f}{N_1^y}
\end{aligned}$$

By setting $\frac{K-K^f}{N_0^o} - \frac{K^f}{N_1^y} = 0$, hence $K^f = \frac{N_1^y}{N_1^y + N_0^o}K = \frac{N_1^y N_0^o}{N_1^y + N_0^o}\eta$, where we have used (3.4), we ensure that consumption of the young and the old responds identically to r^{kn}. Note that the combination $\theta^w = \frac{N_1^y}{N_1^y + N_1^o}$ and $K^f = \frac{N_1^y}{N_1^y + N_0^o}K$ is also necessary to simultaneously ensure identical exposures to w and r^{kn}. The vector of factor prices is a function of the productivity and depreciation shocks. Effectively, we have used the two instruments θ^w and K^f to force identical exposures of all period-1 individuals to these fundamental sources of shocks. This leaves us with the equation

$$
\begin{aligned}
& (1+r)\frac{B^p}{N_0^o} + \left(\frac{N_1^y + N_1^o}{N_1^y}\right)(\theta^p + p) = (1+r)\frac{B^f}{N_1^y} \\
\Leftrightarrow\ & \left(\frac{N_1^y + N_1^o}{N_1^y}\right)(\theta^p + p) = (1+r)\left(\frac{B^f}{N_1^y} + \frac{B^f}{N_0^o}\right) \\
\Leftrightarrow\ & \theta^p + p = (1+r)\frac{N_0^o + N_1^y}{N_0^o\left(N_1^o + N_1^y\right)} B^f
\end{aligned}
$$

where we have used the bonds market equilibrium condition $B^p = -B^f$ to obtain the second line. This completes the proof of part (i).

3.B.2 Part (ii)

Under the DWB system the generational account is given by equation (3.19). Substituting this into the consumption functions (3.21) and(3.22) yields:

$$
\begin{aligned}
c_o &= (1+r)\frac{B^p}{N_0^o} + \left(1+r^{kn}\right)\frac{K^p}{N_0^o} + \frac{(1+r)B^f + \left(1+r^{kn}\right)K^f}{N_1^o} + \tau + \theta^p + \theta^w w \\
&+ \frac{N_1^y}{N_1^o}\theta^{dwb} w - \frac{1}{N_1^o}\left[(1+r)B^f + \left(1+r^{kn}\right)K^f\right] \\
&= (1+r)\frac{B^p}{N_0^o} + \left(1+r^{kn}\right)\frac{K^p}{N_0^o} + \tau + \theta^p + \theta^w w + \frac{N_1^y}{N_1^o}\theta^{dwb} w, \\
c_y &= w + \tau - \frac{N_1^o}{N_1^y}\left\{\theta^p + \theta^w w + \frac{N_1^y}{N_1^o}\theta^{dwb} w - \frac{1}{N_1^o}\left[(1+r)B^f + \left(1+r^{kn}\right)K^f\right]\right\}.
\end{aligned}
$$

Again, a necessary condition for replicating the social planner solution is to set $c_o = c_y$. Doing so, using that $K^p = K - K^f$, and rearranging, we obtain:

$$
\begin{aligned}
& (1+r)\frac{B^p}{N_0^o} + \left(1+r^{kn}\right)\frac{K - K^f}{N_0^o} + \left(\frac{N_1^y + N_1^o}{N_1^y}\right)\left[\theta^p + \theta^w w + \frac{N_1^y}{N_1^o}\theta^{dwb} w\right] \\
= \ & w + \frac{1}{N_1^y}\left[(1+r)B^f + \left(1+r^{kn}\right)K^f\right].
\end{aligned}
$$

Setting $\theta^w + \frac{N_1^y}{N_1^o}\theta^{dwb} = \frac{N_1^y}{N_1^y + N_1^o}$ makes the exposure to wages on both sides of the equation identical. Further, setting $K^f = \frac{N_1^y}{N_1^y + N_0^o}K = \frac{N_1^y N_0^o}{N_1^y + N_0^o}\eta$ results in identical exposure to

r^{kn} for individuals from both generations. We are then left with:

$$(1+r)\,\frac{B^p}{N_0^o} + \left(\frac{N_1^y + N_1^o}{N_1^y}\right)\theta^p = (1+r)\,\frac{B^f}{N_1^y}.$$

Using that $B^p = -B^f$, we rewrite this as:

$$\begin{aligned} \frac{N_1^y + N_1^o}{N_1^y}\theta^p &= (1+r)\left[\frac{B^f}{N_1^y} + \frac{B^f}{N_0^o}\right] \\ \Rightarrow \theta^p &= (1+r)\,\frac{N_0^o + N_1^y}{N_0^o\,(N_1^y + N_1^o)}B^f. \end{aligned}$$

This concludes the proof of part (ii).

3.C Derivatives of expressions in Proposition 1

In this part of the appendix we provide the derivatives of the expressions in Proposition 1.

3.C.1 DRB

$$
\begin{aligned}
\frac{\partial\left(\theta^{p}+p\right)}{\partial N_{1}^{y}} &= (1+r)\,B^{f}\,\frac{N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)-\left(N_{0}^{o}+N_{1}^{y}\right)N_{0}^{o}}{\left[N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)\right]^{2}} \\
&= (1+r)\,B^{f}\,\frac{N_{1}^{o}-N_{0}^{o}}{N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)^{2}} \lesseqgtr 0, \\
\frac{\partial^{2}\left(\theta^{p}+p\right)}{\partial^{2}N_{1}^{y}} &= -2\,(1+r)\,B^{f}\,\frac{N_{1}^{o}-N_{0}^{o}}{N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)^{3}} \lesseqgtr 0, \\
\frac{\partial\left(\theta^{p}+p\right)}{\partial N_{1}^{o}} &= -\,(1+r)\,B^{f}\,\frac{N_{0}^{o}+N_{1}^{y}}{N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)^{2}} \lesseqgtr 0, \\
\frac{\partial^{2}\left(\theta^{p}+p\right)}{\partial^{2}N_{1}^{o}} &= 2\,(1+r)\,B^{f}\,\frac{N_{0}^{o}+N_{1}^{y}}{N_{0}^{o}\left(N_{1}^{y}+N_{1}^{o}\right)^{3}} \lesseqgtr 0, \\
\frac{\partial K^{f}}{\partial N_{1}^{y}} &= \eta\frac{\left(N_{1}^{y}+N_{0}^{o}\right)N_{0}^{o}-N_{1}^{y}N_{0}^{o}}{\left(N_{1}^{y}+N_{0}^{o}\right)^{2}} = \eta\left[\frac{N_{0}^{o}}{N_{1}^{y}+N_{0}^{o}}\right]^{2} > 0, \\
\frac{\partial^{2}K^{f}}{\partial^{2}N_{1}^{y}} &= -2\eta\frac{\left(N_{0}^{o}\right)^{2}}{\left(N_{1}^{y}+N_{0}^{o}\right)^{3}} < 0, \\
\frac{\partial K^{f}}{\partial N_{1}^{o}} &= 0, \\
\frac{\partial^{2}K^{f}}{\partial^{2}N_{1}^{o}} &= 0, \\
\frac{\partial\theta^{w}}{\partial N_{1}^{y}} &= \frac{N_{1}^{y}+N_{1}^{o}-N_{1}^{y}}{\left(N_{1}^{y}+N_{1}^{o}\right)^{2}} = \frac{N_{1}^{o}}{\left(N_{1}^{y}+N_{1}^{o}\right)^{2}} > 0, \\
\frac{\partial^{2}\theta^{w}}{\partial^{2}N_{1}^{y}} &= -2\frac{N_{1}^{o}}{\left(N_{1}^{y}+N_{1}^{o}\right)^{3}} < 0, \\
\frac{\partial\theta^{w}}{\partial N_{1}^{o}} &= -\frac{N_{1}^{y}}{\left(N_{1}^{y}+N_{1}^{o}\right)^{2}} < 0, \\
\frac{\partial^{2}\theta^{w}}{\partial^{2}N_{1}^{o}} &= 2\frac{N_{1}^{y}}{\left(N_{1}^{y}+N_{1}^{o}\right)^{3}} > 0.
\end{aligned}
$$

3.C.2 DWB

$$
\begin{aligned}
\frac{\partial \theta^p}{\partial N_1^y} &= (1+r)\,B^f\,\frac{N_0^o\left(N_1^y+N_1^o\right)-N_0^o\left(N_1^y+N_0^o\right)}{\left[N_0^o\left(N_1^y+N_1^o\right)\right]^2},\\
&= (1+r)\,B^f\,\frac{N_1^o-N_0^o}{N_0^o\left(N_1^y+N_1^o\right)^2} \lesseqqgtr 0,\\
\frac{\partial^2 \theta^p}{\partial^2 N_1^y} &= -2\,(1+r)\,B^f\,\frac{N_1^o-N_0^o}{N_0^o\left(N_1^y+N_1^o\right)^3} \lesseqqgtr 0,\\
\frac{\partial \theta^p}{\partial N_1^o} &= -\,(1+r)\,B^f\,\frac{N_0^o+N_1^y}{N_0^o\left(N_1^y+N_1^o\right)^2} \lesseqqgtr 0,\\
\frac{\partial \theta^p}{\partial N_1^o} &= 2\,(1+r)\,B^f\,\frac{N_0^o+N_1^y}{N_0^o\left(N_1^y+N_1^o\right)^3} \lesseqqgtr 0,\\
\frac{\partial K^f}{\partial N_1^y} &= \eta\frac{\left(N_1^y+N_0^o\right)N_0^o-N_1^yN_0^o}{\left(N_1^y+N_0^o\right)^2} = \eta\left[\frac{N_0^o}{N_1^y+N_0^o}\right]^2 > 0,\\
\frac{\partial^2 K^f}{\partial^2 N_1^y} &= -2\eta\frac{\left(N_0^o\right)^2}{\left(N_1^y+N_0^o\right)^3} < 0,\\
\frac{\partial K^f}{\partial N_1^o} &= 0,\\
\frac{\partial^2 K^f}{\partial^2 N_1^o} &= 0,\\
\frac{\partial\left[\theta^w+\left(N_1^y/N_1^o\right)\theta^{dwb}\right]}{\partial N_1^y} &= \frac{N_1^o}{\left(N_1^y+N_1^o\right)^2} > 0,\\
\frac{\partial^2\left[\theta^w+\left(N_1^y/N_1^o\right)\theta^{dwb}\right]}{\partial^2 N_1^y} &= -2\frac{N_1^o}{\left(N_1^y+N_1^o\right)^3} < 0,\\
\frac{\partial\left[\theta^w+\left(N_1^y/N_1^o\right)\theta^{dwb}\right]}{\partial N_1^o} &= -\frac{N_1^y}{\left(N_1^y+N_1^o\right)^2} < 0,\\
\frac{\partial^2\left[\theta^w+\left(N_1^y/N_1^o\right)\theta^{dwb}\right]}{\partial^2 N_1^o} &= 2\frac{N_1^y}{\left(N_1^y+N_1^o\right)^3} > 0.
\end{aligned}
$$

Chapter 4

Voluntary Participation and Intergenerational Risk Sharing in a Funded Pension System

4.1 Introduction

All over the world countries introduce funded pension systems in anticipation of the projected rise in aging costs. Often those systems are of the defined-contribution (DC) type, which means that workers save through a pension fund and at retirement receive whatever they have accumulated in their own account. Hence, under such an arrangement much of the potential benefit from intergenerational risk sharing, in particular risk sharing between workers and retirees, will be lost. However, this need not be the case if the funded pension scheme is appropriately designed. For example, in the Netherlands, a country with a large pillar of sector and company pension funds that pay out benefits defined in nominal terms, risks are shared among generations of workers and retirees through changes in the contributions paid by the workers and changes in the indexation of pension benefits and pension rights to wage or price inflation. In the case of underfunding (low buffers), reduced indexation implies a reduction in the purchasing power of all generations' benefits or rights, while, in addition, contributions paid by working generations may be raised. The opposite tends to happen when the ratio of assets to liabilities becomes very large. Participation of employees in the system

This chapter is joint work with Roel Beetsma and Ward Romp, and has been published in European Economic Review 56 (2012), pp. 1310-1324.

is mandatory.[2] This is important, because a risk-sharing arrangement that is not legally enforced is only viable when there exists no ex-post incentive to quit the arrangement whenever a transfer has to be made. In a situation of underfunding of the pension fund, the young would be tempted to leave the system instead of guaranteeing the pensions of the retired through higher contributions. If the buffers become too large, the old will be tempted to close the fund. If participation in a funded pension system is voluntary, the ex-ante benefits of intergenerational risk sharing may be forgone.

In this chapter, the feasibility and welfare consequences of a funded pension system with voluntary participation and intergenerational risk sharing is expored. So far, there hardly exists any formal analysis addressing this issue. Such an analysis is of substantial policy relevance, because the countries that currently increase or plan to increase the funded component of their pension systems need to take informed decisions about its design. In particular they need to decide whether to introduce certain defined-benefit elements and whether to make participation obligatory.

Ongoing discussions or even reforms take place in many countries. Of particular relevance for this chapter is the discussion in the U.K. The Department for Work and Pensions (DWP, 2012) is preparing a revision of the Pensions Act 2011 with the aim of making 'auto-enrolment' of employees in pension schemes provided by the employer obligatory. At the same time, the European Commission is revising the Directive for Institutions of Occupational Retirement Provision (IORPs) and plans to have draft legislation before the summer of 2013 (Barnier, 2012). The current IORP directive provides minimum standards for supervision and governance of occupational pension plans. Whether the revision will support or discourage moving towards funded second-pillar pensions in EU member states will depend to a large extent on the exact details of the proposal. In the Netherlands, pressure on mandatory pension fund participation has been growing. The current generosity of funded occupational pensions is unsustainable due to the ageing of the population, while the financial crisis has left many pension funds underfunded. Young generations see their contributions rise, while they are concerned that insufficient assets will remain in the pension funds to secure

[2] More precisely, participation is mandatory when there is a collective labour agreement between the social partners (trade unions and firms or employers' organizations). Except for the self-employed this is the case for most of the workers.

their own future pension benefits. Hence, their reluctance to participate in the system grows. Therefore, social partners, supported by the government, have been working on redesigning current pension contracts, which should preserve mandatory participation and the risk-sharing benefits that this brings to all generations involved.

Our analysis takes place in the context of a simple infinite horizon model with two overlapping generations. Voluntary participation is modelled through a participation constraint that requires the expected utility from participation to be at least as large as expected utility under autarky, which is defined as a situation without any form of pension system and, hence, without any risk sharing between generations. When the return on the pension fund portfolio is low, the current young make a transfer to the current retirees. Individuals always have the option to not participate after the current shock has materialised. Participation yields benefits in terms of intergenerational risk sharing. However, numerical results show that the ex-post option to not participate in the system renders a funded system unfeasible for low degrees of risk aversion and when portfolio returns have low variance. In those circumstances the risk-sharing benefits will be lost. Increases in risk aversion and uncertainty about the returns raise the likelihood of a sustainable funded system based on voluntary participation. However, risk sharing would still be less than under the optimal arrangement with obligatory participation. Raising risk-aversion and volatility further, we find that the optimal solutions under voluntary and obligatory participation coincide.

This chapter draws from several strands of the literature. First, there is a literature on intergenerational risk sharing in pension systems. Bohn (1999b, 2003) analyses such risk sharing in PAYG pension systems. Krueger and Kubler (2006) find that PAYG pension systems can only be welfare improving when markets are incomplete, while, moreover, their welfare effects depend on the degrees of risk aversion and intertemporal substitution. Gottardi and Kubler (2011) extend the analysis of Krueger and Kubler (2006) and Ball and Mankiw (2007) compare risk sharing of an overlapping generations economy under autarky and in the presence of a full set of state-contingent assets. De Menil and Sheshinski (2003), Hassler and Lindbeck (1997) and Matsen and Thøgersen (2004) explore the trade off between a PAYG and a funded pension system. The optimal relative sizes of the two components of the pension system depend on the preferences and the characteristics of the stochastic processes driving wages and returns. Beetsma and Bovenberg (2009) and Beetsma et al. (2011) explore risk sharing through a combination of a PAYG and a fully funded system. Finally, Cui et al. (2011) compare risk sharing in various types of funded pension systems such as an individual DC scheme, a collective DC scheme and a collective defined-benefit (DB) scheme.

Second, this chapter is linked to the literature on participation constraints. Kocherlakota (1996) and Thomas and Worrall (1988) explore risk-sharing between two types of infinitely-lived agents in which transfers can go both ways between the agents, who always have the possibility to walk away from the risk-sharing agreement. Krueger and Perri (2011) allow for a continuum of infinitely-lived agents and explore participation with limited enforcement when public insurance is introduced into a market in which private insurance contracts are not fully enforcable. They show that public insurance can crowd out private insurance, because agents who break the private contract have the option of joining the public insurance contract. Our setup differs from these other contributions in that our agents only live for two periods and transfers always go from the young to the old. The young are prepared to make such transfers because they expect the future young to also honour the risk-sharing arrangement.

The third strand of the literature to which this chapter is related is that on discontinuity risk in pension systems. Demange and G.Laroque (2001) study risk-sharing in a PAYG pension system with voluntary contributions, while Demange (2009) explores political sustainability risk in PAYG systems. In Teulings and De Vries (2006) the young may be exposed to equity risk already before they enter the labour market, implying that they may have accumulated losses even before they start working. Under those circumstances young individuals may prefer not to participate in the pension fund. Bovenberg et al. (2007) also discuss the problem of negative buffers that make it unattractive for new young generations to enter a pension fund. Other articles exploring discontinuity risk in funded pension systems are van Ewijk et al. (2009), Gollier (2008), Westerhout (2009) and Molenaar et al. (2011). The latter two articles are closest to our analysis. Westerhout (2009) employs an infinite horizon model with two overlapping generations. He quantifies the feasible amount of risk-sharing under the assumption that the old are bound by their pension contract and the young are free to choose whether they will join the fund. They will not join when the financial position of the fund is weak. Hence, risk sharing is possible only for a limited set of states. An important assumption is that the return on the pension funds' assets exceeds that on private savings. This makes it relatively attractive to join a pension fund. We do not need to make such an assumption. Molenaar et al. (2011) explore the break-even ratio of pension fund assets over liabilities at which it is optimal for a participant to quit the pension fund. They model a Dutch DB pension fund and in this and other respects their model differs substantially from our model. Moreover, in contrast to our analysis they only optimise the static asset allocation under the alternative of no participation, while there is no fully-fledged analysis of the existence and characterisation of equilibria with voluntary participation.

The remainder of the chapter is structured as follows. In Section 2, the model is set up and solved for the autarky solution. Section 3 introduces a pension fund and solves for the optimal transfer rule in the absence of a participation constraint. In Section 4, we introduce the participation constraint and characterise the various equilibria. This section also solves for the optimal pension fund rule. Section 5 works out a numerical example. Finally, Section 6 concludes the main text of the chapter.

4.2 Model and autarky solution

We set up an infinite-horizon overlapping generations model. In each period, a new, young generation is born. The generation born in period t will be referred to as the "period-t generation". Each generation consists of identical individuals and lives for two periods. In the first period of its life, the generation works, consumes and saves for its old age. When old in the second period of its life, the generation is retired and consumes all of its savings. Savings can be invested in a single risky asset. We assume that subsequent returns on the asset are identically and independently distributed. All generations are of the same size, which we normalise to unity.

The preferences of an individual born in period t are given by:

$$U_t = u\left(c_{t,t}\right) + \beta E_t\left[u\left(c_{t+1,t}\right)\right], \tag{4.1}$$

where $c_{t,t}$ denotes consumption in the first period of its life and $c_{t+1,t}$ denotes consumption in the second period of its life. Hence, the first subscript denotes the period in which consumption takes place and the second subscript denotes the period in which the individual is born. Function $u(.)$ is increasing and concave on $[0,\infty)$ and twice differentiable.

In the absence of any form of pension system, that is, under "autarky", consumption of the period-t generation in the two periods of its life is:

$$c_{t,t} = w_t - s_t, \tag{4.2}$$

$$c_{t+1,t} = (1 + r_{t+1}) s_t. \tag{4.3}$$

where w_t is the exogenous wage rate, s_t are private savings under autarky and r_{t+1} is the return on savings. The individual solves the intertemporal consumption allocation problem:

$$\max_{s_t} \quad u(w_t - s_t) + \beta E_t \{u[(1 + r_{t+1}) s_t]\}, \tag{4.4}$$

which implies the first-order condition:

$$u'\left(c_{t,t}^a\right) = \beta E_t \left[(1 + r_{t+1}) u'\left(c_{t+1,t}^a\right)\right]. \tag{4.5}$$

Utility under autarky is given by:

$$U_t^a \equiv u\left(c_{t,t}^a\right) + \beta E_t \left[u\left(c_{t+1,t}^a\right)\right], \tag{4.6}$$

where $c_{t,t}^a$ and $c_{t+1,t}^a$ are the optimal consumption levels under autarky.

4.3 Introduction of a pension fund

We introduce a pension system with a simple funding rule. This introduces the possibility of intergenerational risk sharing. A young person privately saves an amount s_t and contributes an amount θ to the pension fund. The fund invests this contribution in financial assets and pays out the gross return as a pension benefit one period later. In addition, if the return on the fund's assets in period t is too low to provide a "decent" pension benefit, each individual of the period-t generation pays an additional re-funding contribution τ_t, which is used to supplement the benefit to the current old generation. The transfer τ_t is constrained to be non-negative.

4.3.1 Individuals

The consumption levels of a period-t generation member are:

$$c_{t,t} = w_t - s_t - \theta - \tau_t, \tag{4.7}$$

$$c_{t+1,t} = (1 + r_{t+1})(s_t + \theta) + \tau_{t+1}. \tag{4.8}$$

The individual takes as given the known policy function set by the pension fund and now maximises (4.1) subject to (4.7) and (4.8). This implies the following first-order condition for his intertemporal consumption trade-off:

$$u'\left(c^p_{t,t}\right) = \beta E_t\left[(1 + r_{t+1})\, u'\left(c^p_{t+1,t}\right)\right], \tag{4.9}$$

where $c^p_{t,t}$ and $c^p_{t+1,t}$ are the optimal consumption levels under participation in period t.

4.3.2 The pension fund

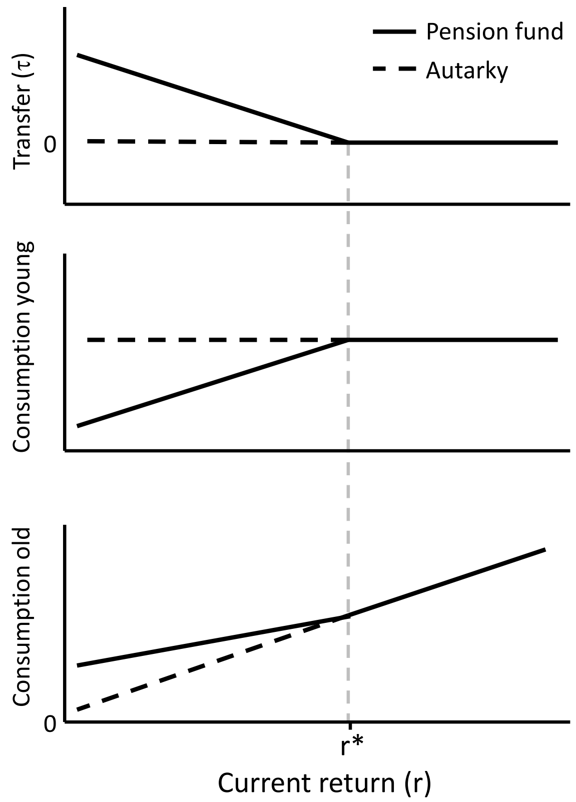
We assume that the pension fund applies the very simple funding rule given by:

$$\tau_t = \begin{cases} (r^* - r_t)\,\theta & \forall\, r_t < r^*, \\ 0 & \forall\, r_t \geq r^*. \end{cases} \tag{4.10}$$

We choose this rule, because it is appealing from a practical point of view, as it is intuitive and it would be easy to implement. It features two parameters to be set by the pension fund, r^* and θ. In the remainder of the chapter, we assume that θ is given and that the pension fund uses r^* as its policy instrument.[3] Figure 4.1 shows the size of

[3] An alternative would be to vary θ and keep r^* fixed. The analysis is similar to the analysis presented here. Because the contribution rate is easier to observe and, hence, to calibrate we choose to vary r^*. If we also allow for different values of θ, we get a two dimensional feasible region in r^*

Figure 4.1: Transfer and consumption of young and old



the transfer τ_t as a function of the current asset return and the effect that this rule has on consumption of the young (in the middle panel) and the old (in the bottom panel). The transfer is rising in the short-fall of the actual market return from r^* (i.e., as we move to the left along the horizontal axis in Figure 1). This dampens the sensitivity of the old's consumption to fluctuations in financial market returns when compared to the autarky situation (the dashed line).

The objective of the pension fund is to maximise from an ex-ante point of view the

and θ and the optimal contract would be a combination of these two pension parameters for which welfare is maximised given that these parameters lie within this feasible region.

sum of the utilities of the current and all future generations:

$$\begin{aligned} V &= E_0 \{u(c_{1,0}) + [u(c_{1,1}) + \beta u(c_{2,1})] + \beta [u(c_{2,2}) + \beta u(c_{3,2})] + \ldots\} \\ &= E_0 u(c_{1,0}) + \sum_{t=1}^{\infty} \beta^{t-1} E_0 [u(c_{t,t}) + \beta u(c_{t+1,t})] \\ &= E_0 u(c_{1,0}) + \sum_{t=1}^{\infty} \beta^{t-1} E_0 [U_t^p], \end{aligned} \tag{4.11}$$

where E_0 denotes the expectation taken at the start of period 1, before r_1 has materialised, and $U_t^p \equiv u(c_{t,t}) + \beta u(c_{t+1,t})$. Notice that the transfer is not a function of the savings of the previous young generation, but of the time-invariant pension fund rule parameters r^* and θ. This implies that all young generations are affected in an identical way by the funding rule and in expectation the decisions of all young generations will be identical. Hence, $E_0 U_1^p = E_0 U_2^p = \cdots \equiv E_0 U^p$. Using this, we can rewrite (4.11) as:

$$V = E_0 u(c_{1,0}) + \frac{1}{1-\beta} E_0 [U^p].$$

Aggregate welfare is maximised by setting r^* such that the derivative of the objective function with respect to r^* equals 0:

$$\frac{\partial V}{\partial r^*} = E_0 \left[\frac{\partial u(c_{1,0})}{\partial r^*}\right] + \frac{1}{1-\beta} E_0 \left[\frac{\partial U^p}{\partial r^*}\right] = 0.$$

We call the value for r^* implied by this first-order condition $r^{*,opt}$. Hence, $r^{*,opt}$ yields the optimal transfer rule in the absence of a participation constraint.

4.4 The participation constraint

In the previous section we optimised welfare assuming that participation is mandatory. In this section we relax this assumption. This gives rise to a participation constraint that needs to be fulfilled to sustain the collective pension scheme. We characterise the equilibria that arise in the presence of this constraint. Finally, it solves for the optimal value for r^* under this participation constraint.

For any period t, we denote the set of possible realisations of the state of the world by $R \equiv [-1, \infty)$. The particular state that actually materialises in period t is exhaustively described by the asset return r_t in that period, with the continuously differentiable density function $p(r_t)$ on R.

We define participation or lack of participation in the pension fund as:

Definition 1: A young generation in period t is said to *participate* in the pension fund if it chooses to follow the funding rule $\tau = \tau(r_t)$ for the current realization r_t. It does *not participate* if it does not follow this funding rule.

Before we continue the analysis we make the following assumptions for an arbitrary period $t \geq 1$:

Assumption 1: All preceding young generations as of period 1 have participated in the pension fund. Each period-t young individual is free to choose whether or not to participate in the pension system after the state r_t has materialised.

Assumption 2: If the period-t young decide not to participate, the economy shifts to autarky and remains in autarky forever after. Hence, none of the future young generations will ever participate.

In the remainder of this chapter we assume that Assumptions 1 and 2 hold. Using these assumptions we can write the participation constraint for the period-t young individual as:

$$u\left(c_{t,t}^{p}\right) + \beta\left\{P_t^{in} E_t\left[u\left(c_{t+1,t}^{in}|\text{in}\right)\right] + P_t^{out} E_t\left[u\left(c_{t+1,t}^{out}|\text{out}\right)\right]\right\} \geq U_t^a, \quad \forall\, r_t \in R, \quad (4.12)$$

with $P_t^{out} = 1 - P_t^{in}$ and $E_t(\cdot|\text{in/out})$ the expected value given that the next generation participates (in) or not (out). For the sake of readability in our notation we have dropped the dependence of consumption levels on the current return r_t. Next period's consumption of the current young depends on whether the next young generation participates or not. If the next young generation participates, the current young receive $c_{t+1,t}^{in}$ in the next period and $c_{t+1,t}^{out}$, otherwise. P_t^{in} and P_t^{out} denote the current young's perceived probabilities that the next young will participate, respectively opt out.

4.4.1 Recursive formulation of the participation constraint

Since subsequent generations are identical, and assuming that the economy has not shifted to autarky in the past, the participation constraint for each state of the world is identical in any period. This means that we can drop the time subscripts and write (4.12) in terms of current and next-period variables (which are denoted by a prime) as:

$$u\left(c^{p}\right)+\beta\left\{P^{in}E\left[u\left(c^{in\,\prime}|\text{in}\right)\right]+P^{out}E\left[u\left(c^{out\,\prime}|\text{out}\right)\right]\right\}\geq U^{a},\quad\forall\, r\in\ R,\qquad(4.13)$$

where a prime denotes next-period variables.

Hence, in our OLG setting, there is a participation constraint analogous to that in the literature that deals with participation constraints in settings with two types of infinitely-lived agents, such as Thomas and Worrall (1988) and Kocherlakota (1996). However, because in our setting the old have no incentive to walk away from the contract, the pension fund does not need to ensure that it delivers at least the utility promised in the previous period. Instead, the fund should ensure that it does not violate the promised funding rule, so that the insurance it promises the current young for the next period remains credible. Otherwise, the economy falls into autarky (Assumption 2). In addition, in contrast to models with infinitely-lived agents in which transfers can go both ways, in our model transfers always go from young to old individuals. While they may have to make a transfer to the current old, the current young depend on the next period's young for their own insurance. That implies that we do not have a repeated game between agents who can punish each other when one of them deviates from the rule. Hence, the beliefs about the willingness of the next period's young to make a transfer are crucial for the current young's willingness to pay a transfer, as will become clear below.

The next period's young will decide not to participate if the transfer they have to pay is relatively large, i.e. if $r' < \tilde{r}'$, where $\tilde{r}'$ denotes the threshold value for the next-period return r' that makes next period's young indifferent between participating and not participating. Using Bayes' rule we obtain

$$P^{in}p(r'|\text{in}) = P(\text{in}|r')p(r') = \begin{cases} 0 & \text{for } r' < \tilde{r}' \\ p(r') & \text{for } r' \geq \tilde{r}' \end{cases},$$

so we can rewrite the participation constraint (4.13) as:

$$\begin{aligned} U^a \leq U^p(r, \tilde{r}') \equiv\ & u\left(w - s^p(r) - \theta - \tau(r)\right) \\ & + \beta \left\{ \begin{array}{c} \int_{-1}^{\tilde{r}'} p(r')\, u\left[(1+r')\left(s^p(r) + \theta\right)\right] dr' + \\ \int_{\tilde{r}'}^{r^*} p(r')\, u\left[(1+r')\, s^p(r) + (1+r^*)\,\theta\right] dr' + \\ \int_{r^*}^{\infty} p(r')\, u\left[(1+r')\left(s^p(r) + \theta\right)\right] dr' \end{array} \right\}, \end{aligned} \quad (4.14)$$

where the right-hand side is the utility to the young when the current return is r and the next period's cut-off return for participating is $\tilde{r}'$.

4.4.2 Equilibrium definition

We explore equilibria that are defined as follows:

Definition 2: A recursive equilibrium with participation is an autarky savings decision s^a, an autarky value U^a, a pension funding rule $\{\tau(r)\}$, a set of savings decisions under participation $\{s^p(r)\}$, values under participation $\{U^p(r, \tilde{r}')\}$, a current cut-off return $\tilde{r}$ and expectations about the next period's cut-off return $\tilde{r}'$, such that

1. For any r, given the funding rule and expectations about future participation, the savings decisions $\{s^p(r)\}$ solve the young generation's optimization problem.

2. For $r < \tilde{r}$, $U^p(r, \tilde{r}') < U^a$, while for $r \geq \tilde{r}$, $U^p(r, \tilde{r}') \geq U^a$.

3. The cut-off return for the current young, $\tilde{r}$, computed given the expectation about the future cut-off return $\tilde{r}'$, equals the cut-off return for the next period's young, $\tilde{r}'$.

4. For at least one element $r \in R$, the funding rule sets $\tau(r) > 0$ and has $U^p(r, \tilde{r}') \geq U^a$.

4.4.3 Solutions for $\tilde{r}$

The cut-off return $\tilde{r}$ depends on the size of the current transfer prescribed by the funding rule and the current belief about the cut-off return of the young in the next period. If $r \geq r^*$, the current young would certainly want to participate, so the cut-off return cannot exceed r^*. Because subsequent young generations are identical in all respects under the assumption that all preceding young generations have participated in the pension fund, the cut-off return must be the same for every young generation that still has the option to participate. This implies that $\tilde{r} = \tilde{r}' = (\tilde{r}')' = \ldots$.

An equilibrium solution for $\tilde{r}$ requires that if the current young believe that the next young generation has the same threshold, then these current young want to participate if confronted with a return higher than this threshold and want to opt out if confronted with a lower return. That is, $\tilde{r}$ is such that

$$\begin{array}{l} U^p(r, \tilde{r}' = r) < U^a \text{ for all } -1 \leq r < \tilde{r} \\ U^p(r, \tilde{r}' = r) \geq U^a \text{ for all } \tilde{r} \leq r \leq r^* \end{array}. \tag{4.15}$$

Here, $U^p(r, \tilde{r}' = r)$ is the utility from participation under the assumption that the next generation uses the current portfolio return as the cut-off return.

As the Appendix shows, the derivative of $U^p(r, \tilde{r}' = r)$ with respect to r is the sum of a positive and a negative term and its sign is generally indeterminate. The negative term measures the utility cost of the extra transfer the current generation has to make this period. The positive term measures the benefit from extra insurance, because the future young have a higher threshold. This indeterminate derivative complicates our analysis. However, as the Appendix also shows, $U^p(r, \tilde{r}' = r)$ is equal to U^a and upward sloping in $r = r^*$. That is, it approaches U^a from below as r approaches r^*. In the examples below, we will work with a constant relative risk aversion utility specification. Plots of the resulting function $U^p(r, \tilde{r}' = r)$ show that it is always convex on $r \in [-1, r^*]$. Henceforth, we restrict ourselves to the case in which $U^p(r, r' = \tilde{r})$ is convex in r on this interval. Under this assumption, depending on whether $U^p\left(r, \tilde{r}' = r\right)$ is larger or smaller than U^a at $r = -1$, there are three possibilities as illustrated by Figure 4.2, which as a function of r plots the function $\Delta^p\left(r\right) \equiv U^p\left(r, \tilde{r}' = r\right) - U^a$:

Situation 1: If $\Delta^p\left(r\right) < 0$ at $r = -1$, then $U^p\left(r, \tilde{r}' = r\right) < U^a$ for all $-1 \leq r < r^*$. This is the situation depicted by the lowest curve in Figure 4.2. In this case, $\tilde{r} = r^*$ is the only threshold that is consistent with our equilibrium definition. This threshold implies that the pension fund collapses as soon as $r < r^*$, so there can effectively be no risk sharing via a pension fund.

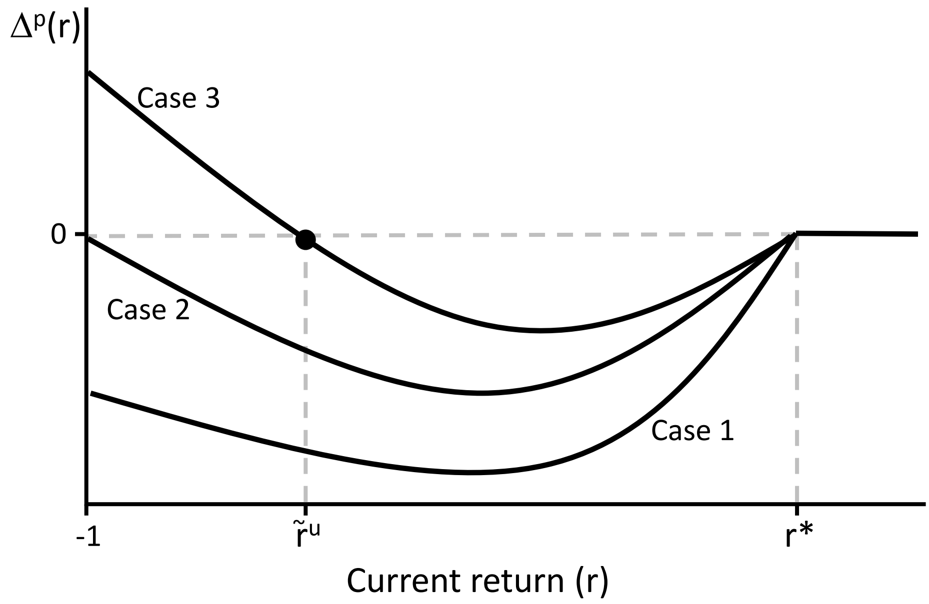
Situation 2: If $\Delta^p\left(r\right) = 0$ at $r = -1$, then $U^p\left(r, \tilde{r}' = r\right) < U^a$ for all $-1 < r < r^*$. In this case, depicted by the middle curve in Figure 4.2, equation (4.15) has exactly two solutions for $\tilde{r}$, namely $\tilde{r} = -1$ and $\tilde{r} = r^*$. This first solution implies that if the current young believe that the next period's young will participate in the pension system with certainty, then in the worst scenario today (i.e. $r = -1$), the current young are indifferent between participating and not participating. Hence, the pension system continues to exist forever.

Situation 3: If $\Delta^p\left(r\right) > 0$ at $r = -1$, as depicted by the upper curve in Figure 4.2, $\tilde{r} = -1$ is a corner solution, because under the belief that the next period's young will always participate the current young have higher utility from participating in the pension system than under autarky, even if they have to pay the highest possible transfer. Again, $\tilde{r} = r^*$ is also a solution. Finally, there exists a third solution $-1 < \tilde{r} < r^*$, such that $U^p\left(r, \tilde{r}' = r\right) = U^a$.

4.4.4 Properties of the solutions for $\tilde{r}$

We shall now further analyse Situation 3, because Situation 1 effectively excludes the possibility of risk sharing via a pension fund, while Situation 2 corresponds to a very specific parameter combination. In Situation 3, if the current young, given their belief that the next period's young will participate in every state ($\tilde{r}' = -1$), are also willing to participate, there exist three solutions, namely $\tilde{r} = -1$, $\tilde{r} = r^*$ and a solution $-1 < \tilde{r} < r^*$. The first two solutions are stable, while the third is a knife-edge solution that will never be realised unless the current young start with an initial belief at exactly

Figure 4.2: Net benefit from participation



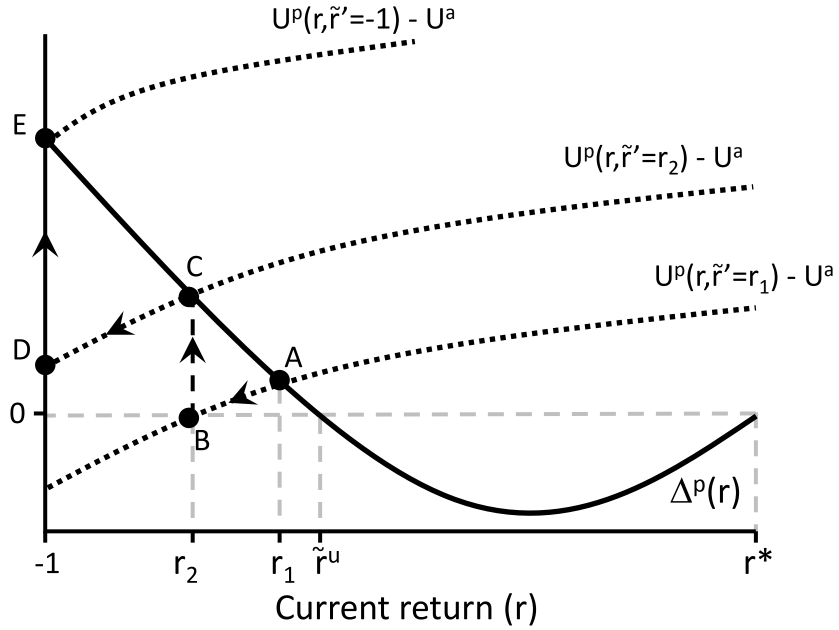
that point. Let us denote this solution by $\tilde{r}^u$. Which one of the two stable equilibria is reached depends on the initial belief of the current young.

Initial belief lies between -1 and unstable solution

If the current young initially believe that the cut-off return $\tilde{r}'$ of the next young generation lies between -1 and $\tilde{r}^u$, then the economy will end up at the equilibrium with $\tilde{r} = -1$. We illustrate this case using Figure 4.3, in which we plot the solid curve $\Delta^p(r)$ as a function of r and the dashed curves $U^p(r, \tilde{r}') - U^a$ for three different values $(r_1 > r_2 > -1)$ of $\tilde{r}'$. Note that there exist an infinite number of such curves, one for each possible belief about next-period young's cut-off value $\tilde{r}'$. Since U^a does not depend on $\tilde{r}'$ and the insurance value of participation is lower as the next generation's threshold is higher, $U(r, \tilde{r}' = r_1) < U(r, \tilde{r}' = r_2) < U(r, \tilde{r}' = -1)$ on the whole interval $r \in [-1, r^*]$.

Take an arbitrary starting value r_1 between -1 and $\tilde{r}^u$ for $\tilde{r}$ and assume that the next generation uses the same threshold. This corresponds to point (A) in Figure 4.3 where the lower dashed line crosses the solid line. This return r_1 cannot be the cut-off return $\tilde{r}$ of the current young since there are lower returns for which the current young

Figure 4.3: Stable solution with participation



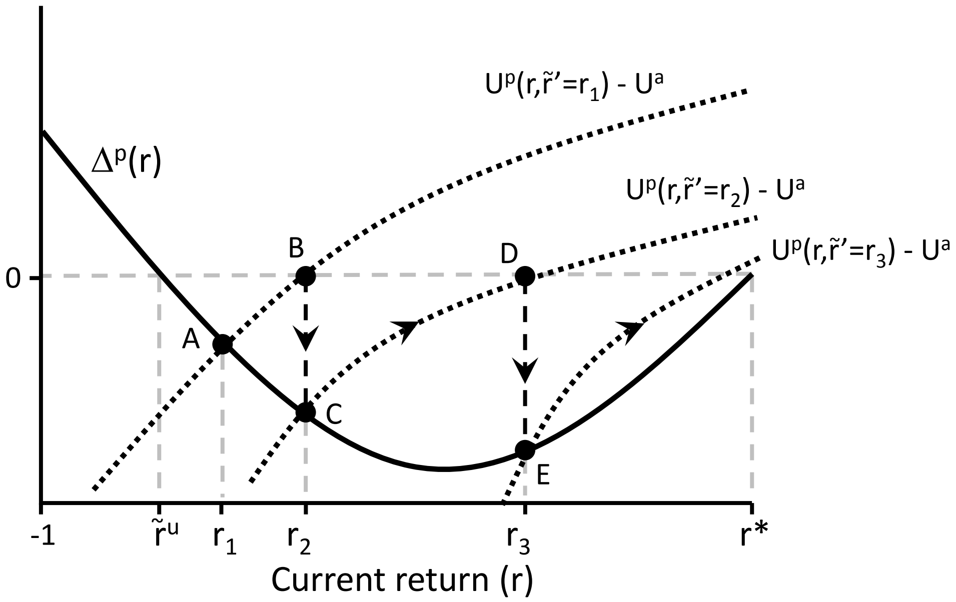
are also willing to participate given their beliefs about the future's young generation's threshold. While keeping $\tilde{r}'$ fixed at r_1, we decrease r, thereby raising the required transfer and reducing expected utility from participation $U^p\left(r, \tilde{r}' = r_1\right)$ until we reach the value of r such that $U^p\left(r, \tilde{r}' = \tilde{r}_1\right) = U^a$ at $r = r_2$ in point (B). This defines the cut-off return $\tilde{r}_2$ of the current young given their initial belief about future participation. But, $\tilde{r}_2 < r_1$, implying that the belief $\tilde{r}' = r_1$ cannot be an equilibrium belief, as in equilibrium the cut-off returns of all subsequent young generations must be identical. Hence, the current young update their belief about the next young's cut-off return to $\tilde{r}' = r_2 < r_1$ to make their threshold consistent with their own (point C). However, at $r = \tilde{r}_2$, $U^p\left(r, \tilde{r}' = r_2\right)$ again exceeds U^a. Lowering r again, we move along the next-to-lowest dashed line to the left until we hit the vertical axis. This yields the cut-off return $\tilde{r} = -1$ of the current young which implies that they will never want to opt out. If opting out is never optimal for the current young, then it is also never optimal for future generations, so the threshold consistent with our equilibrium definition is the stable equilibrium $\tilde{r} = \tilde{r}' = -1$.

Initial belief lies between unstable solution and r^*

We illustrate this case using Figure 4.4, in which we again plot the solid curve $\Delta^p(r)$ as a function of r and the dashed curves $U^p(r, \tilde{r}') - U^a$ for three different values of $\tilde{r}'$. As before, there exist an infinite number of these curves, one for each possible value of $\tilde{r}'$. All these curves are upward sloping since the required transfer decreases and the insurance value stays constant along each line as $\tilde{r}'$ is constant. By definition the curve $U^p(r, \tilde{r}') - U^a$ crosses $\Delta^p(r)$ at $r = \tilde{r}'$. Finally, these curves are positive (or zero) at $r = r^*$ since $U^p(r, r^*) - U^a = 0$ for $r = r^*$ and all curves with $\tilde{r}' < r^*$ lie above this curve.

To show that every initial belief about the next generation's threshold between $\tilde{r}^u$ and r^* results in the autarky solution r^* start in point A in Figure 4.4. At $r = r_1$, we have $U^p(r, \tilde{r}' = r_1) < U^a$ so that the current young do not want to participate. However, there are also higher returns r for which they do not want to participate, but according to their initial belief $\tilde{r}' = r_1$ the next generation does. So, r_1 cannot be the cut-off return $\tilde{r}$ of the current young. While keeping $\tilde{r}'$ fixed at r_1, we raise r, thereby reducing the required transfer and raising expected utility from participation $U^p(r, \tilde{r}' = r_1)$ until we reach the value of r such that $U^p(r, \tilde{r}' = r_1) = U^a$ in point B. This defines the cut-off return r_2 of the current young given their initial belief about future participation. However, $r_2 > r_1$, implying that the belief $\tilde{r}' = r_1$ cannot be an equilibrium belief, as in equilibrium the cut-off returns of all subsequent young generations must be identical. Hence, the current young update their belief about the next young's cut-off return to $\tilde{r}' = r_2$ (point C). However, at $r = r_2 = \tilde{r}'$, one has $U^p(r, \tilde{r}' = r_2) < U^a$. Raising r, we move along the middle dashed line to the right until we hit the horizontal dashed line again. This yields the new cut-off return $r = r_3$ of the current young. Because r_3 exceeds the next generation's threshold r_2, the latter cannot be an equilibrium cut-off return. We repeat the updating procedure until the current generation's cut-off and the beliefs about the next generation's cut-off converge to the stable equilibrium value $\tilde{r} = \tilde{r}' = r^*$.

Figure 4.4: Stable solution without participation



4.4.5 Assumption about initial beliefs

The analysis thus far has shown that the only stable equilibrium can be one in which $\tilde{r} = \tilde{r}' = -1$ or $\tilde{r} = \tilde{r}' = r^*$. The latter case is effectively ruled out by requirement (iv) of Definition 2 of an equilibrium. Hence, in the sequel we limit ourselves to the former case and we assume that each young generation starts with the belief that the next-period young will participate in all states of the world, i.e $\tilde{r}' = -1$. If, given this belief, the current young are prepared to participate for $r = -1$, i.e. when the transfer they have to make is at its largest, we have a sustainable pension fund that continues to operate in all periods under any state of the world. If for this initial belief the current young are unwilling to participate for $r = -1$, then, as the analysis in the previous subsection has shown, there exists no equilibrium with participation and the pension system breaks down.

4.4.6 Feasible pension fund rules

Until now we have fixed the contract at an arbitrary r^* and varied the cut-off value $\tilde{r}$ for young generations. Next, we will vary r^* to find the set of feasible funding rules by the pension fund. This part of the analysis closely follows the analysis by Ljunqvist and Sargent (2004, pp. 724-726) and Kocherlakota (1996).

We evaluate U^p for the worst state of the world ($r = -1$) under the assumption that the next period's young will participate irrespective of the next period's state of the world (i.e., $\tilde{r}' = -1$):

$$U^p(-1,-1) = u(w - s^p - \theta - (1+r^*)\theta) + \beta \left\{ \begin{array}{c} \int_{-1}^{r^*} p(r')\, u((1+r')\, s^p + (1+r^*)\,\theta)\, dr' \\ + \int_{r^*}^{\infty} p(r')\, u((1+r')(s^p+\theta))\, dr' \end{array} \right\}.$$

We can check for all possible values of r^* whether the current young are indeed prepared to participate or not. Start with $r^* = -1$. In this case, the young will never make a transfer, hence their consumption is constant across all states of the world and the pension fund effectively operates as an individual DC system. To see what happens when we increase r^*, we differentiate $U^p(-1,-1)$ with respect to r^*. Applying Leibniz' integral rule, this yields,

$$\frac{dU^p(-1,-1)}{dr^*} = -\theta u'(w - s^p - \theta - (1+r^*)\theta) + \beta\theta \int_{-1}^{r^*} p(r')\, u'((1+r')\, s^p + (1+r^*)\,\theta)\, dr'. \tag{4.16}$$

The first term on the right-hand side of (4.16) is negative: raising r^* from -1 means that the young individual has to pay a transfer with certainty because $r = -1$. The second term on the right-hand side is positive, except at $r^* = -1$. It arises from the fact that the increase in the transfer from the future young for given r' raises future consumption and, hence, future utility to the current young. At $r^* = -1$, this term is exactly zero, because the probability of receiving the higher future transfer is zero in this case. In differentiating $U^p(-1,-1)$ two terms have cancelled out. A marginal increase in r^* expands the interval $[-1, r^*)$ over which a transfer is received in the future. However, this positive contribution to $U^p(-1,-1)$ cancels out against the reduction of utility from the shrinkage of the interval $[r^*, \infty)$ over which no transfer is received.

Figure 4.5 plots the gain from participating $\Delta^p(-1) = U^p(-1,-1) - U^a$ for various constant relative aversion utility functions. As shown above, $\Delta^p(-1)$ initially always falls below 0. Further from $r^* = -1$, the utility loss from the initial transfer is still low, but the probability of receiving income protection in the worst case scenarios in the second period grows. For some parameter combinations (as in cases A and B), this second effect, measured by the term on the right-hand side of (4.16) starts to dominate and the function increases. For high values of r^*, increasing r^* further always has a negative effect on the gain from participating as the insurance value does not offset the relatively large initial transfer. For values of r^* close to $w/\theta - 1$, participation is never attractive, because the initial transfer is so large that positive consumption in the first period may lead to bankruptcy in the second. To ensure positive consumption this value of r^* requires that $s^p + \theta < 0$, so the young are privately heavily indebted. But, if $r' \geq r^*$ then the payout from the pension fund is insufficient to pay the principal plus interest rate on this debt. In our numerical analysis below, the gain from participating always shows the same rotated S-pattern.

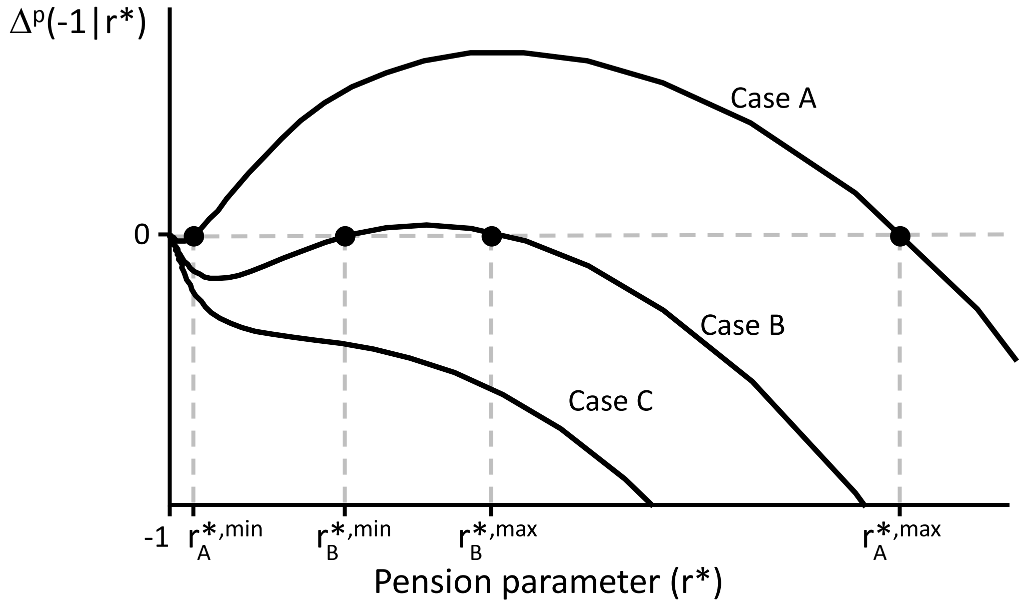
Figure 4.5 shows that there are intervals for r^* on which the gain from participating is positive. In this case, we define $r^{*,\max}$ as the maximum value for r^* for which a pension fund is sustainable and $r^{*,\min}$ as the largest value of $r^* < r^{*,\max}$ for which $\Delta^p(-1) = 0$. However, there are also instances (as in Case C in Figure 4.5) for which no such interval exists.

4.4.7 The optimal pension fund rule

We continue to focus on the stable equilibrium with participation in Situation 3 and assume that the funding rule is in place at the start of period 1. Writing out the terms in the pension fund objective function, equation (4.11), we obtain

$$\begin{aligned} E_0 u\left(c_{1,0}\right) &= \int_{-1}^{r^*} p\left(r\right) u\left[\left(1+r\right) s^p + \left(1+r^*\right)\theta\right] dr \\ &+ \int_{r^*}^{\infty} p\left(r\right) u\left[\left(1+r\right)\left(s^p+\theta\right)\right] dr, \end{aligned}$$

Figure 4.5: Feasible pension schemes when varying r^*



and

$$
\begin{aligned}
E_0 U^p &= \int_{-1}^{r^*} p(r)\, u\left[w - s^p - \theta - (r^* - r)\,\theta\right] dr \\
&+ \int_{r^*}^{\infty} p(r)\, u\left[w - s^p - \theta\right] dr \\
&+ \beta \left\{ \begin{array}{c} \int_{-1}^{r^*} p(r)\, u\left[(1+r)\, s^p + (1+r^*)\,\theta\right] dr \\ + \int_{r^*}^{\infty} p(r)\, u\left[(1+r)\,(s^p + \theta)\right] dr \end{array} \right\}.
\end{aligned}
$$

Substituting these terms into (4.11), it is straightforward to write the pension fund objective as:

$$
\begin{aligned}
V(r^*) &= \frac{1}{1-\beta} \left\{ \begin{array}{c} \int_{-1}^{r^*} p(r)\, u\left[w - s^p - \theta - (r^* - r)\,\theta\right] dr \\ + \int_{r^*}^{\infty} p(r)\, u\left[w - s^p - \theta\right] dr \end{array} \right\} \\
&+ \frac{1}{1-\beta} \left\{ \begin{array}{c} \int_{-1}^{r^*} p(r)\, u\left[(1+r)\, s^p + (1+r^*)\,\theta\right] dr \\ + \int_{r^*}^{\infty} p(r)\, u\left[(1+r)\,(s^p + \theta)\right] dr \end{array} \right\}.
\end{aligned} \quad (4.17)
$$

If an interval $\left[r^{*,\min}, r^{*,\max}\right]$ exists, we denote the value for $r^* \in \left[r^{*,\min}, r^{*,\max}\right]$ that maximises (4.17) subject to the participation constraint by $r^{*,pc}$.

4.5 A numerical example

In this section we work out a simple numerical example of the analysis we have presented in the previous sections. The purpose of this section is to explore under what circumstances a funded pension arrangement with voluntary participation exists and, conditional on this existence, to investigate under what circumstances the participation constraint leads to welfare losses because it is binding.

We assume that period utility is given by,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where parameter γ captures the constant degree of relative risk-aversion. The baseline values of the model parameters are set as follows. Consistent with standard calibrations in the macro-economic literature, we set $\gamma = 5$. Further, we assume that $\beta = 0.5$. With one generation spanning 30 years, this corresponds to an annual discount factor of 0.977. The wage rate is simply a parameter that determines the scale of the economy and we fix it at $w = 100$. Given the wage rate w, we set $\theta = 10$. Hence, this amounts to a realistic pension contribution rate of 10%. We assume that portfolio returns are lognormally distributed and independent over time. In setting the parameters for the portfolio returns, we make use of Campbell et al. (2003) (see Appendix). Specifically, the average annual portfolio return is set at $\mu = 0.089$, which over a period of thirty years translates into an average return of 2.14. The standard deviation of the annual equity return in Campbell et al. (2003) is $\sigma = 0.182$, which implies a standard deviation of 0.91 over a thirty-year period. We will consider a wide range of standard deviations that comfortably covers this value. Given that the degree of risk aversion is somewhat on the high side, the conditions for a recursive equilibrium with participation are relatively favourable.

First, we solve for $r^{*,opt}$, the optimal choice of the pension fund for r^* in the absence of participation constraints. Subsequently, we solve the autarky problem, which yields expected utility U^a. Given U^a, we can then solve for the minimum and maximum values $r^{*,\min}$ and $r^{*,\max}$ of r^* for which our stable equilibrium in the presence of participation constraints still exists as well as the optimum value $r^{*,pc}$ for r^* in the interval $\left[r^{*,\min}, r^{*,\max}\right]$. If there is no interval $\left[r^{*,\min}, r^{*,\max}\right]$ for the specific parameter constellation under consideration, then there is no sustainable pension contract under voluntary participation as indicated by the non-existence of $r^{*,\max}$ and $r^{*,pc}$ in the tables. In these instances we indicate $r^{*,\max}$ and $r^{*,pc}$ in the tables by "–". Obviously, if $r^{*,\max}$ exists, then also $r^{*,\min}$ and $r^{*,pc}$ exist. However, we do not separately report $r^{*,\min}$, because numerically it always lies extremely close to -1. Finally, we compute for the constrained optimum the ex-ante welfare gains Ω^{init} for the initial old generation and Ω for the current young and all future generations. These ex-ante welfare gains are defined as follows:

$$\begin{aligned} E_0 u\left(c_{1,0}^p\right) &= E_0 u\left[\left(1+\Omega^{init}\right) c_{1,0}^a\right], \\ E_0 U^p &= u\left[\left(1+\Omega\right) c_{t,t}^a\right] + \beta E_0 u\left[\left(1+\Omega\right) c_{t+1,t}^a\right]. \end{aligned}$$

We report the results in Table 4.1 for different values of the risk aversion parameter γ, the *annual* standard deviation of the portfolio return σ and the pension contribution θ. The results show that the higher is the degree of relative risk aversion and the higher is the standard deviation of the asset returns, the more likely it becomes that participation in the pension fund is beneficial because of the risk-sharing gains that it provides.

Consider first the case when the pension contribution is at its baseline of $\theta = 10$. Then, for a relatively low degree of risk aversion $\gamma = 3$ the pension fund is viable for high volatilities of 0.25 and larger, while for a relatively high degree of risk aversion $\gamma = 7.5$ it is viable already for a moderate volatility of 0.15. For a given degree of risk aversion, the benefit from future risk sharing and thus the attractiveness of participation increases with the uncertainty about the future asset returns. Vice versa, for a given variance of the asset returns, future risk sharing and thus participation becomes more attractive when risk aversion rises. For $\gamma = 7.5$, when $\sigma = 0.20$ or larger, the young are even prepared to make a transfer that is larger than under the optimum without the participation constraint, i.e. $r^{*,\max} > r^{*,opt}$. In this case, the participation constraint is not binding at the unconstrained optimum and, hence, the constrained optimum coincides with the unconstrained optimum, i.e. $r^{*,pc} = r^{*,opt} > r^{*,\min}$. For

$\gamma = 3$ the pension fund is viable for $\sigma = 0.25$ and for $\gamma = 5$ it is viable for $\sigma = 0.20$, although in these cases it produces less risk sharing than under the unconstrained optimum. In this case, $r^{*,pc} = r^{*,\max} < r^{*,opt}$.[4]

In Table 4.1 we also vary the size of the pension contribution. If $\theta = 20$, then for two risk aversion – volatility combinations, $(\gamma = 3, \sigma = 0.25)$ and $(\gamma = 7.5, \sigma = 0.15)$, the participation constraint now prohibits any risk sharing, while it did not do so before when $\theta = 10$. We also see that whenever there is voluntary participation under $\theta = 20$, the values for $r^{*,\max}$ and $r^{*,pc}$ have dropped relative to the corresponding ones under $\theta = 10$, hence the scope for risk sharing has become smaller. The intuition for this is the following. With a transfer by the current young of the format $(r^* - r_t)\,\theta$, an increase in θ implies a larger transfer for given $r_t < r^*$ and, hence, to avoid violation of the participation constraint for given future transfers, $r^{*,\max}$ needs to fall. However, this reduces the number of states in the future for which the current young will receive a transfer back and, hence, the insurance value of the pension falls. For some parameter combinations under consideration the insurance value falls so much that participation by the young is no longer beneficial to them. A reduction in θ from $\theta = 10$ to $\theta = 5$ yields the opposite effects and the scope for risk sharing increases.

The degree to which the participation constraint is binding is reflected in the welfare gain associated with participation. If voluntary participation is not optimal then, obviously, there can be no welfare gain, while once participation has become beneficial, further increases in risk aversion and the variability of the returns raise the welfare gains from participation. The welfare gain of the initial old is always higher than that of all subsequent generations, because they reap the benefits from the potential transfer, without ever having to pay a transfer themselves.

We also vary the discount factor. A higher discount factor corresponds to a lower time preference rate, implying that future events become relatively more important. Hence, voluntary participation becomes more attractive to the young. After all, any benefit of participation in the form of a transfer only materialises in the future. This

[4] We have also done a more elaborate numerical analysis in which we varied σ by small steps of 0.02 and extended the range beyond the one reported in the Table 4.1. In all instances the results were qualitatively identical to those reported here, while for volatilities larger than 0.3 we always find that $r^{*,pc}$ and $r^{*,opt}$ are equal. These results are available upon request.

Table 4.1: Results for 100% equity

	γ	3					5					7.5				
θ	σ	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3
	$r^{*,opt}$	0.057	0.057	0.061	0.066	0.070	0.063	0.063	0.067	0.071	0.073	0.066	0.066	0.069	0.073	0.075
	$r^{*,max}$	-	-	-	0.068	0.090	-	-	0.074	0.095	0.103	-	0.058	0.094	0.103	0.105
5	$r^{*,pc}$	-	-	-	0.066	0.070	-	-	0.067	0.071	0.073	-	0.058	0.069	0.073	0.075
	Ω^{init}	0	0	0	0.018	0.036	0	0	0.019	0.047	0.094	0	0.011	0.043	0.103	0.192
	Ω	0	0	0	0.009	0.022	0	0	0.012	0.036	0.080	0	0.009	0.035	0.093	0.181
	$r^{*,opt}$	0.053	0.049	0.048	0.049	0.050	0.057	0.052	0.050	0.050	0.051	0.057	0.051	0.050	0.050	0.051
	$r^{*,max}$	-	-	-	0.037	0.064	-	-	0.045	0.070	0.077	-	0.020	0.069	0.078	0.079
10	$r^{*,pc}$	-	-	-	0.037	0.050	-	-	0.045	0.050	0.051	-	0.020	0.050	0.050	0.051
	Ω^{init}	0	0	0	0.015	0.036	0	0	0.018	0.047	0.093	0	0.007	0.043	0.103	0.192
	Ω	0	0	0	0.009	0.022	0	0	0.013	0.037	0.081	0	0.006	0.036	0.094	0.181
	$r^{*,opt}$	0.046	0.039	0.035	0.032	0.031	0.049	0.039	0.034	0.030	0.029	0.047	0.037	0.031	0.029	0.028
	$r^{*,max}$	-	-	-	-	0.037	-	-	0.012	0.044	0.053	-	-	0.043	0.053	0.055
20	$r^{*,pc}$	-	-	-	-	0.031	-	-	0.012	0.031	0.029	-	-	0.031	0.029	0.028
	Ω^{init}	0	0	0	0	0.036	0	0	0.013	0.046	0.092	0	0	0.042	0.102	0.190
	Ω	0	0	0	0	0.022	0	0	0.010	0.036	0.080	0	0	0.036	0.094	0.180

is borne out by the results in Table 4.2, in which we vary the discount factor and, for convenience, repeat the middle panel with $\beta = 0.5$. Discount factors of $\beta = 0.3$ and $\beta = 0.7$ correspond to annual time preference rates of roughly 4% and 1%, respectively. In all instances in which the participation constraint is binding, we see that a higher discount factor raises $r^{*,pc}$ and $r^{*,\max}$. The qualitative effects of an increase in risk aversion or in the standard deviation of the portfolio return that we described earlier are confirmed for different values of the discount factor β.

So far, the benchmark level of riskiness of the pension and individual investment portfolios was based on the implicit assumption of a 100% stake in equity. We will now make the implicit assumption that both the individual and the pension portfolio are invested half in equity and half in a risk-free asset. This implies that both the standard deviation and expected value of the overall portfolio return fall. The reason why we consider this case is that pension funds often invest in a mixture of risk-free assets and risk-bearing assets (mostly equity) and that we want to see what is the scope for risk sharing under voluntary participation in this case. The 50-50 division between the two categories is quite realistic for pension funds in the Netherlands. Obviously, the optimal individual savings rate in our model takes account of the new situation. Following Campbell et al. (2003), we calibrate the risk-free rate at 2.1% per year (see Appendix). Table 4.3 reports the numerical outcomes, where $r^{*,opt}$, $r^{*,\max}$ and $r^{*,pc}$ all refer to returns on the total portfolio. For relatively low aversion ($\gamma = 3$), participation under the constraint is only attractive when the volatility of the equity returns is at its highest value $\sigma = 0.3$. Again, for higher degrees of risk aversion participation can be attractive at lower levels of volatility. Hence, for relatively high risk aversion and medium and high equity volatility, the pension fund is still viable under voluntary participation, although, not surprisingly, the welfare gains relative to autarky have dropped due to the reduced risk-sharing benefits that the fund provides.

Table 4.2: Results for 100% equity, $\theta = 10$ and varying β

	γ	3					5					7.5				
β	σ	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3
$\beta = 0.3$	$r^{*,opt}$	0.055	0.051	0.050	0.051	0.052	0.058	0.053	0.047	0.051	0.052	0.058	0.052	0.051	0.051	0.051
	$r^{*,max}$	-	-	-	0.032	0.061	-	-	0.043	0.069	0.076	-	0.016	0.068	0.078	0.079
	$r^{*,pc}$	-	-	-	0.032	0.052	-	-	0.043	0.051	0.052	-	0.016	0.051	0.051	0.051
	Ω^{init}	0	0	0	0.014	0.036	0	0	0.012	0.046	0.092	0	0.007	0.043	0.102	0.191
	Ω	0	0	0	0.008	0.020	0	0	0.017	0.035	0.078	0	0.006	0.035	0.092	0.179
$\beta = 0.5$	$r^{*,opt}$	0.053	0.049	0.048	0.049	0.050	0.057	0.052	0.050	0.050	0.051	0.057	0.051	0.050	0.050	0.051
	$r^{*,max}$	-	-	-	0.037	0.064	-	-	0.045	0.070	0.077	-	0.020	0.069	0.078	0.079
	$r^{*,pc}$	-	-	-	0.037	0.050	-	-	0.045	0.050	0.051	-	0.020	0.050	0.050	0.051
	Ω^{init}	0	0	0	0.015	0.036	0	0	0.018	0.047	0.093	0	0.007	0.043	0.103	0.192
	Ω	0	0	0	0.009	0.022	0	0	0.013	0.037	0.081	0	0.006	0.036	0.094	0.181
$\beta = 0.7$	$r^{*,opt}$	0.052	0.048	0.046	0.047	0.049	0.056	0.051	0.049	0.049	0.050	0.057	0.051	0.049	0.050	0.051
	$r^{*,max}$	-	-	-	0.040	0.065	-	-	0.047	0.070	0.078	-	0.022	0.069	0.078	0.079
	$r^{*,pc}$	-	-	-	0.040	0.049	-	-	0.047	0.049	0.050	-	0.022	0.049	0.050	0.051
	Ω^{init}	0	0	0	0.016	0.036	0	0	0.019	0.047	0.094	0	0.008	0.043	0.104	0.192
	Ω	0	0	0	0.010	0.024	0	0	0.013	0.038	0.082	0	0.007	0.037	0.095	0.182

Table 4.3: Results for 50% invested in risk free and 50% in equity

γ	3					5					7.5				
σ	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3	0.1	0.15	0.2	0.25	0.3
$r^{*,opt}$	0.038	0.039	0.041	0.044	0.047	0.042	0.042	0.045	0.047	0.049	0.044	0.044	0.046	0.048	0.050
$r^{*,max}$	-	-	-	-	0.051	-	-	-	0.059	0.071	-	-	0.057	0.072	0.077
$r^{*,pc}$	-	-	-	-	0.047	-	-	-	0.047	0.049	-	-	0.046	0.048	0.050
Ω^{init}	0	0	0	0	0.020	0	0	0	0.024	0.044	0	0	0.022	0.047	0.087
Ω	0	0	0	0	0.010	0	0	0	0.016	0.034	0	0	0.016	0.039	0.078

4.6 Conclusion

In this chapter we have explored the feasibility and welfare consequences of a funded pension system with voluntary participation and intergenerational risk sharing. Our numerical results showed that the ex-post option to not participate obviates the existence of such a system when both risk aversion and financial market uncertainty are relatively low. Under those circumstances mandatory participation in the system is needed to reap the benefits of intergenerational risk sharing. Increases in these parameters raise the benefits from risk sharing and make the existence of an equilibrium with voluntary participation more likely. For intermediate values of these parameters, risk sharing would still be less than under the optimal arrangement with obligatory participation. However, raising risk-aversion and volatility further, the optimal solutions under voluntary and obligatory participation coincide.

Many countries that are currently trying to expand the funded pillar of their pension system need to decide on its design. In particular, will it be a defined-contribution system or will it contain certain defined-benefit elements that allow for intergenerational risk sharing? In the latter case, the question whether participation is obligatory becomes important. Giving individuals too much freedom in deciding whether to participate or not may lead to a breakdown of the system and, hence, to a loss of the risk-sharing benefits. The Dutch funded pillar has long and successfully operated under mandatory participation by the employees and employers and by collectively sharing risks among all the fund participants. However, the recent shrinkage of the pension buffers has undermined the confidence in the system, especially among the younger participants. Once large groups start losing their confidence in the system, the pressure to abolish mandatory fund participation will intensify. To prevent a collapse of the system and a loss of the benefits from intergenerational risk sharing, it is important that its planned overhaul leaves young participants with sufficient guarantee that they will also receive a benefit when they are old.

APPENDICES

4.A Details on first-order condition pension fund without participation constraint

The first-order condition is given by

$$\frac{\partial V}{\partial r^*} = E_0\left[\frac{\partial u\left(c_{1,0}\right)}{\partial r^*}\right] + \frac{1}{1-\beta}E_0\left[\frac{\partial U^p}{\partial r^*}\right] = 0,$$

where, using Leibniz' integral rule,

$$\begin{aligned}
E_0\left[\frac{\partial u\left(c_{1,0}\right)}{\partial r^*}\right] &= p\left(r^*\right)u\left[\left(1+r^*\right)\left(s_0^p+\theta\right)\right] - p\left(r^*\right)u\left[\left(1+r^*\right)\left(s_0^p+\theta\right)\right] \\
&\quad + \int_{-1}^{r^*} p\left(r\right)u'\left[\left(1+r\right)s_0^p + \left(1+r^*\right)\theta\right]\theta dr \\
&= \int_{-1}^{r^*} p\left(r\right)u'\left[\left(1+r\right)s_0^p + \left(1+r^*\right)\theta\right]\theta dr,
\end{aligned}$$

and

$$\begin{aligned}
E_0\left[\frac{\partial U^p}{\partial r^*}\right] &= -\int_{-1}^{r^*} p\left(r\right)u'\left[w - s^p - \theta - \left(r^* - r\right)\theta\right]\left(\frac{\partial s^p}{\partial r^*} + \theta\right)dr \\
&\quad - \int_{r^*}^{\infty} p\left(r\right)u'\left[w - s^p - \theta\right]\frac{\partial s^p}{\partial r^*}dr \\
&\quad + \beta\int_{-1}^{r^*} p\left(r\right)u'\left[\left(1+r\right)s^p + \left(1+r^*\right)\theta\right]\left[\left(1+r\right)\frac{\partial s^p}{\partial r^*} + \theta\right]dr \\
&\quad + \beta\int_{r^*}^{\infty} p\left(r\right)u'\left[\left(1+r\right)\left(s^p+\theta\right)\right]\left(1+r\right)\frac{\partial s^p}{\partial r^*}dr.
\end{aligned}$$

4.B Details on $U^p\left(r, \tilde{r}' = r\right)$

4.B.1 $U^p\left(r^*, \tilde{r}' = r^*\right)$ approaches U^a from below as $r \uparrow r^*$

Recall that $U^p\left(r, \tilde{r}' = r\right) = U^a$. Since $\Delta(r)$ was defined as $U^p\left(r, \tilde{r}' = r\right) - U^a$, all we need to show is that Δ^p approaches 0 from below. Differentiating $\Delta^p(r)$, with respect to r, and using Leibniz' integral rule, we obtain:

$$\frac{d\Delta(r)}{dr} = u'\left[w - s^p(r) - \theta - (r^* - r)\theta\right]\left(\theta - \frac{\partial s^p(r)}{\partial r}\right)$$
$$+ \beta p(r) u\left[(1+r)\left(s^p(r) + \theta\right)\right]$$
$$+ \beta \int_{-1}^{r} p(r') u'\left[(1+r')\left(s^p(r) + \theta\right)\right](1+r')\frac{\partial s^p(r)}{\partial r} dr'$$
$$- \beta p(r) u\left[(1+r)s^p(r) + (1+r^*)\theta\right]$$
$$+ \beta \int_{r}^{r^*} p(r') u'\left[(1+r') s^p(r) + (1+r^*)\theta\right](1+r')\frac{\partial s^p(r)}{\partial r} dr'$$
$$+ \beta \int_{r^*}^{\infty} p(r') u'\left[(1+r')\left(s^p(r) + \theta\right)\right](1+r')\frac{\partial s^p(r)}{\partial r} dr',$$

which we can write as:

$$\frac{d\Delta(r)}{dr} = u'\left[w - s^p(r) - \theta - (r^* - r)\theta\right]\left(\theta - \frac{\partial s^p(r)}{\partial r}\right)$$
$$+ \beta \frac{\partial s^p(r)}{\partial r}\left\{\begin{array}{c} \int_{-1}^{r} p(r') u'\left[(1+r')\left(s^p(r)+\theta\right)\right](1+r')dr' \\ + \int_{r}^{r^*} p(r') u'\left[(1+r') s^p(r) + (1+r^*)\theta\right](1+r')dr' \\ + \int_{r^*}^{\infty} p(r') u'\left[(1+r')\left(s^p(r)+\theta\right)\right](1+r')dr' \end{array}\right\}$$
$$+ \beta p(r)\left\{u\left[(1+r)\left(s^p(r)+\theta\right)\right] - u\left[(1+r)s^p(r) + (1+r^*)\theta\right]\right\}$$

where the term between big curly brackets measures $E[(1+r')u'\left(c^p_{t+1,t}\right)]$. According to the Euler equation (4.9) this equals $u'\left(c^p_{t,t}\right)$, so all terms involving $\frac{\partial s^p(r)}{\partial r}$ cancel out and we are left with:

$$\frac{d\Delta(r)}{dr} = u'\left[w - s^p(r) - \theta - (r^* - r)\theta\right]\theta$$
$$+ \beta p(r)\left\{u\left[(1+r)\left(s^p(r)+\theta\right)\right] - u\left[(1+r)s^p(r) + (1+r^*)\theta\right]\right\} \quad (4.18)$$

Evaluated in $r = r^*$ this equals:

$$\frac{d\Delta(r)}{dr} = u'\left[w - s^p(r^*) - \theta\right]\theta > 0.$$

4.B.2 Second-order derivative of $U^p(r, \tilde{r}' = r)$

The second-order derivative of $U^p(r, \tilde{r}' = r)$ equals the second derivative of $\Delta(r)$ since the difference is the constant U^a. Differentiating (4.18) once more gives:

$$\begin{aligned} \frac{d^2\Delta}{d^2r} &= u''\left[w - s^p(r) - \theta - (r^* - r)\theta\right]\theta\left(\theta - \frac{\partial s^p(r)}{\partial r}\right) \\ &+ \beta p(r)\left\{\begin{array}{c} u'\left[(1+r)\left(s^p(r)+\theta\right)\right]\left[s^p(r) + \theta + (1+r)\frac{\partial s^p(r)}{\partial r}\right] \\ -u'\left[(1+r)s^p(r) + (1+r^*)\theta\right]\left[s^p(r) + (1+r)\frac{\partial s^p(r)}{\partial r}\right] \end{array}\right\} \\ &+ \beta p'(r)\left\{u\left[(1+r)\left(s^p(r)+\theta\right)\right] - u\left[(1+r)s^p(r) + (1+r^*)\theta\right]\right\}. \end{aligned}$$

The first line is of ambiguous sign, since $u''(\cdot)$ is negative and θ and $\partial s^p(r)/\partial r$ are both positive (see below). The second line is positive or zero since $p(r) \geq 0$ and since $u'\left[(1+r)\left(s^p(r)+\theta\right)\right] > u'\left[(1+r)s^p(r) + (1+r^*)\theta\right]$, while the sign of the third line is ambiguous, since we do not know the sign of $p'(r)$. This means that we can not make any general statements about the shape of $U^p(r, \tilde{r}' = r)$.

The sign of $\partial s^p(r)/\partial r > 0$

Define

$$\begin{aligned} EU(r, s^p) \equiv\ & u'\left[w - s^p - \theta - (r^* - r)\theta\right] \\ & - \beta\left\{\begin{array}{c} \int_{-1}^{r} p(r')\, u'\left[(1+r')\left(s^p+\theta\right)\right](1+r')dr' \\ + \int_{r}^{r^*} p(r')\, u'\left[(1+r')s^p + (1+r^*)\theta\right](1+r')dr' \\ + \int_{r^*}^{\infty} p(r')\, u'\left[(1+r')\left(s^p+\theta\right)\right](1+r')dr' \end{array}\right\}, \end{aligned} \tag{4.19}$$

so we can write the Euler equation as $EU(r, s^p) = 0$. Implicit differentiation gives:

$$\frac{\partial s^p(r)}{\partial r} = -\frac{\partial EU/\partial r}{\partial EU/\partial s^p}. \tag{4.20}$$

Working out, we have:

$$\frac{\partial EU}{\partial r} = -u''\left[w - s^p - \theta - \left(r^* - r\right)\theta\right]\theta$$
$$+ \beta p(r)(1+r)\left\{u'\left[(1+r)\left(s^p+\theta\right)\right] - u'\left[(1+r)s^p + (1+r^*)\theta\right]\right\},$$

The first line of this derivative is positive, since $u''(\cdot)$ is negative, while the second line is greater than or equal to zero, because $r \leq r^*$ on the relevant domain for $\Delta(r)$. Further,

$$\frac{\partial EU}{\partial s^p} = u''\left[w - s^p(r) - \theta - \left(r^* - r\right)\theta\right] +$$
$$\beta\left\{\begin{array}{c} \int_{-1}^{r} p\left(r'\right)u''\left[\left(1+r'\right)\left(s^p+\theta\right)\right]\left(1+r'\right)^2 dr' \\ + \int_{r}^{r^*} p\left(r'\right)u''\left[\left(1+r'\right)s^p + (1+r^*)\theta\right]\left(1+r'\right)^2 dr' \\ + \int_{r^*}^{\infty} p\left(r'\right)u''\left[\left(1+r'\right)\left(s^p+\theta\right)\right]\left(1+r'\right)^2 dr' \end{array}\right\} < 0.$$

Both lines of this derivative are negative, since $u''(.)$ is negative. Hence, by (4.20), we have $\partial s^p\left(r\right)/\partial r > 0$.

4.C Calibration of the returns process

The portfolio returns follow a lognormal process and are independently and identically distributed over time. A period in our model corresponds to one generation, which we take here to span 30 years. We use the Campbell *et al.* (2003) estimates of annual returns on stocks and bonds to construct corresponding 30-year figures. Their sample period covers annual returns over the period 1890 - 1998. We assume the return on T-bills to correspond to that on a risk-free investment. The average annual real return on T-bills is 2.101%. Hence, this is the calibrated value for the risk-free rate of return in our model. Campbell *et al.* (2003) calculate an average annual equity risk premium of 6.797%. Hence, the average annual real equity return is $2.101\% + 6.797\% = 8.898\%$. Further, it has a standard deviation of 18.192%. Hence, the variance σ_{30}^2 of lognormally distributed thirty-year equity returns is calculated as $\sigma_{30}^2 = 30\ln\left(1 + \frac{\mathrm{var}(r)}{[E(r)]^2}\right) = 30\ln\left(1 + \frac{0.18192^2}{1.08898^2}\right) = 0.027525 * 30 = 0.8258$. The corresponding standard deviation is $\sqrt{0.825756} = 0.908711$. Next, the mean return over a period of thirty years is calculated as $\ln\left\{[E(r)]^{30}\right\} - \frac{1}{2}\sigma_{30}^2 = \ln(12.9002) - \frac{1}{2} * 0.8258 = 2.1443$. To conclude, we calibrate the logarithm of the equity return to follow a normal process with mean 2.1443 and variance 0.8258 over a thirty-year period.

Chapter 5

Redesigning the Dutch occupational pension contract: Simulation of alternative contracts involving soft and hard entitlements

5.1 Introduction

Dutch pension arrangements are generally praised for their generosity, their wide coverage and their high degree of funding. The system consists of a public pay-as-you-go first pillar and a funded second pillar with employees participating in a company pension fund, an industry-wide fund or an occupation-related scheme for self-employed. However, the existing pension contract has come under pressure for a number of reasons. First, life expectancy continues to rise, implying that existing commitments to older workers and retirees are underfunded. Further, the economic and financial crisis of 2008-2009 produced a simultaneous slump in equity markets and a sharp decrease in the interest rate on high-quality public debt implying that existing liabilities had to be discounted at a lower rate. For many pension funds both factors have reduced funding ratios (the ratio of assets over liabilities) to levels that force them to undertake restoration measures. The main steering instruments in this regard are the pension

This chapter is joint work with Roel Beetsma and Ward Romp. The basis on which this chapter builds is the report 'Doorrekening van verschillende varianten van het nieuwe pensioencontract'written for MN. The authors gratefully acknowledge the use of data provided by MN for this chapter.

contribution rate and rate of indexation to nominal wage or price rises. The aforementioned developments have raised doubts about the robustness of the current collective pension contract for demographic and financial markets developments. Those doubts have been made explicit in the Goudswaard Report (Goudswaard, 2010) and have subsequently led to a headline agreement between the "social partners", i.e. the representatives of employees and employers, to revise the pension contract. This agreement (called the pension agreement or pension deal) specifies that social partners and pension funds can choose to stay in the existing contract with some minor changes, or move to the new contract in which all entitlements are 'soft' or conditional, so that no guarantees are given to participants with respect to their future pension benefits.

In this chapter we present some alternatives to the pension agreement, in which a participant can have both unconditional 'hard' entitlements - as in the old contract - and conditional 'soft' entitlements - as in the new contract of the pension agreement. We will quantify the consequences of redesigning the pension contract in such a way at the level of the pension fund and the consequences at the level of the individual fund participant. The main question to be answered for the pension fund concerns the impact of the new contract on its funding ratio under regular and extreme circumstances, while the main questions concerning the individual participants pertain to the distribution of replacement rates under the new contract and the evolution of the transition phase from the old to the new contract. We investigate various formats for the new pension contract allowing for financial market shocks. However, we mostly abstract from demographic shocks, because there is a consensus that the best way to deal with life expectancy shocks is to change the labor market participation of the elderly and the retirement age (see Bovenberg and Knaap (2005), Conesa and Garriga (2007) and Bohn (1999b)).

The various pension contracts considered below feature a number of common elements. Each year, active participants make a contribution to the fund. In return, they accumulate entitlements to their future pension. The way the accrual of entitlements takes place is currently under review and constitutes the focal point of this chapter. Under the current contract, there exists only one type of entitlement. This is a promise of a constant annual nominal payment as of retirement age. This payment is supposed to be very safe in the sense that, once it has been promised to participants, it can only be reduced when the degree of underfunding is so large that it can no longer be undone through deployment of the regular instruments, a situation that is supposed to be very exceptional. The guaranteed nominal benefits are the main reason why pension liabilities are to be discounted against a risk-free interest rate.

The new pension contracts are intended to be robust against financial market shocks. In general terms, this is done by making the contract more complete in the sense that it will be made clearer ex ante how the consequences of unexpected shocks will be allocated across the various participants. Practically speaking, this takes place through the introduction of two types of entitlements. The first type is a very safe entitlement that we will refer to as a "hard entitlement". As described above, all entitlements accumulated until now were intended to be certain in nominal terms, although in reality they may be less safe than expected, as recent events have made clear. The second type of entitlement we will refer to as a "soft entitlement". Its role is to act as a buffer against financial shocks, thereby protecting the safety of the hard entitlements. Of course, it is conceivable that soft entitlements may evaporate completely due to a sequence of particularly unfortunate realizations in the financial markets. In that case also the hard entitlements have to be reduced, implying that even hard entitlements can never be completely safe.

The three new pension contracts differ in the way hard and soft entitlements are acquired. In the first contract, the 'rolling window' contract, newly accrued entitlements are soft. After a fixed period, these entitlements are then converted into hard entitlements. In the second contract, the 'fraction' contract, newly accrued entitlements consist of a fixed fraction of soft entitlements and hard entitlements. In the third contract, the 'split' contract, all newly accrued entitlements are soft. If the funding ratio of the pension fund rises above a certain level, a fraction of the soft entitlements is transformed into hard entitlements.

Obviously, to guarantee a sufficiently high degree of safety of hard entitlements, it is necessary to have a sufficiently large buffer of soft entitlements. In fact, we are particularly interested in computing the likelihood that the stock of soft entitlements is insufficient to absorb all the financial shocks. The different proposals we consider will lead to different distributions of the stocks of hard and soft entitlements across the various generations. Obviously, different allocations of the two types of entitlements will have important consequences for the incidence of the shocks across the various generations. In addition, they may have serious implications for the financial situation of the pension fund and the extent to which hard entitlements are truly safe.

Under the new proposals, indexation is higher and more readily provided than under the current system. Hence, pension buffers are lower, while current retirees benefit from the switch from the old contract to the new contract. Out of the three new proposals, only the Fraction contract succeeds in effectively guaranteeing hard entitlements and providing relatively stable indexation and buffers over time without inducing substantial intergenerational transfers. The results suggest that in a system with both hard and soft entitlements, the only way to prevent substantial intergenerational transfers is to have all generations accumulate both types of entitlements. Because financial shocks are absorbed through the soft entitlements, this way the costs and benefits of those shocks can be spread over all the generations.

The remainder of the chapter is structured as follows. In Section 5.2 we lay out our model. In Section 5.3, we describe the different designs of the new contract that we explore. In Section 5.4 we describe the calibration of our model and in Section 5.5 we present the results for the various contracts we consider. Section 5.6 concludes this chapter.

5.2 The Model

This section describes the set-up of the model.

5.2.1 Demographics

Each period t, there are a total number of N_t individuals, divided over J generations and I types. Further, $N_{i,j,t}$ is the number of type i of age j individuals alive at time t. Hence, $N_t = \sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j,t}$. Differences in types of individuals arise because of differences in skills or in gender. Those differences lead to differences in income within a cohort.

We denote by ψ_j the age-dependent probability for a person that is currently of age j to survive until age $j+1$. We assume that it is constant over time and across the various types of agents i. Hence, the probability of an individual of age j to survive until some age $l \geq j+1$ is then given by $\prod_{k=0}^{l-j-1} \psi_{j+k}$.

Each year a new generation of size $(1+n_t)\sum_{i=1}^{I} N_{i,1,t-1}$ is born, where n_t is the growth rate in the size of the newborn generations. We assume that n_t is constant. Hence, $n_t = n_{t+1} = \cdots \equiv n$. The maximum age an individual can reach is J. Hence, an individual of age J dies for sure the coming year, implying that $\psi_J = 0$. We neglect the childhood of individuals and assume that they enter the labour force and become employed immediately after birth at age 1. Further, retirement happens at the mandatory age R. We denote by $N_{i,t}^{w} = \sum_{j=1}^{R-1} N_{i,j,t}$ the number of working individuals of type i at time t. Hence, the total number of working individuals is given by $N_t^w = \sum_{i=1}^{I} N_{i,t}^{w} = \sum_{i=1}^{I}\sum_{j=1}^{R-1} N_{i,j,t}$.

5.2.2 Wage income and pension fund contributions

Depending on its type i, an individual in period t earns a wage income $w_{i,t}$. Over time, the wage income of all types grows at the exogenous nationwide nominal growth rate g_t:

$$
\begin{aligned}
w_{i,t} &= (1+g_t)\, w_{i,t-1}, \quad \forall i \in I,\ \forall t \geq 1, && (5.1)\\
g_t &= g + \epsilon_t^g, && (5.2)
\end{aligned}
$$

where ϵ_t^g is a shock to aggregate wages in period t with properties detailed in Section 5.2.5.

Each working individual (i.e., individual of age $1 \leq j < R$) of type i pays a contribution $p_{i,t} \geq 0$ to the pension fund. Collectively, pension funds constitute the "second pillar" of the retirement benefit system. The contribution equals a fraction θ of income $w_{i,t}$ above the so-called franchise level fr_t for those who earn more than fr_t and is zero otherwise:

$$
p_{i,t} = \max\left[0, \theta\left(w_{i,t} - fr_t\right)\right], \qquad \forall i \in I,\ \forall t \geq 1, \tag{5.3}
$$

The franchise is the part of wage income over which no pension contributions are paid and over which no pension entitlements are accrued. The existence of the franchise originates in the fact that in the Netherlands the government provides a basic first-pillar PAYG pension (the "AOW" in Dutch) for everyone. The second-pillar pension acquired through participation in the pension fund comes on top of the AOW. In other words, contributions to and accrual of entitlements in the second pillar apply only to the part of the wage above the franchise. Note that since all individuals of type i earn the same wage, irrespective of their age j, the pension contribution does not depend on j. The franchise is indexed to inflation each year, since the AOW is indexed to inflation under our baseline:

$$fr_t = (1 + \pi_t)\, fr_{t-1},$$

where π_t is the period t inflation rate.

5.2.3 Second-pillar entitlements and liabilities under the old contract

Entitlements

The baseline of the old contract features only one type of pension entitlement. In each period t, the accrual of new pension entitlements $m_{i,t}$ of an individual of type i equals the accrual rate μ times the difference between the nominal wage and the franchise:

$$m_{i,t} = \mu \max\left[0, w_{i,t} - fr_t\right]. \tag{5.4}$$

Note that, because the wage is independent of the age j of an individual, the accrual of new entitlements is also independent of the individual's age. The new entitlements built up in a given period t are added to the individual's beginning-of-period stock of entitlements $M_{i,j,t}$. Depending on the financial situation of the pension fund, the fund can apply either uniform indexation $\omega_t \geq 0$ or a uniform reduction $\omega_t < 0$ to the existing stock of entitlements accumulated in the pension fund.[2] Hence, the law of

[2]The fund's decision rules will be described in Section 5.3.

motion for the stock of entitlements of a working individual is given by:

$$M_{i,j,t+1} = (1+\omega_t)\left(M_{i,j,t} + \mu \max\left[0, w_{i,t} - fr_t\right]\right),$$

while the law of motion for the stock of entitlements of retirees is given by:

$$M_{i,j,t+1} = (1+\omega_t)\, M_{i,j,t}. \tag{5.5}$$

The period t second-pillar pension benefit paid out to a type i age j retiree equals his stock of entitlements at the beginning of the period:

$$b_{i,j,t} = M_{i,j,t}. \tag{5.6}$$

Liabilities

The present value of the pension benefits of all participants together constitutes the liabilities of the pension fund. Because $b_{i,j,t} = M_{i,j,t}$, the liability to each individual of age j and type i at the beginning of period t is given by:

$$L_{i,j,t} = \sum_{l=R-j}^{J-j} \frac{\prod_{k=1}^{l} \psi_{j+k-1}}{(1+r_{l,t})^l} M_{i,j,t}, \quad j < R, \tag{5.7}$$

$$L_{i,j,t} = \left(1 + \sum_{l=1}^{J-j} \frac{\prod_{k=1}^{l} \psi_{j+k-1}}{(1+r_{l,t})^l}\right) M_{i,j,t}, \quad j \geq R, \tag{5.8}$$

where $r_{l,t}$ is the l-year interest rate on bonds in period t. The total liabilities of the pension fund L_t equals the sum of the liabilities to all individuals in period t:

$$L_t = \sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j,t} L_{i,j,t}. \tag{5.9}$$

Finally, the indicator of the financial health of the pension fund that is relevant for the fund's supervisor is the funding ratio F_t given by:

$$F_t = \frac{A_t}{L_t}, \tag{5.10}$$

where A_t is the value of the fund's assets at the start of period t.

5.2.4 Assets

The total value of the pension fund's assets at the start of period t is given by:

$$A_t = \left(1 + r_t^f\right) A_{t-1}^{end}, \tag{5.11}$$

where r_t^f is the return to the pension fund's asset portfolio from period $t-1$ to period t and A_{t-1}^{end} is the value of the fund's assets at the end of period $t-1$, i.e. after contributions for period $t-1$ have been collected and benefits have been paid out. We assume that a share α of the pension fund's total asset portfolio is invested in equity and a share $(1-\alpha)$ is invested in bonds:

$$E_{t-1} = \alpha A_{t-1}, \quad B_{t-1} = (1-\alpha) A_{t-1}, \tag{5.12}$$

where E_{t-1} and B_{t-1} are the fund's equity and bond investments, respectively. The bond portfolio is constructed to match the maturity structure of the liabilities. Specifically, if a share ζ^k of total discounted benefit payments (i.e. liabilities of the fund) is expected to take place after k years time, then the fund invests a share ζ^k of its bond portfolio in bonds maturing after k years:[3]

$$b_{k,t-1} = \zeta^k B_{t-1}, \qquad \forall k \in [1, J], \tag{5.13}$$

where $b_{k,t-1}$ is the amount invested in bonds maturing after k years. Assuming that the fund invests only in zero-coupon bonds, the return on a maturity k bond at time t is given by:

$$1 + Ret_{k,t}^b = \frac{1/(1 + r_{k,t})^k}{1/(1 + r_{k+1,t-1})^{k+1}} = \frac{(1 + r_{k+1,t-1})^{k+1}}{(1 + r_{k,t})^k} \tag{5.14}$$

where $r_{t,k}$ is the yield to maturity on a zero-coupon bond with k years to go until the face value is paid out. The return on equity is given by $1 + r_t^{eq}$, so that we can write

[3] Alternatively interest rate swaps could be used with the same pay-off structure as the bond portfolio.

the total gross portfolio return as:

$$1 + r_t^f = (1 + r_t^{eq})\,\alpha + \sum_{k=1}^{K} \left(1 + Ret_{k,t}^b\right) \zeta^k (1 - \alpha)\,. \tag{5.15}$$

Plugging this expression into the law of motion for the fund's assets we obtain:

$$A_t = (1 + r_t^{eq})\,\alpha A_{t-1}^{end} + \sum_{k=1}^{K} \left(1 + Ret_{k,t}^b\right) \zeta^k (1 - \alpha)\, A_{t-1}^{end} \tag{5.16}$$

Then, in period t, after the returns on the equity and bond holdings have materialised, contributions and benefits are paid. After this, the pension fund rebalances its portfolio so that the original investment shares α in equity and $1 - \alpha$ in bonds are restored (and the bond portfolio again matches the maturity structure of its liabilities):

$$E_t = \alpha \left(A_t + \sum_{i=1}^{I} N_{i,t}^w p_{i,t} - \sum_{i=1}^{I} \sum_{j=R}^{J} N_{i,j,t} b_{i,j,t} \right), \tag{5.17}$$

$$B_t = (1 - \alpha) \left(A_t + \sum_{i=1}^{I} N_{i,t}^w p_{i,t} - \sum_{i=1}^{I} \sum_{j=R}^{J} N_{i,j,t} b_{i,j,t} \right). \tag{5.18}$$

End-of-period asset holdings of the pension fund are then given by:

$$A_t^{end} = E_t + B_t. \tag{5.19}$$

Note that because financial market risk is not the only source of risk, even with "matching investments" ($\alpha = 0$) the retirees' benefits still run risk. Moreover, if the total bond portfolio is smaller than the total value of the liabilities, the pension fund still faces some financial market risk, because it can not fully match its liabilities with bonds.

5.2.5 Economic shocks

The modelling of the economic shocks follows closely Beetsma and Bucciol (2011). There are four stochastic macro-economic variables: the inflation rate (π_t), the nominal wage growth rate (g_t), the yield on bonds with a maturity of one year ($r_{1,t}$) and the equity return (r_t^{eq}). These variables obey the following processes:

$$\begin{pmatrix} \pi_t \\ g_t \\ r_{1,t} \\ r_t^{eq} \end{pmatrix} = \begin{pmatrix} \pi \\ g \\ r^b \\ r^{eq} \end{pmatrix} + \begin{pmatrix} \epsilon_t^{\pi} \\ \epsilon_t^{g} \\ \epsilon_t^{b} \\ \epsilon_t^{eq} \end{pmatrix}, \tag{5.20}$$

where the first vector on the right-hand side contains the averages of the respective stochastic variables and the second vector on the right-hand side contains the (mean-zero) shocks to these variables. This vector of shocks follows a VAR(1) process:

$$\begin{pmatrix} \epsilon_t^{\pi} \\ \epsilon_t^{g} \\ \epsilon_t^{b} \\ \epsilon_t^{eq} \end{pmatrix} = B \begin{pmatrix} \epsilon_{t-1}^{\pi} \\ \epsilon_{t-1}^{g} \\ \epsilon_{t-1}^{b} \\ \epsilon_{t-1}^{eq} \end{pmatrix} + \begin{pmatrix} \eta_t^{\pi} \\ \eta_t^{g} \\ \eta_t^{b} \\ \eta_t^{eq} \end{pmatrix}, \tag{5.21}$$

where B is a matrix filled with constants (including zeroes). Thus, each individual shock ϵ_t^i, $i \in \{\pi, g, b, eq\}$ is a combination of the vector of shocks in the previous year $\left[\epsilon_{t-1}^{\pi}, \epsilon_{t-1}^{g}, \epsilon_{t-1}^{b}, \epsilon_{t-1}^{eq}\right]'$ plus an innovation η_t^i. The vector of innovations $\left[\eta_{t-1}^{\pi}, \eta_{t-1}^{g}, \eta_{t-1}^{b}, \eta_{t-1}^{eq}\right]'$ follows a normal process with mean zero and variance-covariance matrix Σ_f.

The full term structure $\{r_{k,t}\}_{k=1}^{K}$ is constructed as follows: each yield to maturity in the term structure $r_{k,t}$ has a gross mark-up factor ν_k on the one year interest rate $r_{1,t}$. After realization of the one-year interest rate for a given year, we construct the full term structure by multiplying the mark-up factor of each maturity with the one-year yield:

$$r_{k,t} = \nu_k r_{1,t}, \quad \forall k. \tag{5.22}$$

5.2.6 The timing

Within any period t, events take place in the following order:

1. Participants become one year older or die, a new generation of participatants of age 1 enters the pension fund and economic shocks and returns materialise.

2. New pension entitlements are accrued, contributions are paid into the fund and benefits are paid out to the retirees.

3. The indexation policy is implemented.

4. The asset portfolio is rebalanced to ensure that a share α is invested in equity and to match the liability structure after indexation.

5.3 Pension fund policy

This section describes the various pension contracts we consider and discusses how these contracts differ. First, we describe the current contract. Then, we describe three proposals for the new contract. All three proposals for the new contract split entitlements in two types: hard and soft. The idea of this division of entitlements is that the hard entitlements provide the participants with some minimum benefit level during retirement. These entitlements should be very safe in the sense that individuals can only lose these entitlements in exceptional circumstances. The soft entitlements form the buffer around the hard entitlements: shocks to the asset position of the pension fund are absorbed first by the soft entitlements, so that these fluctuate with the value of the assets.

For each version of the new contract we consider, we will spell out a policy rule that addresses precisely how soft and hard entitlements are accumulated by participants in the pension fund. The total value of entitlements of an individual under the proposals for the new contract equals the sum of the stocks of hard and soft entitlements:

$$M_{i,j,t} = M^h_{i,j,t} + M^s_{i,j,t} \quad t \geq 0, \tag{5.23}$$

where $M^h_{i,j,t}$ is the stock of hard entitlements and $M^s_{i,j,t}$ is the stock of soft entitlements of individual i of type j at time t.

5.3.1 The current contract

Under the current contract, there is only one type of entitlement, as described above. Whether one can view this entitlement as a hard or a soft entitlement depends on the interpretation of the pension contract (see also Bovenberg and Nijman (2011) and Bovenberg and Ewijk (2011)). The supervisor, the Dutch Central Bank (DNB), considers the entitlements as hard and allows a reduction in those entitlements only in extreme circumstances.

Contributions are such that they cover the expected future cost of the pension in terms of benefit payments. We now describe the rule that the pension fund uses to deal with underfunding and indexation.

Underfunding

Underfunding is defined as a situation in which the funding ratio is below some level F^{lb}. When the pension fund is underfunded, it has to set up a plan to restore the funding ratio to at least F^{lb} within three years by using its policy instruments, which consist of a reduction in indexation, higher contributions and, as a last resort, a reduction in entitlements.

A reduction in indexation means that the pension fund does not provide sufficient indexation for the existing stock of entitlements to grow at the same rate as the price level or the nominal wage rate. This can go so far as to not provinding any indexation at all, so that $\omega_t = 0$. The contribution rate θ can be raised up to a maximum of $\bar{\theta}$. If setting $\omega_t = 0$ and $\theta = \bar{\theta}$ for a period of three years is still not sufficient to bring the funding ratio back to F^{lb} after these three years, the pension fund reduces existing entitlements ($\omega_t < 0$) by as much as is needed to achieve this goal.

Indexation

If the funding ratio exceeds F^{lb}, the pension fund uses the following indexation rule for the existing entitlements. If the funding ratio exceeds $F^{ub} > F^{lb}$, then the fund provides full indexation on all entitlements to either the price level or nominal wage growth (depending on the indexation goal of the pension fund). Moreover, in this case the fund also provides catch-up indexation to compensate for missed indexation in previous years to the extent that all missed indexation is compensated and the funding ratio does not fall below F^{ub}. If the funding ratio is in between F^{lb} and F^{ub} then indexation is a fraction κ_t of full indexation according to:

$$\kappa_t = \frac{F_t - F^{lb}}{F^{ub} - F^{lb}}. \tag{5.24}$$

Hence, the rate of indexation provided is

$$\omega_t = \kappa_t g_t, \tag{5.25}$$

if the fund aims at keeping benefits in line with the nominal wage rate, or by

$$\omega_t = \kappa_t \pi_t, \tag{5.26}$$

if the fund aims at keeping the purchasing power of existing entitlements constant.

5.3.2 Proposals for the new contract

We investigate three different proposals for the new pension contract: the Rolling Window contract, where participants accrue soft entitlements based on a rolling window formula, the Fraction contract, where participants accrue a fixed fraction of hard and soft entitlements each year and the Split proposal, where all existing entitlements at the moment of transition are hard and all newly accrued entitlements are soft.

5.3.3 The Rolling Window contract

The first variant of the new pension contract was suggested by FNV Bondgenoten, the largest labor union in the Netherlands. We will call it the "Rolling Window" proposal because soft entitlements are accrued based on a rolling window formula that we describe below.

The existing stock of entitlements at the moment $t = 0$ of the transition from the old to the new contract will be divided into a share ξ of hard and a share $1 - \xi$ of soft entitlements:

$$M^h_{i,j,0} = \xi M^o_{i,j,0} \tag{5.27}$$

$$M^s_{i,j,0} = (1-\xi) M^o_{i,j,0} \tag{5.28}$$

where $M^o_{i,j,0}$ is the stock of 'old' pension entitlements of individual type i and age j at the start of period 0 just before the transition to the new system is implemented.

All new entitlements accrued through working are soft entitlements. After Q years, these entitlements including their accumulated indexation (positive or negative) are transformed into hard entitlements. Hence, in period t, the stock of soft entitlements evolves as follows:

$$M^s_{i,j,t+1} = \begin{cases} (1+\omega^s_t)\left(M^s_{i,j,t} + \mu \max\left[0, w_{i,t} - fr_t\right]\right), \quad j \leq Q, \\ (1+\omega^s_t)\left(M^s_{i,j,t} - \tilde{m}^s_{i,j,t-Q} + \mu \max\left[0, w_{i,t} - fr_t\right]\right), Q+1 \leq j < R, \\ (1+\omega^s_t)\left(M^s_{i,j,t} - \tilde{m}^s_{i,j,t-Q}\right), R \leq j < R+Q, \\ 0, \quad j \geq R+Q, \end{cases}$$

where ω^s_t is the indexation rate of soft entitlements in year t and $\tilde{m}^s_{i,j,t-Q}$ is the amount of soft entitlements that were earned by working exactly Q years ago, including the indexation that has been accumulated on those entitlements in the intervening years:

$$\tilde{m}^s_{i,j,t-Q} = \prod_{q=t-Q}^{t-1} \left(1+\omega^s_q\right) m^s_{i,j,t-Q}, \tag{5.29}$$

where $m^s_{i,j,t-Q}$ is the amount of new soft entitlements that was earned by working in period $t-Q$. The stock of hard entitlements evolves as follows:

$$M^h_{i,j,t+1} = \begin{cases} 0, \quad j \leq Q, \\ \left(1+\omega^h_t\right)\left(M^h_{i,j,t} + \tilde{m}^s_{i,j,t-Q}\right) \quad Q < j < R+Q, \\ \left(1+\omega^h_t\right) M^h_{i,j,t}, \quad j \geq R+Q, \end{cases}$$

where ω^h_t is the indexation rate of hard entitlements in year t.

Hence, at his retirement age R an individual has a stock of $R-Q-1$ years of accumulated hard rights and Q years of accumulated soft rights. After Q years into retirement a retiree has a stock of $R-1$ years of accumulated hard rights and has no soft rights left. As before, the calculation of the liabilities and the funding ratio follows equations (5.7)-(5.10), given that the total stock of rights of an individual is given by equation (5.23).

We define *full* indexation of *hard* entitlements as $\omega_t^h = \pi_t$, if the fund tries to maintain the purchasing power of the hard entitlements, and as $\omega_t^h = g_t$ if it tries to track nominal wage growth. Full indexation of soft entitlements is defined as $\omega_t^s = \pi_t + \nu^\omega$, in the case that the fund tries to track price inflation, and as $\omega_t^s = g_t + \nu^\omega$, in the case that the fund tries to track nominal wage growth. Here, $\nu^\omega \geq 0$ is a mark-up to compensate for the higher riskiness of the soft entitlements.

Underfunding

The Rolling Window proposal is designed to adjust entitlements such that a situation of underfunding never occurs. This is accomplished as follows. If at the start of some period t, after asset returns have materialised and new pension entitlements have been earned, the pension fund has a funding ratio below F^{lb}, which serves as a lower bound on the funding ratio, it marks down the value of its soft entitlements (by setting $\omega_t^s < 0$) such that $F_t = F^{lb}$. If, after having completely marked down the stock of soft entitlements (i.e., $\omega_t^s = -1$), the funding ratio is still below F^{lb}, hard rights will be marked down (by setting $\omega_t^h < 0$) until $F_t = F^{lb}$.

This underfunding rule highlights the distinction between the soft and hard entitlements: the soft entitlements are the flexible part of the stock of entitlements that can be adjusted on an annual basis. They act as a buffer against the financial market risk that the pension fund faces in its asset portfolio. The hard entitlements are intended to provide a relatively safe guarantee for a minimal pension benefit at retirement and can only be adjusted downward once the entire stock of soft entitlements has been wiped out already.

How well the stock of soft entitlements performs its buffer role obviously depends on its size relative to the stock of hard entitlements. It also depends on the volatility of the financial market shocks hitting the pension fund and on the pension fund's asset mix. A fund with a relatively safe asset mix can likely guarantee its hard entitlements with a relatively small stock of soft entitlements.

Indexation

If the funding ratio exceeds F^{lb}, the pension fund provides indexation with the hard entitlements receiving priority over the soft entitlements. Specifically, the indexation rule obeys the following sequencing:

1. First, as long as F_t does not fall below F^{lb}, the pension fund provides restoration of previous (unrestored) reductions in hard rights and missed indexation on hard rights. Missed indexation occurs when less than full indexation is given to the target (price or nominal wage inflation) pursued by the fund.

2. Next, if after applying the aforementioned restorations the funding ratio still exceeds F^{lb}, and as long as F_t does not fall below F^{lb}, the pension fund provides the new indexation (to a maximum of full indexation) of hard entitlements. For this purpose it potentially marks down the value of the soft entitlements.

3. If the funding ratio still exceeds F^{lb}, the pension fund indexes the soft entitlements until either the funding ratio is F^{lb} or soft entitlements have been fully indexed, including the mark-up ν^{ω} of 0.5%. However, neither soft entitlements that were marked down earlier, nor missed indexation on these soft rights are restored. Soft entitlements function as a 'memory-less' buffer against shocks, absorbing both the good and the bad shocks while no track is kept of what happened in earlier years.

4. If after the full indexation including the mark-up of soft entitlements the funding ratio exceeds F^{ub}, the pension fund provides additional indexation to the soft rights, until the funding ratio is exactly F^{ub} again. This is done to prevent the pension fund buffer from exploding and to award the holders of soft rights some of the upward potential of the pension fund investments as a compensation for the downward risk they are exposed to.

5.3.4 The Fraction contract

Under the second type of new contract, the "Fraction contract", the transition from old to new entitlements is handled in the same way as in the Rolling Window contract: a fraction ξ of the old entitlements is transformed into hard entitlements, while a fraction $1-\xi$ is transformed into soft entitlements:

$$M^h_{i,j,0} = \xi M^o_{i,j,0}, \tag{5.30}$$

$$M^s_{i,j,0} = (1-\xi)\, M^o_{i,j,0}. \tag{5.31}$$

However, the accrual of hard and soft entitlements as a result of working takes place in the same proportion as the transformation of the old entitlements: each year a fraction ξ of the new entitlements is accumulated as hard entitlements, while the remaining fraction $1-\xi$ is accumulated as soft entitlements. Hence, for working generations ($j < R$) we have:

$$M^h_{i,j,t+1} = \left(1+\omega^h_t\right)\left(M^h_{i,j,t} + \mu \max\left[0, \xi\left(w_{i,t} - fr_t\right)\right]\right), \tag{5.32}$$

$$M^s_{i,j,t+1} = \left(1+\omega^s_t\right)\left(M^s_{i,j,t} + \mu \max\left[0, (1-\xi)\left(w_{i,t} - fr_t\right)\right]\right), \tag{5.33}$$

while for retired generations ($j \geq R$) we have:

$$M^h_{i,j,t+1} = \left(1+\omega^h_t\right) M^h_{i,j,t}, \tag{5.34}$$

$$M^s_{i,j,t+1} = \left(1+\omega^s_t\right) M^s_{i,j,t}. \tag{5.35}$$

The total entitlements of each individual are again the sum of the stocks of the soft and hard entitlements as in equation (5.23), while as before the values of the liabilities and the funding ratio are calculated according to (5.7)-(5.10). Notice that, in contrast to the Rolling Window proposal, retirees continue to hold soft entitlements and so they continue to share in the risks affecting the pension fund.

Underfunding

The rule for dealing with underfunding is exactly the same as under the Rolling Window contract.

Indexation

The rule for indexation is exactly the same as under the Rolling Window contract.

5.3.5 The Split contract

The Split contract focuses on a separation between hard and soft entitlements in the transition from the old to the new system. The stock of old entitlements becomes the stock of new hard entitlements in a one-to-one conversion:

$$M^h_{i,j,0} = M^o_{i,j,0}, \tag{5.36}$$

$$M^s_{i,j,0} = 0. \tag{5.37}$$

Hence, at the moment of the transition to the new system, there will be only hard entitlements. Subsequently, all newly accrued entitlements are soft entitlements. As soon as the funding ratio exceeds F^{ub}, some of the soft entitlements are transformed into hard entitlements. For working generations ($j < R$) entitlements evolve as:

$$M^h_{i,j,t+1} = \left(1 + \omega^h_t\right) M^h_{i,j,t} + T_t, \tag{5.38}$$

$$M^s_{i,j,t+1} = (1 + \omega^s_t) \left(M^s_{i,j,t} + \mu \max\left[0, (w_{i,t} - fr_t)\right]\right) - T_t, \tag{5.39}$$

where T_t is the potential transformation of soft entitlements into hard entitlements in period t, as will be described below.

For retired generations ($j \geq R$) we obtain:

$$M^h_{i,j,t+1} = \left(1 + \omega^h_t\right) M^h_{i,j,t} + T_t, \tag{5.40}$$

$$M^s_{i,j,t+1} = (1 + \omega^s_t) M^s_{i,j,t} - T_t. \tag{5.41}$$

Underfunding

The policy in the case of underfunding is identical to that under the Rolling Window and Fraction arrangements.

Indexation

As long at the funding ratio is below F^{ub}, indexation policy is identical to that under the Rolling Window and Fraction proposals. However, when the funding ratio exceeds F^{ub}, step 4 of the indexation rule becomes as follows. First, soft entitlements are uniformly increased until the funding ratio has fallen to F^{ub}. After that some of the soft entitlements may be transformed into hard entitlements. If soft entitlements as a share of the total (hard plus soft) entitlements are smaller than ϕ, nothing happens. If soft entitlements as a share of the total exceed ϕ, soft entitlements are transformed into hard entitlements on a one-for-one basis, until the soft entitlements as a share of the total entitlements are exactly equal to ϕ. The transformation of entitlements is thus given by:

$$T_t = \max\left\{0, \left[\frac{(1+\omega_t^s)\, M_{i,j,t}^s}{\tilde{M}_{i,j,t}} - \phi\right] \tilde{M}_{i,j,t}\right\}. \tag{5.42}$$

where $\tilde{M}_{i,j,t} = \left(1+\omega_t^h\right) M_{i,j,t}^h + (1+\omega_t^s)\, M_{i,j,t}^s$. Notice that indexation is awarded first, before the transformation takes place.

5.3.6 Contribution policy

The contribution rate is fixed and constant at its cost-covering level. Hence, the contribution rate is not used as a policy instrument by the pension fund. The cost-covering contribution rate θ to the pension fund is defined as the contribution rate such that in expectation the discounted sum of all contributions equals the discounted sum of all benefits payed by the pension fund:

$$E_1 \sum_{t=1}^{R-1} \frac{\theta\,(\bar{w}_t - fr_t)}{\left(1+r_l^f\right)^{t-1}} = E_1 \sum_{t=R}^{J} \frac{\bar{b}_t}{\left(1+r_l^f\right)^{t-1}} \tag{5.43}$$

where $\bar{w}_t$ and $\bar{b}_t$ are the population averages of wages and benefits of the pension fund participants.

5.4 Calibration and simulation

5.4.1 Calibration

Individuals enter the labour market at age 25 and work for 42 years, so that they retire at age 67. We normalise the entry age to 1, hence $R = 43$ in the model. The maximum age that an individual can reach and live is 99 years (as soon as the individual turns 100, he dies for sure), which corresponds to a maximum of 75 years of life of the model ($J = 75$). Survival probabilities from year to year are taken from the latest projections by the Dutch Actuarial Society (AG, 2010). Data for the distribution of wages over age groups and gender specific income classes are provided by MN for the PMT pension fund for the year 2010. These figures pin down the initial situation for our simulations.

We set the pension fund parameters so as to mimick the situation for PMT. The pension fund starts with a funding ratio of exactly 100%. The contribution rate is $\theta = 18.6\%$, which equals the cost-covering contribution rate in 2010 plus a solvency mark-up. The accrual rate of pension entitlements is $\mu = 2.236\%$, which is very close to the legally allowed maximum of 2.25%. We set $\alpha = 0.5$, hence the pension fund invests half of its portfolio in equity and the other half in bonds. The lower and upper bounds of the indexation schedules are set at $F^{lb} = 100\%$ and $F^{ub} = 140\%$. The choice for the lower bound is obvious, as a funding ratio below 100% means that the pension fund is technically insolvent. The choice for the upper bound is loosely based on observed indexation policies in the past and the idea that we do not want the buffers to become unrealistically large. For instance, participants might pressure the fund into spending some of the buffer in the form of lower contributions or higher benefits or the government might start taxing away very high pension buffers. Finally, we need to pin down some parameters for whose values we do not have any a priori guidelines. We set the mark-up for the indexation of soft entitlements at $\nu^{\omega} = 0.5\%$, the length of the rolling window of soft entitlements Q in the Rolling Window proposal to 10 years, the fraction of hard entitlements in the Fraction proposal at $\xi = 0.5$ and the fraction of

Table 5.1: Base case parameter values

Parameter	Value
R	43
J	75
α	0.5
θ	0.186
μ	0.02236
F_0	1
F^{lb}	1
F^{ub}	1.4
ν^{ω}	0.005
Q	10
ϕ	0.2
ξ	0.5
π	0.02
g	0.03
r^b	0.03
r^{eq}	0.068

soft entitlements after transformation of entitlements in the Split contract at $\phi = 0.2$.

Further, we set the average value of inflation at 2%, average nominal wage growth at 3%, the average one-year bond return at 3% and the average equity return at 6.8%. By taking these values we take the parameter values prescribed by the Parameters (2009) for Dutch pension funds. The values for the covariance matrix Σ_f of the innovations and the VAR coefficient matrix B are taken from Beetsma and Bucciol (2011). Their estimates are based on times series over the period 1976-2005 for the Dutch consumer price index, Dutch hourly wages - both taken from OECD (2009) - U.S. one-year bond yields, taken from Reserve (2009) and the MSCI U.S. equity index, taken from Datastream (2009). The mark-ups for bond maturities longer than one year are based on the average mark-ups in the Dutch term structure over the period 2001-2010 as published by Bank (2011).

5.4.2 The simulation setup

We start the simulation by generating an initial distribution of individuals and entitlements based on the PMT data. This initial situation is identical in all versions of

Table 5.2: Outcomes under wage indexation

	Current	RW	Fraction	Split
median funding ratio	1.237	1.148	1.195	1.202
standard deviation funding ratio	0.162	0.162	0.161	0.162
median indexation rate hard entitlements	0.019	0.031	0.032	0.032
stand. dev. indexation rate hard entitlements	0.120	0.051	0.017	0.044
median indexation rate soft entitlements		0.034	0.038	0.038
stand. dev. indexation rate soft entitlements		0.834	0.454	0.790
% of time reduction of hard entitlements	0.216	0.067	0.003	0.031
% of time reduction of soft entitlements		0.292	0.198	0.223
median share soft entitlements		0.258	0.575	0.221
stand. dev. of share of soft entitlements		0.194	0.181	0.110
median replacement rate	0.998	1.037	1.015	0.838
stand. dev. of replacement rate	0.376	0.602	0.370	0.274
median share soft entitlements retirees		0.103	0.349	0.157
stand. dev. share soft entitlements retirees		0.040	0.158	0.092

the model, as it reflects the situation as it currently is, which does not depend on the form the new contract takes. Next, we draw 1000 sets of 50 (or 250 to check if the pension scheme is viable in the very long run) realisations for the economic shocks. We then use these realisations to compute a scenario for each set of shock realisations. Thus we obtain 1000 scenario's for all variables in the model. We use these scenarios to compute the relevant statistics for each version of the model, such as medians and standard deviations of the funding ratio, the share of soft entitlements, indexation of hard and soft entitlements and replacement rates of the retiring cohorts.

5.5 Results

For each scenario we present a table with numerical results and a graph. We start with the baseline parameter combination as reported in Table 5.1, first considering the case of indexation to nominal wage growth, followed by the case in which the pension fund indexes entitlements to price inflation.

5.5.1 Indexation to wages

Table 5.2 and Figures 5.1, 5.2 and 5.3 show the consequences for the financial position of a pension fund that targets wage indexation under both the current contract and the three different versions of the new pension contract. Notice that in the current contract there is only one kind of entitlement (hard), so there are no table entries for soft entitlements under the current contract.

The median values of the funding ratios differ across the contracts. The current contract features the highest median funding ratio of 123.7%, while in the rolling window contract the funding ratio is lowest at 114.8%. This corresponds to the fact that in the current contract the median indexation of hard and, hence, total entitlements is 1.9%. This is substantially below the median indexation of hard entitlements in the proposals for the new contract, which are 3.1% under the Rolling Window contract and 3.2% under the Fraction and Split contracts. It is also substantially below the median indexation of soft entitlements of 3.4% under the Rolling Window proposal and 3.8% under the Fraction and Split proposals. Hence, the overall median indexation rate under the new proposals is substantially higher than the median indexation rate under the current contract.

We see that the standard deviation of the funding ratio is virtually identical across contracts. The reason is that it is almost entirely driven by the financial shocks, which are the same for each contract.

The standard deviation of the indexation of hard entitlements is highest under the current contract. This is exactly as expected, since there is no buffer of soft entitlements under the current contract. Of the proposals for the new contract, the standard deviation under the Fraction proposal is notably below the standard deviation under the other two proposals, indicating that hard entitlements are more stable under this scenario. The percentage of time that the hard rights have to be reduced differs materially across the scenarios: the Fraction scenario does best in this regard. Only in 0.3% of the observed outcomes are hard entitlements reduced. The Split proposal is intermediate at roughly 3%, while under the Rolling Window proposal in more than 6% of the cases hard entitlements are reduced. The main reason for these differences is the size of the buffer of soft entitlements. Under the Fraction proposal the median share of soft entitlements as a share of total entitlements is 57.5% , under the Rolling Window scenario it is only 25.8%, while under the Split scenario it is even lower at 22.1%. The substantially larger buffer of soft rights under the Fraction proposal implies that hard entitlements are safer than under the other two proposals.[4]

Finally, the median replacement rates under the Rolling Window and Fraction scenarios are quite close (1.037 versus 0.995). However, under the Rolling Window proposal the replacement rate is much more volatile. The reason is that under the Rolling Window proposal the share of the soft entitlements held by the retired is much lower than under the Fraction proposal; the median shares of soft entitlements are 10.3% and 34.9%, respectively. Hence, under the Rolling Window proposal the elderly share less in the risks affecting the pension fund and, hence, the accumulation of entitlements by the workers is more uncertain.

At 0.838, the median replacement rate under the Split proposal is substantially lower than under the other two proposals. This is caused by the fact that the share of hard entitlements is relatively large, while the young possess all or most of the soft entitlements, especially in earlier periods. The small share of soft entitlements serves as the buffer for the hard entitlements and the chance that a large share of the soft entitlements is wiped out through the shocks that hit the pension fund is relatively high. Because the soft entitlements process is memoryless, these losses will not be restored. The burden of these losses thus falls primarily on the young who will on average be confronted with a lower funding ratio at retirement than under the other two alternatives.

If we look at Figures 5.1, 5.2 and 5.3, we see the same picture emerging in a slightly more detailed way. The figures depict the funding ratio, indexation of hard entitlements, the share of soft entitlements in total entitlements, and the replacement rate of the retiring cohort for each of the three proposals for the new contract, respectively. In Figure 5.1 we see that in the Rolling Window version of the contract, the median funding ratio goes up in the first few years of the simulation period and then steadily

[4]Note that the share of soft entitlements is higher than $\xi = 0.50$, because of the higher indexation awarded to the soft entitlements when the funding ratio is between 100 and 140%, and the additional indexation on top of that when the funding ratio is higher than 140%.

declines. We see that indexation of hard entitlements is pretty stable for the first 20 years of the simulation period, but after 20 years the 5% and 95% intervals strongly diverge: the standard deviation of the indexation of hard entitlements goes up sharply. In the lower left panel we see why this happens: in the first 20 years the pension fund 'burns' its buffer of soft entitlements, so that after 20 years the stock of soft entitlements is not large enough anymore to absorb most financial shocks.

In Figure 5.2 we see that for the Fraction contract, both the volatility and the levels of all four variables depicted are relatively stable. In Figure 5.3 we see that for the Split proposal, indexation of hard entitlements in the first 5 years is highly uncertain (because the stock of soft entitlements is then very small). After the first 5 years, the situation is more or less stable. In the lower right panel, which depicts the replacement rate of the retiring cohort, we see that after about 30 years, the median replacement rates of the retiring cohorts start declining. As described above, this is caused by the fact that these are the generations that posessed few or no hard entitlements at the moment of transition and thus absorbed most of the financial shocks the pension fund faced.

5.5.2 Indexation to prices

We now repeat the simulations of the previous subsection for indexation to price inflation. The results are presented in Table 5.3. We see that in all three scenarios the median funding ratio is higher while its volatility is unchanged, a direct consequence of the fact that price inflation tends to be lower than nominal wage growth. That means that the pension fund provides on average less indexation to its participants, as can be seen when we compare median rates of indexation of the hard and soft entitlements for the two cases. The counterpart of lower indexation is a higher buffer. Since buffers are slightly higher under price indexation, the percentages of times that hard and soft rights have to be reduced are lower than under wage indexation. Since hard entitlements are on average indexed by less under price indexation, soft entitlements can be indexed more often and, hence, the median share of soft entitlements is higher in this case.

The median replacement rates of the retiring cohorts are virtually identical under

Table 5.3: Outcomes under price indexation

	Current	RW	Fraction	Split
median funding ratio	1.252	1.189	1.229	1.236
standard deviation funding ratio	0.161	0.162	0.159	0.160
median indexation rate hard entitlements	0.014	0.022	0.021	0.022
stand. dev. indexation rate hard entitl.	0.120	0.044	0.014	0.038
median indexation rate soft entitlements		0.027	0.029	0.031
stand. dev. indexation rate soft entitlements		0.783	0.300	0.706
% of time reduction of hard entitlements	0.204	0.055	0.001	0.025
% of time reduction of soft entitlements		0.241	0.176	0.184
median share soft rights		0.285	0.626	0.228
standard deviation share soft entitlements		0.191	0.159	0.106
median replacement rate	1.000	1.058	0.996	0.836
standard deviation replacement rate	0.367	0.588	0.351	0.279
median share soft retired		0.108	0.364	0.169
standard deviation share soft retired		0.042	0.157	0.092

wage growth and price inflation indexation for the Fraction and Split proposals, while it is slightly higher under price indexation than under wage indexation for the Rolling Window proposal. This might seem counterintuitive, because higher indexation should imply a higher replacement rate. However, we obtain this result, because under wage indexation the retiring generations are relatively well off in the first years, leading to a lower pension buffer and, hence, increasing the chances of low or negative indexation for later generations. On balance, over the 50-years horizon we consider here, the median replacement rate of the retiring cohort is lower under wage indexation.

5.5.3 Robustness checks

To save space, we report the robustness checks only for wage indexation. The robustness checks for price indexation are qualitatively very similar.

Varying the volatility on asset returns

In this subsection we vary the standard deviations of the returns on bonds and equity. In the "low volatility" scenario, the standard deviation of the return on bonds is 0.85% and that of the return on equity is 10%. In the "high volatility" scenario, the standard

Table 5.4: Varying the volatility of the asset returns

	$\sigma^{eq} = 0.1$	$\sigma^{b} = 0.0085$		$\sigma^{eq} = 0.175$	$\sigma^{b} = 0.02$	
	RW	Fraction	Split	RW	Fraction	Split
median funding ratio	1.177	1.196	1.266	1.121	1.192	1.186
st. dev. funding ratio	0.151	0.149	0.146	0.167	0.165	0.167
median ind. rate hard	0.031	0.031	0.031	0.031	0.032	0.032
st. dev. ind. rate hard	0.022	0.014	0.020	0.073	0.020	0.062
median ind. rate soft	0.035	0.036	0.041	0.031	0.038	0.037
st. dev. ind. rate soft	0.325	0.159	0.357	1.879	0.711	1.411
% time reduction hard	0.020	0.001	0.008	0.101	0.005	0.054
% time reduction soft	0.189	0.143	0.101	0.347	0.227	0.266
median share soft ent.	0.282	0.516	0.242	0.244	0.625	0.200
st. dev. share soft ent.	0.143	0.135	0.122	0.217	0.195	0.105
median replacement r.	0.899	0.914	0.755	1.165	1.090	0.906
st.dev. replacement r.	0.296	0.244	0.164	0.943	0.491	0.396
median share soft retired	0.108	0.322	0.187	0.102	0.357	0.141
st. dev. share soft retired	0.033	0.138	0.094	0.044	0.167	0.096

deviations become 2% for the bond returns and 17.5% for the equity returns.

Not surprisingly, increasing the volatility of the assets returns raises the standard deviations of all variables and the percentage of the time that soft and hard entitlements are reduced increases. Further, under both the Rolling Window and the Split contracts, raising the volatility of asset returns leads to a fall in the median funding ratio and a rise in the median replacement rate. When volatility is low, the funding ratio is almost always between 100% and 140%. Hence, the pension fund almost never provides additional indexation on soft rights, so that the median replacement rate of the retiring cohort is lower, but at the same time this ensures that the buffer is on average a bit higher.

The asset portfolio

We now consider more defensive mixes of the fund's asset portfolio. In particular, we consider the cases of 10% and 25% equity shares.

With more defensive asset mixes, the impact of the equity returns diminishes and the impact of the bond returns increases. Because the fund invests its bond portfolio so as to match its liability structure, a larger share of total assets invested in bonds implies

Table 5.5: Varying the equity share in the fund's poprtfolio

	$\alpha = 0.1$			$\alpha = 0.25$		
	RW	Fraction	Split	RW	Fraction	Split
median funding ratio	1.215	1.239	1.340	1.212	1.237	1.306
st. dev. funding ratio	0.150	0.151	0.136	0.151	0.150	0.144
median ind. rate hard	0.031	0.032	0.032	0.031	0.031	0.032
st. dev. ind. rate hard	0.014	0.014	0.014	0.017	0.014	0.017
median ind. rate soft	0.037	0.039	0.046	0.037	0.039	0.044
st. dev. ind. rate soft	0.139	0.081	0.260	0.291	0.127	0.425
% reduction hard ent.	0.000	0.000	0.001	0.010	0.000	0.006
% reduction soft ent.	0.128	0.107	0.043	0.155	0.119	0.077
median share soft ent.	0.269	0.534	0.240	0.263	0.568	0.242
st. dev. share soft ent.	0.111	0.120	0.166	0.138	0.129	0.122
median replacement r.	0.952	0.951	0.770	1.015	1.001	0.799
st. dev. replacement r.	0.323	0.242	0.174	0.362	0.264	0.196
median share soft retired	0.113	0.346	0.227	0.111	0.361	0.220
st. dev. share soft retired	0.028	0.142	0.114	0.031	0.147	0.105

that the fund is able to better match its assets to the liabilities. This decreases the risk of underfunding. However, it also decreases the potential for indexation, because the average portfolio return is lower.

The standard deviations of most variables under consideration become lower as the equity share is reduced. Hard entitlements can be guaranteed more effectively. The fraction of time hard entitlements are reduced decreases substantially under all three schemes, to essentially never if equity is only 10% of total assets. However, soft entitlements are still reduced regularly under the three schemes, implying that the benefits are still uncertain.

The disadvantage of reducing the equity share is that the median replacement rates fall. Under the Rolling Window proposal, a reduction from 50% to 25% equity share reduces the median replacement rate of the retiring cohort from 1.04 to 1.02, while with a 10% equity share the median replacement rate falls to 0.95. Under the Fraction scenario, the median replacement rate goes down in a comparable way. For the Split contract the median replacment rate was lower to start with. However, the reduction is similar as the equity share goes down.

Summarising, we find that reducing the equity share in the fund's portfolio is a

Table 5.6: Wage indexation, different retirement ages

	$R = 38$			$R = 41$		
	RW	Fraction	Split	RW	Fraction	Split
median funding ratio	1.133	1.168	1.203	1.142	1.185	1.223
st. dev. funding ratio	0.160	0.160	0.161	0.162	0.160	0.161
median ind. rate hard	0.031	0.031	0.032	0.031	0.031	0.031
st. dev. ind. rate hard	0.042	0.025	0.046	0.049	0.021	0.045
median ind. rate soft	0.032	0.035	0.038	0.033	0.036	0.039
st. dev. ind. rate soft	0.528	0.459	0.720	0.667	0.436	0.875
% reduction hard ent.	0.049	0.012	0.035	0.064	0.008	0.032
% reduction soft emt.	0.297	0.240	0.216	0.296	0.217	0.201
median share soft ent.	0.299	0.489	0.212	0.270	0.537	0.201
st. dev. share soft ent.	0.179	0.192	0.109	0.188	0.185	0.099
median replacement r.	0.743	0.768	0.623	0.931	0.927	0.753
st. dev. replacement r.	0.326	0.258	0.163	0.479	0.324	0.235
median share soft retired	0.225	0.406	0.257	0.147	0.374	0.184
st. dev. share soft retired	0.052	0.152	0.097	0.046	0.157	0.098

good way of guaranteeing hard entitlements. However, it comes at the cost of lower or zero median indexation and thus lower median replacement rates for retiring cohorts.

Varying the retirement age

This subsection varies the retirement age. It considers retirement ages of 62 years, which corresponds to a retirement age in the model of $R = 38$, and 65 years, which corresponds to $R = 41$.

If workers retire earlier, they pay contributions for a fewer number of years and, hence they accumulate fewer entitlements. Hence, median replacement rates are lower under all contract types. The effects on other variables are not very pronounced, because the lower lifetime contributions and the lower benefits during retirement have offsetting effects on the funding ratio.

Policy parameters

This subsection varies a number of policy parameters. Under the Rolling Window contract, we vary the rolling window phase of soft entitlements to $Q = 5$ and $Q = 15$ years. Under the Fraction contract, we vary the fraction of hard entitlements to

Table 5.7: Varying the policy parameters

	RW		Fraction		Split	
	$Q = 5$	$Q = 15$	$\xi = 0.75$	$\xi = 0.25$	$\phi = 0.1$	$\phi = 0.3$
median funding ratio	1.097	1.174	1.181	1.196	1.227	1.228
st. dev. funding ratio	0.162	0.161	0.162	0.161	0.163	0.160
median ind. rate hard	0.030	0.031	0.031	0.032	0.031	0.032
st. dev. ind. rate hard	0.079	0.033	0.047	0.014	0.055	0.038
median ind. rate soft	0.028	0.036	0.036	0.038	0.039	0.040
st. dev. ind. rate soft	1.739	0.546	2.193	0.171	1.320	0.897
% reduction hard ent.	0.141	0.029	0.049	0.000	0.058	0.022
% reduction soft ent.	0.379	0.243	0.243	0.190	0.215	0.186
median share soft ent.	0.082	0.399	0.387	0.788	0.100	0.302
st. dev. share soft ent.	0.141	0.206	0.222	0.091	0.076	0.113
median replacement r.	1.056	1.024	1.028	0.994	0.786	0.848
st. dev. replacement r.	0.679	0.510	0.412	0.362	0.267	0.284
median share soft retired	0.057	0.153	0.302	0.363	0.093	0.185
st. dev. share soft retired	0.023	0.060	0.194	0.142	0.069	0.117

$\xi = 0.25$ and $\xi = 0.75$, while under the Split contract, we vary the threshold for the transformation of soft into hard contracts to $\phi = 0.1$ and $\phi = 0.3$.

For the Rolling Window model, we see that if the build up phase of soft entitlements is only 5 years, the buffer of soft entitlements becomes much smaller and is much less effective at insulating the hard entitlements to shocks: the standard deviation of the indexation rate to hard entitlements rises to 0.08, while hard entitlements are more often reduced, namely in 14% of all scenarios.

The median share of soft rights is only slightly more than 8%. The median replacement rate of the retiring cohorts is actually slightly higher (1.06 versus 1.04) than with the 10 year rolling window, because (older) workers have a larger share of hard entitlements for which the soft entitlements act as a buffer. This is a benefit for the older workers in the earlier years: the first 35 generations that retire under the 5 year rolling window are better off, while all generations retiring after that are worse off, since they absorbed most shocks in their soft entitlements when young. The standard deviation of the replacement rate rises to 0.68. In case the build up phase is lengthened to 15 years, the effects are the opposite of a shortening of the rolling window phase.

Under the Fraction proposal, if the share of hard entitlements is raised to $\xi = 0.75$

Table 5.8: Outcomes under wage indexation with 250 periods

	RW	Fraction	Split
median FR	1.106	1.193	1.000
s.d. FR	0.161	0.159	0.156
median hard indexation	0.031	0.032	0.030
s.d. hard indexation	0.065	0.019	0.151
median soft indexation	0.029	0.037	-0.104
s.d. soft indexation	1.001	0.434	6.083
% of time reduction of hard rights	0.104	0.005	0.220
% of time reduction of soft rights	0.355	0.193	0.515
median Share soft	0.170	0.592	0.029
s.d. share soft	0.195	0.188	0.119
median replacement rate	1.145	1.186	1.471
s.d. replacement rate	1.112	0.587	3.985
median share soft retired	0.114	0.423	0.146
st. dev. share soft retired	0.041	0.136	0.103

the hard entitlements will be harder to guarantee, because the buffer of soft entitlements is lower than in the base case. The standard deviation of hard entitlements goes up from 0.017 to 0.047, while the fraction of time the hard entitlements are reduced goes up from 0.003 to 0.049. Not surprisingly, when the fraction of hard entitlements is reduced, the effects go into the opposite direction. Finally, increasing the threshold for the transformation of soft into hard entitlements to $\phi = 0.3$ under the Split proposal, the result will be a larger median fraction of soft entitlements and the consequences are qualitatively the same as those of raising the share of soft entitlements under the Fraction proposal. Similarly, decreasing the threshold to $\phi = 0.1$ has qualitatively similar consequences as reducing the share of soft entitlements under the Fraction proposal.

Simulation length

Running the simulations for 250 periods instead of 50 periods, so that all transition dynamics in both the build up phase and demographics disappear produces the results in Table 5.8. We see that for the Rolling Window contract, the median funding ratio is slightly lower, but not that much. The main difference for the Rolling Window contract is that in the long run the stock of soft entitlements is not nearly large enough to provide a meaningful buffer for the hard entitlements, which essentially become soft themselves. In over 10% of the scenarios, hard entitlements have to be reduced. Also, under the Rolling Window contract there are particularly lucky generations and particularly unlucky generations: the standard deviation of the replacement rate of the retiring cohort is quite high (1.112).

For the Fraction contract, the differences between the 50 periods and the 250 periods simulation are very small, indicating that there were hardly any transitional dynamics in the first place.

For the Split contract, we see large differences between the 50 and 250 periods simulations. The median funding ratio is 100% in the 250 periods simulation, whereas it was over 120% in the 50 periods simulation. Median indexation of soft entitlements is -10.4%, because the buffer of soft entitlements gradually vanishes. The median share of soft entitlements over the entire time horizon is 2.9%. Closer inspection reveals initially this is higher (over 22% for the first 50 years), but that after 100 periods the median share of soft entitlements falls to 0. The median replacement rate is quite high, but also the standard deviation is high, implying some generations are well off, while others suffer. A generation's luck in this regard depends to a large extent on the size of the buffer of soft entitlements at the moment of entering the pension fund.

5.6 Conclusion

In this chapter we have simulated the consequences for benefits and pension funding ratios of several proposals to replace the current Dutch occupational contract. The common feature of all proposals was the replacement of the current system of only hard entitlements by a system with both hard and soft entitlements, where the latter acted as a buffer for the former. That is, negative shocks were first absorbed by the soft entitlements and only once the buffer of soft entitlements was depleted hard entitlements would be affected. Our simulations were carried out both for the case of indexation of entitlements to price inflation and indexation to wage inflation.

Under the proposals considered here, indexation is higher and more readily provided than under the current system. This implies that the currently retired generations benefit more under the new proposals than under the current contract. Out of the three new proposals, only the Fraction contract succeeds in effectively guaranteeing hard entitlements and providing relatively stable indexation and buffers over time. The results suggest that in a system with both hard and soft entitlements, the only way to prevent substantial intergenerational transfers is to have all generations accumulate both types of entitlements. Because financial shocks are absorbed through the soft entitlements, this way the costs and benefits of those shocks can be spread over all the generations.

A possible extension of the analysis in this chapter would be to include additional sources of shocks. Obvious candidates are unexpected changes in survival probabilities and fertility. Another extension would to vary the parameters of the pension contract. For example, it would be interesting to vary the recovery period. Under the current proposals, a fall of funding ratio below 100% is immediately offset through a cut in entitlements. The resulting volatility in the entitlements could be dampened by considering longer recovery periods. We would expect disposable income to become more stable, which would benefit the fund's retirees to the extent that they have difficulties in smoothing consumption.

APPENDICES

5.A Derivation of (5.14)

We assume zero-coupon bonds for simplicity. (Using coupon bonds would be conceptually equivalent, but would complicate the notation needlessly). The return on a zero-coupon bond in year t with k years to maturity at the end of the year equals the capital gain in year t:

$$1 + Ret^b_{k,t} = \frac{P_{k,t}}{P_{k+1,t-1}} \tag{5.44}$$

where $P_{k,t}$ is the price of the bond at the end of the year and $P_{k+1,t-1}$ is the price at the beginning of the year, when the time to maturity is $k+1$. For a zero coupon bond, we can write these prices as:

$$P_{k,t} = \frac{FV}{(1 + r_{k,t})^k}, \tag{5.45}$$

$$P_{k+1,t-1} = \frac{FV}{(1 + r_{k+1,t-1})^{k+1}}, \tag{5.46}$$

where FV is the face or principal value of the bond. Plugging these expressions into (5.44) yields (5.14):

$$1 + Ret^b_{k,t} = \frac{FV/(1 + r_{k,t})^k}{FV/(1 + r_{k+1,t-1})^{k+1}} = \frac{(1 + r_{k+1,t-1})^{k+1}}{(1 + r_{k,t})^k}. \tag{5.47}$$

5.B Indexation Policy Rolling Window proposal

In this appendix we provide the exact formulas for the indexation policy under the Rolling Window proposal as laid out in Section 5.3.3. Throughout this appendix we assume price-indexation, so that full indexation of hard entitlements is defined as $\omega^h_t = \pi_t$ and full indexation of soft entitlements is defined as $\omega^s_t = \pi_t + \nu^\omega$. For a pension fund instead following a policy of wage indexation, we would simply need to replace π_t by g_t in all instances.

5.B.1 Reduction of hard entitlements

We keep track of the stock of reductions and missed indexation of hard entitlements as follows:

$$M^m_{i,j,t} = \left(1 + \omega^h_t\right)\left(M^m_{i,j,t-1} - \tilde{m}^h_{i,j,t}\right) + m^m_{i,j,t}, \tag{5.48}$$

where $\tilde{m}^h_{i,j,t}$ is the amount of previously missed entitlements that is now awarded to the participants of the pension fund through a catch-up indexation and $m^m_{i,j,t}$, the addition to the stock of missed entitlements, is given by:

$$m^m_{i,j,t} = \left(\pi_t - \omega^h_t\right) M^h_{i,j,t} \quad \text{if} \quad \omega^h_t < \pi_t,$$

and zero otherwise. The amount $\tilde{m}^h_{i,j,t}$ is determined as:

$$\tilde{m}^h_{i,j,t} = \min[1, \max(0, \frac{F_t}{F^{ub}} - 1)] * M^m_{i,j,t-1}. \tag{5.49}$$

This expression says that the amount of previously missed entitlements that are restored through catch-up indexation $\tilde{m}^h_{i,j,t}$ is a fraction of previously missed entitlements. This fraction depends on the actual funding ratio F_t relative to the funding ratio at which full indexation is awarded, F^{ub}. It is capped at 1 because it is at most equal to the total stock of missed entitlements. Its minimum value is 0 because catch-up indexation is not allowed to be negative.

5.B.2 Indexation of hard entitlements

If, after catch-up indexation, $A_t > L^h_t$, the pension fund provides indexation of hard entitlements until either $A_t = L^h_t$ or $\omega^h_t = \pi_t$:

$$\omega^h_t = \min[\pi_t, \max(0, \frac{A_t - L^h_t}{L^h_t})]. \tag{5.50}$$

Note that the fact that the pension fund only considers its asset position and the amount of hard liabilities means that it can potentially reduce soft entitlements to provide indexation of hard entitlements.

5.B.3 Indexation of soft entitlements

If, after indexation of hard entitlements, $A_t > L_t$, the pension fund provides indexation of soft entitlements until either $A_t = L_t$ or $\omega_t^s = \pi_t + \nu^\omega$:

$$\omega_t^s = \min[\pi_t + \nu^\omega, \max(0, \frac{A_t - L_t}{L_t^s})]. \tag{5.51}$$

In the case that after awarding indexation to the soft entitlements the funding ratio is above its upper bound, $F_t > F^{ub}$, then the pension fund awards extra indexation to the soft entitlements until $F_t = F^{ub}$. In that case the expression for the total indexation of the soft entitlements becomes:

$$\omega_t^s = \frac{L^{ub} - L_t}{L_t^s} \tag{5.52}$$

where L_t and L_t^s denote the values of total entitlements and total soft entitlements, respectively, after indexation to hard entitlements has been awarded, but before any indexation to soft entitlements has been awarded. Further, L^{ub} is the value of liabilities such that $F_t = F^{ub}$. Hence, $L^{ub} = A_t/F^{ub}$.

5.B.4 Figures for base scenario

Figure 5.1: Rolling Window wage indexation

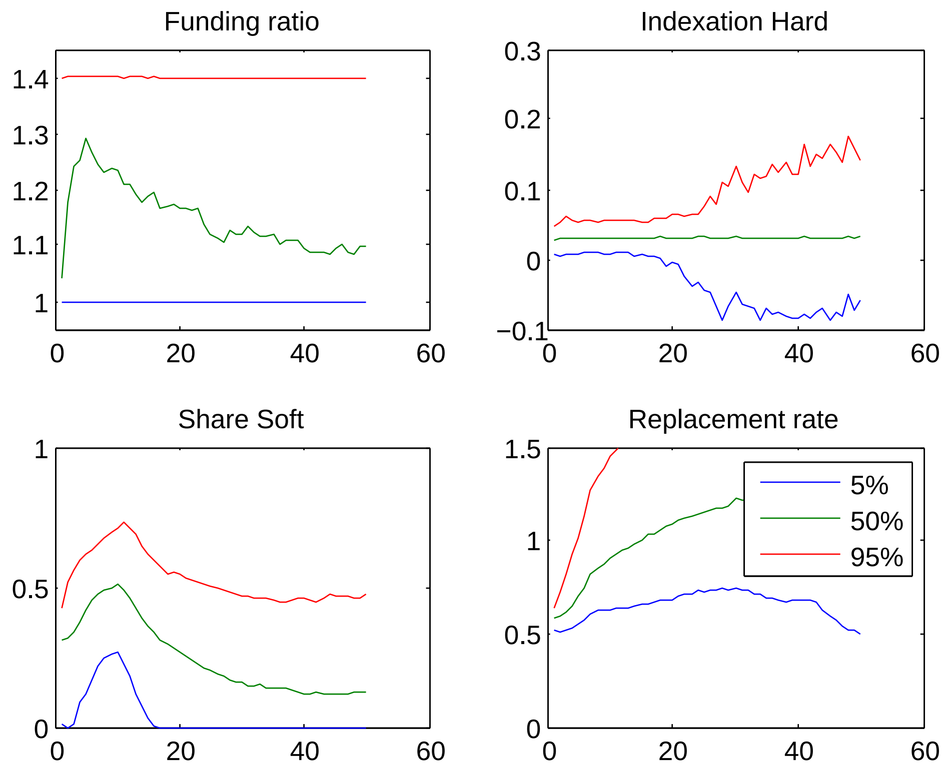


Figure 5.2: Fraction wage indexation

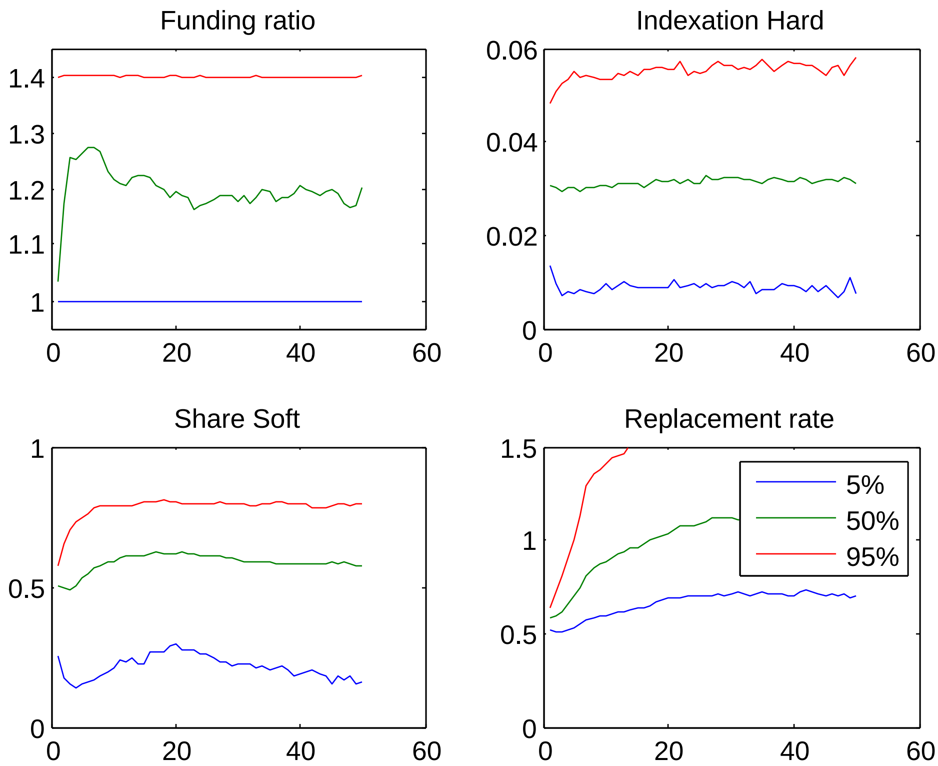
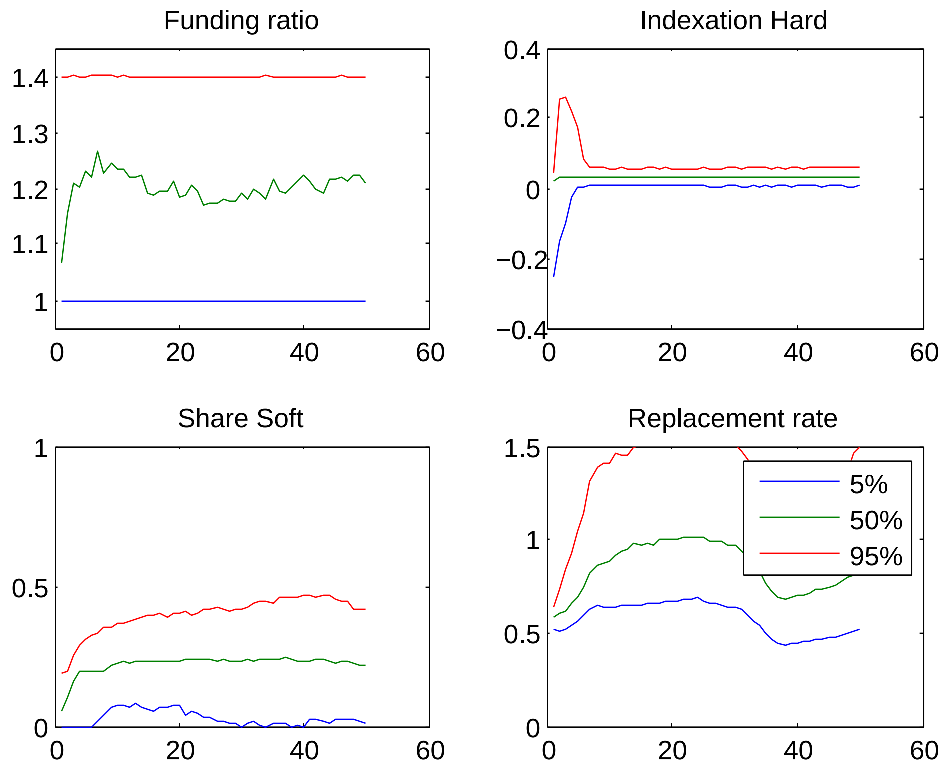


Figure 5.3: Split wage indexation



5.B.5 Figures for replacement rates base scenario

Figure 5.4: Replacement Rates Rolling Window version

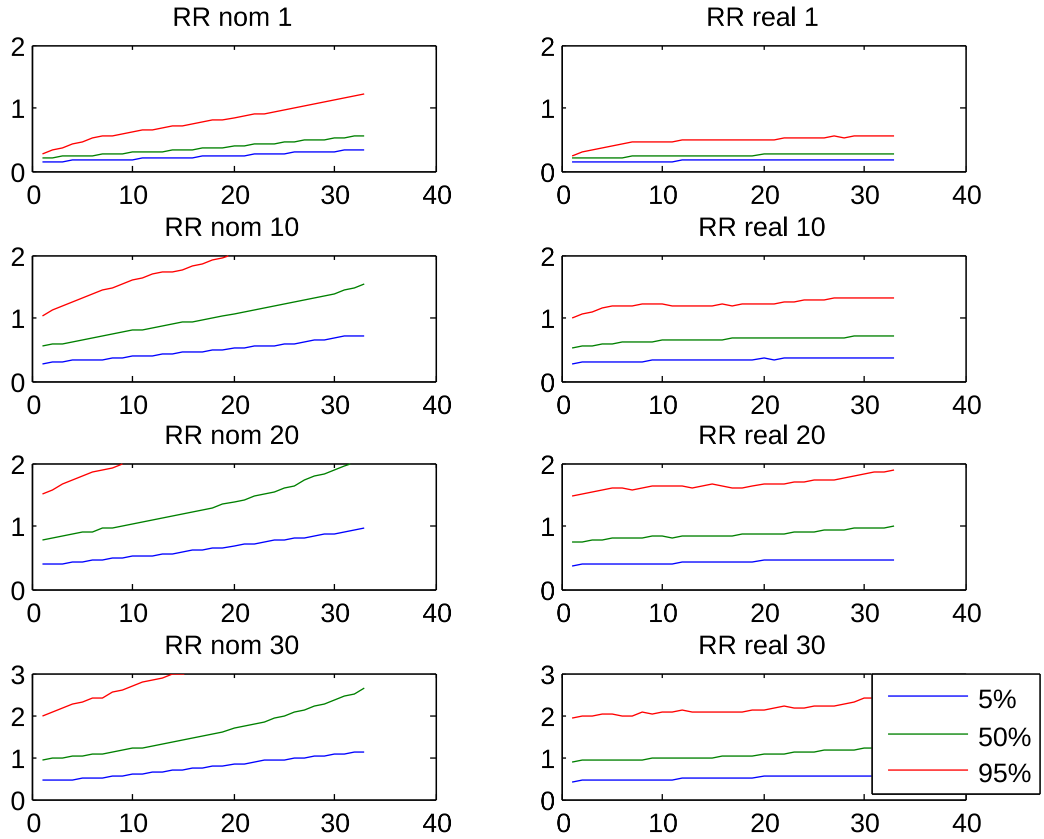


Figure 5.5: Replacement Rates Fraction version

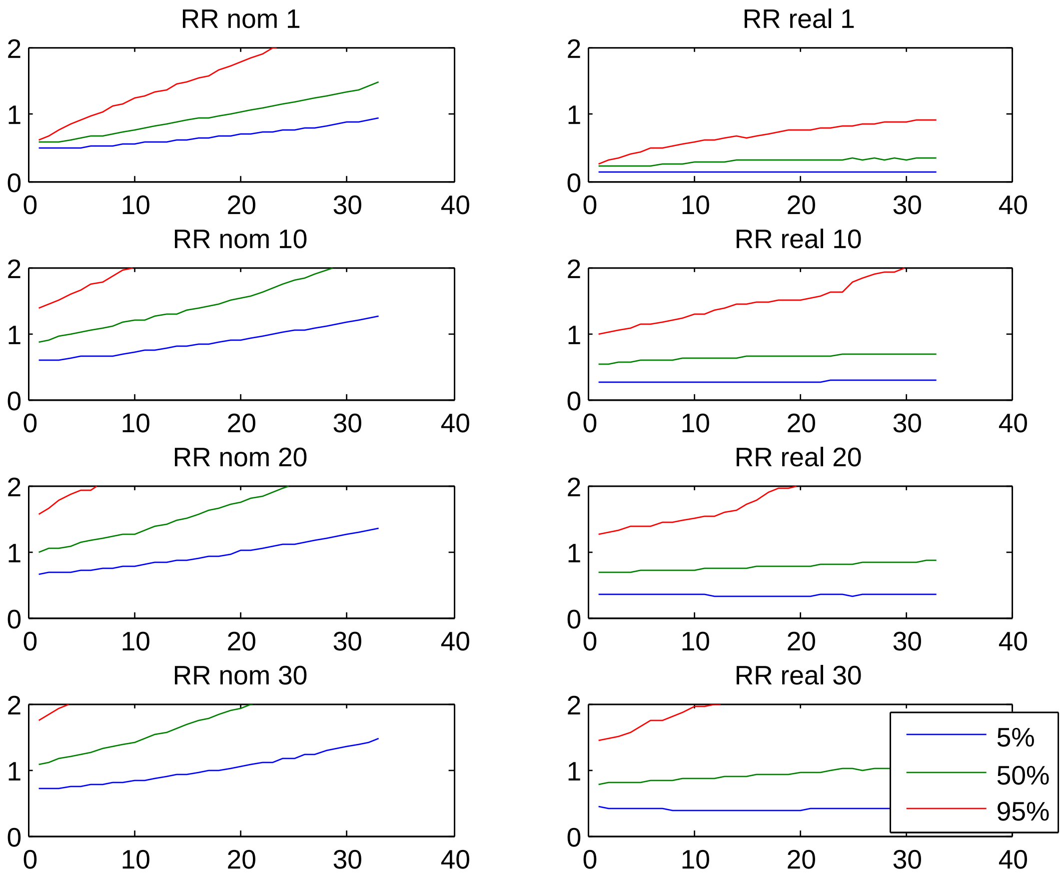
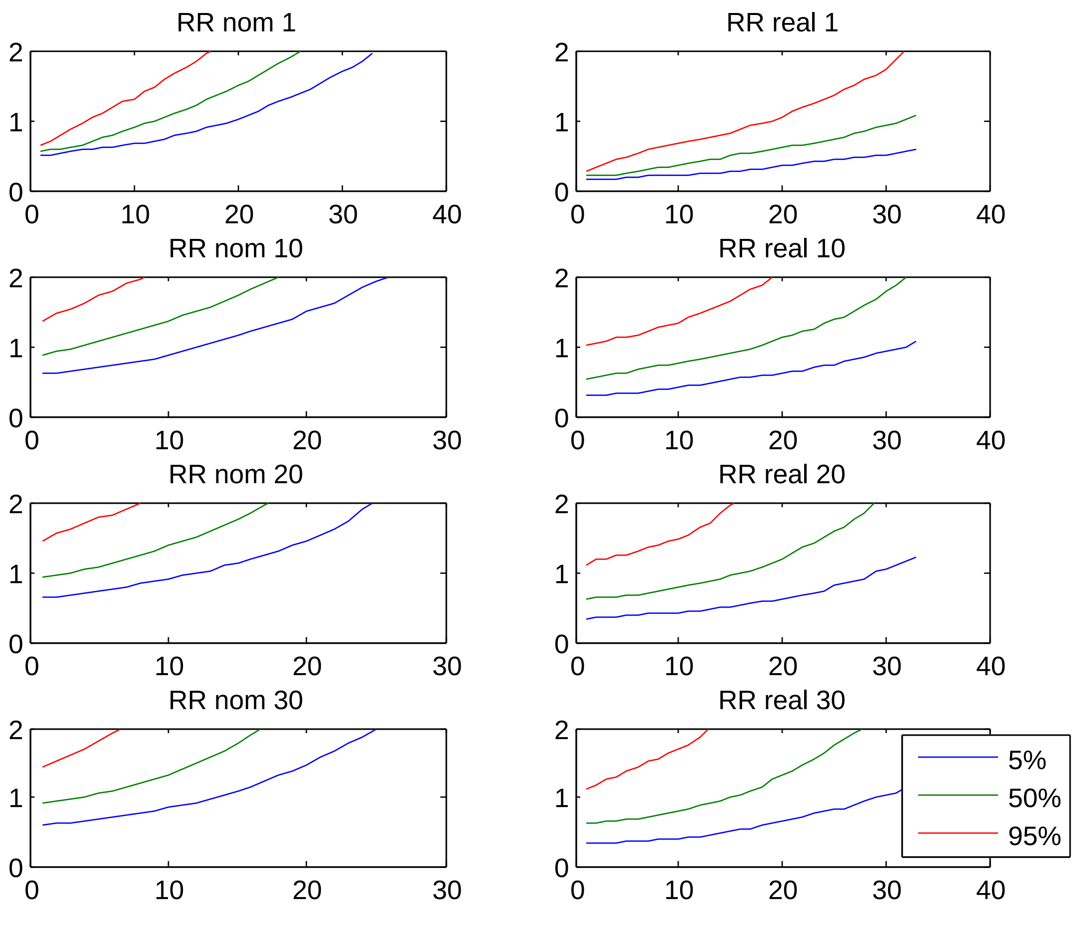


Figure 5.6: Replacement Rates Split version



Samenvatting

In dit proefschrift analyseer ik het ontwerp van pensioen systemen, waarbij de aandacht met name uitgaat naar het intergenerationele risicodelings-aspect van pensioenen. In hoofdstukken 2 en 3 wordt de optimale inrichting van pensioensystemen die op twee pijlers (omslagfinanciering en kapitaaldekking) gebaseerd zijn behandeld, waarbij rekening wordt gehouden met meerderen schokken en verstoringen, waarbij deelname in het pensioensysteem verplicht is. In hoofdstuk 4 wordt de beslissing om al dan niet deel te nemen in een collectieve regeling onderzocht, en de impact daarvan op het ontwerp van collectieve pensioenregelingen indien deelname niet verplicht is. Hoofdstuk 5 geeft een gedetailleerde analyse van enkele opties die de revue gepasseerd zijn bij de recente discussie rondom de vernieuwing van aanvullende pensioencontracten in Nederland.

Hoofdstuk 2 gaat in op optimaal ontwerp van het pensioensysteem, waarbij rekening gehouden wordt met de verstorende effecten hiervan op de arbeidsaanbodsbeslissingen van individuen. De centrale vraag hierbij is: 'Hoe dient een pensioensysteem met twee pijlers - een omslaggefinancierde eerste pijler en een kapitaalgedekte tweede pijler - eruit te zien als er rekening wordt gehouden met endogene arbeidsaanbod beslissingen'? In dit hoofdstuk zijn er twee bronnen van risico, productiviteitsrisico en risico op financiële markten.
In principe zou de ontwerper van het pensioensysteem dit zodanig in willen richten dat alle soorten risico's optimaal door de verschillende generaties gedeeld worden. Het pensioensysteem kan echter verstorend werken op de arbeidsmarkt. Het precieze mechanisme is als volgt: als de premiebetalingen een percentage van het loon bedragen, gaat het nettoloon van het individu omlaag. Hierdoor kan de arbeidsaanbodbeslissing veranderen. Indien alle werknemers deze zelfde prikkel hebben vanuit het pensioensysteem,

kan het totale arbeidsaanbod op een suboptimaal niveau uitkomen. Dit kan lonen, rendement op fysiek kapitaal en de totale nationale productie verstoren en het welvaartsniveau verlagen. In dit hoofdstuk laat ik zien dat als een dergelijke verstoring bestaat, dat het optimale ontwerp van het pensioensysteem dan verandert zodanig dat de sociaal optimale allocatie van productiefactoren toch bereikt kan worden. De oplossing die in dit hoofdstuk gevonden wordt is het koppelen van de premies in de tweede pijler aan de geaggregeerde loonsom in plaats van aan individuele lonen. Hierdoor verandert de premie voor de werkende generaties de facto in een lump-sum premie, waardoor de verstoring van de arbeidsmarkt niet plaatsvindt, terwijl de risico's die actieven via hun loon lopen nog steeds gedeeld kunnen worden met de gepensioneerden.

Hoofdstuk 3 gaat eveneens over het optimale ontwerp van pensioen systemen, maar vanuit een geheel andere invalshoek. In plaats van naar gedragsverstoringen wordt in dat hoofdstuk naar onzekerheid rondom demografische ontwikkelingen gekeken. In aanvulling op de twee schokken die in hoofdstuk 2 een rol speelden wordt in dit hoofstuk onzekerheid rondom de geboorte- en sterftecijfers meegenomen. Hierdoor zijn er vier bronnen van onzekerheid in hoofdstuk 3. Demografische onzekerheid beïnvloedt vrijwel alle macroeconomische relaties; arbeidsaanbod, lonen, rendement op kapitaal, nationaal inkomen, private en pensioenbesparingen, erfenissen en de relatieve grootte van de overdrachten via de eerste pijler zijn er allemaal van afhankelijk. Hoewel het model zeer gestileerd is zorgt de aanwezigheid van demografische risico's ervoor dat optimale risicodeling met constante pensioensysteem-parameters onmogelijk is. Om die reden wordt een gedetailleerde numerieke analyse gedaan om vast te stellen welke mate van risicodeling bereikt kan worden in de aanwezigheid van demografische risico's en constante systeemparameters. Bovendien is onderzocht hoe ver deze oplossing verwijderd ligt van de sociaal optimale oplossing die een 'sociale planner' zou kiezen. Hoewel met het pensioensysteem de oplossing van de sociale planner niet gerepliceerd kan worden, blijkt uit hoofdstuk 3 dat in een systeem met een tweede pijler gebaseerd op een uitkeringsovereenkomst het welvaartsverlies ten opzichte van deze optimale oplossing slechts zeer klein is. Of dit resultaat ook in een realistischer set-up geldt is een vraag voor verder onderzoek.

Hoofdstuk 4 gaat in op een zeer belangrijk aspect van collectieve kapitaalgedekte pensioenregelingen: verplichte deelname. In hoofdstukken 2 en 3 werd aangenomen dat alle individuen verplicht deel moeten nemen aan de pensioenregelingen. In hoofdstuk 4 wordt de vraag onderzocht of, en zo ja onder welke omstandigheden, vrijwillige deelname in collectieve kapitaalgedekte pensioenregelingen mogelijk is. Het is een bekend resultaat dat vanuit een ex-ante perspectief collectieve pensioenregelingen grote welvaartswinsten voor de deelnemers bieden door de risicodeling die binnen die regelingen plaatsvindt.

Nieuwe toetreders tot een regeling kunnen echter geconfronteerd worden met een ongunstige financiële positie van de regeling waar zij tot toetreden vanwege ongunstige schokken in het verleden. In dat geval kan van hen een aanvullende bijdrage gevraagd worden om bij te dragen aan het herstel van de financiële positie, zonder dat hier direct aanspraken tegenover staan. Hierdoor rijst de vraag of voor deze nieuwe toetreders het niet gunstiger zou zijn om, indien zij de keuze hadden, niet toe te treden tot de collectieve regeling, maar voor zichzelf pensioen op te bouwen. De vervolgvraag is dan of het toch mogelijk is om een collectieve regeling zodanig te ontwerpen dat potentiële nieuwe toetreders nooit de prikkel hebben om uit het systeem te stappen, terwijl er toch enige risicodeling in het systeem blijft.

Het model in hoofdstuk 4 is een oneindige-horizon model met twee overlappende generaties, waarbij de jonge generatie kan kiezen om toe te treden tot een collectieve kapitaalgedekte pensioenregeling, of om zelf te sparen voor pensioen. Zodra een jonge generatie besluit niet toe te treden tot het collectieve systeem houdt dit op te bestaan. De uitdaging voor de ontwerper van de collectieve regeling is om dit zodanig te doen dat jonge generaties toe willen blijven treden, wat inhoudt dat het verwachte nut van instappen minimaal gelijk moet zijn aan het verwachte nut van niet toetreden. Of het mogelijk is om een dergelijke regeling te ontwerpen hangt af van zowel de volatiliteit van de schokken waartegen het pensioensysteem bescherming biedt (in het geval van hoofdstuk 4 risico op de financiële markten), als de mate van risico-aversie van de jonge generatie. Hoofdstuk 4 laat zien dat collectieve regelingen ophouden te bestaan indien volatiliteit en risico-aversie relatief laag zijn, terwijl voor wat hogere volatiliteit en risico-aversie het wel mogelijk is om een collectief systeem gaande te houden, maar dat de mate van risicodeling in deze regelingen minder is dan de sociaal optimale mate van risicodeling. In deze omstandigheden is het alleen mogelijk om de optimale hoeveelheid risicodeling te bereiken door deelname aan collectieve pensioenregelingen verplicht te stellen.

Hoofdstuk 5 tenslotte is relevant voor het huidige debat rondom de hervormingen in de aanvullende pensioencontracten in Nederland. Bij deze hervorming is een aantal mogelijke nieuwe varianten voor het inrichten van de aanvullende pensioencontracten overwogen. Een variant, die in dit hoofdstuk uitgebreid onderzocht wordt, is het 'combi-contract'. In een combi-contract worden de aanspraken van deelnemers gesplitst in een 'hard' en een 'zacht' deel, waarbij de zachte aanspraken als een buffer dienen om de op papier volledig gegarandeerde harde aanspraken te beschermen. In hoofdstuk 5 worden 3 manieren om een dergelijk combi-contract te implementeren met elkaar en met het huidige contract vergeleken. De analyse biedt inzicht in hoe in ieder van de varianten schokken verdeeld worden over de verschillende generaties deelnemers.
In de eerste variant zijn nieuw opgebouwde rechten aanvankelijk zacht, maar worden deze na een bepaald aantal jaar omgezet in harde rechten. In de tweede variant is een bepaald percentage van de nieuwe opbouw hard, terwijl het andere deel van de nieuwe opbouw zacht is. In de derde variant zijn nieuw opgebouwde rechten zacht, maar worden deze bij een voldoende hoge dekkingsgraad omgezet in harde rechten. Met behulp van een Asset-Liability Management (ALM) model van een pensioenfonds simuleren we de ontwikkeling van het pensioenfonds, toegekende indexatie, en de ontwikkeling van harde en zachte rechten in een groot aantal scenario's.
De resultaten laten zien dat voor alle drie de vormen van het combi-contract, indexatie sneller verleend wordt dan in het huidige contract. Hierdoor zijn de huidige generaties gepensioeneerden beter af, maar is de kans op lagere pensioenen voor de huidige en toekomstige generaties werkenden groter. Van de drie verschillende combicontracten is alleen de tweede variant (waarbij harde en zachte rechten in vaste verhouding worden opgebouwd) in staat om te garanderen dat harde rechten ook echt hard zijn en vrijwel nooit afgestempeld hoeven te worden. Bovendien zijn het in beide andere voorstellen

met name jongere generaties die zachte rechten hebben, en dus het meeste risico lopen. Hierdoor kunnen grote intergenerationele overdrachten plaatsvinden. De resultaten suggereren dat effectieve risicodeling tussen alle deelnemers in een regeling vereist dat ofwel alle aanspraken van dezelfde soort zijn (zoals in het huidige contract) of dat alle deelnemers beide soorten aanspraken in een vaste verhouding opbouwen, zoals in de tweede variant van het combi-contract.

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