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# Economic Growth and Longevity Risk with Adverse Selection

# Economic growth and longevity risk with adverse selection[*]

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**Abstract**

We study adverse selection on annuity markets in a general-equilibrium model of the closed economy. Agents differ in their health type and invest their assets on the annuity market. Without informational asymmetries the model features a separating equilibrium in which each agent obtains an actuarially fair insurance. If the individual health types are unobservable to the annuity firms there exists a pooling equilibrium in which all agents annuitize at a common pooling rate. At this common rate unhealthy agents would eventually like to borrow but this would reveal their true health type. As a consequence, they face a self-imposed borrowing constraint. Surprisingly, the welfare and growth effects of the informational asymmetries are small.

**JEL codes**: D52, D91, E10, J10.

**Keywords**: Annuity markets, adverse selection, endogenous growth, overlapping generations, demography.

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# 1 Introduction

Economic theory suggests that life annuities are very attractive insurance instruments in the presence of longevity risk. This result was first articulated in the seminal paper by Yaari (1965) and was recently recast in a much more general setting by Davidoff, Brown, and Diamond (2005). The intuition behind this result is not very difficult: annuities insure against the risk of outliving one's assets.

Over the last few decades, however, more and more evidence has been unearthed hinting at the existence of an *annuity puzzle*: in reality very few individuals purchase life annuities despite their theoretical attractiveness. Friedman and Warshawsky (1990, pp. 136-7) give the following potential explanations for the low participation in private annuity markets. First, individuals may want to leave bequests to their offspring. Second, individuals may hold other types of annuities, e.g. in the form of social security and private pensions (social annuities). Third, private annuities may be priced unattractively because of transaction costs and taxes, excessive monopoly profits earned by annuity firms, and adverse selection.[1] A fourth explanation is that family risk sharing may act an as incomplete annuity market, a result first proposed by Kotlikoff and Spivak (1981).

In this paper we restrict attention to the adverse selection channel.[2] Intuitively, adverse selection arises because individuals who believe themselves to be healthier than average are more likely to buy annuities, i.e. the high-risk types are overrepresented in the clientele of annuity firms and annuity pricing cannot be based on average population mortality.

The objective of our paper is to study the growth and welfare implications of adverse selection in the annuity market. Our core model is built on the following assumptions. First, we postulate a simple general equilibrium model of a closed economy featuring endogenous growth. Second, we assume that the economy is populated by overlapping generations of heterogeneous finitely-lived agents. Individual agents know their own death probability profile, but annuity firms cannot observe an agent's health type (neither directly nor indirectly). The mortality process is modeled realistically and closely tracks existing demo-

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[1] Following the initial research by Friedman and Warshawsky (1988, 1990), a large subsequent literature has emerged. See for example Mitchell *et al.* (1999), Finkelstein and Poterba (2002, 2004), and Finkelstein *et al.* (2009).

[2] Heijdra and Mierau (2009) study the general equilibrium implications of imperfect annuities under the excess monopoly profit interpretation.

graphic data. We distinguish two types of agents, namely healthy and unhealthy, and we assume that their respective population shares are constant.

Third, we assume perfectly competitive annuity markets. Our informational assumptions imply that firms offer *linear* annuity contracts, cf. Pauly (1974) and Abel (1986). Under such contracts the insurer can only choose the price of the annuity and cannot achieve complete market separation by offering non-linear price-quantity contracts.[3]

The main findings of our analysis are as follows. First, if health status were observable or could be "credibly signaled" to insurers, then there would be a separating equilibrium (SE) in which each health type would get actuarially fair perfect insurance. We consider the case of a patient economy in which all types would be net savers during life. In the SE, however, healthy individuals have a huge incentive to misrepresent their health status ("by cheating" and claiming to be a low-risk type) thus destroying market separation. The SE is thus a hypothetical case acting as a benchmark.

Second, with health being an unobservable attribute, perfect competition in the annuity market will result in a pooling equilibrium (PE). The equilibrium pooling rate is an asset-weighted average of individual mortality rates, a result derived in a partial equilibrium context by Sheshinski (2008). In the PE, the unhealthy (low-risk types) get a less than actuarially fair rate (as stressed in the literature), but the healthy (high-risk types) get a *better than actuarially fair rate*. This result shows that Friedman and Warshawsky (1990, pp. 147-152) only consider one side of the coin by restricting attention to individuals facing less than actuarially fair annuity returns (see their Tables V and VI). In contrast, in our general equilibrium model the healthy annuitants benefit for part of their life from the presence of unhealthy annuitants.

Third, in the PE the unhealthy encounter a "self-imposed" borrowing constraint if they live long enough. Intuitively, as the unhealthy get close to their maximum attainable age, the pooling rate prompts such individuals to become net borrowers. But borrowing would reveal their health status, so the best the unhealthy can do is to impose a borrowing constraint on themselves during their autumn years. It must be stressed that this asset depletion result

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[3]Alternative equilibrium concepts that can be used to deal with adverse selection are the ones suggested by Rothschild and Stiglitz (1976) and Wilson (1977). See Eichenbaum and Peled (1987) for an application of the Rothschild-Stiglitz concept. See Walliser (2000, pp. 376-7) and below for a defense of the linear pricing assumption.

is not exogenously imposed (as in the partial equilibrium studies of Friedman and War-shawsky (1990, p. 147) and Walliser (2000, pp. 378-9)) but follows from the internal logic of the model. Hence, our model yields a consistent explanation why not everybody annuitizes in a general equilibrium model with risk pooling.

Fourth, for a plausibly calibrated version of the core model we find that the SE is only slightly better in growth and welfare terms than the PE. Hence, the underlying information asymmetry and the resulting adverse selection effects in the annuity market do not seem to cause quantitatively large growth and welfare effects in a general equilibrium setting.

Fifth, all of these findings are robust to (a) alternative assumptions regarding labour market participation and retirement and (b) to a different specification of the economic growth process.

The structure of our paper is as follows. Section 2 presents the model. Section 3 states the key informational assumptions and studies the balanced growth path for the (hypothetical) separating equilibrium and the pooling equilibrium. This section also presents a plausible calibration and visualization of the different equilibria as well as their welfare properties. Section 4 reports some robustness checks. In particular it examines the role of endogenous versus exogenous growth and it incorporates a pay-as-you-go pension system and labour force retirement. Finally, section 5 restates the main results and presents some possible extensions. The paper also contains two brief mathematical appendices.

## 2    Model

### 2.1    Consumers

#### 2.1.1    Individual behaviour

Individuals differ according to their health status acquired at birth. From the perspective of birth, the expected remaining lifetime utility function of a health type $j$ individual is given by:

$$\Lambda_j(v,v) = \int_v^{v+\bar{D}_j} \frac{\bar{c}_j(v,\tau)^{1-1/\sigma} - 1}{1 - 1/\sigma} e^{-\rho(\tau-v) - M_j(\tau-v)} d\tau, \tag{1}$$

where $v$ is the birth date, $\bar{D}_j$ is the maximum attainable age for this type of agent, $\bar{c}_j(v,\tau)$ is consumption, $\sigma$ is the intertemporal substitution elasticity ($\sigma > 0$), $\rho$ is the pure rate of time

preference, and $e^{-M_j(\tau-v)}$ is the probability that the agent is still alive at some future time $\tau$ $(\geq v)$.[4] Here, $M_j(\tau-v) \equiv \int_0^{\tau-v} \mu_j(s)ds$ denotes the cumulative mortality rate and $\mu_j(s)$ is the instantaneous mortality rate of an agent of age $s$, where $0 \leq s \leq \bar{D}_j$. This rate is strictly increasing and convex in age, $\mu_j'(s) > 0$ and $\mu_j''(s) > 0$, and features $\lim_{s \to \bar{D}_j} \mu_j(s) = +\infty$.

The agent's budget identity is given by:

$$\dot{\bar{a}}_j(v,\tau) = \left[r + p_j(\tau-v)\right]\bar{a}_j(v,\tau) + w(\tau) - \bar{c}_j(v,\tau), \tag{2}$$

where $\bar{a}_j(v,\tau)$ is real financial wealth, $r$ is the interest rate (a constant, see below), and $w(\tau)$ is the wage rate. In the spirit of Yaari (1965), we assume that agents can purchase continuous annuities to insure against longevity risk. Annuity contracts are recontracted at each moment in time. Without a bequest motive, financial wealth is fully annuitized so $\bar{a}_j(v,\tau)$ is also the agent's demand for annuities. Since an agent's age at time $\tau$ is directly observable to the insurer, the net return on annuities, $p_j(\tau-v)$, depends on it. Labour supply is exogenous and each agent supplies a single unit of labour throughout life, i.e. in the main part of the paper we abstract from retirement.[5]

At time $v$, the agent chooses paths for consumption and financial assets in order to maximize lifetime utility (1) subject to the flow budget identity (2) and a solvency condition, taking as given the initial level of financial assets, $\bar{a}_j(v,v) = 0$. In the absence of borrowing constraints, the agent's optimal plans for $v \leq t \leq v + \bar{D}_j$ are fully characterized by:

$$\frac{\dot{\bar{c}}_j(v,t)}{\bar{c}_j(v,t)} = \sigma\left[r + p_j(t-v) - \mu_j(t-v) - \rho\right], \tag{3}$$

$$\bar{c}_j(v,v) = \frac{\int_v^{v+\bar{D}_j} w(\tau)e^{-r(\tau-v)-P_j(\tau-v)}d\tau}{\int_v^{v+\bar{D}_j} e^{-(1-\sigma)\left[r(\tau-v)+P_j(\tau-v)\right]-\sigma\left[\rho(\tau-v)+M_j(\tau-v)\right]}d\tau}, \tag{4}$$

$$\bar{a}_j(v,t)e^{-r(t-v)-P_j(t-v)} = \int_v^t w(\tau)e^{-r(\tau-v)-P_j(\tau-v)}d\tau$$
$$-\bar{c}_j(v,v)\int_v^t e^{-(1-\sigma)\left[r(\tau-v)+P_j(\tau-v)\right]-\sigma\left[\rho(\tau-v)+M_j(\tau-v)\right]}d\tau, \tag{5}$$

where $P_j(\tau-v) \equiv \int_0^{\tau-v} p_j(s)ds$ is the cumulative net annuity return factor. Equation (3) is the 'consumption Euler equation', relating the optimal time profile of consumption to

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[4]For a detailed derivation of the lifetime utility function in the presence of mortality risk, see Heijdra and Romp (2008, pp. 91–92). The assumption of a maximum attainable age is made for computational convenience only.

[5]In section 4.1, however, we introduce a simple pension system and mandatory retirement.

the difference between the annuity rate of interest $(r + p_j (\tau - v))$ and the total rate of felicity discounting due to impatience and mortality $(\rho + \mu_j (\tau - v))$. Equation (4) shows that consumption at birth is proportional to human wealth (the numerator), consisting of the annuitized value of wages. Finally, the planned path of financial wealth is defined in (5). It is easy to see that financial assets are zero at birth and at the date of certain death, $\bar{D}_j$. The exact form of the wealth profile depends on the specific type of annuity market equilibrium (see section 3).

Below we encounter equilibria in which type $j$ agents experience a binding borrowing constraint from age $\bar{S}_j$ onward. In that case equations (3) and (5) are valid only for $0 \le t - v \le \bar{S}_j$, $\bar{a}_j (v, t) = 0$ and $\bar{c}_j (v, t) = w(t)$ for $\bar{S}_j \le t - v \le \bar{D}_j$, and $\bar{S}_j$ replaces $\bar{D}_j$ in (4).

### 2.1.2 Demography

We allow for a non-zero rate of population growth but impose that the relative population proportion of people of different health types is constant. Since health groups are distinguished by their mortality process, this requirement furnishes the following condition:

$$\beta_j \int_0^{\bar{D}_j} e^{-ns - M_j(s)} ds = 1, \tag{6}$$

where $\beta_j$ is the crude birth rate of type $j$ cohorts, and $n$ is the growth rate of the population. For a given value of $n$ and a given mortality process $M_j (s)$, equation (6) defines the coherent solution for $\beta_j$. The newborn cohort of type $j$ at time $v$ is given by $L_j (v, v) = \pi_j \beta_j L(v)$ where $L(v)$ is the total population at time $v$ and $\pi_j$ is the fraction of type $j$ people in the population ($\sum_j \pi_j = 1$). Finally, the relative cohort size of type $j$ agents of age $t - v$ evolves according to:

$$l_j (v, t) \equiv \frac{L_j (v, t)}{L(t)} = \begin{cases} \beta_j \pi_j e^{-n(t-v) - M_j(t-v)} & \text{for } 0 \le t - v \le \bar{D}_j \\ 0 & \text{for } t - v > \bar{D}_j \end{cases} \tag{7}$$

Intuitively, the *relative* size of the type $j$ cohort declines with age because the aggregate population grows over time (first cause) and cohort members die (second cause).

### 2.1.3 Aggregate household behaviour

Armed with equation (7), it is possible to compute per capita values for consumption and assets. We restrict attention to the balanced growth path along which wages grow at a constant

exponential rate, $g$ (see section 3 below). It follows that:

$$w(t) = w(v) e^{g(t-v)}. \tag{8}$$

Allowing for a borrowing constraint at age $\bar{S}_j$ and using (8) we find that per capita consumption of type $j$ agents, $c_j(t) \equiv \int_{t-\bar{D}_j}^{t} l_j(v,t) \bar{c}_j(v,t) dv$, can be written as:

$$\frac{c_j(t)}{w(t)} = \beta_j \pi_j \left[ \frac{\bar{c}_j(v,v)}{w(v)} \int_0^{\bar{S}_j} e^{-(n+g)s-(\sigma+1)M_j(s)+\sigma(r-\rho)s+\sigma P_j(s)} ds + \int_{\bar{S}_j}^{\bar{D}_j} e^{-ns-M_j(s)} ds \right], \tag{9}$$

where $\bar{c}_j(v,v)/w(v)$ is independent of the generations index $v$. By aggregating over all health types, per capita consumption is obtained, i.e. $c(t) \equiv \sum_j c_j(t)$.

In a similar fashion we find that per capita asset holdings of type $j$ agents, $a_j(t) \equiv \int_{t-\bar{D}_j}^{t} l_j(v,t) \bar{a}_j(v,t) dv$, evolves over time according to:

$$\dot{a}_j(t) = (r-n) a_j(t) + \pi_j w(t) - c_j(t) + \int_{t-\bar{D}_j}^{t} \left[ p_j(t-v) - \mu_j(t-v) \right] l_j(v,t) \bar{a}_j(v,t) dv. \tag{10}$$

It follows that per capita assets, $a(t) \equiv \sum_j a_j(t)$, satisfy the following differential equation:

$$\dot{a}(t) = (r-n) a(t) + w(t) - c(t) + \Xi(t), \tag{11}$$

where $\Xi(t)$ is defined as:

$$\Xi(t) \equiv \sum_j \int_{t-\bar{D}_j}^{t} l_j(v,t) \left[ p_j(t-v) - \mu_j(t-v) \right] \bar{a}_j(v,t) dv. \tag{12}$$

## 2.2 Firms

In the spirit of Romer (1989), we assume that there exist strong external effects between private firms in the economy. The economy features a large and fixed number, say $N_0$, of identical, perfectly competitive firms. The technology available to firm $i$ is given by:

$$Y_i(t) = \Omega(t) K_i(t)^\varepsilon L_i(t)^{1-\varepsilon}, \qquad 0 < \varepsilon < 1, \tag{13}$$

where $Y_i(t)$ is output, $K_i(t)$ is the capital input, $L_i(t)$ is the labour input, and $\Omega(t)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$w(t) = (1-\varepsilon) \Omega(t) k_i(t)^\varepsilon, \tag{14}$$

$$r(t) + \delta = \varepsilon \Omega(t) k_i(t)^{\varepsilon-1}, \tag{15}$$

where $k_i(t) \equiv K_i(t)/L_i(t)$ is the capital intensity. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity such that $k_i(t) = k(t)$ for $i = 1, \ldots, N_0$. This feature enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Romer (1989) to a non-constant population, we assume that the inter-firm externality takes the following form:

$$\Omega(t) = \Omega_0 k(t)^{1-\varepsilon}, \tag{16}$$

where $\Omega_0$ is a positive constant, $k(t) \equiv K(t)/L(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $L(t) \equiv \sum_i L_i(t)$ is aggregate employment. According to (16), total factor productivity depends positively on the economy-wide capital intensity. Hence if an individual firm $i$ raises its capital intensity, then *all* firms in the economy benefit somewhat as a result because the general productivity indicator rises for all of them. Using (16), equations (13)–(15) can now be rewritten in aggregate terms:

$$Y(t) = \Omega_0 K(t), \tag{17}$$

$$w(t)L(t) = (1-\varepsilon)Y(t), \tag{18}$$

$$r(t) = r = \varepsilon\Omega_0 - \delta, \tag{19}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $\varepsilon\Omega_0 > n + \delta$. The macroeconomic technology is linear in the capital stock and the interest rate is constant and exceeds the rate of population growth.

## 3 Balanced growth path

In this section we study the steady-state features of the general equilibrium growth model. We adopt the following set of assumptions regarding the market for annuities.

**Assumption (A1)** The annuity market is perfectly competitive. A large number of firms offer annuity contracts to individuals. Firm entry and exit is unrestricted.

**Assumption (A2)** Annuity firms do not use up any real resources.

**Assumption (A3)** The annuitant's health status is private information and cannot be observed by the annuity companies. Annuity firms know all the features of the mortality process of each health group.

**Assumption (A4)** The annuitant's age is public information and can thus be observed by the annuity companies.

**Assumption (A5)** Annuitants can buy multiple annuities for different amounts and from different annuity firms. Individual annuity firms cannot monitor an annuitant's holdings with their competitors.

The existence of a pooling equilibrium depends critically on the joint validity of assumptions (A3) and (A5). Under these assumptions, annuity firms cannot distinguish their customers' health type. Even though healthy annuitants are richer than unhealthy annuitants (both in reality and in our model), and thus feature a higher total demand for annuities, they can nevertheless hide this fact by buying small amounts from several companies. Consequently, annuity firms are forced to apply *linear pricing*: they can only control the rate of return on annuities and cannot set prices and quantities simultaneously in order to induce full information revelation. Intuitively, the annuity market differs from regular insurance markets in that the death of an annuitant ends the insurer's liability instead of creating it. This rules out a system of withholding payments and investigating contract compliance that is often used in markets for fire- or car insurance (Walliser, 2000, pp. 376-7).

By assumption (A4), annuity firms can observe each annuitant's age so in the pooling equilibrium there is a single pooling rate, $\bar{p}(u)$, for healthy and unhealthy annuitants of age $u$. There is market segmentation in the sense that the annuity market consists of separate submarkets for each age group or cohort. By assumption (A1), the expected profit in each submarket is zero. With large cohorts, probabilities and frequencies coincide so that actual profit in each submarket is also zero. Finally, assumption (A2) ensures that there is no loading factor on annuities.

Before turning to a detailed study of the pooling equilibrium in subsection 3.2, we first discuss the benchmark case for which assumption (A3) is violated and annuity firms can observe each annuitant's health type. This is the separating equilibrium studied in subsection 3.1.

(a) surviving fraction $e^{-M_j(u)}$

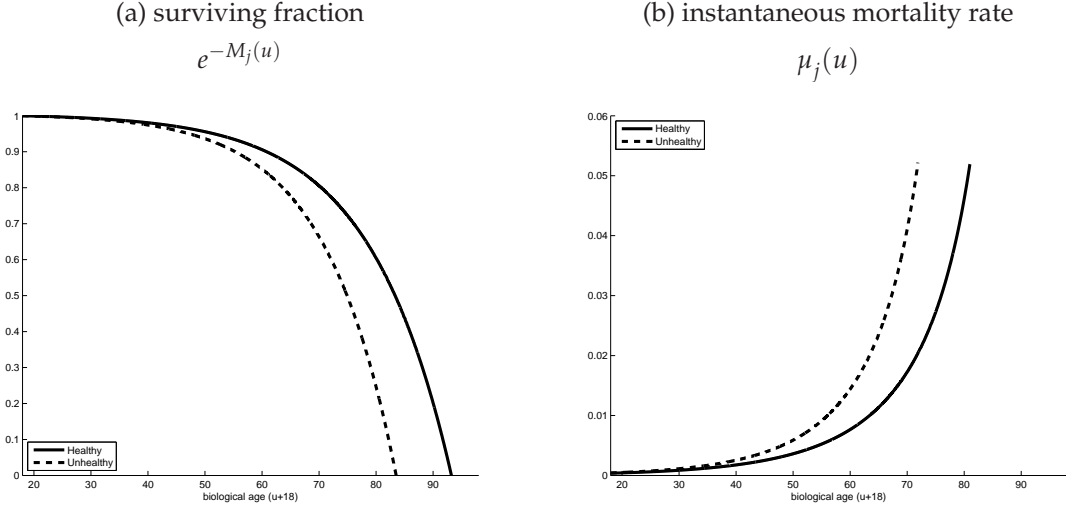(b) instantaneous mortality rate $\mu_j(u)$

Figure 1: Demographics

Although the model has been constructed to allow for an arbitrary number of health types, we simplify the discussion from here on by distinguishing only two health groups, namely healthy agents ($j = H$) and unhealthy agents ($j = U$). We furthermore adopt the mortality process of Boucekkine *et al.* (2002) which takes the following form:

$$e^{-M_j(s)} \equiv \frac{\eta_0 - e^{\eta_{1j}s}}{\eta_0 - 1}, \qquad 0 \le s \le \bar{D}_j \equiv \frac{1}{\eta_{1j}} \ln \eta_0, \tag{20}$$

where $\eta_0 > 1$ and $\eta_{1j} > 0$. The implied instantaneous mortality rate is given by:

$$\mu_j(s) \equiv M_j'(s) = \frac{\eta_{1j}e^{\eta_{1j}s}}{\eta_0 - e^{\eta_{1j}s}}. \tag{21}$$

This mortality process satisfies the assumption made in the text below equation (1). To capture the relative health status of the two groups we set $\eta_{1U} = \lambda\eta_{1H}$ with $\lambda > 1$. This parameterization implies that the maximum attainable age for the healthy exceeds the one for the unhealthy, i.e. $\bar{D}_H = \lambda\bar{D}_U$. Furthermore, the instantaneous mortality rate is uniformly higher for the unhealthy, i.e. $\mu_U(u) > \mu_H(u)$ for $0 \le u \le \bar{D}_U$. See Figure 1 below for a visualization of these results. In that figure and throughout the paper an "economic age" of $u = 0$ corresponds to a biological age of 18 years, i.e. we assume that independent economic decision making starts at the age of maturity.

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## 3.1 Separating equilibrium

If annuity firms are able to observe an annuitant's health type, then they will set the net return on annuities equal to the relevant mortality rate, $p_j(\tau - v) = \mu_j(\tau - v)$ and thus also $P_j(\tau - v) = M_j(\tau - v)$. It follows from (3), (5), and (8) that:

$$\frac{\dot{\bar{c}}_j(v, v+u)}{\bar{c}_j(v, v+u)} = \sigma[r - \rho], \tag{22}$$

$$\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - M_j(u)} = \int_0^u e^{-(r-g)s - M_j(s)} ds - \frac{\bar{c}_j(v, v)}{w(v)} \int_0^u e^{-(1-\sigma)rs - \sigma\rho s - M_j(s)} ds, \tag{23}$$

where $u \equiv t - v$ is the agent's age at time $t$. The instantaneous mortality rate does not feature in (22) because households fully insure against the unpleasant effects of lifetime uncertainty (Yaari, 1965). It is easy to see from (23) that financial assets are positive throughout the agent's life.

**Proposition 1.** *Consider the separating equilibrium (SE) in which annuity firms can observe the health type of annuitants. Provided $\sigma r - g > \rho$, agents of all health types are net savers throughout life, i.e. $\bar{a}_j(v, v) = \bar{a}_j(v, v + \bar{D}_j) = 0$ and $\bar{a}_j(v, v + u) > 0$ for $0 < u < \bar{D}_j$.*

*Proof.* See Appendix A. □

The key equations of the general equilibrium model with a separated annuity market are collected in Table 1. The expressions in (T1.1) follow in a straightforward fashion from (4) and (8) above. Equation (T1.2) is obtained by setting $\bar{S}_j = \bar{D}_j$ in (9) and noting the definition of $c(t)$. The growth expression, equation (T1.3), follows readily from (11) by noting two features of the model. First, since claims on the capital stock are the only financial assets available, capital market equilibrium ensures that $a(t) = k(t)$. Second, since $p_j(t-v) = \mu_j(t-v)$ it follows from (12) that $\Xi(t) = 0$ for all $t$. There is no redistribution between health groups because each group receives the net return befitting its mortality profile. Finally, equation (T1.4) is obtained by combining equations (17)–(18).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. On the one hand, for a given macroeconomic growth rate $g$, (T1.1) determines scaled consumption at birth for the two health types. On the other hand, for given values of scaled consumption at birth, (T1.2)–(T1.4) yield general equilibrium solutions for $c(t)/w(t)$, $g$, and $w(t)/k(t)$.

Table 1: Balanced growth in the separating equilibrium[a]

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*(a) Microeconomic relationships:*

$$\frac{\bar{c}_j(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_j} e^{-(r-g)s - M_j(s)} ds}{\int_0^{\bar{D}_j} e^{-(1-\sigma)rs - \sigma\rho s - M_j(s)} ds}, \qquad j \in \{H, U\} \tag{T1.1}$$

*(b) Macroeconomic relationships:*

$$\frac{c(t)}{w(t)} = \sum_{j \in \{H,U\}} \beta_j \pi_j \frac{\bar{c}_j(v,v)}{w(v)} \int_0^{\bar{D}_j} e^{-(n+g)s - M_j(s) + \sigma(r-\rho)s} ds \tag{T1.2}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r - n + \left[1 - \frac{c(t)}{w(t)}\right] \frac{w(t)}{k(t)} \tag{T1.3}$$

$$\frac{w(t)}{k(t)} = (1 - \varepsilon)\Omega_0 \tag{T1.4}$$

**Note**. [a]Endogenous are scaled newborn consumption by type $j$, $\bar{c}_j(v,v)/w(v)$, the steady-state growth rate, $g$, the wage-capital ratio, $w(t)/k(t)$, and the consumption-wage ratio, $c(t)/w(t)$. There are two types of agents, healthy (subscript $H$) and unhealthy (subscript $U$). $M_j(s)$, $\bar{D}_j$, $\beta_j$, and $\pi_j$ stand for, respectively, the cumulative mortality rate at age $s$, the maximum attainable age, the crude birth rate, and the population fraction of type $j$ agents. $n$ is the population growth rate, $\rho$ is the rate of time preference, $\varepsilon$ is the capital coefficient in the technology, $\sigma$ is the intertemporal substitution elasticity, and $\Omega_0$ is the scale factor in the technology. The interest rate is $r \equiv \varepsilon\Omega_0 - \delta$, where $\delta$ is the depreciation rate of capital.

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In order to visualize the properties of the model and to quantify the effects of informational asymmetries and adverse selection on the general equilibrium allocation, we calibrate the model in a plausible fashion. We assume that the rate of population growth is one percent per annum ($n = 0.01$). We use data from biological age 18 onward for the cohort born in the Netherlands in 1960 and estimate the following model:

$$
\begin{aligned}
SURV_i = {} & \mathbf{d}_H(u_i) \frac{\beta_H \pi_H}{\beta_H \pi_H + \beta_U(1-\pi_H)} \frac{\eta_0 - e^{\eta_{1H} u_i}}{\eta_0 - 1} \\
& + \mathbf{d}_U(u_i) \frac{\beta_U(1-\pi_H)}{\beta_H \pi_H + \beta_U(1-\pi_H)} \frac{\eta_0 - e^{\eta_{1U} u_i}}{\eta_0 - 1} + \varepsilon_i,
\end{aligned}
\tag{24}
$$

where $SURV_i$ is the actual population fraction surviving up to age $u_i$, $\varepsilon_i$ is the stochastic error term, and $\mathbf{d}_j(u_i)$ is a dummy variable such that $\mathbf{d}_j(u_i) = 1$ for $0 \leq u_i \leq (1/\eta_{1j}) \ln \eta_0$ and $\mathbf{d}_j(u_i) = 0$ otherwise. Equation (24) is estimated with nonlinear least squares under the restrictions that $\eta_{1U} = \lambda \eta_{1H}$ and that the $\beta_j$ parameters satisfy (6). This gives the following estimates (with robust t-statistic in brackets): $\hat{\eta}_0 = 187.865$ (16.18) and $\hat{\eta}_{1H} = 0.06961$ (79.63), $\hat{\pi}_H = 0.50767$ (15.54), and $\hat{\lambda} = 1.14793$ (177.63). It follows that $\hat{\eta}_{1U} = 0.0799$ (66.14), $\hat{\beta}_H = 0.0221$ (244.30), $\hat{\beta}_U = 0.0244$ (274.55), $\hat{D}_H = 75.215$ (194.68), and $\hat{D}_U = 65.522$ (176.04). There is a substantial difference between the maximum attainable age for the two health types of about 9.69 years. Similarly, whereas (economic) life expectancy at birth[6] is 61.25 years for the healthy, it is only 53.36 years for the unhealthy types. Figure 1 shows the key features of the mortality processes of the two health types.

The interest rate and capital depreciation rate are set at, respectively, five and seven percent per annum ($r = 0.05$ and $\delta = 0.07$). The efficiency parameter of capital is fixed at $\varepsilon = 0.3$ so that the constant in the production function is equal to $\Omega_0 = (r + \delta)/\varepsilon = 0.4$ and the capital-output ratio attains the plausible value of $K/Y = 2.5$. We postulate that in the (hypothetical) separating equilibrium (SE) the economy features a steady-state growth rate of two percent per annum ($g = 0.02$). For the intertemporal substitution elasticity we use $\sigma = 0.7$, a value often reported in empirical studies – see, e.g., Skinner (1985) and Attanasio and Weber (1995). Using the pure rate of time preference as a calibration parameter we find that it is a shade over half a percent per annum ($\rho = 5.316 \cdot 10^{-3}$). In summary, the SE has the features reported in column (a) of Table 2.

Figure 2 visualizes a number of life-cycle features of the SE. Panel (a) depicts the age

---

[6]Life expectancy at birth for type $j$ individuals is equal to $\int_0^{\bar{D}_j} e^{-M_j(s)} ds$.

Table 2: Endogenous growth and adverse selection in the annuity market: quantitative effects[a]

|  | (a) SE | (b) PE | (c) SE | (d) PE |
|---|---|---|---|---|
|  | No retirement[b] | | With retirement[c] | |
| $\dfrac{\bar{c}_H(v,v)}{w(v)}$ | 0.7668 | 0.7396 | 0.8267 | 0.8147 |
| $\dfrac{\bar{c}_U(v,v)}{w(v)}$ | 0.7858 | 0.7775 | 0.8492 | 0.8476 |
| $\bar{S}_H$ (years) | $\bar{D}_H$ | $\bar{D}_H$ | $\bar{D}_H$ | $\bar{D}_H$ |
| $\bar{S}_U$ (years) | $\bar{D}_U$ | 61.63 | $\bar{D}_U$ | 63.93 |
| $\dfrac{c(t)}{w(t)}$ | 1.0714 | 1.0753 | 0.8966 | 0.8979 |
| $g$ (%year) | 2.00 | 1.89 | 2.00 | 1.96 |
| $\dfrac{w(t)}{k(t)}$ | 0.2800 | 0.2800 | 0.3346 | 0.3346 |
| $\Lambda_H(v_0,v_0)$ | 27.4619 | 26.4639 | 11.9477 | 11.8281 |
| $\Lambda_U(v_0,v_0)$ | 21.8047 | 20.6189 | 10.3314 | 9.7873 |

**Notes**. [a]SE is the separating equilibrium and PE the pooling equilibrium. $\bar{S}_j$ is the age from which type $j$ faces a borrowing constraint. Maximum attainable ages are $\bar{D}_H = 75.22$ and $\bar{D}_U = 65.52$. [b]The participation rate is $l = 1$. [c]In this case $\bar{R} = 47$, $\zeta = 0.3952$, $\theta = 0.07$, and $l = 0.8368$. See also Table 1.

profiles for scaled consumption. The paths for the two health types are virtually on top of each other. As is clear from (22), consumption grows exponentially with age at a rate equal to $\sigma [r - \rho]$. Panel (b) of Figure 2 shows the life-cycle pattern of scaled cohort assets (individual assets display a rather similar pattern). As is to be expected, the healthy cohort is also the wealthiest of the two health types. The difference is more pronounced from middle age onward because the instantaneous mortality rates start to deviate strongly (see Figure 1(b)).

## 3.2 Pooling equilibrium

If we reinstate assumption (A3), so that annuity firms are not able to observe an annuitant's health type, then the best such a firm can do is to set the net return on annuities equal to a common, age-dependent, *pooling rate* $\bar{p}(u)$. This pooling rate takes the following form:

$$
\bar{p}(u) = \begin{cases} \dfrac{\mu_H(u)\, a_H(v, v+u) + \mu_U(u)\, a_U(v, v+u)}{a_H(v, v+u) + a_U(v, v+u)} & \text{for } 0 < u \le \bar{D}_U \\[2mm] \mu_H(u) & \text{for } \bar{D}_U < u \le \bar{D}_H \end{cases} \tag{25}
$$

Annuity firms know that the unhealthy cannot live beyond age $\bar{D}_U$ so for $\bar{D}_U < u \le \bar{D}_H$ no risk pooling is possible and $\bar{p}(u)$ coincides with the instantaneous mortality rate of the healthy individuals. For $0 < u \le \bar{D}_U$, however, both health types are alive and (potentially) active on the annuity market. The zero-profit condition for annuity firms furnishes the expression for the pooling rate in that case. It is the cohort-asset weighted sum of instantaneous mortality rates – see Sheshinski (2008, p. 71).

Using (3), (5), and (8) and *ignoring borrowing constraints for the time being*, we obtain the age profiles for consumption and assets for the two health types:

$$
\frac{\dot{\bar{c}}_j(v, v+u)}{\bar{c}_j(v, v+u)} = \sigma \left[ r + \bar{p}(u) - \mu_j(u) - \rho \right], \tag{26}
$$

$$
\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - \bar{P}(u)} = \int_0^u e^{-(r-g)s - \bar{P}(s)} ds - \frac{\bar{c}_j(v, v)}{w(v)} \int_0^u e^{-(1-\sigma)[rs + \bar{P}(s)] - \sigma[\rho s + M_j(s)]} ds, \tag{27}
$$

where $\bar{P}(u) \equiv \int_0^u \bar{p}(s)\, ds$ for $0 \le u \le \bar{D}_U$ and $\bar{P}(u) \equiv \bar{P}(\bar{D}_U) + \int_{\bar{D}_U}^u \mu_H(s)\, ds$ for $\bar{D}_U \le u \le \bar{D}_H$. Figure 2(c) provides a strong hint that this is not a complete description of the pooling equilibrium. To construct Figures 2(c)-(d), we use the cohort asset paths for the SE to compute the "implied" pooling rate $\bar{p}(u)$. Note that this is not an equilibrium rate because it is not consistent with the assumptions. Panel (d) shows that the healthy benefit from pooling

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(a) scaled individual consumption

$$\frac{\bar{c}_j(v, v+u)}{w(v)}$$

(b) scaled cohort assets

$$\frac{a_j(v, v+u)}{w(v)}$$

(c) "implied" relative pooling rate (U)

$$\bar{p}(u) - \mu_U(u)$$

(d) "implied" relative pooling rate (H)
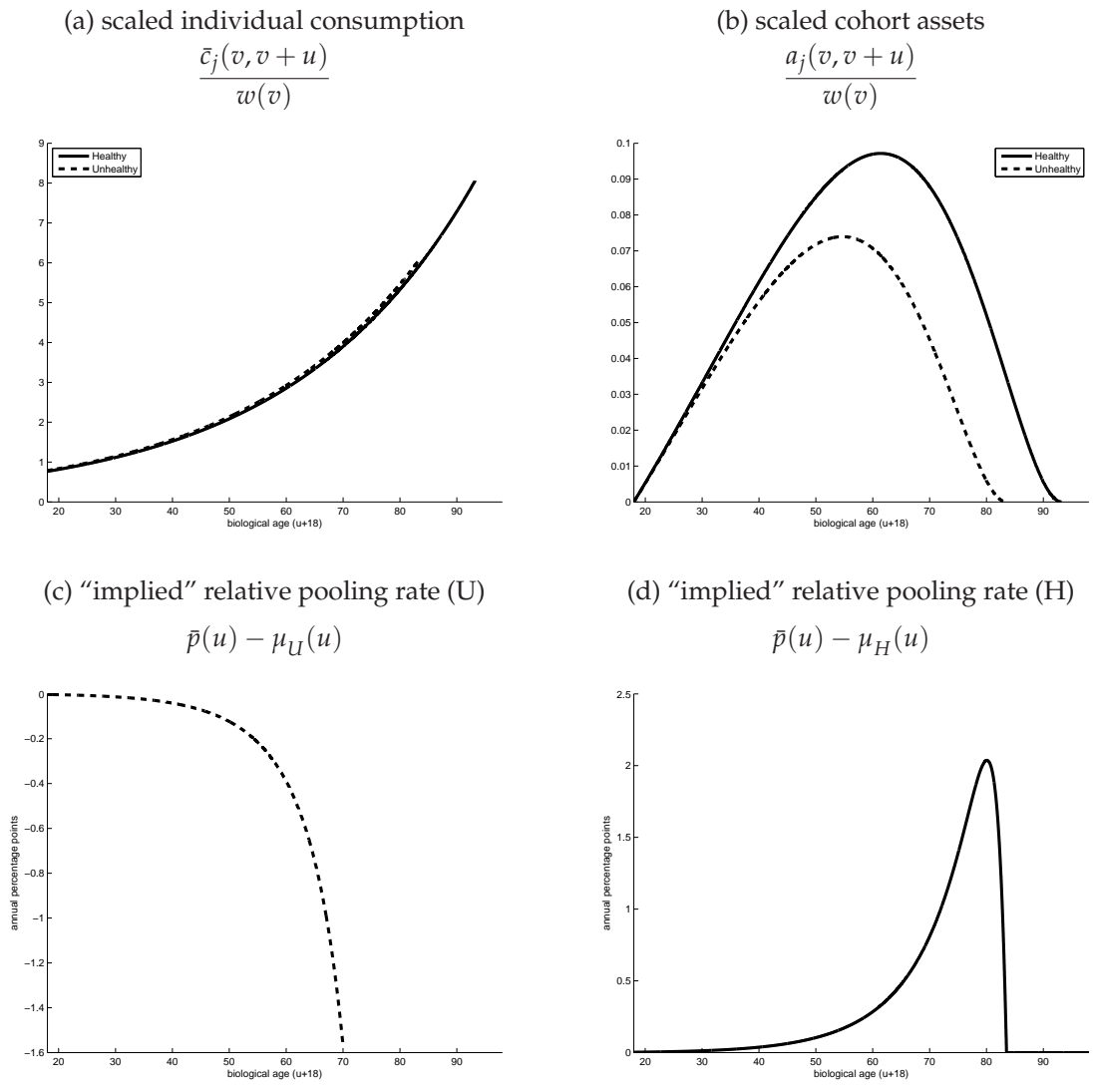
$$\bar{p}(u) - \mu_H(u)$$

Figure 2: Separating equilibrium

– their excess rate peaks at about 2 percentage points per annum around biological age 78. In contrast, as panel (c) shows, the unhealthy lose out as a result of pooling. For the unhealthy the pooling rate becomes so low that they want to borrow at that rate. But in doing so, they would reveal their health status to annuity firms who would only be willing to lend them the funds at a punitively high rate equal to their mortality rate. But at that rate they would like to be savers, as the SE suggests. It follows that the best option for the unhealthy is to impose a binding borrowing constraint on themselves from age $\bar{S}_U < \bar{D}_U$ onward. We summarize as follows.

**Proposition 2.** *Consider the case in which annuity firms are unable to observe the health type of annuitants. Assume that a pooling equilibrium (PE) exists and that $\sigma r - g > \rho$. Then: (i) healthy agents are net savers throughout life, i.e. $\bar{a}_H(v,v) = \bar{a}_H(v, v + \bar{D}_H) = 0$ and $\bar{a}_H(v, v + u) > 0$ for $0 < u < \bar{D}_H$; (ii) unhealthy agents are net savers until age $\bar{S}_U < \bar{D}_U$ after which they adopt a self-imposed borrowing constraint, i.e. $\bar{a}_U(v,v) = 0$, $\bar{a}_U(v, v + u) > 0$ for $0 < u < \bar{S}_U$, and $\bar{a}_U(v, v + u) = 0$ and $\bar{c}_U(v, v + u) = w(v + u)$ for $\bar{S}_U \leq u \leq \bar{D}_U$.*

*Proof.* See Appendix B. □

In the pooling equilibrium, we must redefine $\bar{P}(u) \equiv \bar{P}(\bar{S}_U) + \int_{\bar{S}_U}^{u} \mu_H(s) \, ds$ for $\bar{S}_U \leq u \leq \bar{D}_H$. Equations (26)–(27) are valid for the healthy throughout life ($0 \leq u \leq \bar{D}_H$), and for the unhealthy only until they hit the self-imposed borrowing constraint ($0 \leq u < \bar{S}_U$). Beyond age $\bar{S}_U$ the unhealthy simply consume their wage income.

The key equations of the pooling general equilibrium model are collected in Table 3. Equations (T3.1)–(T3.2) are obtained by using (8) in (4) and noting that the integrals only run up to age $\bar{S}_U$ for the unhealthy. Equation (T3.3) is the smooth-connection condition: consumption at age $\bar{S}_U$ must connect without discontinuity with the level implied by the solved Euler equation under pooling.[7] Equations (T3.4a)–(T3.4c) are the cohort asset paths under pooling, taking account of the self-imposed borrowing constraint for the unhealthy. Equation (T3.5) states the expression for the pooling rate. Equation (T3.6) is obtained from (9) by setting $\bar{S}_H = \bar{D}_H$, $P_j(s) = \bar{P}(s)$, and noting the definition of $c(t)$. The growth expression,

---

[7]Solving equation (26) for the unhealthy gives (for $0 \leq u \leq \bar{S}_U$): $\bar{c}_U(v, v + u) = \bar{c}_U(v, v) e^{\sigma[(r-\rho)u - M_U(u) + \bar{P}(u)]}$. For $\bar{S}_U \leq u \leq \bar{D}_U$ we have: $\bar{c}_U(v, v + u) = w(v) e^{gu}$. For $u = \bar{S}_U$ these two expressions must coincide. This furnishes equation (T3.3) in Table 3.

equation (T3.7), again follows readily from (11) because $a(t) = k(t)$ and $\Xi(t) = 0$. In the PE, redistribution between health groups takes place, something which affects the equilibrium allocation. However, the redistributive term $\Xi(t)$ nevertheless vanishes from the expression for aggregate growth because the annuity firms break even. Hence, the growth equation is the same as in the SE. Finally, equation (T3.8) is the same as before.

Using the parameter values discussed above, we can compute the pooling equilibrium using an iterative solution algorithm.[8] The results are reported in column (b) of Table 2. Relative to the SE, newborn consumption for both health types is lower in the PE. Similarly, the economic growth rate is somewhat less in the PE – 1.89 percent per annum instead of 2 percent. Interestingly, the unhealthy encounter the borrowing constraint fairly early on in old age, namely at economic age 61.63 which is 3.89 years less than their maximum attainable age. Recall, however, that life expectancy at birth of the unhealthy is only 53.36 years, so during youth individual agents are only moderately worried about encountering the borrowing constraint.[9]

It must be stressed that this asset depletion result is not exogenously imposed (as in the partial equilibrium studies of Friedman and Warshawsky (1990, p. 147) and Walliser (2000, pp. 378-9)) but follows from the internal logic of the model. In our model annuitization opportunities vanish *endogenously* for the unhealthy but remain in place for the healthy. Hence, our model yields a consistent explanation why not everybody annuitizes in a general equilibrium model with risk pooling.

It is not difficult to see that our asset depletion result also holds if there are more than two health types, provided $\rho$ is the same for all types. Indeed, with three types (healthy, normal, $N$, and unhealthy), $\bar{p}(u)$ is an asset-weighted average of the respective mortality rates, $\mu_H(u)$, $\mu_N(u)$, and $\mu_U(u)$. In this setting the normal group will feature $\bar{D}_H > \bar{S}_N > \bar{S}_U$. It may be the case that the $U$-types want to borrow at a slightly better rate than without

---

[8]We drop equation (T3.3) and perform a grid search over $\bar{S}_U$ which solves the remaining general equilibrium system. To get the iterations started we use the pooling rate "implied by" the SE. See Figures 2(c)-(d). The iterations are stopped once the unique value for $\bar{S}_U$ is found which solves (T3.3). In all computations we obtain unique solutions for the PE.

[9]From the perspective of birth, the probability of reaching age $\bar{S}_U = 61.63$ is equal to $e^{-M_U(\bar{S}_U)} = 0.2687$. The discount factor due to pure time preference is given by $e^{-\rho \bar{S}_U} = 0.7206$. It follows that the agent attaches a non-trivial weight of 0.1936 to felicity at age $\bar{S}_U$.

Table 3: Balanced growth in the pooling equilibrium[a]

*(a) Microeconomic relationships:*

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds} \tag{T3.1}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{\int_0^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds} \tag{T3.2}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = e^{-(\sigma(r-\rho)-g)\bar{S}_U+\sigma[M_U(\bar{S}_U)-\bar{P}(\bar{S}_U)]} \tag{T3.3}$$

$$\frac{a_H(v,v+u)}{w(v)} = \beta_H \pi_H e^{(r-n)u-M_H(u)+\bar{P}(u)} \left[ \int_0^u e^{-(r-g)s-\bar{P}(s)}ds \right.$$
$$\left. -\frac{\bar{c}_H(v,v)}{w(v)} \int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds \right], \qquad (0 \le u \le \bar{D}_H) \tag{T3.4a}$$

$$\frac{a_U(v,v+u)}{w(v)} = \beta_U \pi_U e^{(r-n)u-M_U(u)+\bar{P}(u)} \left[ \int_0^u e^{-(r-g)s-\bar{P}(s)}ds \right.$$
$$\left. -\frac{\bar{c}_U(v,v)}{w(v)} \int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds \right], \qquad (0 \le u < \bar{S}_U) \tag{T3.4b}$$

$$\frac{a_U(v,v+u)}{w(v)} = 0, \qquad (\bar{S}_U \le u \le \bar{D}_U) \tag{T3.4c}$$

$$\bar{p}(u) = \frac{\mu_H(u) a_H(v,v+u) + \mu_U(u) a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} \tag{T3.5}$$

*(b) Macroeconomic relationships:*

$$\frac{c(t)}{w(t)} = \beta_H \pi_H \frac{\bar{c}_H(v,v)}{w(v)} \int_0^{\bar{D}_H} e^{(r-n-g-\rho^*)s-(1+\sigma)M_H(s)+\sigma\bar{P}(s)}ds$$
$$+\beta_U \pi_U \left[ \frac{\bar{c}_U(v,v)}{w(v)} \int_0^{\bar{S}_U} e^{(r-n-g-\rho^*)s-(1+\sigma)M_U(s)+\sigma\bar{P}(s)}ds + \int_{\bar{S}_U}^{\bar{D}_U} e^{-ns-M_U(s)}ds \right] \tag{T3.6}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r-n + \left[1 - \frac{c(t)}{w(t)}\right] \frac{w(t)}{k(t)} \tag{T3.7}$$

$$\frac{w(t)}{k(t)} = (1-\varepsilon)\Omega_0 \tag{T3.8}$$

**Note.** [a]Endogenous are $\bar{c}_j(v,v)/w(v)$, $a_j(v,v+u)/w(v)$, $\bar{p}(u)$, $\bar{S}_U$, $g$, $w(t)/k(t)$, and $c(t)/w(t)$. $\bar{P}(s)$ is the cumulative pooling rate at age $s$, $u \equiv t - v$, and $\rho^* \equiv (1-\sigma)r + \sigma\rho$. See also Table 1.

the *N*-types, but the *N*-types get a worse rate as a result of the presence of the *U*-types. So they will not borrow for sure, and the *U*-types still cannot borrow without revealing themselves. In general, each health type *j* which encounters the borrowing constraint ends up enforcing the borrowing constraints for all types *k* that are unhealthier than *j*.

Figure 3 visualizes the key life-cycle features of the PE for the two health groups. Panel (a) shows that scaled consumption for the unhealthy reaches a local peak just before encountering the borrowing constraint at $\bar{S}_U$. This is because the pooling rate is rather low (due to the predominance of the healthy in the annuity market), and the mortality rate of the unhealthy starts to rise. In terms of (26), consumption falls for a while because the gross annuity rate, $r + \bar{p}(u)$, falls short of the "effective impatience rate" due to time preference and mortality, $\rho + \mu_U(u)$, for *u* near $\bar{S}_U$. At $u = \bar{S}_U$, the surviving unhealthy reach the Keynesian part of their consumption profile and simply consume their wage income.

Comparing the asset paths for the SE and PE cases in, respectively, Figures 2(b) and 3(b), we observe that the healthy save more and the unhealthy save less in the PE than in the SE. This is in part because the relative pooling rate, $\bar{p}_j(u) - \mu_j(u)$, is positive for the healthy and is negative for the unhealthy – see panels (c)–(d) in Figure 3. In a sense, the healthy benefit from the presence of the unhealthy in the annuity market and are thus able to obtain an annuity rate of interest on their assets that is more than actuarially fair.

### 3.3 Welfare analysis

In the previous subsection we have shown that the growth effects of "pooling versus separation" are small even though the difference in mortality risks faced by healthy and unhealthy individuals is rather large, especially at older ages (see Figure 1(b)). But economic growth is not the only relevant indicator. A key question is, to what extent does it matter to individuals whether or not the annuity markets are perfect (SE) or imperfect (PE)?

To address this question, the last two rows of Table 2 report the lifetime utility scores for newborns (of both health type) at some base year $v_0$. We normalize the wage rate for that generation to unity, $w(v_0) = 1$. As panels (a)-(b) in Table 2 reveal, welfare is higher under the SE than under the PE, i.e. $\Lambda_j^{SE}(v_0, v_0) > \Lambda_j^{PE}(v_0, v_0)$ for $j \in \{H, U\}$. In order to obtain some feel for the significance of these differences, we compute *lost growth years* under

(a) scaled individual consumption

$$\frac{\bar{c}_j(v, v+u)}{w(v)}$$

(b) scaled cohort assets

$$\frac{a_j(v, v+u)}{w(v)}$$

(c) relative pooling rate (U)

$$\bar{p}(u) - \mu_U(u)$$

(d) relative pooling rate (H)
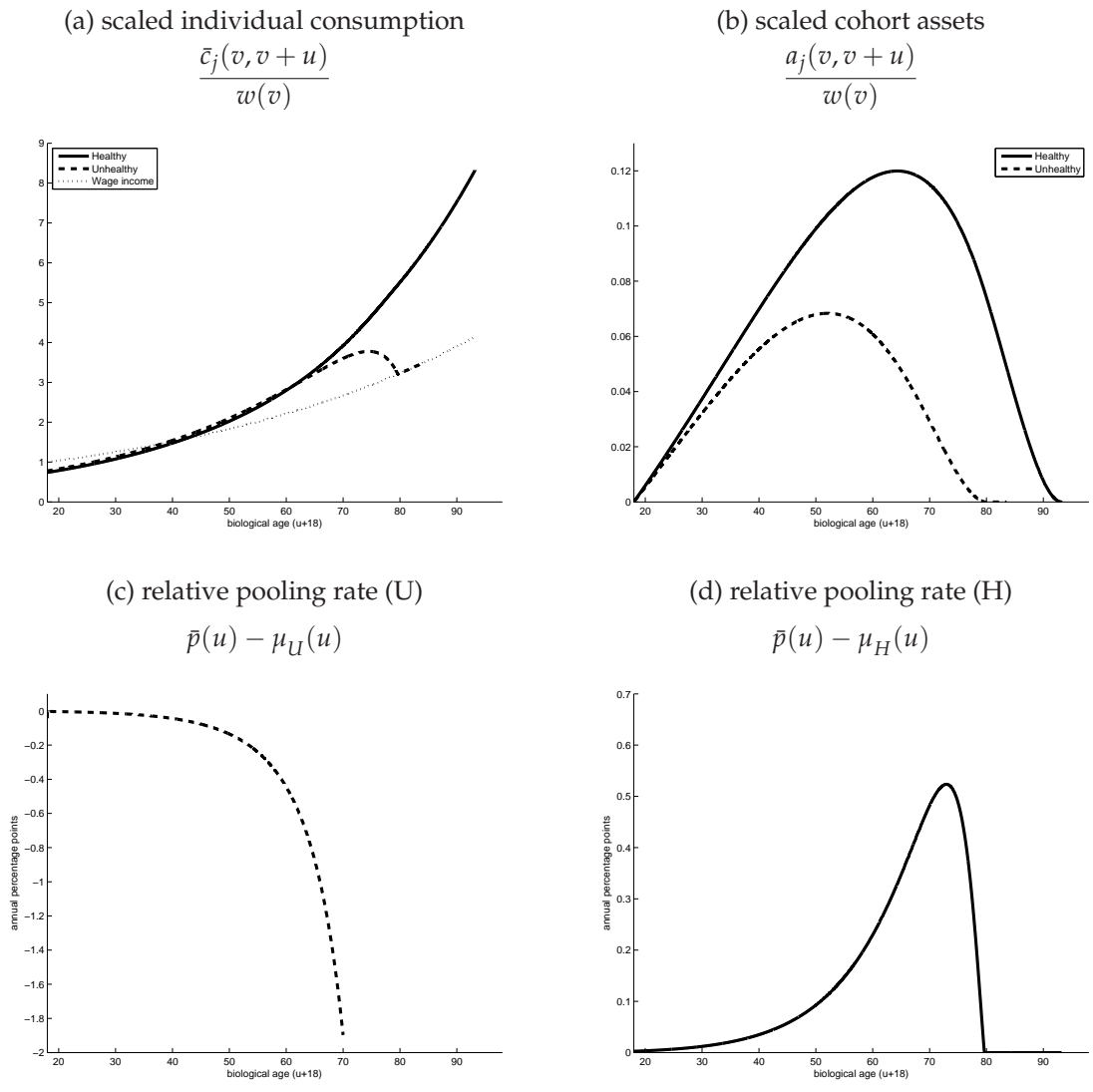
$$\bar{p}(u) - \mu_H(u)$$

Figure 3: Pooling equilibrium

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annuity market equilibrium $i$ relative to the separating equilibrium as follows:

$$LGY_j^i = \frac{1}{g^i} \frac{\Lambda_j^{SE}(v_0, v_0) - \Lambda_j^i(v_0, v_0)}{\int_0^{D_j} e^{-\rho s - M_j(s)} ds}. \tag{28}$$

Intuitively, $LGY_j^i$ is equal to $v_1 - v_0$ such that $\Lambda_j^{SE}(v_0, v_0) = \Lambda_j^i(v_1, v_1)$. How far into the future must a newborn arrive under annuity market $i$ in order to be equally well off as a base-year newborn in the SE? Since wage growth explains why newborn lifetime utility increases over time, the macroeconomic growth rate under annuity market $i$ features in (28).

For $i = PE$ the comparison is between the PE and the SE and we find that $LGY_H^{PE} = 1.0166$ years and $LGY_U^{PE} = 1.3582$ years. By all accounts the annuity market imperfection due to pooling is rather small in welfare terms. At birth, the unhealthy have an economic life expectancy of 53.36 years and for them the lost growth years amount to about 16.3 months. For the healthy the results are 12.2 months of lost growth on an expected economic lifetime at birth of 61.25 years. These are not of an order of magnitude that should worry the policy maker.

At first sight it might appear as though the above results imply that the pooling equilibrium does not exist. Both the unhealthy agents and the healthy agents *as a group* are better off by truthfully signaling their health status to the annuity firms. As a separating equilibrium gives them higher utility, this announcement would be credible. However, each healthy agent *as an individual* has an incentive to deviate from the optimal group strategy. Once the separating equilibrium is realized, posing as an unhealthy (low-risk) agent and receiving the higher annuity premium is optimal given that the other agents are honest in their health claim. Indeed, a healthy individual who cheats during the interval $(v_0, v_0 + \bar{T})$ would attain a welfare level of $\Lambda_H^{SE,C}(v_0, v_0) = 36.5047$ (for $\bar{T} = 1$) and $\Lambda_H^{SE,C}(v_0, v_0) = 37.9725$ (for $\bar{T} = 2$). Indeed, the longer such an individual cheats, the higher is welfare. So cheating clearly dominates the truth-telling strategy for healthy individuals (recall that under truth telling $\Lambda_H^{SE}(v_0, v_0) = 27.4619$). In short, there exists a free-rider problem: as each healthy agent has an incentive to cheat and they cannot coordinate their actions, the pooling equilibrium will be the inevitable, yet suboptimal, outcome.

# 4 Robustness checks

The analysis conducted thus far has yielded a number of tentative conclusions.

**Conclusion (C1)** Unhealthy individuals adopt a self-imposed borrowing constraint fairly early on during old age.

**Conclusion (C2)** The welfare effects of the annuity market imperfection due to asymmetric information are rather modest.

In the remainder of this section we study the robustness of conclusions (C1)–(C2) to alternative assumptions regarding labour market participation (in subsection 4.1) and the macroeconomic growth mechanism (in subsection 4.2).

## 4.1 Retirement and pensions

In the model considered up to this point, agents are assumed to supply one unit of labour in each period until they die. Here we replace this unrealistic assumption by postulating an exogenously imposed mandatory retirement age, $\bar{R}$, i.e. in the augmented model agents supply one unit of labour for ages $u$ such that $0 \leq u < \bar{R}$ and are fully retired for $\bar{R} \leq u \leq D_j$. The retirement age does not feature a type index because the policy maker, like annuity firms, lacks information about an individual's health status.

In addition we postulate a simple pay-as-you-go (PAYG) pension scheme which imposes a wage-indexed tax of $\theta w(\tau)$ (featuring $0 < \theta < 1$) on workers ($0 \leq \tau - v < \bar{R}$) and provides wage-indexed benefits of $\zeta w(\tau)$ (with $\zeta > 0$) to retirees ($\bar{R} \leq \tau - v \leq D_j$). We refer to $\zeta$ as the replacement rate. The PAYG system is run on a balanced budget basis (see below).

Table 4 states the key equations defining the PE model. The model is based on the presumption – which is verified in the parameterization adopted below – that the retirement age falls well short of the asset depletion age of the unhealthy, i.e. $\bar{R} < \bar{S}_U$. The pension system affects the model via the following channels. First, whilst the tax rate $\theta$ reduces the part of human wealth accumulated during the working period, the replacement rate $\zeta$ increases human wealth accumulated during the retirement phase. Both affect scaled consumption at birth and the asset accumulation paths – see equations (T4.1)–(T4.2) and (T4.4a)–(T4.4e). Second, the replacement rate also enters the smooth connection condition (T4.3). With the

*(a) Microeconomic relationships:*

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{(1-\theta)\int_0^{\bar{R}} e^{-(r-g)s-\bar{P}(s)}ds + \zeta \int_{\bar{R}}^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds} \qquad (\text{T4.1})$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{(1-\theta)\int_0^{\bar{R}} e^{-(r-g)s-\bar{P}(s)}ds + \zeta \int_{\bar{R}}^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds} \qquad (\text{T4.2})$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \zeta e^{-(\sigma(r-\rho)-g)\bar{S}_U + \sigma[M_U(\bar{S}_U)-\bar{P}(\bar{S}_U)]} \qquad (\text{T4.3})$$

$$\frac{a_H(v,v+u)}{w(v)} = \beta_H \pi_H e^{(r-n)u - M_H(u) + \bar{P}(u)}\left[(1-\theta)\int_0^u e^{-(r-g)s-\bar{P}(s)}ds\right.$$
$$\left. -\frac{\bar{c}_H(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds\right], \qquad (0 \le u < \bar{R}) \quad (\text{T4.4a})$$

$$\frac{a_H(v,v+u)}{w(v)} = \beta_H \pi_H e^{(r-n)u - M_H(u) + \bar{P}(u)}\left[-\zeta \int_u^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds\right.$$
$$\left. +\frac{\bar{c}_H(v,v)}{w(v)}\int_u^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds\right], \qquad (\bar{R} \le u \le \bar{D}_H) \quad (\text{T4.4b})$$

$$\frac{a_U(v,v+u)}{w(v)} = \beta_U \pi_U e^{(r-n)u - M_U(u) + \bar{P}(u)}\left[(1-\theta)\int_0^u e^{-(r-g)s-\bar{P}(s)}ds\right.$$
$$\left. -\frac{\bar{c}_U(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds\right], \qquad (0 \le u \le \bar{R}) \quad (\text{T4.4c})$$

$$\frac{a_U(v,v+u)}{w(v)} = \beta_U \pi_U e^{(r-n)u - M_U(u) + \bar{P}(u)}\left[-\zeta \int_u^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds\right.$$
$$\left. +\frac{\bar{c}_U(v,v)}{w(v)}\int_u^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds\right], \qquad (\bar{R} \le u < \bar{S}_U) \quad (\text{T4.4d})$$

$$\frac{a_U(v,v+u)}{w(v)} = 0, \qquad (\bar{S}_U \le u \le \bar{D}_U) \qquad (\text{T4.4e})$$

$$\bar{p}(u) = \frac{\mu_H(u)\, a_H(v,v+u) + \mu_U(u)\, a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} \qquad (\text{T4.5})$$

(Table 4, continued)

*(b) Macroeconomic relationships:*

$$\frac{c\left(t\right)}{w\left(t\right)} = \beta_H \pi_H \frac{\bar{c}_H\left(v,v\right)}{w\left(v\right)} \int_0^{\bar{D}_H} e^{(r-n-g-\rho^*)s-(1+\sigma)M_H(s)+\sigma\bar{P}(s)} ds$$

$$+\beta_U \pi_U \left[\frac{\bar{c}_U(v,v)}{w\left(v\right)} \int_0^{\bar{S}_U} e^{(r-n-g-\rho^*)s-(1+\sigma)M_U(s)+\sigma\bar{P}(s)} ds + \zeta \int_{\bar{S}_U}^{\bar{D}_U} e^{-ns-M_U(s)} ds \right] \quad \text{(T4.6)}$$

$$g \equiv \frac{\dot{k}\left(t\right)}{k\left(t\right)} = r - n + \left[1 - \frac{c\left(t\right)}{w\left(t\right)}\right] \frac{w\left(t\right)}{k\left(t\right)} \quad \text{(T4.7)}$$

$$\frac{w\left(t\right)l}{k\left(t\right)} = \left(1-\varepsilon\right)\Omega_0 \quad \text{(T4.8)}$$

$$l \equiv \sum_j \beta_j \pi_j \int_0^{\bar{R}} e^{-ns-M_j(s)} ds \quad \text{(T4.9)}$$

$$0 = \theta \sum_j \beta_j \pi_j \int_0^{\bar{R}} e^{-ns-M_j(s)} ds - \zeta \sum_j \beta_j \pi_j \int_{\bar{R}}^{\bar{D}_j} e^{-ns-M_j(s)} ds \quad \text{(T4.10)}$$

**Note**. [a]See also Table 3. The additional endogenous variables are the labour force participation rate, $l$, and the replacement rate, $\zeta$.

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pension system in place, for age $\bar{S}_U$ onward the unhealthy consume their *pension* income $\zeta w\left(v+u\right)$ rather than their wage income $w\left(v+u\right)$ as was the case in the base model without mandatory retirement. Third, the macroeconomic labour force participation rate $l$ falls short of unity under mandatory retirement. This rate is defined in equation (T4.9) and also shows up in the growth equation (T4.7) and the expression for the wage-capital ratio (T4.8).

We parameterize the augmented model as follows. For the economic structural parameters ($r$, $n$, $\varepsilon$, $\Omega_0$, $\delta$, and $\sigma$) and the demographic parameters ($\eta_0$, $\eta_{1j}$, $\pi_j$, $\beta_j$ and $\lambda$) we use the values discussed above (below equation (24)). We set the mandatory retirement age at sixty-five biological years ($\bar{R} = 47$) and assume that the tax rate equals seven percent of wage income ($\theta = 0.07$). It follows that the replacement rate is about forty percent ($\zeta = 0.3952$) and the participation rate is almost eighty-four percent ($l = 0.8368$). These values broadly capture the main features of the Dutch PAYG system. We assume that the macroeconomic growth rate is two percent per annum in the SE ($g = 0.02$) and use the pure rate of time preference as a calibration parameter ($\rho = 0.0179$).[10]

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[10]In the interest of brevity we do not provide a summary table for the SE model with the pension system incorporated. It is easily deduced from Table 4, however, by comparing Tables 2 and 3.
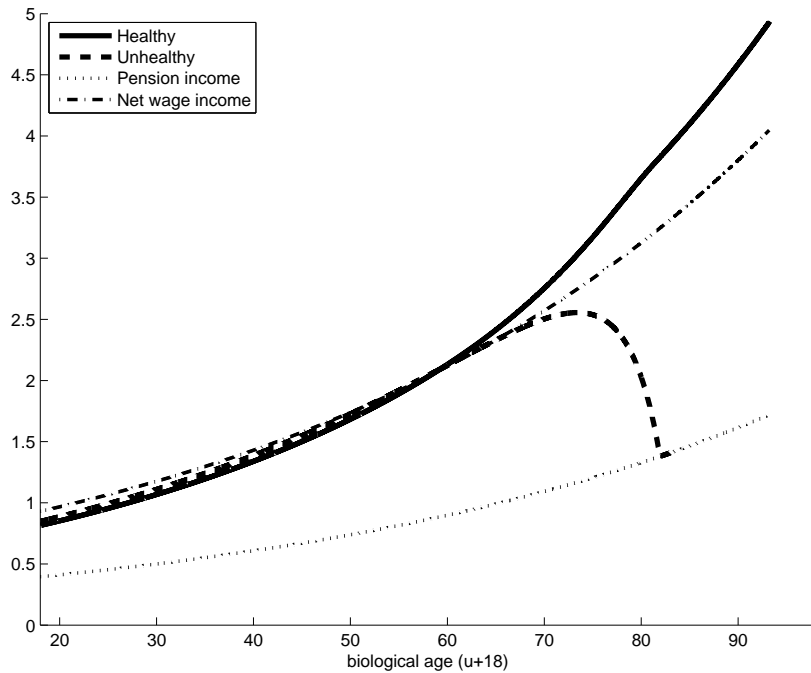
Figure 4: Self-imposed borrowing constraint under PAYG pensions

The key features of the different equilibria (SE and PE) are reported in columns (c)–(d) of Table 2. It is clear from the table that conclusions (C1)–(C2) still hold under a system of mandatory retirement and PAYG pensions. Comparing columns (b) and (d) we observe that the self-imposed borrowing constraint occurs later on in life under the PAYG system.[11] The intuition behind this result is that with a replacement rate of less than unity, the smooth connection point occurs later on in life. In terms of Figure 4, asset depletion occurs at the point where the consumption Euler path (dashed line) meets the pension income path (dotted line) which lies well below the after-tax wage income path.

## 4.2 Endogenous versus exogenous growth

In this section we consider the role of the economic growth process. Are conclusions (C1)–(C2) still valid if growth is exogenous rather than endogenous? To study this issue we postulate an alternative model featuring exogenous labour-augmenting technological change

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[11]Of course, the results in columns (a)–(c) in the table are strictly speaking not directly compatible to those in columns (d)–(f) because they are based on a different value of $\rho$, and thus on structurally different individuals.

Table 5: Exogenous growth and adverse selection in the annuity market: quantitative effects[a]

| | (a) SE | (b) PE | (c) SE | (d) PE |
|---|---|---|---|---|
| | No retirement[b] | | With retirement[c] | |
| $\dfrac{\bar{c}_H\,(v,v)}{w\,(v)}$ | 0.7668 | 0.7428 | 0.8267 | 0.8157 |
| $\dfrac{\bar{c}_U\,(v,v)}{w\,(v)}$ | 0.7858 | 0.7803 | 0.8492 | 0.8485 |
| $\bar{S}_H$ (years) | $\bar{D}_H$ | $\bar{D}_H$ | $\bar{D}_H$ | $\bar{D}_H$ |
| $\bar{S}_U$ (years) | $\bar{D}_U$ | 61.60 | $\bar{D}_U$ | 63.93 |
| $\dfrac{c\,(t)}{w\,(t)}$ | 1.0714 | 1.0755 | 0.8966 | 0.8980 |
| $\dfrac{k\,(t)}{Z\,(t)}$ | 2.5000 | 2.4593 | 2.5000 | 2.4827 |
| $r$ (%year) | 5.00 | 5.14 | 5.00 | 5.06 |
| $\dfrac{w\,(t)}{Z\,(t)}$ | 0.7000 | 0.6966 | 0.8365 | 0.8348 |
| $\Lambda_H(v_0,v_0)$ | 27.4619 | 27.6194 | 11.9477 | 12.1563 |
| $\Lambda_U(v_0,v_0)$ | 21.8047 | 21.5796 | 10.3314 | 10.0692 |

**Notes**. [a]See the notes in Tables 2 and 3. [b]The participation rate is $l = 1$. [c]In this case $\bar{R} = 47$, $\zeta = 0.3952$, $\theta = 0.07$, and $l = 0.8368$.

yielding a long-run growth rate of $g$. Of course, in a closed-economy exogenous growth model the interest rate $r(t)$ is an endogenous variable. In the model employed here, the representative firm faces an intensive-form production function of the form $y(t) = \Omega_0 k(t)^\varepsilon$ $[lZ(t)]^{1-\varepsilon}$, where $\dot{Z}(t)/Z(t) = g$ is the exogenous rate of change in labour-augmenting technology. Instead of (18)–(19) the marginal productivity conditions are given by:

$$\frac{w(t)}{Z(t)} = (1-\varepsilon)\,\Omega_0 \left(\frac{k(t)}{lZ(t)}\right)^\varepsilon, \qquad r(t) + \delta = \varepsilon\Omega_0 \left(\frac{k(t)}{lZ(t)}\right)^{\varepsilon-1}. \tag{29}$$

These expressions replace equation (T4.8) in Table 4. The remainder of the model is unchanged except for the fact that $r(t)$ is endogenous and $g$ is exogenous. In the steady state, $k(t)/Z(t)$ and $r(t)$ are time-invariant constants, and $c(t)$, $w(t)$, $k(t)$, and $y(t)$ all grow at the exponential rate $g$.

We analyze the model both without and with a pension system. We parameterize the two versions of the model as follows. For the economic structural parameters ($n$, $\varepsilon$, $\delta$, $\sigma$, $\bar{R}$, $\theta$, and $\zeta$) and the demographic parameters ($\eta_0$, $\eta_{1j}$, $\pi_j$, $\beta_j$ and $\lambda$) we use the values discussed above (below equation (24)). We set $g = 0.02$, assume that the steady-state capital-output ratio equals $K(t)/Y(t) = 2.5$, and postulate that the interest rate equals five percent per annum in the SE ($r = 0.05$). We use the scale parameter in production and the pure rate of time preference as calibration parameters and find $\Omega_0 = 0.7597$ and $\rho = 5.31610^{-3}$ for the case without retirement, and $\Omega_0 = 0.8606$ and $\rho = 0.0179$ for the case with retirement.

In Table 5 columns (a)–(b) and (c)–(d) present the quantitative results for, respectively, the model without and with retirement. Apart from the changed roles played by $r$ and $g$, columns (a) and (c) in Table 5 by construction coincide with, respectively columns (a) and (c) in Table 2.

Two main conclusions can be drawn. First, it is clear from Table 5 that conclusions (C1)–(C2) still hold in an exogenous growth setting. Second, the comparison between Tables 2 and 5 reveals that the quantitative effects are virtually identical for the endogenous and exogenous growth models. Whereas the difference between SE and PE results in different growth rates in Table 2, it shows up in the form of different interest rates in Table 5. But the growth-corrected interest rates ($r - g$) for the two growth models are very close for the different equilibria.

# 5 Conclusions

We have constructed a consistent general equilibrium model featuring overlapping generations of heterogeneous agents distinguished by health status. An individual's health status and annuity purchases are private information. Under our set of assumptions about the annuity market, competitive firms offer linear contracts so that a risk pooling equilibrium emerges. In this equilibrium the healthy (high-risk) individuals benefit from the market presence of unhealthy (low-risk) annuitants. The model gives a partial explanation for the annuity puzzle. At high ages, low-risk individuals cease to purchase annuities and impose a borrowing constraint on themselves.

Interestingly, the growth and welfare effects of the annuity market imperfection due to adverse selection are rather small. The fact that information is asymmetric in this market may thus be quantitatively unimportant after all.

In future work we hope to pursue the following extensions. First, we wish to endogenize the labour supply decision in order to investigate the retirement effects of annuity market imperfections. In that context we will also introduce social annuity schemes such as funded and PAYG pension systems. Second, we wish to model the optimal schooling decision by individuals in an adverse selection setting and study the effects on aggregate human capital formation and macroeconomic growth. Finally, we want to extend the model to include agents who differ both in health type and in their degree of impatience as measured by the rate of time preference. In this context we will study the emergence of joint pooling equilibria for annuities and life-insurance.

## Appendix A: Proof of Proposition 1

In a separating equilibrium we have $M_j(u) = P_j(u)$ for all $0 \leq u \leq \bar{D}_U$ as $p_j(u) = \mu_j(u)$ for all $0 \leq u \leq \bar{D}_j$. Define $\rho^* \equiv (1 - \sigma) r + \sigma \rho$. For $\sigma r - g > \rho$ it follows that $r - g > \rho^*$ and thus:

$$\frac{\bar{c}_j(v, v)}{w(v)} = \frac{\int_0^{D_j} e^{-(r-g)u - M_j(u)} \, du}{\int_0^{\bar{D}_j} e^{-\rho^* u - M_j(u)} \, du} < 1.$$

Let $u \in [0, \bar{D}_j]$ be the age of the consumer. Then we can write:

$$\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - M_j(u)} = \Gamma_j(u),$$

where $\Gamma_j : [0, \bar{D}_j] \to \mathbb{R}$ is defined by:

$$\Gamma_j(u) = \int_0^u e^{-(r-g)s - M_j(s)} ds - \frac{\bar{c}_j(v, v)}{w(v)} \int_0^u e^{-\rho^* s - M_j(s)} ds.$$

As $\Gamma_j$ is a continuous function defined on a closed and bounded interval $[0, \bar{D}_j]$, we know that $\Gamma_j$ has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_j(0) = \Gamma_j(\bar{D}_j) = 0$ as $\bar{a}_j(v, v) = \bar{a}_j(v, v + \bar{D}_j)$ by the initial condition and the property of non-saturation.

Using Leibnitz' rule, we find that the first order derivative of $\Gamma_j$ is given by:

$$\Gamma_j'(u) = e^{-M_j(u)} \left[ e^{-(r-g)u} - \frac{\bar{c}_j(v, v)}{w(v)} e^{-\rho^* u} \right].$$

The unique interior root of this equation is:

$$u^* \equiv -\frac{1}{r - g - \rho^*} \ln\left( \frac{\bar{c}_j(v, v)}{w(v)} \right),$$

where $u^* > 0$ as $\bar{c}_j(v, v)/w(v) < 1$ and $r - g > \rho^*$ by assumption. We find that $\Gamma_j'(u) > 0$ for $0 \leq u < u^*$ and $\Gamma_j'(u) < 0$ for $u^* < u < \bar{D}_j$. We conclude that $\Gamma_j$ has a global maximum at $u^*$ and a global minimum at $0$ and $\bar{D}_j$. As this global minimum equals zero, we find $\bar{a}_j(v, v + u) > 0$ for all $u \in (0, \bar{D}_j)$. □

## Appendix B: Proof of Proposition 2

We assume that there exists a pooling equilibrium in the annuity market. This is only possible if the asset holdings of both health groups have the same sign everywhere. However, they cannot both be negative as that would imply that nobody saves in this closed economy. In that case there is no capital stock and wages are zero, which contradicts the existence of an equilibrium. Hence the asset holdings of the healthy and unhealthy agents will both have to be nonnegative: $\bar{a}_H(v, v + u) \geq 0$ and $\bar{a}_U(v, v + u) \geq 0$ for $0 \leq u \leq \bar{D}_U$. The corresponding

pooling premium is given by:

$$\bar{p}(u) = \begin{cases} \dfrac{\mu_H(u)a_H(v,v+u) + \mu_U(u)a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} & \text{for } 0 \le u \le \bar{D}_U \\ \mu_H(u) & \text{for } \bar{D}_U < u \le \bar{D}_H \end{cases}.$$

Write $\bar{P}(u) \equiv \int_0^u \bar{p}(s)\,ds$. It follows that:

$$\mu_H(u) \le \bar{p}(u) \le \mu_U(u), \qquad \text{for } 0 \le u \le \bar{D}_U,$$

$$M_U(u) \ge \bar{P}(u), \qquad \text{for } 0 \le u \le \bar{D}_U,$$

$$M_H(u) \le \bar{P}(u), \qquad \text{for } 0 \le u \le \bar{D}_H.$$

Now consider the two statements made in the proposition.

Item (i). Take a healthy agent. Define $f : [0, \bar{D}_H] \to \mathbb{R}$ by:

$$f(u) = e^{\sigma[M_H(u) - \bar{P}(u)]}.$$

It follows that $f$ is a differentiable function, that $f(0) = 1$ and that $f(u) \le 1$ for all $u \in (0, \bar{D}_H]$. The first-order derivative of $f$ is given by:

$$f'(u) = [\mu_H(u) - \bar{p}(u)]f(u) = \begin{cases} \sigma[\mu_H(u) - \bar{p}(u)]f(u) & \text{for } 0 \le u \le \bar{D}_U \\ 0 & \text{for } \bar{D}_U < u \le \bar{D}_H \end{cases},$$

such that $f'(u) \le 0$ for all $u \in [0, \bar{D}_H]$. Using the function $f$, we can write consumption at birth as:

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_H} f(u)e^{-(r-g)u-(1-\sigma)\bar{P}(u)-\sigma M_H(u)}\,du}{\int_0^{\bar{D}_H} e^{-\rho^* u-(1-\sigma)\bar{P}(u)-\sigma M_H(u)}\,du},$$

where $\rho^* \equiv (1-\sigma)r + \sigma\rho$. By the properties of $f$ and the assumption that $r - g > \rho^*$ it immediately follows that:

$$\frac{\bar{c}_H(v,v)}{w(v)} < 1.$$

We now write:

$$\frac{\bar{a}_H(v,v+u)}{w(v)}e^{-ru-\bar{P}(u)} = \Gamma_H(u),$$

where $\Gamma_H : [0, \bar{D}_H] \to \mathbb{R}$ is defined by:

$$\Gamma_H(u) = \int_0^u f(s)e^{-(r-g)s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}\,ds - \frac{\bar{c}_H(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}\,ds.$$

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As $\Gamma_H$ is a continuous function defined on a closed and bounded interval we know that $\Gamma_H$ has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_H(0) = \Gamma_H(\bar{D}_H) = 0$ as $\bar{a}_H(v, v) = \bar{a}_H(v, v + \bar{D}_H) = 0$.

The first-order derivative of $\Gamma_H$ is given by:

$$\Gamma'_H(u) = e^{-(1-\sigma)\bar{P}(u)-\sigma M_H(u)} \left[ f(u) e^{-(r-g)u} - \frac{\bar{c}_H(v,v)}{w(v)} e^{-\rho^* u} \right].$$

It follows that a stationary point $u_H^*$ of $\Gamma_H$ satisfies:

$$\frac{\bar{c}_H(v,v)}{w(v)} e^{(r-g-\rho^*)u_H^*} = f(u_H^*).$$

Since $r - g > \rho^*$, the left-hand side of this equation is increasing in $u$. Combined with the fact that $\bar{c}_H(v,v)/w(v) < 1$, $f(0) = 1$ and $f'(u) \leq 0$ for all $u \in [0, \bar{D}_H]$ we find that the stationary point is unique. The second-order derivative of $\Gamma_H$ evaluated in $u_H^*$ is:

$$\Gamma''_H(u_H^*) = -e^{-(r-g)u_H^*-(1-\sigma)\bar{P}(u_H^*)-\sigma M_H(u_H^*)} \left[ (r - g - \rho^*) f(u_H^*) - f'(u_H^*) \right].$$

As $f'(u_H^*) \leq 0$, it follows that $\Gamma''_H(u_H^*) < 0$. We conclude that $\Gamma_H$ has a global maximum at $u_H^*$ and a global minimum at $0$ and $\bar{D}_H$. As this global maximum is strict and equals zero, we find $\bar{a}_H(v, v + u) > 0$ for all $u \in (0, \bar{D}_H)$.

Item (ii). Take an unhealthy agent. Define $h : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$h(u) = e^{\sigma[M_U(u) - \bar{P}(u)]}.$$

It follows that $h$ is a differentiable function, that $h(0) = 1$ and that $h(u) \geq 1$ for all $u \in (0, \bar{D}_U]$. The first-order derivative of $h$ is given by:

$$h'(u) = \sigma[\mu_U(u) - \bar{p}(u)]h(u).$$

Since we have shown above that $\bar{a}_H(v, v + u) > 0$ for all $u \in (0, \bar{D}_H)$ and $\mu_U(u) > \mu_H(u)$ for all $u \in [0, \bar{D}_U]$ by assumption, it follows that $\bar{p}(u) < \mu_U(u)$ for all $u \in (0, \bar{D}_U]$. As a consequence, we find that $h'(u) > 0$ on its domain. Using the function $h$, we can write consumption at birth as:

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_U} h(u) e^{-(r-g)u-(1-\sigma)\bar{P}(u)-\sigma M_U(u)} \, du}{\int_0^{\bar{D}_U} e^{-\rho^* u-(1-\sigma)\bar{P}(u)-\sigma M_U(u)} \, du}.$$

If $\bar{c}_U(v, v)/w(v) > 1$ then there exists $\varepsilon > 0$ such that $\bar{a}_U(v, v + \varepsilon) < 0$, which contradicts the assumption that a pooling equilibrium exists. Hence:

$$\frac{\bar{c}_U(v, v)}{w(v)} < 1.$$

It follows that $\bar{a}_U(v, v + u)$ is positive for small values of $u$. Now suppose to the contrary that there does not exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v + u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$. Then we would have $\bar{a}_U(v, v + u) > 0$ for $u \in (0, \bar{D}_U)$. In that case we can write:

$$\frac{\bar{a}_U(v, v + u)}{w(v)} e^{-ru - \bar{P}(u)} = \Gamma_U(u),$$

where $\Gamma_U : [0, \bar{D}_U] \to \mathbb{R}$ is defined by:

$$\Gamma_U(u) = \int_0^u h(s) e^{-(r-g)s - (1-\sigma)\bar{P}(s) - \sigma M_U(s)} \, ds - \frac{\bar{c}_U(v, v)}{w(v)} \int_0^u e^{-\rho^* s - (1-\sigma)\bar{P}(s) - \sigma M_U(s)} \, ds.$$

As $\Gamma_U$ is a continuous function defined on a closed and bounded interval we know that $\Gamma_U$ has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ as $\bar{a}_U(v, v) = \bar{a}_U(v, v + \bar{D}_U) = 0$.

The first-order derivative of $\Gamma_U$ is given by:

$$\Gamma_U'(u) = e^{-(1-\sigma)\bar{P}(u) - \sigma M_U(u)} \left[ h(u) e^{-(r-g)u} - \frac{\bar{c}_U(v, v)}{w(v)} e^{-\rho^* u} \right].$$

It follows that a stationary point $u_U^*$ of $\Gamma_U$ satisfies:

$$\frac{\bar{c}_U(v, v)}{w(v)} e^{(r-g-\rho^*)u_U^*} = h(u_U^*).$$

Both the left-hand side and the right-hand side of this equation are increasing and convex in $u$, opening the possibility of multiple stationary points. Define $g : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$g(u) = \frac{\bar{c}_U(v, v)}{w(v)} e^{(r-g-\rho^*)u}.$$

As $\bar{c}_U(v, v)/w(v) < 1$ it follows that $g(0) < h(0)$. Since both functions are strictly increasing and $\lim_{u \to \bar{D}_U} h(u) > g(\bar{D}_U)$, it follows that if $g$ and $h$ cross on $[0, \bar{D}_U]$ then they cross exactly twice. Hence, we conclude $\Gamma_U$ has two critical points on its domain.

The second-order derivative of $\Gamma_U$ evaluated in $u_U^*$ is:

$$\begin{aligned}
\Gamma_U''(u_U^*) &= -e^{-(r-g)u_U^* - (1-\sigma)\bar{P}(u_U^*) - \sigma M_U(u_U^*)} \left[ (r - g - \rho^*) h(u_U^*) - h'(u_U^*) \right] \\
&= -h(u_U^*) e^{-(r-g)u_U^* - (1-\sigma)\bar{P}(u_U^*) - \sigma M_U(u_U^*)} \left[ (r - g - \rho^*) - [\mu_U(u_U^*) - \bar{p}(u_U^*)] \right] \gtreqless 0.
\end{aligned}$$

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where we have used the fact that $h'(u_U^*) = [\mu(u_U^*) - \bar{p}(u_U^*)]h(u_U^*)$. Since $r - g > \rho^*$ and $\mu_U(u) - \bar{p}(u) \approx 0$ for low values $u$, we find that the first stationary point is a maximum. As $[\mu_U(u) - \bar{p}(u)] \to \infty$ for $u \to \bar{D}_U$, we find that the second stationary point is a minimum. As $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ and there are exactly two interior stationary points it follows that the minimum is associated with negative asset holdings. This is a contradiction to the assumption that $\bar{a}_U(v, v + u) > 0$ for all $u \in (0, \bar{D}_U)$. Hence we conclude that there does exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v + u) > 0$ for $u \in (0, \bar{u})$ and $\bar{a}_U(v, v + u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$. $\qquad\square$

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