Dynamic Asset Allocation with predictability and transaction costs

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Based on research with Kent Daniel, Ciamac Moallemi, Mehmet Saglam
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- **Summary**
- **Motivation**
- **The linear-quadratic framework**
- **An Alternative Approach**
- **Preliminary results**
- **Conclusion**
Overview

- Portfolio choice literature started by Markowitz (1958) with one-period mean-variance optimization.
  - Introduces idea of hedging demand.
- Small impact on practitioners, who mostly rely on Mean-variance framework. Why?
  - Dynamic problem difficult to implement and ignores realistic frictions such as T-cost, price impact, size, liquidity constraints.
- Traditional literature on investment under transaction costs limited to iid stock versus cash problem (Constantinides (1986)-Davis-Norman (1990))
  - Introduces idea of No-trade region
- Little work on combining dynamic asset allocation in the presence of time-varying opportunity set and realistic transaction/price impact costs.
- Yet, paradoxically, it is when opportunity set is time-varying that dynamic trading is most beneficial!
  - Propose simple practical approach to strategic asset allocation in the presence of predictability and transaction costs.
Large literature on Strategic Asset Allocation

- Merton (1969, 1971) and Cox-Huang (1989) show that in a dynamic setting investors want to hedge against future changes of the intercept (the risk-free rate) and the slope (Sharpe ratio) of the conditional mean-variance frontier.
- Full generality of this result is now well understood (e.g., Detemple-Rindisbacher (2011)) with representations of portfolio rule under ‘full generality’ using complicated mathematics (‘Malliavin derivatives’).
- Lots of academic work, studying strategic asset allocation focuses on impact of predictability on long-term asset allocation, i.e., on Merton hedging demand: Brennan, Schwartz, Lagnado (1997), Brandt (1999), Kim and Omberg (1996), and in particular Campbell (1999) and Campbell and Viceira (2002).
- Yet, most of that literature seems to be ignored by practitioners (deliberately?). Why?
- Realistic asset allocation needs to impose realistic objective function, constraints, transaction and price impact (‘slippage’) costs.
- Academic literature makes specific assumptions about objective function and dynamics to obtain ‘closed-form’ solutions and mostly ignores issues of model mis-specification and parameter uncertainty (of course, some exceptions...).
Transaction cost literature

- Constantinides (1986) Davis-Norman (1990) prove the optimal policy (of no-trading region) under proportional costs in iid setting.
- Most papers in this literature are mathematically very difficult (e.g., Shreve-Soner (1994)) even though limited to a one risky stock i.i.d. setup.
- Balduzzi and Lynch (1999): study one risky asset and one risk-free asset case with transaction costs and predictability.
- Lynch and Tan (2011): extend to two risky assets, with a very computationally demanding numerical technique.
- Grinold (2006): one period (stationary) approximation to dynamic trading strategy
- Garleanu-Pedersen (2011): Academic version similar to the (unpublished/unknown!) Litterman paper.
How costly is suboptimal rebalancing in the absence of transaction costs?

- Ang, Ahn, Collin-Dufresne (2011) consider cost to long horizon investor choosing an optimal mix of stocks/bonds and cash of various rebalancing frequency rules under different assumptions on return predictability.

- The setup is a two-factor extended affine model of the term structure, where slope predicts future returns (Fama-Bliss (1981), Duffee (2001), Cochrane-Piazzesi (2008)).

- The (log) stock market price also follows an affine process, with price/dividend ratio acting as a predictor (Fama-French (1993), Cochrane 2007)).

- We use monthly data on US Treasuries and on S&P500 prices and dividend yields from 1941 to 2011.

- The four factor affine model (two term structure factors, and two stock price factors) are fit to that data.

- We then consider the performance and utility costs for an investor who follows different rebalancing strategies.

- Main results are
  - In the absence of predictability costs to approximate (‘rule of thumb’) rebalancing rules are very small.
  - When there is predictability: costs of suboptimal strategies can be very large.
Asset allocation with Transaction costs and predictability

- What to (quant) practitioners do?
  - Identify return forecasting factors
  - Identify risk factors
  - Create portfolios with ‘optimal’ exposures to these factors that
  - Maximize expected returns, net of expected trading costs, subject to a risk budget

- As an example consider optimal combination of three signals for large (e.g., 2000 stocks) cross-section of individual stocks
  - Short term reversal (REV: half-life of 7 days)
  - Momentum (MOM: half-life 6 month)
  - Value (VAL: long-term reversal with a half-life of three years)

- Each stock will have a specific exposure to REV, MOM, and VAL. These factors will decay at different rates and are clearly not independent.

- How do we operationalize the ‘optimal trade-off’ when there are transaction costs?
  - If trade more frequently, expect to capture more alpha, but pay more transaction costs.
  - If trade less frequently, may not benefit from high frequency signals at all (should we ignore them?).
  - Trading more frequently also allows to better hedge risk-factors.
  - Should we trade more/less aggressively when signals decay faster/slower?
A reduced-form Practitioner approach

- A one period problem with quadratic transaction costs:

\[
\max_x \left\{ x' \alpha - \frac{\gamma}{2} x' \Sigma x - \frac{\tau}{2} \Delta x \Lambda \Delta x \right\}
\]

- where
  - \( x \) is vector of positions in each stock
  - \( \alpha \) is vector of excess returns on stocks
  - \( \Sigma \) is covariance matrix of returns
  - \( \Lambda \) is matrix of (quadratic) transaction costs

- Optimal trade:

\[
\Delta x^* = (\gamma \Sigma + \tau \Lambda)^{-1} (\alpha - \gamma \Sigma x_{t-1})
\]

- If \( \Lambda = \lambda \Sigma \) (Grinold, GP) and defining \( \hat{\tau} = \frac{\tau \lambda}{\gamma} \):

\[
x^*_{t+1} = \frac{1}{1 + \hat{\tau} (\gamma \Sigma)^{-1} \mu + (1 - \frac{1}{1 + \hat{\tau}})} x_t
\]

⇒ The optimal position is weighted average of first-best \((\gamma \Sigma)^{-1} \mu\) and initial position.

⇒ Trade-off transaction costs and tracking error by only partially moving to first best.

- Practitioners typically:
  - target a level of risk with risk-aversion parameter \( \gamma \),
  - calibrate \( \Lambda \) to match realistic price impact/transaction costs estimates, and
  - optimize over the t-cost aversion parameter \( \tau \) to maximize backtest performance.

- One period model easily extended to allow for linear-proportional t-costs: introduces no-transaction region.
The linear-quadratic framework.

- Litterman (2005), Grinold (2006), Garleanu-Pedersen (2011) propose a linear quadratic framework to ‘endogenize’ t-cost aversion and take into account dynamics explicitly.

- **Returns:** \( r_{t+1} = \mu_t + \beta f_t + u_{t+1} \)

  - \( r \) is a \( n \)-vector of returns.
  - \( \mu_t \) is fair (e.g., CAPM) benchmark expected return.
  - \( f \) is a \( k \)-vector of return predictors which decay at speed \( \Phi \):
    \[
    \Delta f_{t+1} = -\Phi f_t + \epsilon_{t+1}
    \]
  - \( \beta \) is a \((n, k)\) matrix of ‘factor loadings.’
  - \( \text{Cov}_t(r_{t+1}) = \text{Cov}_t(u_{t+1}) = \Sigma \) the covariance matrix of returns is constant.

- **Objective:**
  \[
  \max_{x_0, x_1, \ldots} \sum_{t=0}^{\infty} (1 - \rho)^t \left( x_t' \alpha_t - \frac{\gamma}{2} x_t' \Sigma x_t - \frac{1}{2} \Delta x_t' \Lambda \Delta x_t \right)
  \]

  - This is a classic linear quadratic control problem.
  - GP provide interesting micro-structure foundation to quadratic t-costs. Also discuss equilibrium implications.
Insights from the solution

- When $\Lambda = \lambda \Sigma$ and $\Phi$ is diagonal then the optimal solution is as in the one-period case: $x_{t+1}^* = a(\gamma \Sigma)^{-1} \hat{\alpha}_t + (1 - a)x_t$

- where
  - $a = a(\frac{\gamma}{\lambda}, \rho)$ is endogenous $\Rightarrow$ endogenizes t-cost aversion $\hat{\tau}$!
  - $\hat{\alpha}_t = \beta \hat{f}_t$ where $\hat{f}_t$ is endogenously adjusted.

- The target position is the Markowitz solution where each factor premium is decreased by a factor that depends on its decay speed ($\Phi_{ii}$), the risk-aversion and the ‘endogenously’ derived t-cost aversion $a$.

$\Rightarrow$ The less persistent a factor the more its mean return is lowered when computing the target portfolio.

$\Rightarrow$ As in one-period case, trade less quickly towards target as transactions costs (relative to risk-aversion $\frac{\lambda}{\gamma}$) are higher.

- In general, when $\Lambda$ is not colinear with $\Sigma$ then the solution is more complex:
  - optimal position is a ‘matrix-weighted’ average of the initial and a target position.
  - the target is obtained by a security specific t-cost adjustment of the alpha.
Application to relative value trading

- Suppose each security \( (s = 1, \ldots, n) \) has its own characteristic \( (f^s_k, k = 1, \ldots, K) \): short-term reversal, momentum, value) that predicts future returns, with common constant factor expected returns \( \beta_k \) (Fama-French (1996)):

\[
\alpha^s_t = \sum_{k=1}^{K} \beta_k f^s_k(t) \quad \forall s = 1, \ldots, n
\]

- Each characteristic has same mean reversion speed:

\[
\Delta f^s_k(t + 1) = -\phi_k f^s_k(t) + \epsilon^s_k(t + 1)
\]

- Then optimal strategy is

\[
x_{t+1} = a(\gamma \Sigma)^{-1} \sum_{k=1}^{K} \frac{1}{1 + (1 - \rho) a \frac{\lambda}{\gamma} \phi_k} f_k(t) + +(1 - a)x_t
\]

- GP test this strategy for portfolio of commodity futures and find significant improvement relative to naive myopic strategy.
Shortcomings of the linear-quadratic framework.

- Does not measure positions in dollars but in number of shares.

  ⇒ if expected returns do not change, it is optimal not to trade
  ≠ lessons from i.i.d. models of Constantinides-Davis-Norman, i.e., no rebalancing
  based on ‘drift’ in positions.

- Requires constant covariance matrix of returns (Σ).

  ⇒ Pure characteristics model, where characteristics do no affect the covariance structure
  of returns (≠ Fama-French!).

- Does not allow for common factors (e.g., industry, size) that may be correlated with
  return generating factors.

  ▶ Cannot accommodate any constraints or penalty that depart from linear-quadratic
  form (e.g., short-sale constraints, drawdown constraints, linear-proportional
  transaction costs...).

  ▶ Does not allow for common liquidity factors driving transaction cost matrix \( \Lambda_t \) and
  alpha generating factors.
  ≠ Reversal factor performance correlated with liquidity measures (Nagel (2011)).
How reasonable is the constant covariance structure assumption?

- A few pictures from the ‘Quant crisis’ of 2007 clearly show that covariance structure of factor returns is not constant.
- Even in normal times, the analysis of Fama-French (1996) suggests that there is a factor structure in returns related to the cross-sectional spread in expected returns.
- Market liquidity (e.g., Bid-ask spreads), market volatility (e.g., VIX), and returns to reversal strategy are correlated (Nagel (2011)).

![Figure 1: 3-month moving averages of daily return-reversal strategy returns and the CBOE S&P500 implied volatility index (VIX). Each day $t$, the reversal strategy returns are calculated as the average of returns from five reversal strategies that weight stocks proportional to the negative of market-adjusted returns on days $t−1$, $t−2$, ..., $t−5$, with weights scaled to add up to $1$ short and $1$ long. Returns are calculated from daily CRSP closing transaction](image)
Linear generating rules

- Build on several ideas:
  - ‘Parametric portfolio policies’ (Brandt, Santa-Clara (2006) and Brandt, Santa-Clara, Valkanov (2008)) consider a restricted set of trading rules, that make the problem similar to a static choice among a larger set of assets (each of which can be interpreted as a managed portfolio).

- Specify set of affine-linear-rebalancing rules for a general portfolio problem with:
  - Many stocks,
  - Many predictors with different decay rates,
  - Arbitrary dynamics for the factor exposures,
  - Quadratic transaction costs,
  - Time varying stochastic volatility (driven by factor innovations),
  - Time-varying stochastic liquidity costs (correlated with factor expected returns),
  - Formulated in wealth (dollar) terms rather in units of securities.

- Show that the optimization problem of choosing the coefficients of the (restricted) policy rule solves a closed form deterministic linear quadratic program (given a set of moments that need to be pre-estimated from data or via simulation).

- Performance of the trading approach significantly improves over linear-quadratic rules of Litterman and Garleanu Pedersen and over the myopic (with t-cost aversion) practitioner approach, especially when there is stochastic volatility.
Return generating process

- Assume \( \log(R_i(t + 1)) = B'_{i,t}(\lambda + F_{t+1}) + \epsilon_{i,t+1} - \frac{1}{2}(\sigma^2_i + B_{i,t}\Omega B_{i,t}) \)
  - where
    - \( B_{i,t} \) are the security specific factor exposures/security characteristics (e.g., reversal, momentum, value)
    - Factor realizations form \( K \) dimensional vector \( F_t \) with mean \( \lambda \) and covariance matrix \( \Omega \)
    - idiosyncratic risk is given by \( \epsilon_i \) with \((n, n)\) covariance matrix \( \Sigma_{\epsilon} \).
- Could accommodate arbitrary dynamics for factor exposures, but for illustration:
  \[
  B^k_i(t + 1) = (1 - \phi_k)B^k_{i,t} + \eta_k B'_{i,t}F_{t+1} + \epsilon_i(t + 1)
  \]
  - where
    - \( \phi_k \) captures the decay rate of the factors \( \phi_{VAL} > \phi_{MOM} > \phi_{REV} \).
    - \( \eta_{VAL} = \eta_R = 0 \) to capture the fact that short term reversal factor mostly is due to idiosyncratic shocks (e.g., reversal is less present in industry portfolios),
    - In contrast \( \eta_{MOM} = 1 \) (for example momentum is present in industry portfolios)
    - Allow for a market risk-factor \( (\lambda_M = 0, B_M \) constant)\)
- Note this example
  - Includes time-varying stochastic covariance matrix for returns \( \Sigma_i = B'_{i,t}\Omega B_{i,t} + \Sigma_{\epsilon} \).
  - Exposures move through time to capture average persistence of the signals.
    \( \Rightarrow \) In well-specified model, ‘decay’ of alpha should equal to mean-reversion of exposure.
  - This example could be extended to time-varying factor performance \( (\lambda \) could be time-varying - as a function of liquidity states, or decay over time)
The Wealth dynamics and Objective function

- Wealth dynamics can be written in terms of vector of dollar positions in securities $x_t$:
  \[ x_{i,t+1} = R_{i,t+1} x_{i,t} + u_{i,t+1} \quad i = 1, \ldots, N \]
  \[ w_{t+1} = w_t + \sum_{i=1}^{N} r_{i,t+1} x_{i,t} - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} u_{i,t+1} \Lambda_{t+1}(i,j) u_{j,t+1} \]

- Iterating and assuming wlog $w_0 = x_0 = r_f = 0$ gives:
  \[ w_T = \sum_{t=1}^{T} \left( r(t,T) u_t - \frac{1}{2} u_t \Lambda u_t \right) \]

- Investor max expected final wealth subject to risk penalty:
  \[ E \left[ W_T - \frac{\gamma}{2} \sum_{t=1}^{T} x_t' \Sigma x_t \right] \]

- Plugging in wealth dynamics:
  \[ \max_{x_0,x_1,\ldots,x_T} E \left[ \sum_{t=1}^{T} \left( r(t,T) u_t - \frac{1}{2} u_t \Lambda u_t - \frac{\gamma}{2} x_t' \Sigma x_t \right) \right] \]

- Note: This looks like a LQ-system but it is not!
  - $\Sigma_t$ is stochastic time varying
  - Exposures $B_t$ and returns form jointly Markov, non-linear system.
  - $x_t, u_t$ satisfy joint wealth equation dynamics

- The trick: specify dynamics of position and trades in a restricted set, so that optimization remains tractable, and solution approaches unconstrained optimum.
The linear-generating policy function

- We specify the admissible dollar position policy as follows:

\[
    x_{i,t} = \sum_{u=1}^{t} \left( J_{i,u,t} B_{i,u} \frac{S_{i,t}}{S_{i,u}} + \beta_{i,u,t} \frac{S_{i,t}}{S_{i,u}} \right)
\]

- Then to satisfy the wealth equation: \( x_{i,t+1} = R_{i,t+1} x_{i,t} + u_{i,t+1} \) \( \forall i \) the dollar trade vector is given by:

\[
    u_{i,t} = \sum_{u=1}^{t} \left( E_{i,u,t} B_{i,u} \frac{S_{i,t}}{S_{i,u}} + \alpha_{i,u,t} \frac{S_{i,t}}{S_{i,u}} \right)
\]

where:

\[
    E_{i,s,t} = J_{i,s,t} - J_{i,s,t-1} \quad \text{and} \quad \alpha_{i,s,t} = \beta_{i,s,t} - \beta_{i,s,t-1} \quad \text{for} \ s < t
\]

\[
    E_{i,t,t} = J_{i,t,t} \quad \text{and} \quad \alpha_{i,t,t} = \beta_{i,t,t} \quad \text{for} \ s = t
\]

- Optimizing in that restricted set amounts to picking the matrices of coefficients \( J_{i,u,t} \) and \( \beta_{i,u,t} \) (or \( E_{i,u,t}, \alpha_{i,u,t} \)) subject to the constraints above.

- Note that these policies are fairly general:
  - Specify policy as linear in current and past exposures as well as cumulativs past returns.
  - Nests the special form of Brandt, Santa-Clara and Valkanov as special case (only depends on current exposures with identical loadings across all securities).
  - With transaction costs it seems natural to allow policy to depend on past exposures and past returns.
Solving for the Optimal Linear Strategy

- Objective stated in dollar terms (not in units of securities). Therefore expect two motives to trade:
  - Trading for rebalancing purposes (even holding alpha constant). Intuition from Constantinides, Davis-Norman.
  - Trading for strategic reasons (time varying alpha and alpha decay). Similar to Litterman, Garleanu Pedersen.

- Define vectors $\theta_{u,t} = [E_{1u,t}, \alpha_{1u,t}, \ldots, E_{nu,t}, \alpha_{nu,t}]$ and $\pi_{u,t} = [J_{1u,t}, \beta_{1u,t}, \ldots, J_{nu,t}, \beta_{nu,t}]$

- $F_{u,t} = [B_{1u}R_{1u,t}, R_{1u,t}, \ldots, B_{nu}R_{nu,t}, R_{nu,t}]$ and $G_{u,t} = [B_{1u}R_{1u,T}, R_{1u,T}, \ldots, B_{nu}R_{nu,T}, R_{nu,T}]$.

- Then, $u_t = F_{1,t} \otimes \theta_{1,t} + \ldots + F_{t,t} \otimes \theta_{t,t}$ and $x_t = F_{1,t} \otimes \pi_{1,t} + \ldots + F_{t,t} \otimes \pi_{t,t}$

- Plug into objective function, we can rewrite the problem as:

$$\max_{\theta_t, \pi_t} \sum_{t=1}^{T} \Theta_t^\top (E[F_t] - E[G_t]) - \frac{1}{2} \Theta_t^\top E[P_t] \Theta_t - \frac{\gamma}{2} \Pi_t^\top E[Q_t] \Pi_t$$

subject to $\Pi_t = \begin{pmatrix} \Pi_{t-1} & 0 \end{pmatrix} + \Theta_t$

where

- $\Pi_t, \Theta_t, F_t, G_t$ are the stacked vectors (e.g., $\Pi_t = [\pi_{1,t}, \ldots, \pi_{t,t}]$).
- the matrices, $P_t$ and $Q_t$ are (with $\gamma_t := F_t F_t^\top$):
  - $P_t((u-1)n + i, (s-1)n + j) := \Lambda_t(i,j) \gamma_t((u-1)n + i, (s-1)n + j)$ $1 \leq i, j \leq M$
  - $Q_t((u-1)n + i, (s-1)n + j) := \Sigma_t(i,j) \gamma_t((u-1)n + i, (s-1)n + j)$ $1 \leq i, j \leq M$, 

Explicit solution

- The problem is a standard deterministic Linear Quadratic problem, given the unconditional expectations $E[P_t], E[Q_t], E[F_t], E[G_t]$.
- The latter need to be solved ex-ante (either via Monte-carlo simulation or in closed-form if the dynamics of returns and exposures are known).
- But the optimization component over the coefficients of the policy rule (the optimally managed portfolios) is then obtained in closed-form:

\[
\Pi_t = (E[P_t] + \gamma E[Q_t] + A_{zz}^t)^{-1} \left( E[G_t] + E[P_t] \begin{pmatrix} \Pi_{t-1} \\ 0 \end{pmatrix} + A_z^t \right)
\]

with the recursions given by

\[
\tilde{A}_{zz}^{t-1} = -E[P_t] (E[P_t] + \gamma E[Q_t] + A_{zz}^t)^{-1} E[P_t] + E[P_t]
\]

\[
\tilde{A}_z^{t-1} = E[P_t] (E[P_t] + \gamma E[Q_t] + A_{zz}^t)^{-1} (E[G_t] + A_z^t) - E[G_t]
\]

Due to the changing dimensions of the state variables, $A_{zz}^{t-1}$ is the upper left submatrix of $\tilde{A}_{zz}^{t-1}$ with size $l_{t-1} \times l_{t-1}$. Similarly, $A_z^{t-1}$ is the upper part of $\tilde{A}_z^{t-1}$ with size $l_{t-1} \times 1$. 
Intuition for the strategy

- Impulse response function to a positive return shock, if there is only one value factor (slow decay) generating alpha.
Intuition for the strategy

- Impulse response function to a positive return shock, if there is only one momentum factor (fast decay) generating alpha.
Intuition for the strategy

- Impulse response function to a positive return shock, if there are two negatively correlated factors (momentum and value) generating alpha.
Terminal Wealth Statistics for Different Strategies

- Momentum only without Factor noise \((\Omega = 0)\)

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- Momentum only with Factor noise

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<td>0.22</td>
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Terminal Wealth Statistics for Different Strategies

- Value only without Factor noise ($\Omega = 0$)

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<td>3.15</td>
<td>2.07</td>
<td>3.19</td>
<td>4.41</td>
</tr>
</tbody>
</table>

- Value only with Factor noise

<table>
<thead>
<tr>
<th></th>
<th>GP</th>
<th>Myopic</th>
<th>BL</th>
<th>No TC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>95.48</td>
<td>53.80</td>
<td>51.25</td>
<td>58.00</td>
</tr>
<tr>
<td><strong>Std Err</strong></td>
<td>0.68</td>
<td>0.33</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>104.69</td>
<td>54.80</td>
<td>53.72</td>
<td>59.25</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>-2039.78</td>
<td>-307.41</td>
<td>-434.01</td>
<td>-315.67</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>1647.67</td>
<td>346.03</td>
<td>422.17</td>
<td>348.70</td>
</tr>
<tr>
<td><strong>Sharpe (/yr)</strong></td>
<td>0.63</td>
<td>0.72</td>
<td>0.76</td>
<td>0.77</td>
</tr>
</tbody>
</table>
# Terminal Wealth Statistics for Different Strategies

- Two factors (momentum and value) without Factor noise ($\Omega = 0$)

<table>
<thead>
<tr>
<th></th>
<th>GP</th>
<th>Myopic</th>
<th>BL</th>
<th>No TC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>322.18</td>
<td>263.98</td>
<td>319.20</td>
<td>474.88</td>
</tr>
<tr>
<td><strong>Std Err</strong></td>
<td>0.59</td>
<td>0.74</td>
<td>0.60</td>
<td>0.74</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>329.81</td>
<td>273.77</td>
<td>325.67</td>
<td>484.74</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>-588.24</td>
<td>-555.96</td>
<td>-365.13</td>
<td>-348.79</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>824.84</td>
<td>776.10</td>
<td>841.73</td>
<td>997.56</td>
</tr>
<tr>
<td><strong>Sharpe (/yr)</strong></td>
<td>2.42</td>
<td>1.60</td>
<td>2.38</td>
<td>2.86</td>
</tr>
</tbody>
</table>

- Two factors (momentum and value) with Factor noise

<table>
<thead>
<tr>
<th></th>
<th>GP</th>
<th>Myopic</th>
<th>BL</th>
<th>No TC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>56.50</td>
<td>43.47</td>
<td>47.84</td>
<td>61.61</td>
</tr>
<tr>
<td><strong>Std Err</strong></td>
<td>0.47</td>
<td>0.36</td>
<td>0.29</td>
<td>0.36</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>65.13</td>
<td>43.79</td>
<td>50.31</td>
<td>61.82</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>-1477.66</td>
<td>-343.79</td>
<td>-331.84</td>
<td>-316.28</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>890.04</td>
<td>367.37</td>
<td>304.23</td>
<td>385.20</td>
</tr>
<tr>
<td><strong>Sharpe (/yr)</strong></td>
<td>0.54</td>
<td>0.54</td>
<td>0.73</td>
<td>0.77</td>
</tr>
</tbody>
</table>
Propose a new approach to dynamic portfolio choice under transaction costs and with predictability.

The approach builds on the linear-quadratic framework of Litterman-Garleanu-Pedersen to handle
- Many stocks and many return predictors with different decay rates,
- Arbitrary dynamics for the factor exposures,
- Quadratic transaction costs,
- Time varying stochastic volatility (driven by factor innovations),
- Time-varying stochastic liquidity costs (correlated with factor expected returns),
- Formulated in wealth (dollar) terms rather than in units of securities.

The (preliminary) results suggests that the approach is similar to the LQ approach when there is no (or little) factor risk, but that with factor noise (i.e., stochastic volatility) the benefits of the new approach can be large.

Unlike the LQ approach our policy also trades for pure rebalancing motives (not solely driven by alpha timing). This seems more important when the covariance matrix is stochastic.

More numerical is work needed to confirm these results with a realistic calibration.

Extending the model to allow for time-varying liquidity shocks that affect the profitability of the factor returns, as documented by Nagel (2011) for the Reversal factor, is also an interesting avenue to pursue.