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**Thesis MSc. Econometrics, Operations Research and Actuarial Studies:**

# An Empirical Study of Mean Reversion in International Stock Market Indexes and the Implications for the FTK Continuity Analysis

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Amsterdam, January 24th 2010

# Abstract

This thesis considers three different issues regarding long-term mean reversion in stock prices. First, the literature on long-term mean reversion is reviewed by introducing a dichotomy in which a distinction is made between absolute mean reversion and relative mean reversion. Second, the empirical evidence of relative mean reversion in international stock indexes, found by Balvers et al. (2000), is examined by (i) enlarging the number of years to the interval 1900 to 2008, and (ii) introducing a bootstrap method to correct for the small sample bias in the test statistic. This thesis finds mean reversion, however at a much longer term than suggested by Balvers et al. (2000). Moreover, significant fluctuations of the mean reversion process across time undermines the usefulness of long-term mean reversion. Third, the effect of mean reversion on the FTK continuity analysis is examined. Also an advice regarding the regulation on mean reversion in the continuity analysis is provided.

# Contents

# 1 Introduction

At the beginning of March 2009, many stock markets across the world experienced their lowest value in years. In less than two years the U.S. equity market indexes lost more than 50%, and the Dutch stock market index decreased over 55% in value in this relatively short time period (Yahoo Finance, 2009). At this time, there was an ongoing discussion about the movements of the stock prices in the future. Some argued that if stocks are down over 50% then certainly an increase must follow. And the increase followed. At the end of 2009, stock markets were up more than 30% after March 2009 (Yahoo Finance, 2009).

Looking back, it is tempting to assume that the increase that followed after the large drop was to be expected. An important question in the situation described above is: do stock prices exhibit mean reverting behavior? Over the last two decades a large amount of literature is dedicated to mean reversion of stock prices over long horizons. Mean reversion is the event of stock prices moving around their mean over time. Fama and French (1988b) and Poterba and Summers (1988) provide the literature with the first empirical evidence of mean reversion. The authors of both papers find a mean reversion period of 3 to 5 years, and Fama and French (1988b) claim that 25-40% of the variation in 3-5 year stock returns can be explained by the negative serial correlation. However, due to several statistical and empirical limitations in both studies, strong evidence of mean reversion is still absent. Other studies examine the reverting behavior of stock prices relative to an specified mean. This specification could be a direct relation between stock prices and fundamental indicators, like dividends and earnings. Alternatively, a relation of stock prices with a benchmark portfolio could be assumed to estimate the mean process. Balvers et al. (2000) find significant reversion of international stock market indexes to their intrinsic mean. The authors claim a half-life of the mean reversion process of approximately 3.5 years. This thesis comprehensively summarizes the long term mean reversion evidence and criticism in the literature in order to examine the general view on the existence and usefulness of mean reversion.

A major problem in finding mean reversion over long horizons is the small amount of available data. Fama and French (1988b) and Poterba and Summers (1988) make use of yearly overlapping returns to increase the number of observations and analyze the time period from 1926 to 1985. Balvers et al. (2000) test for mean reversion using a panel data model applied to stock indexes of 18 countries from 1970 to 1996. The first two papers are criticized on statistical and data grounds by many studies, however the mean reversion evidence of Balvers et al. (2000) still holds. This thesis addresses two issues in the work of Balvers et al. (2000) on mean reversion. First of all, the time period is extended from 27 to 109 years; the period from 1900 to 2008. The three decades on which Balvers et al. (2000) base their results might be not representative, because it contains a relatively high growth and no severe recessions (Yahoo Finance, 2009). The longer time period will therefore increase the accuracy of the mean reversion results. Secondly, Balvers et al. (2000) use a Monte Carlo simulation based on normally distributed residuals. In finite samples, the normality assumption could lead to substantial biases in the estimators and their standard errors (Freedman and Peters, 1984). To avoid this small sample bias, a bootstrap method is applied to improve on the value of the estimators and their standard errors.

Early research always examines mean reversion using the complete available set of data, and thereby ignores the fluctuation within the mean reversion process across time. The large amount of observations between 1900 and 2008 in 17 countries available in this thesis allows for testing the stability of the mean reversion process over the years. It is expected that the speed at which stock prices revert to their mean depends highly on the economic and political environment. Therefore, the speed of reversion could fluctuate across time.

The larger time period reveals a substantially longer half-life of the mean reverting process from 3.5 years in the period 1970-1996 to an average of 13.8 years in the full period between 1900 and 2008. Further, evidence of a significant difference between time periods is found, which results in half-lifes varying from a minimum of 2.1 years to a maximum of 23.8 years. Furthermore, in a substantial number of time intervals no significant mean reversion is found. The large fluctuations across time indicates a changing speed at which the stock prices revert to their mean. Moreover, the period used by Balvers et al. (2000) exhibits extreme results in comparison to other time intervals, which indicates that the choice of data contributes substantially to the evidence of mean reversion found by Balvers et al. (2000).

The results in this thesis have their implications to the supervision of pension funds. Dutch pension funds must execute a continuity analysis every three years in order to assess their future financial position. At the end of 2008, the Dutch Central Bank obliged pension funds to construct an extra analysis due to the financial crisis. The last goal of this thesis is to examine the continuity analysis and assess the opportunities for pension funds to insert mean reversion into their analysis. Furthermore, a method to recognize the use of mean reversion is explained. Finally, an advice on incorporating mean reversion into the continuity analysis is presented, using the literature review and results from the empirical analysis.

The thesis is organized as follows. Section 2 reviews the literature on long-term mean reversion and divides the enormous amount of papers in two distinct mean reversion studies. Subsequently, some practical implications of both groups are considered. The following section derives the panel data model that is used by Balvers et al. (2000) and discusses the way to apply the bootstrap. Section 4 first describes the large data set of international stock index returns from 1900 to 2008. Furthermore, the individual countries are considered to examine the differences across countries. Finally, Section 4 applies the panel data model to find the speed of reversion, and the unbiased estimator of the half-life of the mean reversion process in the full period. The fluctuation between time intervals containing 27 yearly returns are considered in Section 5. Also, the implications of a time varying mean reversion process are presented in this section. Section 6 describes the continuity analysis that Dutch pension funds are obliged to perform. Additionally, this section examines the impact of mean reversion on the continuity analysis. And finally, recommendations to regulation are being made, based on the literature review and the results from this thesis. Section 7 concludes the thesis with a short summary of the main results and some ideas for future research.

## 2 Long-term Mean Reversion: 20 Years of History

The literature of long-term mean reversion has a comprehensive history. During the last two decades, studies have shown evidence for and against the event of stock prices reverting to their mean. Several theories are presented to support the mean reverting behavior of stock prices. In essence these economic explanations are strongly related to the discussion of efficient markets. The hypothesis that markets are efficient states that all available information is reflected in the value of a stock (Fama, 1991). In case stock markets would follow a mean reverting process this might stroke with the understanding that markets are efficient. According to Poterba and Summers (1988) mean reversion is caused by irrational behavior of noise traders resulting in stock prices that take large swings from their fundamental value. This irrational pricing behavior might be due to fads (McQueen, 1992; Summers, 1986), overreaction to financial news (De Bondt and Thaler, 1985, 1987) or investor's opportunism (Poterba and Summers, 1988). In addition, Shiller (1981) finds that stock prices fluctuate much more than rationally expected based on real economic events. Finally, Summers (1986) concludes that there are too many implications for markets to be efficient. Therefore, he claims that "the inability of empirical tests to reject the hypothesis of market efficiency does not mean that they provide evidence in favor of its acceptance".

The mean reverting behavior of stock prices may alternatively be explained in accordance with the efficient market hypothesis. Suppose all available information is incorporated into the price of a stock. Then, the price is determined by the expected returns per share of stock, and mean reversion is observed when expected returns are mean reverting (Summers, 1986). Conrad and Kaul (1988) find that the time-varying process of the expected return is stationary, i.e. reverts to its mean over time. Therefore, mean reverting stock prices are expected under the efficient market hypothesis (Fama and French, 1988b). The fluctuation in expected returns is explained to be caused by uncertainty about the survival of a certain economy, due to e.g. a World War, or depression (Kim et al., 1991). Also, rational speculative bubbles might play a role in the fluctuating expected stock value (McQueen, 1992). Furthermore, the change in expected returns might be due to uncertain company prospects.

In a general form, Summers (1986) and Fama and French (1988b) define a mean reverting price process $p(t)$ to be composed out of a permanent component and a temporary component:

$$p(t) = p^*(t) + z(t), \tag{1}$$

where $z(t) = \phi z(t-1) + \eta(t)$ is a stationary AR(1) process. Error term $\eta(t)$ is assumed to be a zero-mean stationary process. In this specification, the permanent component $p^*(t)$ models the intrinsic value of the stock. Any shock to the permanent component at time $t$ is incorporated completely into the future stock price. Furthermore, the temporary component $z(t)$ models the mean reverting part of the price. A price shock through the temporary component will slowly decay towards zero. According to Summers (1986), the slowly decaying part of the price requires a large period of time to revert towards zero. Therefore, the reversion of stock prices can only be observed at long horizons. Moreover, the explicit value of either components is unobservable which leads to implications to test for mean reversion. These two empirical issues are discussed extensively in studies of long-term mean reversion in the last twenty years.

Two approaches are suggested in the literature to test the stationarity of process $z(t)$. One approach is to derive an alternative statistic for $\phi$, which can be used to test against the null hypothesis of a random walk, and can be referred to as absolute mean reversion. Another possibility is to estimate the fundamental value process $p^*(t)$ and derive the estimator for $\phi$ directly from equation (1). This method is referred to as relative mean reversion, since stock prices revert relative to a specified mean value. Both approaches will be discussed below. In either way, it is important to test the null hypothesis that $\phi$ equals one. Namely, when $\phi$ is equal to one, the stationary component follows a random walk and the stock price does not revert to its mean. The alternative hypothesis that is tested against is that $\phi$ is smaller than one, since this would imply mean reverting behavior of stock prices.

### 2.1 Unspecified Mean: The Random Walk

Fama and French (1988b) were the first to find significant results in testing for mean reversion at long horizons. They examine absolute mean reversion, and specify the underlying intrinsic value process $p^*(t)$ as a random walk, defined as $q(t)$.

$$q(t) \quad = \quad q(t-1) + \delta(t).$$

The authors derive a regression model to test whether stock returns are negatively autocorrelated. In the model of equation (1), a value $\phi$ significantly smaller than one implies negative autocorrelation in the stock returns. Since the value of $\phi$ is expected to be close to one, this negative autocorrelation is more likely observable at long horizons. For their analysis Fama and French (1988b) examine several time horizons between one and ten years. The empirical study finds significant mean reversion, which explains 25-40% of the variation in the 3-5 year stock returns.

Poterba and Summers (1988) show in a second paper on long-term mean reversion how to use a specific property of a random walk process to reject no mean reversion. Mean reversion of stock prices implies that the variance of the returns does not grow proportional to time. In case of a random walk this proportionality does holds. Therefore, Poterba and Summers (1988) introduce the variance-ratio test of Cochrane (1988) to find mean reversion. The variance-ratio measures the proportion of variance in the multiple year horizon divided by the number of years, compared to the one year variance divided by one year. In case the ratio equals one, the random walk hypothesis cannot be rejected. Poterba and Summers (1988) find mean reversion over the long horizon. Additionally they conclude the same for several developed countries. The lack of significance in their results is devoted to the absence of more powerful tests to reject the null hypothesis.

Both papers base their results on long horizon returns, with return periods from one up to ten years. To accurately measure the autocorrelation in the returns, the observations require to be independent. However, the sample of independent observations reduces dramatically as the return horizon increases. To increase the power to reject the null hypothesis, both abovementioned papers use monthly overlapping data in order to increase their sample size. The issue of dependency, which is inherent to the use of overlapping observations, is solved applying the method of Hansen and Hodrick (1980). Their method takes into account the moving average structure of the standard errors of estimation, assuming asymptotic normally distributed stock returns. Due to the small sample property of the long-run stock returns, the asymptotics within the model leaves a substantial small sample bias. Richardson and Smith (1991) criticize the adjustment of the standard errors and provide an analytic way to reduce the overlapping sample bias. The authors compare the several adjustments and conclude that their method is superior in adjusting the standard errors. Applying the alternative adjustments to the empirical work of Fama and French (1988b) and Poterba and Summers (1988) results in insignificant mean reversion behavior of stock markets. Moreover, Richardson and Stock (1990) argue that increasing testing power is obtained using a larger overlapping interval at longer horizons. The more powerful statistical test doesnot result in a rejection of the random walk hypothesis. Kim et al. (1991) weaken the evidence of mean reversion by analyzing the bias of assuming asymptotic normally distributed returns. They introduce a simulation technique in order to find more accurate standard errors of the estimated test statistics. Jegadeesh (1991) raises another issue using monthly overlapping stock returns. The seasonal effects of stock price movements are ignored in the initial empirical work of Fama and French (1988b). The model of Jegadeesh (1991) regresses the one-month return on multiperiod returns. The regression coefficients resulting from his research are significant in January only. For all other months he finds that mean reversion does not occur. Therefore, Jegadeesh (1991) concludes that the mean reversion in stock prices is completely concentrated in January.

Ignoring the issues of using monthly overlapping stock returns, the results of Fama and French (1988b) are still under discussion. Richardson (1993) shows that the values of the autocorrelations are strongly correlated across different horizons. The correlation is strong enough to conclude that these results are likely to occur under the null hypothesis of the random walk. Boudoukh et al. (2008) support this drain of thought and add that the parameters are almost perfectly correlated. McQueen (1992) adds to these findings the problem of heteroskedasticity in the observation period. The high volatile years tend to have larger impact on the results. McQueen (1992) finds that these periods posses stronger mean reverting tendencies and that the general evidence of mean reversion is therefore overstated. Also Kim and Nelson (1998), and Kim et al. (1998) criticize the mean reversion evidence of, respectively, Fama and French (1988b) and Poterba and Summers (1988) on these grounds.

The issue of heteroskedasticity is directly linked to another point of criticism. Periods of high volatility might not be representative for current stock price behavior. Poterba and Summers (1988) note that the Great Depression has substantial influence on the mean reversion results. Excluding this period weakens the evidence of mean reversion. Kim et al. (1991) divide the total period in a before and after World War II period and conclude that mean reversion is a pre-war phenomenon only. Furthermore, the post-war period reveals mean aversion[1], indicating a fundamental change in the stock return process.

Fama and French (1988b) mention in their paper the difference in mean reversion evidence across firm size. Small firms have a stronger tendency to revert to the mean value than larger firms. Using a Bayesian approach to temporary stock price components, Eraker (2008) find that small firms display three times as much mean reversion than large firms. A final

[1]Mean aversion is the event of stock prices moving away from their mean over time.

point of criticism is the use of US stock market data only. Although Poterba and Summers (1988) indicate comparable evidence from other country's stock markets, Jorion (2003) find a contrary result. To test mean reversion the author applies the variance-ratio test to 30 countries. Except for the 2 and 3 year returns for Germany, all stock price processes differ insignificantly from a random walk. Furthermore, most countries show a variance-ratio larger than one, indicating mean aversion over the long horizon. The global and non-US index both reveal values of the variance ratio larger than one.

### 2.2 Specified Mean: Fundamental Value Process

Papers described above examine the mean reversion of stock prices without an assumption about the mean process $p^*(t)$. Absolute mean reversion is equivalent to negative autocorrelation in the stock returns. Although early papers find significant absolute mean reversion, the general thought on the subject is that convincing evidence for mean reverting stock prices remains absent until so far. A lot of researchers attribute the lack of evidence to the small sample, in combination with the low power of mean reversion tests. A substantial decrease in variation in the error term would result from replacing the random walk component by a predetermined proxy for intrinsic value[2]. A typical formulation of such a process is

$$p_t \quad = \quad p_{t-1} + a + \lambda(p_t^* - p_{t-1}) + \epsilon_t, \tag{2}$$

where $p_t^*$ indicates the intrinsic value process of one share of stock, $a$ is a positive constant, and $\epsilon_t$ is a zero-mean stationary shock term[3]. Up to a constant, the price at time $t$ equals the price at time $t-1$, adjusted for the deviation in fundamental price at time $t$ and stock price at time $t-1$. Notice that the mean process $p_t^*$ is explicit in this model and that mean reversion is implies the reversion towards the specific mean. This definition differs from the one in the previous subsection, where mean reversion was inherent to negative autocorrelation of the returns[4].

The evidence of mean reversion depends highly on the specification of the mean. Inevitably, the choice of $p_t^*$ will result in a specification error in the results, because the estimator will almost surely mismeasure intrinsic value. However, the choice of a good proxy for the stock

[2]See Footnote 3: $\epsilon_t := (1-\phi)\delta_t + \eta_t$, for $0 < \phi < 1$.

[3]Notice that the representation of equation (2) is directly derived from the initial two component model of equation (1), defining $\lambda := (1-\phi)$ and $\epsilon_t := (1-\lambda)\delta_t + \eta_t$ and substituting the specific incremental value process $p_t^*$ for $p_{t-1}^* + \delta_t$, as follows:

$$\begin{aligned}
p_t &= p_t^* + z_t \\
&= p_{t-1}^* + \delta_t + a + \phi z_{t-1} + \eta_t \\
&= p_{t-1}^* + \delta_t + a + (1-\lambda)\cdot(p_{t-1} - p_{t-1}^*) + \eta_t \\
&= p_t^* + a + p_{t-1} - (p_t^* - \delta_t) - \lambda(p_{t-1} - (p_t^* - \delta_t)) + \eta_t \\
&= a + p_{t-1} + \lambda(p_t^* - p_{t-1}) + (1-\lambda)\delta_t + \eta_t \\
&= p_{t-1} + a + \lambda(p_t^* - p_{t-1}) + \epsilon_t.
\end{aligned}$$

[4]Please notice that the autocorrelation of the stock returns need not to be negative in the model of equation (2), since it depends on the characteristics of the explicit mean process.

price, selected using reliable arguments, increases the power of the tests and thus assures more accurate results. At first, consider the fundamentals of stock prices. According to the Gorden growth model, the value of a stock equals the discounted future cash flows generated by the stock (Gorden, 1959). In practice, these cash flow are the dividend that will be payed out to the owners. As an alternative to estimating future dividends, earnings could be used as a proxy of future cash flows towards investors. To determine parameter $\lambda$ in equation (2), the research of fundamental values is often performed using valuation ratios, like dividend yield or price-earnings ratios. Campbell and Shiller (2001) examine the mean reverting behavior the dividend yield and price-earnings ratio over time. Theoretically, these ratios are expected to be mean reverting, since fundamentals, like dividends and earnings, are determinants for the stock prices. In case stock prices are high in comparison to the company fundamentals, like dividend and earnings, it is expected that an adjustment to either stock price, or fundamental will follow. Campbell and Shiller (2001) find that stock prices rather than company fundamentals contribute most to adjusting the ratios in order to bring them back to an equilibrium level. Coakley and Fuertes (2006) consider the mean reverting behavior of valuation ratios and attribute this to the difference in investor sentiment. The authors conclude that in the long-run financial ratios revert to their mean. In earlier research, Fama and French (1988a) link the dividend yield to the expected returns of a stock. They find that expected returns have a mean reverting tendency.

A second specification of fundamental value is based on asset pricing models. Ho and Sears (2004) link mean reverting behavior of stocks to the Fama-French three factor model and conclude that the models cannot capture the mean reverting behavior of stock prices. Similar results follow from Gangopadhyay and Reinganum (1996), who extract the Capital Asset Pricing Model (CAPM) from the stock price process and analyze the mean reverting behavior of the residuals. However, Gangopadhyay and Reinganum (1996) argue that the mean reversion can be explained by the CAPM when the market risk premium is allowed to vary over time. Note that this fluctuation is in accordance with the theoretical explanation of mean reversion in efficient markets; the expected returns fluctuate in a mean reverting manner (Summers, 1986)

Gropp (2004) argues that valuation ratios are inherently flawed, because information in the company fundamentals cannot be compared to the stock prices due to the delay in adjustment. Expected future dividends and earnings influence fundamental value, which cannot be captured by the current dividend yield or price-earnings ratio. Moreover, the potential loss of information due to the use of proxies might contribute highly to the lack of recognizing of mean reverting behavior. Gropp (2004) suggests to use the proxy of Balvers et al. (2000) which bases its fundamental value process on a benchmark portfolio. According to Balvers et al. (2000), the stationary relation between the fundamental processes of the stock and its benchmark allows for a direct approximation of the mean reversion coefficient. The authors use the country stock indexes of sixteen OECD countries and compare them to the world index benchmark over the period 1970-1996. They find significant mean reversion in the stock indexes and argue that the half-life of the mean reversion process is approximately 3.5 years. The half life measures the period in which half of a shock to stock prices is reverted to zero. Balvers et al. (2000) find a 90% confidence interval for the half life of (2.4, 5.9) years.

## 2.3 Implications of Mean Reversion

This section divides the literature of long-term mean reversion into studies regarding an unspecified and a specified mean. Both groups of mean reversion models have a number of economic implications. An interesting property of absolute mean reversion is that stock returns are negatively autocorrelated over the long-run. On the other hand, in relative mean reversion models, stock prices revert to some specified fundamental value. De Bondt and Thaler (1985) describe the so-called contrarian trading strategy based on the absolute mean reverting behavior of stock prices. In this strategy a number of stocks that underwent a period of negative returns are selected, in order to obtain positive returns in the long future. They find that portfolios of decreased stocks experience exceptional returns in January five years after portfolio formation. De Bondt and Thaler (1987) and Jegadeesh and Titman (1993) apply a similar test and derive similar results based on the negative autocorrelation property of mean reverting stock prices. Jorion (2003) argues that a variance ratio smaller than one in the long-run, has implications to the allocation of assets. In case stock return variation increase less than proportional to the period at long horizons, the risk of stocks is smaller than expected under the random walk. Therefore, risk averse investors investing in the long-run, should allocate an additional amount of money to stocks. Along this argument, company policy and government regulations towards investment strategies could be affected by the evidence of mean reverting stock prices. In a paper on the regulation towards pension funds, Vlaar (2005) mentions that the occurrence of mean reversion would strongly increase the attractiveness of the stock market for pension funds. The author argues that, in case of mean reversion, low returns are followed by higher expected future returns which would motivate a pension fund to invest in equity after a downfall of the market.

In the reversion towards a specified mean, the stock prices could be compared to the mean and could be considered relatively too high or low. An investor investing in equity with a low price relative to the fundamental value, would expect to experience positive future returns. Campbell and Shiller (2001) argue that traditional valuation levels, like dividend and earnings, are a long-term outlook for the stock market. The corresponding valuation ratios, the dividend-price and price-smoothed-earnings ratios, might be used to forecast stock prices (Campbell and Shiller, 2001). Balvers et al. (2000) finds that this trading strategy outperforms buy-and-hold and standard contrarian strategies. Gropp (2004) find similar results applying this so-called parametric contrarian investment strategies to an industry-sorted portfolio benchmark.

The next section considers the model of Balvers et al. (2000) and applies the model to a large data set, containing 109 yearly returns from 1900 to 2008. The first issue with the research of Balvers et al. (2000) is that the analysis is based on a small sample from 1970 to 1996 of only 27 yearly returns. Moreover, the period that the authors choose contains a high growth and no severe recessions (Yahoo Finance, 2009). Partly due to the financial crisis started in the third quarter of 2008 it is expected that the small period is not representative for the general economic state. The second issue in their study is the multivariate normality assumption in the Monte Carlo simulation. Freedman and Peters (1984) argue that the normality of the residuals in a time series model could hold for large samples, however causes substantial small sample biases in finite samples. Therefore, a bootstrap simulation is applied, which does not depend on a parametric distribution but uses the empirical residual distribution.

## 3 Mean Reversion Model

The previous section discusses two methodologies that are used to analyze absolute and relative mean reversion in stock prices. This section derives a parametric model, based on a specified fundamental value process similar to Balvers et al. (2000). Equation (2) is applied to the general stock market index of $N$ developed countries over a time period $T$.[5] Each stock index is expected to revert to their intrinsic value in the long-run. In the model description the ideas and definitions of Balvers et al. (2000) are used. Consider the general formulation of a mean reverting process for each country $i$:

$$r_{t+1}^i = a^i + \lambda^i(p_{t+1}^{*i} - p_t^i) + \epsilon_{t+1}^i, \tag{3}$$

where $r_{t+1}^i$ equals the continuously compounded return of stock index of country $i$ between time $t$ and $t+1$, $p_{t+1}^{*i}$ is the natural logarithm of the intrinsic value of the index of country $i$ at time $t+1$ and $p_t^i$ is the natural logarithmic stock index of country $i$ at time $t$. The error term $\epsilon_t^i$ is considered to be a country specific stationary process with unconditional mean zero. Parameter $a^i$ is a country specific constant and $\lambda^i$ is called the speed of reversion of the index price process of country $i$. The relation between the index return and the deviation from its fundamental value depends entirely on parameter $\lambda^i$. The process in equation (3) is a mean reverting process when $0 < \lambda^i < 1$. Mean aversion occurs in case $\lambda^i$ is smaller than zero.

In order to estimate the parameters $a^i$ and $\lambda^i$ directly, the fundamental value $p^{*i}$ must be obtained. Unfortunately, due to the difficulty of determining the intrinsic equity value of a firm, this value can generally not be measured. Section 2 discusses the options of estimating the fundamental value of a firm. One possibility is to assume that the difference between the intrinsic value of a benchmark portfolio $p_t^{*b}$ and the intrinsic stock index value itself is a stationary process. This assumption cannot be tested empirically since both intrinsic values are unobservable. However, Balvers et al. (2000) justify the assumption using an economic explanation based on the convergence of per capita GDP. Real per capita GDP across 20 OECD countries display absolute convergence, which means that real per capital GDP converges to the same steady state (Barro and Sala-i Martin, 1995). According to the authors, absolute convergence results from the fact that countries catch up in capital and technology. Developed countries are expected to catch up in capital because lower per capita GDP implies a larger marginal efficiency of investment (Barro, 1991). Catching up in technology occurs because adapting an existing technology is cheaper than inventing a new one (Barro and Sala-i Martin, 1995). The connection between stock index convergence and GDP convergence is imposed by Balvers et al. (2000). Validility of this argument can be found in a country index that represents the general state of the stock market. Assuming a direct relation between the intrinsic value of the stock market and companies generating the gross domestic product justifies the assumption of stationarity between the country's fundamental stock price indexes.

[5]The data used in this empirical research of the $N$ countries over a time period 1900 - 2008 is further discussed in Section 4.

Assume that benchmark $b$ is defined such that for all countries $i$ the following holds:

$$p_t^{*i} - p_t^{*b} \;=\; c^i + \xi_t^i, \tag{4}$$

where $c^i$ is a country specific constant and $\xi_t^i$ is a zero-mean stationary process. The stationary process $\xi_t^i$ of equation (4) is possibly serially correlated, as well as correlated between countries. The choice of the benchmark does not affect the theoretical model of Balvers et al. (2000) in case equation (4) holds. As argued above, it is assumed that the difference between country's fundamental stock indexes is stationary. Thereby, an individual country is justified to be a candidate for the benchmark in this model. Moreover, a portfolio of countries satisfies equation (4) and is considered a benchmark candidate as well. In the empirical analysis of Section 4 several benchmarks are considered in order to examine the differences between the possibilities.

Assume that the benchmark of the stock index is chosen such that it follows the mean reverting process of equation (3)[6]. In addition, assume that the speed of reversion parameter $\lambda^i$ is constant across countries. The latter assumption does not indicate that the mean reversion process needs to be synchronized across countries, however the speed at which stock prices revert to their fundamental values are deemed to be similar (Balvers et al., 2000). Constant speed of reversion across countries allows for applying a panel data model. The panel data model assumes a constant value for the variable of interest, allowing for country specific parameters. An advantage of the use of a panel data model is the large increase in sample size, leading to a substantial increase in the power of testing the null hypothesis $\lambda = 0$.

Consider the differences between the returns of country $i$ and benchmark $b$ at time $t+1$:

$$\begin{aligned}
r_{t+1}^i - r_{t+1}^b &= \left(a^i - a^b\right) + \lambda\left(p_{t+1}^{*i} - p_t^i\right) - \lambda\left(p_{t+1}^{*b} - p_t^b\right) + \left(\epsilon_{t+1}^i - \epsilon_{t+1}^b\right) \\
&= \left(a^i - a^b\right) + \lambda\left(c^i + \xi_{t+1}^i\right) - \lambda\left(p_t^i - p_t^b\right) + \left(\epsilon_{t+1}^i - \epsilon_{t+1}^b\right) \\
&= \left(a^i - a^b + \lambda c^i\right) - \lambda\left(p_t^i - p_t^b\right) + \left(\epsilon_{t+1}^i - \epsilon_{t+1}^b + \lambda \xi_{t+1}^i\right) \\
&= \alpha^i - \lambda\left(p_t^i - p_t^b\right) + \psi_{t+1}^i, \qquad (5)
\end{aligned}$$

where $\alpha^i := a^i - a^b + \lambda c^i$ is a country specific constant and $\psi_{t+1}^i := \epsilon_{t+1}^i - \epsilon_{t+1}^b + \lambda \xi_{t+1}^i$ is the error term. Notice that the processes $\epsilon_t^i$ and $\xi_t^i$ are both zero-mean stationary. Process $\psi_t^i$ is composed out of two stationary processes and inherits the same statistical properties (Balvers et al., 2000). In particular, $\psi_t^i$ is allowed to be correlated over time and between countries. All variables in equation (5) are observable from historical data, and therefore this specification allows for estimating the country specific constants and the speed of reversion $\lambda$.

[6] Note that a country's stock index and a portfolio of country's stock indexes satisfies this assumption.

### 3.1 Generalized Least Squares Estimation

Under the assumption of uncorrelated error terms $\psi_t^i$ in equation (5) the ordinary least squares estimator would asymptotically converge to the actual parameter values. However, the assumption of no correlation violates the inherent properties of the process of long-horizon stock returns. To take into account the serial correlation, $k$ lagged return differentials are added to the equation. With the right choice for $k$, a significant purge of autocorrelation in the returns is realized. Define the adjusted model as:

$$r_{t+1}^i - r_{t+1}^b \quad = \quad \alpha^i - \lambda \left( p_t^i - p_t^b \right) + \sum_{j=1}^{k} \phi_j^i \left( r_{t-j+1}^i - r_{t-j+1}^b \right) + \omega_{t+1}^i. \tag{6}$$

The number of lagged return differentials inserted into the equation is determined based on the Bayesian information criteria (BIC). The residuals of equation (6) are now only correlated across countries. Hence, for each country the error term $\omega_t^i$ is considered a white noise process with a time-invariant, country specific variance $\sigma_i^2$.

As abovementioned, after the adjustment to purge the autocorrelation within each country only correlation between countries remains. Yet again, under the assumption of no correlation across countries, a simple ordinary least squares estimation of equation (6) would apply. The correlation between countries is adjusted in the estimation process by using seemingly unrelated regression (SUR) (Johnston and DiNardo, 1997), explained in Appendix A. The idea behind this adjustment to the generalized least squares estimator is that the correlation structure between seemingly independent systems of equations is adjusted for.

To derive an estimator for the parameters in equation (6) consider the following system of equations.

$$\begin{aligned} \boldsymbol{y} := \begin{bmatrix} \boldsymbol{y}^1 \\ \boldsymbol{y}^2 \\ \vdots \\ \boldsymbol{y}^N \end{bmatrix} &= \begin{bmatrix} -\boldsymbol{z}^1 & \boldsymbol{X}^1 & \boldsymbol{0} & \dots & \boldsymbol{0} \\ -\boldsymbol{z}^2 & \boldsymbol{0} & \boldsymbol{X}^2 & \dots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{z}^N & \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{X}^N \end{bmatrix} \begin{bmatrix} \lambda \\ \theta^1 \\ \theta^2 \\ \vdots \\ \theta^N \end{bmatrix} + \begin{bmatrix} \omega^1 \\ \omega^2 \\ \vdots \\ \omega^N \end{bmatrix} \\ &=: \boldsymbol{X}\beta + \omega, \end{aligned} \tag{7}$$

where,

$$\begin{aligned} \boldsymbol{y}^i &= \left[ \left( r_{k+1}^i - r_{k+1}^b \right) \quad \left( r_{k+2}^i - r_{k+2}^b \right) \quad \dots \quad \left( r_T^i - r_T^b \right) \right]', \\ \boldsymbol{z}^i &= \left[ \left( p_k^i - p_k^b \right) \quad \left( p_{k+1}^i - p_{k+1}^b \right) \quad \dots \quad \left( p_{T-1}^i - p_{T-1}^b \right) \right]', \\ \boldsymbol{X}^i &= \begin{bmatrix} 1 & (r_k^i - r_k^b) & \cdots & (r_1^i - r_1^b) \\ 1 & (r_{k+1}^i - r_{k+1}^b) & \cdots & (r_2^i - r_2^b) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (r_{T-1}^i - r_{T-1}^b) & \cdots & (r_{T-k}^i - r_{T-k}^b) \end{bmatrix}, \\ \theta^i &= \left[ \alpha^i \quad \phi_1^i \quad \cdots \quad \phi_k^i \right]', \qquad \text{and} \\ \omega^i &= \left[ \omega_{k+1}^i \quad \omega_{k+2}^i \quad \dots \quad \omega_T^i \right]', \end{aligned}$$

for all countries $i = 1, \ldots, N$. Equation (7) represents the system of the $N$ country specific deviations of equation (6). By assumption each country specific $\omega^i$ is serially uncorrelated. The across-countries' variance-covariance matrix $\boldsymbol{\Sigma}$ is constructed using the approach of Appendix A.

At this point, the linear panel data model and the corresponding variance-covariance matrix is defined in terms of the parameters $\beta$ and $\boldsymbol{\Sigma}$. In case the elements of the covariance matrix $\boldsymbol{\Sigma}$ are known, the generalized least squares estimator for $\beta$ would be

$$\hat{\beta}_{GLS} = \left(\boldsymbol{X}'\boldsymbol{\Sigma}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{\Sigma}^{-1}\boldsymbol{y}. \tag{8}$$

The variance-covariance matrix for the GLS estimator then equals

$$\mathrm{var}(\hat{\beta}_{GLS}) = \left(\boldsymbol{X}'\boldsymbol{\Sigma}^{-1}\boldsymbol{X}\right)^{-1}. \tag{9}$$

In the situation where the long horizon return deviations are considered, the variance-covariance matrix is not predetermined. Zellner (1962) suggests to construct a feasible GLS estimator in case the elements of $\boldsymbol{\Sigma}$ are unknown. This procedure requires to estimate $\beta$ by ordinary least squares, and uses the sample covariance matrix of the residuals as an estimator for $\boldsymbol{\Sigma}$.

The sample variance-covariance matrix asymptotically converges to the actual covariance matrix of $\omega$. However, for a finite sample analysis, the estimator tend to lead to great misleading estimates of $\beta_{GLS}$ and $\boldsymbol{\Sigma}$ (Freedman and Peters, 1984). In this thesis, Freedman and Peters (1984) are followed in using the bootstrap approach to the generalized least squares, in order to find more accurate estimators in the finite sample linear model. In general, the bootstrap approach uses the observed sample to estimate the distribution function of a test statistic. Appendix B considers the bootstrap explanation and the procedure to apply the bootstrap to the time series model of equation (6). In the following section the general model described above will be applied to a large data set of 17 developed countries from 1900 to 2008. The bootstrap method will be considered to adjust the estimated parameters for a small sample bias and to calculate the correct 95% confidence intervals for statistical inference.

## 4 Empirical Evidence of Mean Reversion

This section applies the model to the historical stock index prices of 17 developed countries. One of the goals of this thesis is to compare the results of Balvers et al. (2000), obtained in the period between 1970 and 1996, to a larger time interval. This section considers the largest available set of international stock index data, available for 17 different countries, ranging from 1900 to 2008. Henceforth this period is referred to as the full period. As mentioned in Section 3, the model of Balvers et al. (2000) is improved upon the small sample bias correction, by applying the bootstrap method of Appendix B.

The first subsection provides a comprehensive data description, including some summary statistics. Next, the country specific mean reversion process will be examined, and differences across countries are enlightened. The country specific results are presented for five different benchmarks. Finally, the full period is considered applying the panel data model to 17 countries. The latter results are given for five different benchmarks and will be compared to the findings of Balvers et al. (2000).

### 4.1 Data Description

The study performed in this thesis benefits considerably from accurate data over a long period of time. Short after the second millennium, Dimson et al. (2002) published a book containing the financial history of 17 countries with historically well developed economies and financial markets[7]. The authors carefully collect yearly equity, bond and treasury bill investment data from all over the world over a long horizon. With the aid of many local specialists, the most precise index value of the 101 years, from 1900 to 2000 are determined, and adjusted for inconsistencies with high accuracy. In addition, a world index is constructed out of the values of the country indexes[8]. Each year the authors present an extension of the data, cumulating to 109 yearly stock market returns up until 2008.

The yearly nominal equity returns are of particular interest in this thesis. According to Jegadeesh (1991) the use of monthly returns in the estimation of mean reversion could be influenced substantially by seasonal effects. Section 2 reveals that mean reversion based on monthly returns is completely concentrated in January (Fama and French, 1988b). Using yearly returns, this problem can be avoided. Furthermore, Perron (1991) finds that the power of unit root tests depends mainly on the time span rather than the number of observations. Therefore monthly data provide little additional information for detecting a slowly decaying component in stock prices (Balvers et al., 2000). Thirdly, using yearly in stead of smaller period returns, avoids the problem of ex-dividend price fluctuations.

[7] The Dimson, Marsh and Staunton data covers the stock indexes of the following 17 countries: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, The Netherlands, Norway, South Africa, Spain, Sweden, Switzerland, United Kingdom and the United States.

[8] The world index is a size-weighted portfolio of all country's indexes. The weights before 1968 are determined by GDP due to a lack of reliable data on capitalization prior to that date. The weights from 1968 onwards are based on market capitalization published by Morgan Stanley Capital International (MSCI). The sizes are annually adjusted to the GDP or market capitalization at the beginning of the year. (Dimson et al., 2002, pg. 311)

A second data property is the returns in dollars rather than own country currencies. Due to large fluctuation in the inflation of some countries, the results in own currency could turn out to be misleading. Additionally, the main interest lies in the mean reversion of stock prices. Therefore, it is best to correct for the price fluctuation due to exchange rate movements. Moreover, according to Balvers et al. (2000) the dollar term country indexes are being applied in related studies in this area as well.

Table 1 gives the mean, the standard deviation, the skewness and the kurtosis of the annualized continuously compounded nominal returns on the stock values between 1900 and 2008 in dollars. In addition, the beta coefficient with the world index is shown.

| | Mean | Standard Deviation | Skewness | Excess Kurtosis | Beta with World Index |
|---|---|---|---|---|---|
| Australia | 0.097 | 0.219 | -0.879 | 2.408 | 0.903 |
| Belgium | 0.055 | 0.237 | -0.074 | 1.278 | 0.945 |
| Canada | 0.085 | 0.185 | -0.892 | 2.092 | 0.932 |
| Denmark | 0.081 | 0.210 | -0.281 | 2.699 | 0.804 |
| France | 0.061 | 0.284 | -1.034 | 4.337 | 1.052 |
| Germany | 0.061 | 0.433 | 0.429 | 6.076 | 1.134 |
| Ireland | 0.068 | 0.243 | -0.740 | 4.208 | 0.959 |
| Italy | 0.051 | 0.302 | -0.117 | 1.458 | 1.015 |
| Japan | 0.072 | 0.376 | -3.018 | 18.29 | 0.876 |
| The Netherlands | 0.079 | 0.236 | -0.952 | 4.654 | 0.889 |
| Norway | 0.069 | 0.275 | -0.209 | 2.704 | 0.826 |
| South Africa | 0.088 | 0.251 | 0.234 | 1.246 | 0.788 |
| Spain | 0.066 | 0.258 | -0.229 | 1.005 | 0.710 |
| Sweden | 0.099 | 0.247 | -0.697 | 1.161 | 1.008 |
| Switzerland | 0.078 | 0.187 | 0.066 | 1.016 | 0.753 |
| UK | 0.077 | 0.220 | -0.568 | 2.675 | 1.027 |
| US | 0.088 | 0.196 | -0.879 | 0.741 | 1.026 |
| World | 0.080 | 0.167 | -0.915 | 1.949 | 1.000 |

Table 1: Summary statistics of the continuously compounded yearly index returns for 1900-2008.

The largest historical return over the full period is realized by Sweden, followed closely by the Australian market, generating an average yearly return of 9.9% and 9.7%, respectively. The lowest average over the full period is found for Italy, where a mean return of 5.1% is realized. The most volatile markets are Germany and Japan, however the standard deviations of both markets are extremely sensitive to the exclusion of outliers. In the years at the end of World War I (1919) and at the end of World War II (1945) Germany suffered extreme declines in their market prices of, respectively 73.3% and 79.2%. In either situation, the stock market regained the losses a number of years later; during 1923 and 1948 the German stock market increased by a striking 338% and 700%, respectively. The fluctuations during these historically critical years cause the standard deviations to rise substantially. Similar findings apply for the Japanese equity market index. The smallest volatility is observed in Canada, followed by Switzerland and the US.

The skewness of most countries is negative, which indicates more volatility in negative returns. Especially Japan and France show relatively high volatility when stock markets decline. The excess kurtosis measures the deviation of the kurtosis from three. In case the excess kurtosis equals zero, the tails of the return distribution are comparable to those of a normal distribution. When the excess kurtosis is much larger than zero, the return distribution exhibits fat tails. Notice that the market index of the United States, which is the most frequently studied market (Jorion, 2003), has the lowest kurtosis. A kurtosis of 3.7 for the US implies that their return distribution is comparable to a normal distribution in the tails. The obvious outlier in excess kurtosis is Japan with a kurtosis of 21.3. Germany follows with a kurtosis of 9.1. The latter two values are extremely sensitive to the outliers in the end-of-war years 1919, 1923, 1945 and 1948.

The last column of Table 1 displays the beta coefficient compared to the world index. The value can be interpreted as the correlation between a country's index and the portfolio of all countries' stock indexes. In general, a positive beta indicates positive correlation between a country index and the world index. A value larger than one indicates that the specific country exhibits larger volatility than the world index. In case the beta coefficient is smaller than one, the country fluctuates less than the world index. The smallest value of beta is obtained for Spain and the largest for Germany; respectively, the two countries have a beta coefficient of 0.71 and 1.13.

## 4.2 Individual Countries

Consider the case in which each country's index returns are compared to a benchmark separately. This analysis allows for estimating country specific speeds of reversion $\lambda^i$. Moreover, it is possible to examine the differences across countries in order to justify the assumption of constant speed of reversion in the panel data model of Section 3.

First consider the world index to be the benchmark in the model of Balvers et al. (2000). The first column of Table 2 gives the country specific biased estimators $\hat{\lambda}_0^i$, based on the generalized least squares method applied to equation (6). Also the 95% confidence interval is given based on the critical values used by Balvers et al. (2000). Notice that the issue with the GLS in this equation is the assumption of normality of the residuals. The normality assumption holds asymptotically, however in the situation at hand, the finite sample could cause a substantial bias in the estimates. Therefore, the bootstrap method of Appendix B is applied to the data. This method is based on the empirical distribution of historical returns, and therefore does not assume a parametric distribution. The bootstrapped small sample bias corrected estimators $\hat{\lambda}^i$ are displayed along with their 95% confidence interval in Table 2 as well.

The estimators in the first column are all positive, however insignificantly different from zero. Each biased estimator is substantially larger than the estimator after a bias adjustment by the bootstrap method. The main result from Table 2 is that the unbiased estimators are all insignificantly different from zero. This indicates that the individual country model does not provide evidence in favor of mean reversion for any country. A second result from Table 2 is that the speed of reversion does not differ significantly across countries. The latter motivates the use of a constant speed of reversion in a panel data model.

| BENCHMARK: | WORLD | | | |
|---|---|---|---|---|
| | $\hat{\lambda}_0^i$ | 95% C.I. $\hat{\lambda}_0^i$ | $\hat{\lambda}^i$ | 95% C.I. $\hat{\lambda}^i$ |
| Australia | 0.082 | (-0.011, 0.175) | 0.057 | (-0.010, 0.136) |
| Belgium | 0.038 | (-0.016, 0.092) | 0.014 | (-0.009, 0.065) |
| Canada | 0.069 | (-0.021, 0.159) | 0.041 | (-0.020, 0.114) |
| Denmark | 0.067 | (-0.041, 0.175) | 0.029 | (-0.034, 0.112) |
| France | 0.037 | (-0.026, 0.100) | 0.004 | (-0.015, 0.074) |
| Germany | 0.071 | (-0.025, 0.167) | 0.045 | (-0.016, 0.114) |
| Ireland | 0.066 | (-0.024, 0.156) | 0.036 | (-0.015, 0.110) |
| Italy | 0.029 | (-0.034, 0.092) | 0.003 | (-0.014, 0.057) |
| Japan | 0.046 | (-0.041, 0.133) | 0.014 | (-0.034, 0.083) |
| The Netherlands | 0.065 | (-0.043, 0.173) | 0.028 | (-0.030, 0.111) |
| Norway | 0.045 | (-0.036, 0.126) | 0.008 | (-0.024, 0.075) |
| South Africa | 0.120 | (-0.006, 0.246) | 0.094 | (-0.002, 0.187) |
| Spain | 0.029 | (-0.025, 0.083) | 0.001 | (-0.027, 0.054) |
| Sweden | 0.095 | (-0.013, 0.203) | 0.068 | (-0.008, 0.147) |
| Switzerland | 0.117 | (-0.009, 0.243) | 0.095 | (-0.002, 0.177) |
| UK | 0.073 | (-0.044, 0.190) | 0.029 | (-0.032, 0.123) |
| US | 0.085 | (-0.026, 0.196) | 0.060 | (-0.007, 0.137) |

Table 2: Country specific speed of reversion (world index benchmark)

Theoretically, the country specific estimators should be independent from the choice of benchmark. Table 3 considers four different benchmarks for the stock indexes: the US, Japan, France and Australia. These four countries are arbitrarily picked from four different continents in order to examine the differences between the choice of benchmark. At a 5% confidence level, significant mean reversion is found for South Africa for two out of five benchmarks, and for Belgium, Norway and Sweden for one out of five benchmarks. All other country estimates of the speed of reversion in Table 3 are insignificantly different from zero for all benchmarks. The estimators across the five different benchmark countries do not differ significantly. This result indicates that the choice of benchmark does not affect the estimation of $\lambda$.

Due to the insignificant results, no evidence of mean reversion is found when the countries are considered separately. However, the analysis above supports the use of the panel data model described in Section 3. The next subsection considers this model under the assumption of an identical speed of reversion across countries.

| BENCHMARK: | US | | | JAPAN | | |
|---|---|---|---|---|---|---|
| | $\hat{\lambda}_0^i$ | $\hat{\lambda}^i$ | 95% C.I. | $\hat{\lambda}_0^i$ | $\hat{\lambda}^i$ | 95% C.I. |
| Australia | 0.130 | 0.094 | (-0.014, 0.199) | 0.044 | 0.012 | (-0.022, 0.081) |
| Belgium | 0.040 | 0.014 | (-0.007, 0.069) | 0.076 | 0.046 | (-0.021, 0.127) |
| Canada | 0.077 | 0.058 | (-0.029, 0.137) | 0.045 | 0.015 | (-0.028, 0.076) |
| Denmark | 0.064 | 0.028 | (-0.026, 0.121) | 0.063 | 0.023 | (-0.036, 0.109) |
| France | 0.040 | 0.007 | (-0.014, 0.057) | 0.083 | 0.045 | (-0.030, 0.135) |
| Germany | 0.072 | 0.036 | (-0.012, 0.116) | 0.081 | 0.049 | (-0.025, 0.133) |
| Ireland | 0.065 | 0.042 | (-0.010, 0.117) | 0.089 | 0.063 | (-0.018, 0.142) |
| Italy | 0.031 | 0.005 | (-0.014, 0.053) | 0.063 | 0.036 | (-0.017, 0.103) |
| Japan | 0.045 | 0.020 | (-0.027, 0.087) | | | |
| The Netherlands | 0.072 | 0.046 | (-0.029, 0.105) | 0.052 | 0.006 | (-0.045, 0.098) |
| Norway | 0.044 | 0.006 | (-0.022, 0.067) | 0.071 | 0.034 | (-0.031, 0.115) |
| South Africa | 0.137 | 0.109 | (0.020, 0.199) | 0.053 | 0.018 | (-0.025, 0.098) |
| Spain | 0.029 | 0.004 | (-0.018, 0.049) | 0.077 | 0.051 | (-0.021, 0.117) |
| Sweden | 0.126 | 0.094 | (0.002, 0.189) | 0.054 | 0.020 | (-0.022, 0.090) |
| Switzerland | 0.099 | 0.070 | (-0.008, 0.155) | 0.065 | 0.032 | (-0.024, 0.107) |
| UK | 0.078 | 0.045 | (-0.014, 0.122) | 0.068 | 0.038 | (-0.031, 0.114) |
| US | | | | 0.045 | 0.012 | (-0.030, 0.078) |
| BENCHMARK: | FRANCE | | | AUSTRALIA | | |
| | $\hat{\lambda}_0^i$ | $\hat{\lambda}^i$ | 95% C.I. | $\hat{\lambda}_0^i$ | $\hat{\lambda}^i$ | 95% C.I. |
| Australia | 0.042 | 0.012 | (-0.011, 0.077) | | | |
| Belgium | 0.205 | 0.169 | (0.002, 0.193) | 0.045 | 0.019 | (-0.008, 0.074) |
| Canada | 0.041 | 0.009 | (-0.014, 0.077) | 0.120 | 0.091 | (-0.014, 0.194) |
| Denmark | 0.068 | 0.033 | (-0.022, 0.111) | 0.070 | 0.039 | (-0.010, 0.113) |
| France | | | | 0.042 | 0.012 | (-0.011, 0.073) |
| Germany | 0.072 | 0.039 | (-0.039, 0.125) | 0.059 | 0.031 | (-0.014, 0.098) |
| Ireland | 0.077 | 0.040 | (-0.030, 0.129) | 0.052 | 0.028 | (-0.005, 0.086) |
| Italy | 0.081 | 0.036 | (-0.041, 0.145) | 0.035 | 0.009 | (-0.012, 0.067) |
| Japan | 0.083 | 0.039 | (-0.032, 0.149) | 0.044 | 0.011 | (-0.022, 0.072) |
| The Netherlands | 0.043 | 0.002 | (-0.028, 0.081) | 0.061 | 0.029 | (-0.013, 0.103) |
| Norway | 0.179 | 0.145 | (0.006, 0.195) | 0.045 | 0.013 | (-0.011, 0.085) |
| South Africa | 0.045 | 0.011 | (-0.015, 0.080) | 0.184 | 0.162 | (0.012, 0.195) |
| Spain | 0.136 | 0.105 | (-0.014, 0.199) | 0.036 | 0.012 | (-0.011, 0.061) |
| Sweden | 0.054 | 0.018 | (-0.011, 0.088) | 0.085 | 0.057 | (-0.023, 0.133) |
| Switzerland | 0.050 | 0.009 | (-0.023, 0.088) | 0.077 | 0.054 | (-0.008, 0.117) |
| UK | 0.047 | 0.001 | (-0.027, 0.082) | 0.072 | 0.052 | (-0.010, 0.116) |
| US | 0.040 | 0.008 | (-0.014, 0.074) | 0.130 | 0.095 | (-0.014, 0.197) |

Table 3: Country specific speed of reversion (four benchmarks)

### 4.3 Panel Data Model

Despite the large time interval, the country specific lambdas do not provide significance in the estimation results in the full period. An increase in sample size is required to find significant evidence of mean reversion. To achieve an increase in sample size, the panel data model described in Section 3 is applied to the data of Dimson et al. (2002). In order to apply this model, the assumption of constant speed of reversion is required. The bootstrap method of Appendix B is executed to eliminate the small sample bias.

Table 4 presents the biased estimator $\hat{\lambda}_0$ based on generalized least squares estimation. Also, the unbiased, bootstrapped estimator $\hat{\lambda}$ is included in the table, along with its 95% confidence interval. In addition to the speed of reversion, the half-life of the mean reversion process is determined and displayed with confidence intervals. This half-life is calculated by $\frac{\ln(0.5)}{\ln(1-\lambda)}$, where $\lambda$ is taken to be the unbiased estimator[9]. Similar to the country specific lambda analysis in the last subsection, the sensitivity to considering several benchmarks is examined.

| BENCHMARK | $\hat{\lambda}_0$ | $\hat{\lambda}$ | 95% C.I. | Half-life in years (C.I.) |
|---|---|---|---|---|
| WORLD | 0.0668 | 0.0491 | (0.0323, 0.0661) | 13.8 (10.1, 21.1) |
| US | 0.0699 | 0.0523 | (0.0353, 0.0697) | 12.9 (9.6, 19.3) |
| JAPAN | 0.0714 | 0.0521 | (0.0304, 0.0713) | 13.0 (9.4, 22.5) |
| FRANCE | 0.0708 | 0.0533 | (0.0306, 0.0718) | 12.7 (9.3, 22.3) |
| AUSTRALIA | 0.0703 | 0.0533 | (0.0367, 0.0714) | 12.7 (9.4, 18.5) |

Table 4: Full period speed of reversion based on the panel data model

First consider the situation where the world index is the benchmark. The biased speed of reversion $\hat{\lambda}_0$ over the full period equals 0.0668. Adjusted for the small sample, the speed of reversion $\hat{\lambda}$ is 0.0491. To interpret this number, consider the initial model of equation (3), where the stock returns are assumed to be a function of the deviation from intrinsic value. The positive value of $\hat{\lambda}$ implies that a positive deviation between stock price and intrinsic value in this period will result in a negative expected return in the next period. This relation between stock price, intrinsic value and expected return is intuitively correct. Further, the value of lambda estimates the speed at which the price reverts to its fundamental value. Each year a price correction takes place of 4.91% of the logarithmic price deviation from fundamental value.

In order to measure the speed of reversion in years, the half-life of a gap between stock price and intrinsic value is considered. Notice that a higher speed of reversion implies a shorter half-life. In case of using the world index as a benchmark, the half-life of a price deviation is 13.8 years. In other words, 50% of a shock to the stock price will be offset in 13.8 years after occurrence. The confidence interval displayed in Table 4 gives a range in which 95% of

[9] The half-life of the stationary process in equation (1) is calculated by $\frac{\ln(0.5)}{\ln(\phi)}$. Footnote 3 shows that $\phi = 1 - \lambda$, which leads to the formula to calculate the half-life in this section.

all future estimates over a period of 109 years will fall. The lowerbound implies a half-life of 10.1 years, the upperbound gives a half-life of 21.1 years.

The values corresponding to the four other benchmarks are similar the the world index reference. France and Australia exhibit the largest unbiased estimator of $\lambda$ equal to 0.0533. This value is somewhat higher than the estimate using the world index, and implies a half-life of 12.7 years. For France, the highest speed of reversion within the confidence interval results a half-life of 9.3 years. The slowest mean reversion process within the confidence bounds of the four benchmarks is found for Japan and exhibits a half-life of 22.4 years.

Balvers et al. (2000) estimate the speed of reversion in the same way for the period 1970-1996. The authors adjust the small sample bias by use of a Monte Carlo simulation. They assume multivariate normality of the residuals of the time series model in equation (6). The data they use is similar to the data above, however not the same. Their empirical work is based on the Morgan Stanley Capital International (MSCI) for stock market price indexes of 18 countries[10]. Like the full period, Balvers et al. (2000) find significant speed of reversion within the time interval 1970-1996. The value of the unbiased estimator $\hat{\lambda}$ equals 0.182, which implies a half-life of 3.5 years. Furthermore, 90% of the sample lambda falls within a range of 0.110 and 0.250, i.e. a half-life interval between 2.4 and 5.9 years. The results of the U.S. as a benchmark are of similar magnitude. Here, the unbiased estimator of $\lambda$ equals 0.202 implying a half-life of 3.1 years.

The unbiased estimators of the one hundred year period differ substantially from the three decade results of Balvers et al. (2000). Considering the world index benchmark, the estimated speed of reversion between 1970 and 1996 shows to be nearly four times larger than the speed in the full period. Furthermore, the half-life differs approximately 10 years between the two data sets. Moreover, the 90% confidence interval of the half-life is much larger for the estimators based on the full period. The confidence interval of the half-lifes for the world index benchmark is $(10.8, 20.4)$. The latter interval contains 9.6 years, while the smaller period interval contains only 3.5 years. The US index benchmark shows similar differences in the results.

Apparently, the estimators of the speed of reversion of a stock market index depend on the time interval. Next section considers a rolling window estimation approach to determine the difference between speed reversion across time.

[10]These countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, the Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Notice that in this data set, South Africa is replaced for Austria and Singapore, due to different choices made by the data collectors.

# 5 Time Varying Mean Reversion Process

The main result in the previous section indicates a half-life of the stock price deviation from fundamental value of 13.8 years. This significant number results from the analysis of the full time interval, the period 1900-2008. Several reasons can be thought of questioning the assumption of a constant mean reversion coefficient over the last 109 years. Kim et al. (1991) conclude that mean reversion is a pre-war phenomenon only. Furthermore, Poterba and Summers (1988) conclude that the Great Depression has a significant influence on the results of mean reversion. Also the difference between the full period results and the result from the period considered by Balvers et al. (2000) indicates a changing mean reversion pattern. The power increase caused by the inclusion of 17 countries in a panel data model, allows for analyzing the speed of reversion coefficient over multiple smaller time intervals. For this purpose, a rolling window estimation approach is applied to time intervals of 27 years. This interval is considered in accordance with Balvers et al. (2000), who choose the 27-year interval of returns from 1970 to 1996.

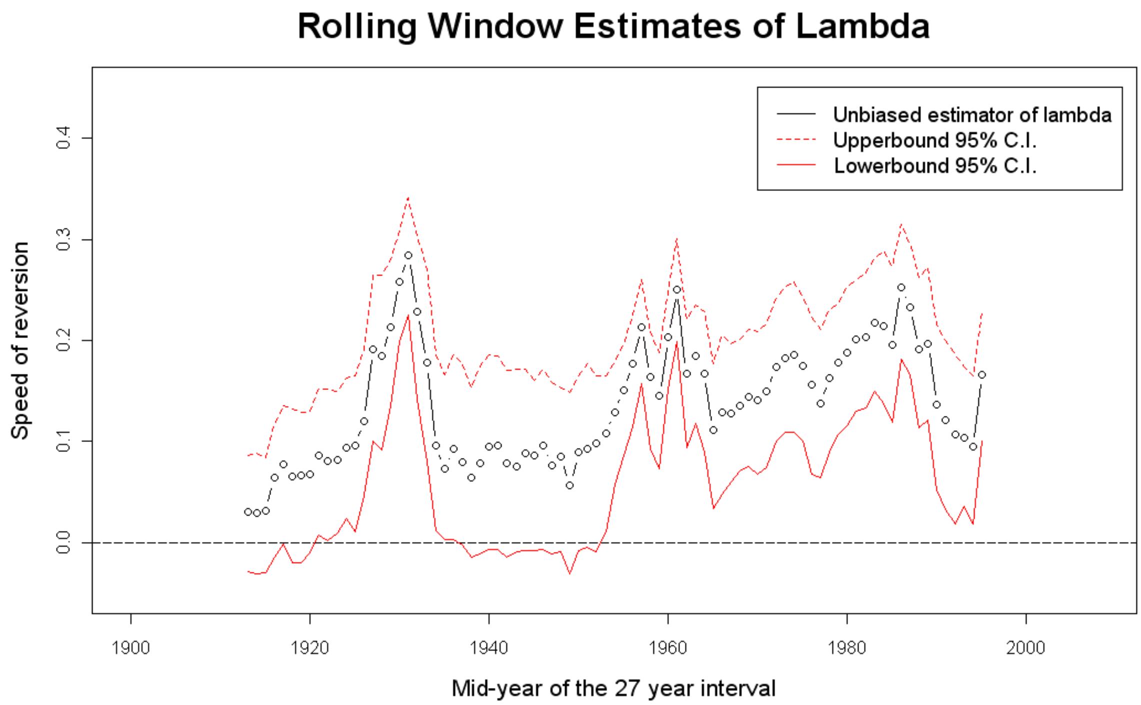
Table 5 displays the biased and unbiased estimators of the speed of reversion, based on 83 rolling windows of 27 yearly returns. The 95% confidence interval is calculated as well, using the bootstrap method of Appendix B. The mean reversion coefficient $\lambda$ is clearly not constant over time. The largest value estimator of the speed of reversion is 0.284, observed in interval 1918-1944. The smallest estimator is obtained for interval 1901-1927 to equal 0.029, which is almost 10 times smaller. Respectively, the two intervals above have a half-life of 2.1 and 23.6 years. Moreover, notice that the extreme intervals overlap in the years 1918 up to 1926. A conclusion about mean reversion in these nine years might be very different, depending on the interval choosen.

An important conclusion to be drawn is whether significant mean reversion occurs. For 24 of the 83 intervals, i.e. 29%, no significant mean reversion is found. The insignificant speeds of reversion are all located in intervals before 1939-1965. The confidence intervals of the time intervals that are considered after this interval are all strictly positive. As mentioned in Section 4, an interesting comparison can be made between the results in this thesis and those of Balvers et al. (2000). The value found for the interval 1970-1996 is one of the largest of all estimators. 93% of the remaining 82 time intervals results in a smaller unbiased estimator of the mean reversion coefficient. Furthermore 59% of all unbiased estimators fall below the 95% confidence interval of the estimated speed of reversion in the period of Balvers et al. (2000). Moreover, at a 2.5% significance level, 11% of the estimators are significantly lower than the 1970-1996 period.

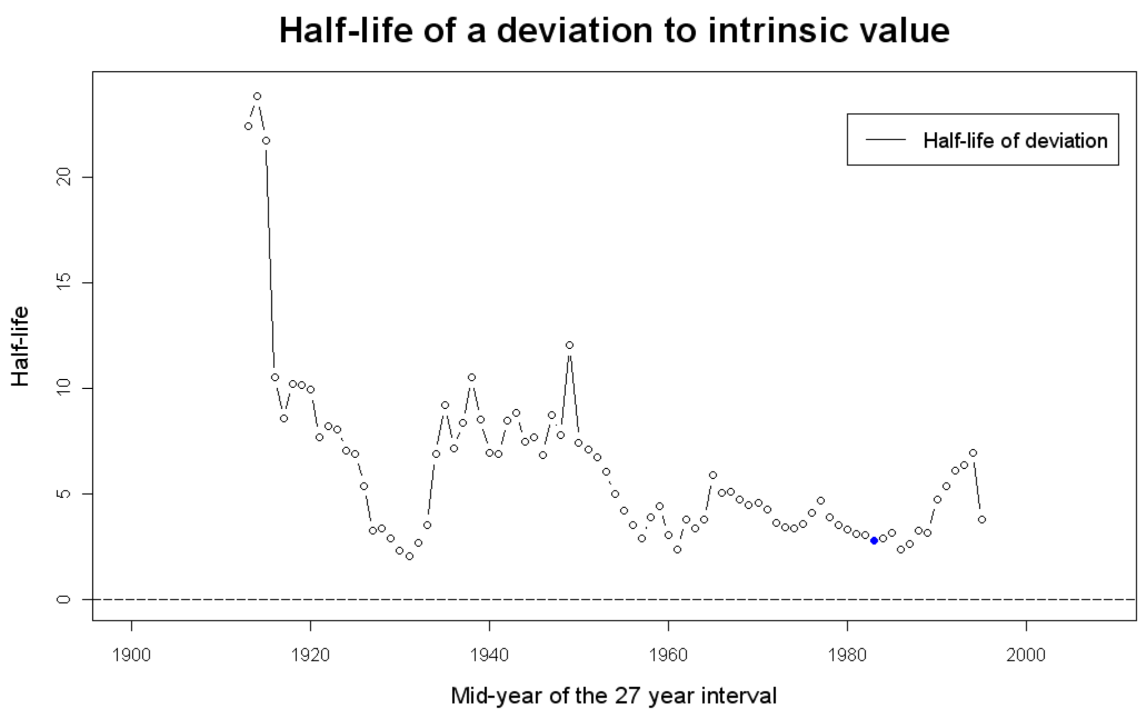
To illustrate the differences between time intervals, consider Figure 1. Figure 1a gives the unbiased estimators of the speed of reversion, along with their 95% confidence intervals. The graph in Figure 1b displays the half-life of a price deviation from intrinsic value. Both graphs resemble the rolling window estimators, hence each point in the graphs is assigned to a 27 year interval. The values are given with respect to the mid-year of such an interval. In Figure 1a, a horizontal line is drawn at critical value zero. All lowerbounds below this line indicate that the speed of reversion insignificantly differs from zero. This implies that during these time intervals no significant mean reversion is found.

| Period | $\hat{\lambda}_0$ | $\hat{\lambda}$ | 95% C.I. $\hat{\lambda}$ | Period | $\hat{\lambda}_0$ | $\hat{\lambda}$ | 95% C.I. $\hat{\lambda}$ |
|---|---|---|---|---|---|---|---|
| 1900 to 1926 | 0.081 | 0.030 | (-0.029, 0.086) | 1942 to 1968 | 0.178 | 0.151 | (0.086, 0.196) |
| 1901 to 1927 | 0.084 | 0.029 | (-0.031, 0.088) | 1943 to 1969 | 0.207 | 0.177 | (0.115, 0.225) |
| 1902 to 1928 | 0.085 | 0.031 | (-0.030, 0.083) | 1944 to 1970 | 0.242 | 0.213 | (0.158, 0.260) |
| 1903 to 1929 | 0.112 | 0.064 | (-0.015, 0.117) | 1945 to 1971 | 0.192 | 0.163 | (0.093, 0.209) |
| 1904 to 1930 | 0.124 | 0.078 | (-0.001, 0.136) | 1946 to 1972 | 0.174 | 0.145 | (0.072, 0.188) |
| 1905 to 1931 | 0.123 | 0.066 | (-0.020, 0.132) | 1947 to 1973 | 0.230 | 0.203 | (0.152, 0.248) |
| 1906 to 1932 | 0.124 | 0.066 | (-0.020, 0.129) | 1948 to 1974 | 0.276 | 0.251 | (0.199, 0.301) |
| 1907 to 1933 | 0.126 | 0.067 | (-0.010, 0.130) | 1949 to 1975 | 0.209 | 0.167 | (0.094, 0.220) |
| 1908 to 1934 | 0.141 | 0.086 | (0.008, 0.152) | 1950 to 1976 | 0.224 | 0.185 | (0.118, 0.234) |
| 1909 to 1935 | 0.139 | 0.081 | (0.001, 0.151) | 1951 to 1977 | 0.213 | 0.167 | (0.089, 0.229) |
| 1910 to 1936 | 0.135 | 0.082 | (0.009, 0.149) | 1952 to 1978 | 0.171 | 0.111 | (0.033, 0.177) |
| 1911 to 1937 | 0.150 | 0.094 | (0.023, 0.163) | 1953 to 1979 | 0.191 | 0.129 | (0.048, 0.205) |
| 1912 to 1938 | 0.148 | 0.096 | (0.011, 0.165) | 1954 to 1980 | 0.190 | 0.127 | (0.060, 0.196) |
| 1913 to 1939 | 0.171 | 0.120 | (0.046, 0.191) | 1955 to 1981 | 0.193 | 0.136 | (0.071, 0.201) |
| 1914 to 1940 | 0.237 | 0.191 | (0.100, 0.265) | 1956 to 1982 | 0.196 | 0.144 | (0.075, 0.211) |
| 1915 to 1941 | 0.237 | 0.185 | (0.092, 0.265) | 1957 to 1983 | 0.191 | 0.141 | (0.067, 0.209) |
| 1916 to 1942 | 0.252 | 0.213 | (0.135, 0.280) | 1958 to 1984 | 0.208 | 0.150 | (0.074, 0.216) |
| 1917 to 1943 | 0.284 | 0.258 | (0.198, 0.308) | 1959 to 1985 | 0.229 | 0.174 | (0.100, 0.242) |
| 1918 to 1944 | 0.315 | 0.284 | (0.226, 0.341) | 1960 to 1986 | 0.236 | 0.183 | (0.109, 0.252) |
| 1919 to 1945 | 0.284 | 0.228 | (0.142, 0.304) | 1961 to 1987 | 0.239 | 0.186 | (0.109, 0.258) |
| 1920 to 1946 | 0.244 | 0.178 | (0.080, 0.271) | 1962 to 1988 | 0.231 | 0.175 | (0.101, 0.243) |
| 1921 to 1947 | 0.169 | 0.096 | (0.012, 0.187) | 1963 to 1989 | 0.215 | 0.156 | (0.067, 0.223) |
| 1922 to 1948 | 0.159 | 0.073 | (0.003, 0.166) | 1964 to 1990 | 0.202 | 0.138 | (0.065, 0.211) |
| 1923 to 1949 | 0.166 | 0.092 | (0.003, 0.186) | 1965 to 1991 | 0.218 | 0.162 | (0.089, 0.230) |
| 1924 to 1950 | 0.158 | 0.080 | (-0.002, 0.177) | 1966 to 1992 | 0.227 | 0.177 | (0.106, 0.236) |
| 1925 to 1951 | 0.146 | 0.064 | (-0.015, 0.153) | 1967 to 1993 | 0.236 | 0.188 | (0.115, 0.253) |
| 1926 to 1952 | 0.162 | 0.078 | (-0.011, 0.175) | 1968 to 1994 | 0.245 | 0.201 | (0.130, 0.259) |
| 1927 to 1953 | 0.175 | 0.095 | (-0.007, 0.186) | 1969 to 1995 | 0.249 | 0.204 | (0.133, 0.266) |
| 1928 to 1954 | 0.174 | 0.096 | (-0.007, 0.185) | <u>1970 to 1996</u> | <u>0.260</u> | <u>0.218</u> | <u>(0.149, 0.281)</u> |
| 1929 to 1955 | 0.163 | 0.079 | (-0.014, 0.170) | 1971 to 1997 | 0.263 | 0.214 | (0.138, 0.288) |
| 1930 to 1956 | 0.153 | 0.075 | (-0.009, 0.171) | 1972 to 1998 | 0.252 | 0.195 | (0.119, 0.273) |
| 1931 to 1957 | 0.156 | 0.089 | (-0.009, 0.171) | 1973 to 1999 | 0.294 | 0.252 | (0.181, 0.315) |
| 1932 to 1958 | 0.150 | 0.086 | (-0.008, 0.160) | 1974 to 2000 | 0.276 | 0.233 | (0.166, 0.295) |
| 1933 to 1959 | 0.151 | 0.096 | (-0.007, 0.171) | 1975 to 2001 | 0.250 | 0.192 | (0.113, 0.262) |
| 1934 to 1960 | 0.141 | 0.076 | (-0.011, 0.158) | 1976 to 2002 | 0.256 | 0.197 | (0.121, 0.272) |
| 1935 to 1961 | 0.143 | 0.085 | (-0.009, 0.153) | 1977 to 2003 | 0.208 | 0.136 | (0.052, 0.215) |
| 1936 to 1962 | 0.133 | 0.056 | (-0.031, 0.148) | 1978 to 2004 | 0.188 | 0.121 | (0.032, 0.199) |
| 1937 to 1963 | 0.146 | 0.089 | (-0.008, 0.165) | 1979 to 2005 | 0.175 | 0.107 | (0.018, 0.186) |
| 1938 to 1964 | 0.155 | 0.093 | (-0.004, 0.177) | 1980 to 2006 | 0.154 | 0.103 | (0.036, 0.173) |
| 1939 to 1965 | 0.149 | 0.098 | (-0.010, 0.165) | 1981 to 2007 | 0.148 | 0.095 | (0.018, 0.165) |
| 1940 to 1966 | 0.148 | 0.108 | (0.010, 0.165) | 1982 to 2008 | 0.205 | 0.166 | (0.100, 0.228) |
| 1941 to 1967 | 0.159 | 0.129 | (0.058, 0.179) | | | | |

Table 5: Interval analysis of the Speed of Reversion $\lambda$




(a) Unbiased estimators of the speed of reversion.




(b) Half-life of a deviation between stock price and fundamental value.

Figure 1: Rolling window estimators for 27-year intervals

The three largest local maxima in the speed of reversion are obtained for the intervals 1918-1944, 1948-1974 and 1973-1999, with values of, respectively, 0.284, 0.251 and 0.252. The two smallest local minima can be observed in the intervals 1901-1927 and 1936-1962, and have respective value 0.029 and 0.056. The lowest value obtained in the 40 most recent intervals equals 0.095, in time interval 1981-2007.

It is difficult to assign a disparity between two time intervals to one specific year. Consider for example the last two intervals, 1981-2007 and 1982-2008. The corresponding half-lives of the intervals are, respectively, 6.9 and 3.8 years. Either the inclusion of the year 2008, or the exclusion of 1981 causes the decrease in half-life. Probably both years contribute to the varying half-life. The decrease in half-life of 3.1 years gives the aggregate effect on the half-life only. This aggregate effect could possibly contain two opposite effects on the speed of reversion, which would be offset by the aggregation.

A more accessible interpretation can be found considering small time intervals. In the first peak in Figure 1a there are seven unbiased estimators remarkably larger than others. These points are assigned to the intervals between 1914-1940 and 1920-1946. The intervals all completely contain the years from 1920 to 1940. These 21 years are likely to contain some years with high speed of reversion. Secondly, one could examine the first three intervals and look at Figure 1b to see that the half-life in these years was extremely high. The time interval from 1902 to 1926, which is contained in all three first intervals, will probably contain a number of years with low speed of reversion. Thirdly, consider the series of insignificant speed of reversion from interval 1924-1950 to 1939-1965. The time interval completely contained within each of these period are the years 1939 to 1950. This implies that within these periods there are some years with extremely low reversion speed, or even without mean reversion.

The difference in the half-life process between the period before mid-year 1953 and after mid-year 1953 are substantial. The post-1953 period exhibits no insignificant mean reversion, in contrary to the earlier period in which over 50% of the estimators are insignificant. The average level of speed of reversion is higher after mid-year 1953, and the fluctuation of the estimates is lower.

Table 5 and Figure 1 show significant deviations. The mean reversion process cannot be considered to have a general speed of reversion across time. Due to the complication to attach the deviations to specific years it is difficult to explain the changes in half-life. However, the graphs show a number of remarkable relations between speed of reversion and historical events. The largest speed of reversion is found during the mid-year 1930. The period around this year includes World War I, the Great Depression and the start of World War II. Another remarkable high speed of reversion period is found for the mid-year 1987; the year in which Black Monday occurred (Yahoo Finance, 2009). Furthermore, the reversion speed in the last interval is remarkably higher than the previous one. This last interval is the only interval which includes part of the financial crisis. From the examples above one can conclude that periods of uncertainty about the sustainability of the economy cause stocks to revert to their fundamental value faster.

This behavior can be justified by the governmental intervention, as well as close monitoring activities of companies during recessions. During uncertainty about the survival of the economy, stock prices might drop in value substantially. Financial institutions, like pension funds and banks, could suffer from these drops in value, which could affect individual companies. Therefore, companies and financial as well as governmental institutions increase their awareness of the negative economic situation and come up with measures to restore their operations. These measures intend to increase the stability on the financial markets, restore trust and lead to an increase in stock markets.

Figure 2 displays the unbiased estimators of the half-life including the 95% confidence bounds for the 37 post World War II time intervals, ranging from interval 1946-1972 to interval 1982-2008. This more recent period provides a better indication about current international stock markets.

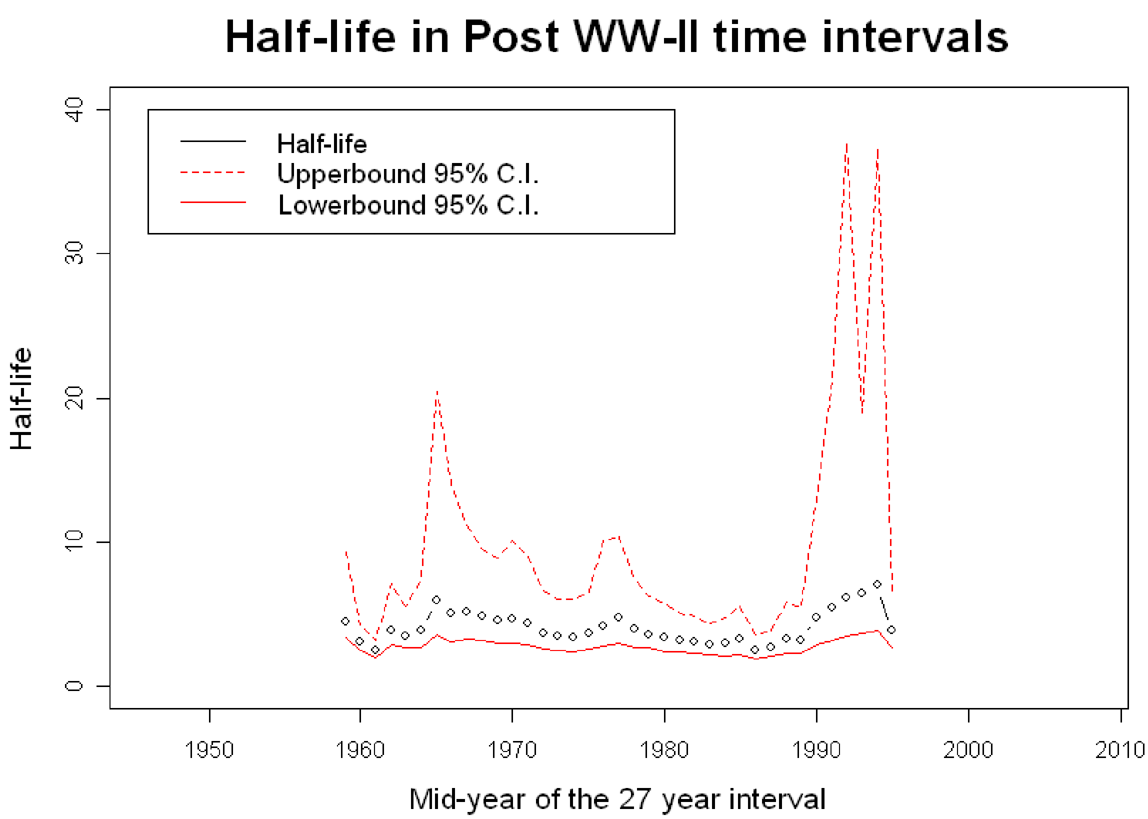



Figure 2: Half-life and confidence interval for 27-year Postwar intervals

The period considered after the World War II displays a larger speed of reversion, and therefore smaller half-lifes. However, substantial differences in the unbiased estimators across time remain to occur. The longest half-life equals 7.0 years while the shortest half-life is 2.4 years. Moreover, the graph displays large uncertainty in the point estimates of the half-life. The upperbound in the time interval 1979-2005 differs 34.4 years from the lowerbound in this time interval. It is remarkable that the uncertainty in the estimation for long half-lifes exceeds the uncertainty in intervals with short half-lifes. This indicates that if the half-life is short, there is more certainty about its estimate. If the half-life is long, the uncertainty about the value of the estimator is larger.

### 5.1 Implications to empirical research

The differences in the mean reversion process across time have their implication on earlier empirical research. Section 2 discusses the two distinct groups of empirical mean reversion studies. In the latter group, Balvers et al. (2000) assumes a speed of reversion that is constant over a 27 year time interval. The results from Figure 1a clearly show that this speed of reversion fluctuates each time interval. Moreover, notice that the speed of reversion is determined each estimation from a 27 year interval. The daily, monthly or yearly fluctuations of speed of reversion within the interval remain unobserved, however are expected to be substantial. The half-life of a shock to the stock price fluctuates between 2.1 and 23.6 years. Hence, an investment strategy cannot be based on mean reversion due to the large amount of risk of assuming a fault speed of reversion.

Another impact of the fluctuation in the mean reversion process is found in absolute mean reversion research. The focus within this group lies on the negative autocorrelation of long-term stock returns. Fama and French (1988b) and Poterba and Summers (1988) both assume a constant period at which stock returns are expected to be negatively correlation. They analyze several terms in order to determine the number of years at which negative autocorrelation is observed. Both papers find these terms to be 3 to 5 years, and find significant mean reversion at these horizons. However, Section 2 mentions as well that the results are criticized extensively, and that convincing evidence remains absent until so far. In case the research allows for fluctuating speed at which prices revert to their mean, stronger evidence of mean reversion is expected.

It is interesting to examine the concept of a fluctuating mean reversion process across time. Theoretically, the fluctuation could be caused by measures taken by individual companies, or governmental and financial institutions. According to Kim et al. (1991), uncertainty causes smaller expected future returns, and therefore lower stock prices. The governmental measures and measures from financial institutions could result in a substantial increase in speed of reversion, since they influence uncertainty of the economic state directly. Therefore, the mean reversion process at some period in time depends highly on the economic state, and the expected measures taken by companies, governments or financial institutions.

In the following section the impact of the finding above on the continuity analysis for Dutch pension funds are considered. First the continuity analysis is explained and the regulations regarding the analysis are summarized. Subsequently the literature review, and the results from the empirical test in this thesis are used to construct an advice on how to assess the use of mean reversion in the continuity analysis.

# 6 Mean Reversion and the Continuity Analysis for Dutch Pension Funds

Mean reversion in stock prices has multiple implications for institutions, active on the financial market. The main focus of the implication of mean reversion in the literature is the predictability that arises from mean reversion (see e.g. Fama and French (1988b)). A second application that is often examined is the relation between long-term risk and return in mean reverting markets, and its impact on asset allocation (see e.g. Jorion (2003)). In this thesis another area of interest is considered. In The Netherlands, pension funds are obliged to monitor their long-term financial position at least every three years. Moreover, the pension fund must perform such a monitoring test in times of concern about their financial sustainability. This so-called continuity analysis involves a 15-year stochastic simulation approach to forecast the financial position of the pension fund.

In this section the implication and desirability of inserting mean reversion into the continuity test is considered. First, the continuity analysis is explained, the regulation regarding the assumptions and the principles of supervision maintained by the Dutch Central Bank are discussed. Next, the implication of mean reversion in a continuity analysis is considered. And finally, this section discusses the desirability of implementing mean reversion into the regulatory framework for pension funds, based on the analysis and the literature review in previous sections.

## 6.1 Continuity Analysis

The central measure in the continuity analysis of a pension fund is the financial position during the next 15 years. The development of this financial position over this forecasting period is evaluated by use of a stochastic simulation approach. In this approach, a large number of scenarios of the financial position is generated, based on underlying stochastic determinants like discount rate, indexation rate and pension premium. Another important determinant of the financial position is the value of the investments of the pension fund. This value is determined by the return on the total portfolio of assets held by the pension fund. This portfolio contains, among others, common stock, fixed income products, real estate and commodities. Therefore, the return on the portfolio is completely determined by the returns on the assets included in the portfolio.

The returns on the portfolio in the next 15 years are unknown, and therefore simulated a large number of times. The process of generating portfolio returns completely depend on the stochastic portfolio return distribution. Notice that this distribution is determined by the return distributions of individual assets within the portfolio. Assumptions in these underlying distributions influence the properties of the distribution of the returns on the portfolio.

Consider the expected value, the variance, the skewness and the kurtosis of the portfolio return distribution. These four distribution properties influence the simulation of the financial position of the pension fund as follows. A high expected portfolio return will positively affect each generated future scenario, leading to a more favorable financial position. A high return variance increases the risk in the portfolio, leading to a larger scale of generated financial positions. Positive skewness implies a larger variance for positive returns, as compared to negative returns. Hence, positive financial positions are relatively more volatile than negative financial positions. Further, a high kurtosis implies fat tails, which leads to larger extreme positive and negative scenarios of the financial position of a pension fund.

| Individual asset return distributions *Assumptions* | | Portfolio return distribution *Properties* |
|---|---|---|
| Expected values | $\Longrightarrow$ | - Expected value of the portfolio return |
| Variances | $\Longrightarrow$ | - Variance of the portfolio return |
| Skewness | $\Longrightarrow$ | - Skewness of the portfolio return |
| Kurtosis | $\Longrightarrow$ | - Kurtosis of the portfolio return |
| Autocorrelations | $\Longrightarrow$ | - Expected value of the portfolio return<br>- Variance of the portfolio return |
| Correlations between assets | $\Longrightarrow$ | - Variance of the portfolio return |
| Correlations with exogenous variables | $\Longrightarrow$ | - Expected value of the portfolio return<br>- Variance of the portfolio return |
| Correlation structure | $\Longrightarrow$ | - Skewness of the portfolio return<br>- Kurtosis of the portfolio return |

Table 6: The effect of assumptions in the individual assets on the properties of the portfolio return distributions. Notice that A $\Longrightarrow$ B indicates that a change in assumption A could influence the value of property B.

As mentioned above, the properties of the portfolio return distribution completely depend on the individual asset return distributions. Asset specific properties, like expected value, variance, skewness, kurtosis and autocorrelation, as well as correlation between assets, correlation with exogenous variables and correlation structure are of keen importance in determining an accurate simulation process. Consider a pension fund that assumes the properties of the individual asset returns, and uses these individual assets to simulate the portfolio returns. The required assumptions influence the properties of the portfolio returns in several ways. Table 6 shows the relation between the assumptions in the individual asset return distributions and the properties of the portfolio return distributions. For each assumption, the table mentions the property which is influenced by a change in assumption. It is important to recognize these influences, since the assumptions in the continuity test are made on an individual asset level, however the financial position of the pension fund is reported in terms of the portfolio value. Thus, examining the plausibility of the simulated portfolio value requires knowledge of the way in which the assumptions affect the results.

The expected value of the portfolio return is influenced by the expected values of the return on individual assets. A high average return could therefore be caused by optimistic assumptions about the expected return on individual assets. Moreover, Table 6 shows that the assumption of autocorrelation could also influence the expected portfolio return. Negative autocorrelation implies that after a period of negative returns, a period of positive returns is expected. With this assumption, pension funds might choose to adjust the first few years of prospects upwards after the occurrence of a market downfall. Notice that this assumption only influences the expected returns in the first few years. Alternatively, changing expected returns might be caused by an assumed relation with exogenous variables, like dividends and earnings. Suppose that the pension fund assumes that the expected returns on individual assets are determined by the expected dividend that is payed out by the company. Then, the fluctuation in expected dividend influences the expected portfolio returns.

The variance of the portfolio return is influenced by the individual asset return variances, the correlation between the assets, and the correlation with exogenous variables. High individual variances imply high portfolio variance. Negative correlation between assets will negatively influence the value of the portfolio variance, leading to smaller portfolio risk. The correlation with exogenous variables influences the variance through the correlation between assets. In case it is assumed that multiple assets depend on the same exogenous variable, the pension fund assumes an implicit correlation between the multiple assets. Hence, in case two assets depend on the same exogenous variables in opposite ways, negative correlation is implied, leading to lower portfolio variance.

The skewness of the portfolio return is influenced by two assumptions. First, the individual asset return skewness plays an important role, since it directly results in skewness in the portfolio return. Second, the correlation structure between assets can have an impact on the skewness of the portfolio returns. Suppose that the lower tail correlation structure differs from the upper tail correlation structure, then it is possible that the correlation between assets influences the portfolio variance in the negative returns, however does not influence the portfolio variance in the positive returns. This event leads to a difference between negative return variance and positive return variance and thus influences the skewness of the portfolio returns.

The kurtosis of the portfolio return is influenced by this correlation structure as well. Suppose that the correlation is such that it diversifies the lower and upper tail risk. Then this could lead to a lower portfolio kurtosis. A second assumption that influences the portfolio kurtosis is the kurtosis of the individual assets.

### 6.1.1 Regulation and Supervision

The impact of assumptions about the return distributions of individual assets on the simulation of portfolio returns is considered above. Pension funds could choose optimistic parameter value in order to improve the forecast of their financial position during the next 15 years. To assure that a realistic view on the future is displayed, the Dutch Pension Act introduces a financial assessment for pension funds[11]. This assessment provides with regulations towards applying the continuity analysis. It refers to a commission, called Commissie Parameters, which developed a ministrial rule[12] in which the maximum expected returns on investments are displayed. To clarify the supervision approach that is applied by the Dutch Central Bank, a policy rule[13] is published in which the principles applied by the supervisor are explained.

The ministrial rule and the policy rule are the two documents for pension funds to check the seven assumptions of Table 6. Only the maximum expected return per investment category is displayed explicitly. The variance, correlation between assets and autocorrelation are mentioned required to be chosen realistically. The other assumptions are left out of the regulatory documents. Table 7 shows the assumptions to be made by the pension fund, along with the document in which rules or principles regarding these assumptions are displayed. Notice that the distribution of the returns is explicitly mentioned as well, since it may have an impact on the result.

| Assumption | Document | Rule or Principle |
|---|---|---|
| Expected values | Ministerial rule | Maximum expected returns |
| Variances | Policy rule | "Realistic assumptions" |
| Skewness | - | N/A |
| Kurtosis | - | N/A |
| Autocorrelations | Policy rule | "Realistic assumptions" |
| Correlations between assets | Policy rule | "Realistic assumptions" |
| Correlations with exogenous variables | - | N/A |
| Correlation structure | - | N/A |
| Distribution of returns | - | N/A |

Table 7: Parameter specification in a continuity analysis

The principle in the policy rule of the Dutch Central Bank involve an analysis of the expected returns, variances, correlations between assets and autocorrelations applied by the pension fund. The supervision described in this policy rule is based on the plausibility of the outcomes of the continuity analysis. Regarding the investment returns, the principles maintained involve examining whether the parameters are consistent and realistic, analyzing the development of the financial position on the long horizon, determining the financial risks and the probability of having to take emergency measures and assumptions of financial returns. The policy rule suggests that pension funds report the development of the coverage rate[14]. The coverage rate is an indicator for the financial position of the pension fund. The development results from the simulation method from which percentiles can be derived as well.

[11] Besluit financieel toetsingskader voor pensioenfondsen.
[12] Regeling parameters pensioenfondsen.
[13] Beleidsregel uitgangspunten beoordeling continuïteitsanalyse van pensioenfondsen.
[14] The coverage rate equals the total assets divided by the technical liabilities of a pension fund.

Another important indicator of the financial position is the probability of additional required capital in combination with the level of the additional capital in case of a deficit. The supervisor can observe from this result the impact of a deficit in case it occurs. Regarding the assumptions of the returns on investment, the policy rule suggests to display the portfolio returns over the next 15 years. From this view the maintained expected value, variance and skewness of the portfolio return is observable. However, as mentioned above, the underlying assumptions that influence these portfolio return distribution properties are still unknown. Moreover, negative autocorrelation is unobservable using this suggestion. In general, the degree in which the pension funds understand the drivers behind the results from the continuity analysis cannot be observed from the principles maintained by the regulator. Since these drivers are unknown it is impossible to supervise whether assumptions are consistent and realistic.

A possible solution to this issue is to supervise the underlying models applied by the pension fund. However, due to the complexity of the models, and the amount of models to be checked, this inevitably leads to overwhelming amount of work. Alternatively, the regulator could choose to require pension funds to report all underlying assumptions in the model. That is, for each asset the expected value, variance, skewness, kurtosis, autocorrelation, correlation with exogenous variables and the maintained probability distribution of the returns, along with the correlation between assets and the correlation structure must be contained in the report. Then, the outcomes of the continuity analysis can perform as a basis of supervision, and the underlying assumptions can be checked to be consistent and realistic.

### 6.2 Mean Reversion

Section 2 describes the distinction between absolute mean reversion, where the mean is unspecified, and relative mean reversion, in which a specified mean is considered. To examine the effect of mean reversion on the continuity analysis for pension funds, both mean reversion groups are considered separately, due to the different effects on the investment portfolio.

Consider a continuity test in which absolute mean reversion is assumed. Fama and French (1988b) argue that absolute mean reversion implies that stock returns are negatively autocorrelated. This negative relation over time has an implication to the assumptions of the stochastic analysis in two ways. The obvious adjustment to the assumptions in the continuity analysis in Table 6 is to assume negative autocorrelation. The effect of this adjustment is that the long-term risk decreases and therefore the coverage rate increases. Alternatively, the property of negative autocorrelation can be applied to the expected returns. In case a pension fund observes negative returns in the past, it can adjust the expected returns upwards in the first upcoming years. Hence, pension funds could model the expected returns as a function of previously observed returns.

Relative mean reversion is applied in accordance with the specified mean that is assumed. Section 2 discusses that many possible proxies can be considered as an estimate for the mean. These specified means can be grouped into exogenous and endogenous means. Endogenous means are defined to directly depend on another asset in the investment portfolio. Suppose the pension fund invests in several country indexes. Then, according to Balvers et al. (2000)

there is a significant reversion relation between the indexes. This implies that the correlation between assets is negative. Hence, reversion to endogenous means implies negative correlation between assets. This negative correlation decreases the portfolio variance, and therefore has a decreasing effect on long-term risk. Define an exogenous mean to be independently determined from the assets in the pension fund's investment portfolio. The effect of reversion to an exogenously specified mean can be found in the changing expected returns. Notice that in case multiple assets depend on the same mean, also correlation between assets might occur, which influences the variance in the portfolio.

The outcome-based approach of supervising the continuity analysis recognizes the use of mean reversion as follows. The outcomes that are possibly influenced are the expected returns and the (auto)correlations. Firstly, fluctuation of the expected return over time is a clear indicator of mean reversion. Two possible explanations apply to this fluctuation; either negative autocorrelation, or dependency to exogenous variables. Negative autocorrelation implies that negative returns in the past indicate positive returns in the future. After a sequence of low returns, the pension fund that assumes absolute mean reversion could argue that the upcoming returns must be higher. With this methodology, it can be observed that the expected returns in the years short after the financial crisis are higher than the long-term expected returns. The fluctuation in expected returns could also arise due to the reversion to an exogenous mean. Suppose that this mean is expected to fluctuate over time, the expected returns conditional on these means, will also fluctuate.

Secondly, the development of the long term variance in the portfolio returns could indicate that absolute mean reversion is assumed. The use of negative autocorrelation can be checked by looking at the variance ratio over the years (Poterba and Summers, 1988). In case the assets have a constant variance ratio of one over the 15 year horizon, no autocorrelation is incorporated. However, when the ratio decreases as the number of years increase the pension fund assumes negative autocorrelation.

Finally, the observed portfolio variance can be analyzed in order to find an assumption of mean reversion. Relative mean reversion to an endogenous mean implies negative correlation between assets. This assumption decreases the variance of the portfolio of assets. Therefore, assuming relative mean reversion could be recognized by a relatively low assumption of portfolio variance.

### 6.3 Advice

The discussion about the implementation of mean reversion into the regulation is twofold, due to the two distinct mean reversion groups. The advice in this subsection will be mainly based on the analysis from the literature on long-term mean reversion, and the results in this thesis. First consider the group of absolute mean reversion models, in which no specific mean is assumed. Fama and French (1988b) and Poterba and Summers (1988) provide evidence of negative autocorrelation in the 3- to 5-yearly returns. However, Section 2 present many papers discussing the issues of this research and concluding that long-term mean reversion cannot be proven. Furthermore, the analysis in this thesis shows that the speed at which the stock prices revert to their mean changes significantly over time. Any assumption about the period in which stock prices would exhibit mean reversion could result in substantial errors in the continuity analysis. Section 5 shows that for some time intervals, the half-life of a shock to the stock prices might take over 20 years before being corrected for. Therefore, in a continuity analysis of 15 future years the assumption of mean reversion might imply severe biased estimation of the actual stock price process. Lastly, Figure 2 shows a large uncertainty in the point estimators of the half-life in some time intervals. The interval 1979-2005 the 95% confidence interval covers over 34 years and on average this range is over 7 years in all time intervals after World War II[15]. The great uncertainty undermines the reliability of the assumption of mean reversion in a predetermined period.

Due to the lack of evidence of negative autocorrelation and the uncertainty about the term in which mean reversion occurs, the advice about inclusion of negative autocorrelation is as follows. The arguments against the inclusion of negative autocorrelation are comprehensive enough to advice not to take into account absolute mean reversion in the continuity analysis for pension funds.

Second, consider the group of relative mean reversion models in which a specific mean is chosen. In the literature some evidence is presented on relative mean reversion. Criticism on this evidence is still absent, however the choice of the mean process remains an important issue. In case the pension fund chooses to use a benchmark portfolio, Balvers et al. (2000) suggest that mean reversion does occur. However, this thesis shows that the evidence found in their paper depend highly on the time interval chosen. Therefore, applying relative mean reversion to the continuity analysis might result in substantial biases due to the changing mean reversion process. A continuity analysis not based on relative mean reversion avoids these biases and is therefore considered preferable.

The issue of determining the mean process for applying relative mean reversion, and the dependency of the mean reversion process on the time period leads to an advise to not take into account relative mean reversion. Until stronger evidence is presented, regulators should assess continuity analysis incorporating mean reversion with extra caution, due to possibly too optimistic and unrealistic future prospects.

[15] The pre World War II period exhibits even much larger confidence intervals.

# 7 Conclusion

This thesis treats three issues regarding long-term mean reversion. The literature on long-term mean reversion is reviewed by introducing a dichotomy in which a distinction is made between absolute mean reversion and relative mean reversion. Although the two processes differ fundamentally, literature until so far confuses both groups by using the general terminology of mean reversion. Considering absolute mean reversion, the literature review of Section 2 concludes that the criticism on the evidence of mean reversion is severe. Therefore, strong evidence of absolute mean reversion in stock prices remains absent.

Reversion to a specific mean is analyzed based on the research of Balvers et al. (2000). The authors find that mean reversion occurs in international stock market indexes, and that the half-life of the reversion period is approximately 3.5 years. The results from Section 4 show that the findings of Balvers et al. (2000), based on a small time interval from 1970 to 1996, is not representative for the general state of the financial markets. Moreover, the assumption of Balvers et al. (2000) based on normally distributed residuals results in too optimistic estimates of mean reversion. Using a bootstrap method, applied to a much larger time interval 1900 to 2008, mean reversion in stock market indexes is still present, however the half-life of the reversion period is on average must larger than Balvers et al. (2000) suggest.

Section 5 applies the model of Balvers et al. (2000) to all 27 year intervals between 1900 and 2008, and finds a large fluctuation in the speed at which stocks revert to their mean. At a large number of intervals, no significant mean reversion is found. The differences between half-lifes are significant and fluctuate between 2.1 years and 23.6 years. Further, the time interval chosen by Balvers et al. (2000) appears to result in extremely optimistic evidence of mean reversion. This indicates that the findings of Balvers et al. (2000) are biased, and the the usefulness of mean reversion in international stock market indexes could be questioned. Furthermore, the significant differences across time indicate that the mean reversion process fluctuates over the years. Research that fail to take this fluctuation into account might fail to recognize mean reverting behavior of stock prices.

The finding in the literature, in combination with the results from Section 4 and Section 5 are used to analyze the implication and desirability of inserting mean reversion into the continuity analysis for Dutch pension funds. It is argued that mean reversion leads to an adjustment of several parameter assumptions in the simulation model used to analyze the future financial position. This adjustment could lead to a biased view of expectations of pension fund performance in the upcoming 15 years. Therefore, it is advised to not take into account mean reversion, and to be extra cautious with pension funds that do. Especially, future expected returns and autocorrelation, as well as correlation between assets and portfolio variance could be assumed in accordance with absolute mean reversion or mean reversion towards a specified mean by the pension fund. It is important to analyze these underlying assumptions, and make sure that pension funds understand the consequences. To increase the transparency, the regulator should obligate pension funds to publish the underlying assumption along with the continuity analysis.

The existence of long-term mean reversion in stock prices is difficult to prove. Evidence of mean reversion has been found, however the criticism about its significance, and more importantly its usefulness remain very substantial. Intuitively, stock prices tend to perform mean reverting behavior; the performance since the beginning of the financial crisis encourages this intuition. However, to prove its occurrence and to demonstrate its usefulness, further research is required. In order to find evidence for absolute mean reversion, the underlying stock price drivers, like governmental intervention and company specific measures, are required to be examine. Obviously, the mean reversion process fluctuates over time, and research of absolute mean reversion ignores this fact. Therefore, an improvement in estimating absolute mean reversion could be realized, assuming a time varying speed at which stock prices revert to their mean.

Improvements on relative mean reversion can be done by improving the specification of the mean process to which stocks revert. Until so far, the main categories of estimators of the mean are current dividends or earnings, and benchmark portfolios. However, stock prices are driven by future cash flows. Therefore, an interesting study would involve a mean estimated by a large number of future dividend payments. A suggestion would be to use the observed dividends that are paid out to the stock holder in the future, and find the mean process for past stock price observations. The fluctuation around this mean process could give a new insight in the mean reverting behavior of stock prices.

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## A Seemingly Unrelated Regression

Seemingly unrelated regression (SUR) can be applied to multiple systems of equations, with nonzero correlation between the systems. The SUR method is feasible for any collection of systems of equations, however this appendix discusses its application to the panel data model defined in Section 3. Consider the model of equation (7), where $N$ systems of equations are considered. For each country $i = 1, \ldots, N$ the system of equations is defined as

$$
\begin{aligned}
\boldsymbol{y}^i &= \begin{bmatrix} -\boldsymbol{z}^i & \boldsymbol{X}^i \end{bmatrix} \begin{bmatrix} \lambda \\ \theta^i \end{bmatrix} + \omega^i \\
&= \begin{bmatrix} (p_k^b - p_k^i) & 1 & (r_k^i - r_k^b) & \ldots & (r_1^i - r_1^b) \\ (p_{k+1}^b - p_{k+1}^i) & 1 & (r_{k+1}^i - r_{k+1}^b) & \ldots & (r_2^i - r_2^b) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (p_{T-1}^b - p_{T-1}^i) & 1 & (r_{T-1}^i - r_{T-1}^b) & \ldots & (r_{T-k}^i - r_{T-k}^b) \end{bmatrix} \begin{bmatrix} \lambda \\ \alpha^i \\ \phi_1^i \\ \vdots \\ \phi_k^i \end{bmatrix} + \begin{bmatrix} \omega_1^i \\ \vdots \\ \omega_{T-k}^i \end{bmatrix}, \quad (10)
\end{aligned}
$$

where $p_t^b$ equals the log price of the benchmark and $p_t^i$ equals the log price for stock index of country $i$, for $t = k, \ldots, T-1$. Further, $r_t^b$ and $r_t^i$ are the continuously compounded returns of the benchmark and the stock index of country $i$, respectively, for $t = 1, \ldots, T-1$, $\lambda$ is the speed of reversion, and $\theta^i = \begin{bmatrix} \alpha^i & \phi_1^i & \ldots & \phi_k^i \end{bmatrix}'$ is a country specific parameter. The set of systems of equations may be written as

$$
\boldsymbol{y} := \begin{bmatrix} \boldsymbol{y}^1 \\ \boldsymbol{y}^2 \\ \vdots \\ \boldsymbol{y}^N \end{bmatrix} = \begin{bmatrix} -\boldsymbol{z}^1 & \boldsymbol{X}^1 & \mathbf{0} & \ldots & \mathbf{0} \\ -\boldsymbol{z}^2 & \mathbf{0} & \boldsymbol{X}^2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{z}^N & \mathbf{0} & \mathbf{0} & \ldots & \boldsymbol{X}^N \end{bmatrix} \begin{bmatrix} \lambda \\ \theta^1 \\ \theta^2 \\ \vdots \\ \theta^N \end{bmatrix} + \begin{bmatrix} \omega^1 \\ \omega^2 \\ \vdots \\ \omega^N \end{bmatrix}
$$

or

$$
\boldsymbol{y} = \boldsymbol{X}\beta + \omega. \quad (11)
$$

The autocorrelation of the residuals in equation (10) is purged due to the inclusion of a number of lagged values of the return deviations. Therefore, it is assumed that the residuals from equation (10) are serially uncorrelated and have an equal variance $\sigma_i^2$, for each country $i$. The covariance between country $i$ and $j$ is considered nonzero and defined as $\mathrm{Cov}(\omega_t^i, \omega_t^j) = \sigma_{ij}$, for all $t = 1, \ldots, T-k$. By definition the variance covariance matrix of $\omega$ in equation (11) equals

$$
\begin{aligned}
\mathbf{\Sigma} = E(\omega\omega') &= \begin{bmatrix} E(\omega^1\omega^{1'}) & E(\omega^1\omega^{2'}) & \cdots & E(\omega^1\omega^{N'}) \\ E(\omega^2\omega^{1'}) & E(\omega^2\omega^{2'}) & \cdots & E(\omega^2\omega^{N'}) \\ \vdots & \vdots & \ddots & \vdots \\ E(\omega^N\omega^{1'}) & E(\omega^N\omega^{2'}) & \cdots & E(\omega^N\omega^{N'}) \end{bmatrix} \\
&= \begin{bmatrix} \sigma_{11}\boldsymbol{I} & \sigma_{12}\boldsymbol{I} & \cdots & \sigma_{1N}\boldsymbol{I} \\ \sigma_{21}\boldsymbol{I} & \sigma_{22}\boldsymbol{I} & \cdots & \sigma_{2N}\boldsymbol{I} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1}\boldsymbol{I} & \sigma_{N2}\boldsymbol{I} & \cdots & \sigma_{NN}\boldsymbol{I} \end{bmatrix} \\
&= \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix} \otimes \boldsymbol{I} \\
&= \mathbf{\Sigma}_c \otimes \boldsymbol{I},
\end{aligned}
$$

where the identity matrix has dimension $(T-k) \times (T-k)$. Notice that the elements of $\mathbf{\Sigma}_c$ are unknown. To apply the SUR method to the generalized least squares estimation these elements must be obtained. Zellner (1962) suggests to apply a feasible GLS by estimating the systems of equations by OLS and subsequently estimate the variance-covariance matrix $\mathbf{\Sigma}$ using the OLS residuals. Then, the estimator for $\mathbf{\Sigma}$ can be used directly to determine $\hat{\beta}_{GLS}$ and $\text{var}(\hat{\beta}_{GLS})$ in equations (8) and (9).

# B Bootstrap Method

This appendix introduces an alternative method of determining the correct elements of $\mathbf{\Sigma}$ in a generalized linear model. This method is called the bootstrap and derives its name from the old saying about pulling yourself up by your own bootstraps. The saying reflects the procedure of the bootstrap method. In a bootstrap simulation, the observed sample residuals are assumed to equal the population of residuals. The distribution of the residuals is obtained by assigning an equal probability of occurrence to each observation. Subsequently, residuals are drawn from this empirical distribution and applied to find bootstrapped observations. These bootstrapped observations can be used to derive a bootstrap estimated parameter. Repeating this procedure a large number of times results in the small sample bias corrected parameter distribution.

This computer intensive simulation method was initially described by Efron (1979), and extended to generalized least squares by Freedman and Peters (1984). Both papers argue that there are two statistical problem that can be tackled by the bootstrap method. First, the bootstrap can be applied when it is impossible to derive the distribution of an estimator analytically. Second, in small samples where asymptotic theory can be questioned, the bootstrap could provide a good estimate for the small sample distribution of a parameter estimate. The second situation applies to the data of long-horizon stock returns.

The bootstrap can be applied to tests for no mean reversion, i.e. the hypothesis that $\lambda = 0$. In the SUR method of Appendix A the estimator for $\beta$, its standard error and the distribution of $\hat{\beta}_{GLS}$ all depend on asymptotic theory. In case the data at hand contains a small number of observations the $p$-values based on these estimated values are often too optimistic (Freedman and Peters, 1984). Therefore, in order to test for mean reversion, $\hat{\beta}_{GLS}$ and its standard errors are generated by use of a bootstrap simulation.

To test for $\lambda = 0$ the best way to proceed is to derive the parameters under the null hypothesis and consider the small sample distribution of $\hat{\lambda}_{GLS}$ given $\lambda = 0$. Then the $p$-value for the sample estimator $\hat{\lambda}_{GLS}$ can subsequently be derived from this bootstrapped empirical small sample distribution of $\hat{\lambda}_{GLS}$ under the null hypothesis. The procedure of finding the empirical small sample distribution under the null hypothesis incorporates taking the following steps:

1. Estimate the empirical values of $\hat{\alpha}$ and $\hat{\phi}$ under the null hypothesis that $\lambda = 0$, i.e. estimate the model in equation (6), and store the residuals $\hat{\omega}$.
2. Use the residuals from Step 1 to generate the bootstrapped residuals $\hat{\omega}^*$ by randomly selecting $T - k$ vectors. Each vector equals a row of error terms from $N$ countries at some time instance between $t = 1$ and $T - k$.
3. Recursively calculate the values of the return deviations using $\hat{\omega}^*$ in the model of equation (6), under the assumption that $\lambda = 0$, to arrive at a bootstrapped set of return deviations: $(r_t^i - r_t^b)^*$, for $t = 1, \dots, T - k$ and $i = 1, \dots, N$.
4. Apply a seemingly unrelated regression to the bootstrapped deviations $(r_t^i - r_t^b)^*$ to find a bootstrapped estimator $\hat{\beta}^*_{GLS}$.
5. Repeat Step 2 to 4 a large number of times, say $B = 500$, to find the empirical small sample distribution of $\hat{\beta}_{GLS}$ under the null hypothesis.

In this thesis, the bootstrap method is applied is to find the small sample bias corrected speed of reversion and its confidence bounds. The goal of estimating the corrected lambdas and confidence bounds is twofold. First, to derive the bias corrected half-life of the stock price deviations and the corresponding confidence bounds in Section 4. Second, to compare the several estimators over time to check the consistency of the speed of reversion and its significance over time in Section 5. The procedure of determining the small sample bias corrected estimator is similar to the procedure described above, however applied to different initial values for $\lambda$. The goal is to specify the initial value $\lambda$ in such a way that the bootstrapped empirical small sample distribution has a median equal to the original estimator $\hat{\lambda}_{GLS}$ from the original data. To find this initial value, a large range of initial speeds of reversion is considered. For each initial value $\lambda$ the bootstrapped median is compared to the empirical estimator of $\lambda$. The initial values considered are $(-0.12, -0.10, \dots, 0.50)$, and linear interpolation is used to find the closest value for the small sample bias corrected estimator of $\lambda$. To find the confidence bounds, the same procedure is executed, however applied to the 2.5 and 97.5 percentile.

Consider the following example to illustrate this methodology. Suppose for the interval 1946-1970 the generalized least squares estimate for $\lambda_{GLS}$ equals 0.200. For each initial values $\lambda$ from $\{-0.12, -0.10, \ldots, 0.50\} \in \mathbb{R}^{32}$, the empirical small sample distribution is estimated. Due to the finite sample, the distribution of $\hat{\lambda}_{GLS}$ is not centered around its real value, but tends to be biased upwards. Therefore, the median of this distribution would give a good approximation on what the empirical estimator would be in case the actual estimator is equal to the initial value. Moreover, whenever one of the initial values displays a median bootstrapped value equal to 0.200, it is most likely that this will be the actual $\lambda$ in the interval 1946-1970. For the confidence bounds, the same 32 small sample distributions are considered, only now for each distribution the upper and lower percentile is compared to the empirical estimated value of 0.200. In case the lower percentile of some distribution equals 0.200, then this would indicate the highest actual value of $\lambda$ that can be expected in this small sample. On the other hand, when the higher percentile of some small sample distribution equals 0.200, this is the lowest actual value with 2.5% confidence.