

# Life-Cycle Decision Making without Backward Induction

## A Model with Feasibility Goals

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# Briefing on backward induction

A utility function

$$\sum_{i=0}^{T-t} \beta^i E_t u(c_{t+i})$$

looks like an innocuous object. Allows for evaluation of any future consumption stream.

But ...

## Briefing on backward induction (cont'd)

... But when it comes to *decision making* ...

- $\max \sum_{i=0}^{T-t} \beta^i E_t u(c_{t+i})$  is only a meaningful model if the agent knows what she (thinks she) will consume in all future periods in all possible states of the world.
- The only consistent way to infer future consumption levels is to work backwards through the event tree starting at the end (“full contingent planning”).
- A 30 year old first has to know how much he will consume during retirement before thinking about his equity investments today ...

# Why quarreling with backward induction?

- This would be fine if “as if” provided a valid argument. But . . .
- Allen and Carroll (2001) provide a staggering result that renders the “as-if” argument more than problematic.
- Lettau and Uhlig (1999) provide more reasons to be skeptical.

# Aims of the paper

- Show that simple, (computationally) tractable and parsimonious models without backward induction *do exist*, contrary to common beliefs.
- Understanding life-cycle savings and asset allocation in a no-backward induction environment.
- Explaining puzzles related to asset allocation.
- Stimulating further research on how to model decision making of real human beings without invoking contrived assumptions.

## Related literature

- Shefrin and Thaler (1988);
- Laibson (1996), Laibson et al. (1998);
- Bowman et al. (1999);
- Gabaix and Laibson (2000);
- Binswanger (2006).

None of these papers has addressed backward induction.

## Feasibility goals

### Idea of this paper

- Giving up the idea that decision makers “know” their future actions and develop a full contingent plan. → Bounded rationality.
- But staying as close as possible to the idea of “preferences.”

→ *Feasibility goals*: Individuals care about feasibility of certain future choices, rather than about actual “utility maximizing” future choices.

## Feasibility goals (cont'd)

Two feasibility goals:

- Feasibility of at least a certain fraction of current consumption in the future *in all states of the world*. → Refers to worst-case scenario. → **Insurance goal**.
- Feasibility of a particular standard of living in the future in case that a “*normal*” scenario evolves. → **Accumulation goal**.

Such *feasibility goals* can be achieved even if the decision maker is completely ignorant about her future *actual* decisions.

## The insurance goal

A consumption/savings/asset-allocation choice fulfills the insurance goal in period  $t$  if

$$\min_t \tilde{c}_{t+i} \geq \alpha_i c_t$$

is *feasible* for all  $i = 1, 2, \dots, T - t$ .

## The insurance goal (cont'd)

### A three-period toy example.

- Period 1 resources are  $w_1$ . No non-financial income in periods 2, 3.
- Bonds pay gross interest of  $r$  in all states of the world.

The present value of future “habit consumption” is then  $\frac{\alpha_1 c_1}{r} + \frac{\alpha_2 c_1}{r^2}$ . Thus, any  $c_1$  for which

$$w_1 - c_1 < \frac{\alpha_1 c_1}{r} + \frac{\alpha_2 c_1}{r^2}$$

will *not* fulfill the insurance goal. → Lower bound on savings.

*No need for projecting future decisions in order to achieve insurance goal!*

## The accumulation goal

The accumulation goal represents a target standard of living that is *feasible* from a future date  $T^*$  on

- In case that a “normal” scenario evolves (all exogenous random variables take on their expected values).
- In case that a particular level of consumption/saving is fixed from now until  $T^*$ .
- Savings are invested such as to maximize expected returns, subject to achieving the insurance goal at each point in time.

→ Target value is a function of current consumption.

# The feasibility-goals algorithm

## The feasibility-goals algorithm

- Step 1. Identify the choice that maximizes  $c_t$  among those which satisfy the insurance goal.
- Step 2. If this maximal level of  $c_t$  falls short of  $\bar{c}$ , then implement choice identified by step 1. Otherwise go to step 3.
- Step 3. Maximize  $\ln(c_t - \bar{c}) + \beta^{T^* - t} \ln[\hat{A}_{T^*}(c_t)]$ .

The parameter  $\bar{c} \geq 0$  denotes a “normal” standard of living, a typical “consumption bundle.” Represents a reference point that has to be achieved before accumulation goal is triggered.  $\bar{c} = 0$  represents a parsimonious special case.

## The feasibility-goals algorithm (cont'd)

- In each period the decision maker solves a similar decision problem afresh. → Procedural rationality.

## Discussion of the Model

- The insurance and accumulation goal include in a stylized way many important savings motives:
  - Precautionary savings
  - Old-age provision
  - To save in order to expand one's standard of living
  - "Becoming rich", speculating
  - To save in order "to enjoy a sense of independence and the power to do things, though without a clear idea or definite intention of a specific action" (Keynes)
  - ...

# Calibrating the model

## Baseline calibration

- $\alpha_i = 0.96^i$ . This is not meant literally. Should rather capture an “intuitive” way of decision making!
- $\bar{c} = 27,000$  year-2001 US dollars
- $\beta = 0.85$ .

These values are chosen such that they correspond to values that fit best cross-sectional data within a *two-period* framework where, by construction, there is *no* variation of savings rates and equity shares over the life-cycle.

## Calibrating the model (cont'd)

- Set  $T^* = (t + T) / 2$ , i.e. the mean of a uniform distribution over the remaining life-time. (It turns out that it doesn't matter ...)
- Standard calibrations for risk-free interest rate and stock returns. Distribution of stock returns truncated at first percentile.

## Calibrating the model (cont'd)

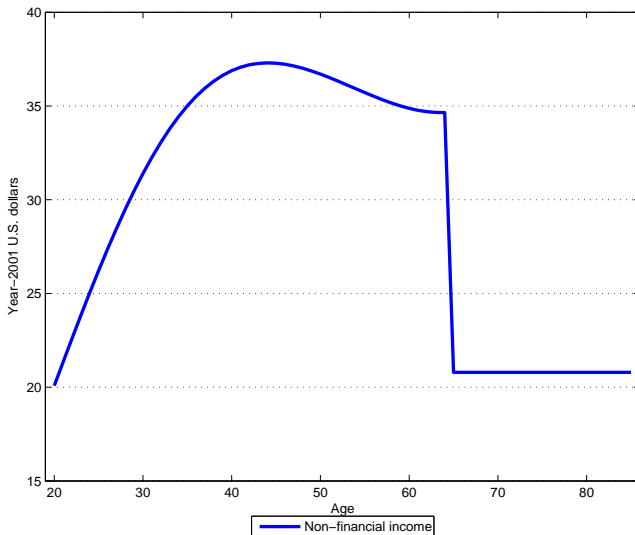
Standard calibrations for non-financial income.

- Hump-shaped in the absence of shocks.
- Transitory and permanent shocks.
- Stylized medical expenditure shock during retirement.
- All distributions of shocks truncated at first percentiles.

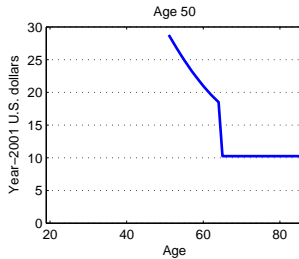
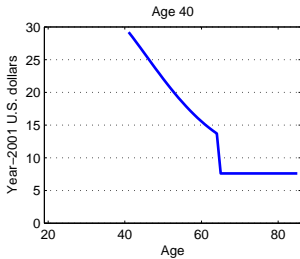
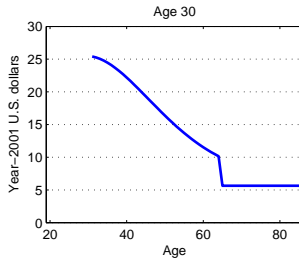
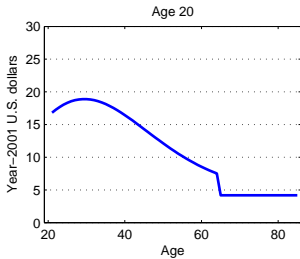
## Calibrating the model (cont'd)

All calibrations are such that, ex post, no shocks occur. It's always expected values that materialize. Makes simulations easier to interpret.

## Non-financial income in the absence of shocks



## Minimum future non-financial income at various ages

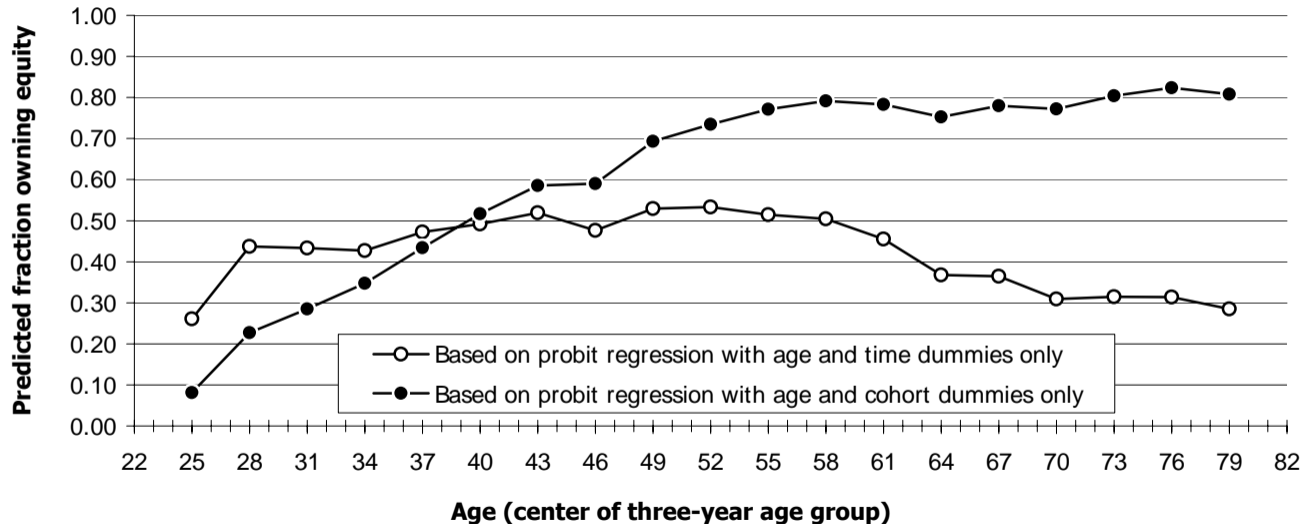


## Bringing the Model to the Data

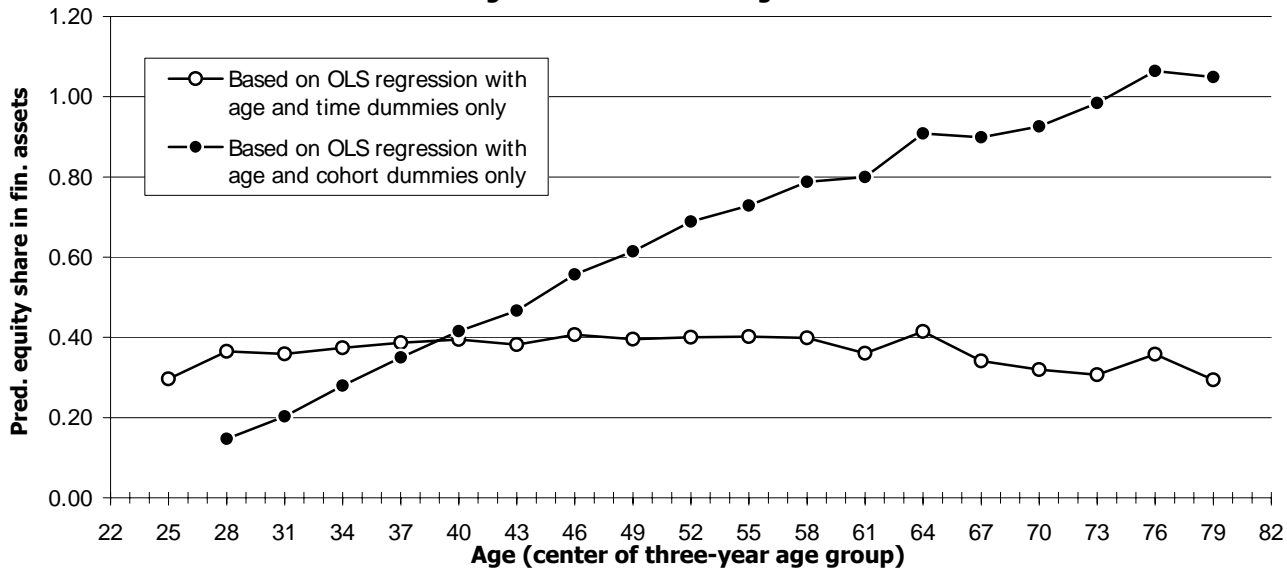
What data are to be understood?

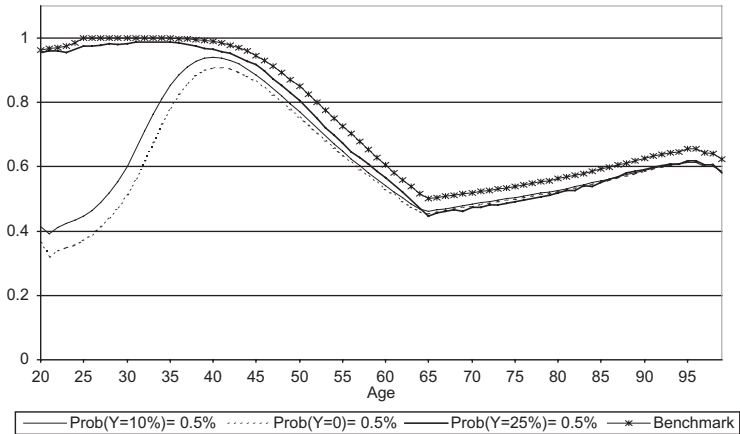
- So called “buffer-stock” models perform relatively well explaining the variation of savings over the life cycle (Carroll, 1997; Gourinchas and Parker, 2001).
- BUT it is notoriously hard for existing preference models to get the right picture concerning stock-market participation and equity shares (Gomes and Michaelides, 2005).

## Regression estimates of age effects



**Regression estimates of age effects**



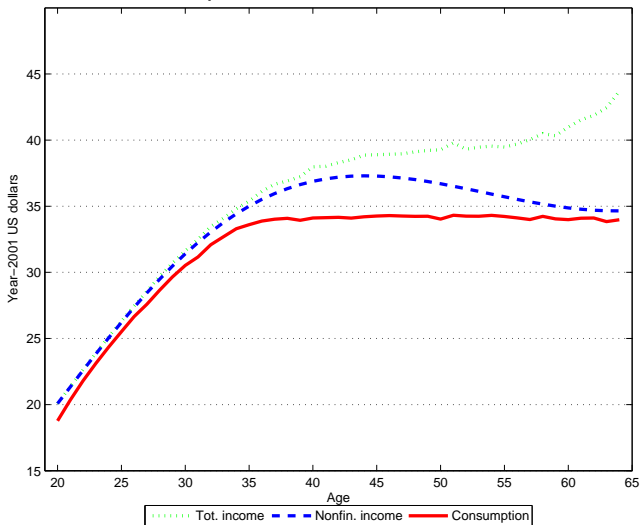


**Figure 6**  
 Simulated portfolio share invested in stocks with a probability of a drop in retirement income to 25% of its value.

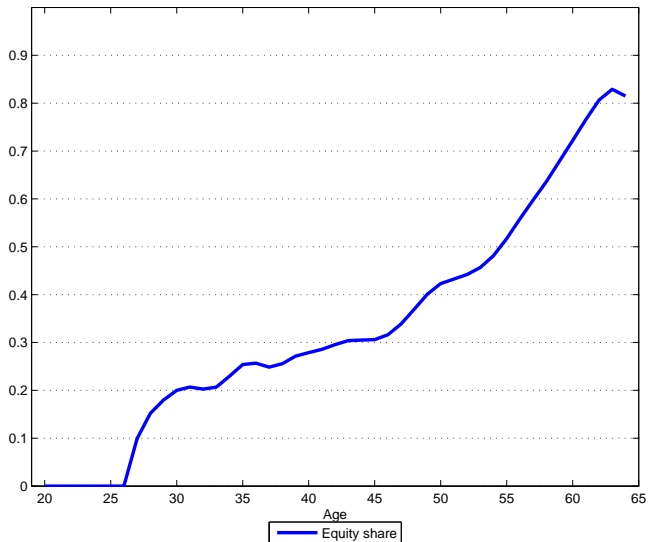
## Median equity shares per income quintile (SCF 2001)

Income quintiles	Age					
	<30	30-39	40-49	50-59	60-69	70+
First	0	0	0	0	0	0
Second	0	0	0	0	0	0
Third	0.28	0.36	0.36	0.26	0.26	0.36
Fourth	0.46	0.56	0.48	0.49	0.44	0.41
Fifth	0.65	0.54	0.60	0.57	0.56	0.55

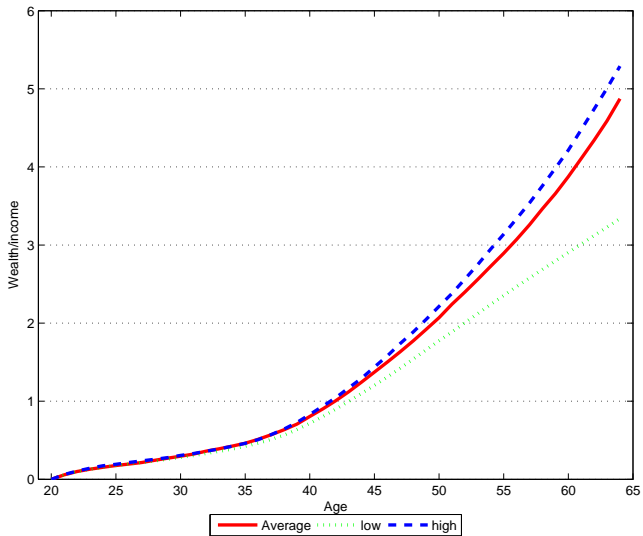
## Consumption and income: Baseline



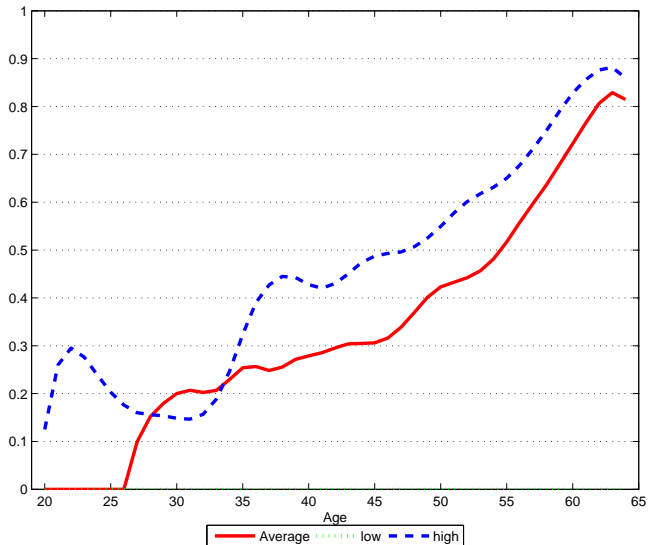
## Equity shares: Baseline



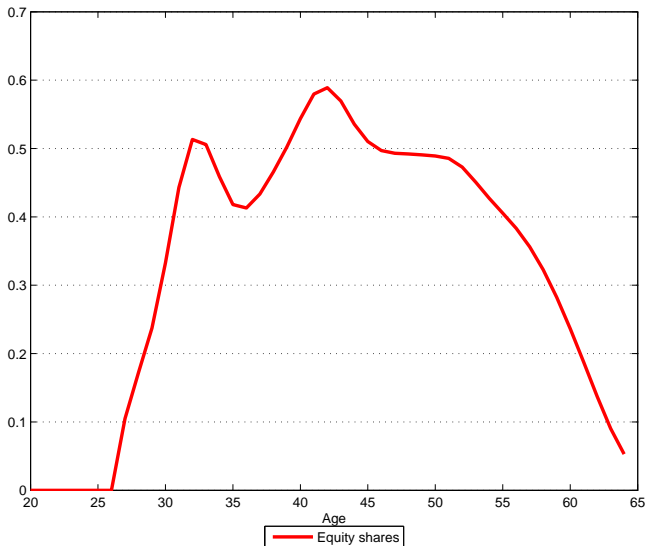
## Wealth-income ratios for different income levels: Baseline



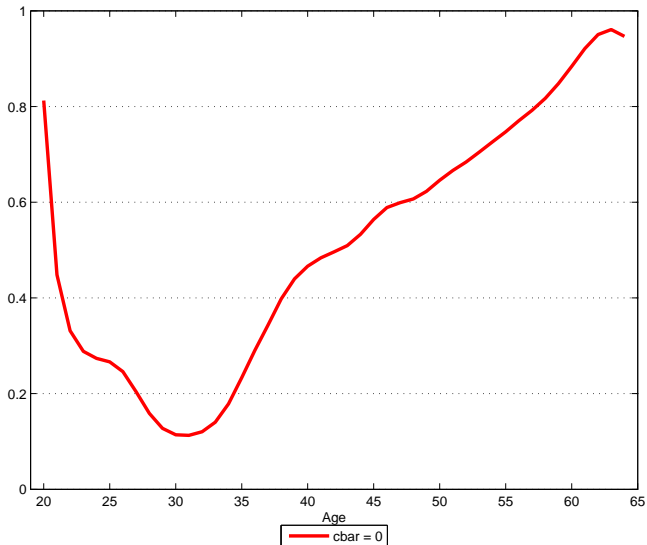
## Equity shares for different income levels: Baseline



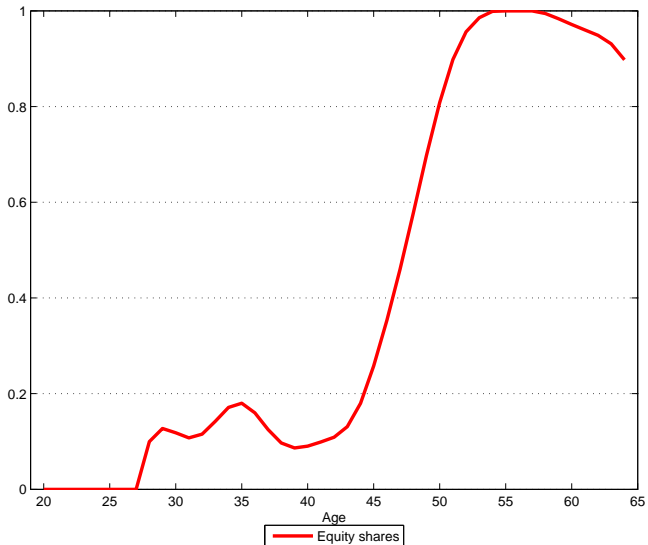
## Equity shares for high $M_t$ -risk ( $\underline{M} = .25$ )



## Equity shares for $\bar{c} = 0$



Equity shares  $\alpha_i = .9$  for  $i = 1, \dots, 10$ , and 0 otherwise



# Conclusion

- Parsimonious and tractable alternatives to the backward induction model *do* exist.
- They allow us to better understand important data.

## Next steps

- Exploring individual decision rules *empirically* using Internet surveys, in-depth interviews and other suitable survey methods.
- Doing so in different countries to learn about how institutional and cultural factors influence decision rules.
- Not only savings decisions, but also early retirement, firm decisions, political decisions . . .