

Aging and the interaction between education, retirement and the working life¹

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Abstract

Population aging and the burden it imposes on state finances is one of the major economic challenges governments around the world face. Responses are formulated in terms of either increasing employment (for example by raising the retirement age) or increasing productivity (investment in education). This paper brings together these two responses in a unified framework and shows how the individual's education and retirement decisions are affected by population aging – caused either by a fall in the population growth rate, or an increase in life expectancy – and the budget balancing mechanism of the public pension systems. We discuss how a budget balancing mechanism can be informed by fairness considerations and we show that early retirement can be the result of the application of Musgrave's rule in response to a fall in fertility.

Key words: aging, fairness, education, retirement
JEL Classification: H55, I38, J22, J24, J26

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I. Introduction

Population aging and the burden it imposes on state finances is one of the major economic challenges facing governments around the world. Responses are formulated in terms of either increasing employment (for example raising the retirement age) or increasing productivity (increasing investment in education). This paper brings these two responses together in a unified framework. Central to this is the way in which individuals will respond to aging itself and the policy response, and how that impacts the policy space of the government and the fairness of the outcome.

The process of demographic change is usually referred to as demographic transition, which is not necessarily the same as aging. Demographic transition is the process whereby a population moves from high mortality and fertility rates to a state with low mortality and fertility rates. Everything else constant, a fertility decline reduces the size of the most recent birth cohorts relative to the previous birth cohorts, hence reducing the size of the youngest age groups relative to that of the older ones. On the other hand a decline in mortality does not necessarily result in aging. A fall in neonatal mortality increases the size of the most recent birth cohorts (and is effectively equivalent to a rise fertility) and actually reduces aging. A mortality decline at older ages, on the other hand, increases the life span and results in aging of society (Gavrilov et al 2003). The sequence of the demographic transition in most developed countries has the following four stages (categorized according to Thompson, 1929):

- 1) Pre modern times characterised by a balance between birth rates and death rates;
- 2) A decline in death rate (primarily childhood mortality) first due to better hygiene and later to vaccination reducing mortality due to infectious diseases such as measles and polio and TB. This decline in neo natal mortality occurred while birth rate remains high, which resulted in the population becoming younger;
- 3) A return to population stability through decline in birth rate, which coincided with urbanisation and the use of contraceptives. This results in aging of society relative to phase 2;
- 4) A fall in mortality of the elderly, the “oldest old”, due to medical progress, primarily improvement in treating chronic disease such as cancer and heart disease, leading to further aging;

Thus stage 3 and 4 is what is commonly referred to as aging - a fall in fertility followed by a rise in life expectancy. The distinction between the causes of aging proves to be an important one, not only for philosophical reasons (see Howse 2006) but also from an economic point of view. In our set-up aging is characterised separately by a decline in population growth and an increase in life expectancy.

In contrast to most models where aging it is presented as a government problem, our approach focuses on aging as a problem for the individual. In particular, we examine the change in the education and retirement choices of the individual. This paper argues that it is increasingly appropriate to treat retirement as a choice variable. The intervening period between completion of education and the start of retirement, the working life, is affected by both these decisions. We demonstrate further that an increase in life expectancy results in an increase of the fraction of life spent working. In contrast, a reduction in fertility increases the old age dependency burden.

The paper is structured as follows. In section II-III we introduce our continuous time model of retirement and consumption in an open economy setting. The key technical element of our model is the inclusion of a cost of working that workers incur while employed, to capture the real or monetarised disutility of working. In section IV we introduce the education decision. Agents can defer the start of work by investing in further educating, foregoing earnings and incurring a quadratic cost. The model shows that an increase in life expectancy will result in an increase in time and money spent in education. This is a remarkable result as most models rely on a change in factor prices (where aging results in a relative scarcity of labour and thus increases the return to education). Here it happens without such general equilibrium effects.

In a Pay-As-You-Go (PAYG) pension system agents pay a contribution while working and receive a pension while retired. Section V show the government equilibrates aggregate contributions with aggregate payments. In our set-up the government has to take into account changes in the education and retirement behaviour of the individual, in response to change in policy. While an increase in life expectancy affects the optimisation decisions of the individual, a decline in fertility only affects the budget balance of the government. This assumes that there are no transfers between parents and their children. In reality the parents have more consumption when they share it with fewer children. The effect of an increase in the pension contribution on the fraction of life spent working is negative compounding the decrease in the working life. In contrast, a reduction in the pension level increases the fraction of life spent working. Section VI examines policy choices a government has, considering the individual's behaviour in response to aging and discusses what constitutes a fair policy choice. Actuarial fairness has limited applicability in guiding the government. But intergenerational fairness is a more appealing concept. In particular we consider the consequences of Musgrave's rule for sharing the burden between the employed and the retired. We show that early retirement can be the result of the application of Musgrave's rule in response to a fall in fertility. Section VII concludes.

II. The model

The general framework of inter generational economics is provided by the benchmark discrete time overlapping generation (OLG) model by Samuelson (1958) and Diamond (1965). We believe that a discrete OLG model is less well suited to modelling the retirement decision, as we require continuous variation in the retirement date. We assume a population of identical agents in continuous time. A continuous time model was first formulated by Yaari (1965), and Tobin (1967) and introduced the concept of life expectancy as a random variable in a continuous time setting. The model was later extended by Blanchard (1985) assuming an exponential density function for the probability of death. In contrast we assume no uncertainty of death. Agents will thus entirely dissave at the time of death and savings (either public or private) are accordingly simply a way of smoothing income over the lifetime of the consumer. We ignore the effect on saving of family structure, precautionary savings and saving with a bequest motive.

We further assume a small open economy setting with free movement of capital but not of labour. By using an open economy set-up we do not take into account the effect of increased savings and reduced labour force - resulting from say a slow down of population growth - on the wage rate and the return on capital. In a closed economy setting a slow down in population growth results in a relative scarcity of labour pushing up the wage rate for the young and reduces the returns to savings for the old. Here this is ruled out by assuming that the interest rate is exogenously given by the world interest rate. Our economy is a single good economy.

We assume that agents maximise their discounted utility over their lifetime, making the dual choices of when to retire and how much to save/consume, subject to their budget constrain. We start by investigating a static model of consumption/saving with constant, known finite life expectancy and turn to the retirement decision in section IV.

Let an agent's preferences be represented by a concave utility function $U(C_t)$, then the agent's objective function is to maximise at time 0.

$$\int_0^T e^{-\delta t} U(C_t) dt \tag{2.1}$$

Where T denotes the certain time of death, δ the subjective rate of time preference and C_t consumption at time t (of an agent born at $t=0$). Workers earn a wage W , which is assumed to be constant initially, but we make it an increasing function of education in section VI.

The budget constraint can be written as

$$\int_0^T C_t e^{-rt} dt = \int_0^R W_t e^{-rt} dt \quad (2.2)$$

where R is the date of retirement and r the exogenously given interest rate and we start life at $t=0$. Equation (2.2) states that the net present value of life time consumption has to equal the net present value of life time earnings. We assume perfect capital markets, with both the borrowing and lending rates equal to r and no borrowing restrictions allowing perfect consumption smoothing. The maximisation problem is solved using optimal control theory.³ We can deduce that the consumption path has to satisfy the Ramsey (1928) rule.

$$\frac{\dot{C}}{C} = r - \delta \quad (2.3)$$

Consumption therefore grows at a constant rate $r - \delta$.

1. State PAYG pension system

We assume that the government operates a Pay-As-You-Go (PAYG) public pension system which pays agents a pension of value P after retirement, and to which they have to contribute a flat pension contribution τ when employed. We have assumed a flat rate of pension, which resembles the situation in the Netherlands and Ireland with a well developed second pillar. Other countries have opted for an earnings related scheme (Rother et. al. 2003). Indeed it might be more realistic to assume that agents pay a percentage of their wage as pension contribution $\bar{\tau} = \frac{\tau}{W}$, as in the UK. However, because we do not assume heterogeneity in wages this is effectively identical, although it does affect their education decision.

Implicit in our set-up is that public pensions are a perfect substitute for private savings and we ignore any asset return uncertainty. The assumption of perfect substitutability is contrary to the not unchallenged view of Feldstein (1974) that the wealth impact of social security amounts to a 50 percent reduction in private savings. The budget constraint can then be expressed as

$$C_0 \int_0^T e^{-\delta t} dt = \int_0^R (W - \tau) e^{-rt} dt + \int_R^T P e^{-rt} dt \quad (2.4)$$

³ See appendix 1 for the derivation

If we assume that the government runs a balanced budget, the following identity has to hold in addition to (2.4)

$$\int_{-T}^{-R} PL_0 e^{nt} dt = \int_{-R}^0 \tau L_0 e^{nt} dt \quad (2.5)$$

where n denotes the growth rate of the population and L_0 denotes the size of the cohort born at time 0. Equation (2.5) says that pensions paid to those (and assumed constant) born between R and T years ago should equal the tax paid by the working, born less than R years ago. We ignore any incidence of unemployment and part-time employment.

The retirement age is chosen by the individual and we assume that the pension contribution and pension level are set to balance the government budget. The individual chosen retirement age is subject a minimum statutory retirement age, R_0 , below which the individual does not receive, a pension. Therefore we add the additional constraint that $R \geq R_0$, allowing the people to choose $R = R_0$. In fact people could choose $R < R_0$ but would not receive a pension upon retirement but upon reaching the statutory retirement age R_0 .

2. Cost of working

Empirical evidence suggests there is a retirement consumption puzzle: consumption at retirement falls even though the retirement date was not unexpected. Liabson (1998) argues that agents exhibit hyperbolic discount functions, which “sets up a conflict between today’s preferences and the preferences which be held in the future, implying that preferences are dynamically inconsistent”. Alternative explanations include the suggestion that resources are less than anticipated (Bernheim, Skinner & Weinberg 2001), that retirement coincides with adverse shocks (Banks, Blundell & Tanner 1998) or that it is caused by the presence of uncertainty and the fact that retirement is a discrete and not a continuous variable. (Blau, 2004). Hurd & Rohweller (2003) suggest that it is caused by the fact that agents no longer incur a cost of working. It is the last of these approaches that we adopt here.

We now introduce a cost of working denoted by $D(t)$ that an agent incurs while he is working but not once retired. The economic rationale for including it is that it captures the real cost or monetarised disutility of working, and provides a trigger point for retirement. It could be interpreted as the cost of travelling, cost of keeping skills up to date or a monetary cost of effort. Thence the budget constraint with the cost of work included is as follows:

$$C_0 \int_0^T e^{-(r-g)t} dt = \int_0^R (W - \tau - D(t)) e^{-rt} dt + \int_R^T P e^{-rt} dt \quad (2.6)$$

We assume the following functional form of the cost of working

$$D(t) = \frac{\alpha t}{T} \quad (2.7)$$

We would like to highlight two aspects of the chosen functional form of the cost of working. First the cost of work is homogeneous of degree zero in life expectancy and age, $D(\theta T, \theta t) = D(T, t)$, so if life expectancy and the age of the agent doubles his cost of working that year is the same. This reflects the assumption that improvements in life expectancy will lead to proportionate improvements in health and therefore ability to work. Second, the cost of working increases with time to reflect the fact that the cost, or disutility, of working rises with age, this could be interpreted that keeping up a certain level of effort or skill increases with age. We have chosen a linear form out of mathematical convenience.⁴ We can write C_0 as (where $r \neq 0$)

$$C_0 = \frac{\delta}{1 - e^{-\delta T}} \left\{ \frac{W - \tau}{r} (1 - e^{-rR}) - \frac{\alpha}{Tr} \left[\frac{1}{r} - e^{-rR} R - \frac{e^{-rR}}{r} \right] + \frac{P}{r} (e^{-rR} - e^{-rT}) \right\} \quad (2.8)$$

For $r = 0$ expression (2.8) for C_0 simplifies to:

$$C_0 = \frac{\delta}{1 - e^{-\delta T}} \left[WR - \tau R - \frac{\alpha R^2}{2T} + PT - PR \right] \quad (2.9)$$

⁴ Using a form like $\frac{\alpha}{T-t}$ does not give an analytical solution for consumption if you want to integrate

$\int \frac{\alpha}{T-t} e^{-rt} dt$, while using e^t does not give an explicit solution for the retirement date.

III. Endogenous retirement decision

The retirement age is frequently viewed as institutionally determined. In many European pension schemes there are strong incentives to retire at, say, 65 and it may even be impossible to carry on working. However, the legal framework is changing such that the mandatory retirement age is progressively being abolished, in particular through the implementation of the European Union Directive covering discrimination in employment.⁵ The Directive was designed to abolish age discrimination and to increase labour force participation of older people in Europe, and proposes the abolition of mandatory retirement. Moreover, three countries (the USA, Australia and New Zealand) and some Canadian provinces have completely abolished mandatory retirement and in Norway mandatory retirement is not permitted before the age of 70 (Meadows 2003). Thus an endogenous retirement is an increasingly realistic modelling assumption.

Burkhauser (1979) showed that the timing of acceptance of a pension depends upon the actuarial value of the plan at different ages, not on payments in any one year. A rich literature has since developed on the determinants of early retirement. It is not just economic incentives that determine the retirement date but also the expected remaining life expectancy. Hurd et. al. (2002) find that those with very low subjective probabilities of survival retire earlier and claim earlier than those with higher subjective probabilities, although the effects are small. And Bloom et. al. (2003) construct a model showing that improvements in life expectancy leads to increased savings even with endogenous retirement.

In our model the agent faces the joint retirement and consumption decisions, which are solved sequentially. First the agent chooses the optimal retirement date R , giving life time income. Subsequently, the agent determines his consumption path given his discount rate, life expectancy and the planned retirement date. In this section we derive an expression for the retirement decision and show the limits it imposes on the PAYG pension system.

The agent chooses a value of R , which maximises the right hand side of the budget constraint:

$$C_0 \int_0^T e^{-(r-g)t} dt = \int_0^R (W - \tau - D(t)) e^{-rt} dt + \int_R^T P e^{-rt} dt \quad (3.1)$$

This yields an expression for the retirement date R .

$$R = \frac{T}{\alpha} [W - \tau - P] \quad (3.2)$$

⁵ Council Directive 2000/78/EC, 27 November 2000

To ensure an interior solution we make the following two assumptions:

1. Agents do indeed work. To ensure that agents do indeed work in the first place ($R > 0$) we assume that $W - \tau > P$ (3.3)

This seems a natural assumption, it states that the replacement rate is less than 100%.

2. Agents would eventually retire even in the absence of a public pension system i.e. $R < T$. We require the following condition to be satisfied for this assumption to hold: $\alpha > W$ (3.4)

I.e. the cost of working exceeds the wage rate W at the end of the natural life T . This seems a reasonable assumption certainly for the developed world and in a world of no uncertainty about the time of death.

1. *The fraction of life spent working*

One of the key determinants of the dependency ratio is the fraction of life that agents work $\frac{R}{T}$. From the expression for the retirement decision we can

deduce that the fraction of life spent working $\frac{R}{T}$ can be expressed as

$$\frac{R}{T} = \frac{W}{\alpha} \left[1 - \frac{\tau}{W} - \frac{P}{W} \right] = \frac{W}{\alpha} \left(1 - \frac{\tau}{W} \right) (1 - \rho) \quad (3.5)$$

Thus the share of life spent working is determined by the average tax rate $\frac{\tau}{W}$ and the replacement rate $\rho = \frac{P}{W - \tau}$. This implies a menu of choices for the government, and bounds on P and τ . We will return to the concept of thinking of life in terms of fractions, when we introduce the partition of life in education, work and retirement phases in the next section.

2. *The menu of options of the government*

The combination of P and τ is not just restricted by our assumption that the replacement rate is less than 100%, implying $W > P + \tau$. The level of pension contribution and the level of pensions affect the retirement date.

$$R = \frac{T}{\alpha} [W - \tau - P]$$

And the retirement date in turn impacts both P and τ . For a balanced budget we require, in the absence of population growth, that the level of pensions is related to pension contribution as follows

$$P = \frac{\tau R}{T - R} \quad (3.6)$$

Substituting the expression for R into the expression for a balanced budget which gives an expression of the pension in terms of τ , and thus the menu of options for the government of pension and pension contributions, taking into account the retirement behaviour of individual agents. The relationship between the pension contribution and the pension level is independent of life expectancy. This means that the highest possible value of pensions is given by

$$P = \frac{W^2}{4\alpha} < \frac{W}{4} \tag{3.7}$$

Thus the higher the coefficient of the cost of working the lower the maximum attainable pension, this is presented graphically in figure III-1.

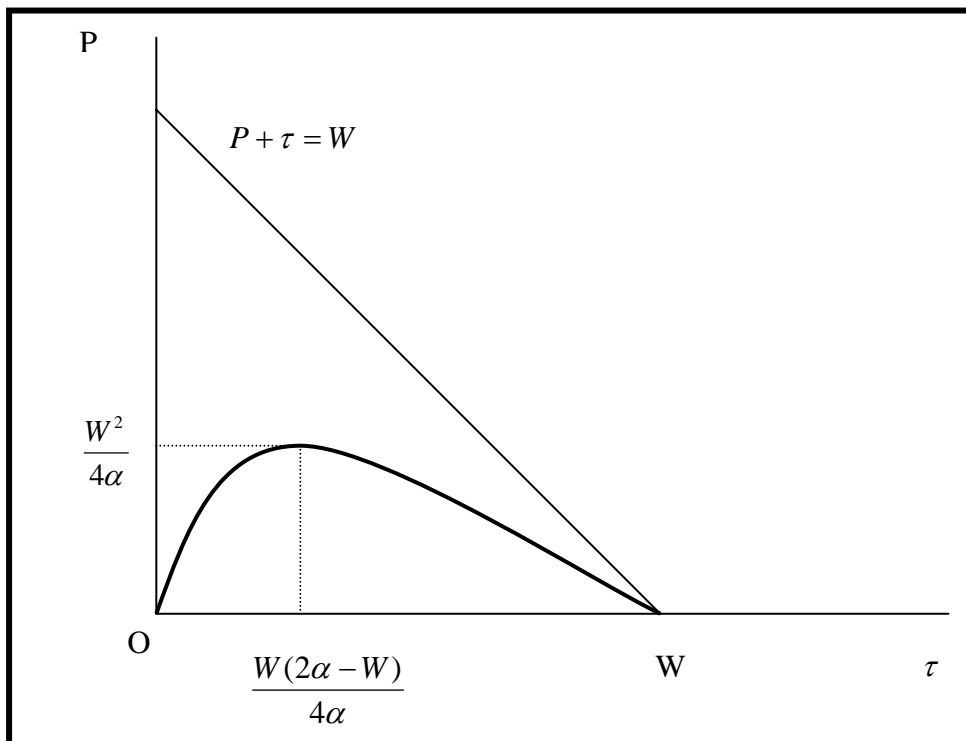


Figure III-1 The menu of options for the government

The economic rationale is as follows: raising the contribution level has two effects: (i) it increases the pension contribution per working agent (ii) it reduces the retirement age. For small values of the pension contribution the first effect dominates the second, enabling the government to increase the pension level. For high values of the contribution levels, the second effect dominates, reducing the aggregate pension contribution paid to the government, which leads to a decrease in the pension level.

IV. Introducing education

Thus far people only had two choice variables to maximise their expected life time utility: the retirement date and the amount of consumption and thus the amount of saving for old age. In this section we introduce the possibility of investment in human capital. This theme is developed in an expanding literature which looks at the consequences of aging on growth, through accumulation of both physical and human capital. Our approach is distinct in a number of ways: investment requires both time (in contrast to, for example, Ozcan et al 2000) and money and we assume an open economy setting. The latter is in contrast to Fougère and Mérette (1999) who develop an endogenous growth model with population aging in a closed economy setting. They show that as a result of population aging the amount of both capital and labour in the economy falls, although the reduction in the latter outstrips the former and thus labour becomes relatively scarce. Thus the return on capital falls and the return on human capital rises increasing the incentive for investment in human capital. They finally argue that population aging has a positive effect on real output. After having added investment in human capital to our model we examine the interaction between the education and retirement decisions.

The assumption that investment requires time reflects the fact that we are ultimately interested in fractions of life spent in work and retirement. Let E denote the amount of time spent in education: an individual is educated from time 0 to time E incurring two costs: foregone earnings, and the direct cost of education, which is assumed to be quadratic in time spent in education. This captures the fact that university education is more expensive than secondary education. The agent then works from E to R earning a wage $W(E)$ which is monotonically increasing with the amount of education E (i.e. better educated agents receive a higher wage). While working the individual incurs a cost of working $\frac{\alpha}{T}[t - E]$; the amended functional form reflects that the cost of working only start when the agent completes his education and starts working. Finally the individual is retired from R to T receiving a pension P .

1. *Special case linear earnings function*

Empirical studies normally find a logarithmic relationship between earnings and education (See Card (1999), Polachek & Siebert (1993)). However, for mathematical convenience we postulate that the wage rate increases linearly with the amount of time spent in further education: $W(E) = \bar{W} + \psi E$ where \bar{W} is the return to compulsory universal education, taking zero time for simplicity, and ψ per unit of time spent in further education. In addition we assume that $r = 0$. The cost of education is quadratic in time spent in education with f denoting the cost parameter: $C(E; f) = fE^2$. Moreover, we

recall that our open economy setting implies that we ignore changes in the returns to the factors as a result of increased savings. The maximand becomes

$$V(E) = -\int_0^E f dt + \int_E^R \left[\bar{W} + \psi E - \tau - \frac{\alpha}{T} [t - E] \right] dt + \int_R^T P dt \quad (4.1)$$

Each of the three parts relates to a phase of the life cycle: education, working life and retirement. We use a change of variable to recast our problem in that way. If we let M denote the working life, $M = R - E$, then we can write

$$V(E) = -f \frac{E^2}{2} + \bar{W}M + \psi EM - \tau M - \frac{\alpha M^2}{2T} + PT - PM - PE \quad (4.2)$$

The first order condition with respect to education requires (for an internal solution):

$$\frac{\partial V(E, L)}{\partial E} = -fE + \psi M - P = 0 \quad \Rightarrow E = \frac{\psi M - P}{f} \quad (4.3)$$

The first order condition with respect to working life requires:

$$\frac{\partial V(E, L)}{\partial M} = \bar{W} + \psi E - \tau - \frac{\alpha}{T} M - P = 0 \quad \Rightarrow M = \frac{T}{\alpha} \left[\bar{W} + \psi E - \tau - P \right] \quad (4.4)$$

The second order conditions can be checked by constructing the Hessian matrix

$$H = \begin{bmatrix} -f & \psi \\ \psi & -\frac{\alpha}{T} \end{bmatrix} \quad (4.5)$$

For a strict local maximum we need the leading principal minors of the Hessian to alternate in sign which obviously holds. The determinant of the Hessian matrix has to be strictly greater than zero:

$$\frac{\alpha f}{T} - \psi^2 > 0 \quad \Rightarrow \frac{\alpha}{\psi T} > \frac{\psi}{f} \quad (4.6)$$

The graphic interpretation of this is that the slope of the education line is less than the slope of the working life line. Moreover it gives bounds on the return

$$\text{to education: } \psi \in \left(0, \sqrt{\frac{f\alpha}{T}} \right) \quad (4.7)$$

Figure IV-1 shows the education and working life equations:

$$\text{Education} \quad E = \frac{\psi}{f} M - \frac{P}{f} \quad (4.8)$$

Working Life

$$E = \frac{\alpha}{\psi T} M + \frac{1}{\psi} [\tau + P - \bar{W}] \quad (4.9)$$

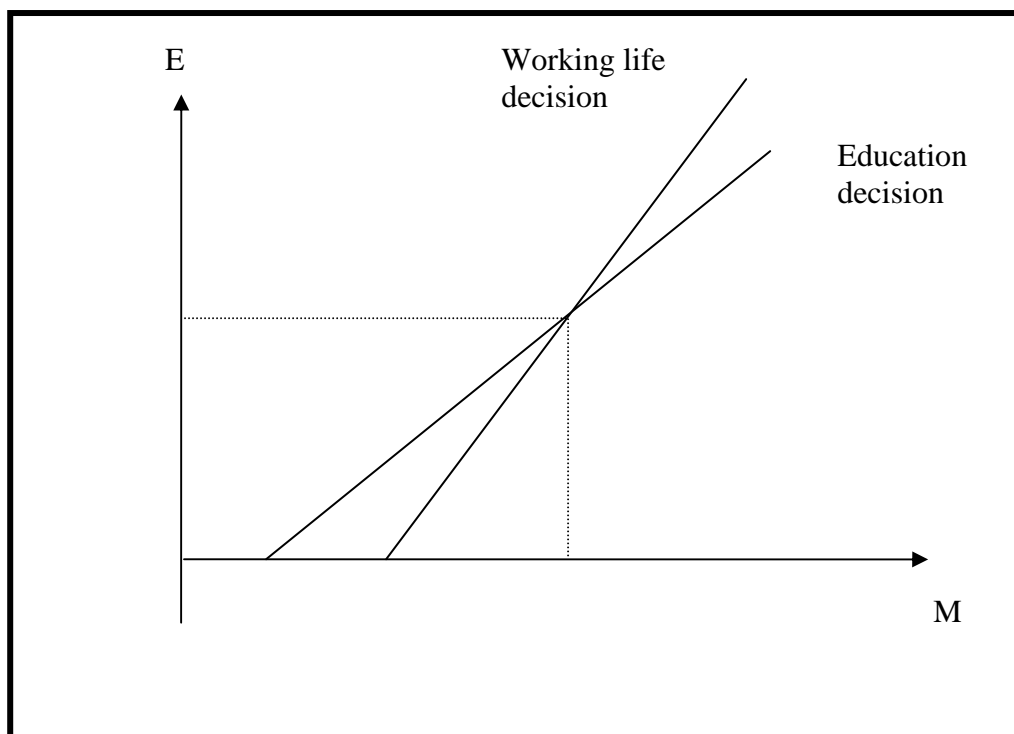


Figure IV-1 The equilibrium of the working life and education decision

The working life line implies that an increase in the working life, requires an increase in education and the education line show that an increase in education leads to an increase in the working life. The equilibrium is at the intersection of these two reaction functions and is characterised by⁶

$$E = \frac{f\bar{W}T\psi - T\psi(fP + \psi P + f\tau) - \frac{P}{f}}{f^2\alpha - \psi^2 fT} \quad (4.10)$$

$$M = \frac{Tf\bar{W} - Tf(P + \tau) - T\psi P}{f\alpha - \psi^2 T} \quad (4.11)$$

$$R = \frac{T[\psi + f][f\bar{W} - \psi P - Pf - f\tau] - \frac{P}{f}}{f^2\alpha - f\psi^2 T} \quad (4.12)$$

B. Comparative statics

We now derive the comparative statics of the education, retirement and working life taking into account feedback effects. It transpires that the secondary effects sometimes dominate to provide interesting results:

⁶ Note that by (4.11) $\alpha f > \psi^2 T$

1. Proposition: the dynamics of the education decision

$$E = \frac{f\bar{W}T\psi - T\psi(fP + \psi P + f\tau)}{f^2\alpha - \psi^2fT} - \frac{P}{f}$$

The level of education is,

- i. decreasing in the pension contribution τ
- ii. decreasing in the coefficient of the cost of education f
- iii. increasing in the returns to mandatory universal education \bar{W}
- iv. increasing in life expectancy T
- v. decreasing in the coefficient of disutility of work α
- vi. increasing in the returns to education ψ
- vii. decreasing in the pension level P

2. Proposition: the dynamics of the working life

$$M = \frac{Tf\bar{W} - Tf(P + \tau) - T\psi P}{f\alpha - \psi^2T}$$

The working life is,

- i. decreasing in the pension contribution τ
- ii. decreasing in the coefficient of the cost of education f
- iii. increasing in the mandatory universal education \bar{W}
- iv. increasing in life expectancy T
- v. decreasing in the coefficient of disutility of work α
- vi. increasing in the returns to education ψ
- vii. decreasing in the pension level P

3. Proposition: the dynamics of the retirement decision

$$R = \frac{T[\psi + f][f\bar{W} - \psi P - Pf - f\tau]}{f^2\alpha - f\psi^2T} - \frac{P}{f}$$

The retirement date is,

- i. decreasing in the pension contribution τ
- ii. decreasing in the coefficient of the cost of education f
- iii. increasing in the mandatory universal education \bar{W}
- iv. increasing in life expectancy T
- v. decreasing in the coefficient of disutility of work α
- vi. increasing in the returns to education ψ
- vii. decreasing in the pension level P

These comparative statics are noteworthy for a number of reasons:

Whereas Fougère and Mérette (1999) argued that aging causes increased investment in education as a result of a change in factor prices, we replicate this finding in an open economy setting, thus without a change in factor prices (i.e. aging results in a relative scarcity of labour, pushing up the wage rate and thus providing an incentive to spend more time in education).⁷

An increase in the return to compulsory education leads to an increase in the investment in higher education. The economic rationale is as follows: If the returns to compulsory education rise, *ceteris paribus* the wage rate rises and this increases the working life. An increase in the working life increases the amount of higher education. This result highlights the complementarity between compulsory education and further education.

Education is increasing in the returns to education ψ . The economic rationale is as follows: If the returns to compulsory education increase, *ceteris paribus* the wage rate rises and this increases the working life. An increase in the working life and an increase in the returns to education ψ increase the amount of further education.

What is important, however, is what happens to the *fractions* spent in education, work and retirement respectively.

4. Proposition: education vs working life, the impact of an increase in life expectancy

The amount of time in education compared to the working life increases with an increase in life expectancy.

Proof

Immediate and illustrated in figure IV-2

Proposition 4 implies that individuals spend more time in education per year in work as life expectancy increases, providing an engine for growth.

5. Proposition: life expectancy and the fraction of life spent in education

In a model with education an increase in life expectancy leads to an increase of the *fraction* of life spent in education.

⁷ The idea that education is increasing in life expectancy has recently be confirmed in a Brazilian data sample. Soares (1995) shows that higher fertility is systematically associated with higher schooling (and lower fertility).

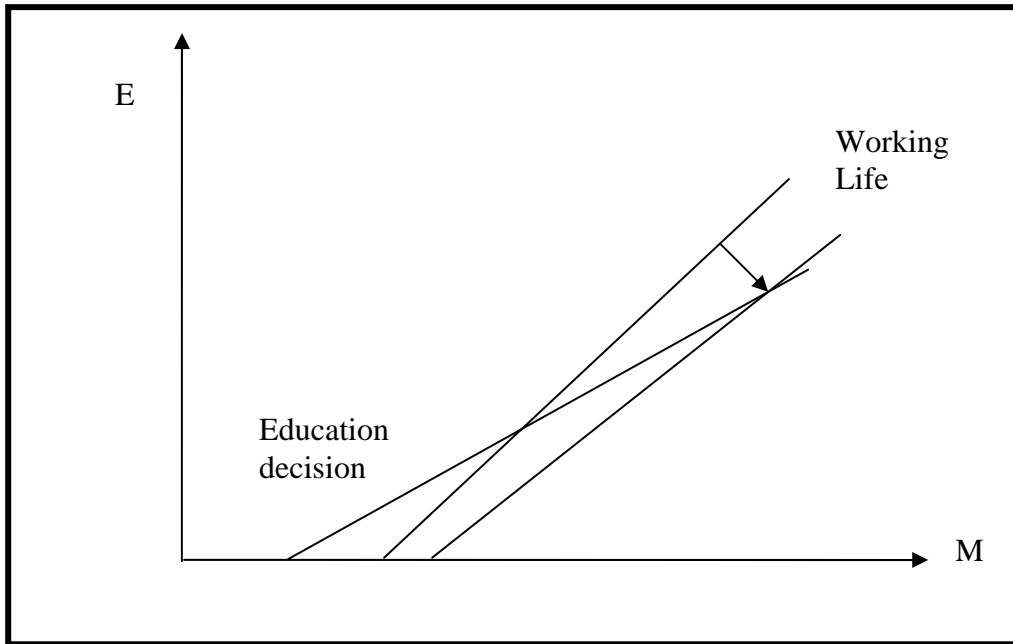


Figure IV-2 The effects of an increase in life expectancy

Proof

$$\frac{E}{T} = \frac{\psi}{f} \frac{[f\bar{W} - \psi P] - f(P + \tau)}{f\alpha - \psi^2 T} - \frac{P}{Tf}$$

$$\frac{\partial}{\partial T} \left[\frac{E}{T} \right] = \psi^2 \frac{\psi [f\bar{W} - \psi P] - f(P + \tau)}{(f\alpha - \psi^2 T)^2} + \frac{P}{T^2 f} > 0$$

As indicated above there are two opposite effects of further education: (i) further education is time not worked so reduces working life, while (ii) being educated results in a higher wage and thus delays retirement and increases the working life. This leads us to the question what happens to the fraction of life spent working $\frac{M}{T}$?

6. Proposition: life expectancy and the working life

An increase in life expectancy increases the fraction of life spent working

Proof

Immediate by propositions 4 and 5.

7. Corollary: life expectancy and the retired life

In a model with education an increase in life expectancy results in a decrease of the fraction of life spent retired.

Proof

Follows straightforwardly from proposition 3 and 4.

V. Demographic shock and restoring a balanced budget

A common claim is that aging leads to an increase in the old-age dependency ratio. To examine this claim we expand our model to include a balanced budget constraint and look at the effects of both an increase in life expectancy and a fall in fertility respectively.

A. Increasing life expectancy and the dependency burden

We assume no population growth for simplicity and concentrate on a scenario where population aging is solely caused by rising life expectancy. The balanced budget condition is given by

$$P(T - R) = \tau(R - E) \quad E = T - \frac{(\tau + P)M}{P} \quad (5.1)$$

The balanced budget gives the combination of M and E that balances the budget for given P and τ and it is shown in figure V-1. Note that the slope of the Balanced Budget line is smaller than -1. The government has a deficit to the left of the balanced budget line and a surplus to the right.

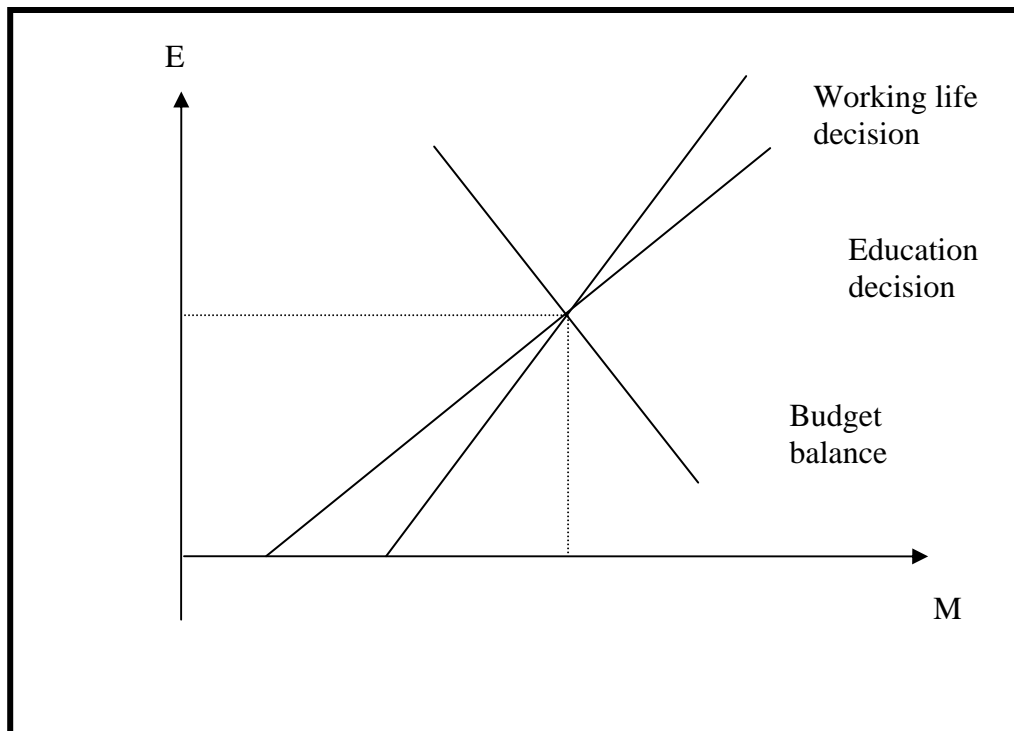


Figure V-1 The equilibrium of the working life and education decisions and balanced budget

The old age dependency ratio is the number of individuals over the retirement age, usually 65, compared to those of working ages, usually those between the ages of 15 and 65. However, in our model both the retirement age and the education are endogenous and change as a result of an increase in life expectancy. Thus the boundaries of the old-age dependency ratio shift. In

a scenario without population growth ($n = 0$) the old-age dependency ratio is given by

$$ODR = \frac{T - R}{R - E} = \frac{T - R}{M} = \frac{(T - R)/T}{M/T} \quad (5.2)$$

Proposition 6 says that denominator increases while corollary 7 says that numerator falls in response to an increase in life expectancy, causing an overall *fall* in the old age dependency rate. Thus an increase in life expectancy raises the retirement age more than sufficiently to counter the increase education and the rise in life expectancy leading to an increase in the *fraction* of life spent working and a *decrease* in the fraction of life spent retired. This is illustrated in the graph below. Following the increase in life expectancy the budget balance curves shift and the working life line rotates. As a consequence there will be a budget surplus. It should be noted that this might be different if people are constrained at $R \geq R_0$.

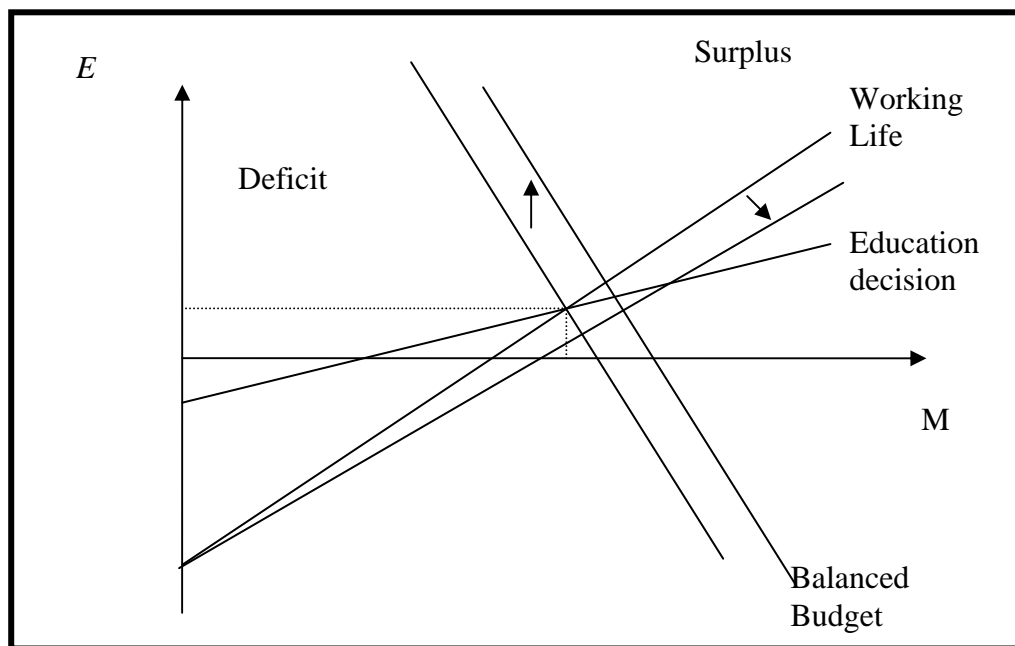


Figure V-2 The effects of an increase in life expectancy

This seems a remarkable result, and is based on two assumptions:

1. There is no statutory retirement date, individual adjust in response increases in life expectancy
2. The cost of working is proportional to life expectancy – this reflects the assumption that health status improves proportionately with rising life expectancy. This latter might not be reflecting the latest research, which shows that increases in total expected years of life have not been accompanied by an increases in expected years of, or proportional of, disability-free life or active life.⁸

⁸ Crimmens, Hayward and Saito 1994, and see Robine, Bucquet and Ritchie (1991) for a meta-analysis

Proposition 4 states that the young dependency ratio actually increases in this model. But because this model does not include any transfers from the working to those in education there is no young age dependency burden.

$$YDR = \frac{E}{R-E} = \frac{E}{M} = \frac{E/T}{M/T} \quad (5.3)$$

B. Alternative aging: fall in fertility

Previously we have characterised aging by an increase in life expectancy, which reflects phase four of the demographic transition. Aging in the recent history would be better characterised by phase three - a fall in fertility. Fertility rates (number of children per women) have fallen below the replacement rate of 2.1: the fertility rate in UK has fallen from 2.44 in 1970 to 1.70 in 1997, in Italy from 2.24 to 1.2 (World Bank 2000). Indeed the fall in fertility is not confined to Europe, the fertility rate in Russia is 1.3 and in East Asian economies it lies between 1.1 and 1.4. This section models a fall in fertility and it is instructive to see the difference in response depending on the stage of demographic transition. A change of the fertility rate affects neither the education nor the working life line. It is purely through the budget balancing mechanism that the equilibrium adjusts.

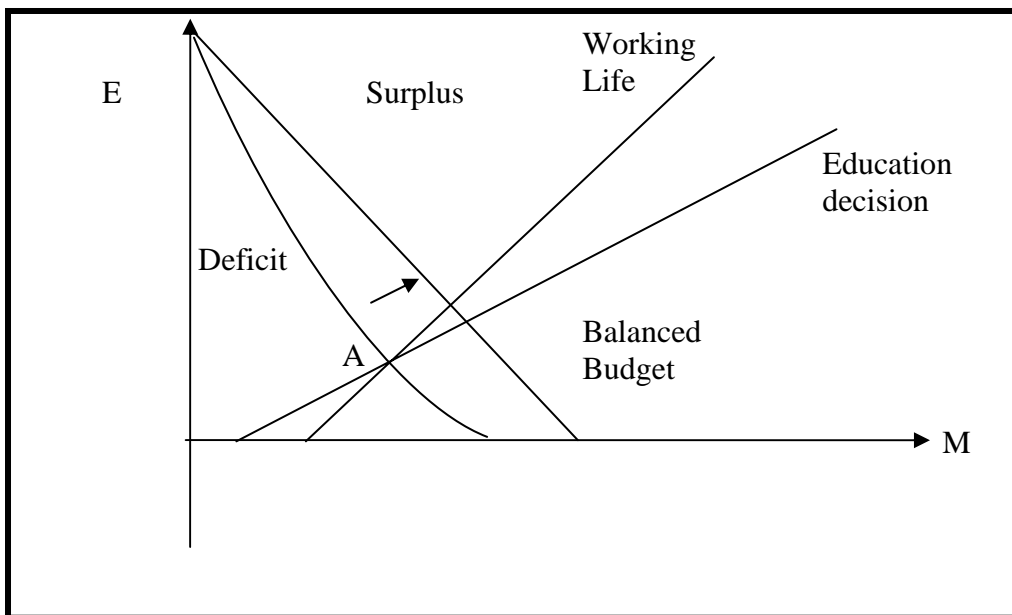


Figure V-3 The effects of a fall in fertility from positive to zero.

The budget balance condition can be expressed as follows:

$$\int_{-R}^{-E} PL_0 e^{nt} dt = \int_{-R}^{-E} \tau L_0 e^{nt} dt \quad \Rightarrow \quad \frac{(P + \tau)}{P} e^{n(T-L)} - \frac{\tau}{P} e^{nT} = e^{nE}$$

Figure V-3 shows a decline in fertility from positive population growth ($n > 0$) to a stagnant population ($n = 0$), which shifts the budget line out. In contrast to an increase in life expectancy, a fall in fertility does not affect the education and working life decisions; the equilibrium remains at point A. However as a result there is an increase in the dependency burden and a deficit in the PAYG system. The government can either decrease the pension level P or increase pension contributions τ to regain a balanced budget.

An increase in the pension contribution – shifting the working line upwards – reduces both the fraction of life spent in education and the fraction of life spent working. In contrast a reduction in the pension level – shifts the working line downward and the education line upwards – increases the fraction of life spent working and the time in education.

C. Understanding early retirement

One of the interesting phenomena in a number of aging societies is the decline in old age labour participation. The OECD Economic Outlook 2002 reports that the effective retirement age in France has fallen from 64 years in 1970-75 to 59 years in the late 1990's. Over the same period the effective retirement age in West Germany declined from 63 to 61 years. Interestingly the fall was not uniform across OECD countries: the effective retirement age has stayed constant at 62 years in the UK and has seen a slight upward trend to 65 years in the US. (OECD 2002, Table V.1) An alternative way to assess labour force participation is by looking at the employment rates of older workers which has fallen across the board, although more steeply in some countries (e.g. France from 74% to 38.5%) than in others (UK from 62.6% in 1980 to 59.8% in 2000). (OECD 2002, Table V.2)⁹ The fall in the effective retirement age is remarkable considering that in our model retirement date is an increasing function of life expectancy and a fall in fertility does not directly affect the retirement date.

In the context of our model early retirement can be explained through a combination of phasing in of the pension system and economic growth. The introduction of public pension systems have varied histories: whereas in the UK it was designed to prevent chronic poverty amongst the old, in most countries it was a manifestation of solidarity with the elderly in effect it was an insurance against old age, state pensions have been phased in and have become more generous over time and the rise has outstripped the rise in wages, effectively increasing the replacement rate¹⁰. It is straightforward to see that the retirement date drops if the level of the pension increases.

⁹ Employment of male workers at age 55 to 64 as a percentage of male populations of the same age

¹⁰ The replacement rate is the pension an agent receives as a percentage of the working income prior to retirement

Maturing occupational pension systems have the same effect resulting in an increase in the replacement rate. An increase in the coverage of occupational pensions also contributed to a fall in the average effective retirement age.

Atkinson (1995) explains the UK experience with two factors. In the UK unemployment rates increased from 4.1% in 1975 to 11.8% a decade later, which spurred the governments regard early retirement as part of labour market policy that keeps unemployment low. In addition improvements in the standard of living increased the preference for leisure and early retirement by choice.

VI. Policy Choice and Fairness

In order to examine the policy options of the government we draw the retirement and budget balance lines in the policy space. In section V we showed that a fall in fertility caused the old age dependency burden to rise and either the pension contribution or the pension level has to adjust to maintain a balanced budget. In this section we look at a combination of the policy instruments and look at the fairness implications.

A. Policy choice: population growth decline

An increase in life expectancy does not cause a deficit on the pension account. We therefore concentrate on a fall in fertility as is illustrated in figure VI-1. The mechanism chosen such that the retirement date remains the same involves a combination of increased pension contributions and reduced pensions. The initial equilibrium is point A where the government runs a balanced budget. It can be shown¹¹ that the slope of the iso-retirement curves is $-\frac{(Tf + T\psi)}{(2T\psi + Tf + \alpha)}$. The economic rationale is as follows: A fall in the pension level raises the working life and the amount of education directly. In contrast an increase in the pension contribution only impact the working life decision directly.

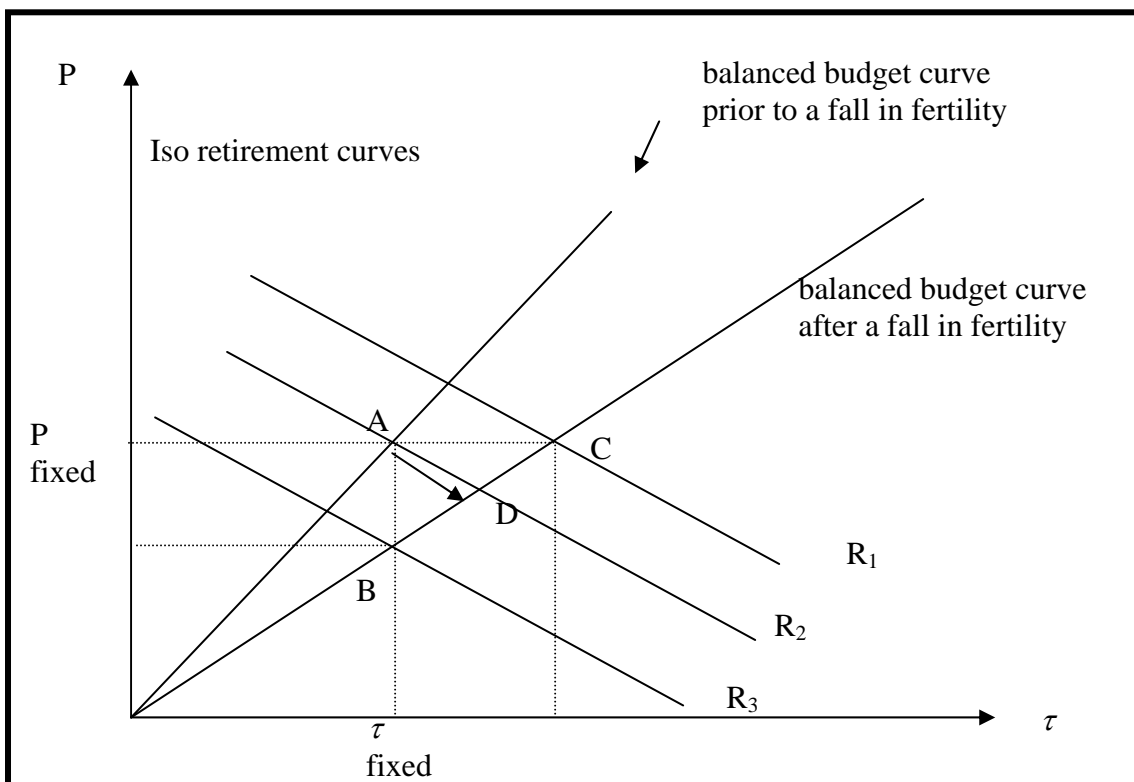


Figure VI-1 The options of a government following a fall in fertility

¹¹ See appendix for derivation

If the constraint of the mandatory retirement age binds the government moves from A to D. The overall policy space is given by DC. In contrast with an endogenously determined retirement age the government has a continuum of options on the BDC line that all satisfy the balanced budget condition. The process of pension reform poses the problem of how to fulfil the implicit promise stemming from the fact that the old generation has paid pension contributions, while not imposing the burden on the young generations by making them pay now without the promise of their own public pension and thus paying double also setting up their own second pillar. Thus, the problem is how to distribute the consequences of a demographic shock across generations.

B. Applying the Musgrave (1986) rule

Musgrave formulated a solution to this problem, allocating the consequences of an unpredictable shock over the entire life cycle keeping constant the ratio of the income of the retirees to the net income of those working. Our model does not incorporate growth so this means a constant net replacement rate $\frac{P}{W - \tau} = \rho$. Thus the slope of the Musgrave line is $-\rho$.

1. Proposition: replacement rate, Musgrave's rule and retirement

The application of Musgrave's rule in response to a fall in fertility, leads to fall in the retirement date if the replacement rate is less than

$$\frac{(Tf + T\psi)}{(2T\psi + Tf + \alpha)} = \frac{1}{2} \left[1 + \frac{\left(\frac{Tf}{2} - \frac{\alpha}{2} \right)}{T\psi + \frac{Tf + \alpha}{2}} \right].$$

For example in the case of no cost of education and no disutility of work, The application of Musgrave's rule in response to a fall in fertility, leads to fall in the retirement date if the replacement rate is less than $\frac{1}{2}$.

The budget condition curve changes as a result of either an increase in life expectancy or a fall in population growth rate. The application of the Musgrave rule in the case of an increase in life expectancy is illustrated in figure VI-2. In this case the effect on the retirement date of applying Musgrave rule is not immediately obvious. This depends on the starting position and the constant net replacement rate. It can only lead to an increase in the retirement date if there is no binding mandatory retirement date. When applying Musgrave's rule to the situation of a fall in the population growth it will lead to a reduction in the retirement age as illustrated in the figure VI-2 below.

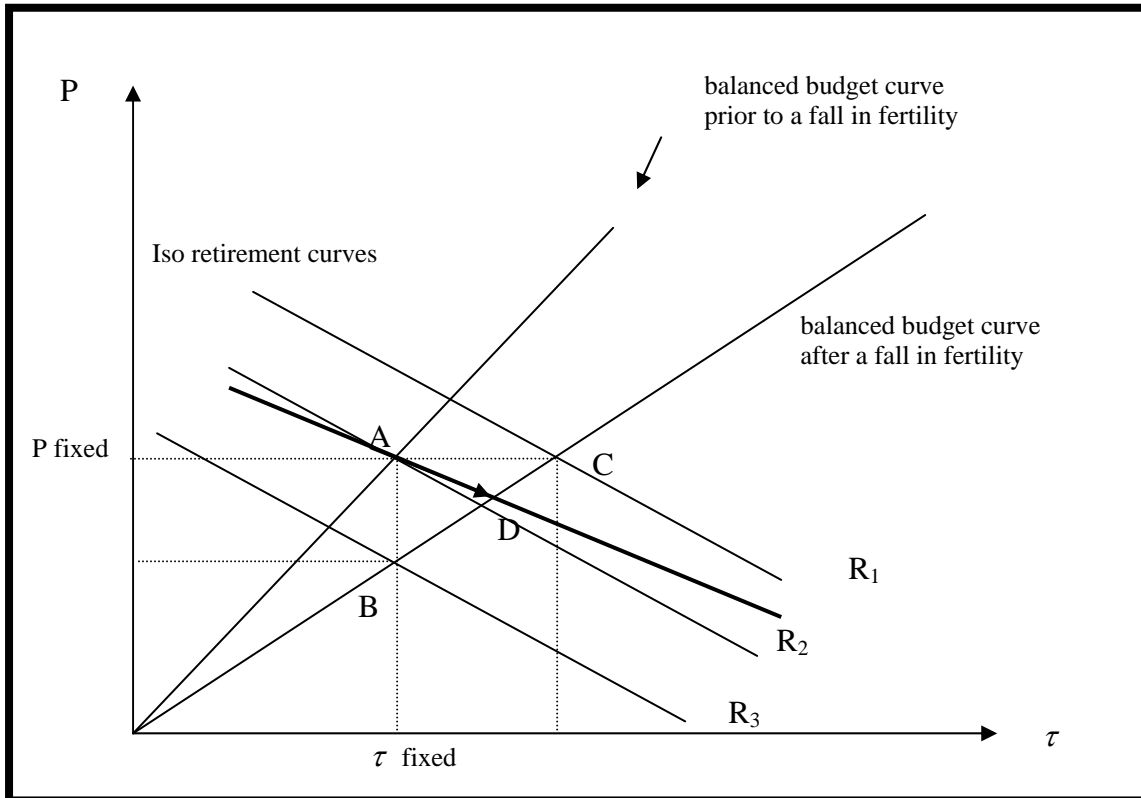


Figure VI-2 Application of Musgrave's rule with the replacement rate less than $\frac{1}{2}$ leads to a fall in retirement date.

C. Alternative Strategies

An alternative solution to the budget imbalance would be to increase the return to compulsory education.

1. Proposition: returns to compulsory education and the working life

The fraction of life spent working increases with an increase in the return to compulsory education.

Proof

By straightforward inspection of $\frac{M}{T} = \frac{[f\bar{W} - \psi P] - f(P + \tau)}{f\alpha - \psi^2 T}$ ■

Graphically, this shifts the working life line out and increases both the amount of further education (privately financed) and the working life, thus reducing the retired life. This is illustrated in figure VI-3 below.

This is increasingly advocated by the European Community. Lisbon (2000) and Stockholm (2001) and Barcelona (2002) all called for increased expenditure in education. In addition to a plea to increase resources there is

an appeal that resources be well targeted and managed in the most efficient way. Indeed a recent Communication from the European Commission identifies a number of common signs of inefficiencies in expenditure, such as excessive duration of studies, low attainment levels and high drop out rates.¹² It needs to be emphasised that this is not equivalent to increasing the *time* spent in compulsory education as this *ceteris paribus* reduces the working life, but an increased return to existing duration of compulsory education, by having more skill based education. Moreover as the further education system in this model is de facto privatised and the compulsory education is assumed to be costless.

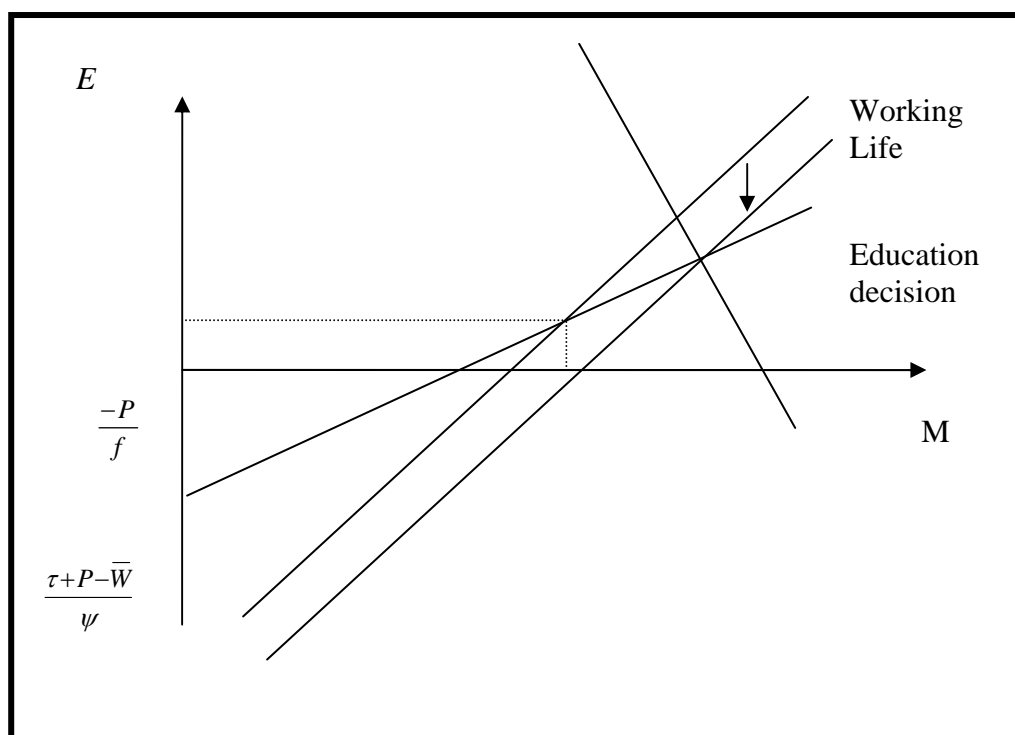


Figure VI-3 Increasing the return to compulsory education

D. Intra-generational fairness.

Approaching the pension policy as an *inter* generational problem we have explicitly side-stepped the issue of *intra* generational fairness., and focussed on *inter* generational fairness. *Intra* generational fairness relates to how resources are distributed among agents of the same generation. To study this we need heterogeneity in our model, which, for easy of exposition, we have excluded this from the analysis, but which will be introduced in the next chapter.

¹² Commission of the European Communities, Investing efficiently in education and training: an imperative for Europe, Communication from the Commission, Com(2002) 779 final, Brussels, 10.01.2003

VII. Conclusion

This paper introduces three new elements to the debate on old age finance. First it views retirement as a choice variable and not a fixed. Second, we assume education – the accumulation of skills – takes time and is a choice variable. Third, a technical assumption, an individual incurs a rising cost of working. This set up brings together two responses to aging in a unified framework: increasing employment (for example by raising the retirement age) or increasing productivity (investment in education). Moreover the model incorporate aging a much richer form: rising life expectancy, falling infertility and relative health status. Our specific model derives a maximum attainable public PAYG pension of less than a quarter of gross wage, at which point the net replacement rate is strictly smaller than a third.

We conclude that rising life expectancy does not lead to an increase in the old-age dependency burden, but to an increase in the “young-age dependency rate” and increases the amount of education per year in the working life. It thus requires the young to pre-finance more education. Thus the burden is shifted from the old to the young not through the pension system but through the education system.

A fall in fertility on the other hand does raise the old-age dependency burden. An increase the pension contribution reduces both the fraction of life spent in education and the fraction of life spent working. In contrast a reduction in the pension level increases the fraction of life spent working and the time in education. The choice of instruments chosen can be informed by fairness considerations, such as Musgrave’s rule. This gives a new explanation for early retirement. The application of Musgrave’s rule in response to a fall in fertility, leads to fall in the retirement date if the replacement rate is less than $\frac{1}{2}$.

A more complete analysis would include notions intra generational fairness through the introduction of heterogeneous agents.

VIII. Appendix 1:

1. Setting up the Hamiltonian

We can solve our problem by using optimal control theory. Whereas in section II we postulated a constant growth rate g of consumption, that assumption is justified here as the outcome of utility maximisation.

The representative agent chooses to maximise lifetime utility discounted at the subjective rate of time preference δ . The maximand is given by

$$\int_0^T e^{-\delta t} U(C_t) dt \quad (5.1)$$

We assume that utility is logarithmic $U(C_t) = \log C_t$. If we let $S(t)$ denote the agent's savings at time t then the following equation of motion of savings of the individual agent has to hold:

$$\dot{S}(t) = r(t)S(t) + (W - \tau - D(t))I(t) + P(t)(1 - I(t)) - C(t) \quad (5.2)$$

Where $I(t)$ is an indicator function such that

$$I(t) = \begin{cases} 1, & t < R \\ 0, & t > R \end{cases} \quad (5.3)$$

Where R denotes the retirement date, which can be either exogenously imposed or endogenously given by the model. The boundary condition for this problem is given by the assumption that there is no intergenerational transfer of wealth - agents do not derive utility from transferring wealth to their children, nor is there any uncertainty about the time of death. Therefore agents will have totally dissaved at the time of death T .

$$S(T) = 0 \quad (5.4)$$

We can now construct the Hamiltonian

$$H = \log(C_t) + \lambda(t) \{ r(t)S(t) + (W - T - D(t))I(t) + P(t)(1 - I(t)) - C(t) \} \quad (5.5)$$

where λ is the co-state variable. The first order conditions for this problem are:

$$\frac{\partial H}{\partial C} = 0 \quad (5.6)$$

$$\frac{\partial H}{\partial S} = \delta\lambda - \dot{\lambda} \quad (5.7)$$

$$\dot{S}(t) = r(t)S(t) + (W(t) - T - D(t))I(t) + P(t)(1 - I(t)) - C(t) \quad (5.8)$$

Rearranging the equations gives the following

$$\frac{\partial H}{\partial C} = 0 \Rightarrow \frac{1}{C} - \lambda = 0 \Rightarrow \frac{1}{C} = \lambda$$

$$\frac{\dot{C}}{C^2} = -\dot{\lambda}$$

$$\frac{\partial H}{\partial S} = \delta\lambda - \dot{\lambda} \Rightarrow \lambda(t)r(t) = \delta\lambda - \dot{\lambda}$$

$$\frac{\dot{\lambda}}{\lambda} = \delta - r$$

We can therefore deduce that the consumption path has to satisfy Ramsey (1928) rule.

$$\frac{\dot{C}}{C} = g = r - \delta \quad (5.9)$$

Hence the consumption function can be written as:

$$C_t = C_0 e^{(r-\delta)t} \quad (5.10)$$

Thus we replace the growth rate g by, $r - \delta$ where

$$C_0 = \frac{\delta}{1 - e^{-\delta T}} \left\{ \frac{W - \tau}{r} (1 - e^{-rR}) - \frac{\alpha}{Tr} \left[\frac{1}{r} - e^{-rR} R - \frac{e^{-rR}}{r} \right] + \frac{P}{r} (e^{-rR} - e^{-rT}) \right\} \quad (5.11)$$

We note that the retirement date R does not affect the consumption growth over time. Neither does the life expectancy T , the wage level W , Pension P or the pension contribution level τ enter into the consumption growth equation. These variables merely have the following level effects.

2. Deriving the retirement decision

The agent chooses a value of R , which maximises the right hand side of the budget constraint expressed in equation (4.1)

$$C_0 \int_0^T e^{-(r-g)t} dt = \int_0^R (W - \tau - D(t))e^{-rt} dt + \int_R^T P e^{-rt} \quad (4.1)$$

Denote the right hand side by:

$$J = \left\{ \frac{W - \tau}{r} (1 - e^{-rR}) - \frac{\alpha}{Tr} \left[\frac{1}{r} - e^{-rR} R - \frac{e^{-rR}}{r} \right] + \frac{P}{r} (e^{-rR} - e^{-rT}) \right\} \quad (6.1)$$

$$\frac{\partial J}{\partial R} = \left\{ (W - \tau) e^{-rR} - \frac{\alpha}{Tr} \left[-e^{-rR} + r e^{-rR} R + e^{-rR} \right] - P e^{-rR} \right\} = \left\{ W - \tau - \frac{\alpha R}{T} - P \right\} e^{-rR} \quad (6.2)$$

For maximisation we need $\frac{\partial J}{\partial R} = 0$ which yields the following expression

$$W - \tau - \frac{\alpha R}{T} = P \quad (6.3)$$

The agent will retire if the marginal benefit/return of working (LHS in 6.3) is equal to the marginal benefit of retiring (RHS in eq 6.3). Rearranging (6.3) gives an expression for the retirement date R.

$$R = \frac{T}{\alpha} [W - \tau - P] \quad (6.4)$$

3. Derivation of the menu of options of the government

The combination of P and τ is not just restricted by our assumption that the replacement rate is less than 100%, implying $W > P + \tau$. The level of pension contribution and the level of pensions affect the retirement date.

$$R = \frac{T}{\alpha} \{W - \tau - P\}$$

And the retirement date in turn impacts both P and τ . For a balanced budget we require, in the absence of population growth, that the level of pensions is related to pension contribution as follows

$$P = \frac{\tau R}{T - R}$$

Substituting the expression for R into the expression for a balanced budget which gives an expression of the pension in terms of τ , and thus the menu of options for the government of pension and pension contributions, taking into account the retirement behaviour of individual agents.

$$P = \frac{\tau \frac{T}{\alpha} \{W - \tau - P\}}{T - \frac{T}{\alpha} \{W - \tau - P\}} \quad \Rightarrow \quad P^2 + P(\alpha - W + 2\tau) + \tau(\tau - W) = 0 \quad (6.8)$$

The relationship between the pension contribution and the pension level is independent of life expectancy and can be characterized as follows. For positive pension P we need:

$$P = \frac{-(\alpha - W + 2\tau) + \sqrt{(\alpha - W + 2\tau)^2 + 4\tau(W - \tau)}}{2} \quad (6.9)$$

The value of τ at which the maximum value of pension is attained is given by the solution to $\frac{\partial P}{\partial \tau} = 0$

$$\frac{\partial P}{\partial \tau} = \left(\frac{1}{2}\right) \left[-2 + \frac{1}{2} \frac{(4(\alpha - W + 2\tau) + 4W - 8\tau)}{\sqrt{(\alpha - W + 2\tau)^2 + 4\tau(W - \tau)}} \right] \Rightarrow \tau = \frac{W(2\alpha - W)}{4\alpha} \quad (6.10)$$

This means that the highest possible value of pensions is given by

$$P = \frac{-\left(\alpha - W + \frac{W(2\alpha - W)}{2\alpha}\right) + \sqrt{\left(\alpha - W + \frac{W(2\alpha - W)}{2\alpha}\right)^2 + \frac{W(2\alpha - W)}{\alpha} \left(W - \frac{W(2\alpha - W)}{4\alpha}\right)}}{2}$$

$$\Rightarrow P = \frac{W^2}{4\alpha} \quad (6.11)$$

Thus the higher the coefficient of the cost of working the lower the maximum attainable pension.

4. Derivation of the iso retirement line

$$E = \frac{\psi Tf(P + \tau) + T[\psi P - f\bar{W}]}{f \psi^2 T - f\alpha} - \frac{P}{f}$$

$$M = \frac{Tf(P + \tau) + T[\psi P - f\bar{W}]}{\psi^2 T - f\alpha}$$

$$R = \frac{\psi Tf(P + \tau) + T[\psi P - f\bar{W}]}{f \psi^2 T - f\alpha} - \frac{P}{f} + \frac{Tf(P + \tau) + T[\psi P - f\bar{W}]}{\psi^2 T - f\alpha}$$

Following straightforward manipulation this can be expressed as

$$R = \frac{T\bar{W}(\psi + f) - P(2T\psi + Tf + \alpha) - (Tf + T\psi)\tau}{f\alpha - \psi^2 T}$$

$$aR = b - P(2T\psi + Tf + \alpha) - (Tf + T\psi)\tau$$

$$P = \frac{b}{(2T\psi + Tf + \alpha)} - \frac{a}{(2T\psi + Tf + \alpha)} R - \frac{(Tf + T\psi)}{(2T\psi + Tf + \alpha)} \tau$$

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