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Longevity Risk Pricing

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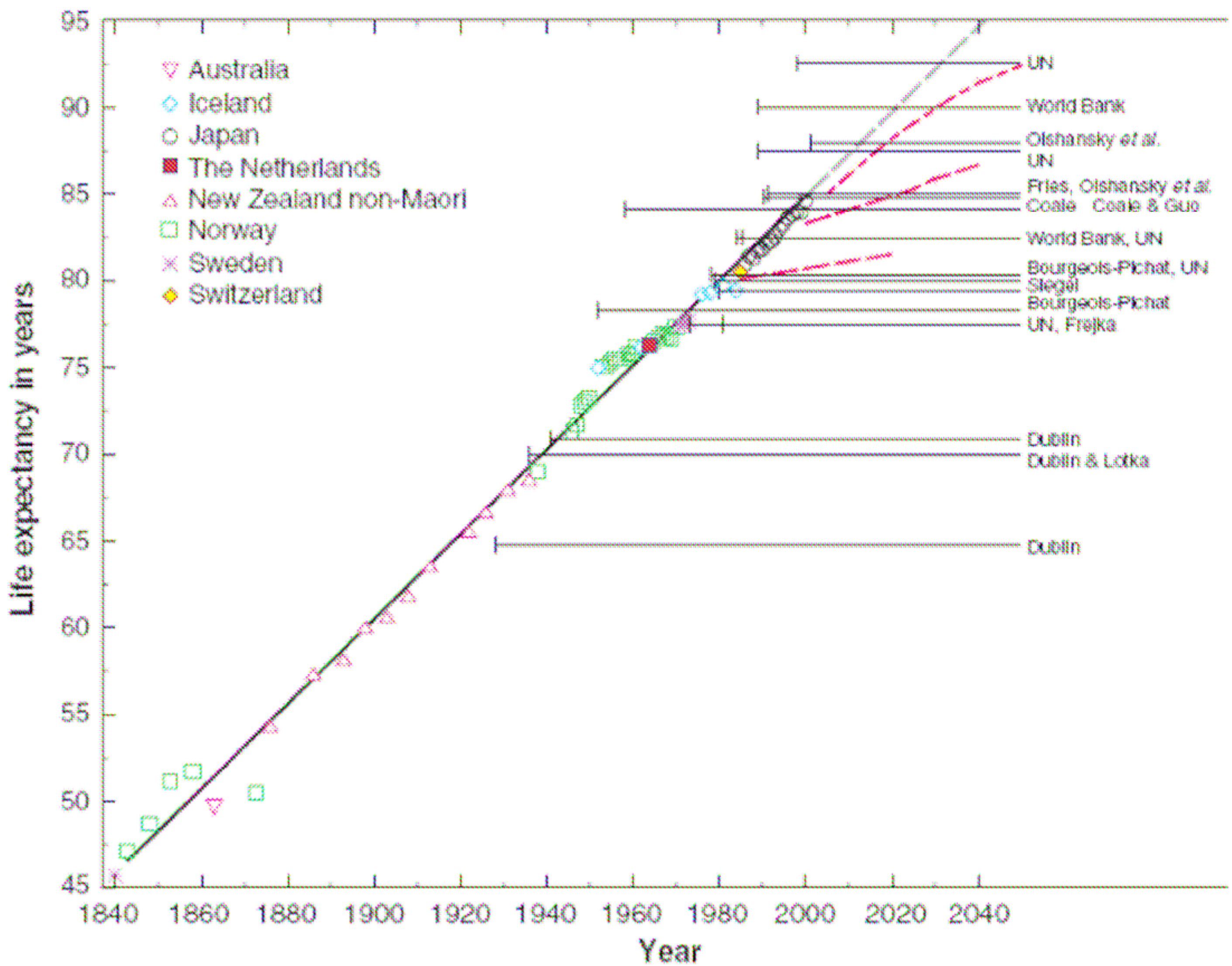
"Broken Limit to Life Expectancy"

Oeppen & Vaupel (2002), Science



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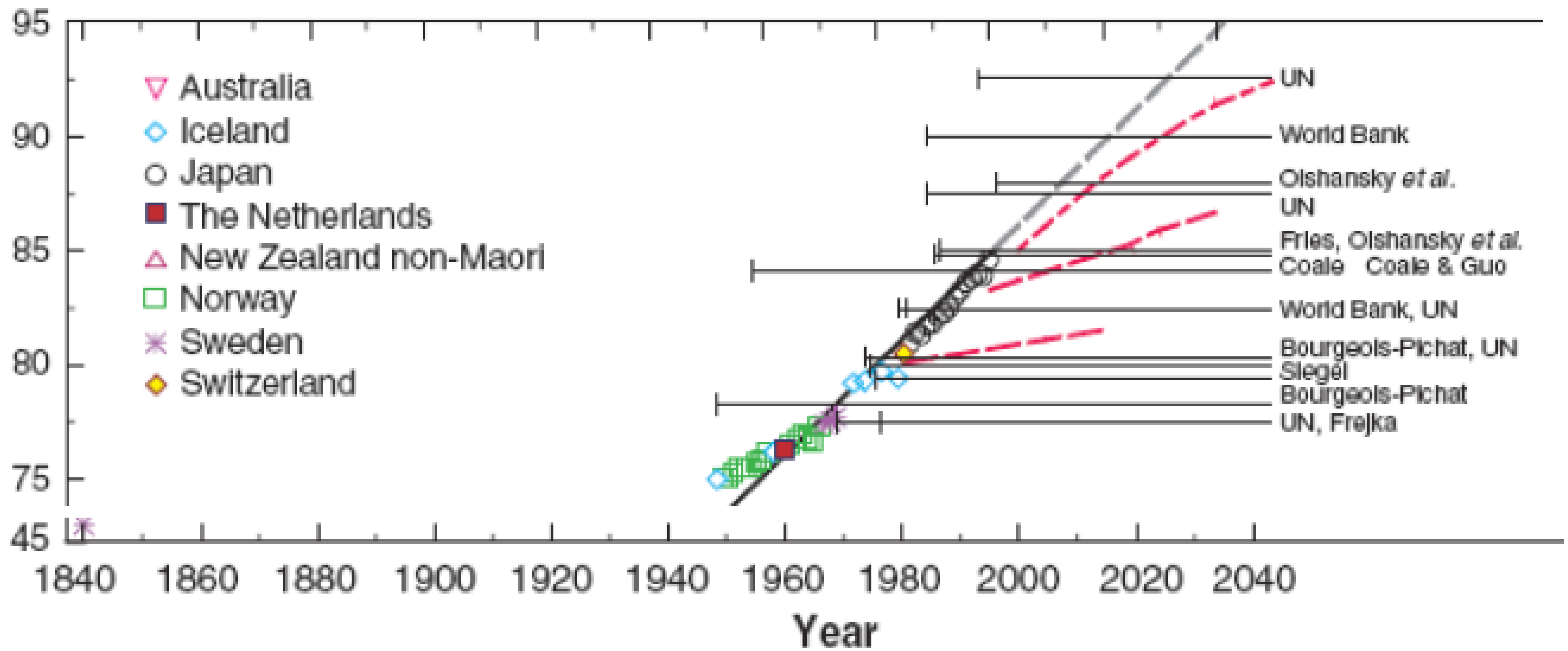
Longevity risk



- Uncertain mortality trend (longevity risk)
- Longevity risk is macro risk
- Difficult to project accurately

Motivation & Goal

- EIB/BNP longevity bond (20 bp)
  - *A 'coupon-based' bond; Maturity: 25 years*
  - *First announced in Nov 2004;*
  - *Withdrawn for redesign in late 2005;*
  - Obstacles: Design, pricing, institutional issues (Blake, Cairns and Dowd (2006))
- No clear view on the 'right' price
  - Incomplete market, unhedgable risk
  - Rule-of-thumb risk loading system

➢ Goal: Quantify longevity risk premiums

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Overview

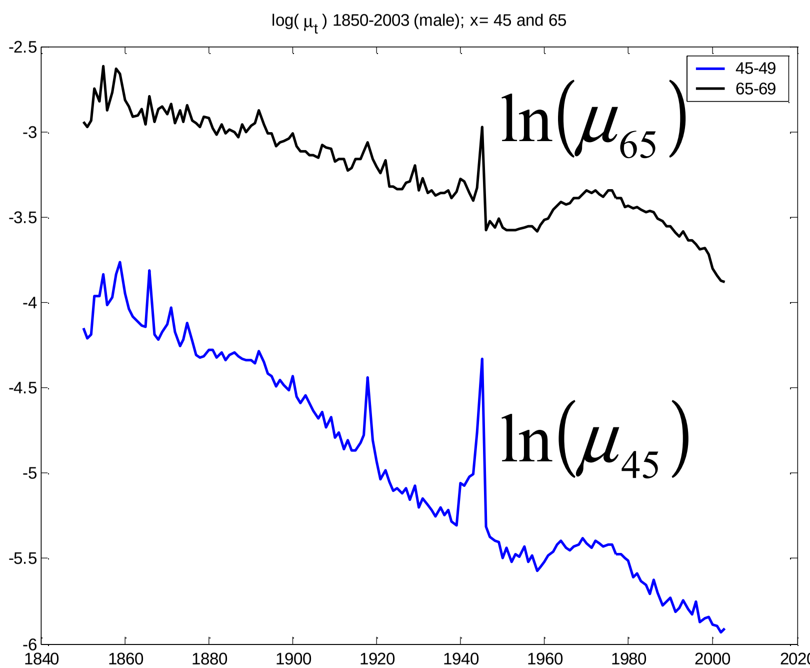
- Motivation & Goal
- Three building blocks
  - Stochastic mortality modeling
  - Longevity linked securities
  - (incomplete market) Pricing principles
- Pricing longevity bonds & derivatives
  - Bonds, swaps, floors, …
- Impact of natural hedgeing and basis risk
- Conclusion

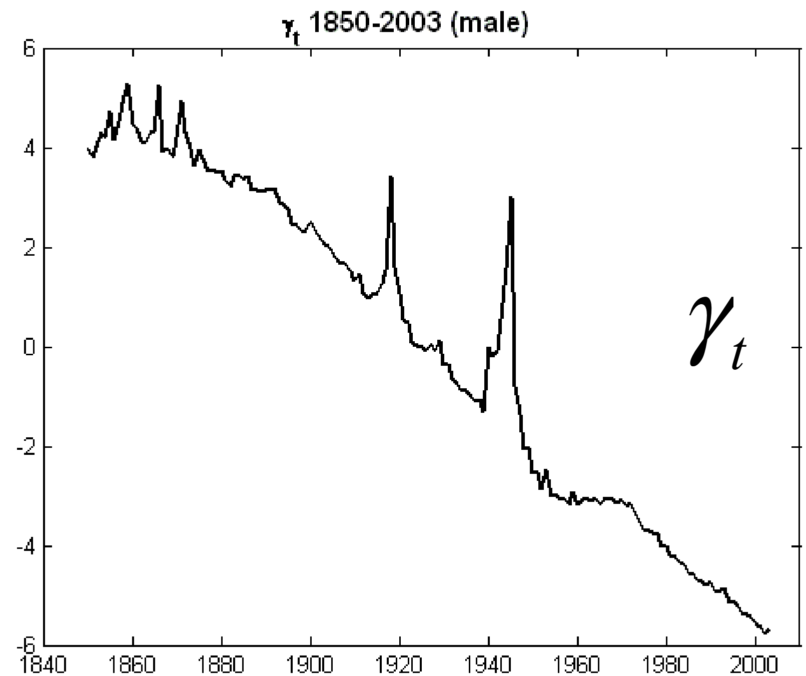
Stochastic mortality (Lee-Carter 92)

- model

$$\ln(\mu_{x,t}) = \alpha_x + \beta_x \gamma_t + \delta_{x,t}$$

$$\gamma_t = c + \gamma_{t-1} + \varepsilon_t$$





Longevity linked securities

- Longevity Zeros
- Coupon based longevity bonds
- Swaps
- Deferred starting longevity bonds
- Floors (cap)

Pricing principles

- CAPM (Friedberg and Webb (2006))

$$E_t[R_b] - R_f = \beta_b \left[E_t(R_m) - R_f\right]$$

  - 75 bp (= 0.15 * 5%), with c.i. [-75, 230] bp

- CCAPM (Friedberg and Webb (2006))

$$E_t(R_{b,t+1}) - R_f = -\frac{Cov(U'(C_{t+1}), R_{b,t+1})}{E_t(U'(C_{t+1}))}$$

  - 2 bp

- Sharpe Ratio (Milevsky, Promislow and Young (2006))

$$SR^{Ins} = \frac{N(1+L) - E(W_N)}{\sigma(W_N)}$$

Equivalent Utility Pricing

- Compensate the bond seller (e.g. EIB/BNP), such that, he is indifferent between bearing risk and not bearing risk.
  - Seller’s minimum price

- The bond buyer (e.g. life annuity providers) pays, such that, she is indifferent between bearing risk and not bearing risk.
  - Buyer’s maximum price

Utility function assumption

- Preference
  - Risk aversion depends on financial position
  - Additive in independent risk factors
  - b = 0, …, 1 (include CARA to CRRA)

$$u(X) = -\frac{1}{\alpha(W_0)}\exp(-\alpha(W_0)X)$$

$$\textit{where} \qquad \alpha(W_0) = \overline{\alpha}(W_0)^{-b}$$

Pricing longevity bond: Seller

- Without longevity risk

$$V_0 = \max_{\{D_t, x_t\}} E\left[\int_0^T \beta^{-t} u(D_t) dt\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t dt\right] = W_0$$

- With risk (seller)

$$V^{\pi}{}_0 = \max_{\{D^{\pi}{}_t, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D^{\pi}{}_t + E(S_t) - S_t\right) dt\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D^{\pi}{}_t dt\right] = W_0 + \pi$$

- Indifferent

$$V^{\pi}{}_0 = V_0$$

Pricing longevity bond: Buyer

- Without longevity risk

$$V^{\pi}{}_0 = \max_{\{D^{\pi}{}_t, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D^{\pi}{}_t\right) dt\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D^{\pi}{}_t dt\right] = W_0 - \pi$$

- With risk (seller)

$$V_0 = \max_{\{D_t, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D_t + E\left(S_t\right) - S_t\right) dt\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t dt\right] = W_0$$

- Indifferent

$$V^{\pi}{}_0 = V_0$$

Longevity bond risk premium (1)

- Results
  - Risk loading ($)

$$\pi = \frac{1}{\alpha} E\left[\int_0^T e^{-rt} \ln G_t dt\right]$$

$$where \qquad G \equiv E[\exp(-\alpha(E[S_t] - S_t))]$$

  - Risk premium R_p

$$\int_0^T e^{-rt} E[S_t]dt + \pi = \int_0^T e^{-(r+R_p)t} E[S_t]dt$$

Longevity bond risk premium (2)

equity	w0 = 10000				w0 = 1000				w0 = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	-1	0	0	0	-1	0	0	-1	-1
10	0	0	-1	-3	0	-1	-1	-3	0	-1	-2	-3
15	0	-1	-2	-7	0	-1	-3	-7	0	-2	-4	-7
20	0	-1	-4	-11	0	-2	-5	-11	0	-4	-7	-11
25	0	-2	-5	-15	0	-3	-6	-15	0	-5	-8	-15
30	0	-2	-5	-16	0	-3	-7	-16	0	-5	-9	-16
35	0	-2	-5	-16	0	-3	-7	-16	0	-5	-9	-16

Implications

- Longevity risk premium depends on the financial position of the issuer.
  - Stronger issuer requires lower risk premium
  - Smaller issues require lower risk premium
- Implication for market development:
  - More financial institutions to issue moderate amount of longevity bonds, which are linked to same survivor index.

Deferred starting longevity bonds

# year	w0 = 10000				w0 = 1000				w0 = 100			
defer	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
0	0	-2	-5	-16	0	-3	-7	-16	0	-5	-9	-16
5	0	-2	-6	-17	0	-3	-8	-17	0	-6	-10	-17
10	0	-2	-7	-21	0	-4	-9	-21	0	-7	-12	-21
15	0	-3	-9	-26	0	-5	-12	-26	0	-9	-15	-26
20	0	-3	-10	-30	0	-6	-13	-30	0	-10	-18	-30

Longevity floors & Caps

	w0 = 10000			w0 = 1000			w0 = 100		
maturity	bond	floor	cap	bond	floor	cap	bond	floor	cap
5	0	-3	3	0	-3	3	-1	-3	3
10	-1	-5	4	-1	-5	4	-2	-5	4
15	-2	-7	5	-3	-7	5	-4	-8	5
20	-4	-9	7	-5	-10	6	-7	-11	6
25	-5	-11	8	-6	-12	7	-8	-14	7
30	-5	-12	8	-7	-13	8	-9	-15	8
35	-5	-12	9	-7	-13	8	-9	-15	8

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The impact of natural hedgeing

- Term insurance is a natural hedge for annuity
- Natural hedgeing may have significant impact

equity	w0 = 10000				w0 = 1000				w0 = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	-1	0	0	0	-1	0	0	0	-1
15	0	0	-1	-2	0	0	-1	-2	0	-1	-1	-2
20	0	0	-1	-5	0	-1	-2	-5	0	-1	-3	-5
25	0	-1	-2	-6	0	-1	-3	-6	0	-2	-3	-6
30	0	-1	-2	-7	0	-1	-3	-7	0	-2	-4	-7

The impact of basis risk

- Basis risk: a discrepancy between the reference population and the annuitant population
- EIB/BNP survivor bond is linked British survivor index. Is it a good deal for Dutch pension fund?
- Buyer’s view

Dutch buyer's price to EIB/BNP bonds

- Without basis risk (bond linked to Dutch mortality)

equity	w0 = 10000				w0 = 1000				w0 = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	-1	-2	0	0	-1	-2	0	-1	-1	-2
10	0	-1	-2	-7	0	-1	-3	-7	0	-2	-4	-7
15	0	-2	-5	-15	0	-3	-7	-15	0	-5	-9	-15
20	0	-3	-8	-23	0	-5	-11	-23	0	-8	-14	-23
25	0	-3	-10	-28	0	-6	-13	-28	0	-10	-17	-28
30	0	-3	-10	-30	0	-6	-14	-30	0	-10	-18	-30

- With basis risk (bond linked to British mortality)

equity	w0 = 10000				w0 = 1000				w0 = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	-1	0	0	0	-1	0	0	0	-1
10	0	0	-1	-2	0	0	-1	-2	0	-1	-1	-2
15	0	-1	-2	-6	0	-1	-3	-6	0	-2	-4	-6
20	0	-1	-4	-11	0	-2	-5	-11	0	-4	-7	-11
25	0	-2	-6	-16	0	-3	-8	-16	0	-6	-10	-16
30	0	-2	-7	-18	0	-4	-9	-18	0	-7	-12	-18

Conclusion

- Longevity risk is potentially large
- Longevity linked securities offer a solution
- This paper tackles the pricing difficulty
- Provide design implications for longevity linked securities
  - Market calls for more issuers,
  - each issues a moderate amount of longevity bonds linked to same survivor index.
- Natural hedgeing and basis risk may have significant impacts on pricing.