ALM Modeling for Dutch Pension Funds: Indexation and New Regulatory Rules

Willem K. Klein Haneveld   Matthijs H. Streutker*

Maarten H. van der Vlerk

Department of Econometrics & OR
University of Groningen
PO Box 800, 9700 AV Groningen
The Netherlands

March 2007

Abstract

This paper describes a multistage recourse Asset Liability Management (ALM) model for pension funds, which utilizes defined benefit contracts and is tailored to the Dutch situation. The model deals with the developments currently faced by the pension funds. The paper is focused on the modeling side of the problem. The contributions of this paper are the accurate and yet compact modeling of indexation decisions, the implementation of the new Dutch regulatory rules for pension funds, which are called Financieel Toetsingskader, and an economically interpretable objective function.

1 Introduction

Pension fund management in the Netherlands is on the move, which sets ALM modeling for pension funds for new challenges. Years of apparent stability abruptly ended with the severe drop of the stock indices at the beginning of the new millennium. This unrest, also fed by the ageing discussion, encouraged a broadly conducted debate on the Dutch pension system. In the Netherlands, the vast majority of the pension contracts are defined benefit contracts, where the pension rights are predetermined, as they are based on the salary of the participant. In a final earnings (FE) scheme the rights are based on the final salary before retirement, while in an average earnings (AE) scheme they are based on

*Corresponding author: m.h.streutker@rug.nl

the career's average salary. Throughout his working years the participant and his employer make contributions to the pension fund. In general, the correction of the pension rights for inflation, which is called indexation, is conditional.

Due to the pension debate, firmly rooted foundations were reconsidered and altered. For example, one agreed that the use of a single actuarial interest rate for discounting the future expected benefit payments is outdated and needs to be replaced by an interest rate term structure. Other important outcomes of the debate are a shift from FE to the more modest AE schemes and the development of new regulatory rules for pension funds. We will deal with these two issues subsequently. First, with the shift to AE schemes[1] the indexation decisions of the pension fund board become very important, since solid indexation is indispensable for a substantial pension in an average earnings scheme. However, the standard practice of full indexation of pension entitlements came to an end[2]. Second, the new regulatory rules, which are called the Financieel Toetsingskader (FTK) and are scheduled to be implemented as of January 2007, are based on risk based supervision and show strong affinity with the Solvency [8] and Basel [2] accords.

Multistage Recourse ALM for Dutch Pension Funds

An upcoming ALM methodology is multistage stochastic programming. The headway made is described in surveys by Mulvey [15] and Sodhi [17]. ALM for Dutch pension funds with stochastic programming starts with the hybrid simulation/optimisation model of Boender [3]. In this paper a first step toward optimization is made, away from the standard practice of simulation. The first genuine multistage optimization model for Dutch pension funds is described by Dert [6]. This model contains accounting and some straightforward policy constraints. The most striking feature of the model is the presence of chance constraints, i.e. VaR constraints, which bound the probability of underfunding for the next year. However, these chance constraints limit the solvability of the model. Bogentoft et al. [4] provide with conditional value at risk (CVaR) constraints an alternative to the chance constraints. The CVaR model is convex, which makes it much more tractable than the chance-constrained model of Dert [6]. Kouwenberg [14] chooses to emphasize the need for a healthy fund not by restricting the solvency of the fund, but by penalizing deficits in the objective. The model was applied to real data of a Dutch pension fund by Gondzio and Kouwenberg [10]. All models mentioned above have rather sketchy policy constraints. Drijver [7] on the other hand formulates policies in great detail.

[1] Looking at the type of scheme of active participants in the Netherlands, we see the following development - FE: from 58.7% (2000) to 10.6% (2005), AE: from 30.6% (2000) to 74.3% (2005), source: DNB [5].

[2] The indexation of ABP, which is one of the largest pension funds in the Netherlands, for the past two years did not equal the wage inflation. It was 79% (2004) and 45% (2005) of the wage inflation, source: ABP [1].

The afore-mentioned models are not adequate in the current situation of new regulatory rules and incomplete indexation. As discussed in [12], the model of Drijver [7] has the most affinity with the FTK rules. Nevertheless, the model is deficient on the market valuation of liabilities, and the solvency requirements. Concerning indexation, only Drijver [7] models the indexation policy, though in a rough way. The other models resort to fully-indexed final earnings schemes. Moreover, the objective functions of most models are difficult to interpret economically, since penalty or risk parameters without a clear interpretation need to be specified.

We designed a multistage recourse ALM model tailored to the current Dutch situation (defined benefit, average earnings scheme). The model does not only generate decisions for the traditional asset policies of investment and financing, but also for the liability policy of indexation. The detailed modeling of the indexation decisions is a novelty. We believe that these decisions will become of vital importance in the near future. Another key feature is that it covers the regulatory rules dictated by the FTK: market valuation of the liabilities is applied, and risk-based solvency conditions are included. Special attention is paid to the objective function, which is a weighted sum of the specific interests of the three stakeholder groups, which are in our case the sponsor, active participants, and passive participants. The interests of a stakeholder are assumed to be the discounted expected cash flows resulting from the decisions made in the model.

Organization

The organization of the rest of the paper is as follows. In the next section, it is explained how pension rights are built up in an average earnings scheme. Beginning at the individual employee level, we aggregate the calculations to a pension fund as a whole. Section 3 gives the model description comprising the indices, variables, and parameters used, the objective function, and the constraints. Moreover, some shortcomings and possible extensions are discussed. The last section deals with the prerequisites for data (scenario) generation for the model. The paper ends with a conclusion.

Notational Issues

Since the notion of time plays an important role in a multistage model, we pay some special attention to it. All quantities except for deterministic parameters are related to a certain point in time or period of time. We will use t as the main time index. Since we have a discrete model, t either specifies the point in time t or the period of time $(t-1, t]$. This is illustrated in Figure 1. The values of quantities specified for a period t will be observed at time t.

Besides the current time, we also need to refer to events in the past (e.g., accrual of rights) and future events (e.g., expected benefit payments). The time

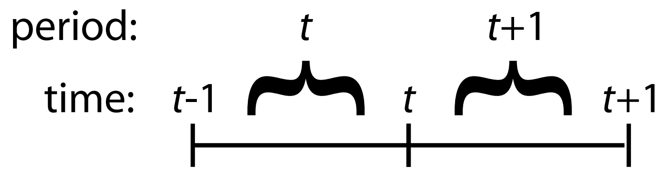

Figure 1: Notion of time in the model.

notation used is as follows:

$$x_t, x_t^s, x_t(u), x_t^s(u),$$

with x a variable or stochastic parameter. The t refers to the point/period of time that the quantity is observed. A superscript s refers to the past. A postfix u between brackets refers to the future. Additionally, r is used whenever another past index is needed.

2 Build-up in an Average Earnings Scheme

In this section, we will explain how pension calculations are executed in an average earnings scheme, where we restrict the discussion to retirement pension. First, the characteristics of an average earnings scheme are discussed with special attention to indexation. Next, it is shown how a single participant builds up his benefit rights. Finally, we will pursue the single participant calculations to the pension fund as a whole.

2.1 Average Earnings Schemes and Indexation

The underlying idea of an average earnings scheme is to provide the participant with a pension that is 70% of his average earnings when he retires. The rights accrued in each year equal the accrual rate times the pension basis of that year and are to be paid from the age of 65 till death. The pension basis is usually determined yearly, and is based on the pensionable wages of a certain year, which comprises both the structural and incidental wage components. The accrual rate is mostly 1.75% or 2% corresponding to, respectively, 40 and 35 years of service needed to reach the 70% of average earnings goal.

Due to inflation, the rights accrued during working years will have a much lower purchasing power upon retirement. Pension funds strive to correct the pension rights for inflation so as to provide the participants an inflation-proof pension. However, in general, this indexation of rights is conditional, mostly depending on the financial position of the fund. Indexation decisions are of

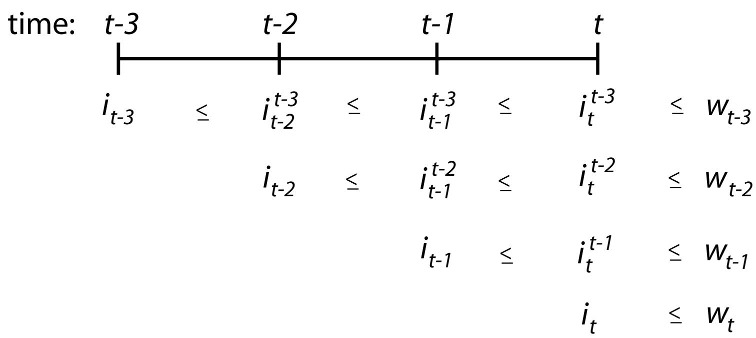

Figure 2: Indexation process.

utter importance in average earnings schemes, since they both affect active and passive participants. As Van Ewisk [18] points out, this is in contrast with a final earnings scheme where only those who are already retired or otherwise inactive suffer from incomplete indexation. As mentioned in the introduction of this paper, in the recent years no full-indexation was given as opposed to the past. Throughout this manuscript we will treat the indexation decisions explicitly. Therefore, we consider three types of rights. First, the guaranteed *nominal rights*, which are the rights as they are accrued during working years. Second, the desired *fully-indexed rights*, which are the nominal rights fully corrected for past inflation. Third, the *actual rights*, which are based on the past indexation decisions of the fund.

Indexation Decisions

At the end of each year, the board of the pension fund decides by how much the accrued rights will be indexed, with a maximum of the wage inflation of that year. So, we take the wage inflation to be the inflation measure. Let i_t be the *initial indexation* decision for year t, i.e. the accrued rights multiplier, then it must hold that

$$1 \le i_t \le w_t$$

with w_t the wage increase factor for year t.

Besides initial indexation decisions concerning the last year, it is also possible that incomplete indexation of the past is repaired. Let Δi_t^s be the *repair* at time t of the initial indexation decision taken at time s with $s < t$. Suppose last year's initial indexation i_{t-1} did not cover the wage inflation and at time t there is a repair Δi_t^{t-1}, then it needs to hold that

$$i_{t-1} + \Delta i_t^{t-1} \le w_{t-1},$$

with $\Delta i_t^{t-1} \ge 0$. The total indexation may not exceed the wage inflation. The nonnegativity condition on the repairs expresses an important feature, namely that once granted indexation cannot be taken back. The *indexation* decision over year s at time t ($s < t$), which we denote by i_t^s, equals the initial indexation decision plus the sum of all repairs:

$$i_t^s = i_s + \sum_{r=s+1}^{t} \Delta i_r^s$$

for which the following conditions hold

$$1 \leq i_t^s \leq w_s, \quad (1)$$

$$i_t^s \geq i_{t-1}^s. \quad (2)$$

The indexation process is illustrated in Figure 2.

2.2 Single Participant

We consider a single participant of a pension fund (either a company or an industry pension fund), who we denote by j. In this section it is shown how j's pension entitlements are built up during his working years and how these rights are valued. First, the accrued rights are considered. Next, we explore the (expected) benefit payments. Finally, the value of the liabilities to j is calculated.

Accrued Rights

The rights accrued in year t are calculated using the so-called franchise method. From the pensionable wages W_{jt} the franchise F_t is deducted to get the pension basis P_{jt}, with the franchise being ten seventh of the state old age provision AOW_t in year t. The newly accrued rights R_{jt}^+ equal now the accrual rate a times the pension basis. So, the newly accrued rights in year t are calculated using the following formula:

$$R_{jt}^+ = a(W_{jt} - \frac{10}{7} AOW_t).$$

This process is summarized and exemplified in Table 1. The accrual of pension rights over the years is also illustrated in Figure 3.

In order to be able to calculate the nominal rights, we need to specify j's first year of participation in the pension fund. We name this year s_j. The total nominal rights of the participant at time t, denoted by $\underline{R}_{jt}$, are the sum over all newly accrued rights during his working years so far:

$$\underline{R}_{jt} = \sum_{s=s_j}^{t} R_{js}^+.$$

The nominal rights are guaranteed by the pension fund. However, they strive to provide an inflation-proof pension. The value of the fully-indexed newly accrued

Terms	Calculation	Example
Pensionable wages	W_{jt}	100,000
AOW	AOW_t	7,000
Franchise	$F_t = \frac{10}{7} AOW_t$	10,000
Pension basis	$P_{jt} = W_{jt} - F_t$	90,000
Accrual rate	a	0.02
Newly accrued rights	$R^+_{jt} = aP_{jt}$	1,800

Table 1: Calculation of newly accrued rights using the franchise method.

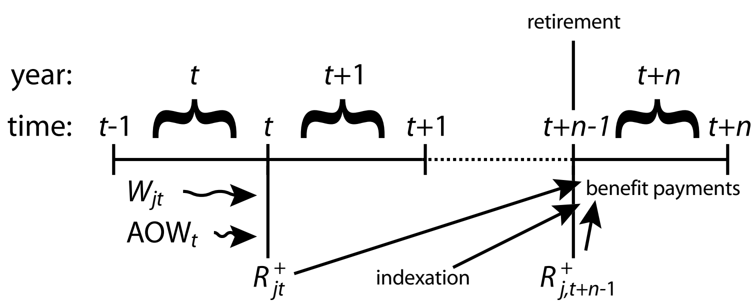

Figure 3: Accrual of pension rights.

rights of year s in year t, where $s_j \leq s < t$, equals $\prod_{r=s+1}^{t} w_r R^+_{js}$. The value of the fully-indexed rights, denoted by $\overline{R}_{jt}$ is given by the newly accrued rights in year t plus the sum of wage inflation corrected formerly accrued rights:

$$\overline{R}_{jt} = R^+_{jt} + \sum_{s=s_j}^{t-1} \prod_{r=s+1}^{t} w_r R^+_{js},$$

or recursively,

$$\overline{R}_{jt} = R^+_{jt} + w_t \overline{R}_{j,t-1}.$$

The value of the actual rights depends on the indexation decisions i^s_t. The actual rights are given by:

$$R_{jt} = R^+_{jt} + \sum_{s=s_j}^{t-1} \prod_{r=s+1}^{t} i^r_t R^+_{js}.$$

Figure 4 illustrates the calculation of the actual rights at time t after five years of accrual. In Table 2 we elaborate the example of Table 1.We added four years, where we let the initial indexation in each year equal half of the wage increase and no repairs are made. It is assumed that the AOW follows the wage inflation.

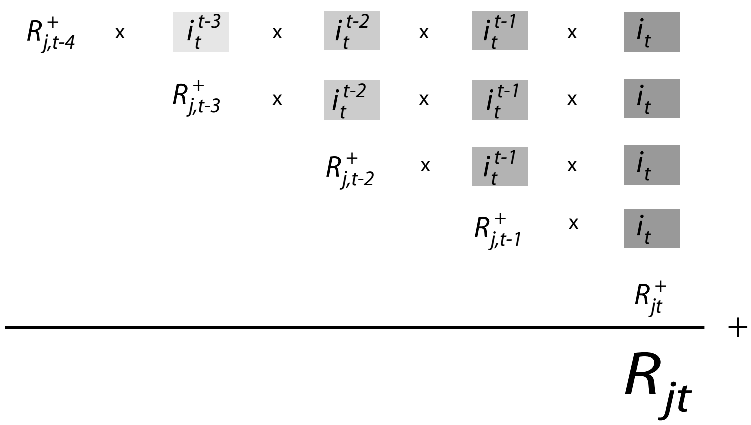

Figure 4: Actual pension rights.

t	s_j	s_j+1	s_j+2	s_j+3	s_j+4
W_{jt}	100,000	102,000	103,020	108,171	110,334
AOW_t	7,000	7,140	7,211	7,572	7,723
F_t	10,000	10,200	10,302	10,817	11,033
P_{jt}	90,000	91,800	92,718	97,354	99,301
R^+_{jt}	1,800	1,836	1,854	1,947	1,986
$\underline{R}_{jt}$	1,800	3,636	5,490	7,437	9,423
w_t	-	1.02	1.01	1.05	1.01
$\overline{R}_{jt}$	1,800	3,672	5,563	7,788	9,852
i_t	-	1.01	1.005	1.025	1.005
R_{jt}	1,800	3,654	5,527	7,612	9,636

Table 2: Example continued.

Benefit Payments

The value of the benefit payments to the participant depends on whether the participant is retired and alive in year t. Let $\mathbf{1}^r_{jt}$ indicate whether participant j is retired or not in year t, so

$$\mathbf{1}^r_{jt} = \begin{cases} 1 & \text{if participant } j \text{ is retired in year } t; \\ 0 & \text{if participant } j \text{ is not retired in year } t. \end{cases}$$

Let $\mathbf{1}^a_{jt}$ indicate whether participant j is alive or not in year t, so

$$\mathbf{1}^a_{jt} = \begin{cases} 1 & \text{if participant } j \text{ is alive in year } t; \\ 0 & \text{if participant } j \text{ is dead in year } t. \end{cases}$$

The nominal, fully-indexed, and actual benefit payments to the participant in year t are now, respectively, given by:

$$\underline{B}_{jt} = \mathbf{1}^r_{jt}\mathbf{1}^a_{jt}\underline{R}_{j,t-1},$$
$$\overline{B}_{jt} = \mathbf{1}^r_{jt}\mathbf{1}^a_{jt}\overline{R}_{j,t-1},$$
$$B_{jt} = \mathbf{1}^r_{jt}\mathbf{1}^a_{jt}R_{j,t-1}.$$

Whether the accrued rights are capitalized and during which period is uncertain. The *expected benefit payments* to the participant are determined using underwriting principles like mortality tables, mortality trends, and longevity risk. The complex calculation of the expected benefit payments is omitted, since it is beyond the scope of this paper. We denote the expected nominal, fully-indexed, and actual benefit payments to participant j in year u as estimated at time t by $\underline{B}_{jt}(u)$, $\overline{B}_{jt}(u)$, and $B_{jt}(u)$.

Value of Liabilities

The value of the liabilities is determined by discounting the expected benefit payments. Until recently, the discount factors were based on a single actuarial interest rate. Currently, however, an interest rate term structure is used at the supervisor's instigation. Introducing $d_t(u)$ as the discount factor for discounting payments in year u to time t, the expected value of the nominal, fully-indexed, and actual liabilities to the participant at time t are, respectively

$$\underline{L}'_{jt} = \sum_{u=t+1}^{\infty} d_t(u)\underline{B}_{jt}(u),$$
$$\overline{L}'_{jt} = \sum_{u=t+1}^{\infty} d_t(u)\overline{B}_{jt}(u),$$
$$L'_{jt} = \sum_{u=t+1}^{\infty} d_t(u)B_{jt}(u).$$

2.3 Pension Fund

In the previous section, we explained the pension calculations for a single participant. Aggregation over all participants gives the results for the pension fund as a whole. Again we make a distinction between nominal, fully-indexed, and actual pension rights. Of interest for our ALM model is the value of the liabilities and benefit payments and to relate them in a correct way to the indexation decisions.

Let $P = \{1, \ldots, J\}$ be the set of participants (both active and passive) of the fund during the decision period. Let $W_t(P) \subseteq P$ be the set of active participants in year t. Let $A_t(P)$ be the set of persons that are alive in year t, i.e.,

$$A_t(P) = \{j \in P : \mathbf{1}^a_{jt} = 1\}.$$

Participants having entitlements at time t must be alive at t and have been active in the past. Let $E_t(P) \subseteq P$ be the set of participants that have pension entitlements at time t. So,

$$E_t(P) = \{j \in A_t(P) : \exists s \leq t \text{ s.t. } j \in W_s(P)\}.$$

Let $R_t(P) \subseteq P$ be the set of retirees, i.e.,

$$R_t(P) = \{j \in P : \mathbf{1}^a_{jt} = \mathbf{1}^r_{jt} = 1\}.$$

The total unconditional expected benefit payments, liabilities, and benefit payments are:

$$\begin{aligned}
\underline{B}_t(u) &= \sum_{j \in E_t(P)} \underline{B}_{jt}(u), \\
\underline{L}_t &= \sum_{j \in E_t(P)} \underline{L}_{jt}, \\
\underline{B}_t &= \sum_{j \in R_t(P)} \underline{B}_{jt}.
\end{aligned}$$

The fully-indexed and actual (expected) benefit payments and liabilities are calculated in a similar way.

In our ALM model, we are mostly concerned with the value of the liabilities ($\underline{L}_t$, $\overline{L}_t$, and L_t) and the benefit payments ($\underline{B}_t$, $\overline{B}_t$, and B_t). The nominal ones and fully-indexed ones will be data. The values of the actual ones are decisions, since they follow directly from the indexation decisions. The actual liabilities at time t are

$$L_t = \sum_{j \in E_t(P)} L_{jt}.$$

Enumerating this expression gives

$$L_t = \sum_{u=t+1}^{\infty} d_t(u) \sum_{j \in E_t(P)} p_{jt}(u) \sum_{s=s_j}^{t} \prod_{r=s+1}^{t} i^r_t R^+_{js}. \tag{3}$$

By changing the order of summation, and introducing suitable auxiliary parameters $\underline{L}^s_t$ and s_t, we get the equivalent representation

$$L_t = \sum_{s=s_t}^{t} \prod_{r=s+1}^{t} i^r_t \underline{L}^s_t,$$

where $\underline{L}^s_t$ is the value of the nominal liabilities at time t stemming from rights accrued in year s and s_t is the year the oldest rights stem from at time t. Similarly, we have for the benefit payments in year t:

$$B_t = \sum_{s=s_t}^{t} \prod_{r=s+1}^{t} i^r_t \underline{B}^s_t,$$

where $\underline{B}^s_t$ are the nominal benefit payments in year t stemming from rights accrued in year s.

3 Model Description

In this section we describe our ALM model, which is a multistage recourse model. The stochastics and time structure involved are dealt with implicitly. The emphasis is on the choice of suitable variables and specification of the objective and constraints.

3.1 Introduction

We will explain the assumptions we make concerning the pension fund, and what outcomes our model will give, i.e., the pension policy.

Setting

The pension fund we consider is a company pension fund. Therefore, in case of financial distress the company can/must act as a sponsor. Moreover, we use an average earnings scheme. The nominal rights, as accrued, are guaranteed by the fund. The indexation of the rights is conditional. However, once granted indexation cannot be taken back. We only consider retirement pension.

Pension Policy

The pension policy generated by the model contains three components: investment, financing, and indexation. The investment policy concerns the purchase and selling of assets. Pension funds invest, in general, in a number of different asset classes, which all have their own return and risk characteristics. At the beginning of the decision period a certain asset portfolio is given. At each decision moment the portfolio can be changed by selling or buying assets.

The regular inflow of funds comes from the contributions made to the fund. Both the employees and the employer contribute each period to the pension fund. The amount contributed is related to the wage of the employee. The contribution rate (the part of the wages that is contributed to the pension fund) for the next period is decided at each decision moment in the model. Besides these regular contributions, it is possible that a remedial contribution of the sponsor is needed.

The indexation policy concerns the indexation decisions. In section 2.1 we stated that the board makes decisions i_t^s, which are the indexation decisions taken at time t for the wage inflation in period s ($s < t$). The actual value at time t of liabilities originating in year s is given by

$$L_t^s = \prod_{r=s+1}^{t} i_r^s \underline{L}_t^s$$

with $\underline{L}_t^s$ the value of the nominal liabilities at time t originating in year s. As direct implementation of the indexation decisions would result in nonlinear constraints for the actual value of the liabilities, we choose to work with accumulated indexation decisions. The *accumulated indexation decision* at time t for rights accrued in year s is

$$I_t^s = \prod_{r=s+1}^{t} i_t^r,$$

and conversely

$$i_t^s = \frac{I_t^{s-1}}{I_t^s}.$$

As a consequence, we need to translate the constraints on the indexation decisions (1) and (2) to constraints on accumulated indexation decisions, which is implemented in section 3.4.

The policy decisions are steered in the objective, while limitations are expressed in constraints on the decisions. Before we can state the objective and constraints we need to specify the indices, variables, and parameters.

3.2 Indices, Variables, and Parameters

This section describes the indices, variables, and parameters used in the ALM model.

Indices

The indices used throughout the model relate to time, asset class, stakeholder type, or risk type.

t	time index for decision scope, either the point in time t with $t \in \{0, 1, \ldots, T\}$ or period $[t-1, t]$, with $t \in \{1, \ldots, T\}$;
u	time index for future benefit payments scope, with $u \in \{1, \ldots, T, \ldots, U\}$;
s	time index for accrual scope with $s \in \{s_0, \ldots, 1, \ldots, T\}$;
i	asset class index, with $i \in \{1, \ldots, I\}$;
k	stakeholder index, with $k = 1 :=$ active participants, $k = 2 :=$ retirees, and $k = 3 :=$ sponsor;
r	risk index.

Variables

The variables of the model are the policy instruments and financial healthiness indicators of the fund. We distinguish decision and state variables.

The decision variables are the actual decisions that are made by the board. They are the pension fund policy.

A_{it}^{+}	value of assets bought of class i at time t;
A_{it}^{-}	value of assets sold of class i at time t;
c_t	contribution rate for period $t+1$;
C_t^{rem}	value of remedial payment at time t;
I_t^s	accumulated indexation at time t of rights accrued in period s.

State variables are important indicators of the financial healthiness of the pension fund.

A_{it}	value of asset class i at time t;
A_t^*	total asset value at time t before decisions;
A_t	total asset value at time t after decisions;
B_t (B_t^s)	value of benefit payments in period t (due to rights accrued in period s);
L_t (L_t^s)	value of the liabilities at time t (due to rights accrued in period s);
S_t	solvency at time t;
S_{rt}	required solvency for risk type r at time t;
u_t	underfunding indicator (binary) at time t.

Parameters

The parameters are the data of the model. In section 4, we will explain how these data will be obtained. Some of the parameters are stochastic, others are deterministic.

The stochastic parameters will have multiple realizations in the model. Directions for the generation of these parameters are given in section 4.

$\underline{L}_{kt,0}^s$	value of nominal liabilities at time t of stakeholder k ($k=1,2$) originating in rights accrued in period s using the discount factors of the beginning of the decision period ($t=0$);
$\underline{L}_t$ ($\underline{L}_t^s$)	value of nominal liabilities at time t (due to rights accrued in period s);
$\overline{L}_t$ ($\overline{L}_t^s$)	value of fully-indexed liabilities at time t (due to rights accrued in period s);
$\underline{B}_t$ ($\underline{B}_t^s$)	value of nominal benefit payments at time t (due to rights accrued in period s);
$\overline{B}_t$ ($\overline{B}_t^s$)	value of fully-indexed benefit payments at time t (due to rights accrued in period s);
r_{it}	return on asset class i in period t;
W_t	pensionable wages in period t;
w_t	wage increase factor in period t;
$d_t(u)$	discount factor for discounting payments in year u to time t.

The deterministic parameters have fixed values in the model.

T	horizon of the model;

I	number of asset classes;
U	time of last expected benefit payment;
s_t	year the oldest rights stem from at time t;
tc_i	transaction costs for asset class i;
w_i^l	proportional lower bound on asset class i;
w_i^u	proportional upper bound on asset class i;
c^l	lower bound on the contribution rate;
c^u	upper bound on the contribution rate;
Δc^l	lower bound on the contribution rate change;
Δc^u	upper bound on the contribution rate change;
γ	sponsor part of regular contributions;
α_k	weight of stakeholder k in the objective;
n	length of recovery period;
φ	integrated chance constraint risk parameter.

3.3 Objective Function

Which decisions to make is steered by the objective of the model, which should reflect the main goals of the pension fund board. Some minor goals are modeled as (soft) constraints, like, e.g., the wish for a stable contribution rate. The objective should reflect the interests of the three stakeholders involved: sponsor, active participants, and passive participants (retirees and sleepers). We will formulate the subobjectives of the stakeholders separately and later combine them to one objective. In order not to compare apples and oranges we will formulate all three subobjectives in terms of discounted expected cash flows. The discount factors are those known right now ($d_0(t)$). In this way, we formulate a risk-neutral objective function. Risk aversion is expressed in the constraints.

Sponsor

The sponsor of the pension fund is the company the fund is allied to. Looking at cash flows, the subobjective of the sponsor is to minimize its discounted contributions, which consist of regular contributions and remedial contributions. Therefore, its objective is:

$$\min C_0^{rem} + \sum_{t=1}^{T} d_0(t)\Big(\gamma c_{t-1} W_t + C_t^{rem}\Big), \tag{4}$$

with γ the part of the regular contributions paid by the company.

Active Participants

The active participants are interested in the contributions they have to pay and the additional pension rights they gain with it. So the first component of the objective function of the active participants is the value of the participants part of the discounted contributions:

$$\sum_{t=1}^{T} d_0(t)\Big(1-\gamma\Big)c_{t-1}W_t.$$

The second component consists of the increase in pension rights during the decision period. The pension right gains stem from two sources, namely newly accrued rights and indexed existing rights. The expected cash flows to the active participants based on the new rights are the expected benefit payments coming from these rights. These expected benefit payments are discounted using the discount factors of the beginning of the decision period. The component due to newly accrued rights is thus:

$$\sum_{t=1}^{T}\sum_{u=t+1}^{U} d_0(u)\underline{B}_{1t}^{t}(u), \tag{5}$$

which is a constant. Introducing $\underline{L}_{kt,0}^{s}$ as the value of the nominal liabilities of stakeholder k ($k=1,2$) originating in rights accrued in period s using the discount factors at the beginning of the decision period, i.e.,

$$\underline{L}_{kt,0}^{s} = \sum_{u=t+1}^{U} d_0(u)\underline{B}_{kt}^{s}(u),$$

component (5) becomes

$$\sum_{t=1}^{T}\underline{L}_{1t,0}^{t}. \tag{6}$$

The second cause of increased pension rights is the indexation of existing rights. The additional discounted value due to indexation decisions is

$$\sum_{t=1}^{T}\sum_{s=s_t}^{t-1}\Big(I_t^s - I_{t-1}^s\Big)\underline{L}_{1t,0}^{s}. \tag{7}$$

Remember that we let $I_t^t := 1$. Further, we let $I_t^{t+1} := 0$. Now we can combine (6) and (7) to

$$\sum_{t=1}^{T}\sum_{s=s_t}^{t}\Big(I_t^s - I_{t-1}^s\Big)\underline{L}_{1t,0}^{s}.$$

Thus, the subobjective of the active participants is given by:

$$\min\sum_{t=1}^{T}\left(d_0(t)\Big(1-\gamma\Big)c_{t-1}W_t - \sum_{s=s_t}^{t}\Big(I_t^s - I_{t-1}^s\Big)\underline{L}_{1t,0}^{s}\right). \tag{8}$$

Passive Participants

The passive participants of the pension fund are the retirees and sleepers. Since they do not pay contributions or accrue new rights, they are merely concerned with indexation. Therefore, their subobjective is given by:

$$\min -\sum_{t=1}^{T}\sum_{s=s_t}^{t}\Big(I_t^s - I_{t-1}^s\Big)\underline{L}_{2t,0}^s. \tag{9}$$

Total

The total objective, i.e., the pension fund objective is the weighted sum of the three stakeholder subobjectives (4), (8), and (9). The weights $\alpha_1, \alpha_2, \alpha_3$ indicate the influence on the decision making of, respectively, active participants, passive participants, and sponsor. Therefore, the objective of the model is

$$\begin{aligned}\min \quad & \sum_{t=1}^{T}\Bigg\{d_0(t)\Bigg(\Big(\alpha_1 + (\alpha_3 - \alpha_1)\gamma\Big)c_{t-1}W_t + \alpha_3 C_t^{rem}\Bigg) \\ & - \sum_{k=1,2}\sum_{s=s_t}^{t}\alpha_k\Big(I_t^s - I_{t-1}^s\Big)\underline{L}_{kt,0}^s\Bigg\} + \alpha_3 C_0^{rem}.\end{aligned}$$

3.4 Constraints

Now that we defined our model grammar (indices, variables, and parameters) and the objective, it is time to specify the decision space. The model comprises the observation, decision, balancing, policy, and supervisory constraints. First, after the realizations of the uncertain parameters are observed, the observation constraints update the the state variables. Next, the decisions for the coming period have to be made and these are subject to some basic constraints. The balancing constraints show the relation between the new decisions and a number of key state variables. The decision and state variables are subject to policy and supervisory constraints.

Observation Constraints

The observation constraints define values of the actual benefit payments and asset value state variables. This happens just after the realizations of the stochastic parameters are observed.

The actual benefit payments in period t (observed at time t) due to rights accrued in period s are based on the indexation decisions and the nominal benefit payments:

$$B_t^s = I_{t-1}^s\underline{B}_t^s, \qquad 1 \leq t \leq T, s_t \leq s \leq t-1.$$

The total benefit payments in period t are the sum of the different benefit payments, i.e.,

$$B_t = \sum_{s=s_t}^{t-1} B_t^s \qquad 1 \leq t \leq T.$$

After the observations of the returns on the asset classes, the total wages, and benefit payments in period t and before the new decisions, the total value of the assets is

$$A_t^* = \sum_{i=1}^{I} (1 + r_{it}) A_{i,t-1} + c_{t-1} W_t - B_t \qquad 0 \leq t \leq T.$$

Decision Constraints

After all information is revealed at observation time t, decisions concerning investment, financing, and indexation policies (for period $t+1$) have to be made. The investment decisions concern the buying and selling of assets, which is modeled by the non-negative variables A_{it}^+ and A_{it}^- respectively. Since we do not allow short positions in assets, it must hold that

$$A_{it}^- \leq (1 + r_{i,t}) A_{i,t-1} \qquad 0 \leq t \leq T.$$

The financing policy is reflected by the contributions made to the fund by the participants and the sponsor. The decisions to be made are the contribution rate c_t and the possible remedial contribution C_t^{rem}, which are both non-negative variables. For now, we do not allow money to be given back to the sponsor and participants. The indexation policy is modeled by accumulated indexation decisions I_t^s for each year of accrual of rights. The basic constraint on the accumulated indexation decisions is that the nominal rights are guaranteed, i.e.,

$$I_t^s \geq 1 \qquad 0 \leq t \leq T,\ s_t \leq s \leq t-1.$$

Balancing Constraints

The decisions influence the state variables. The asset balancing constraints update the amounts invested in the asset classes. Furthermore, the value of the liabilities will be updated with the indexation decisions.

The next constraint expresses the positions in the asset classes for the coming period:

$$A_{it} = (1 + r_{it}) A_{i,t-1} + A_{it}^+ - A_{it}^- \qquad 0 \leq t \leq T-1.$$

All funds must be reinvested in one of the asset classes. This includes a possible remedial contribution. Introducing tc_i as the transaction costs for trading (either selling or buying) one unit of asset class i, the following equation must hold:

$$\sum_{i=1}^{I} A_{it} = A_t^* + C_t^{rem} - \sum_{i=1}^{I} tc_i (A_{it}^+ + A_{it}^-) \qquad 0 \leq t \leq T-1.$$

The total value of the assets after the decisions is thus

$$A_t = \sum_{i=1}^{I} A_{it} \qquad 0 \leq t \leq T-1.$$

On the liability side, the value of the liabilities has to be updated with the indexation decisions for each year of accrual:

$$L_t^s = I_t^s \underline{L}_t^s \qquad 0 \leq t \leq T, s_t \leq s \leq t-1.$$

The total value of the liabilities is now

$$L_t = \sum_{s=s_t}^{t} L_t^s \qquad 0 \leq t \leq T.$$

Policy Constraints

The decisions are subject to a number of policy constraints. These constraints stem both from rules specified in the pension contract, as well as from views of the board of the pension fund. We treat the three decisions (investment, financing, and indexation) successively.

In the decision constraint section we already saw that the investment decisions were restricted to long positions. Here we formulate some additional constraints. It is common that the pension fund board limits the proportional amount invested in a certain asset class. We utilize proportional lower and upper bounds w_i^l and w_i^u to realize this:

$$w_i^l A_t \leq A_{it} \leq w_i^u A_t \qquad 0 \leq t \leq T-1.$$

If the bandwidths $(w_i^u - w_i^l)$ for the asset classes are small, then the set of bandwidths can be seen as the FTK risk-profile of the pension fund.

Financing policy constraints concern the contribution decisions, both regular and remedial. The board of our fund would like to control the behavior of the contribution rate. First, unrealistically low or high contribution rates are not allowed to occur. To this end, lower and upper bounds on the contribution rate are specified:

$$c^l \leq c_t \leq c^u \qquad 0 \leq t \leq T-1.$$

Second, the contribution rate should not vary too much from period to period, because this might foster social upheaval. That's why also bounds on the change of the contribution rate are present:

$$\Delta c^l \leq c_t - c_{t-1} \leq \Delta c^u \qquad 0 \leq t \leq T-1.$$

The indexation policy deserves some special attention. As explained in the introduction of this section, we will not work with indexation decisions i_t^s, but with accumulated indexation decisions I_t^s with

$$I_t^s = \prod_{r=s+1}^{t} i_t^r$$

and conversely

$$i_t^s = \frac{I_t^{s-1}}{I_t^s}.$$

As a consequence, we need to translate the constraints on the indexation decisions to constraints on the accumulated indexation decisions. As described in section 2.1, the constraints on the indexation decisions were:

$$1 \leq i_t^s \leq w_s, \tag{10}$$
$$i_t^s \geq i_{t-1}^s. \tag{11}$$

Constraint (10) leads to

$$I_t^s \leq I_t^{s-1} \leq w_s I_t^s \qquad 0 \leq t \leq T, s_t \leq s \leq t,$$

which is linear in the accumulated indexation decisions. However, constraint (11) translates to

$$\frac{I_t^{s-1}}{I_t^s} \geq \frac{I_{t-1}^{s-1}}{I_{t-1}^s}, \tag{12}$$

which is clearly nonlinear in the accumulated indexation decisions. Therefore, we adjust this constraint. The accumulated indexation can be seen as 1 plus a small amount, so we rewrite (12) to

$$\frac{1+\epsilon_1}{1+\delta_1} \geq \frac{1+\epsilon_0}{1+\delta_0},$$

or

$$(\epsilon_1 - \delta_1) - (\epsilon_0 - \delta_0) \geq \epsilon_0\delta_1 - \epsilon_1\delta_0.$$

By neglecting the products of small terms, we get the following linear approximation of constraint (12)

$$I_t^{s-1} - I_t^s \geq I_{t-1}^{s-1} - I_{t-1}^s \qquad 0 \leq t \leq T,\ s_t \leq s \leq t.$$

Supervisory Constraints

As mentioned in the introduction, as of January 2007 new regulatory rules for pension funds will be enforced. Since we aim at applicability of our model in practice, we strive to accurately model the FTK. Moreover, we would like to contribute to the discussions on the FTK. Many believe that the FTK is too strict and might result in a destruction of the defined benefit system in favor of a defined contribution system. We believe that we can provide valuable quantitative contributions, since in an optimization model the effects of new constraints can be analyzed explicitly. In this section, we will outline the new rules and list our implementations.

The FTK [16] states that it is intended to improve the insight of both the supervised institution as well as the supervisory authority into the institution's financial position and its possible development over the short and medium term. Its main goal is to make the ins and outs of pension funds more transparent. The FTK is involved with the subjects of valuation of liabilities, solvency of the fund, contribution rate, and indexation. Below, we will outline these four fields, for more information see [12].

First, the FTK prescribes market valuation of the liabilities. As long as there are no international standards, this concerns discounting the expected benefit payments, which are to be based on prudent underwriting principles, with a term structure of risk-free discount rates. Second, the FTK has two conditions on the solvency (assets minus liabilities), which we denote by S_t:

$$S_t := A_t - L_t.$$

The solvency should at least equal the so-called *minimum required solvency*, which is to be calculated as prescribed by the European Union [9]. Since this minimum required solvency approximates 5 percent of the value of the liabilities, in practice the condition boils down to:

$$S_t \geq 0.05 L_t.$$

If the condition is not satisfied, a recovery period of 1 or 3 years is allowed, which depends on the characteristics of the recovery plan. In addition, the solvency should be such that the probability of underfunding in the next year is at most 2.5%:

$$P(S_{t+1} < 0 \mid S_t) \leq 0.025. \tag{13}$$

If not, a recovery period of 15 years is allowed. This recovery must be steady ex ante. Third, the contribution rate must be cost-effective, and moreover the FTK would like the contribution rate to be stable in order to prevent social upheaval. Fourth, the indexation policy must be in line with the indexation ambition communicated to the participants. Depending on this ambition, funds must construct buffers and can ask contribution for indexation. Further, in case of financial distress funds must be cautious with indexation.

The expected value of the liabilities is calculated using a government bond yield curve for discounting. In this way, we comply with the prescribed market valuation of liabilities.

The most stringent requirements in the FTK are on the solvency (also called actuarial surplus) of the pension fund. We model the short term solvency requirement by the use of binary variables u_t that indicate underfunding. The following constraints are used to this end:

$$
\begin{aligned}
S_t + M u_t &\geq 0.05 L_t, \\
S_t + M(u_t - 1) &\leq 0.05 L_t - \frac{1}{M}, \\
\sum_{s=t-n}^{t} u_s &\leq n.
\end{aligned}
$$

with $u_t \in \{0, 1\}$, $n = 1$ or 3 the recovery period allowed, and the modeling parameter M sufficiently large. The first two constraints imply that u_t equals 1 whenever there is underfunding and 0 otherwise. Underfunding in $n+1$ subsequent years is excluded by the third constraint. For modeling the medium term solvency requirement we consider several possibilities. The direct implementation of constraint (13) would result in chance constraints. The use of chance constraints in ALM for pension funds was introduced by Dert [6]. These constraints are, however, computationally hard. The use of integrated chance constraints would be an alternative, see [7, 11, 13]. In this case we would limit next year's expected shortage, e.g., proportional to the value of the liabilities:

$$\mathbb{E}[S_{t+1}]^- \leq \varphi L_t$$

with φ a risk parameter. However, it is difficult to specify the relationship between the integrated chance constraint and condition (13). A third method is provided by the FTK. According to the FTK (13) comes down to

$$S_t \geq \sqrt{S_{1t}^2 + S_{2t}^2 + 2\rho_{12} S_{1t} S_{2t} + S_{3t}^2 + S_{4t}^2 + S_{5t}^2 + S_{6t}^2}, \tag{14}$$

with S_{rt} the required solvency for risk of type r and ρ_{12} the correlation between effects of interest risk and market risk. The different risks associated are interest rate risk (S_{1t}), market risk (S_{2t}), foreign exchange risk (S_{3t}), commodities risk (S_{4t}), credit risk (S_{5t}), and underwriting risk (S_{6t}). A complication is that constraint (14) is a nonlinear function of the risk types. Moreover, the calculation of the required solvency for some risk types involves nonlinearities as well. No matter which alternative we choose, we still need to deal with the ex ante steady 15-year recovery period. The time horizons of ALM optimization models, which usually have yearly decision moments, are usually much shorter than 15 years. Horizons of more than, say, 5 years make these complex models intractable. How to incorporate an ex ante steady 15-year recovery period in a model with a much shorter horizon is still open.

The contribution rate will be cost-effective as stipulated, since our model is an optimization model. Now that the liabilities are valued using an interest rate term structure the volatility of the solvency is increased, which could incur volatile contribution rates. In order to preserve a stable contribution rate, we bound the contribution rate and changes in the rate, as written in the policy section.

The indexation regulatory rules have not been crystallized out yet. The supervisor and pension funds are searching for accurate formulations of indexation ambitions. For now we do not include any constraints on indexation besides the ones already mentioned in the policy section.

3.5 Discussion

The model presented above should be seen as a starting point. Important topics as the build-up in an average earnings scheme, the indexation process, and new regulatory rules are dealt with, but it is assumed that computational results will inspire reformulations of the model. Anticipating this, we mention a possible extension and a shortcoming.

Unlinking Decision and Accrual Periods

In the current model the decision period and the accrual period have equal length. It can be beneficial to brake this synchronism. If the period between two decision periods is large, the accrual of rights and accordingly the indexation process is too aggregated. For example, in the current model a 5-year decision period implies that indexation decisions concern a period of 5 years as well. A logical extension of the model would be to unlink the decision period and the accrual period.

Limitations to Remedial Contributions

Assuming that the weights of the three subobjectives are about equal, i.e., each stakeholder has the same level of influence in the board of the pension fund, optimization under the current constraints will probably result in a large amount of remedial contributions by the sponsor, even if there are no financial problems. This can be expected, since remedial contributions are only disadvantageous for the sponsor (one third of the board) and advantageous for the passive and active participants (two third of the board). In reality, remedial contributions will only occur in case of financial distress.

4 Scenario Generation

In this section it is described how we will generate realizations of the stochastic parameters of our model. For the specification of these parameters we need two generators, namely a macro generator that produces the future economic environment and a micro generator that produces the nominal liabilities.

The scope of the model is:

$$\{s_0, \ldots, t_0, \ldots, T, \ldots, U\}$$

with

s_0	the year the oldest rights stem from;
t_0	beginning of the decision period, i.e., 'now', also notated as $t = 0$;
T	the horizon of the decision period;
U	last year of expected benefit payments.

In addition, we utilize

s_I	the last year that all rights were fully-indexed,

which is defined as

$$s_I := \max_{s_o \leq s \leq 0} \{s : i_s^r = w_r \ \forall r, s_0 \leq r \leq s\}.$$

All rights accrued before s_I can be interpreted as accrued in year s_I. In this way we limit the number of parameters, variables, and equations. The scope of the model now becomes

$$\{s_I, \ldots, t_0, \ldots, T, \ldots, U\}$$

As introduced in section 3.2, the stochastic parameters we need for $1 \leq t \leq T$ are

r_{it}	return on asset class i in period t;
$d_t(u)$	discount factor for discounting payments in year u to time t;
w_t	wage increase factor in period t;
W_t	pensionable wages in period t;
$\underline{L}_{kt,0}^s$	value of nominal liabilities at time t of stakeholder k ($k = 1, 2$) originating in rights accrued in period s using the discount factors of the beginning of the decision period ($t = 0$);
$\underline{L}_t$ ($\underline{L}_t^s$)	value of nominal liabilities at time t (due to rights accrued in period s);
$\overline{L}_t$ ($\overline{L}_t^s$)	value of fully-indexed liabilities at time t (due to rights accrued in period s);
$\underline{B}_t$ ($\underline{B}_t^s$)	value of nominal benefit payments at time t (due to rights accrued in period s);
$\overline{B}_t$ ($\overline{B}_t^s$)	value of fully-indexed benefit payments at time t (due to rights accrued in period s).

The returns, discount factors, and wage increases are produced by the macro generator. The other, liabilities related, stochastic parameters are based on the outcomes of a micro generator and the wage increases of the macro generator.

4.1 Economic Parameters

The macro generator will produce the returns r_{it}, which probably concern bond and stock returns and depositor's interest, the yield curve $d_t(u)$, and the wage increase w_t. We intend to base the yield curve and bond return in an arbitrage-free way on a factor model, with as factors the depositor's interest and the wage increase. The depositor's interest and wage increase are generated using a parametrized VAR-model. An open question is how to relate the stock returns to this factor model.

4.2 Liability Parameters

At $t = 0$ the pension fund under consideration has a known liability portfolio. It is important whether all rights are fully-indexed at the beginning of the decision period. The first characteristic we need to know is s_I, the last time that all rights were fully indexed. If $s_I < 0$, we also need to know w_s and i_0^s for $s_I < s < 0$. Further, the pension fund specifies:

$\underline{B}_{k0}^s(u)$	at time 0 expected nominal benefit payments in year u to stakeholders k ($k = 1, 2$) due to rights accrued in year s;
$\underline{B}_0^s$	nominal benefit payments in year t due to rights accrued in year s.

The micro generator will provide us, based on the participant file and the above mentioned existing liabilities, with the following data:

$\tilde{B}_{kt}^s(u)$	at time t expected benefit payments in year u to stakeholders k ($k = 1, 2$) due to rights accrued in year s at wage level of t_0 for $s > 0$;
$\tilde{B}_t^s$	benefit payments in year t due to rights accrued in year s at wage level of t_0 for $s > 0$;
$\tilde{W}_t$	pensionable wages in year t at wage level of t_0;

Notice that these quantities are inflation-free for $s > 0$. The wage inflation was modeled in the macro-generator. Using this wage inflation, we get the following for the nominal expected benefit payments:

$$\underline{B}_{kt}^s(u) = \begin{cases} \tilde{B}_{kt}^s(u) & \text{if } s \leq 0; \\ \prod_{r=0}^s w_r \tilde{B}_{kt}^s(u) & \text{if } 0 \leq s \leq T. \end{cases}$$

$$\underline{B}_t^s = \begin{cases} \tilde{B}_t^s & \text{if } s \leq 0; \\ \prod_{r=0}^s w_r \tilde{B}_t^s & \text{if } 0 \leq s \leq T. \end{cases}$$

$$W_t = \prod_{s=0}^t w_s \tilde{W}_t,$$

from which follows

$$\underline{B}_t^s(u) = \sum_{k=1,2} \underline{B}_{kt}^s(u).$$

Using the discount factors we get:

$$\underline{L}_t^s = \sum_{u=t+1}^U d_t(u) \underline{B}_t^s(u),$$

$$\underline{L}_{kt,0}^s = \sum_{u=t+1}^U d_t(u) \underline{B}_{kt}^s(u).$$

The fully-indexed liabilities are

$$\overline{L}_t^s = \prod_{r=s_I} w_r \underline{L}_t^s.$$

The total nominal/fully-indexed liabilities and (expected) benefit payments are retrieved by summation over all years of accrual.

5 Conclusion

In this paper we have proposed an ALM model for Dutch pension funds. First, it is explained how pension rights are built up in the now dominant average earnings pension contracts. Special attention is payed to the indexation of rights, which is crucial for a substantial pension in average earnings contracts. Second, we formulated a multistage recourse model. Besides the adequate modeling of the indexation process, key features of the model are an economically interpretable objective and the modeling of the new regulatory rules for pension funds. Third, we gave directions for the generation of data (scenario's) for the proposed model.

We have not completely captured the new regulatory rules yet. How to include the 15-year recovery period for the medium term solvency requirement is still an open question. For modeling the requirement itself, we consider various options. Another shortcoming of the model is the lack of remedial contribution limiting constraints. It is to be expected that given the current set of constraints and objective unrealistically many remedial contributions will occur. A further possibility for improvement might be the unlinking of the decision and accrual period. In the current model, if the decision period is large, the accrual of rights and accordingly the indexation process is too aggregated.

Further research will be mainly on the generation of scenario's for the model. The macro generator is to provide the future economic environment in a consistent way, while the micro generator concerns the nominal liability portfolio. Once the model is running, the shortcomings mentioned above are to be investigated.

We believe that our multistage optimization model will provide new insights in ALM for pension funds. The outcomes of our models are dynamic strategies, which are probably different from the current strategies used in the common-practice ALM simulation models. With these dynamic strategies the costs of the Dutch defined benefit system might remain affordable and the risks acceptable. Another contribution to the pension debate could be to give quantitative feedback on the not undisputed new regulatory rules. In an optimization model, the effects of new supervisory constraints can be analyzed explicitly.

References

[1] Algemeen Burgerlijk Pensioenfonds (ABP). http://www.abp.nl.

[2] Basel Committee on Banking Supervision. Basel accords. http://www.bis.org/bcbs/index.htm.

[3] C.G.E. Boender. A hybrid simulation/optimisation scenario model for asset/liability management. *European Journal of Operational Research*, 99:126–135, 1997.

[4] E. Bogentoft, H.E. Romeijn, and S. Uryasev. Asset/liability management for pension funds using CVaR constraints. *The Journal of Risk Finance*, 3(1):57–71, 2001.

[5] De Nederlandsche Bank (DNB). http://www.dnb.nl.

[6] C.L. Dert. *Asset Liability Management for Pension Funds: A Multistage Chance Constrained Programming Approach*. PhD thesis, Erasmus University Rotterdam, 1995.

[7] S.J. Drijver. *Asset Liability Management for Pension Funds Using Multistage Mixed-Integer Stochastic Programming*. PhD thesis, University of Groningen, 2005.

[8] European Insurance and Occupational Pensions Committee. Solvency accords. http://ec.europa.eu/internal_market/insurance/solvency_en.htm.

[9] European Parliament and the Council of the European Union. Directive 2003/41/EC of the European Parliament and of the Council of 3 June 2003 on the activities and supervision of institutions for occupational retirement provision. *Official Journal of the European Union*, L 235:10–21, 2003.

[10] J. Gondzio and R. Kouwenberg. High performance computing for asset liability management. *Operations Research*, 49(6):879–891, 2001.

[11] W.K. Klein Haneveld, M.H. Streutker, and M.H. van der Vlerk. An ALM model for pension funds using integrated chance constraints. Research Report 05A03, SOM, University of Groningen, http://irs.ub.rug.nl/ppn/274795531, 2005.

[12] W.K. Klein Haneveld, M.H. Streutker, and M.H. van der Vlerk. Implementation of new regulatory rules in a multistage ALM model for Dutch pension funds. SOM research report, University of Groningen, forthcoming.

[13] W.K. Klein Haneveld and M.H. van der Vlerk. Integrated chance constraints: reduced forms and an algorithm. *Computational Management Science*, 3(4):245–269, 2006.

[14] R. Kouwenberg. Scenario generation and stochastic programming models for asset liability management. *European Journal of Operational Research*, 134:279–292, 2001.

[15] J.M. Mulvey. Introduction to financial optimization: Mathematical programming special issue. *Mathematical Programming*, 89:205–216, 2001.

[16] Pensioen- & Verzekeringskamer. Financial assessment framework consultation document. `http://www.dnb.nl/dnb/bin/doc/FTK%20Consultation%20Document%20English%20translation_tcm13-47968.pdf`, October 2004.

[17] M.S. Sodhi. LP modeling for asset-liability management: A survey of choices and simplifications. *Operations Research*, 53(2):181–196, 2005.

[18] C.D. van Ewisk. Reform of occupational pensions in the Netherlands. *De Economist*, 153(3):331–347, 2005.